

A SIMPLE AND EFFICIENT WAY TO INTRODUCE BOUNDED RANGES IN PARAMETERS AND FUNCTIONAL CONSTRAINTS IN A DIMENSIONAL SYNTHESIS PROBLEM

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1. INTRODUCTION

A very common approach to a mechanism dimensional synthesis problem is performed by minimizing an objective function and solving the problem with various optimization methods [1]. These methods may allow the introduction or not of constraints for the problem variables, among which are the dimensional parameters of the mechanism.

The approach of Avilés et al [2], generalized by García de Jalón and Bayo [3] proposes the formulation of the objective function using a kinematic modeling of the mechanism by means of generalized coordinates and solving the optimization problem by means of a quasi-Newton method. This method is highly efficient but has the drawback that it does not allow bounding the range in which the dimensional parameters vary.

In this paper we propose an elegant and simple way to introduce a condition that allows to bound a dimensional parameter and/or a functional constraint to a given range by adding an equation and an additional variable to the kinematic model.

2. METHODOLOGY

The formulation proposed in [3] for a classical kinematic dimensional synthesis problem is as follows: at each design position, we will have a set of geometric constraint equations of the mechanism to be designed and a set of functional constraint equations, which are those defining the desired synthesis conditions at that design position. If all design positions are considered, the global set of equations is as shown in Eq. 1.

$$\begin{bmatrix} \Phi^1(\mathbf{q}^1, \mathbf{b}) \\ \Phi^2(\mathbf{q}^2, \mathbf{b}) \\ \vdots \\ \Phi^{n_p}(\mathbf{q}^{n_p}, \mathbf{b}) \end{bmatrix} = \mathbf{0} \rightarrow \Phi(\mathbf{q}, \mathbf{b}) = \mathbf{0} \quad \text{with} \quad \mathbf{q} = \begin{bmatrix} \mathbf{q}^1 \\ \mathbf{q}^2 \\ \vdots \\ \mathbf{q}^{n_p} \end{bmatrix} \quad (1)$$

The objective function shown in Eq. 2 can be posed:

$$\Psi(\mathbf{q}, \mathbf{b}) = \frac{1}{2} \cdot \Phi^T(\mathbf{q}, \mathbf{b}) \cdot \Phi(\mathbf{q}, \mathbf{b}) \quad (2)$$

and obtain an optimal solution of the synthesis problem, in the least-squares sense, by minimizing that objective function with respect to the vector \mathbf{q} and \mathbf{b} . By calculating the gradient of the objective function and equaling it to zero, the expression shown in Eq. 3 is obtained:

$$\mathbf{J}(\mathbf{q}, \mathbf{b}) \cdot \Phi(\mathbf{q}, \mathbf{b}) = \mathbf{0} \quad \text{with} \quad \mathbf{J}(\mathbf{q}, \mathbf{b}) = \begin{bmatrix} \Phi_{\mathbf{q}}(\mathbf{q}, \mathbf{b}) & \Phi_{\mathbf{b}}(\mathbf{q}, \mathbf{b}) \end{bmatrix} \quad (3)$$

Doing a Taylor series expansion and substituting we obtain the iterative Eq. 4:

$$\begin{bmatrix} \mathbf{q} \\ \mathbf{b} \end{bmatrix}_{k+1} = \begin{bmatrix} \mathbf{q} \\ \mathbf{b} \end{bmatrix}_k - \left(\mathbf{J}(\mathbf{q}, \mathbf{b}) \cdot \mathbf{J}^T(\mathbf{q}, \mathbf{b}) \right)_k^{-1} \cdot \mathbf{J}(\mathbf{q}, \mathbf{b})_k \cdot \Phi(\mathbf{q}, \mathbf{b})_k \quad (4)$$

Since the components of \mathbf{b} represent physical quantities, it may be desirable to constrain them to a particular range. This can be done by adding an additional equation to the original set of geometric and functional constraint equations. This additional equation is based on the sigmoid function and must be scaled to the desired range, as shown in Fig. 1. It employs a new auxiliary variable that is added to the set of variables of the synthesis problem and can take any real value. This is a fundamental difference from the variables representing physical quantities of the problem. Moreover, the additional equation has a non-negative, continuous and very simple derivative.

This strategy can be generalized to functions constructed with variables of \mathbf{q} and \mathbf{b} vectors so that, for example, the desired assembly modes in the mechanism at each design position can be constrained. In that case, an additional equation will have to be included for each design position.

$$l \leq \mathbf{b}_j \leq u \rightarrow \mathbf{b}_j = 1 + (u-1) \cdot a \rightarrow \mathbf{b}_j - 1 - (u-1) \cdot a = 0 \quad \text{with} \quad a = \frac{1}{1 + e^{-v_{\text{aux}}}} \quad \text{and} \quad \frac{da}{dv_{\text{aux}}} = a \cdot (1-a)$$

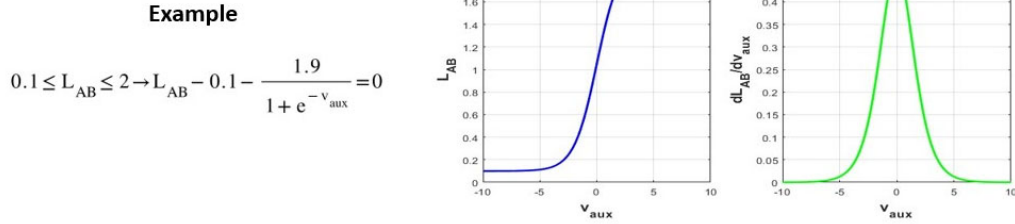


Figure 1. Sigmoid function and additional equation

3. RESULTS

A connecting rod guiding problem (red) in an RRRR mechanism, imposing three precision positions. It has been solved starting from the same initial approximation of the solution in two situations: without imposing any additional constraint and imposing a constraint at each position to maintain the desired assembly mode between the connecting rod and the rocker arm (black). Fig. 2 shows the two solutions obtained.

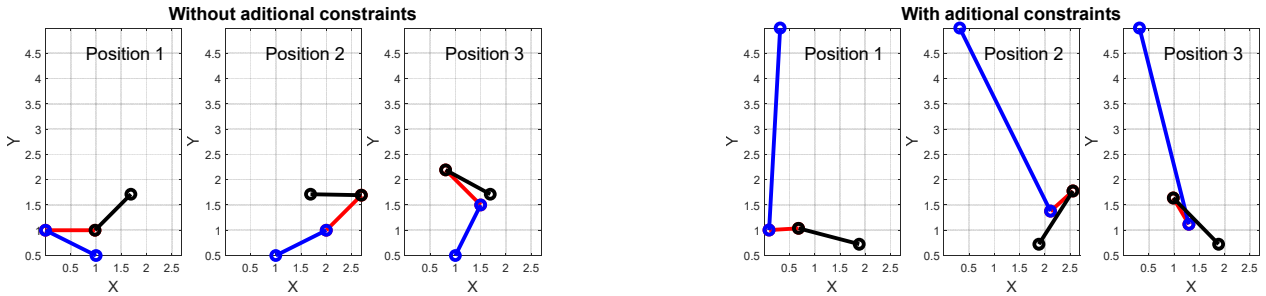


Figure 2. Solutions of the synthesis problem without and with additional restrictions

4. CONCLUSIONS

The proposed method allows introducing bounded variables in a dimensional synthesis problem in a very simple way, which avoids a series of very common problems such as obtaining impossible solutions (with negative lengths) or different configurations of the mechanism in each synthesis position. This method allows the value taken by the variables or functions of variables of the synthesis problem to be bounded. In this way, it is possible to provide a highly efficient and local method with characteristics of global search methods such as genetic algorithms without the need to alter the original algorithm.

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