Chapter 18
Teachers, Resources, Assessment Practices: Role and Impact on the Curricular Implementation Process

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A broad perspective must be taken while discussing teachers, resources and assessment practices in relation to reform and this is what we have attempted in the present chapter. We consider factors in curriculum implementation including physical materials, technologies, but also processes such as classroom and system assessment practices and, in a privileged way, the role of teachers. Curricular change can become just a proposal printed on an official paper if it does not actively involve teachers and their practices, and if it does not secure the needed resources for teachers. In focusing in this chapter on the implementation of curriculum reform,

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we have the classroom firmly in focus. We begin considering issues around the intended curriculum, including the preparation and professional development of teachers and then consider what is attained, which takes us into issues around assessment.

The chapter suggests different roles played by teachers, resources and assessment in helping and/or restraining success in curriculum reforms, so that a reader can identify common elements relating to their own context and, we hope, take forward ideas about how to act and/or not to act when implementing a curriculum reform. The distinction between the *breadth* of reform and its *depth* is particularly significant when it comes to considering teachers, resources and assessments. By *breadth* we mean the number of schools or teachers affected, which could range from all the teachers in a country, or all the teachers in publicly-funded schools in a country, to more regional or local innovations. *Depth* of reform is concerned with the extent to which teachers are brought into the reform process, for instance whether pedagogical innovation is envisaged as part of reform and, if so, whether there is any theory of change driving what takes place and the extent to which changes are harmonious.

To draw on the issue of assessment raised at the start of this chapter, for instance, depth of reform will concern the extent to which assessment practices fit with the new curriculum innovation or not. A deep reform will bring assessment innovation with it, alongside opportunities for teachers to develop and learn in relation to innovation. A shallow reform might be a change in curriculum documentation with little else by way of changes to assessment structures and little consideration given to supporting the envisaged pedagogy.

In this chapter, we have largely drawn from contributions to the ICMI Study 24 Conference that took place in Japan in November 2018. The rationale here is that the Study Conference was the result of a global call and all contributions were peer-reviewed. However, we have supplemented our review with other publications where, as authors, we felt that some important perspective was not represented. We end the chapter by bringing into focus an issue which was not represented at the ICMI Study 24 Conference, which we feel is nonetheless vital to consider, and this is the extent to which curriculum reform pays attention to local or global concerns and challenges.

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Teachers in the Curricular Implementation

Teachers are the most important agents of change during curriculum reforms. The role of teachers in a period of implementation of curriculum reforms is not the same as the one that develops when there is absence of these changes. In this section, we explore the aims and general contexts for teachers' participation in curriculum reform implementation. Teachers' participation can be significant in the process of policymaking and formulation, as well as in the process of design, implementation, reflection, ongoing development and evaluation of pedagogical innovations.

Aims and General Contexts for Teacher Participation in Curriculum Reform Implementation

The basic choices that characterise curriculum reforms emphasise recovery and improvement actions in the school system by attempting to optimise all existing resources, then by sharing and involving all their actors. At the centre of this general interest of the community is the crucial role of the teacher, an aware protagonist, promoter and facilitator of processes of change that are usually involved in curriculum reforms. The renewal process triggered by curriculum reforms often presents different dimensions, among which are the institutional, the organisational and the pedagogical.

Moreover, the role of teacher professionalism in a reform develops, from being within a particular perspective, to opening to potentially new tasks and the feasibility of innovation. We assume that the effectiveness of the laws that introduce novelties in the educational systems depends, to a decisive extent, on the cultural and professional training of those who are called upon to transform these innovations into formative action, an appropriate methodology and a new teaching practice.

Different school systems can involve mathematics teachers in curriculum reform in different ways:

- When teachers are involved before the drafting of the reform, this can be through formally or informally collecting opinions, critical points and good practices. Sometimes teachers get involved through government or other agencies, other times through teacher consultations.
- Teachers could be involved during the preparation of curriculum reform in two ways: by involving teacher-researchers or teachers in the commissions or groups of teachers who produce specific reform documents. In some contexts, this process can actively involve almost all teachers in a region or country.
- Teachers can be contacted immediately before the start-up of reform, by consulting them on a large scale about the possible desired outcomes, or during the implementation of reform; the process could consider trials followed by teachers' opinion or evaluation.

– Finally, teachers can be involved after the implementation of the reform: what are teachers' professional development initiatives? At a global or local level? This phase naturally implies a different involvement, because at the local level there are possibilities for different interactions and comparisons. Does the process foresee the involvement of teachers in evaluation and revision of the reform?

In any case, curriculum reforms are (and must be) important moments, both for teachers' awareness and for the revision and self-analysis of their professionalism. As underlined by Brodie (2018) it is now internationally accepted that curriculum reforms should be accompanied by aligned teacher professional development to support teachers in working with curricula they may not have previously experienced (Borko et al., 2015; Zaccarelli et al., 2018). In fact, as Karsenty (2018) points out, even when most teachers have been positively impressed by new curriculum reforms, few may be willing to implement new practices in their classes. A willingness to implement change depends both on external factors and on the influence of internal beliefs that are often implicit.

An important point to consider when analysing curriculum reform is what kind of discussion between teachers it stimulates. Discussions may take place in institutional contexts, at school level, geographic area, in institutional forms, in teachers' professional training courses, and at an informal level (teacher blogs, chats, etc). Such discussion is important for developing professional awareness that can arise from a new curriculum reform. The aims and role of teachers are affected by national or even regional contexts, which may vary widely across the world. At the same time, teachers' skills become an issue in the progress of curriculum reform implementation.

Finally, what role can teachers take in reforms? Recent approaches (e.g. Lozano et al., 2018) bestow upon teachers a prominent role, since teachers are considered at the same level as the designers of the curriculum. With this in mind, instead of worrying about teachers' fidelity in implementing reformed practices and resources, efforts will be focused on providing support to teachers by guiding them in making informed curricular decisions, based on insights derived from both research and practice.

Certainly, one of the main objectives of a curriculum reform is to ensure that all teachers know the curriculum and have the opportunity to discuss and interpret it. We can, in fact, conclude that one of the fundamental elements for the implementation of a new curriculum is making sure that the teachers feel confident teaching it. Therefore, it is impossible to neglect a considerable commitment to continuous professional development and training inherent to new curricular ideas, with the aim of making teachers aware and competent, in such a way that they feel able and comfortable in the implementation of any new curriculum.

Preparation of Prospective Teachers

A central part of mathematics teaching is the curriculum, which may provide tasks and activities that constitute the instructional core, influencing the sequencing of mathematics topics and the ways mathematical ideas and processes are made available to students. With reformed school curricula, in-service mathematics teachers may need to cope with implementation gaps if their prospective teacher education preparation was based on a past curriculum. Thus, continuing professional development of in-service teachers is an urgent need in the early years of implementation of reform (Reston, 2018). Mathematics teacher preparation and development for implementing a curriculum can be viewed as comprising two stages, the prospective and in-service stages which are generally regarded as a continuum rather than discrete phases (Reston, 2018). In this sub-section, we explore the question: how are prospective teachers connected to a reformed curriculum?

In the scope of the ICMI Study 24 Proceedings, we only find some ideas related to the question above. From the studies of Thailand (Inprasitha, 2018, p. 349; Changsri, 2018, p. 341) and Vietnam (Tran et al., 2018, p. 405), some suggestions can be realised in courses of teacher education programmes at university, such as prospective teachers having opportunities to analyse the new curriculum, new textbooks, or watch and analyse videos of mathematics classrooms. In Thailand, Inprasitha (2018) started an initial teacher education programme with two innovations, the use of Lesson Study and what he labels an Open Approach.

These innovations have been adapted and gradually implemented in all programme and collaboration schools, since 2002 (Changsri, 2018). To meet the demands of the reformed school curriculum following a competence-based learning model, which was implemented from the start of 2020 in Vietnam, Tran et al. (2018) conducted an innovation project, focusing on developing PCK for secondary mathematics prospective teachers, by offering them opportunities to experience mathematical literacy as active learners. The results showed that the prospective teachers began to develop an understanding of mathematical literacy and tried to integrate mathematical literacy into their teaching plans at an increased level of sophistication.

Prospective teachers might be required to connect with the reformed curriculum in their time of field experience. Prospective teachers can be expected to have significant learning from the ways in-service teachers teach in schools. There is evidence that teachers view the internship or field experience as the most valuable and beneficial part of their teaching experience (Behm & Lloyd, 2009). We assume that the learning opportunities arising from the interaction between prospective teachers and university lecturers, or researchers, are also important. Thus, it is desirable to have a variety of approaches or strategies to incorporate a reformed school curriculum across teacher education programmes at universities. Schools and teacher education institutions should ideally cooperate, in order to have coherent and aligned programmes and this has implications for the content and form of teacher preparation programmes. Alternatively, teacher education at university may take a “meta”

approach (e.g., developing skills of reflection) which can be applied to a range of curriculum designs.

Along with the challenges of curriculum implementation, Lloyd (2006) suggested that prospective teachers should be experienced with school curriculum materials at university (although we note that not all teacher education, globally, takes place at universities). Through critical analysis of curriculum materials, prospective teachers would have opportunities to develop sophisticated views of the curriculum. These experiences could prepare prospective teachers to make a reasoned match between practices in their classroom and the curriculum that will continue to emerge in the future. Moreover, the teachers' field experience could ideally allow for the involvement of prospective teachers in the organisational aspects of their work, related to any new school programme.

Professional Development and Implementation of Curriculum Reforms

As noted in the previous sub-section, there is a wide consensus about the idea that success of curriculum reforms is linked to professional development (PD) processes that teachers undergo before and during the implementation of these reforms. Kilpatrick related to this idea when interviewed at the ICMI Study 24 conference:

> In a way, it is artificial for us to think of the curriculum as being separate from the teacher's professionalism, because it completely depends on that, and we cannot talk about reforming the curriculum, getting it in a new form, if the teachers are not with us. (interview with Dr. Jeremy Kilpatrick, in Shimizu & Vithal, 2018, p. 41)

We see such a view as being shared among scholars from diverse cultures and contexts. For instance, when describing a curriculum reform in Ireland, O'Meara et al. (2018) maintain that, "without explicit professional development […] any efforts to align revised curricula with existing curricula will result in reform efforts not realizing their full potential" (p. 155). Similarly, Brodie (2018), who works in the South African context, argues that curriculum reforms and ambitious teaching require ambitious professional development if they are to succeed.

Aims of Reform-Oriented Teacher Professional Development

Whilst the overarching goal of reform-oriented teacher professional development is to support teachers in implementing the reformed curricula, different local aims are documented for such programmes in different places internationally. We can identify two main categories: resource-centred aims, and student-centred aims. In the former group, the focus is on helping teachers be acquainted and work with, reformed materials, texts, mathematical concepts at the heart of the reform, and

other resources. In this group, we find, for example, the aims of the PD programme held in Luxembourg during the 1960s and 1970s, towards the implementation of the *New Math* reform (Nadimi & Siry, 2018).

In the massive preparation courses conducted for in-service Luxembourgian teachers, the central aim was to introduce teachers to the notions of modern mathematics, so that they would be able to integrate *New Math* in their classrooms. A more recent example is the Champion Programme, a part of the Australian national project *reSolve: Mathematics by Inquiry*, which aims to develop teachers' pedagogical design capacity (Thornton et al., 2018). Champions support teachers in their work with the new curriculum ideas, assist them in understanding the material and human resources as well as using these resources to develop the learners' mathematics.

The second category, student-centred aims, is focused on the work of teachers around student input and student learning. This group is represented, for example, by the South African *Data-Informed Practice Improvement Project (DIPIP)*. The aim of *Professional Learning Communities* in this project was to discuss the reasoning behind learners' errors as a means to better value and understand learner thinking, teachers' own mathematical knowledge, and their teaching practices. It was hoped that teachers' practices will improve in relation to responsiveness to learners' input (Brodie, 2018).

In Lao People's Democratic Republic, the aim of a PD programme was to develop teachers' knowledge and orientations towards a student-centred reformed curriculum, based on task design through Lesson Study (Inthavongsa et al. 2018). Evidently, the aims of a reform-oriented PD programme do not necessarily fall into being just resource-centred or just student-centred. For instance, the *SHLAV* PD programme in Israel had both a resource-centred aim, i.e., to acquaint teachers with the reformed materials for use with students of low prior attainment, but also a student-centred aim – to enhance teachers'confidence in those students' ability to successfully handle matriculation[1] test items (Karsenty, 2018).

Models Suggested for Reform-Oriented Professional Development

Exploring approaches suggested in different countries for supporting in-service teacher learning around the implementation of a reformed curricula, reveals that a wide range of models exist for shaping PD. These models differ in terms of their content; pedagogical orientation (i.e., views about how teachers best learn); and organisational form (school-based, university-based, MOOC-based, etc.). We

[1] Matriculation certificate is a prerequisite examination for entering universities and colleges in Israel.

describe several examples below to demonstrate the spectrum of documented possibilities.

At one end of the spectrum we find individual initiatives for teacher development towards a new curriculum. Mok and Sung (2018) describe a case in Hong Kong, where a lead teacher designed, implemented and disseminated a 3 year enrichment programme for mathematically gifted primary students, around the idea of inquiry-based learning under the umbrella of the school vision and in alignment with the broader context of curriculum reform ideas promoted by the education system. The result was a reformed school-based curriculum that was adopted (with adaptations) by other STEM teachers in the school. This model of reform-oriented teacher professional development thus used a bottom-up method, centering on the experimental design approach, as a powerful way of teacher learning.

Another model that also focuses on individual teacher learning, yet not through a bottom-up approach, is the personalised PD model presented by Karsenty (2018). In this model, professional counsellors provide school-based PD for secondary mathematics teachers of low attaining students, around reformed materials designed to encourage students' learning through doing and understanding. Although the materials are defined by the Matriculation test topics, and not designed by teachers as in the Hong Kong case described above (Mok & Sung, 2018), the orientation behind this model is that the teachers learn best when they feel that the reformed materials are relevant for them. Thus, the endeavour in this model is to tailor the reformed curriculum to the specific context of low-track realities, and furthermore, to adapt it to local practices and constraints, through counselling that reaches out to schools.

Further on in this spectrum we find models that combine university-based and school-based PD. Inprasitha (2018) linked a prospective teacher education programme with an in-service teacher programme. Building up this idea through the implementation of Lesson Study as a professional learning community, experienced school teachers worked collaboratively with prospective teachers and both groups formed habits of 'teachers learning together' and formed a long-term professional learning community.

Inthavongsa et al. (2018) describe another model, implemented in Lao People's Democratic Republic, for the dissemination of Lesson Study. The teachers participating in the project first received instruction from university experts, followed by organised school visits. Then, the next phase of the PD was school-based, and included 3 months of practice within the teachers' own classrooms, directed by Masters' degree students. The implied (although non-explicit) pedagogical orientation here is that teachers learn to implement reformed ideas when they are given institutionalised opportunities to practice these ideas in their classrooms.

As the scale of the PD gets larger, the model offered may include several strategies and modalities. For instance, in the *Project Mathematics Education Reform in Costa Rica* (PMRECR), a large-scale PD for thousands of mathematics teachers was carried out between 2011 to 2017, using both face-to-face sessions and online independent work (Hernández-Solís & Scott, 2018; Ruiz, 2015). The model involved a two-tier PD: one for teacher leaders and the other for large populations

of teachers, conducted by the trained leaders (Ruiz, 2015). At a later stage, MOOCs and mini-MOOCs were utilised, serving not only the need to reach a massive number of teachers, but also the hope to bring teachers closer to the use of technology and promote a modern vision of the educator (Hernández-Solís & Scott, 2018).

In the Philippines, the *Enhanced Basic Education Act* reform has shifted from the cascading model (a top-down process moving from the national level to the regional, division, and finally school level) and the cluster-based model (i.e., teachers from several schools attending the same training programme conducted by invited subject specialists), towards more innovative models such as Lesson Study, the Learning Action Cell and the needs-based PD model (Reston, 2018). A Learning Action Cell comprises a group of teachers who engage in collaborative learning sessions to solve shared challenges encountered in the school, facilitated by the school head or a designated Learning Action Cell leader. In the needs-based PD model, the content of the PD is determined following survey research, that identifies and prioritises teachers' needs.

A model similar to the Learning Action Cell, described by Brodie (2018), is the use of Professional Learning Communities as a platform to enhance changes in teacher practices around curriculum and pedagogical reforms. Brodie describes how in the South African project *DIPIP*, a set of activities designed for teachers' collaborative work was applied to support teachers' understanding of the reasoning behind learners' errors. Brodie suggests that Professional Learning Communities can be a useful model for reform-oriented PD, "particularly when curriculum reforms are seen as requiring ongoing interpretation and reinterpretation by teachers in relation to their local contexts, rather than once-off, fragmented inputs by outsiders" (Brodie, 2018, p. 334). This view emphasises once more the pedagogical orientation of *relevance*.

Types of On-Going Support Provided to Teachers

In different cases of curriculum reform implementations around the world, various types of ongoing support offered to teachers are reported, which include digital, textual, and face-to-face modes of support. Digital support may take the form of e-learning systems and communication platforms for participating teachers, as reported in Reston (2018). Through these online systems, teachers can access additional resources and share best practices. Another role that digital platforms can play is to support teachers' interactions with curriculum materials, so that they can become co-designers of the intended curriculum by actions such as tagging (Olsher & Yerushalmy, 2018). One of the advantages of such activities is that they support teachers' ownership of the reformed materials.

Textual support can be provided in the form of Teacher Guides and other materials designed specifically for teachers, as in the Mexican and the UK contexts reported by Lozano et al. (2018). These materials are explicitly designed to support teachers in making informed curriculum decisions, by providing them with

guidance on conceptual learning, errors and misconceptions, strategies for differentiation, and more. Face-to-face modes of support are documented in several projects. In Lao People's Democratic Republic, support is given in the form of close collaboration with Masters' students coming regularly to the schools (Inthavongsa et al., 2018).

Similarly, in the Israeli *SHLAV* project, personalised support is provided weekly in schools by specialised counsellors (Karsenty, 2018). In the Australian *reSolve* project, recruited volunteering Champions work with teachers in Professional Learning Communities (Thornton et al., 2018), and the South African *DIPIP* programme is organised around Professional Learning Community weekly meetings with facilitators (Brodie, 2018).

One tool that has been gaining increasing exposure in schools and in research about teachers and PD, is the use of video, i.e., lesson videos for teacher reflection or learning and, in some cases, evaluation (see Gaudin & Chalies, 2015; Major & Watson, 2018). Finally, in relation to ongoing support for teachers, Zehetmeier and Krainer (2011) propose the content-community-context dimensions, which determine the sustainability of PD outcomes into the classroom fitting to context, quality, practicality (content); opportunities for collaborative reflection and discussion, teacher ownership and empowerment, an inquiry stance for teachers (community); administrative support, school-based support and resources (context). The categories provide a further perspective on (and a potential framework for) the planning required for effective on-going PD for mathematics teachers during a time of curriculum change.

Empowering Teachers' Voices in Reform-Oriented PD Programmes

One of the important questions that can be asked, regarding PD programmes designed towards curriculum reforms, is the following: to what degree, if at all, do teachers have opportunities to express their perspectives about the reform and be heard by policy makers? Moreover, do teachers' voices have an effect on the reform?

In some reform-oriented PD programmes, teachers' perspectives are taken into account only for evaluation purposes, e.g. in the Lao People's Democratic Republic Lesson Study reform (Inthavongsa et al., 2018), teachers were interviewed as part of an evaluative research, but this is not reported to have an effect on the programme. Yet, in several documented cases, efforts were made to include teachers' input as a kind of 'formative assessment'for the programme. Osta (2014) pointed out that other models of mathematics curriculum evaluation use more flexible approaches that include the close relationship between teachers and other actors, such as, principals and educational authorities. For instance, an example from Canada (Bednarz et al. 2012, quoted in Osta, 2014) is a hybrid model characterised by its long-term

span, formative continuous development, regulated by the roles of the actors with the involvement of teachers and school personnel.

Pegg and Krainer (2008) reported on four examples of large-scale national reform initiatives in mathematics, where teachers were involved in rich professional learning experiences. Teachers were perceived not only as participants but as collaborators and change agents, and their communications with university practitioners formed the basis for fruitful contributions.

Within three of the projects mentioned earlier, opportunities seem to be provided for teachers' voices to be heard. In the South African *DIPIP* programme, teachers communicate their needs in the Professional Learning Communities, and the facilitators bring them back for discussion and consideration. Similarly, in the Israeli *SHLAV* project teachers can define their needs and receive personalised support tailored to these inputs. In the Philippines, teachers' perspectives on the reformed curriculum were documented, in specific studies, within PD initiatives where teachers expressed their opinions in either written or oral forms. We note, however, that these are only sporadic examples; there is an apparent necessity for more empirical evidence on the types and extent of impact that teacher input might have on the design, or the re-design, of reform-oriented PD programmes.

Evaluating the Work with Teachers in Reform-Oriented PD Programmes

A central issue for researchers on reformed curricula dissemination and designers of PD initiatives is the evaluation of the degree to which a reform-oriented PD programme was successful. Two kinds of criteria can be discussed: teacher-related criteria, and student-related criteria.

Teacher-related criteria refer to the evaluations of the PD initiative itself, and to the evaluations pertaining to the teachers' practices. Examples of questions to be posed for PD programmes' evaluations are: were the teachers engaged? did they find the PD initiative relevant? could they point to new things they have learnt? was collaborative work taking place? Whereas examples of questions to be posed for teacher practices' evaluations are: could changes be tracked in the teachers' lessons? which parts of the reformed curriculum were implemented, and how?

Various means are suggested for collecting teacher data: self-reports; surveys; written feedbacks; interviews; questionnaires; analyses of teacher conversations in PD sessions; direct lesson observations; longitudinal tracking of shifts in teachers' choices, and more. For example, in the Israeli *SHLAV* project, teachers' feedback was collected through interviews and questionnaires, showing positive views of the reformed curriculum for low-track students, and high satisfaction of the teachers regarding the personalised support they have received (Karsenty 2012, 2018). Data collection can be used also to better understand difficulties in implementing the PD

programme, for instance, the *DIPIP* team (Brodie, 2018) conducted interviews with teachers who left the programme.

Student-related criteria refer to evaluations of student performance under assessments coherent with the curriculum reform. These can be local, national and international tests. In the Philippines, for example, the National Achievement Test (NAT) is used as a basis for ranking school and teacher performance; it is taken by students at the end of each academic year at the end of elementary (Grade 6), Junior High School (Grade 10) and Senior High School (Grade 12). Reston (2018) makes links between the low performance of Filipino students in this test, as well as in international assessments such as TIMSS, and the decision to initiate a *K to 12* basic education reform in the Philippines as of 2012 (as well as to expand the Basic Education cycle from 10 to 12 years). Thus, there is an expectation that the implementation of the reform will be manifested in improvement of student outcomes. However, since complex interrelations exist between student-related and teacher-related criteria, the use of student outcomes to evaluate the success of reform-oriented PD programmes is intricate, to say the least.

In the next section, we move on to consider many kinds of resources that are relevant to the implementation of a curriculum reform, not directly linked to prospective teacher education or in-service professional development. Of course, the separation is in some sense artificial, in that the role of resources is connected with, and in some sense, dependent on, the initial teacher education and the professional development opportunities for teachers. We recognise that adequate training and on-going PD opportunities are critical for teachers, in order to look critically at resources available and make judgments about when to adhere to them and when not.

Resources in the Curriculum Reforms Implementation

Apart from teachers, many resources can intervene in curriculum implementation. We first want to set out what we will and will not be considering in this section. Among the resources of an institutional and social nature, is the role of principals in educational units, the systems of inspection of the classroom action, and the role of parents. Depending on the magnitude of the reform, its educational or social impact, these can play roles of greater or lesser importance. These factors, however, will be touched on here only in an instrumental or tangential way. In what follows, we will focus on 'traditional' textbooks and 'physical' materials for teachers, as well as resources based on digital technology, but also other resources of a more social nature such as counselling and guidance processes for teachers, and guidance from leaders, experts and educational communities.

When talking about resources, we need to consider the context. In particular, textbooks: in some countries they are mandatory in the sense that the school or the teacher must follow one textbook; sometimes there is only one national textbook or several ones, but they must be certified; in other countries, textbooks are a market product without certification and only dependent on editors, with schools making

their own decisions about what to buy. A similar range of structures and considerations apply in the case of other types of resources, but especially for assessments, when they are external to the school.

The resources for curriculum reform considered in this section can all be considered as 'tools' for teachers (textbooks, tasks, digital technology, and advice and guidance around their use) within the assumptions that are shared by the authors. We accept the notion of 'situated abstraction' (Noss et al., 2002, p. 207) in our belief that, in most cases, mathematical abstractions cannot be separated from the context of their creation or application. In this sense, any 'transfer' of knowledge from one context to another, takes place "not as a static entity, but rather through a reconfiguration and re-expression of mathematical relationships as they are reconstituted within different discourses" (p. 207).

Tools, technologies and contexts are, therefore, not 'ready-mades' (Rabardel, 2003, p. 641). Rather users adapt tools and are adapted by tools in a bi-directional relationship (Hollebrands et al., 2010, p. 325). The sensitivity of tool use to context must make us wary of how we read and what we synthesise from reports about reforms in different countries. To re-iterate points made earlier, it is important to keep in mind that the 'same' tool may, in different contexts, afford widely different possibilities and constraints.

Textbooks, Tasks, Teaching Resources

When analysing textbooks as a resource and examining their role in the implementation of curricular changes, it is necessary to consider different facets that vary among countries. These are the process of elaborating the textbooks, the use that textbooks have in the reform, particularly how they are connected to the new curriculum that has to be implemented, and the influence that high-stakes assessments have over textbook content. Moreover, the role of the textbook as a resource varies among countries; the type and the intensity of its use is closely related to other structures and infrastructures, such as the quantity and quality of material resources in the classrooms (including power relationships involved in the process of decision making about the use of resources), how professional development is organised, how the school is organised, what is the role of the principal and/or the teachers in the selection of the materials they use, or what are the working conditions of teachers. For example, in many Latin American countries, for teachers, the number of class hours per week can be more than 30–35, which leaves no room for PD and makes teachers highly dependent on resources such as textbooks.

Learning from different countries, we find that textbooks can be conceived as a public initiative or as a business opportunity. For instance, in Mexico (Lozano et al., 2018) textbooks are created by a core government funded team of mathematics educators and teachers, and the textbooks are distributed by the administration to every student across the country, because they are conceived both as teaching

resource and as a tool for general literacy, not only mathematical literacy (especially in the poorer areas, textbooks could be the only books in a family home).

The alignment between curriculum reform and textbooks, when created and delivered in a centralised way, is high in general because of government-driven strategy, despite there being a lack of connections with real life problems or the concerns of local communities. Examples from Tunis (Artigue, 2018) and South Africa (Volmink, 2018) also show how different curriculum reforms were accompanied by the introduction of one textbook.

A certification process for creating textbooks can be found in China (Cao, 2018), where the government decided to move from a former unique national textbook (as it is still in Iran, see Gooya & Gholamazad, 2018) to the possibility of using other textbooks, but only with a prior authorisation. That is a kind of middle point between the government creating a textbook and the government stepping aside and leaving all the process in the hands of publishers. In Thailand (Inprasitha, 2018), Japanese textbooks were translated and introduced to the system within a university and government led process. Any certification process allows the alignment of textbooks with curricular changes, not only regarding contents but also methodologies. Japanese reform (Namikawa, 2018) is a paradigmatic example of this, after the introduction of the methodology of structured problem solving, this way of working in the classroom configured the textbook skeleton.

In other countries, textbooks are created by publishers, without any kind of certification process. This freedom means that, in some countries, books may not reflect the reform, since publishers tend to save elements from previous versions and even, for example in Spain, keep contents that no longer belong to the new curriculum. Moreover, the focus of curricular changes regarding procedures and methodologies can be missed or biased by retaining content from earlier frameworks.

For instance, Statistics in Spanish high school textbooks (Rodríguez-Muñiz et al., 2018) tends to be treated as a summary of rules and calculation procedures, barely insisting on the main ideas about statistical competence, specifically the notion of variability (GAISE project, see Franklin & Garfield, 2006). This fact was also pointed out by McCallum (2018) in his ICMI Study 24 plenary talk, who considered that textbooks are often not faithful to mathematics as it is intended by mathematicians, but they present mathematics as a closed set of concepts, rules and procedures. However, some examples were pointed out (e.g. the Netherlands in van Zanten et al. 2018) in which commercial bookmakers have progressively adopted reform-based approaches.

High-stakes assessments, or curricular-based exit exams, can also have a strong influence in the process of creating textbooks. The case of Lebanon (Osta, 2018) is an instance of a low degree of coherence between assessment practices and curricular objectives, especially regarding reasoning and communication competences, since textbooks are made to be coherent with the standardised high-stakes exams which, themselves, are not aligned with the curriculum reform. This role of external assessments, having more influence on textbooks than curricular changes in their philosophy, goals and objectives, has been also pointed out in other countries like Italy (Martignone et al., 2018) and Spain (Rodríguez-Muñiz et al., 2018).

Thus, we have observed different models about textbooks, from a unique textbook created or endorsed by government, to an authorisation or certification process driven by the government for publishers, up to the complete freedom of distributing and adopting textbooks. In the latter case, the decision variables for many schools not only depend on the quality of the textbooks but also on the commercial offers by publishers. Overall, even assuming the differences, textbooks are still a leading resource in implementing curricular changes, but we have noticed a pattern consisting of: the lower the level of development of continuous training and supporting resources for teachers, the higher the influence of textbook on teachers' practices, and, thus, the higher risk of failure of the curriculum reform if textbooks are not aligned with it.

We now move on to consider, when analysing curriculum reforms, resources in the form of manipulatives, classroom tasks and activities designed for being used by teachers and students, relating to curricular changes. These resources can be more influential than textbooks themselves, depending on the cultural contexts of the countries. That is the case of curriculum reforms in France (Artigue, 2018) and Denmark (Niss, 2018), where banks of resources, classroom activities or methodological guidelines and advice were generated as a support for implementing reform.

As with textbooks, the creation of these resources admits a wide variety of processes that can be built bottom-up, by communities, or top-down, led by authorities, as well as different grey-scales between both extremes.These examples underline that the curriculum is not only a set of contents or an official document. On the contrary, learning from examples, it becomes clear that there is validity in the notion of curriculum as a 'six-dimensional vector' (see, Niss, 2018), consisting of contents, but also of goals, materials, forms of teaching, student activities, and assessment.

Digital Technology

We pointed out the diversity of cultural, social and, therefore, educational contexts in the previous section. If technology is considered, this diversity often turns into inequality, producing huge gaps that become difficult to overcome. Thus, digital technology and its role as an educational resource is an issue to be considered separately. This chapter was conceptualised and mainly written before the Covid-19 pandemic. We want to acknowledge this fact, since we are not able to report on the massive and unexpected shift to online learning which took place during 2020 in many parts of the world.

This shift on-line did not necessarily provoke or entail any change in written curriculum documents, but inevitably resulted in a huge change in teachers' and students' experience of the curriculum. And the inequalities mentioned just now were, it seems clear, exacerbated. Concerns have been expressed in many parts of the world over the differential access of students to technology and support for learning, during times of school closures, as well as over teachers' unequal skills on

technology in general and educational technology in particular. It remains to be seen whether the enforced uptake of online learning will have a lasting impact more widely, on technology use in schools.

Literature shows evidence of how and why digital technology improves the learning of mathematics (Drijvers, 2015) under different conditions and by different functionalities, but the focus in this chapter is on how digital technology contributes to curricular change. There is little evidence about this question, further than highlighting the role of inequality and comparing different environments from the technological point of view. No new evidence was found within the *ICMI Study 24 Proceedings* about this question, further than comments remarking that, as Northcote et al. (2010) pointed out, in many cases, technology such as the interactive whiteboard is used as a traditional whiteboard. Hence, the first idea we learn is that the technological resource is not enough, by itself, to produce a change in curriculum or teaching approach. On the contrary, its role strongly depends on its use.

Despite some of the examples presented in the *ICMI Study 24 Conference* mentioning the use of digital textbooks, such as in Serbia where they are going to be mandatory (Milinkovic, 2018, p. 146), no evidence about their role in curricular change was found. Moreover, there are no large-scale analyses about whether digital textbooks significantly change, or not, the format, and more importantly the role, of a traditional paper textbook. So, a key (unsolved) question arises: what are the differences between the use of a paper-based, a digitalised and a digital textbook?

Having answers to this question would lead to posing further ones: what is the role of a digital textbook in changing a curriculum? How can a digital textbook change the methodologies, activities, tasks and ways of learning mathematics? It is not clear at all that textbook makers are taking advantage of the multiple possibilities of integrating technologies in digital textbooks, by embedding different types of software and using technology in supporting learning rather than as using it only as a digital support.

Further than digital-*ised* textbooks, there is an abundance of research on the role of some products (e.g. dynamic geometry software, computer algebra systems) and the ways they are spreading in many classrooms, but they are conceived much more as an innovation resource rather than an instrument for developing and supporting curriculum changes. Developing digital text-book-like resources seems to be a teacher-led movement, with a lack of co-ordination.

In general, we have found there is little top-down guidance about how to connect technology with curricular change, beyond brief comments suggesting the use of technology in the curricular guidelines of some countries, or allowing the use of such technology in high-stake national-wide exams. We could say that the use of digital technology is an example of autonomous organisation of teachers, but its connection with curricular changes remains unstudied.

Nevertheless, some examples can be highlighted. For instance, in Ruiz (2015) we found a relevant experience from Costa Rica of the way technology can support curriculum reform. There, due to the teachers' mobility problems and intensive work days it was difficult for them to attend face-to-face courses, therefore, a PD programme based on e-learning was created in order to overcome these difficulties.

Hence, a set of blended online courses, MOOCs, and also less intensive short-duration Mini-MOOCs, were developed for qualifying teachers, allowing them to follow the courses according to their free time, without commuting, which is an important issue under difficult geographical and transportation conditions.

A further innovation in Costa Rica, with a special focus on independent study by students, is the development of *Mathematics Free Resources*.[2] Ruiz (2020) provides a detailed report on this experience in Costa Rica of technology as a curricular instrument. Such experiences are potentially transferable to other countries in Latin America.

Another highlighted example was found in England (Lozano et al., 2018), where the government supports a National Centre for Excellence in the Teaching of Mathematics (NCETM), which hosts a website with a wide variety of freely available resources, including teachers' materials, guidance, presentations, that can be used by teachers, supporting the implementation of a curriculum, renewed a few years ago, which is being re-interpreted in terms of these resources.

We also note an Australian example, based on the use of technology to overcome huge distances. Under the umbrella of the *reSolve: Mathematics by Inquiry* programme funded by the government (Thornton et al., 2018), selected 'champion' teachers follow a training programme, which is both online and face-to-face, to empower them in the use of different resources and approaches, in order to promote a challenging way of learning mathematics.

We conclude that, at the current time, technology is not a driver of curriculum change, nor are current curricula provoking significant uptake of new technologies for teaching and learning mathematics. And we contrast this conclusion with reports from a decade or so in the past (Sinclair et al., 2009) at which time there were several examples, from across the world, of national implementation of technology. Sinclair and colleagues reported on national projects taking place in Mexico, USA, Italy, Lithuania and Iran, some supporting the existing curriculum and teaching approaches with new technology and, in other cases, pushing the boundaries of curriculum and pedagogy. We have found it instructive to follow up, where possible with some of the instigators of these projects, what was the fate of those with a curriculum innovation element.

The Mexican programme, *Enciclomedia*, was an ambitious project led by the Mexican government to equip a great number of schools with free online textbooks and activities for primary education, provided by internet connection and supported with computers and projectors (Trigueros et al., 2006). The project was not endorsed with a capacitation programme for users and there was lack of technical supplies. Results were consequently not as good as expected. Sadly, a common sight on a recent visit by one of the authors to Mexican schools was the *Enciclomedia* hardware, covered and not being used, in the corner of each classroom.

The USA project, *Sketchpad for Young Learners*, aimed to help primary age students explore and understand key mathematical concepts through the dynamic

[2] See: https://recursoslibres.reformamatematica.net

visualisations possible within *The Geometer's Sketchpad*. The technology, at that time (in 2009), was unfamiliar to many teachers and students and, in common with the Mexican experience, there was a lack of resource for teacher support.

In Italy, the m@t.abel project started in 2006, as a follow up of another project 'La matematica per il cittadino' – Mathematics for the citizen – which developed from 2000 to 2005. M@t.abel lasted until 2012 and had some influence on the national curriculum, which was elaborated in those years (through many political changes). The original project focused around problem situations for which no clear cut or routine procedure is available for solution. A cascading model of training was built into this project, with the aim, within a few years, of reaching almost all teachers in grades 6–10. This extensive training model is interesting to reflect on, in light of the project's longevity as it seems to provide a contrast to the USA and Mexican experiences.

The experience in Lithuania was focused on the implementation of the use of *Geometer's Sketchpad* in grades 9 and 10, as a way to promote dynamic visualisation across the entire curriculum. The project included the development of over 800 sketches, for covering almost all the curricular issues. Apart from specific teacher training, the project highlighted the potential for making an explicit link between curriculum and technology.

In reflecting on the challenges of incorporating digital technology into the school mathematics curriculum, Ruthven (2017) identified *ecological*, *epistemological* and *existential* challenges. This framework seems relevant to considering more broadly the up-take of new resources in a time of curriculum change. In brief, ecological challenges relate to constraints of time, space and infrastructure, in adapting everyday practice in the classroom. Epistemological challenges relate to the requirements for new knowledge and skills in using new resources; and existential challenges relate to the way that values and identities associated with the whole project of school mathematics influence the adoption (or not) and understanding of new tools. There seems evidence, above, that for resources to meet the ecological and epistemological challenges, is a prerequisite for successful implementation (i.e., their absence seems to mean digital projects cannot thrive and grow). We suggest the existential challenge is likely as significant as the other two.

Support Agents for the Use of Resources

There are wide differences across countries in the use made of advisory documents and guidance for teachers, either linked to textbooks or particular resources, or more broadly to curriculum reform. In this section, we offer two examples of country-specific innovations which give a sense of past practice and ways that the two countries are attempting to drive reform through the development and promotion of resources. The two countries are Mexico and England, which were chosen in part because of a similarity in what is taking place across very different contexts but also

because they perhaps signalled some unusual imagined relationships between users and creators of new resources.

In 2017, a Mexican curriculum reform was presented through a new 'educational model' which emphasises quality in education for all students. In addition to the new educational model and the National Curriculum, the Secretary of Public Education (SEP) set about developing different materials, including nation-wide textbooks and accompanying teaching guides[3] for each subject. These textbooks are meant to provide, "a common ground for education in the country […], and are conceived as instruments which facilitate diverse and pertinent educational practices" (SEP, 2017, p. 126).

Through the new nation-wide textbooks and teachers' guides, new ways of working are being introduced, with a stronger emphasis than previously on guidance given to teachers. The previous versions of nation-wide textbooks gave general recommendations for each area of mathematics (number, geometry and measurement, data handling), and included brief suggestions specific to the chapters.

In the new materials, specific guidance is provided for each chapter and particular attention is given to: intentions related to conceptual learning; questions that can be asked to promote reflection; common mistakes and misconceptions; strategies for problem solving; strategies for differentiation; manipulatives or models that can be used. Relating this innovation to the previous section, we can see that the teacher guides are being used in an attempt to prompt and provoke classroom innovation, related to curriculum reform. The textbooks in Mexico are mandatory and there is a high degree of consistency in terms of curriculum reform and textbook guidance. Our second example comes from England.

In England, a new curriculum was introduced in 2014. In 2015, there was an explicit government agenda to alter the practice of mathematics teaching, drawing on practices from East Asia, particularly Shanghai, towards what is labelled as a 'mastery' approach. The introduction of a new vision for mathematics teaching took place without a change in curriculum (the 2014 curriculum does not mention mastery). The official government body tasked with promoting and developing the new approach is the National Centre for Excellence in the Teaching of Mathematics (NCETM). The NCETM defines a mastery approach as meaning, "Pupils are taught through whole-class interactive teaching, where the focus is on **all** pupils working together on the same lesson content at the same time" (NCETM, 2016, p. 1; bold in original). In contrast, an organising principle in relation to typical primary school teaching would previously have been that of 'personalisation' a concept at the centre of a past reform of mathematics teaching in England (DfE, 2011, p. 26).

The 2014 National Curriculum in England specifies learning outcomes for each year of study but is deliberately neutral about how these might be achieved. New guidance materials and resources being produced by the NCETM offer an ordering of content alongside themes, such as "equivalence", which are introduced at the start of grade 1, and developed in a systematic manner throughout the primary

[3] See: http://www.snie.sep.gob.mx/descargas/estadistica/SEN_estadistica_historica_nacional.pdf

school years. Students are introduced to the number line and two models for conceptualising part-whole relationships. Having core representations of additive structure that are introduced early in primary schooling, and used consistently and repeatedly, is an innovation in England.

A final distinction to note, compared to previous guidance, is the explicit distinctions made about how "number" is conceptualised, to ensure a balance of cardinal and ordinal or measure-based approaches. Current practice in England would have worked on number in an almost exclusively cardinal manner in grades 1 and 2 (Coles & Sinclair, 2017). According to the new guidance, students' very first introduction to number work will be in the context of measures (drawing inspiration from the work of, for example, Dougherty, 2008).

We notice a similarity across the Mexico and England examples, which is that resources are explicitly aimed at provoking professional development activity. In other words, in neither country does guidance aim to define precisely what teachers should do. Rather, guidance aims to support teachers in re-thinking their practice and making use of resources in a way that allows for innovation - at least, this is the clear intention of the authors of the guides. The resources, for instance, can be used as part of a collaborative planning process in schools.

We see an interesting development, in these examples, in that teachers are being imagined as co-designers of the curriculum, rather than as implementers of someone else's design. Of course, whether the resources will be experienced by teachers in a spirit of co-design will depend on a myriad of factors including local leadership and opportunities for continuing professional development related to the new guides and materials.

Final Remarks on the Use of Resources

In looking back across this section, we aim now to distil some of the features of the research cited, which pertain to the question of how and why teachers use resources, in a context of curriculum change. We identify areas in which there is quite significant variation of practice across counties and areas, which could lead to further research.

Control of, and access to, resources ranges from resources mandated for teacher use such as a national textbook, and resources, either freely or commercially available, which teachers can choose to use or not. In-between these extremes there are varying levels and levers of active promotion of resources. For example, where resources are linked closely to high-stakes assessments, there may be a sense of needing resources to avoid disadvantaging students (e.g. in England, Pearson's Publishing group runs an examination board and also publishes textbooks linked to those examinations).

Government inspection regimes may also have particular areas of focus related to the use of particular resources or pedagogies that are linked to resources. The kinds of resources available range from being freely available online, to being

online behind a paywall, to blended resources combining digital and physical versions, to physical artefacts. Again, there is a range of practices in terms of the costs of physical resources. In Mexico, schools are provided textbooks for each child at no cost to the school. In England, schools must meet the full cost of any textbooks they purchase.

We see a range of variation across the creation of resources in terms of the extent of teacher involvement. In some cases, resources are created by 'experts' (who might be teacher educators, mathematicians, publishers), perhaps with trialling and testing in classrooms. In other cases, there is a process of co-creation with teachers and in some cases the resources are created by teachers, perhaps shared via social media. It is beyond the scope of this chapter to offer any evaluation of these different processes and variations but we see such evaluation as a fruitful area of future research.

Similar kinds of variations to those discussed above are apparent in relation to students' use of resources. In some cases, it is mandated by government that students use particular textbooks, in some other cases students access resources decided on by their teacher or school. There are, of course, resources freely available to students online that may be accessed as part of school lessons or independently.

The actual effectiveness of resources in the implementation of a reform is linked to the factors described above. There are examples across the world of innovative resources being made available but not being taken up. As described above, factors at play here include whether particular resource use is mandated or not by local or national government and the extent of training and support provided to teachers. One general question we would like to raise is the extent to which new resources and tools are a good *fit* to new pedagogies implied within curriculum reform. We intuitively feel such a fit, or its absence, must be a significant factor in the use of resources linked to reform, but we are unable to draw on any evidence for this idea. To return to the framework, we suggest that Ruthven's (2017) *ecological*, *epistemological* and *existential* challenges, in the take up of digital resources, can be applied more broadly, to help consider the range of challenges in using resources related to curriculum change.

Role of Assessment in Curriculum Reform Implementation

Assessment has a complex role in teaching-learning processes as well as in the process of curriculum implementation. This section deals with some aspects concerning the role of classroom assessment and external assessment in curriculum implementation. In particular, we focus on how assessment has been used, not only as a source of information about the attained curriculum, but also as a way to perceive the intended curriculum and as a resource for curriculum implementation.

International, National and Classroom Assessment

Depending on its aims, assessment takes many forms and involves students and teachers in different ways. Assessment can be carried out by specific tasks, being part of classroom teaching and learning activities, or of a process of system evaluation. In literature, we find studies in mathematics education on the relationships between the processes of curriculum implementation and assessment that focus on classroom activities carried out by teachers. In particular, the classroom assessment can be identified by, "the activities undertaken by teachers in eliciting and interpreting evidence of the students' learning and using this evidence to inform subsequent action" (Goos, 2014, p. 413). This type of assessment should be distinguished from the external assessment, usually developed for summative or evaluative purposes, which often involves large scale standardised tests.

The complex relations and the interweaving between assessment (classroom and external assessment) with curriculum implementation can be analysed from different perspectives. An issue that has been identified and studied in the last years, concerns the influence of some international surveys (both in terms of framework and in terms of results) on national curricula around the world. The results of these international surveys must be read and interpreted within the different cultural systems. The connection between international assessment and national curricular changes was also pointed out in some papers in ICMI Study 24 Conference focusing on the influence of TIMSS or PISA surveys on design and changes of Mathematics Curriculum (Kadijevich, 2018; Milinkovic, 2018; see Chap. 22).

TIMMS and PISA surveys aim at evaluating education systems worldwide by testing knowledge and skills of students in different school grades. These surveys use standardised tests because they can be administered on a large scale, but standardisation causes some criticisms from the academic world. Moreover, there are also concerns about the contents and the tasks proposed: one of the most commonly criticised aspects is that they focus on calculation skills rather than on mathematical thinking. Unfortunately, many, but not all, standardised tests were constructed following a view of learning (perhaps implicit), according to which learning is most effective when knowledge and skills are broken down into many small steps that can be taken sequentially by learners.

Such a fragmented view of evaluation is not aligned with the more current socio-constructivist and historical-cultural perspectives, which are at the basis of many educational practices suggested by research and new curricula around the world. Although it is certainly true that in a standardised test it is not possible to find certain types of problems, which are however very important for assessing mathematical competences (conjecture, exploration, long and complex problem-solving activities), it must also be said that tests can be made up not only of questions which require the recollection of notions or the production of calculations. It is important to define their aims and what they actually show. External assessment is usually used for summative and evaluative purposes and seen only as related to the analysis of the attained curriculum, despite a range of other possible uses.

We would like to point out some of the other aspects of an external assessment. It can also impact on the complex process of curriculum implementation. For example, changes in national assessments can lead to a change in teaching practices, as has happened in some European countries. In particular in Italy, the National standardised assessment tests, called INVALSI tests, have become a means for teachers and students to deal with tasks that are constructed according to the goals explicitly stated in the National Guidelines. Therefore, national standardised tests can be used by teachers to reflect on curriculum demands and then to perceive the intended curriculum (Martignone et al., 2018). Standardised tests, especially when they are strictly linked to country standards and curricula, can become a resource for teachers who can use them to reflect on curriculum requirements and thus on changes and differences in practices.

Even if the aim of this external assessment is to contribute to a system evaluation, we argue that an analysis of these test tasks can also be a tool to modify the system itself and to carry messages that influence the implementation of the curriculum by leading, for example, the attention on topics rarely developed or on particular types of tasks. In this way, assessment can become a resource influencing the curriculum reform. Still, in the case of Italy, the analysis of national assessment tests is often used in teacher educational programmes because, in order to become agents of the reform, teachers have to be supported by means of educational projects.

During these programmes teachers focus their attention on the understanding of where, how and why students have difficulties in particular problems. This can lead teachers to analyse and reflect on these problems and also to use them in the design of classroom activities. This is not "teaching to the test", but developing formative activities that consider the curricular requirements and that may also have been inspired by problems from standardised tests. Thus, it is possible to build a bridge between external assessment and the activities carried out in the classroom.

In contrast to the case of Italy above, Osta (2007) presents the example of the Lebanese, where the high-stakes national examinations set implicit boundaries for the implementation of the reformed curriculum when they were not aligned with the intended reformed curriculum, and still carried the "assessment culture" rooted in the educational community's understandings and practices, through years of pre-reform stereotyped examinations. Osta highlights the consequent formation of a 'mini-curriculum' consisting of a limited set of stereotyped test items repeatedly included in the national examinations: "This stability and stereotyped structure make teachers and students adopt that mini-curriculum as their set of guidelines instead of the curriculum" (p. 194).

Osta contends that the written, or intended, curriculum is static once a reform is set on paper, while the implemented and the tested curricula are dynamic and variable. In the case where assessments are aligned with a new curriculum, the set of tested curriculum items moves within the intended curriculum by including, each time, different topics, in a way to ultimately cover the intended curriculum. However, in the case where the assessments focus on a small part of the intended curriculum, it is the taught or implemented curriculum that is variable, gradually shrinking closer and closer to the tested curriculum, thus forming a mini-curriculum.

Classroom assessment is usually identified as part of curriculum implementation. It can be different from class to class, from school to school and, of course, from country to country. Therefore, about class practices and implemented curriculum, we can only make general considerations or quote specific examples. Classroom assessment can be carried out for both summative and formative purposes, mainly by the teachers of the classroom. It is often formative because it aims at supporting students' learning and informing teachers' instructional decisions: "Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction" (Black & Wiliam, 2009, p. 9).

The typical activities in formative assessment processes are those through which students have the opportunity to verify their own learning levels, to plan and implement, to interact with the teacher and the classmates, and to develop the strategies necessary to achieve the set learning objectives. Therefore, the formative assessment is carried out during day-to-day practice, in-class exercises, homework, projects, etc. Teachers should develop tasks and methodologies that can give evidence of students' learning by promoting mathematical thinking (abstraction, contextualisation, connection between different concepts, argumentation, problem solving, etc.) and by using different appropriate representations (verbal, tabular, graphic, symbolic, etc.). Like all types of assessment, formative assessment is based on the interpretation of observable variables, which can allow a judgment on the quality of learning and on the effective curriculum implementation.

In this chapter, we have already dealt with teachers in the process of curriculum reform implementation, in addition, in this paragraph, we focus on their fundamental role in the assessment process. This type of assessment may or may not be consistent with curriculum goals. It considers different activities developed at school. Studies on the relationships between curriculum and assessment might take into account components of teacher knowledge of curriculum and assessment methods and how they can be improved (Santos & Cai, 2016).

Teachers use their professional competences to develop teaching strategies that translate the written curriculum into the implemented curriculum. Often, however, teachers are strongly influenced both by textbooks and by external assessment more than by what is written in the official curriculum. For example, if some topics or a specific type of problem are not present in the textbooks or are not assessed, no matter what is written in the curriculum, teachers and students may not focus on those or even, a teacher may decide to skip, or not to assess these topics. There may also be another effect: if a topic or certain tasks are present in the external assessment tests, then the attention of teachers and students will turn to these. Therefore, it is fundamental for effective reform that the assessment, as well as textbooks, be aligned (Schmidt et al., 2005) with curricular goals.

Conclusion

Different roles can be played by teachers as agents of curriculum reform. From getting directly or indirectly involved in drafting the reform, to participating in producing specific documents or materials, or designing or organising professional development programmes associated with the reform. Teachers' engagement with the curriculum reform, and the way professional development is organised and aligned with it, has been pointed out as one of the most important factors in the implementation of a new curriculum.

The acquisition of an adequate mathematical content knowledge during initial training may support teachers in fostering a close connection with future reforms. But, also, specialised knowledge, and specifically, training with school materials, will prepare prospective teachers to match theories underlying the curriculum reform with possible classroom practices.

Reform-oriented PD programmes can focus not only on teachers but also on teachers' resources or even on students. Examples show how helping teachers in working with a new curriculum and supporting them with materials and shared discussions helps in the implementation process. We have seen different implementation experiences from different countries and different cultural contexts, which underline that there are several ways to approach the problem and that just translating one successful case is not enough, because an adaptation to each country's circumstances must be made to transpose any model.

The wide variety of implementation models seems to become even greater when considering the results. We have seen that there are no clear patterns regarding the use of resources during implementation processes. Sources of variation go from the creation to the control of the resources and their availability. New digital technologies are gaining space, not only as supporting resources but also as digital and digitalised books. The way teachers use resources is also a source of great variation. From different examples, it seems that the absence of an authority-driven control of textbooks produces bad effects in their quality and alignment with the reforms.

A similar case occurs with the assessment. Some examples have been pointed out about how external assessment influences teachers' classroom activity as well as PD activities. Particularly, international assessments such as PISA or TIMMS produced changes in the curriculum of some countries. It is difficult to find a prior alignment in such cases, but the most common problem emerges when the assessment is a national one, not aligned with curricular changes, thus weakening its implementation.

From reflections and discussion presented in this chapter, we have summarised, in Table 18.1 some of the key dimensions of variation, as well as questions for the future.

We want, finally, to raise an issue of pressing importance which, however, does not appear in any of the documents presented at the ICMI Study 24 Conference, that is the issue of when and how curriculum reform will pay attention to the global crises of climate change, biodiversity loss, mass migration, access to water and other worldwide issues. In other words, we see daily evidence of the precarious

Table 18.1 Variations and questions identified relating to resources and curriculum reform

Domain	Sub-domain	Range of practices
Teachers	Teacher participation in reform	Drafting; preparation; consultancy; implementation; evaluation; co-designing.
	Teacher preparation	The role of teacher preparation in curriculum reform seems unexplored.
	PD aims	Resource-centred; student-centred.
	Models of PD	Content (the curriculum itself); pedagogical orientation (views on how to teach); organisational form (national; school-led/university-led; face-to-face/online; personalised/collective; cascading/cluster-based/needs-based).
	Teacher voice in PD	Participant; evaluator; collaborator; change agent.
	Evaluation	Teacher-related criteria; student-related criteria.
Resources	Textbooks	Government mandated for all students; limited government certified range of options; commercial marketplace.
	Tasks and teaching resources	Resources as a support for implementing reform; top-down lead; bottom-up lead.
	Digital technology	The connection between take-up of digital technology and curriculum change seems largely unexplored.
	Support agents	Curriculum change; teacher guidance around pedagogy; teacher guidance around content knowledge.
Assessment	International	TIMSS or PISA influence on national curricula; assumption in these tests that learning comes from breaking knowledge into small steps.
	National	Changes in national assessment leading to changes in teaching practice; national assessments perceived as the intended curriculum.
	Classroom	Summative purposes; formative purposes.

nature of life on earth and yet the mathematics curriculum appears to continue, to some extent, as though nothing immediate and different is taking place beyond the school walls.

We have little to say on this point, except to point towards the relative absence of research, within mathematics education, that is considering, for example, a curriculum for an age of climate emergencies. We would, however, like to use this publication to urge scholars to consider *now* what reforms will be needed when catastrophic events force re-thinking the way we live. We want to suggest that there be an urgent need to develop a *curriculum in waiting*, for example, or to find ways of connecting the school curriculum with the concerns of school communities (Coles & Sinclair, 2017). Reflecting on the past implementation of reforms and what we can learn about resources that occasion change is equally vital work, in terms of looking towards an uncertain future.

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