TOWARDS A SOLUTION TO THE IDENTIFICATION OF CONCENTRIC CIRCLES IN PERSPECTIVE

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ABSTRACT

In order to speed up the location and identification of circles centres in perspective views/pictures and to draft ellipses in perspective view more consciously, new observations have been introduced and clarified, sustained by concepts coming from the theory of projective geometry. The basis for the implementation of a tool has been pointed out, and new outlooks of investigation have been proposed.

KEY WORDS: concentric circles, perspective, projective geometry, reverse engineering.

1. INTRODUCTION

The representation of a circle in perspective has been achieved for a long time by several approximate solutions. The real solution is defined, employing the projective geometry concepts, by an ellipse. This in perspective can be drafted, almost roughly, by means of a certain number of relevant points of a square circumscribed to the circle to be projected. The random lying of the circle with respect to the view point generates a lot of problems related to the heavy graphic elaboration required to identify the exact representation of the ellipse. The true draft of the ellipse in perspective could be easily obtained once the centre and the axes, minor and major were known. In any case, it should be realized that the "centre" of the ellipse, even obtained by approximation as junction of both minor and major axis, does not correspond to the centre of the circle from which it originates.

The problem becomes more intriguing in the applications of the computer vision technology (i.e. camera calibration), when targets of circular forms, or as concentric circles, are employed. From an empirical point of view these kinds of targets seem to be the most suitable for the identification of a point in the space. But when one tries to identify the centre of a concentric circles target he/she realizes that the ellipses in which the target degenerates have two distinct centres, that is the intersection points of both axes, minor and major, do not coincide, even if the axes remain parallel among them.

The paper aim is to point out the problem and to solicit a conjoint collaboration among different competencies: graphic engineering, projective geometry, computer science, in order to attain an easily implementable solution, both in the graphic ambit as in reverse engineering.

2. RESULTS OF THE RESEARCH

The representation of a circle in the space has a double aspect: in perspective a simple method is required to sketch the ellipse in which degenerates the circle; in computer vision it is necessary to identify the true position of the projected circle centre as a prerequisite to reconstruct the true geometry of a scene, when circle targets are employed for the acquisition.

Both problems find the basis on the theoretical findings of projective geometry, mainly the so-called cross-ratio, that is the relation in which must lye four aligned points (or the corresponding angles among four adjacent convergent lines).

2.1 CIRCLE IN PERSPECTIVE

The problem of perspective is well described in books of descriptive geometry [1] and several applications for all kinds of relative position between circle in the space and point of view are given. The true position of the circle centre is well determined by the intersection of the lines crossing the mid-points of square opposite sides after the distortion in projective. This construction allows to draft the ellipse as interpolation of a certain number of points.

The true position of the ellipse centre can be identified employing the harmonic cross-ratio along the line that connects the pole and the circle centre. The ellipse centre, to respect the law of the harmonic cross-ratio must lay in the middle point of the ellipse conjugate diameter aligned with that line.

Generally, a set of conjugate diameters can be drawn on the ellipse, but not immediately the minor and major axes. This latter identification might greatly simplify the ellipse representation.

In Figure 1 a draft of circles in perspective is represented [2]. As can be seen a sufficiently clear representation is made, even if no one intrinsic characteristic of the ellipse is given: the centre of the ellipse, the major and minor axes. Both ellipses on the vertical planes have these axes almost horizontal, because the point of view is positioned near the cube mid height, and vertical, because no distortion is considered due to this position (really a two focus representation has been adopted). The ellipses contained on the horizontal planes (face $A_0B_0C_0D_0$) have the axes major and minor exactly horizontal and vertical, considering that the point of view is aligned to the true circle centre.

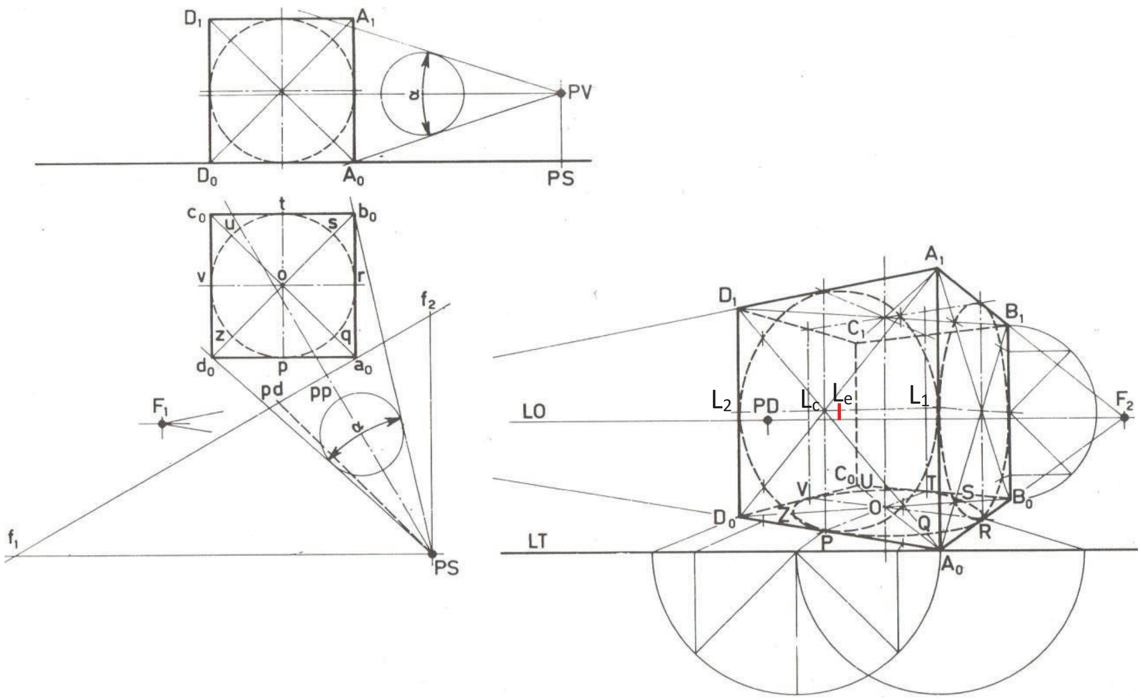


Figure 1. Circles in perspective, constructed with the measurement points

The cross-ratio is satisfied. Considering for instance the ellipse on the face $A_0A_1D_1D_0$ the cross-ratio is, indicating with L_c the centre of the projected circle:

$(L_1\text{-}L_c)/(L_2\text{-}L_c) : (L_1\text{-}F_1)/(L_2\text{-}F_1)$

And, as can be easily verified, measuring the lengths on the drawing

$(L_1\text{-}L_c)/(L_2\text{-}L_c) = -\ 1.342$

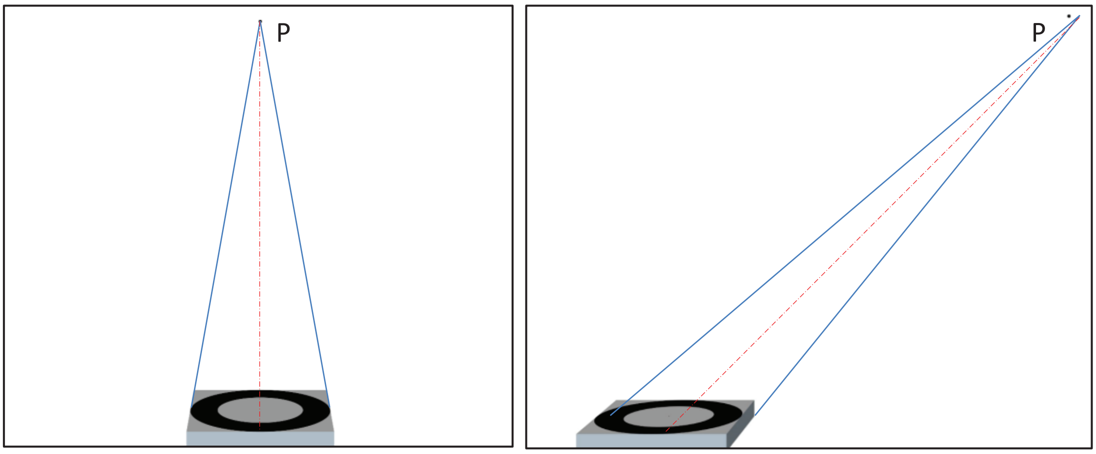
$(L_1\text{-}F_1)/(L_2\text{-}F_1) = 1.348$

the cross ratio is a harmonic ratio, being ≈ -1.

The position L_e of the ellipse centre is determined as middle point of the minor axis L_1-L_2.

2.2 ELLIPSE IN SCENE RECONSTRUCTION

The reconstruction of a scene from several camera acquisitions is performed by the employment of targets. The circular shape seems to be naturally the most suitable for this purpose, considering its intrinsic symmetry around the centre. Unfortunately, the circle in perspective becomes an ellipse and the centres, of both circle and ellipse, does not coincide [3]. This is the reverse of the previous problem. Now we have a scene with the “true” form as taken by a camera, and we try to reconstruct the true dimensions. As reported in Figure 2 the most complex image can be reduced to search the relation about a form (circular item) and a point of view (the projected position of the camera on the 2D representation, as a picture or one frame of a movie). An ellipse aligned with the point of view is represented in Figure 2.a, and an ellipse not aligned with the point of view is represented in Figure 2.b, as the projection of two concentric circles.



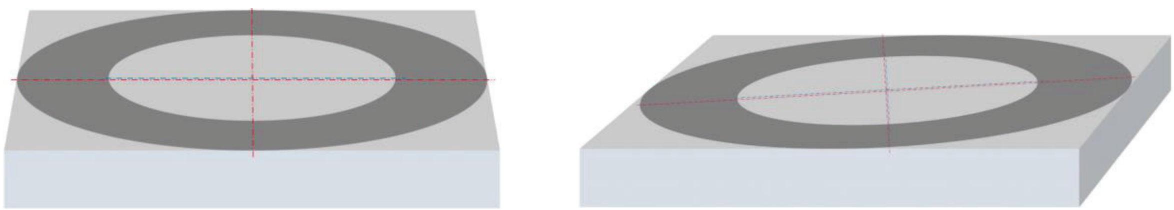
a) b)

Figure 2. Concentric circles in perspective: a) aligned with the point of view; b) unaligned with the point of view P.

In a certain sense the problem appears more simple than the previous, because no graphical construction is required. The true form of the ellipses can be extracted from the image by means of least squared method, after some image enhancement of the sharp boundaries of the shape recorded. Some efficient algorithms have been coded for this purpose. Therefore, the intrinsic shape of the ellipses and their fundamental parameters can be determined: the centres and directions of minor and major axes.

The geometries of Figure 2 were obtained modelling a target of two concentric circles of 11 and 18 cm by means of a 3D CAD. The relative position of the camera from the origin of the 3D space was: {100, 0, 50} [cm] for the aligned target (Figure 2a) and {100, 50, 50} [cm] for the unaligned target (Figure 2b). The camera was simulated with a 35 mm objective.

After reconstruction, the position of centres and the major and minor axes of the targets appears as in Figures 3 a) and b). In Figures 3 c) and 3d) a zoom of the central zones are reported.



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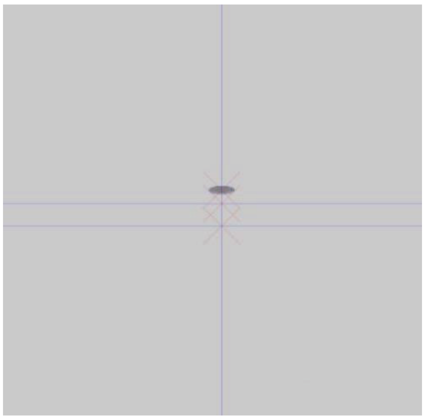
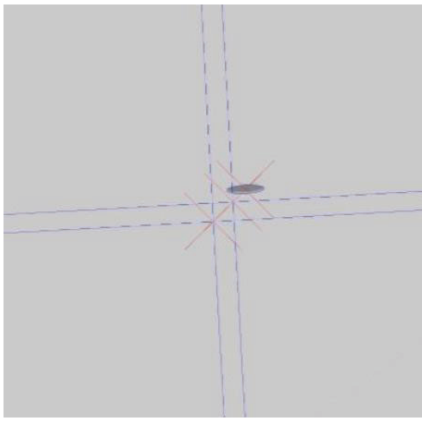


b)



d)

Figure 3. a) Ellipses reconstructed with the indication of centres and minor and major axes; b) zoom in the central zone with the enhancement of ellipses centres and projected circle centre point.

As it can be seen the direction along which the centres are aligned coincides with the line passing through the projection point P. This can be verified comparing Figure 2 and Figure 3 (c and d), being intrinsically contained in the main assumptions of projective geometry. Along this line (the red line in Figure 2) the cross-ratio can be evaluated either for inner and outer ellipses. The centre of the projected circle can then be computed from one of the two ratio if the position of the point of view P is determined on the image. Generally, this is identifiable as the centre of the image on the picture. It is important to note that the projected centre is inside the segment between the ellipse centre and point of view.

When higher precision is required, as in the case of camera calibration, the exact position of the point of view can be stated in a setup in which the mutual position of a set of concentric circle targets and the camera position is checked.

3. CONCLUSION

The problem of the correct representation of a circle in perspective has been pointed out. The problem is presented either during drafting, with the correct identification of the ellipse that appears on the drawing. The problem has a reverse aspect when, on the basis of a picture, it is necessary to identify the centre of the projected circle, for 3D reconstruction of a scene or the camera calibration.

A set of problems remains to be solved, as the identification of the orientation of minor and major axes during drafting, and further the possibility to identify the centre point of the projected circle on the basis of only the intrinsic parameter of both ellipses: centres and major and minor axes.

The paper can be a stimulus to implement a software module for which several competencies are required on different topics.

4. REFERENCES

[1] Migliari R., Geometria Descrittiva, Ed. Città Studi, 2009
[2] Filippi F., Disegno di Macchine, Hoepli, 1977.
[3] Yang S., Liu M., Yin S., Guo Y., Ren Y., Zhu J., An improved method for location of concentric circles in vision measurement, Measurement, 2017 (100) 243-251.

5. RESEARCH TEAM

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NUEVOS MODELOS DE INVESTIGACIÓN Y COLABORACIÓN EN INGENIERÍA GRÁFICA

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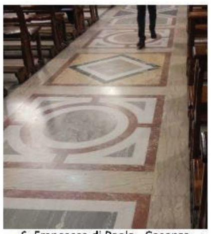
Evidences



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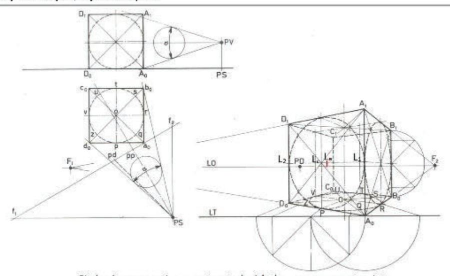

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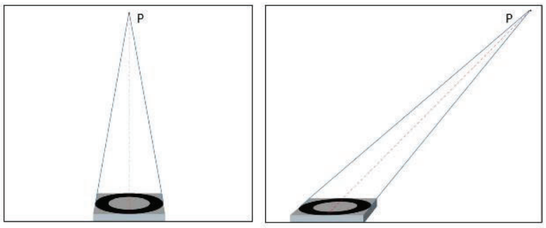

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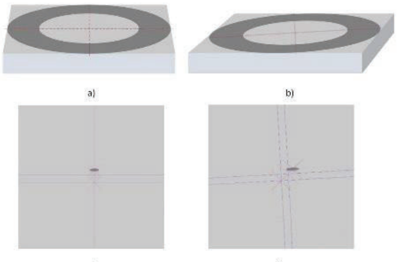

a, b) Ellipses reconstructed with the indication of centres and minor and major axes;
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