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Automatic machine learning vs. human knowledge-based models, property-based models and the fatigue problem

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ABSTRACT

This paper is devoted to emphasize the importance of human-based knowledge and to present the original property-based models. The main idea is that models are built in terms of equations that reproduce and guaranty their satisfaction, which leads to non-arbitrary parametric models. The methods based on data alone are not sufficient to be applied to fatigue S–N and GRV-N models: first, the required high number of results in machine learning, is not attained in fatigue; second, a black box cannot supply a comprehension of the fatigue phenomenon; third, the absence of a supporting model impedes extrapolation beyond the scope of experimentation and fourth, many other data are required to include the stress ratio, R, while robust models are already available. These models are AI mixed models, where its main part is human-based whereas the parameter estimation is solved based on data. The fatigue problem is used to illustrate the methodology and show that its generalization is applicable to other real problems. A detailed analysis of the fatigue model properties is done to show the readers how to extend those properties to other cases. Some future lines of research are suggested, followed by some conclusions.

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1. Introduction and motivation

In this paper the importance of human knowledge based models in artificial intelligence (AI) is emphasized and the consequences of its ignorance is discussed. A special and original contribution is the property-based models, which provide a deep information about the problem behavior, analytical equations, which is superior to most used models in AI.

In addition, due to the uncertain character of real phenomena, the need of incorporating random or stochastic models better than deterministic ones is defended and justified. The problem of random fatigue, as an interesting example, is used to present the proposed methodology, which must be based not only on data, but on properties that a model must have.

Bayesian networks are introduced as the best, simpler and compatible tool to define the joint distribution of all the involved variables, which in addition can be defined locally by means of conditional distributions, and allows well established explanation methods to be used.

Furthermore, Bayesian methods permit converting the model parameters into new variables, generating mixtures of distributions, that lead to completely general models, i.e., to distribution free models in practice.

Otherwise, most current fatigue models used in practice utilize only one stress as primary variable, when at least two variables must be used, for example, the maximum stress σ_{max} and the stress ratio R, whereas the frequency should also be included. These three variables constitute the smallest set required for damage accumulation models to be valid in the design. We note that automatic learning models allow the inclusion of several variables easily, but cannot explain clearly their relation with the material behavior. Since most fatigue models based on human knowledge do not use sufficient variables to define the problem adequately, new methods must be used to add them, as those proposed by the authors in this paper.

Finally, the consideration of extreme value distributions, see [Freudenthal and Gumbel, 1956], [Bolotin, 1981] and [Castillo, 1988], is crucial in design, because very large (loads) or very low (strengths) values of the variables play the central roll. In this context, it is necessary to know that three types cover all possible extreme models: Weibull, Gumbel and Fréchet types. Many of the used models, normal, gamma, log-gamma, etc. have tails of a Gumbel type, implying the risk of ignoring the Weibull and Fréchet types, which are much more general (Gumbel is only a limiting case of the other two). We note that, in minimum problems, bounded tails correspond to Weibull, and then, unbounded ones can be Gumbel or Fréchet.

This work is motivated by the authors' long experience in developing fatigue models and using them in practice. The above problem requires a solution based on human knowledge, because otherwise the understanding of the fatigue behavior becomes incomplete. These ideas also appear, as the current trend to achieve fatigue characterization, both as S–N field and crack growth rate approaches, based on machine learning methods, [Zhu et al., 2023], sometimes combined with human models, as proven with very recent publications, [Wang et al., 2023a], [Wang et al., 2023b], [Zhou et al., 2023] and [Awd, 2023]. This entails new challenges in an attempt to determine consistent guidelines to spare wrong future applications and orientations in this new field of fatigue assessment.

Based on big-data of a TC17 titanium alloy for various loading conditions, [Zhu et al., 2023] introduce a multi-algorithm integration machine learning (ML) approach as an effective tool to predict high cycle fatigue (HCF) life. A comparative analysis is made with two other ML models and the experimental results using R^2 and root-mean square error (RMSE) values, whereas the influence of the specimen rather than mechanical one and the experimental conditions is found to be the most significant on the fatigue life.

To investigate the importance of the random defects inherent to additive manufacturing and their influence on the fatigue life scatter, [Wang et al., 2023a] developed two frameworks, which combine physics-based and ML algorithms. After evaluating and comparing three additive materials, they achieved improved prediction ability while avoiding overfitting when insufficient data are available.

[Wang et al., 2023a], after a comprehensive review of physics-based and data-driven approaches for fatigue life prediction, draw attention to the detrimental lack of physical interpretation and the insufficient training samples for limited fatigue data of very different type, when data-driven approaches are used alone. On the contrary, using the proposed hybrid physics-informed and data-driven model (HPDM), hybrid models, as an emerging paradigm for fatigue life prediction, complementary advantages and synergistic effects are attained from both sides contributing to promote scientific knowledge.

[Zhou et al., 2023] proposed a probabilistic neural network to include the physics knowledge in the fatigue analysis of three different metallic materials. They consider the scatter of the fatigue life in the analysis to parameterize the fatigue life distribution with some physical constraints using failure data with run-outs to prevent overfitting and to provide an effective generalization of the stress amplitudes not used in the experimental program, i.e., to predict lifetimes in the VHCF region beyond the experimental frame. In fact, the definition of the dispersion law along the S–N field, but particularly in the VHCF zone, becomes critical when using machine learning due to the limited number of results in this zone as a result of the long duration of tests and their higher scatter.

In favor to the human knowledge-based models approach, can be argued that a valid S–N model fulfils the statistical conditions. First, they provide a supplementary analytical support in the two limiting zones (LCF and fatigue limit) where the data are customarily scarce, due to technical reasons or increasing test duration, and second, they allow the extrapolation or prediction of the lifetimes beyond the domain of the experimental campaign because their general validity based on the model robustness.

Hybrid models as a combination of physics-informed and data-driven models (HPDM) are proposed by [Wang et al., 2023a], [Wang et al., 2023b] and [Zhou et al., 2023] for fatigue life prediction. Since the data assessment is, ultimately, a statistical problem, the hybrid models should additionally fulfil statistical conditions to support the extrapolation of the fatigue prediction beyond the test scope and to avoid the arbitrary definition of the variability field, as proven in [Castillo and Fernández-Canteli, 2009].

In the cumulative damage calculation of structures under real loads, fatigue damage process monitoring, is studied, separately, using structural health monitoring and continuous fatigue monitoring whereas a combination of both techniques, with possible application of machine learning, is still in development, see [García-Fernández et al., 2023].

The probabilistic compatible S–N models developed by the authors, [Castillo and Fernández-Canteli, 2009], [Fernández-Canteli et al., 2021] and [Castillo et al., 2021] and [Alvarez-Vázquez, 2020], based on the theory of functional equations and extreme value statistics, allow a reliable data assessment as percentiles curves of the S–N field fatigue characterization with definition of the asymptotic fatigue limit as a model parameter, of the low cycle fatigue (LCF) region, of the influence of the stress ratio, R, and of the test frequency (in this case, partially depending on a fundamental experimental data).

The substitution of the conventional stress range or stress amplitude by the maximum value of a suitable generalized reference variable (GRV) in the S–N field in conjunction with these advanced models allows a unified methodology to be established for the fatigue characterization of materials and their application to the practical design according to the structural integrity concept. To this point, the recurrence of artificial intelligence methods to define a probabilistic S–N field is superfluous. It is only in the final stage of the statistical definition of the S–N field when the application of the machine learning method contributes to determine the confidence intervals, o more properly, the distributions of the model parameters by applying, for instance, Bayesian techniques, [Castillo et al., 2019].

The paper is organized as follows. In Section 2 we present some different methods for working with artificial intelligence. In Section 3 the S–N fatigue models based on human knowledge is introduced. In Section 4 the compatibility conditions in the derivation of S–N and GRV–N models, where GRV is an acronym for generalized reference variable, are listed and justified.

In Section 5 we introduce the mixed fatigue models, that is, human knowledge combined with machine learning based models and we present the Maennig's fatigue example. In Section 6 some interesting future study directions are indicated to extend the property-based models to the case of many other problems. Finally, in Section 7 we provide a list of conclusions. In addition, the appendix Section A is devoted to Bayesian networks, where its definition and important properties are described, and the appendix Section B lists the table of notations used in this work.

2. Different methods for working with artificial intelligence

In the machine learning process, three different viewpoints can be found: Automatic learning, intelligent human learning, and mixed models. In the first case, model fitting of big data and reasonable predictions can be achieved in the data region, but nothing is attained about comprehension or deep learning of the real properties, such as, dimensionality, compatibility, normalization, etc., related to the fatigue phenomenon. The second provides a valuable information about the insides of the problem being dealt with. Finally, the third combines the advantages of both methods. Table 1 provides a list of some advantages and disadvantages of the three methods. As already pointed out in the Introduction, some examples of physics-guided machine learning frameworks for fatigue life prediction and a combination of neural networks with physical models can be seen in [Wang et al., 2023a], [Wang et al., 2023b] and [Zhou et al., 2023].

Table 1. A comparison of the advantages and shortcoming of the three different methods used in AI.

Method	Advantages	Shortcomings
Automatic learning	They are based only on data. No other info than an initial variable list or data base is needed. Standard methods are directly applicable.	They do not provide info about the model properties and physical behavior. Explanation difficult to be made. Too many computer resources required. Extrapolation possibly invalid outside the data region.
Human knowledge based	They provide information about model properties and behavior. They produce the only formulas that reproduce the associated properties. Explanation easy to be done, based on physical meaning of variables. Extrapolation valid inside the properties validity region and possibly outside data region. Least computer resources needed. Bayesian methods and Bayesian network methods easily applicable and very convenient.	Identification of relevant variables is required. Model properties must be identified. Humen resources needed. Functional and differential equations knowledge is required.
Mixed models	They allow to choose the advantages of the two methods above and avoid the associated shortcomings.	They need the knowledge of both methods.

There are several methods for working with artificial intelligence. For simplicity, we will distinguish those based on neural networks, those based on Bayesian networks (the interested reader is referred to the Appendix A for a clear definition of Bayesian networks and their important properties), those based on human intelligence, and the mixed ones, which combine several of these three alternatives. In addition a powerful support vector machine method for large samples and big data is given in [Castillo et al., 2015].

(a) Variable selection

An initial, important and unavoidable stage is the selection of the relevant variables that intervene in the fatigue problem. In principle, when selecting the database, a decision is already being made, since it is assumed that the relevant variables are part of that database. Apart from using the generalized Buckingham theorem [Sonin, 2004], discussed in Section 4, in a second stage of learning, it will be necessary to identify them and discard the irrelevant ones. These two stages are part of the methods already indicated, although neural and Bayesian networks offer automatic selection methods, that are based on data, while human intelligence uses the prior knowledge of experts for this selection and the 'a priori' distributions in the Bayesian methods.

In addition, probabilistic S–N models to be valid simply require the consideration of the Buckingham theorem, which answers two important questions: (a) is the set of initially selected variables sufficient to describe our problem? if not, (b) which variables must be added to the initial set to obtain an extended set satisfying the (a) condition?, and (c) which set of dimensionless variables must be used to reproduce our problem without loss of the information contained in the extended set of variables?.

(b) Black boxes and models based on human knowledge

From here, the neural and Bayesian networks use the data to finish the learning process, which is based on data, so it is very important to have enough data to obtain a satisfactory result. No good results should be expected with lack of data.

However, in the case of fatigue, at least two variables related to stresses or one to one stress and another to the stress ratio effect must be considered, in which case, the need to increase the number of tests is intensified in the application of data-based machine learning, while there is already a model that, based on compatibility conditions, allows offering a solution to the consideration of this new (or secondary) variable, see [Castillo and Fernández-Canteli, 2009], and also other parameters which appear in well-known and recognized models.

(c) Deterministic and random models

There is the need to consider the problem as random or stochastic, which is the most common case and frequently ignored. For example, most differential equations-based models are deterministic and only a few are treated as random or stochastic processes, when this is the real case, as it occurs in the fatigue problem.

In the characterization of materials in fatigue, three possible procedures are traditionally distinguished, namely, the stress-, strain- and fracture mechanics based approaches. In this work, only the S–N field will be addressed as the fatigue characterization related to the GRV- or extended stress-based approach (see Section 5(b)) due to space limitations and the specificity required by the applications to machine learning and to "ad hoc" models. The strain energy-based fatigue approaches are considered to represent a particular case of the unified GRV-based approach, since the underlying compatible model, based on physical and probabilistic requirements, continues being still applicable. Note that in accordance with [Castillo and Fernández-Canteli, 2009, Fernández-Canteli et al., 2022], the strain-based approach follows the same modeling pattern, based on compatibility, as that applied to a extended stress-based approach, which includes the LCF, HCF and VHCF domains, as well as the influence of the stress ratio, R, see [Castillo and Fernández-Canteli, 2022], but even including the influence of the test frequency [Castillo et al., 2021] and [Blasón, 2019]. Consideration of the different fatigue zones is also analyzed in [Kant and Harmain, 2021] and [García-Fernández et al., 2023].

The determination of the S–N field is one of the possible ways to characterize a material under fatigue loading consisting of evaluating the random and independent results of the number of cycles until failure resulting from the application of different values of a certain driving force, here denoted generalized reference variable (GRV), [Castillo and Fernández-Canteli, 2009, Correia, 2015, Fernández-Canteli et al., 2022, Schijve, 2009], or more precisely of the two reference variables that describe properly the fatigue relationships, such as $\Delta\sigma$ and σ_{mean} or the maximum stress, σ_M, and stress ratio, R. For the sake of simplicity, the basic version of the S–N field, referred to as the primary reference variable, σ_M, will be used in the illustrative examples handled in the following S–N probabilistic models that are random models in whose evaluation the parameters are estimated as deterministic values that allow the, equally deterministic, percentile curves of the S–N field to be determined. In this way, they represent a particular or unique solution of the whole family of the S–N random field, among all possible ones that result from that material sample.

(d) Bayesian methods

It is also convenient to distinguish Bayesian methods from those that are not, especially when the parameters of the base models are considered new variables. In this case, they contribute to knowledge and endow them with a random character opening the solutions to become mixtures of given classes of models that can be completely general. Thereafter, the confidence intervals or, more precisely, the percentiles of the percentiles of the S–N field, treated as a stochastic field are determined by applying the Bayes technique. In fact, the whole density functions of any set of variables can be obtained based on simulated very large samples.

A suitable combination of the conventional fatigue S–N approaches using advanced fatigue models complemented with the application of the Bayes technique allows us to extend the probabilistic definition at a higher level, represented by the joint density of any subset of variables, which surpasses the concept of confidence intervals. This allows the percentiles of the percentiles of the S–N field to be obtained. Thus, the assessment can be contemplated as a machine learning procedure.

However, the application of the Bayesian technique, considering these parameters as random variables, extends the family of possible models as a combination of all the models of the previously indicated parametric family, which in practice converts it into a completely general model, since any of the possible solutions can be generated from these combinations or mixtures. This allows us, for example, to determine the percentiles of the percentiles of that S–N field, or what is the same, to consider these as stochastic processes. It also allows obtaining the uni-variate or multivariate marginal distributions of any subset of parameters.

Therefore, the confidence intervals or regions are exceeded by these distributions that provide much more information. In this case, it can be said that the final model represents the combination of multiple possible solutions of the S–N field for that sample (or coupon) of the material.

In this case, the program facilitates learning and allows transforming the initial distribution (prior distribution) from the data to deduce the posterior distribution. Bayes' theorem provides the 'law of mixtures' by means of weights from the simulations. The troubles in machine learning arises when the number of the available results is meagre, demonstrating that the application based only on data, is unreliable, and the use of human models becomes essential to obtain a reliable prediction.

3. S–N fatigue models based on human knowledge

Human intelligence provides important knowledge in the form of properties or models that relate the different variables in the fatigue analysis. As mentioned above, only a generalized stress-based approach, represented by the S–N field or using the modified GRV-S model, in which the GRV variable is a transformation of the S variable used to satisfy extra properties, will be analysed as a suitable case of fatigue characterization.

At this point, several currently open questions arise:

(i) Does automatic machine learning contribute to improve the parameter estimation of probabilistic S–N fields, traditionally evaluated with models based on human knowledge?
The identification of variables is relevant to fatigue problems and an important part of machine learning. Automatic machine learning also exists in the processes of estimating the parameters of the neural and Bayesian networks or the estimation of the parameters of the parametric models provided by human intelligence. In fact, a posteriori distributions of the variables resulting from the prior distributions and the data, provides the corresponding predictive distributions using the Bayes' theorem, which result from the mixtures. In fact, this is the way Bayesian methods learn, combining an initial knowledge, the prior one, with the data, that results in one combined 'a posterior' knowledge.

(ii) Is it advisable to ignore any machine learning application when there are consistent and valid compatible knowledge-based models?
If the contribution of human knowledge were extensive and sufficient, it could be dispensed with. However, in the parameter estimation and prediction processes, the other AI methods can be used to an advantage.

(iii) Which reliability can be guaranteed from an evaluation with machine learning, compared to that provided by a model, which allows prediction outside the experimental environment based on compatibility?
Data-driven models can only be valid if the predictions are made in the regions of the data, but they can be very ineffective and even invalid if we go outside of these regions. Contrary, knowledge-based models, permits this extrapolation.

(iv) Is it possible to establish criteria that allow an optimized joint application of both procedures?
It does seem possible to establish these criteria, especially for specific cases, as in the definition of probabilistic S–N fields.
Previous mathematics or physical knowledge cannot be discarded for a good machine learning program according to the objective of assessing probabilistic lifetime prediction as in the case of the S–N field.

It seems to be necessary to distinguish between models arising from arbitrary hypotheses and valid models based on properties that provide a deeper knowledge. Most of the currently proposed S–N and ε-N fields models are defined as regression functions related to the mean fatigue life in terms of stress or strain, see [Pascual and Meeker, 1999].

The percentile curves are subsequently defined a posteriori entailing the assumption of a gratuitous fatigue lifetime distribution along the S–N median or mean curve because the compatibility condition between the cdfs of lifetime and driving force,here denoted generalized reference variable (GRV), is not implicitly fulfilled [Fernández-Canteli et al., 2022].

Alternatively, the fatigue model derivation should be based on physical and statistical requirements such as the acceptation of the weakest link theory, or principle, and dimensional consistency, statistical independence among the experimental results, stability against location and scale transformation, limit behaviour as required in the asymptotic trend of the S–N field, and compatibility condition between lifetime and GRV distributions [Castillo and Fernández-Canteli, 2009].

The probabilistic S–N model proposed is a parametric random model that defines the number of cycles to failure, owning to a terminal state, as a function of a suitable GRV, without providing information about the damage evolution in the fatigue process from the nucleation stage to the propagation one. In its general version, the model ensures a friendly application to the HCF and VHCF domains, based on the software program ProFatigue, see [Fernández-Canteli et al., 2014], with extension to the LCF region [Fernández-Canteli et al., 2022], including the influence of the stress ratio, R [Castillo and Fernández-Canteli, 2022], and even that of the test frequency [Castillo et al., 2021].

Lastly, the consideration of the scale effect, which is a fundamental and critical point in the transferability of the material characterization results and, consequently, in its application in the design of components, turns out to be a subject of diffuse treatment in the machine learning, since it would require new specific data to be evaluated. On the contrary, in the case of valid models, the problem is solved by applying robust concepts within the framework of a scientific probabilistic methodology, based on the theory of the weakest link and on the two conditions of compatibility and normalization [Muñiz-Calvente, 2017, Muñiz-Calvente et al., 2015]. Moreover, the methodology is contrasted in the case of fracture and fatigue.

4. The compatibility conditions in the derivation of S–N and GRV–N models

The human based models require the use of a list of properties that the models must satisfy, which immediately provides the list of associated compatibility conditions. In the case of our fatigue example we have used the properties, leading to the following compatibility conditions:

(a) Variable reduction using the generalized Buckingham's theorem must be used ahead of other considerations.

According to [Sonin, 2004], the generalized Buckingham's theorem states, [Buckingham, 1914, 1915]:

"**Generalized Pi-Theorem.** *If a quantity Q_0 is completely determined by a set of n independent quantities, of which k are dimensionally independent, and if n_F of the independent quantities have fixed values in all the cases being considered, a number k_F of which are dimensionally independent, then a suitable dimensionless Q_0 will be completely determined by (n-k)-(n_F-k_F) dimensionless similarity parameters, where $k_F \leq n_F$*".

Accordingly, this must be used ahead of other considerations, because using dimensionless ratios among the variables involved in the problem, the number of them can be reduced without loosing anything and with the extra advantage of reducing the problem to a set of dimensionless variables, which has many advantages. For example, dimensional variables cannot be used as arguments of trigonometric functions, logarithms, exponential, etc.

This theorem allows not only the selected set of variables to be confirmed as sufficient or the need of it to be extended, but to identify a new reduced set of dimensionless variables that must be used to avoid problems with the dimensions of the variables involved, see [Nogal, 2011].

(b) The equality of the conditional F(GRV;N) and F(N; GRV) distributions

The second compatibility condition deserves particular attention as a transcendental model property, usually disregarded in the model construction, although recognized as a 'one-to-one pdf transformation' for a monotonically increasing function $y = F(x)$ in elementary statistical texts [Benjamin and Cornell, 1970]. In the derivation of the GRV–N model one could imagine a virtual sample function as resulting from a unique specimen being tested at different levels of the GRV (for the moment, the GRV could be identified as σ_M, see [Fernández-Canteli et al., 2022]).

Any specimen would provide a stochastic GRV–N curve, as following the particular random micro-structural characteristics of the specimen tested while the resulting GRV–N field for the sample consists of several GRV–N curves inter-crossing each other, see Figure 1a), that, despite this, can be probabilistically defined as percentiles curves, see Figure 1b).

At this point, the compatibility between the lifetime and GRV distributions is unequivocally established, since the set of percentile curves entering at a certain GRV_i level up to a certain number of cycles, N_j, are the same one issuing at N_j, up to the GRV_i level, irrespective of them inter-crossing each other, see Figure 1c).

Accordingly, the compatibility requirement referred to the conditional lifetime equality distributions becomes

$$F(GRV;N) = F(N;GRV) \tag{4.1}$$

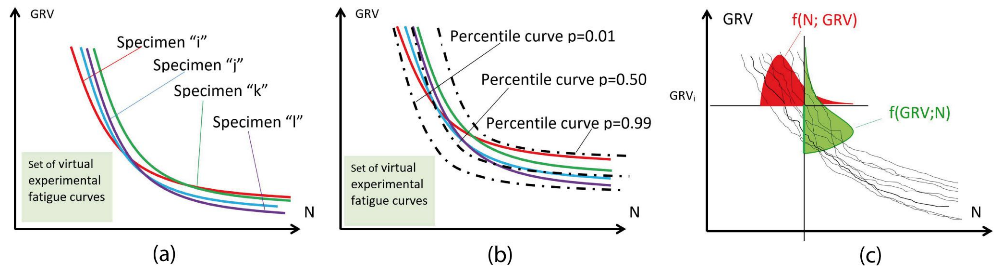

Figure 1. Schematic representation of: a) Virtual set of GRV–N curves from different specimens; b) Definition of the percentile curves in the GRV–N field; c) Illustration of the compatibility condition between $F(GRV;N)$ and $F(N;GRV)$ distributions.

(c) Location-scale families independent of measure units used

The selected family of distributions must be valid for any units used for the different variables (scale invariant property) and any possible location of asymptotic (location invariant property), and this implies a location-scale family. According to condition (1) and the monotonous increasing character of the F() function it can be written as:

$$(N - \lambda(GRV))/\delta(GRV) = (GRV - \lambda(N))/(\delta(N)), \tag{4.2}$$

where $\lambda(GRV), \delta(GRV), \lambda(N)$ and $\delta(N)$ are the four unknown functions of the functional equation (4.2) that represent the location and scale parameters of a certain GRV value as a function of the number of cycles, N, and of a certain number of cycles, N, as a function of the GRV, respectively [Aczél, 1966, Castillo et al., 2004]. Solving Equation 4.2 we discover that the family of distributions has to be a location-scale family.

(d) The weakest link theory

Since the failure of the elements must satisfy the weakest link theory, a minimum distribution statistical law must be used, which must be Weibull if the random variable is bounded (the Gumbel laws are limit distributions of Weibull laws), [Bolotin, 1981, Castillo, 1988, Castillo and Fernández-Canteli, 2009, Freudenthal and Gumbel, 1956].

Thus, the compatibility condition associated with all the previous conditions becomes:

$$\begin{aligned} F(GRV;N) &= 1 - \exp\left[-\left(\frac{N-\lambda(GRV)}{\delta(GRV)}\right)^{\beta(GRV)}\right] \\ &= 1 - \exp\left[-\left(\frac{GRV-\lambda(N)}{\delta(N)}\right)^{\beta(N)}\right], \end{aligned} \tag{4.3}$$

which is a functional equation where $\beta(), \lambda()$ and $\delta()$ are functions that represents the Weibull shape, location and scale parameters of a Weibull family as a function of the associated variables, respectively. Its solution provides two different models, from which only the one shown in Figure 2, see [Castillo and Fernández-Canteli, 2009], can be accepted as the valid one according to the experimental and physical evidence. It represents the complete description of the GRV–N field for constant stress ratio R and frequency f:

$$p = 1 - \exp\left[-\left(\frac{\log\frac{GRV}{GRV_0}\log\frac{N}{N_0} - \lambda}{\delta}\right)^{\beta}\right]; \quad \log\frac{GRV}{GRV_0}\log\frac{N}{N_0} \geq \lambda, \tag{4.4}$$

where λ, δ and β, are, respectively, the location, scale and shape Weibull parameters, GRV_0 is the conventional fatigue endurance limit and N_0, the limit number of cycles, below which no fatigue failures occurs.

Because the latter assumes the validity of the model for GRV values far over the monotonic static material strength, it follows that the GRV, being the stress range, $\Delta\sigma$, or the maximum stress, σ_M, does not represent adequately the non-linear relation between the σ-ε relationship.

Consequently, it should be replaced by an alternative $GRV = \psi\sigma_M$, able to follow the asymptotic trend of the model ordinates, where $\psi = Ed\epsilon/d\sigma$ represents a magnification factor, such as that proposed by the authors in [Fernández-Canteli et al., 2022, Fernández-Canteli et al., 2023].

Other possible alternatives are presently being studied. Now we can justify the use of the

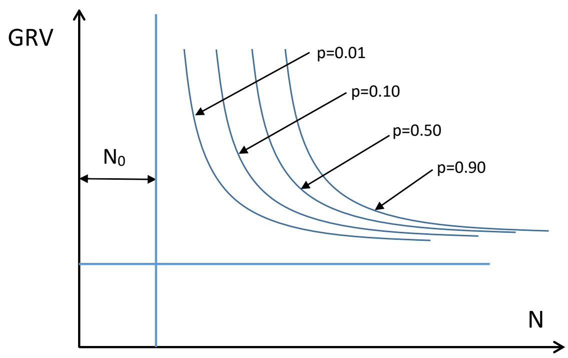

Figure 2. The GRV–N field resulting from the compatible model.

GRV instead of the σ_M variable, because model (4.3) gives a vertical asymptote for $N = N_0$ if GRV is used in the LCF a condition that is not satisfied by σ_M, which in fact shows a horizontal asymptote.

(e) The normalizing reference variable

The analytical definition of the percentiles curves in the GRV–N model in (4.3) entails the definition of a normalizing reference variable:

$$V = \log\frac{GRV}{GRV_0}\log\frac{N}{N_0} \tag{4.5}$$

and, as an extension, the equivalence of the probability of failure for the test results owning to the same V value, i.e. to the same percentile curve, irrespective of the GRV level, see Figure 3.

It follows that the distribution of the normalizing variable, V, can be assessed encompassing all the GRV–N results as pertaining to a unique sample, with the consequent improvement of the reliability in the parameter estimation. Furthermore, the variable V allows to check and interpret, at a simple glance, the quality of the assessed GRV–N model by observing unexpected clustering of the data points as potential bias of results from particular GRVs in the cdf of the normalizing

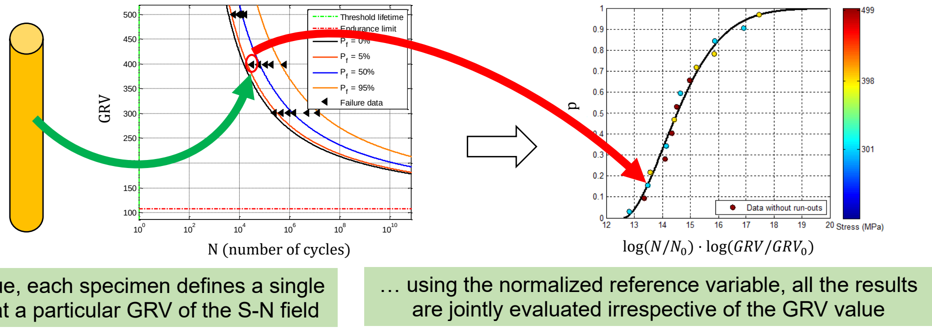

Figure 3. Schematic representation of one test result in the GRV–N field, its location at the percentile curve, and the representation of the results for that percentile curve as a probability in the cdf of the normalizing reference variable V.

variable. This would point out anomalies in the data evaluation and the possible need of re-assessment of the GRV–N field.

This provides a final support in the justification of the GRV–N field as the concept of accelerated test, and supports a logical conversion law in the cumulative damage calculation. In fact, the V variable provides the damage accumulation rule.

The fatigue calculation of real components under variable load requires the definition of an GRV–N field that includes the consideration of the maximum and minimum stresses σ_M and σ_m. The proposals currently used of Smith, Goodman, Morrow, SWT, Walker, Haigh, etc. [Lu et al., 2018, Papuga et al., 2018] do not represent a consistent solution, since they propose empirical solutions that guarantee, in the best case, its application referred to a specific material, while its definition for a particular number of cycles evidences its limitations.

Consequently, the probabilistic and stochastic behavior of the resulting models can be referred to the normalizing reference V variable, which, based on human knowledge, arises in a natural form, i.e. it can be immediately identified from the parametric model, (see the rectangular boxes in Section 4). In addition, this variable becomes crucial in the damage accumulation that can be written in terms of this variable.

(f) The uniqueness of the fatigue endurance limit

The consideration of the pair of reference variables, σ_M and $R = \sigma_m/\sigma_M$, allows the uniqueness of the fatigue endurance limit, irrespective of the R value, to be postulated, and a simplification in the solution of the S–N field to be achieved compared with the previous three-dimensional model developed in [Castillo and Fernández-Canteli, 2009].

(g) The three-dimensional model including the stress ratio effect

The model in Figure 2, referred to the $\Delta\sigma$-N field, see [Castillo and Fernández-Canteli, 2009], is valid for tests performed at a constant maximum stress σ_M and for tests performed at a constant minimum stress σ_m, as illustrated in the upper and left sides of the Figure 4, because all the previous constraints are valid for both cases.

Consequently, we have two different families of percentile curves with their associated $\Delta\sigma$-N fields, as illustrated in Figure 5.

In a similar way, the last model version (4.3), referred to the $\Delta\sigma$–N field, can be applied to define the associated percentile curves of the tests performed at a constant maximum stress σ_M (subscripts M_1 and M_2) and of those performed at a constant stress ratio R (subscripts R_1 and R_2) see Figure 3 [Castillo and Fernández-Canteli, 2022], because the same constraint conditions are also applicable leading to two different families of percentile curves with their associated $\Delta\sigma$-N fields. Again, we have two different families of percentile curves with their associated $\Delta\sigma$-N fields.

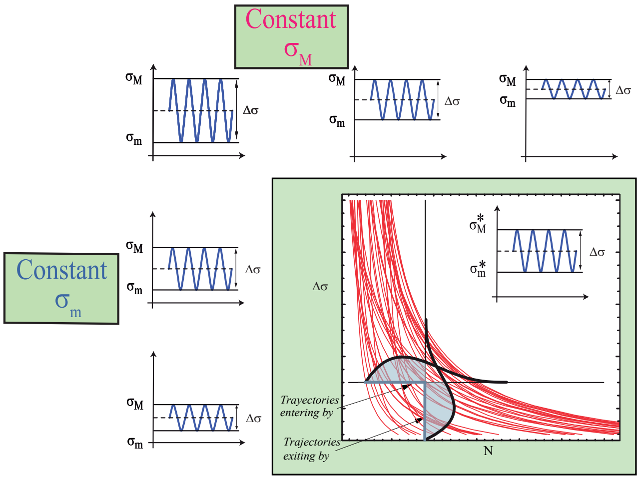

Figure 4. Illustration of the two associated type of alternating loads considered in this compatibility condition, indicated in the upper and left sides, for constant σ_M and constant σ_m, respectively.

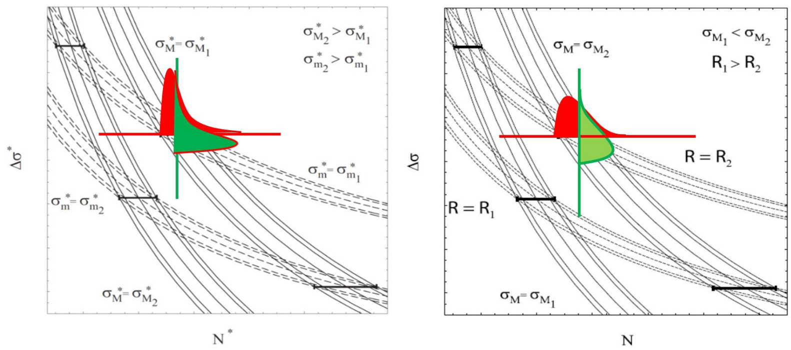

Figure 5. Illustration of the cross compatibility between the $\Delta\sigma$-N given σ_M and $\Delta\sigma$-N given σ_m fields, from [Castillo and Fernández-Canteli, 2009], and the $\Delta\sigma$-N given σ_M and $\Delta\sigma$-N given R fields, from [Castillo and Fernández-Canteli, 2022]

Since a particular test belonging to the family M_i performed at a constant stress ratio R_j coincides with another particular test of the family R_j performed at a constant maximum stress M_i, the statistical distributions $H(N;\Delta\sigma,R)$ and H(N;$\Delta\sigma$, σ_M) must be the same, see [Ruiz-Ripoll, 2009]. This can be extended to the cdfs H($\Delta\sigma$;N,R) and H($\Delta\sigma$;N,σ_M) due to the compatibility condition. The cross compatibility condition is applied to both $\Delta\sigma$-N models, i.e., that referred to σ_M=constant:

$$H(\Delta\sigma,N;\sigma_M)=((N-B(\sigma_M))(\Delta\sigma-C(\sigma_M))-\lambda(\sigma_M))/(\delta(\sigma_M)) \tag{4.6}$$

and that referred to $R=constant$:

$$H(\Delta\sigma,N;R)=((N-B(R))(\Delta\sigma-C(R))-\lambda(R))/(\delta(R)) \tag{4.7}$$

By replacing $\Delta\sigma=(1-R)\sigma_M$, it leads to the following functional equation [Aczél, 1966, Castillo et al., 2004]:

$$\begin{aligned} H(N,\sigma_M,R) &= ((N-B_M(\sigma_M))(\sigma_M(1-R)-C_M(\sigma_M))-\lambda_M(\sigma_M))/(\delta_M(\sigma_M)) \\ &= ((N-B_M(R))(\sigma_M(1-R)-C_M(R))-\lambda_M(R))/(\delta_M(R)) \end{aligned} \tag{4.8}$$

whose solution is:

$$p = 1 - \exp\left[-\frac{(1-R)\left[\left(\log\left(\frac{N}{N_0}\right) - \alpha\frac{R}{(1-R)}\right)\left(\frac{\sigma_M - \sigma_{M_0}}{\sigma_u}\right) - \gamma\right] - \lambda}{\delta}\right]^{\beta}, \quad (4.9)$$

as given in [Castillo and Fernández-Canteli, 2022].

Expression 4.10 immediately suggests a normalizing variable, V, i.e., that allows the probabilistic σ_M-R-N field to be jointly evaluated with test results for different R values as pertaining to a unique sample with unique Weibull cdf, is given as:

$$V(N;\sigma_M,R) = (1-R)\left[\left(\log\left(\frac{N}{N_0}\right) - \alpha\frac{R}{(1-R)}\right)\left(\frac{\sigma_M - \sigma_{M_0}}{\sigma_u}\right) - \gamma\right]. \quad (4.10)$$

which, facilitates the cumulative damage conversion in a varying loading process. A complete discussion of the model and its suitability when applied to different materials and R values, can be found in [Castillo and Fernández-Canteli, 2022].

5. Application of combining human knowledge and machine learning to the Maennig's fatigue program

It seems not advisable to renounce to enhance the reliability provided by models related to human knowledge in the evaluation of the S–N field when it comes to applying automatic machine learning. Their combination contributes with a component that can hardly be obtained directly through other means, [Awd, 2023, Wang et al., 2023a,b, Zhou et al., 2023].

The application of the Bayesian technique, considering deterministic parameters of the S–N field as random variables, extends the family of possible models to combinations or mixtures of all the models of the previously indicated parametric family, which in practice convert it into a completely general model, since any of the possible solutions can be generated from these combinations or mixtures.

This allows, for example, the percentiles of the percentiles of that S–N field to be determined, or what is the same, to consider these as stochastic processes. It also allows the uni-variate or multivariate marginal distributions of any subset of parameters to be obtained.

Therefore, the confidence intervals or regions are exceeded by these distributions that provide much more information. In this case, it can be said that the final model represents the combination of multiple possible solutions of the S–N field for that sample (coupon) of the material.

The human knowledge-based approach combined with Bayesian networks permits us to discover the causes of the model properties and provide us some exact and property compatible formulas to derive the conditional distributions of the nodes given their parents, which is very important. These formulas are the only ones consistent with the model properties, because they are the solutions of the corresponding functional equations.

(a) Improvement of the Castillo-Canteli model by applying Bayesian networks

In this section, we deal with the application of Bayesian techniques to the Castillo-Canteli probabilistic S–N field model, referring to Eq. 4.4, treating the model's five parameters as stochastic variables. The tool utilized for this purpose is the OpenBUGS or WinBUGS software suite, which produces extensive posterior model samples, given prior distributions and experimental result samples, leveraging the principles of Markov Chain Monte Carlo (MCMC). As a result, comprehensive samples of any statistic can be achieved, offering an exceptional estimation of its probabilistic distribution. Particularly, we introduce the incorporation of the

Bayesian Weibull fatigue Model within OpenBUGS [Lunn et al., 2009] (See Figure 6 that shows the acyclic graph of the Bayesian network of the model,[Castillo et al., 2019]).

We note that the Castillo–Canteli model is a mixed model, because the properties part becomes from human knowledge, but later the use of Bayesian networks and Bayesian methods, using OpenBUGS or WinBUGS [Lunn et al., 2009], is a type of automatic learning.

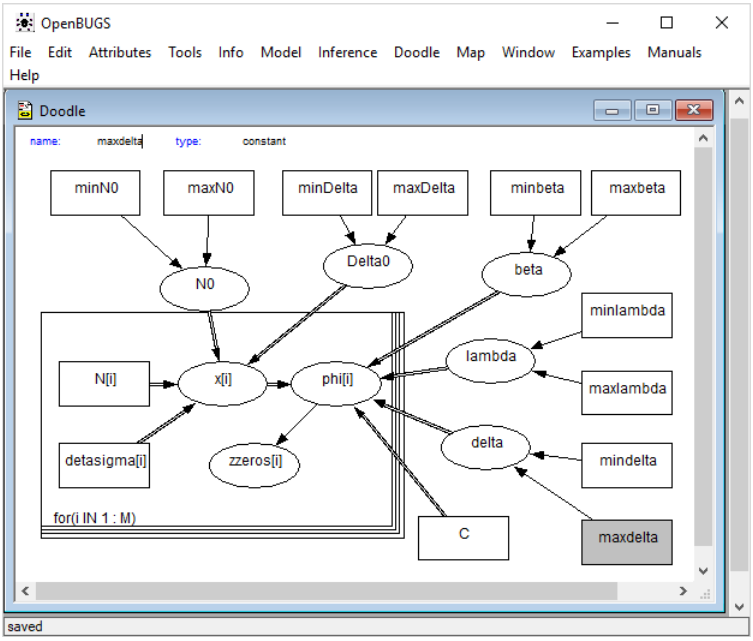

Figure 6. Acyclic Doodle Bayesian network graph used in this example.

(b) The Maennig's fatigue example

On this section, the former models are applied to the results of the Maennig experimental campaign [Maennig, 1967, 1970].

The solution of the application of the human based knowledge model provides the solution of the σ_M-N field as given in Table 2, using the ProFatigue software [Aenlle, 2001, Castillo et al., 2004, Fernández-Canteli et al., 2014, Przybilla, 2014], see Figure 7(a-b), where only the percentile curves 0.01, 0.10 and 0.50 have been represented, because higher percentiles are not relevant to a minimum problem, as the fatigue one. Note that in this case, the tests are carried out for constant R. An improvement of the assessment quality can be achieved by considering $GRV = \psi\sigma_M$, as proposed in [Benjamin and Cornell, 1970], see Figure 7(c-d)) and Table 2. An extension to include the R influence in the assessment of S–N fields is straightforward using 4.10, as proven in [Castillo and Fernández-Canteli, 2022], and could be considered in the next future.

After the capabilities of human knowledge models based on ProFatigue and Castillo-Canteli model [Aenlle, 2001, Castillo et al., 2004, Fernández-Canteli et al., 2014] has been shown, we used the Maennig's fatigue data [Maennig, 1967, 1970] to demonstrate the benefits of using mixed models and its deployment in OpenBUGS [Castillo et al., 2019]. Despite this fatigue program incorporating a large set of results, it does not challenge or doubt the applicability of the Bayesian approach to fatigue programs with smaller datasets. The choice of this data set simply serves as an exceptional case to observe and discuss the variation in confidence boundaries in relation to

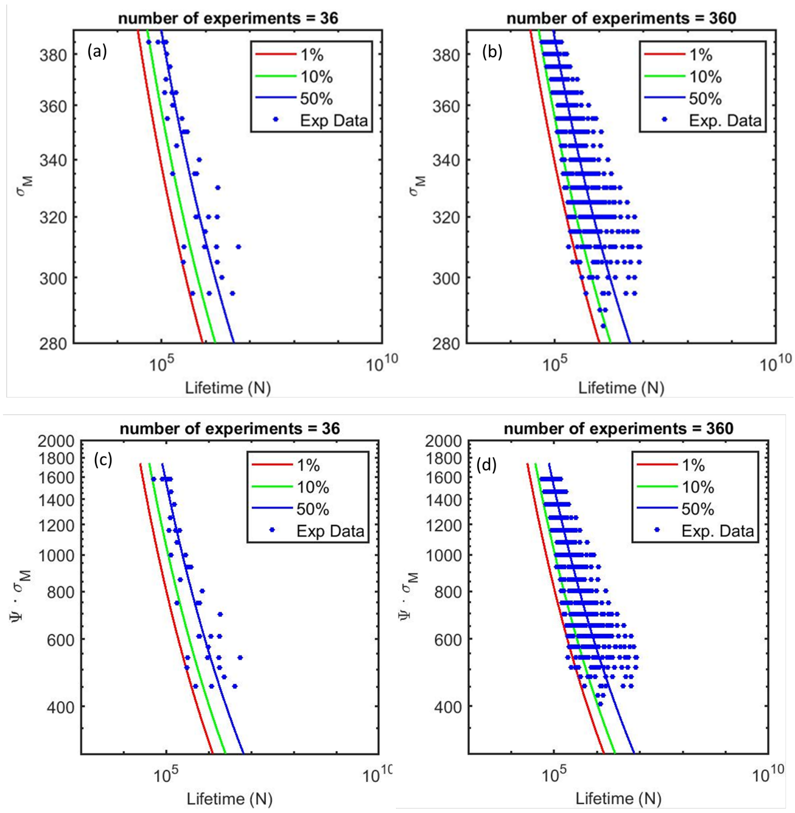

Figure 7. S–N field for the Maennig results using ProFatigue software: a) 10% of data and σ_M Model; b) full data and σ_M Model; c) 10% of data and $\psi\sigma_M$ Model; d) full data and $\psi\sigma_M$ Model.

Table 2. Assessment of the Maennig campaign. Model parameters considering: a) $GRV = \sigma_M$; b) $GRV = \psi\sigma_M$

$Results$	$Model$	GRV_0	N_0	λ	δ	β
All results	σ_M	4.69	7.18e-8	12.66	2.19	2.70
	ψ-σ_M	1.40	9.48e-6	59.15	10.20	2.75
10% Results	σ_M	4.63	2.22e-14	13.03	2.59	3.49
	ψ-σ_M	1.19	2.22e-14	59.93	12.10	3.67

the volume of available results. Since the campaign is performed for constant stress ratio, R, the influence of this parameter has not been investigated.

As previously indicated, the quantity of tests considerably impacts the outcome of a p–S–N curve fitting process. Therefore, the experimental scheme devised by Maennig, which is nearly an ideal fatigue data scenario in literature, may not adequately demonstrate the effectiveness of the S–N evaluation using the novel probabilistic model.

Regardless, the applicability of the Bayes model and the proposed fatigue model can be examined by imagining a virtual test program that includes limited but steadily increasing sets of test results, arbitrarily drawn from the original Maennig's test program. After fitting these sets, the change in the fatigue program assessment in terms of failure percentiles and fatigue limit distribution is explored.

To achieve this, a random reordering of the original outcomes in the vector is created, mimicking the experimental test sequence as virtually conducted by Maennig. Following this, the Bayesian Weibull fatigue model is employed to initially fit only the 5% of the experimental results (13 tests), as can be seen in Figure 8. After that, the 10%, 50% and 100% of the experimental results

(36, 130 and 360 tests, respectively) are fitted in order to observe how the confidence intervals contract with the increase in the number of tests.

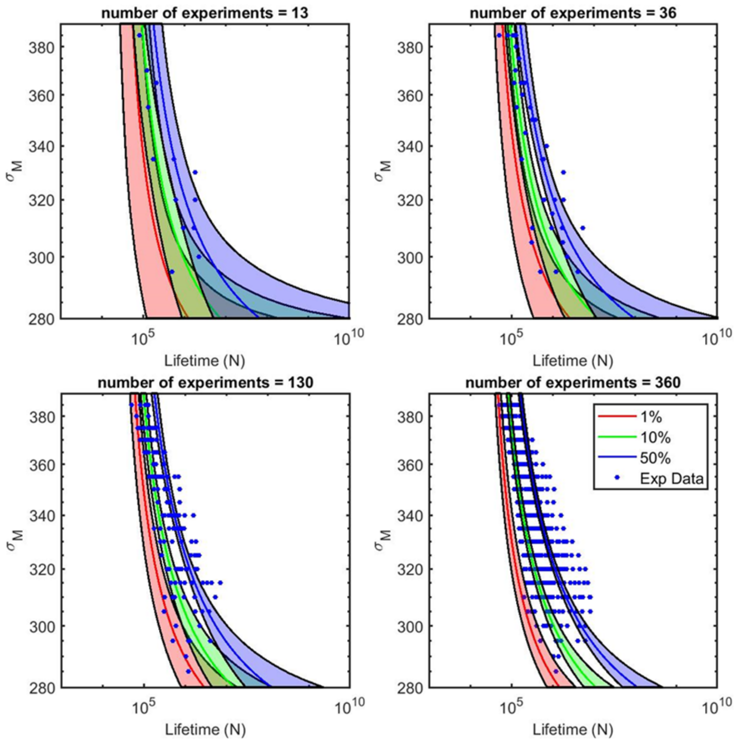

Figure 8. Results of p–S–N curves and evolution of confidence intervals has a function of number of test evaluated by the Castillo-Canteli model in combination with Bayesian networks (using 5% (13 tests), 10% (36 tests), 50% (130 tests), 100% of tests involved on the fitting).

All confidence intervals, shrink with decreasing number of the tests included in the fitting process, regardless of the percentile and stress ratio evaluated. The construction of these graphs assists researchers in determining the optimal decision and at which moment the current fatigue experimental program should be interrupted, based on a criterion of maximizing the reliability-cost ratio, i.e., when an additional test does not yield a worthwhile increase in reliability.

6. Future study directions

Due to the main aims of our work and the lack of space, we will limit this section to the case of knowledge based on properties.

Taking a look to the example of fatigue, developed in Section 4, we can see that it is not easy to identify the properties of the model to be developed in order to apply the property-based technique. Thus, given a real problem, one of the future research directions consists of looking for ways of identifying those properties of the associated model to be used.

Next, the fatigue example is used to identify the properties that are of general application and those that are associated with the particular problem of interest. It is shown that dimensional and statistical analysis are of general applicability, and the rest are problem dependent. When models for a set of variables is available, if new variables are incorporated, the compatibility conditions must be considered, as explained when the variable R was incorporated to our fatigue model.

Consequently, the following future lines of research are considered:

- *Variable reduction using the generalized Buckingham's theorem must be used ahead of other considerations.* This is a general property that can and must be used in all models. The dimensional consideration and analysis is fundamental to obtain simple models.
- *The equality of the conditional F(GRV;N) and F(N; GRV) distributions.* This is a property that comes from relating the expected model trend with statistical considerations, as the definition of pdf and cdf. Note however, that this property is applicable to all cases in which the trends of the expected models are similar to the one in fatigue.
- *Statistical properties (mean or extreme).* The selection of the statistical model must consider if we are in front of a mean or extreme value (minimum or maximum) and if the associated selected distributions can be derived from the physical meaning of the random variables involved. We note here that statistical models, such as Poisson, binomial, multinomial, gamma, beta, geometric, hyper-geometric, normal, etc. have clear physical interpretations. This helps to justify the use of these models which must be used if the conditions implied by them are applicable to the problem being considered.
- *Location-scale families independent of measure units used.* Any variable must belong to given families, independently of the unit used to measure it. For example, if the variable is measured in given units, changing the units, the resulting variable must belong to the same family of distributions. If the variable can be subject to translations or the model contemplate asymptotes, the location property is relevant and cannot be avoided. This justify the relevance of location families of distributions, which solve this problem in general.
- *The weakest link theory.* The behavior of the left and right tails of a random variable can be known just by knowing the range in which it is defined and the heaviness of the associated tail. For example, bounded tails can have only Weibull type limit distributions, light unbounded tails can have only Gumbel and Fréchet types limit distributions with infinite mean and finite variance, and heavy unbounded tails can have only Fréchet type limit distributions with infinite mean and variance. This is very relevant when reproducing variables and assigning them a distribution function, specially when the tails are the relevant part of the design, what happens frequently in Engineering, where failures occur with the presence of very high or very low values. Even when using families of distributions not of the extreme families, the tails of the distributions must be considered. We note that normal, gamma, and many other usual families, etc, have tails of the Gumbel type, what limits too much the valid models to be used.
- *The normalizing reference variable.* This reference variable allows us to reduce the analysis to a transformation, the one associated with the normalizing reference variable V explained above and the identification of the random cdf of V. We note that this variable can be identified once the model equations have been obtained and represent the failure criteria and for random variables, their random behavior.
- *The uniqueness of the fatigue endurance limit.* Some properties, as this one, come from the knowledge about the fatigue problem itself. Similar properties can be identified by consulting of experts in the area.
- *The three-dimensional model including the stress ratio effect.* When incorporating new variables to a model, the associated compatibility conditions must be considered when different test conditions are used, as in the case of the fatigue example.

Another possible line of research consists of incorporating the automatic learning technologies to learn the model parameters. Here the neural and Bayesian networks and the Bayesian methods have a good set of possibilities for the future.

Finally, it would be interesting to develop computer programs, specially working in symbolic form, to solve functional and differential equations. Examples of this for the case of functional equations are in [Castillo and Iglesias, 1997]. This will facilitate the human knowledge avoiding the experts to solve these equations.

7. Conclusions

The main aim of this work was to present the property-based methodology with special emphasis on the human knowledge-based models contribution on AI. The results related to this aim are presented in this Section as follows:

- (i) The relevance of human knowledge-based in AI has been emphasized. A new original and powerful paradigm, arising from the property-based methodology to build valid models, is presented and illustrated in connection with the S-N fatigue problem, which provides analytical equations of the model. It comprises: (a) the identification of properties to be satisfied by the model, and (b) its implementation as functional or differential equations, which, once solved, allow the only valid models to be recognized. In addition, valid models frequently ensure reliable extrapolation beyond the scope of the test data. In fact, the validity region depends on the associated model with the set of properties, but not on the data set.
- (ii) The probabilistic random or stochastic behavior of the resulting models is related to the normalizing reference variable, V, which arises in a natural form from the first step, based on human knowledge. This variable, identified from the formulae of the parametric models, becomes crucial in the probabilistic analysis of the cumulative damage calculation.
- (iii) The evaluation of the S-N field for the well-known Maennig's data on C35 steel using the proposed compatible model and the ProFatigue software, allows a robust estimation of the model parameters to be achieved irrespective of the overabundance or scarcity of data available. These estimates are very useful in defining the initial values for the OpenBUGS software
- (iv) The size of the Maennig's experimental campaign overwhelms by far the data amount of the great majority of the fatigue programs available in the literature, both, in the number of tests performed and the number of stresses covered, in which the application of the automatic machine learning based on data would be not recommendable.
- (v) The strategy applied to the fatigue experimental program based on the selected probabilistic S-N model, enables the optimal test setup to be strived for the GRV sampling and the number of tests. This determines, ultimately, the quality of the assessment in terms of cost minimization and reliability maximization.
- (vi) Bayesian networks together with Bayesian methods and extreme value analysis are the most adequate tools to define and treat random and stochastic problems, as the case of fatigue, specially to reproduce adequately the tail behavior of variables. Contrary to black box automatic learning methods, a physical interpretation of the model parameters and results is possible due to the existing correspondence between the phenomenon and the suggested set of variables when the proposed compatible model is used.
- (vii) In the particular case of fatigue, models to be useful and able to deal with damage accumulation must include two stresses and loads frequencies. Most used models fail to satisfy this condition.

8. Acknowledgments

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A. Introduction to Bayesian networks

An important model is that of Bayesian networks, which are specially designed for random models. As shown in the example of Figure 9, a Bayesian network consists of two parts:

- **A directed acyclic graph.** This graph must be directed and with no cycles, that is, following the arrows, we can never return to any node. The graph must include the set, $\{X_1, X_2, \dots, X_n\}$, of all n relevant variables of a given problem.
- **A set of conditional distributions.** This set, $\{f_1(x_1|pa(X_1)), f_2(x_2|pa(X_2)), \dots, f_n(x_n|pa(X_n))\}$, contains, n, as many as variables, where $f_i(x_i|pa(X_i))$ is the conditional density or distribution of variable X_i and $pa(X_i)$ is its set of parents, or variables on which the variable X_i is directly dependent.

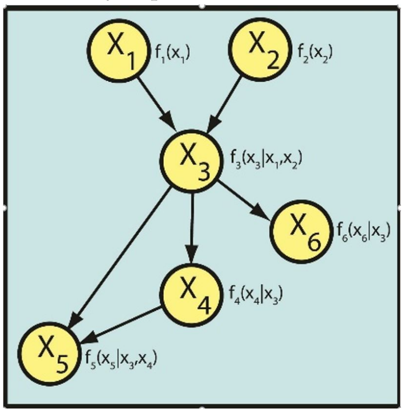

Figure 9. Example of a Bayesian network with 6 nodes and the direct dependencies of all variables indicated by the corresponding arrows.

This Bayesian network contains 6 variables $X_1, X_2, \dots, X_6$ and you can see that variables X_1 and X_2 have no parents, $pa(X_1) = pa(X_2) = \phi$, X_4 and X_6 have only one parent, $pa(X_4) = pa(X_6) = \{X_3\}$, and X_3 and X_5 have two parents, $pa(X_3) = \{X_1, X_2\}$ and $pa(X_5) = \{X_3, X_4\}$, respectively.

The interesting think is that the joint density or distribution of all variables is the product of all these conditional densities or distributions, that is,

$$f(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f_i(x_i|pa(X_i)), \qquad \text{(A 1)}$$

which in the case of independence of all variables becomes the product of all marginals:

$$f(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f_{0i}(x_i), \qquad \text{(A 2)}$$

where $f_{0i}(x_i)$ is the marginal of variable X_i.

We note the importance of these formulas, which permit answering any probability related question about these variables individually considered or as any possible subset of them. Of course, this joint probability can be defined of different forms, but Bayesian networks have very relevant properties, that position them at the top of the most efficient and consistent methods, such as:

(i) They use local information. They allow their joint probability distribution to be defined using local information, consisting of the conditional probabilities of the variables to those of their parents.

(ii) They are products of simpler densities or distributions. The joint distribution is a product of the distributions of all individual variables conditional on their parents no matter they are independent or not.
(iii) Parsimony. They constitute the simplest way of defining the joint probability of all the variables, that is, they comply with the property of parsimony or simplicity.
(iv) Generality. They allow any joint probability to be defined, that is, none of the possible ones is excluded.
(v) Consistency. They are consistent, in the sense that the conditional probabilities used in the definition are precisely the corresponding conditional ones obtained from the resulting joint distribution. This does not happen with other alternative methods, which can lead to the non-existence of the joint distribution, due to an excess of conditional distributions or their incompatibility.
(vi) Physical interpretation. Normally, Bayesian networks use variables whose physical interpretation is known, which facilitates the interpretation of results.
(vii) Easy and automatic explanation is possible. The explanation of the results of the Bayesian network models is solved in a very efficient and clear way.
(viii) Can be easily converted to Bayesian methods. In fact, converting the model parameters into random and new variables, adding them to the initial Bayesian network converts the model into a Bayesian method.

B. Table of notations

The notation used in this paper is as follows:

- AI : artificial intelligence.
- $B(\sigma_M)$: location parameter function associated with N for test subject to constant σ_M.
- $C(\sigma_M)$: location parameter function associated with $\Delta\sigma$ for test subject to constant σ_M.
- $B(\sigma_m)$: location parameter function associated with N for test subject to constant σ_m.
- $C(\sigma_m)$: location parameter function associated with $\Delta\sigma$ for test subject to constant σ_m.
- $B(R)$: location parameter function associated with N for test subject to constant R.
- $C(R)$: location parameter function associated with $\Delta\sigma$ for test subject to constant R.
- $F()$: cumulative distribution function (cdf).
- $f_i(x_i)$: marginal density of variable X_i.
- $f(x_1, x_2, \dots, x_n)$: cumulative distribution function of $X_1, X_2, \dots, X_n$.
- $f_{01}(x_i)$: marginal density of the random variable X_i.
- GRV : generalized reference variable.
- $H()$: cdf.
- HCF : high cycle fatigue.
- k : number of independent quantities.
- k_F : corrected number of independent quantities.
- LCF : low cycle fatigue.
- N_0 : vertical asymptote for N.
- N : number of cycles.
- n : number of independent quantities of variables.
- n_F : corrected number of independent quantities of variables.
- $pa(X_i)$: set of parents of variable X_I.
- OpenBUGS : free computer software for the Bayesian analysis of complex statistical models that uses Markov chain Monte Carlo (MCMC) methods.
- Q_0 : set of quantities or variables used in the Buckingham theorem.
- $R = \sigma_m/\sigma_M$: stress ratio.
- S : generic stress.
- X_i : the i-th random variable.
- V : normalizing reference variable.

- VHCF : very high cycle fatigue.
- β : shape parameter of the Weibull distribution.
- δ : scale parameter of the Weibull distribution.
- λ : location parameter of the Weibull distribution.
- ψ : magnification factor.
- $\Delta\sigma = \sigma_M - \sigma_m$: stress range.
- σ_m : minimum stress in a fatigue cycle.
- σ_M : maximum stress in a fatigue cycle.

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C. Figure and table captions

- **Caption of Figure 1:** Schematic representation of: a) Virtual set of GRV–N curves from different specimens; b) Definition of the percentile curves in the GRV–N field; c) Illustration of the compatibility condition between $F(GRV;N)$ and $F(N;GRV)$ distributions.
- **Caption of Figure 2:** The GRV–N field resulting from the compatible model.
- **Caption of Figure 3:** Schematic representation of one test result in the GRV–N field, its location at the percentile curve, and the representation of the results for that percentile curve as a probability in the cdf of the normalizing reference variable V.
- **Caption of Figure 4:** Illustration of the two associated type of alternating loads considered in this compatibility condition, indicated in the upper and left sides, for constant σ_M and constant σ_m, respectively.
- **Caption of Figure 5:** Illustration of the cross compatibility between the $\Delta\sigma$-N given σ_M and $\Delta\sigma$-N given σ_m fields, from [Castillo and Fernández-Canteli, 2009], and the $\Delta\sigma$-N given σ_M and $\Delta\sigma$-N given R fields, from [Castillo and Fernández-Canteli, 2022].
- **Caption of Figure 6:** Acyclic Doodle Bayesian network graph used in this example.
- **Caption of Figure 7:** S–N field for the Maennig results using ProFatigue software: a) 10% of data and σ_M Model; b) full data and σ_M Model; c) 10% of data and $\psi\sigma_M$ Model; d) full data and $\psi\sigma_M$ Model.
- **Caption of Figure 8:** Results of p–S–N curves and evolution of confidence intervals has a function of number of test evaluated by the Castillo-Canteli model in combination with Bayesian networks (using 5% (13 tests), 10% (36 tests), 50% (130 tests), 100% of tests involved on the fitting).
- **Caption of Figure 9:** Example of a Bayesian network with 6 nodes and the direct dependencies of all variables indicated by the corresponding arrows.
- **Caption of Table 1:** A comparison of the advantages and shortcoming of the three different methods used in AI.
- **Caption of Table 2:** Assessment of the Maennig campaign. Model parameters considering: a) $GRV = \sigma_M$; b) $GRV = \psi\sigma_M$.