Distributed Real-Time OPF and State Estimation Architecture for Active Distribution Networks

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***Abstract*—This work proposes an online distributed control architecture that utilizes dynamic optimal power flow (DOPF) and time of use energy prices alongside state estimation and linearized AC-power flow to ensure optimal and safe operation of active distribution networks, where distributed energy resources (DERs) are embedded. The DOPF problem is formulated as Mixed-Integer Non-linear Programming (MINLP) to maximize the net profit of an active distribution network assuming different buying and selling time of use energy prices. After that the DOPF formulation is relaxed to a non-linear programming problem to be solved with off-the-shelf solvers like IPOPT. The proposed framework is introduced showing how the algorithms are distributed between the cloud and edge devices to allow online control of distribution networks. Finally, the proposed control architecture is validated using real data from a distribution grid located in Spain.**

***Index Terms*—distributed energy resources,energy storage,optimal power flow, state estimation**

I. Introduction

Conventionally, the electrical network was mainly managed through optimal power flow (OPF) done by the transmission system operator (TSO) [1]. Distribution grids were designed on a “fit-and-forget” basis, which would hinder the progression towards electrification. However, generation centers are shifting towards the distribution grid and the incorporation of smart Distributed Energy Resources (DERs) like energy storage systems (ESS) and manageable loads have created an urgent need to also optimally operate the distribution grid [2]. Accordingly, optimal operation of the newly introduced DERs in the distribution grid is crucial to allow electrification and ensure grid flexibility.

OPF allows the grid operator to optimize a certain objective function, while representing nodal power balance and the network equations as constraints. Moreover, physical limits imposed by the network components can be taken into account like: voltage magnitude, branch power flow and generation limits [1]. OPF for distribution networks was mainly explored for optimal placement and sizing of capacitor banks to maximize the net savings from the DSO perspective [3], [4]. The net savings function is formulated as the sum of the weighted power losses, energy losses, cost of the fixed capacitors and cost of the switched capacitors. Accordingly, this problem presents a Mixed-Integer Non-linear Programming (MINLP) formulation of the OPF, since the decision variables of enabling a certain capacitor bank are binary.

Moreover, with the integration of energy storage devices, the OPF problem became more complicated as the OPF has to be run over a certain period where the profit is to be maximized. Accordingly, the constraints of not overcharging or over-discharging an ESS in the grid depend on the preceding time steps. The OPF that is solved over several time steps is called Dynamic Optimal Power Flow (DOPF) [5] or Multi-Period Optimal Power Flow (MPOPF) [6], which is usually solved using interior point algorithms. The general concept of DOPF was introduced in [5] with a general formulation of the problem, which was solved for different active networks to show the importance of incorporating the time factor in the OPF formulation. In [6], the authors present an efficient interior point algorithm, called BELTISTOS, that is specially designed for MPOPF problems achieving less convergence time and lower memory footprint compared to other interior point packages like IPOPT, KNITRO, and MIPS. A comprehensive survey was presented in [7], [8] reviewing the different OPF formulations and ways to solve them.

Most of the literature focuses on formulating and solving the OPF problem, but not on the real-time use of the OPF as a part of the control architecture to operate the distribution grid. A real-time control framework was developed in [9] to take into account the temporal difference between the time of solving the OPF problem and the actual time of executing the control command. The control architecture relies on linearizing the AC-OPF problem to solve it every control cycle. However, this framework does not take into account the effect of preceding control cycles in its OPF formulation which only guarantees an optimized operation in one control cycle. Other formulations have deviated from the OPF in managing DERs and chose to just deal with behind-the-meter power balance like [10], [11]. Even though these formulations allow real-time operation, they do not take into account the physical constraints imposed by the distribution network.

OPF formulations to optimize the net profit in a distribution network, taking into account different selling and buying energy prices and self consumption of the generated or stored energy, are lacking in the literature. However, the use of semi-

The present work has been partially supported by the European Union’s H2020 Research and Innovation Program under Grant Agreement No 864459 (UE-19-TALENT-864459) and by the Spanish Ministry of Innovation and Science under Grants PID2019-111051RB-I00 and PID2021-122704OB-I00.

definite programming to solve certain OPF formulations for radial distribution networks was presented in several publications [12] [13]. Measurements from across the distribution network would be the main driver of any real-time control architecture, yet it is known that measurements are never 100% accurate but usually are accurate within a certain covariance. State estimation helps grid operators to estimate the grid status as accurate as possible, given a certain set of measurements [14]. Combining OPF and state estimation to have a reliable real-time control framework to manage DERs in distribution grids is lacking in the literature. The contributions of this paper addresses this gap in literature through three main points:

- A MINLP DOPF formulation to optimize the net profit across a 24-h period for an active distribution grid that takes into account the state of charge limits for ESS and the opportunity cost of selling or buying the energy at each control cycle.
- A relaxed DOPF formulation that converts the MINLP problem to a non-linear programming problem, which would facilitate solving the DOPF problem using off-the-shelf solvers like IPOPT.
- An introduction of a real-time control framework for active distribution networks that is distributed among the cloud and edge devices allowing to combine the DOPF formulation with real-time state estimation and real-time DER set points alterations based on a linearized AC power flow.

The proposed grid-control architecture is tested by simulating the operation of a realistic 76-bus distribution grid [15].

II. DOPF Formulation

This section presents the DOPF formulation used to optimize the net profit of an active distribution grid across a 24-h period and the relaxation employed to facilitate the solution using off-the-shelf solvers.

A. AC power flow formulation

The main constraints of an OPF problem are the power flow equations. For a distribution network, the power flow equations have to take into account the unbalanced nature of the distribution grids. The main difference between balanced power flow and its unbalanced counterpart is that each bus in the distribution network would be treated as four different nodes because of the unbalanced load. Accordingly, the power flow formulation of the unbalanced power flow is similar to that its balanced counterpart after properly constructing the Y_{bus} matrix. Each element of the Y_{bus} matrix is generally a complex number that is denoted as $G+jB$, where G is the conductance and B is the susceptance. Voltages are represented as phasors, thus each one given by two unknowns, voltage magnitude V_i and its angle δ_i. The active and reactive power injected at each node are given by (1) and (2) respectively. i is an index that maps all the non-neutral nodes in the system.

$$P_i = \sum_{j=1}^{4N_{bus}} V_i V_j (G_{ij}\cos(\delta_i - \delta_j) + B_{ij}\sin(\delta_i - \delta_j) \tag{1}$$

$$Q_i = \sum_{j=1}^{4N_{bus}} V_i V_j (G_{ij}\sin(\delta_i - \delta_j) - B_{ij}\cos(\delta_i - \delta_j) \tag{2}$$

For a 4-wire network, power injection at the neutral point, as shown in (3) has to be considered. The fourth phase or wire would be denoted as N and p denotes non-neutral phases.

$$S_i = v_{p-N} \cdot i_p^* = v_p \cdot i_p^* + v_N \cdot i_N^* = S_p + S_N \tag{3}$$

Accordingly, the active and reactive power injected to the neutral wire in terms of voltage phasors are shown in (4) and (5) respectively.

$$P_{N_i} = \frac{V_{N_i}}{V_i}(-P_i\cos(\delta_{N_i} - \delta_i) + Q_i\sin(\delta_{N_i} - \delta_i)) \tag{4}$$

$$Q_{N_i} = \frac{V_{N_i}}{V_i}(-P_i\sin(\delta_{N_i} - \delta_i) - Q_i\cos(\delta_{N_i} - \delta_i)) \tag{5}$$

the power flow equations can be formulated as shown in (6)-(7), where P_{S_i} and Q_{S_i} are the scheduled active and reactive power at node i.

$$fP = P_i + P_{N_i} - P_{S_i} = 0 \tag{6}$$

$$fQ = Q_i + Q_{N_i} - Q_{S_i} = 0 \tag{7}$$

B. DER modelling

For this paper, two main DERs would be considered: PV generators and energy storage systems. PV generation is modelled as a function of irradiance which changes overtime, as shown in (8).

$$P_{PV}(t) = PV_{rating} * Irradiance(t) \tag{8}$$

Energy storage systems, in contrast to PV arrays, can be fully controlled up to their ratings and their injected power does not directly depend on the environmental conditions. Accordingly, energy storage elements can be considered as flexibility agents in the distribution grid and can be modelled also as a controllable power source. Since ESS can both inject and absorb power to/from the grid, the ESS model should include an element that can act as either a power source or a power sink. The OPF solver should decide whether an energy storage element injects or absorbs energy into the grid, based on a given cost function. The limits on P_{ESS} are that its absolute value should not exceed the rated value of the ESS and that the energy stored in the energy storage element complies with the minimum and maximum energy limits ($E_{ESS_{min}}$ and $E_{ESS_{max}}$, respectively), which are part of the OPF constraints and expressed in (9) and (10).

$$|P_{ESS}| \leq P_{ESS_{rated}} \tag{9}$$

$$E_{ESS_{min}} \leq \int P_{ESS} \cdot dt \leq E_{ESS_{max}} \tag{10}$$

The scheduled active power would be also denoted as P_S, which is equal to the net power at each node taking into account the DERs and the load, as shown in (11).

$$P_{S_i} = -P_{D_i} + P_{PV_i} + P_{ESS_i} \tag{11}$$

C. MINLP DOPF formulation

Since the objective is to optimize the net profit of the active distribution network across a 24-h period, the cost function has to take into account different buying and selling energy prices that are a function of time $f_{Buy}(t)$ and $f_{Sell}(t)$ respectively. The net profit, which is the objective function, across the day of a distribution network can be calculated as shown in (12), where $\delta_{Sell_i}(t)$ and $\delta_{Buy_i}(t)$ are boolean variables that specify whether each node is selling or buying energy at each time instant. It is important to note here that the time step taken is 1 hour.

$$\begin{aligned} f(x) = \sum_{i=1}^{3N_{PQbuses}} \sum_{t=1}^{24} & P_{S_i}(t) \cdot \delta_{Sell_i}(t) \cdot f_{Sell}(t) \cdot \Delta t \\ & + P_{S_i}(t) \cdot \delta_{Buy_i}(t) \cdot f_{Buy}(t) \cdot \Delta t \end{aligned} \tag{12}$$

The complete formulation of the MINLP DOPF problem is shown in (13), assuming that the voltage magnitude is allowed to deviate by 10% from the base voltage (V_{base}).

$$\begin{aligned} \max_{V,\delta,P_{ESS}} \quad & f(x) \\ \text{s.t.} \quad & (6),(7),(9),(10) \\ & 0.9V_{base} \le |V_i| \le 1.1V_{base} \\ & |I_{ij}| \le I_{ij_{rated}} \end{aligned} \tag{13}$$

D. DOPF relaxation

As previously introduced in section II-C, the OPF formulation to optimize the net profit for a distribution network with high DER penetration is a MINLP, which means that solving the problem usually takes a long time, convergence is not guaranteed and also global optimality is not guaranteed. Accordingly, a relaxation to the objective function can be employed to eliminate the need for binary variables. The relaxation employed makes sure to penalize the objective function when the node starts to buy energy and reward the objective function when the node sells energy. In addition to that, it has to capture the change of price across the day. Consequently, the two price curves would be combined into one price curve through a weighted difference scheme that is governed by the expression (14).

$$f_{comb}(t) = a \cdot f_{Selling}(t) - b \cdot f_{Buying}(t) \tag{14}$$

However, the value of the objective function using the combined price curve does not reflect the actual net profit of the network, but this can be easily determined through post processing. This relaxation would slightly change the OPF formulation, easing the solving while guarantying global optimality and convergence [16]. The relaxed OPF formulation is shown in (15).

$$\begin{aligned} \max_{V,\delta,P_{ESS}} \quad & \sum_{i=1}^{3N_{PQbuses}} \sum_{t=1}^{24} P_{S_i}(t) \cdot f_{comb}(t) \cdot \Delta t \\ \text{s.t.} \quad & (6),(7),(9),(10) \\ & 0.9V_{base} \le |V_i| \le 1.1V_{base} \\ & |I_{ij}| \le I_{ij_{rated}} \end{aligned} \tag{15}$$

It is important to remark that a duality between both formulations was not mathematically established, but simulations have showed that the relaxed problem converges to a minimum point of net profit.

III. Online distributed control architecture

The proposed control architecture is distributed between the cloud and edge devices, where the more computationally expensive day-ahead DOPF problem is solved on the cloud to produce the power set points for each of the DERs. The DOPF problem is solved based on day-ahead generation, load and price predictions. The day-ahead power set points and the predicted system states (voltage magnitudes and angles) are sent to the edge device. In the edge, a real-time state estimation problem is solved based on the augmented Weighted Least-Square (WLS) method [12] to estimate the current grid state based on the available measurements. After that, the difference between the predicted system states and the actual estimated states is used to compute a small-signal disturbance in the DER power set point through a linearized form of the AC power flow. These perturbations are tested to ensure that there are no constraint violations, and the resulting modified set points are sent to the DERs.

The linearized AC power flow is presented as a Newton-Raphson linearization by calculating the jacobian of the system based on the Y_{bus} and the current operating point, as shown in (16), where J_p is the jacobian of the electrical grid based on the current operation point, ΔP is the perturbation in the scheduled power and ΔX is the difference between the predicted states and the estimated states.

$$\Delta P = J_p \Delta X \tag{16}$$

The complete control architecture is illustrated by Fig. 1. In order to normalize the angle difference, expression (17) based on the dot product is used.

$$\cos(\Delta\delta) = \frac{a \cdot b}{|a||b|} \tag{17}$$

IV. Case study

Fig. 2 shows the 76-bus 4-wire LV distribution network used in this study [13]. This network has 54 different single-phase customers, which means that 18 of the 76 buses are load buses with 3 customers each. All load buses are monitored by smart meters. The data regarding the active and reactive power demand and network topology was made available in [17]. The sample time of the loading data is 1 second for a total period of 24 hours, which means that there are 86400 different data points available for this network. These data points were averaged to obtain a 1-h sample dataset. The grid was modelled in openDSS [18] using the Python interface to extract the needed data like the grid measurements and the Y_{bus} matrix. On the other hand, the relaxed DOPF formulation was modelled using Pyomo [19] and IPOPT [20] was used to solve the relaxed DOPF problem.

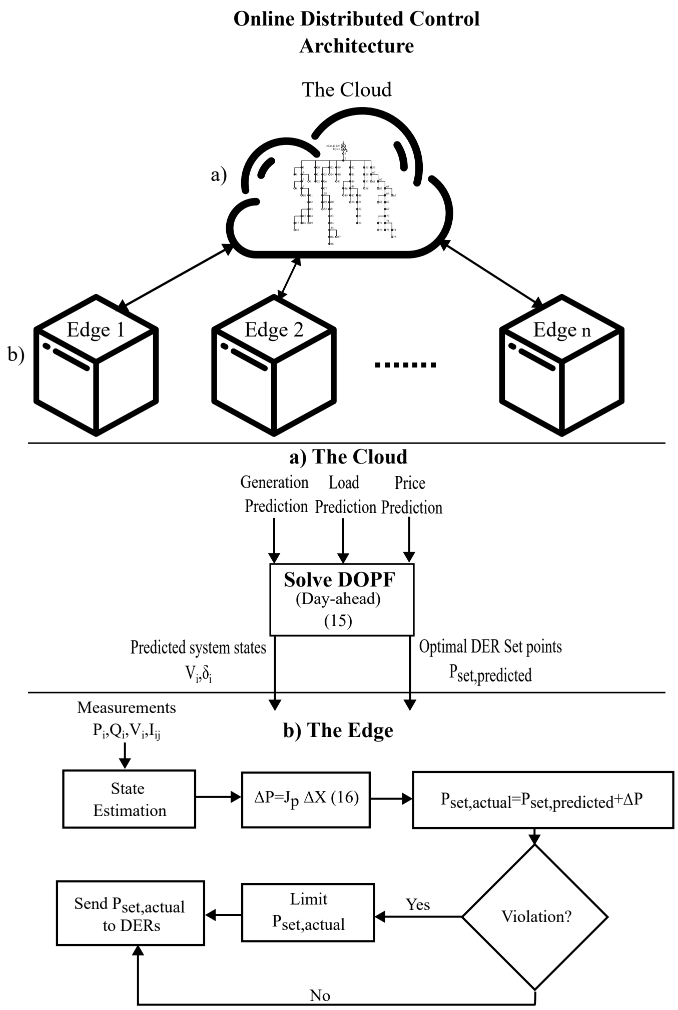

Fig. 1. Proposed real-time control architecture for active distribution grids

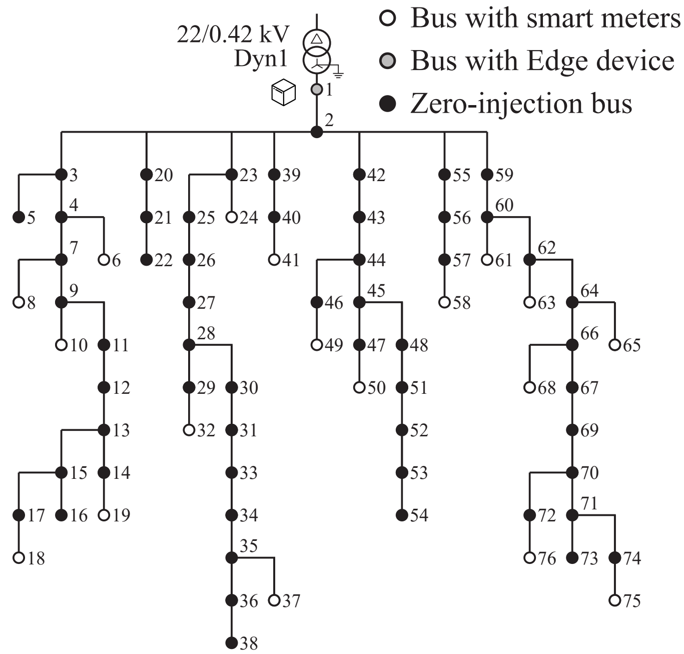

Fig. 2. 76-bus 4-wire LV distribution network

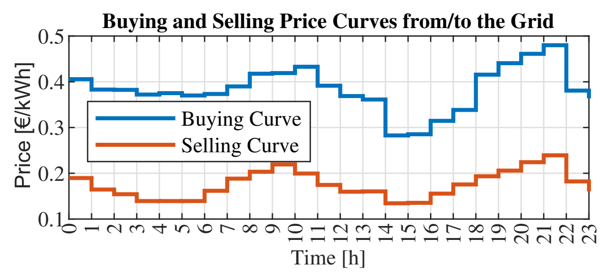

Fig. 3. Buying and selling price curves

A. DOPF solution for the base case

For the day-ahead DOPF solution, the irradiance was assumed to be perfectly sinusoidal starting from 8 a.m. until 8 p.m. While the price curves were extracted from the eSios API [21], the buying and selling curves are shown in Fig. 3. Furthermore, the DOPF model was fed day-ahead active and reactive power predictions. The solution of the DOPF problem is shown in Fig. 4 presenting the state of charge (SOC) of the ESS at selected buses along the loading profiles at the same buses with the weighted difference curve. As shown, the weighted difference curve is always positive so it penalizes buying energy from the grid (negative power) and awards selling energy to the grid (positive power) and it includes the dynamics exhibited by both curves. As expected, the algorithm prompts the ESS to store energy when the buying price is lower than the peak in the morning to prevent buying energy at peak time. During PV peak, the ESS charges to use the energy at night when the buying prices are high and also selling energy does not produce a high revenue due to the low selling prices.

B. Test cases for the proposed control architecture

Deviations from the predicted load profiles (base case) were introduced to test the proposed control framework. Three different test cases were developed: a -5% deviation from the predicted profile, a 5% deviation from the predicted profile and a random Gaussian distributed load profile that has the prediction as its average but with a standard deviation of 5%. Fig. 5 shows the net savings across a 24-h period for the three test cases alongside the base case. The net savings are almost the same when using the proposed control architecture with the base case as the prediction compared to the case of 100% accurate prediction.

Table. I shows a summary for a one year simulation that was setup to test the economic performance of the proposed control architecture. As shown, the control architecture with DER introduction into the distribution grid has achieved savings in the electrical energy cost of 85.86%. A complete financial analysis should be done with payback period and capital cost consideration when designing the ESS and PV systems for the distribution grid, but this analysis is an indication of the economic competitiveness of the proposed architecture.

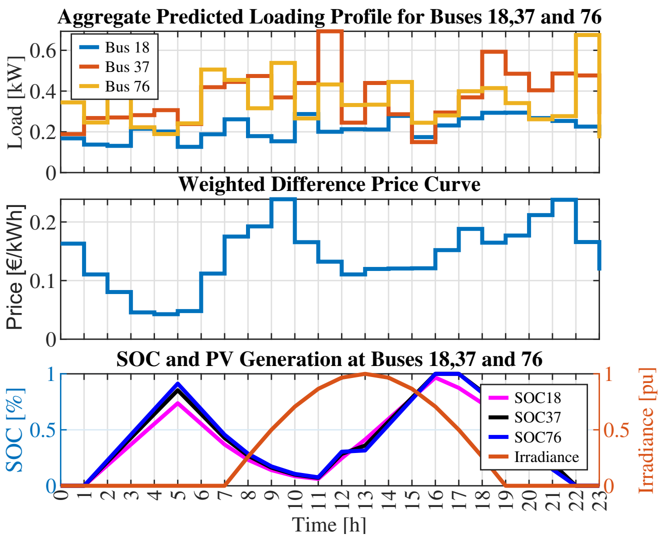

Fig. 4. SoC of the batteries for buses 18, 37 and 76 in the 76 bus system compared with the PV generation (day-ahead DOPF solution)

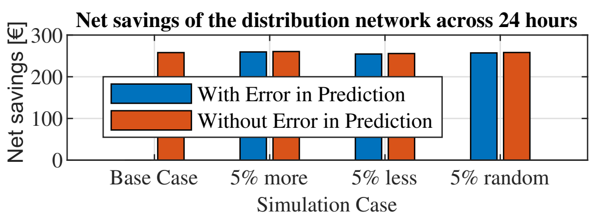

Fig. 5. Net profit comparison between the case with error in prediction and no errors in prediction

TABLE I
SUMMARY OF THE FINANCIAL PERFORMANCE FOR A 1-YEAR

Without DER	With DER		
Energy cost(€)	Energy cost(€)	Net savings(€)	% Saved
98,550	13,973	84,613	85.86%

V. CONCLUSION

This paper presents an online distributed control architecture that can be used for optimal operation of active distribution networks. Firstly, a MINLP DOPF formulation is introduced by formulating the constraints and a cost function that models the net profit of the whole distribution grid across a 24-h period. Secondly, the MINLP problem is relaxed to a non-linear programming problem to solve the problem using off-the-shelf optimizers like IPOPT. A duality between both problems is not mathematically established, but simulations have shown that the relaxed problem converges to a global optimum of the net profit function. After that, an online control architecture distributed between the cloud and edge devices is presented. The proposed architecture solves the computationally expensive DOPF based on day-ahead predictions on the cloud, then utilizes state estimation and online measurements to estimate the grid state on the edge devices and sends real-time DER set points accordingly. The proposed control architecture is then tested on a 76-bus realistic distribution grid to evaluate the DOPF solution and the effectiveness of the proposed architecture. The results of the different test cases show that the proposed controllers achieve net savings for the grid while allowing the integration of DERs. In future works, it is important to conduct further analysis to test the proposed architecture under different load profiles and grid architectures.

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