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Review

Some considerations on assessing the importance of a coefficient

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A B S T R A C T

In input-output models, a technical coefficient is considered important when its variation causes significant changes in output, thus having great capacity to influence the economy. This concept of importance has limitations, some of which are resolved in this paper. Thus, a generalization of this concept is proposed by including in the analysis households as another sector to be considered. A new conceptualization of the notion of importance is also raised, in which changes in the total output are considered rather than the sectoral changes. Finally, a connection is established and formulated between an important coefficient and the intensity of the field of influence.

1. Introduction

Economic development is strongly influenced by the nature and pace of technological change that may be reflected in modifications in the productive structure and the associated changes produced in the network of intersectoral relations. If the productive branches experience a transformation in the technology that they use, the relationships of the exchanges between them will also change; therefore, a change in the technical coefficients reflects not only modifications in production method, but also the existence of technical progress and transformations in the structure of the economy. In this sense, structural changes can be considered as an element that can impact economic development. Further, we can point to the fact that transformations in technology and in the structure of the productive system are primarily observed in those sectors with a greater capacity to influence economic activity [1–3].

The challenge is to identify the most important coefficients for an economy; this set of coefficients is understood to contain those for which, if a small variation occurs, they generate important changes in output. In other words, they are the coefficients that have a great capacity to influence the structure and structural changes in an economy. The term *sensitivity of coefficients* has been coined in order to define these coefficients [4]. However, there are many cases for which this information is not available. Moreover, in those economies in which the information is available, there is normally a considerable gap between the latest table published and the current year. This often makes it necessary to estimate the matrices, which will be all the more useful if the sectors receiving the most attention are the most important for the economy, thus ensuring the overall accuracy of the matrix.

In this paper, we propose some extensions of the concept of important coefficients. The first is to include households as another "sector" of the economy, as it is influenced by the consumption habits and income of the families. Furthermore, consumption by households represents between 60 and 75 % of Gross Domestic Product on the expenditure side in most countries; hence, it is likely that changes in the allocation of income (i.e., consumption shares) could have an analytically important impact on an economy. With the aim of incorporating households into this analysis, the Miyazawa model (1976) will also be included, given that it is an extended input-output formulation that provides greater analytical insights into the relationship between sectors and households.

This paper is structured as follows: following the Introduction in Section 1, Section 2 provides a brief review of the relevant literature. Section 3 presents the methodology beginning with a review of the basic concepts of input-output analysis as a prelude to presentation of some extensions of the concept of important coefficients. The main results and discussion are found in Section 4. Finally, some conclusions and extensions are provided in Section 5.

2. literature review

This concept of important coefficient is related with those of sensitivity and elasticity. These issues have been a continuing source of discussion throughout the history of input-output analysis. This review provides some summary perspectives grouped into three categories, with a final section focusing on recent work.

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2.1. Early initiatives

Different authors, such as Sherman and Morrison [5], have considered and analyzed the problem of sensitivity in the context of uncertainty in matrices in general. However, their work was not highlighted until later by Bullard and Sebald [6,7]. Prior to this time, attention was directed to changes in coefficients and their impact on the associated Leontief inverse matrix (see Refs. [8–10]). Evans [8], for example, evaluated the effect of error or changes in individual coefficients on the elements of the Leontief inverse matrix. The general effects of coefficient change induced by technology, changing markets, structural change and the general effects of economic growth and development can be found in the work of Sevaldson [11], Carter (1970) and Tilanus [12]. Tilanus [12] in fact proposed a distinction between average and marginal coefficients; while Lahiri [13] suggested that the choice of input coefficient might be a function of the level of demand facing an industry, an echo of Sraffa's [14] ideas of switching and re-switching in joint production.

2.2. Error analysis

Theil's (1957, 1972) pioneering work in entropy decomposition analysis provided a useful way of examining error or change in input structures. He suggested that change could be decomposed into a set of additive components, an approach followed by Jackson et al. [15]. On the other hand, West [16] has approached error analysis from a relative change perspective, focusing, in particular, on the effects of coefficient error on the multipliers of the associated inverse matrix. Closely allied with this approach is that adopted by Jackson [17] who developed the notion of a probability density distribution for each coefficient and showed how this "uncertainty" could lead to serious problems in the utilization of the input-output model [18,19]. The relative change approach has also been explored by Xu and Madden [20].

Lawson [21] has approached the problem conceptually by considering various forms of error - additive and multiplicative - and the ways in which these might be used in a "rational approach" to modeling. Closely allied with this line of reasoning would be the work of Stevens and Trainer [22], Burford and Katz [23] and Giarratani and Garhart [24] who have developed some propositions about the major sources of error. The notion of some "rationality" in the error or in the structure of coefficient change of course underlies the widespread application of the RAS or bi-proportional technique in the context of updating (especially at the national level) and estimation (at the regional level, where a national table is often used as a base). Bacharach's [25] work revealed a strong link between the RAS technique and the assumptions explicit in linear and nonlinear programming. Matuszewski et al. [26] did in fact propose an LP-RAS technique; in their applications, several coefficients were "blocked out" in the updating algorithm because their true values were either known or could be estimated with what Jensen and West [27] have referred to as "superior data." To this point, (early 1970s), however, no attempt had been made to assess the degree to which errors in individual coefficients could be ranked or rated in terms of their importance. West [28] provided some important directions in this regard, suggesting a relationship between coefficient size and the associated multiplier. Several of the techniques and approaches developed for error analysis were subsequently modified to perform sensitivity analysis; these are described in the next section.

2.3. Sensitivity analysis

Using a little-known theorem developed by Sherman and Morrison [5], Bullard and Sebald [6,7] were able to show that, in energy terms, only a very small number of the input coefficients in the US input-output model were analytically important. In applications at the regional level, Hewings [29] referred to these as inverse important coefficients. In a similar fashion, Jensen and West [27] found that the removal of a large percentage of the entries in an input-output table could be accomplished with little appreciable effect on the results from the use of the model for impact analysis. Subsequently, West [16] noted that the size and location of the coefficient within the input-output table provided the major determinant of an individual coefficient's importance. Further work by Morrison and Thumann [30] and Hewings and Romanos [31] has extended the sensitivity notions to suggest that the censal mentality characterizing the developments of many input-output models (namely, that all entries need to be estimated with the same degree of accuracy) is probably misplaced. This is especially true in the cases in which regional tables are derived from national tables or in the process of updating tables. The results of the sensitivity analysis in combination with statistical estimation techniques suggest that a more "rational" approach to coefficient change could be developed [18].

2.4. Recent work

Hondo et al. [32] analyzed the sensitivity of the total CO_2 emission intensities, using the Japanese IO table, to identify the elements that significantly influence these emissions. In addition, they assessed how much the total CO_2 emission intensities changed due to the variation of these influential elements. Sonis and Hewings [33–35] generalized the concept of the sensitivity of the coefficients in terms of the identification of the field of influence, analyzing the changes that a row, column or complete table of coefficients generate on the output of an economy. Tarancón et al. [36] developed an approach that combined with an analysis of sensitivity to analyze the direct and indirect consumption of electricity of eighteen manufacturing sectors in fifteen European countries. Subsequently, Tarancón and del Río [37] provided a general vision of the principal applications of sensitivity analysis to the study of CO_2 emissions and energy, classifying them and evaluating on their main advantages and disadvantages. Mattila et al. [38] address the use of sensitivity analysis to analyze environmentally extended input-output models with the aim of constructing scenarios of sustainable development. Meng et al. [39] identified the economic sectors and regions with the greatest potential for electricity savings, proposing two indicators: the elasticity coefficient and the price sensitivity coefficient. Yuan and Zhao [40] employed an approach that combines input-output analysis and sensitivity analysis to explore the impact of technological changes on CO_2 emissions in high energy consuming industries. Yan et al. [41] implemented a sensitivity analysis based on the Leontief demand model and the Ghosh supply model to study the factors that lead to the greatest changes in CO_2 emission intensities in energy-intensive industries. Liu et al. [42] identify the relationships of the sectors with the highest CO_2 emissions; this paper estimates the elasticity of technical coefficients and final demand in relation to emissions. The results show that the greatest emission-coefficient elasticities are those related to the transactions between the construction and manufacture of non-metallic mineral products, between gross fixed capital formation and construction, and between agriculture and processing and manufacture of food, and production and supply of electric power and heat power and mining and washing of coal. Guan et al. [43] estimated the energy incorporated into the construction sector in China, using a hybrid Life Cycle Assessment method and a sensitivity analysis with the aim of identifying the key links between the sectors that significantly affect said the construction sector. To evaluate the CO_2 emissions of China, Li et al. [44] used a sensitivity analysis from which they were able to identify the key sectors and the main productive links between the different branches of activity. For their part, Zhang et al. [45] identified the key sectors as those with the highest intensity of consumption of metals and study the effect of the technical coefficients and final demand on their consumption using, among others, sensitivity analysis.

Furthermore, as has already been pointed out in Tarancón et al. [1], one of the limitations of the concept of important coefficients is that these coefficients may fundamentally influence a sector with little weight in the economy, probably meaning that this coefficient has only limited analytical importance. We propose, following other authors but using a different formulation, to focus on consideration of the economy-wide impacts, not the output of a sector, in the definition of the importance of a coefficient.

Finally, and given the existing link between the concepts of importance of a coefficient and that of the intensity of the field of influence of the first order (see Ref. [34]), we propose an alternative formulation that makes possible the formalization of this relationship. In addition, with the exception of West [16], little attention has been paid to the position of the coefficient in the matrix whereas in the evaluation of value chains, this feature has assumed considerable importance in the context of both position and participation (see Ref. [46] for a recent review and application).

3. Methodology

The analysis will be based on the familiar Leontief demand model [47] which can be expressed in the following manner:

$$x = Ax + y \tag{1}$$

where **x** represents the vector of the total sectoral production of the economy, **A** is the matrix of the regional technical coefficients and **y** is the vector of final demands and the solution for equation (1) yields:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \tag{2}$$

$\mathbf{(I\text{-}A)^{-1}}$ is the known Leontief inverse matrix. Considering changes in the economy, we can obtain:

$$\Delta\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{y}^1 - \mathbf{y}) + \left[(\mathbf{I} - \mathbf{A}^1)^{-1} - (\mathbf{I} - \mathbf{A})^{-1}\right]\mathbf{y} \tag{3}$$

where the elements that have been modified appear with the superscript 1. The first addition, $(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{y}^1 - \mathbf{y})$, refers to the analysis of multipliers and the second, $[(\mathbf{I} - \mathbf{A}^1)^{-1} - (\mathbf{I} - \mathbf{A})^{-1}]\mathbf{y}$, to the analysis of the coefficient sensitivity. In this paper, we will focus on the latter.

3.1. Important coefficients

In this section, we present the proposed extensions.

3.1.1. Consideration of households in the analysis

The extended model of Miyazawa [48] will be used in order to add households to the analysis. This model is one of the first that proposed the analysis of the "demographic-economic" impacts; subsequently, many other options have been considered to link the demographic and economic aspects. This link makes it possible to show, for example, the effects of changes of economic activities on the distribution of income, the workforce situation, migration behavior or, the effects of changes in consumer expenditure, the employment situation, etc., on economic activity ([49,50] and for a recent review, see Ref. [51]). Some initial explorations along these lines were proposed by Hewings [52] linking the notions of analytical importance with the identification of key sectors. Hewings and Romanos [31] also explored the role of households in terms of analytical importance in a regional application; in lieu of the Miyazawa formulation, a simple extended input-output model was used with the $(n+1)^{\text{th}}$ sector being household income (row) and consumption (column).

The model of Miyazawa [48] takes the following form:

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{V} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} + \begin{bmatrix} \mathbf{f} \\ \mathbf{g} \end{bmatrix} \tag{4}$$

where **x** is a vector that reflects the total output of the economy, **y** represents the income of the households classified by income groups, **A** is the matrix of technical coefficients, **V** is the matrix of value added of households classified into income groups, **C** is the matrix of coefficients of consumption, **f** is a vector of final demand excluding household consumption and **g** is a vector of exogenous income, excluding families.

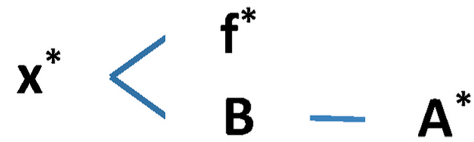

Fig. 1. Relationships between variables.

We denote by means of compact notation equation (4)

$$\boldsymbol{x}^* = \boldsymbol{A}^*\boldsymbol{x}^* + \boldsymbol{f}^* \tag{5}$$

where **A*** represents the Leontief extended matrix that reflects household activity and, analogously, **x*** and **f*** represent the production and income vectors.

From equation (2) and clearing the "extended" output (**x***), we obtain:

$$\boldsymbol{x}^* = (\boldsymbol{I} - \boldsymbol{A}^*)^{-1}\boldsymbol{f}^* = \boldsymbol{B}\boldsymbol{f}^* \tag{6}$$

where **B** is the Leontief extended inverse matrix. Therefore, it is possible to establish the following mathematical relationships **x*=h(B, f***) and **B=g(A***), that is to say, the extended output (**x***) is a function of the Leontief inverse matrix and of the income (**f*)**. Meanwhile, matrix **B** is a function of the matrix of extended technical coefficients**.** These relationships can be represented by Fig. 1:

Hence, changes in the coefficients of matrix **A*** cause changes in the Leontief inverse (**B**) and in the extended output.[1] On the other hand, modifications in **f'** also generate changes in **x***.

If we consider very small changes, we obtain:

$$\boldsymbol{dx}^* = \frac{\partial \boldsymbol{x}^*}{\partial \boldsymbol{B}}\frac{\boldsymbol{dB}}{\boldsymbol{dA}^*}\boldsymbol{dA}^* + \frac{\partial \boldsymbol{x}^*}{\partial \boldsymbol{f}^*}\boldsymbol{df}^* \tag{7}$$

The total change experienced by **x*** ($\boldsymbol{dx}^*$) may be due to both modifications in matrix A^* and in f^*.

If we assume that income f^* does not change, the modifications of the coefficients of the matrix **A*** will lead to repercussions in sectoral output, through **B** and, therefore:

$$\boldsymbol{dx}^* = \frac{\partial \boldsymbol{x}^*}{\partial \boldsymbol{B}}\frac{\boldsymbol{dB}}{\boldsymbol{dA}^*}\boldsymbol{dA}^* \tag{8}$$

In order to determine if a coefficient can be defined as important, we will consider modifications in an element of the matrix A*, for example a^*_{kl} and the changes that it generates in the output of a specific sector, the *i*th sector. In this sense, we can establish Fig. 2:

Therefore, the expression that follows can be established:

$$dx^*_i = \left(\frac{\partial x^*_i}{\partial b_{i1}}\frac{db_{i1}}{da^*_{kl}} + \frac{\partial x^*_i}{\partial b_{i2}}\frac{db_{i2}}{da^*_{kl}} + \ldots + \frac{\partial x^*_i}{\partial b_{in}}\frac{db_{in}}{da^*_{kl}}\right)da^*_{kl} \tag{9}$$

That is to say, the total change experienced by the output of the *i*th sector is due to the modifications of a coefficient (a^*_{kl}) whose change affects the elements of the Leontief inverse ($b_{ij}\ \forall j = 1, 2..n$).

Similarly, if we consider changes in relative terms, the concept of output-coefficient elasticity can be introduced with the following expression:

$$\eta(x^*_i, a^*_{kl}) = \frac{dx^*_i i / x^*_i i}{da^*_{kl} / a^*_{kl}} \tag{10}$$

The output-coefficient elasticity can be interpreted as the change experienced by the output of a sector due to modifications in a technical coefficient, in relative terms.

[1] From now on, with the aim of simplification, in the text we will refer to extended output, extended final demand and Leontief extended inverse as output, final demand and Leontief inverse, respectively.

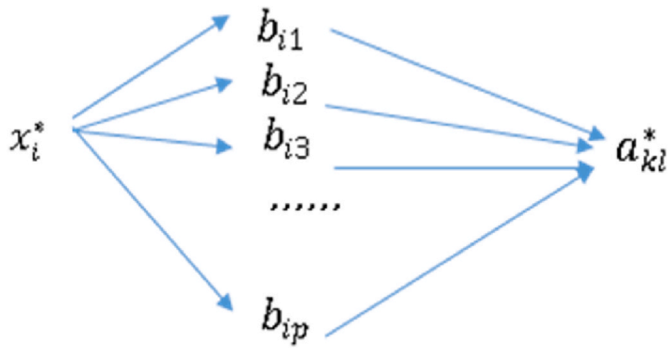

Fig. 2. Relationship between the output of a sector and a technical coefficient.

Returning to equation (9) and operating conveniently, we obtain:

$$\eta(x_i^*, a_{kl}^*) = \left(\frac{\partial x_i^*}{\partial b_{i1}}\frac{b_{i1}}{x_i^*}\frac{db_{i1}}{da_{kl}^*}\frac{a_{kl}^*}{b_{i1}} + \frac{\partial x_i^*}{\partial b_{i2}}\frac{b_{i1}}{x_i^*}\frac{db_{i2}}{da_{kl}^*}\frac{a_{kl}^*}{b_{i1}} + \ldots + \frac{\partial x_i^*}{\partial b_{in}}\frac{b_{i1}}{x_i^*}\frac{db_{in}}{da_{kl}^*}\frac{a_{kl}^*}{b_{i1}}\right) = \sum_j \eta(x_i^*, b_{ij})\eta(b_{ij}, a_{kl}^*) \tag{11}$$

The expression $\eta(x_i^*, a_{kl}^*)$ can be broken down into the sum of n factors, the first term of each measures the relationship between production and the elements of the Leontief inverse matrix, $\eta(x_i, b_{ij})\ \forall j$, and the second considers the influence of a change in the technical coefficients on the elements of the inverse, $\eta(b_{ij}, a_{kl}^*)$.[2]

From equation (14) and given that $x_i^* = \sum_j b_{ij} f_j^*$, we obtain:

$$\eta(x_i^*, b_{ij}) = \frac{b_{ij} f_j^*}{x_i^*}\ \forall j = 1, 2..n \tag{12}$$

Following the work of Evans [8] [3],which the following expression is proposed that allows determination of the changes in the matrix derived from modifications in the technical coefficients:

$$\Delta b_{ij} = \frac{b_{ik} b_{lj} \Delta a_{kl}}{1 - b_{lk}\Delta a_{kl}} \tag{13}$$

The elasticity between technical coefficients and inverse Leontief coefficients can be expressed as:

$$\eta(b_{ij}, a_{kl}^*) = \frac{b_{ik} b_{lj} a_{kl}^*}{(1 - b_{lk}\Delta a_{kl}^*) b_{ij}}\ \forall j = 1, 2..n \tag{14}$$

Substituting and operating

$$\eta(x_i^*; a_{kl}^*) = \frac{a_{kl}^* b_{ik}\ x_l^*}{(1 - b_{lk}\Delta a_{kl}^*) x_i^*} \tag{15}$$

Equation (15) reveals that the sectoral production-technical coefficient elasticity depends on the coefficient itself (and its change), on the elements of the Leontief inverse, and on sectoral production. The higher the sectoral output-technical coefficient elasticity, the greater the change experienced by output due to a change in the coefficient. In essence, in the change experienced by the output when the technical coefficient is modified, the direct effects (a_{kl}^*), the totals (elements of the Leontief inverse) and the sectoral production contribute.

3.1.2. Considering the total output

As has been pointed out, one of the limitations of the concept of important coefficient is that if the coefficient influences a sector with little weight in the economy as a whole, it will not generate relevant synergies throughout the economy, and therefore, instead of considering the output of a sector, total (economy-wide) output will be taken into consideration[4] [1].

From equation (11) and considering the total, economy-wide output, we obtain:

$$\frac{dx^*}{da_{kl}^*} = \sum_j \frac{\delta x}{\delta b_{ij}}\ \frac{db_{ij}}{da_{kl}^*} \tag{16}$$

from where

$$\eta_{x^*, a_{kl}^*} = \frac{a_{kl}^*}{x^*}\sum_j \frac{\delta x^*}{\delta b_{ij}}\ \frac{db_{ij}}{da_{kl}^*} = \sum_j \eta(x^*, b_{ij})\eta(b_{ij}, a_{kl}^*) \tag{17}$$

From equation (19) and given that $x^* = \sum_i x_i^*$, we can state:

$$\eta(x^*; b_{ij}) = \sum_i \frac{b_{ij} f_j^*}{x^*} \tag{18}$$

Therefore, the expression for the total output-technical coefficient elasticity is:

$$\eta_{x^*.a_{kl}^*} = \frac{\sum_i \Delta x_i^*}{\sum_i x_i^*}\ \frac{a_{kl}^*}{\Delta a_{kl}^*} = \frac{a_{kl}^*}{1 - b_{lk}\Delta a_{kl}^*}\ \frac{\sum_i\sum_j b_{ik} b_{lj} a_{kl}^*}{\sum_i x_i^*} \tag{19}$$

This expression of the elasticity is very similar to that presented in Tarancón et al. [1]. The higher the value of elasticity, the greater the change in the output caused by the variation of a technical coefficient and, therefore, the more important it can be considered.

On the other hand, defining the quotient $\frac{a_{kl}^*}{\Delta a_{kl}^*}$ and given that $\sum_i\sum_j b_{ij} f_j^* = \sum_i x_i^*$, we obtain:

$$\frac{a_{kl}^*}{\Delta a_{kl}^*} = \frac{a_{kl}^* \sum_i\sum_j b_{lj} b_{ik} f_j^*}{(1 - \Delta a_{kl}^* b_{lk})\sum_i \Delta x_i^*} \tag{20}$$

Further, define $p = \frac{\sum_i \Delta x_i^*}{\sum_i x_i^*}$ and ordering the terms, we can state:

$$\frac{\Delta a_{kl}^*}{a_{kl}^*} = \frac{p}{a_{kl}^*\left(\frac{p b_{lk}}{100} + \frac{\sum_i\sum_j b_{lj} b_{ik} f_j^*}{\sum_i\sum_j b_{ij} f_j^*}\right)} \tag{21}$$

This expression is related to one proposed by Schintke and Stäglin [53] within the methodology of tolerable limits (referring to the output of a sector) that is defined as:

$$r_{kl}(p) = \frac{p}{a_{kl}\left(\frac{b_{lk} p}{100} + b_{kk}\frac{X_l}{X_k}\right)} \tag{22}$$

This relationship quantifies the sensitivity of the coefficient a_{kl} to changes of the p%. Therefore, a technical coefficient will be more important the lower the r_{kl}, as this means that “small” variations of the coefficient will lead to relatively large changes in the production of an economic sector.

[2] A similar expression where a single element of the Leontief inverse matrix is considered can be seen in Pulido and Fontela (1993).

[3] Evans [8] developed a formalism similar to the idea of Sherman and Morrison [5] who proposed the expression below to quantify the changes in the elements of a generic inverse matrix (**D**), due to modifications in the coefficients ($a_{kl} + \Delta a_{kl}$) of an initial matrix (**A**), without having to recalculate the inverse matrix itself.$D_{ij} = d_{ij} - \frac{d_{ik} d_{lj}\Delta a_{kl}}{1 + d_{lk}\Delta a_{kl}}$.

[4] Note that the Fields of Influence concept [35] exceed this limitation in their own definition.

3.1.3. Considering fields of influence and their intensity

Closely related to the seminal works of Evans [8] and Sherman and Morrison [5], Sonis and Hewings [33,34] proposed the notion of a *field of influence* that allows the determination of the changes in the Leontief inverse matrix generated by modifications of one or several technical coefficients. This can be considered as a generalization of previous formulations, as changes in all the technical coefficients can be evaluated simultaneously.

Let us suppose a change (ϵ) in the coefficient a_{kl}, that is to say, $a_{kl}(\epsilon) = a_{kl} + \epsilon_{kl}$, and therefore the new Leontief inverse will have changed and taken the form $\mathrm{B}(\epsilon) = \{\mathrm{b_{ij}}(\epsilon)\}$, where $\epsilon_{kl} \neq 0$ and the rest of the elements $\epsilon_{ij} = 0\ \forall i,j \neq k,l$. If a modification is produced in a single coefficient, the following expression is obtained:

$$B(\epsilon) = B + \frac{\epsilon_{lk}}{1 - b_{lk}\epsilon_{lk}} F(l,k) \tag{23}$$

F(l,k) represents the field of influence, the mathematical formulation of which is

$$F(l,k) = \begin{bmatrix} b_{1k} \\ \cdots \\ b_{nk} \end{bmatrix} [\, b_{l1} \quad \ldots \quad b_{ln} \,] \tag{24}$$

Therefore, if we refer to a single element, we obtain:

$$\Delta b_{ij} = \frac{\Delta a_{kl}}{1 - b_{lk}\Delta a_{kl}} b_{ik} b_{lj} \tag{25}$$

where $b_{ik}b_{lj}$ represents the field of influence due to the modification of Δa_{kl}. This expression coincides with that derived by Evans [8].

Continuing this idea, Sonis and Hewings [35] establish the concepts of the intensity of a field of influence and of total intensity using the following formulation:

$$IntF(k,l) = \sum_{ij} b_{ik} b_{lj} = B_{.k} B_{l.} \tag{26}$$

where $IntF(k,l)$ represents the intensity of the field of influence.

Normalizing the previous expression from the global intensity of the Leontief inverse matrix: $T = \sum_{ij} b_{ij}$, we obtain:

$$\mathrm{M} = \frac{1}{T}[B_{.k} B_{l.}] \tag{27}$$

From this expression, Sonis and Hewings [35], derive the multiplier product matrix (MPM) that can be expressed as:

$$\boldsymbol{MPM} = \{m_{ij}\} = \frac{1}{T} B_{i.} B_{.j} = \frac{1}{T} \begin{bmatrix} B_{1.} \\ \vdots \\ B_{n.} \end{bmatrix} [\, B_{.1} \quad \ldots \quad B_{.n} \,] \tag{28}$$

If $\sum_i \sum_j b_{ij} f_j^* = T^d$ is denoted, where $\mathbf{T}^{\mathrm{d}}$ represents the total intensity of matrix **B** weighted by the final demand with $\sum_i \sum_j b_{lj} b_{ik} f_j^* = \sum_i b_{ik} \sum_j b_{lj} f_j^* = (B_{.k} B_{l.})^d$ and with $(B_{.k} B_{l.})^d$ representing the element (*l, k*) of the **MPM** weighted by the volume of the final demand. This weighting allows to reflect the importance of the branch in terms of the final demand (Rasmussen, 1954 proposed a similar idea in the context of key sector analysis).

If $\frac{\Delta a_{kl}^*}{a_{kl}^*}$ is re-written as r_{kl}^{MPM}, following the notation of Schintke and Staglin [53], supposing that *p* takes the value 1 % and substituting in (22), we obtain:

$$r_{kl}^{MPM}(p) = \frac{1}{a_{kl}^* \left(\frac{b_{lk}}{100} + \frac{(B_{.k} B_{l.})^d}{V^d} \right)} = \frac{1}{a_{kl}^* \left(\frac{b_{lk}}{100} + \frac{(B_{.k} B_{l.})^d}{V^d} \right)} = \frac{1}{a_{kl}^* \left(\frac{b_{lk}}{100} + m_{kl}^d \right)} \tag{29}$$

In other words, $r_{kl}^{MPM}(p)$ represents the measure of the importance of a coefficient from the point of view of the intensity of the field of influence (Matrix **MPM**).

This definition of important coefficient is determined from the intensity matrix of the field of influence generated by the variation in a technical coefficient. The higher the intensity (weighted by the final demand) of the field of influence generated by the modification of the technical coefficient, the greater its importance in mobilizing the global output of the economy.

4. Empirical analysis

The empirical analysis will focus on comparing the new approach based on the most important coefficients, with other traditional approaches, using the Spanish IOT for year 2016; this table is the most recent one that is available in this moment. Additionally, for the construction of the extended matrix, microdata from the Spanish Households Budget Survey (EPF) for the year 2016 will be used; these data are provided by the National Statistical Institute (INE).

4.1. Statistical information

The Spanish IOT for year 2016 is defined in producer's prices and disaggregated into 64 industries; this level of disaggregation will be modified to integrate with other statistical sources. The next step is to construct a wage matrix which will be disaggregated according to four income groups based on the net monthly income declared by households in the afore mentioned EPF for year 2016. In this way, the disaggregated wage matrix will be constructed according to the following expression:

$$\boldsymbol{Wq}_{n,4} = \widehat{\boldsymbol{W}}_{n,n}\, \boldsymbol{PI}_{n,4} \tag{30}$$

Where $\boldsymbol{Wq}_{n,4}$ represents the disaggregated wages by income quartiles, $\widehat{\boldsymbol{W}}_{n,n}$ is the diagonalized vector of total wages, and $\boldsymbol{PI}_{n,4}$ is a matrix of shares of income in each quartile and in each industry.

Finally, it is necessary to disaggregate household's consumption to match the income quartiles by industries. In the EPF, this information is available for more than 27,000 observations that are representative of all Spanish households with further information about income level and other socio-economic characteristics. However, the consumption expenditure is disaggregated by purpose of consumption according to the COICOP[5] classification in more than 100 categories, while the Spanish IOT classifies the products by activity (CPA classification); hence, it is necessary to integrate the two datasets through a bridge matrix (BM). In this integration, the 2015 BM built by Cai and Vandyck [54] will be used using through the following equation:

$$\boldsymbol{Cq}_{n,4} = \widehat{\boldsymbol{C}}_{n,n} \boldsymbol{BM}_{n,m} \boldsymbol{HE}_{m,4} \tag{31}$$

Where $\boldsymbol{Cq}_{n,4}$ is the consumption matrix of the households classified by income quartiles, $\widehat{\boldsymbol{C}}_{n,n}$ is a diagonalized vector of the household's consumption and classified according to the CPA classification, $\boldsymbol{BM}_{n,m}$ is the bridge matrix that reconciles the COICOP and the CPA classifications, and $\boldsymbol{HE}_{m,4}$ represents the household expenditure by income quartile classify according to the COICOP classification. The final aggregation used in this paper is the defined in the Statistical Classification of Economic Activities (NACE) which defines 21 industries specified in the Appendix.

[5] The Classification of individual consumption by purpose, abbreviated as COICOP, is a classification developed by the United Nations Statistics Division to classify and analyze individual consumption expenditures incurred by households, non-profit institutions serving households and general government according to their purpose.

4.2. First results

In this section the important coefficients will be determined through the application of the proposed methodology in this paper; thereafter, they will be compared with the traditional methodology has been normally used in this kind of literature. First, it is assumed that an important coefficient is the one which is situated over the 9th decile of the distribution, that is, a coefficient is described as important when it is among the highest 10 % of coefficients. The results are shown in Table 1.

Each coefficient is designated through two letters, the first one refers to row industry, and the second one refers to the column industry. In this way, CD represents the cross coefficient belonging to the row where the Manufacturing industry (C) is situated with the column where the Electricity industry (D) is situated. The coefficients are ordered from smallest (MJ and EQ) to largest (HH and BC).

Table 2 shows those coefficients that are considered important according to both methodologies, that is, the coincidences between the two methods, which amount to 40 %.

There are two numbers within the brackets; the first one refers to the importance ranking according to the proposed methodology, and the second one according to the traditional methodology. If the important coefficients calculated with the proposed methodology are considered, then it can be seen that the higher they are, the greater the number of matches; this outcome could be explained by the fact that there are a group of coefficients whose importance remains whatever the methodology applied.

This result could be related with the notion of "fundamental economic structure" (FES) of Hewings et al. [55,56]. According to the authors this FES is characterized in three important ways: stability, that is the degree to which certain elements are present across different samples; predictability, the degree to which the size of some elements may be predictable using some aggregate measures of an economy (e.g., gross national product, the degree of industrial concentration by sector); and importance, the degree to which the elements of the FES are part of a set of components of the economic structure which may be important, in the sense that change in these elements would likely create the most potential for system-wide change [55,56].

The greatest differences between the two methodologies, from the perspective of the proposed methodology, are those derived from the coefficients belonging to the Manufacturing industry, which are mostly important. This is due, on one hand, to the fact that those coefficients have a significant weight in the total economic output, and, on the other hand, because those coefficients are weighted by final demand which is very high for the Manufacturing industry. These two characteristics are not taken into consideration by the traditional methodology, and, therefore, they are not considered as important coefficients, even though they are very important for the Spanish economy.

A sector is considered as "important" if it is formed by many important coefficients. In this sense, it can be defined an index which measures the sectoral importance degree (SIDI) if is considered both the number of important coefficients and the degree of importance. This indicator is defined as:

$$SIDI_i = \frac{n_i}{n} \sum_j R_j \tag{36}$$

Where n_i is the number of important coefficients of the ith sector, n is the number of total coefficients, and $\sum_j R_j$ is the sum of the ranks of the important coefficients. Then, the first term of the expression $\left(\frac{n_i}{n}\right)$ represents the proportion of the important coefficients in the sector, and the second term $\left(\sum_j R_j\right)$ the degree of importance. In Table 3 the values of the index computed according to the two methodologies are shown.

If the proposed methodology is applied, the most important seller's industries are Manufacturing (C), Professional, scientific and technical activities (M), Electricity, gas, steam and air conditioning supply (D) and Transporting and storage (H). All of them have a strong impact in the national economy, especially the Manufacturing (C) industry that presents a very high value in the index (13.56); one reason could be the aggregation level used in this work. If the results are compared with the traditional methodology, it can be seen that there are some matches in the Professional, scientific and technical activities (M) industry. In

Table 1
Coefficients over the 9th decile according to both methodologies.

Order	Proposal	Traditional	Order	Proposal	Traditional
1	MJ	EQ	21	AA	MG
2	NH	EJ	22	CQ	NC
3	NI	QQ	23	GG	NN
4	ME	SR	24	CD	JJ
5	HG	SS	25	CA	EG
6	NE	NM	26	QQ	EH
7	LL	IN	27	SS	HG
8	CG	MJ	28	CI	NG
9	DE	NJ	29	FC	HC
10	II	KG	30	CE	BD
11	CO	KC	31	RR	FF
12	CQ	RR	32	NN	MM
13	GE	MF	33	JJ	GC
14	SC	LG	34	FF	KK
15	CR	NH	35	DD	CC
16	NC	KL	36	MM	HH
17	CJ	EI	37	KK	ED
18	CG	DC	38	EE	EC
19	MC	MC	39	CC	AC
20	CH	DE	40	HH	BC

Source: own elaboration

Table 2
Important coefficients according to both methodologies.

Concordances	
MJ (1–8)	RR (31-12)
NH (2–15)	NN (32-23)
HG (5–27)	JJ (33-24)
DE (9–20)	FF (34-31)
NC (16–22)	MM (36-32)
MC (19-19)	KK (37-34)
QQ (26-3)	CC (39-35)
SS (27-5)	HH (40-36)

Source: own elaboration

Table 3
Important seller's and buyer's industries according to both methodologies.

—	Proposal		Traditional	
	Seller	Buyer	Seller	Buyer
A	0.0525	0.23	0.001	0
B	0	0	0.060	0
C	13.56	1.235	0.015	7.1
D	0.22	0.295	0.220	0.335
E	0.095	0.15	1.785	0.05
F	0.085	0.315	0.025	0.22
G	0.33	0.345	0.020	1.875
H	0.225	0.465	0.233	0.578
I	0.065	0.19	0.085	0.043
J	0.0825	0.3825	0.043	0.43
K	0.0925	0.0925	0.930	0.085
L	0.0175	0.0175	0.068	0.04
M	0.3075	0.275	1.400	0.19
N	0.12	0.1825	2.145	0.15
O	0	0.03	0.000	0
P	0	0.0275	0.000	0
Q	0.065	0.24	0.095	0.02
R	0.0775	0.23	0.073	0.08
S	0.0675	0.205	0.365	0.013

Source: own elaboration

addition, the calculated index presents high values in the following sectors: Administrative and support service activities (N), Water supply; sewerage; waste management and remediation activities (E) and Financial and insurance activities (K); however, not all these industries could be considered as important sectors due to their modest impact on the national economy.

The most important buyer's sectors determined according to the proposed methodology are Manufacturing (C), Transporting and storage (H), Wholesale and retail trade; repair of motor vehicles and motorcycles (G), Construction (F) and Information and communication (J), and it can be observed many similarities between the two methodologies, except for the Construction (F) sector. All of them are very significant industries in the economy and with a high level of final demand.

Using the extended Leontief matrix in which the proposed methodology is based, and where households are considered as an additional sector in the economy, the results are shown separately in Table 4; since there have been limited attempts to explore analytical significance with such extended matrices, no comparison will be possible.

Considering the household's sector in columns implies that the consumption patterns are being analyzed in terms of their impacts on the productive industries, with household disaggregation by income level. Table 4 shows the most important coefficients obtained by the proposed methodology, that is, those that are above the 9th decile of the distribution. In this table (Table 4) Q1, Q2, Q3 and Q3+ represent those households which income level is situated in the first, second, third and above third quartile of the income distribution, respectively. In Table 4, it can be observed that the most important group is the one with the highest income level (Q3+). These households consume goods and services of Education (P), Manufacturing (C), Human health and social work activities (Q), among others. In addition, households situated in the third quantile of the income distribution (Q3) present an important coefficient related to their consumption in the Manufacturing (C) industry. Households whose income level is situated above the third quantile of the distribution (Q3+) are considered as an important sector in the economy, meaning that a variation in their consumption patterns will lead to significant changes in the economy. Using equation (36), the values for the upper two quartiles are $SIDI_{Q3+}, = 0.356$, and $SIDI_{Q3} = 0.013$.

Next attention, will focus on the household rows of the value added entires, also disaggregated by income quartiles. In Table 5, the most important coefficients are shown (those being in the 90 %):

Once again, the most important coefficients are found in the value added of the highest income households, specifically those situated over the third quartile (Q3+). This value added is generated in Education (P), Public Administration (O), and Human health and social work activities (Q). Also, there are important coefficients relative to the value added of households situated in the third quartile of the income distribution in the following industries: Education (P), Public Administration (O); and relative to the value added of those households situated in the second quartile there is an important coefficient in Education (P). Table 6 shows the results of SIDI indicator.

As can be seen from this table, the highest value belongs to households situated over the third quartile in the income distribution, meaning that those households are an important sector in the economy.

Table 4
Most important coefficients. Household's consumption.

Order	Coefficients
1	CQ3
2	IQ3+
3	GQ3+
4	NQ3+
5	QQ3+
6	CQ3+
7	PQ3+

Source: own elaboration

Table 5
Most important coefficients. Household's value added.

Order	Coefficients
1	Q3+G
2	Q2P
3	Q3+N
4	Q3O
5	Q3P
6	Q3+Q
7	Q3+0
8	Q3+P

Source: own elaboration

Table 6
SIDI Indicator values.

Sector	SIDI
Q3+	0.329
Q3	0.118
Q2	0.026

Source: own elaboration

5. Conclusions

Obtaining important coefficients is very relevant in terms of potential contributions to the formation and evaluation of economic policies, especially those related to addressing spatial and personal inequalities. The more important a coefficient, the greater the technical output-coefficient elasticity. Therefore, policies affecting important coefficients will generate greater economy-wide impacts; however, their impact on dimensions such as income inequality would require additional analysis.

The concept of important coefficient presents limitations that could be addressed by extensions and modifications of the methodology. One such extension consists in the analysis based on demographic-economic models [48] that would allow for the introduction of households as another relevant factor to be considered. A good example would be the activity analysis models reviewed in Batey and Hewings [51]. Here status in the labor force (e.g., employed or unemployed) and the impact of movement between those two could be evaluated using the methodology presented in this paper. Moreover, as highlighted by Tarancón et al. [1], a coefficient is not truly important if it mainly affects a sector with little weight in the economy. In this paper, a formulation has been proposed that considers total (system-wide) output rather than individual sectoral output. For that reason, a coefficient will be more relevant if a small change to it causes a strong modification in the total output. The results obtained, although are similar in form to those revealed by Tarancón et al. [1], they have been derived by means of a different procedure to that employed by the latter authors.

Finally, and given the connection between the concept of the field of influence and the changes experienced by the elements of the Leontief inverse matrix [35], a link has been established between the formula that allows to determination of the importance of the coefficients and the intensity of the direct fields of influence. A coefficient will have more importance the greater the intensity, weighted by the final demand, of a direct field of influence.

The proposed methodology has been applied on the Spanish IOT of year 2016 and the results obtained have been compared with those derived from the traditional methodology. A 40 % coincidence between both methodologies has been found in Wholesale and retail trade; repair of motor vehicles and motorcycles (G). This result could be related with the concept of fundamental economic structure; on the other hand, the greatest differences occur in the manufacturing sector.

The importance of the sectors was measured through the SIDI indicator, which considers both the number of important coefficients and the relevance of each coefficient. If the two methodologies are compared, only one match is found if the five most important seller's sectors are considered: Professional, scientific and technical activities (M) sector. Instead, if the five most important buyer's sectors are considered, it can be seen more matches, specifically, four matches.

Additionally, another contribution of the proposed methodology is that it allows households to be considered as one more sector of the economy, both in terms of consumption and in their value added. It has been integrated using the Miyazawa model in an extended Leontief matrix. Households have been classified into income quartiles, to explore which group of households have the highest impact over the economy. Not surprisingly, the group of households whose income is above the third quartile presents the highest values of the SIDI index. Further work will be needed to examine the degree of stability of these outcomes, as a way of uncovering structural changes in the role of income formation and its impact on an economy over time.

Data availability

No data was used for the research described in the article.

Appendix

Table N A.1
Statistical Classification of Economic Activities

A - Agriculture, forestry and fishing
B - Mining and quarrying
C - Manufacturing
D - Electricity, gas, steam and air conditioning supply
E — Water supply; sewerage; waste management and remediation activities
F - Construction
G - Wholesale and retail trade; repair of motor vehicles and motorcycles
H - Transporting and storage
I - Accommodation and food service activities
J - Information and communication
K - Financial and insurance activities
L - Real estate activities
M — Professional, scientific and technical activities
N - Administrative and support service activities
O - Public administration and defense; compulsory social security
P - Education
Q - Human health and social work activities
R - Arts, entertainment and recreation
S - Other services activities
T - Activities of households as employers; undifferentiated goods - and services - producing activities of households for own use
U - Activities of extraterritorial organizations and bodies

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