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AdS_3 vacua realising $\mathfrak{osp}(n|2)$ superconformal symmetry

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Abstract: We consider $\mathcal{N}=(n,0)$ supersymmetric AdS_3 vacua of type II supergravity realising the superconformal algebra $\mathfrak{osp}(n|2)$ for $n>4$. For the cases $n=6$ and $n=5$, one can realise these algebras on backgrounds that decompose as foliations of $AdS_3\times\mathbb{CP}^3$ (squashed $\mathbb{CP}^3$ for $n=5$) over an interval. We classify such solutions with bi-spinor techniques and find the local form of each of them: they only exist in (massive) IIA and are defined locally in terms of an order 3 polynomial h similar to the AdS_7 vacua of (massive) IIA. Many distinct local solutions exist for different tunings of h that give rise to bounded (or semi infinite) intervals bounded by physical behaviour. We show that it is possible to glue these local solutions together by placing D8 branes in the interior of the interval without breaking supersymmetry, which expands the possibilities for global solutions immensely. We illustrate this point with some simple examples. Finally we also show that AdS_3 vacua for $n=7,8$ only exist in $d=11$ supergravity and are all locally $AdS_4\times S^7$.

Keywords: Extended Supersymmetry, Flux Compactifications, Superstring Vacua, AdS-CFT Correspondence

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1 Introduction and summary

Warped AdS_3 solutions of supergravity in 10 and 11 dimensions, "AdS_3 string vacua", play an important role in string theory in a wide variety of contexts. AdS_3 appears in the near horizon limit of black-strings solution, so the embedding of such solutions into higher dimensions enables one to employ string theory to count the micro states making up the Bekenstein-Hawking entropy a la Strominger-Vafa [1]. Through the AdS-CFT correspondence they are dual to the strong coupling limit of CFTs in 2 dimensions. This avatar of the correspondence promises to be the most fruitful as more powerful techniques are available to probe CFT_2s and there is better understanding of how to quantise strings on AdS_3 than in higher dimensional cases. AdS_3 vacua also commonly appear in duals to compactifications of CFT_4 on Riemann surfaces [2–9], a topic of rekindled interest in recent years with improved understanding of compactifications on surfaces of non-constant curvature such as spindles. Some other venues in which AdS_3 vacua have played a prominent role are geometric duals to c-extremisation [10, 11] and dual descriptions of surface defects in higher dimensional CFTs [12–16].

Given the above listed wealth of applications, a broad effort towards classifying supersymmetric AdS_3 vacua is clearly well motivated, but at this time many gaps remain. Generically such AdS_{d+1} vacua can support the same superconformal algebras as CFTs in d dimensions. The possible $d=2$ superconformal algebras are far more numerous than their higher dimensional counterparts, which partially accounts for these gaps. For comparison $d>2$ the possible (simple) superconformal algebras typically[1] come in series depending on a parameter which varies as the number of super charges increase; for instance in $d=3$ one has $\mathfrak{osp}(n|4)$ for CFTs preserving $\mathcal{N}=(n,0)$ supersymmetry, where $n=1,\ldots,8$. CFTs in $d=2$ buck this trend, being consistent with several such series as well as isolated examples such as $\mathfrak{f}(4)$ and $\mathfrak{g}(3)$ — see [17] for a classification of these algebras and [18] for those that can be supported by string vacua. The focus of this work will be AdS_3 vacua supporting the algebra $\mathfrak{osp}(n|2)$ (the $d=2$ analogue of the $d=3$ algebra).

The $\mathcal{N}=(n,0)$ superconformal algebra $\mathfrak{osp}(n|2)$ for arbitrary n was first derived, independently, in [19] and [20] — they are characterised by an $\mathfrak{so}(n)$ R-symmetry with supercurrents transforming in the fundamental representation and central charge

$$c=\frac{k}{2}\frac{n^2+6k-10}{k+n-3}. \tag{1.1}$$

A free field relation was presented in [21] (see also [22]) in terms of a free scalar, n real fermions, and an $\text{SO}(n)$ current algebra of level $k-1$. There are in fact many examples of AdS_3 vacua realising $\mathfrak{osp}(n|2)$ for $n=1,2$ as these are the unique ways to realise $(1,0)$ and $(2,0)$ superconformal symmetries — see for instance respectively [23–32] and [11, 33–43] . Similarly $n=3$ is unique for $\mathcal{N}=(3,0)$, examples are more sparse [44–46], but this is likely not a reflection of their actual rarity. The case of $n=4$ is in fact a degenerate case of the large $\mathcal{N}=(4,0)$ superconformal algebra $\mathfrak{d}(2,1,\alpha)$, where the continuous parameter is tuned to $\alpha=1$ — examples of vacua allowing such a tuning include [47–50], there is also a Janus solution preserving $\mathfrak{osp}(4|2)\oplus\mathfrak{osp}(n|2)$ specifically in [13]. The case of $n=8$ was addressed in [51] where it was shown that the only solution is the embedding of AdS_3 into $\text{AdS}_4\times\text{S}^7$. The status of AdS_3 vacua realising $\mathfrak{osp}(n|2)$ for $n=5,6,7$ has been up to this time unknown — a main aim of this work is to fill in this gap, we will now explain the broad strokes of how we approach this problem.

For the case of $\mathfrak{osp}(7|2)$, with a little group theory [51], it is not hard to establish that the required $\mathfrak{so}(7)$ R-symmetry can only be realised geometrically on the co-set $\text{SO}(7)/\text{G}_2$. The metric on this space is simply the round 7-sphere, which possesses an $\text{SO}(8)$ isometry, but the co-set also comes equipped with a weak G_2 structure with associated 3 and 4-forms that are invariant under $\text{SO}(7)$, but charged under $\text{SO}(8)/\text{SO}(7)$. If these forms appear in the fluxes of a solution then only $\text{SO}(7)$ is preserved. Our results prove that all such solutions are locally $\text{AdS}_4\times\text{S}^7$.

To realise the requisite $\mathfrak{so}(6)$ R-symmetry of $\mathfrak{osp}(6|2)$ one might naively consider including a 5-sphere in solutions, however this supports Killing spinors in the **4** of $\text{SU}(4)$,

[1] $d=5$ is an exception with only one possibility, $\mathfrak{f}(4)$.

which is not the representation associated to the desired algebra. A space supporting both the correct isometry and spinors transforming in its fundamental representation is of course round $\mathbb{CP}^3$, as famously exemplified by the $\text{AdS}_4\times\mathbb{CP}^3$ vacua of type IIA supergravity dual to $\mathcal{N}=6$ Chern-Simons matter theory [53]. This is the smallest space with the desired features, and given that a string vacua has to live in d=10 or 11, it does not take long to realise that the only additional option is to fiber a U(1) over $\mathbb{CP}^3$. Such solutions were ruled out in type II supergravity at the level of the equations of motion in [51] and those that exist in M-theory can always be reduced to IIA.[2] As such we seek to classify solutions of type II supergravity that are foliations of $\text{AdS}_3\times\mathbb{CP}^3$ over an interval, we leave the status of d=11 vacua containing similar foliations over Riemann surfaces to be resolved elsewhere.

For the algebra $\mathfrak{osp}(5|2)$ the 4-sphere is as much of a non-starter to realise the $\mathfrak{so}(5)$ R-symmetry as the 5-sphere was previously. One way to realise this algebra is to start with an existing $\mathfrak{osp}(6|2)$ solution, then orbifold $\mathbb{CP}^3$ by one of the discrete groups $\hat{D}_k$ (the binary dihedral group) or $\mathbb{Z}_2$, as discussed in the context of $\text{AdS}_4\times\mathbb{CP}^3$ in [57].[3] This however only breaks supersymmetry globally, locally such solutions still preserve $\mathfrak{osp}(6|2)$. One way, perhaps the only way[4] to break to $\mathfrak{osp}(5|2)$ locally is to proceed as follows: if one expresses $\mathbb{CP}^3$ as a fibration of S^2 over S^4 and then pinches the fiber, one breaks the SO(6) isometry down to SO(5) locally. The **6** of SO(6) branches as $\mathbf{5}\oplus\mathbf{1}$ under its SO(5) subgroup thereby furnishing us with both the representation and R-symmetry that $\mathfrak{osp}(5|2)$ demands. We shall thus also classify $\mathcal{N}=(5,0)$ AdS_3 vacua of type II supergravity on squashed $\mathbb{CP}^3$ by generalising the previous ansatz to included additional warp factors and SO(5) invariant terms in the flux. We shall in fact classify both these vacua and those supporting $\mathfrak{osp}(6|2)$, or orbifolds there of, in tandem as the latter are special cases of the former.

We find two classes of solutions preserving respectively $\mathfrak{osp}(5|2)$ (locally) and $\mathfrak{osp}(6|2)$ superconformal algebras. We also find for each case that it is possible to construct solutions with bounded internal spaces, which should provide good dual descriptions of CFTs through the AdS/CFT correspondence. The existence of backgrounds manifestly realising exactly the superconformal algebra $\mathfrak{osp}(5|2)$ is interesting in the light of [52], which claims that all CFTs supporting such global superconformal algebras experience an enhancement to $\mathfrak{osp}(6|2)$. Our results cast some doubt of the veracity of the claims of [52], at least naively. It would be interesting to explore what leads to this apparent contradiction and whether this can be resolved, that however lies outside the scope of this work.

The layout of this paper is as follows.

In section 2 we consider AdS_3 vacua of type II supergravity that preserve an SO(5) isometry in terms of squashed $\mathbb{CP}^3$, without making reference to supersymmetry. On symmetry grounds alone we are able to give the local form that the NS and RR fluxes must take in regular regions of their internal space, which we found useful when deriving the results in the subsequent sections.

[2]Either the spinors are not charged under the additional U(1), or some algebra other than $\mathfrak{osp}(6|2)$ is being realised.

[3]See section 3 therein.

[4]One can realise an $\mathfrak{so}(5)$ R-symmetry on a squashing of $\text{S}^3\to\text{S}^7\to\text{S}^4$, but AdS_3 vacua containing this factor only exists in $d=11$ and, when they support $\mathfrak{osp}(5|2)$, they can always be reduced to IIA within the 7-sphere resulting in squashed $\mathbb{CP}^3$ ($\widehat{\mathbb{CP}}^3$) and preserving $\mathcal{N}=(5,0)$.

In section 3 we explain our method for solving the supersymmetry constraints. We reduce the problem to solving for a single $\mathcal{N}=1$ sub-sector of the full $(5,0)$ as the remaining 4 $\mathcal{N}=1$ sub-sectors are shown to be implied by this and the action of $\mathfrak{osp}(5|2)$ which the spinors transform in. This enables us to employ an existing minimally supersymmetric AdS_3 bi-spinor classification [27, 28, 31] to the case at hand.

In section 4 we classify $\mathcal{N}=(5,0)$ vacua of type II supergravities realising the algebra $\mathfrak{osp}(5|2)$ in terms of a foliation of AdS_3 solutions of type II supergravity that are foliations of $AdS_3\times\widehat{\mathbb{CP}}^3$ over an interval, we are actually able to find the local form of all of them. They only exist in type IIA, generically have all possible fluxes turned on and are governed by two ODEs. The first of these takes the form $h'''=-2\pi F_0$, where F_0 is the Romans mass, making h locally an order 3 linear polynomial highly reminiscent of the AdS_7 vacua of [54, 55]. The second ODE defines a linear function u which essentially controls the squashing of $\mathbb{CP}^3$ and hence the breaking of $\mathfrak{osp}(6|2)$ to $\mathfrak{osp}(5|2)$. For generic values of u one has $\mathcal{N}=(5,0)$ supersymmetry, but if one fixes $u=$ constant this is enhanced to $\mathcal{N}=(6,0)$

In section 5 we perform a regularity analysis of the local vacua establishing exactly what boundary behaviour are possible for the interval. We focus on $\mathcal{N}=(6,0)$ in section 5.1 where we find that fixing $F_0=0$ always gives rise to $AdS_4\times\mathbb{CP}^3$ locally, while for $F_0\neq 0$ it is possible to bound the interval at one end with several physical singularities but that the other end is always at infinite proper distance, at least when F_0 is fixed globally. We study the $\mathcal{N}=(5,0)$ case in section 5.2 were conversely we find no AdS_4 limit and that a globally constant F_0 is no barrier to constructing bounded solutions. Many more physical boundary behaviours are possible in this case.

Up to this point in the paper we have assumed F_0 is constant, globally it need only be so piece-wise which allows for D8 branes along the interior of the interval — we explore this possibility in section 6. We establish under what conditions such interior D8s are supersymmetric and explain how they can be used to construct broad classes of globally bonded solutions. We illustrate the point with some explicit examples. All of this points the way to broad classes of duals interesting superconformal quiver we shall report on in [63].

The work is supplemented by several appendices. In appendix A we provide technical details of the construction of spinors on the internal space transforming in the fundamental representation of SO(5) and SO(6). In appendix B we present details of the $d=6$ bi-linears that feature during computations in section 4. Finally in appendix C we additionally show that all $\mathfrak{osp}(7|2)$ preserving AdS_3 vacua experience a local enhancement to $AdS_4\times S^7$ — SO(7) preserving orbifolds of this are a possibility, but such constructions are AdS_4 rather than AdS_3 vacua.

2 SO(2,2)×SO(5) invariant type II supergravity on $AdS_3\times\widehat{\mathbb{CP}}^3$

In this section we consider the most possible vacua of type II supergravity that preserve the full SO(2,2)×SO(5) isometries of a warped product containing AdS_3 and a squashed $\mathbb{CP}^3$ ($\widehat{\mathbb{CP}}^3$). Specifically we construct the full set of SO(5) invariant forms and use them to find the general form of NS and RR fluxes that are consistent with their source free (magnetic) Bianchi identities. Let us stress that this section makes no use of supersymmetry only symmetry, it is none the less useful when we choose to impose the former in the following sections.

In general AdS_3 solutions of type II supergravity admit a decomposition in the following form

$$
\begin{aligned}
&ds^2 = e^{2A}ds^2(\text{AdS}_3) + ds^2(\text{M}_7), \qquad H = e^{3A}h_0\text{vol}(\text{AdS}_3) + H_3,\\
&F = f_\pm + e^{3A}\text{vol}(\text{AdS}_3)\star_7\lambda(f_\pm),
\end{aligned}
\tag{2.1}
$$

where (e^{2A}, f, H_3) and the dilaton Φ have support on M_7 so as to preserve the SO(2,2) symmetry of AdS_3. The $d=10$ NS and RR fluxes are H and F respectively, the latter expressed as a polyform of even/odd degree in IIA/IIB. The function λ acts on a p-form as $\lambda(X_p) = (-1)^{[\frac{p}{2}]}X_p$ — this ensures the self duality constraint $F = \star_{10}\lambda(F)$.

We are interested in solutions where M_7 preserve an additional SO(5) isometry that can be identified with the R-symmetry of the $\mathcal{N}=(5,0)$ superconformal algebra $\mathfrak{osp}(5|2)$. The 4-sphere comes to mind as the obvious space realising an SO(5) isometry, however this supports Killing spinors in the **4** of SP(2), where as we require spinor in the **5** of SO(5) — so we will need to be more inventive.

The coset space $\mathbb{CP}^3$ is a 6 dimensional compact manifold that can be generated by dimensionally reducing S^7 on its Hopf fiber — it appears most famously in the $\mathcal{N}=6$ $\text{AdS}_4\times\mathbb{CP}^3$ solution dual to Chern-Simons matter theory. The 7-sphere supports spinors transforming in the **8** of SO(8) and the reduction to $\mathbb{CP}^3$ preserves the portion of these preserving the **6** of SO(6). Advantageously $\mathbb{CP}^3$ has a parametrisation as an S^2 fibration over S^4 that allows a squashing breaking SO(6)→SO(5) by pinching the fiber — we will refer to this space as $\widehat{\mathbb{CP}}^3$. As the **6** branches as $\mathbf{1}\oplus\mathbf{5}$ under SO(5) ⊂ SO(6) clearly $\widehat{\mathbb{CP}}^3$ supports both the isometry group and spinors we seek. Embedding this SO(5) invariant space into M_7 leads to a metric ansatz of the form

$$
\begin{aligned}
&ds^2(\text{M}_7) = e^{2k}dr^2 + ds^2(\widehat{\mathbb{CP}}^3) \qquad (2.2)\\
&ds^2(\widehat{\mathbb{CP}}^3) = \frac{1}{4}\left[e^{2C}\left(d\alpha^2 + \frac{1}{4}\sin^2\alpha(L_i)^2\right) + e^{2D}(Dy_i)^2\right], \qquad Dy_i = dy_i + \cos^2\left(\frac{\alpha}{2}\right)\epsilon_{ijk}y_jL_k,
\end{aligned}
$$

where y_i are embedding coordinates on the unit radius 2-sphere, L_i are a set of SU(2) left invariant forms and $(e^{2A}, e^{2C}, e^{2D}, e^{2k}, \Phi)$ have support on r only.

To write an ansatz for the fluxes on this space we need to construct the SO(5) invariant forms on $\widehat{\mathbb{CP}}^3$. As explained in appendix A, the S^4 base of this fiber bundle contains an $\text{SO(4)}=\text{SO(3)}_L\times\text{SO(3)}_R$ isometry in the 3-sphere spanned by L_i. In the full space SO(3)_R is lifted to the diagonal SO(3) formed of SO(3)_R and the SO(3) of the 2-sphere. As such the invariants of SO(5) can be expanded in a basis of the $\text{SO(3)}_L\times\text{SO(3)}_D$ invariants on the $\text{S}^2\times\text{S}^3$ fibration (see for instance [56]), namely

$$
\begin{aligned}
&\omega_1 = \frac{1}{2}L_iy_i, \qquad \omega_2^1 = \frac{1}{2}\epsilon_{ijk}y_iDy_j\wedge Dy_k, \qquad \omega_2^2 = \frac{1}{2}L_i\wedge Dy_i,\\
&\omega_2^3 = \frac{1}{2}\epsilon_{ijk}y_iL_j\wedge Dy_k, \qquad \omega_2^4 = \frac{1}{8}\epsilon_{ijk}y_iL_j\wedge L_k,
\end{aligned}
\tag{2.3}
$$

and wedge products there off, leaving only the α dependence of the SO(5) invariants to fix via consistency with the remaining $\mathrm{SO}(5)/(\mathrm{SO}(3)_L\times\mathrm{SO}(3)_D)$ subgroup.

First off when $e^{2C}=e^{2D}=1$ we regain unit radius round $\mathbb{CP}^3$, which is a Kähler Einstein manifold with an SO(6) invariant Kähler form J_2, so we have the following SO(6) invariants on $\mathbb{CP}^3$

$$\mathbb{CP}^3:\ e^{2C}=e^{2D}=1 \quad\Rightarrow\quad \mathrm{SO}(6)\ \text{invariants}:\ J_2,\quad J_2\wedge J_2,\quad J_2\wedge J_2\wedge J_2=6\mathrm{vol}(\mathbb{CP}^3). \tag{2.4}$$

where specifically

$$J_2=\frac{1}{4}\Big(\sin\alpha d\alpha\wedge\omega_1-\sin^2\alpha\omega_2^4-\omega_2^1\Big). \tag{2.5}$$

It is not hard to show that the remaining SO(5) invariants, which are not invariant under the full SO(6) of $\mathbb{CP}^3$, may be expressed in terms of the SU(3)-structure spanned by

$$\tilde{J}_2=\frac{1}{4}\Big(\sin\alpha d\alpha\wedge\omega_1-\sin^2\alpha\omega_2^4+\omega_2^1\Big),\qquad \Omega_3=-\frac{1}{8}\sin\alpha\Big(\sin\alpha\omega_1+id\alpha\Big)\wedge\Big(\omega_2^3+i\omega_2^2\Big), \tag{2.6}$$

These invariant forms obey the following identities

$$\begin{aligned}
&J_2\wedge\Omega_3=\tilde{J}_2\wedge\Omega_3=0, && J_2\wedge J_2\wedge J_2=\tilde{J}_2\wedge\tilde{J}_2\wedge\tilde{J}_2=\frac{3i}{4}\Omega_3\wedge\overline{\Omega}_3,\\
&J_2\wedge J_2+\tilde{J}_2\wedge\tilde{J}_2=2\tilde{J}_2\wedge J_2, &&\\
&dJ_2=0,\qquad d\tilde{J}_2=4\mathrm{Re}\Omega_3, && d\mathrm{Im}\Omega_3=6\tilde{J}_2\wedge J_2-2J_2\wedge J_2,
\end{aligned} \tag{2.7}$$

and as such, they form a closed set under the exterior product and derivative. This is all that is needed to construct the fluxes.

The general form of an SO(5) invariant H_3 obeying $dH_3=0$ is given by

$$H_3=dB_2,\qquad B_2=b(r)J_2+\tilde{b}(r)\tilde{J}_2, \tag{2.8}$$

The general SO(5) invariant $f_\pm$ obeying $df_\pm=H_3\wedge f_\pm$ can be expressed as

$$\begin{aligned}
f_+&=\Big[F_0+c_1J_2+c_2J_2\wedge J_2+c_3\frac{1}{3!}J_2\wedge J_2\wedge J_2+d\Big(p(r)\mathrm{Im}\Omega_3+q(r)\mathrm{Re}\Omega_3\Big)\Big]\wedge e^{B_2},\\
f_-&=d\Big[a_1(r)+a_2(r)J_2+a_3(r)\frac{1}{2}J_2\wedge J_2+a_4(r)J_2\wedge\tilde{J}_2+a_5(r)\frac{1}{3!}J_2\wedge J_2\wedge J_2\Big]\wedge e^{B_2},
\end{aligned} \tag{2.9}$$

giving us an SO(5) invariant ansatz for the flux in IIA/IIB which is valid away from the loci of localised sources.[5] In IIA this depends locally on 4 constants (F_0,c_1,c_2,c_3) and 4 functions of r $(b,\tilde{b},p,q)$ — there is an enhancement to SO(6) when $\tilde{b}=p=q=0$. If we also consider $d(e^{3A}\star_7 f_\pm)=e^{3A}H_3\wedge\star_7 f_\pm$ we find we must in general fix $q=0$. In IIB this depends on 7 functions of r, with an enhancement to SO(6) when $\tilde{b}=a_4=0$.

[5] These need to be generalised in scenarios which allow for sources smeared over all their co-dimensions.

3 Necessary and sufficient conditions for realising supersymmetry

In this section we present the method by which we shall impose supersymmetry on SO(5) invariant ansatz of the previous section.

Geometric conditions for $\mathcal{N}=(1,0)$ AdS_3 solutions with purely magnetic NS flux (ie $h_0=0$) were derived first in massive IIA in [27], then generalised to IIB in [28] with the assumption that $h_0=0$, this assumption was then relaxed in [31] whose conventions we shall follow. These conditions are defined in terms of two non vanishing Majorana spinors $(\hat{\chi}_1,\hat{\chi}_2)$ on the internal M_7 which without loss of generality obey

$$|\hat{\chi}_1|^2+|\hat{\chi}_2|^2=2e^{A},\qquad |\hat{\chi}_1|^2-|\hat{\chi}_2|^2=ce^{-A}, \tag{3.1}$$

for c an arbitrary constant. One can solve these constraints in general in terms of two unit norm spinors (χ_1,χ_2) and a point dependent angle θ as

$$\hat{\chi}_1=e^{\frac{A}{2}}\sqrt{1-\sin\theta}\,\chi_1,\qquad \hat{\chi}_2=e^{\frac{A}{2}}\sqrt{1+\sin\theta}\,\chi_2,\quad c=-2e^{2A}\sin\theta. \tag{3.2}$$

Plugging this into the necessary and sufficient conditions for supersymmetry in [31] (see appendix B therein), we find they become[6]

$$e^{3A}h_0=2me^{2A}\sin\theta,\qquad d(e^{2A}\sin\theta)=0, \tag{3.3a}$$

$$d_{H_3}(e^{2A-\Phi}\cos\theta\Psi_{\mp})=\pm\frac{1}{8}e^{2A}\sin\theta f_{\pm}, \tag{3.3b}$$

$$d_{H_3}(e^{3A-\Phi}\cos\theta\Psi_{\pm})\mp 2me^{2A-\Phi}\cos\theta\Psi_{\mp}=\frac{e^{3A}}{8}\star_7\lambda(f_{\pm}), \tag{3.3c}$$

$$e^{A}(\Psi_{\mp},\ f_{\pm})_7=\mp\frac{m}{2}e^{-\Phi}\cos\theta\,\text{vol}(\text{M}_7), \tag{3.3d}$$

where $(\Psi_{\mp},\ f_{\pm})_7$ is the 7-form part of $\Psi_{\mp}\wedge\lambda(f_{\pm})$ and the real even/odd bi-linears $\Psi_{\pm}$ are defined via

$$\chi_1\otimes\chi_2^{\dagger}=\frac{1}{8}\sum_{n=0}^{7}\frac{1}{n!}\chi_2^{\dagger}\gamma_{a_n\dots a_1}\chi_1 e^{a_1\dots a_n}=\Psi_++i\Psi_- \tag{3.4}$$

for e^a a vielbein on M_7. In the above m is the inverse AdS_3 radius, in particular when $m=0$ we have Mink_3 while when $m\neq 0$ its precise value is immaterial as it can be absorbed into the AdS_3 warp factor, thus going forward we fix

$$m=1 \tag{3.5}$$

without loss of generality.

In this work we will construct explicit solutions preserving (5,0) and (6,0) supersymmetries and for the cases of extended supersymmetry (3.3a)–(3.3d) is not on its own sufficient. If one has $\mathcal{N}=(n,0)$ supersymmetry one has n independent $\mathcal{N}=(1,0)$ sub-sectors

[6] These do not represent a set of necessary and sufficient conditions when $\cos\theta=0$. However as this limit turns off one of $(\hat{\chi}_1,\ \hat{\chi}_2)$ the NS 3-form is the only flux that can be non trivial. The common NS sector of type II supergravity is S-dual to classes of IIB solution with the RR 3-form the only non trivial flux which are contained in the conditions we quote.

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that necessarily come with their corresponding n independent bi-linears $\Psi_{\pm}^{(n)}$. These must all solve (3.3a)–(3.3d) for the same bosonic fields of supergravity. However the AdS_3 vacua we are interested in realise the superconformal algebra $\mathfrak{osp}(n|2)$ which means the internal spinors which define these bi-linears transform in the $\mathbf{n}$ of $\mathfrak{so}(n)$ while the bosonic fields are $\mathfrak{so}(n)$ singlets. Thus the bi-linears decompose into parts transforming in irreducible representations of the tensor product $\mathbf{n}\otimes\mathbf{n}$. Specifically this contains a singlet part that is common to all $\Psi_{\pm}^{(n)}$ and a charged part in the symmetric representation.[7] The charged parts of $\Psi_{\pm}^{(n)}$ are mapped into each other by taking the Lie derivative with respect to the SO(n) Killing vectors, and in particular the bi-linears of a single (1,0) sub-sector + the action of SO(n) is enough to generate the whole set. Then, since the Lie and exterior derivatives commute, it follows that if a single pair of bi-linears, $\Psi_{\pm}^{1}$ say, solve (3.3a)–(3.3d) then they all do.

In summary to know that extended supersymmetry holds on (2.2) it is sufficient to construct an n-tuplet of spinors that transform in the $\mathbf{n}$ of $\mathfrak{so}(n)$, and then solve (3.3a)–(3.3d) for the $\Psi_{\pm}$ following from any $\mathcal{N}=(1,0)$ sub-sector whilst imposing that the bosonic fields are all $\mathfrak{so}(n)$ singlets. In particular this means that we must solve (3.3a)–(3.3d) under the assumption that the warp factors and dilaton only depend on r and the fluxes only depend on r and SO(n) invariant forms. We deal with the bulk of the construction of these SO(n) spinors in appendix A where we construct spinors in relevant representations on $\mathbb{CP}^3$. Below we present the embedding of these spinors into (2.2).

$\mathcal{N}=6$ spinors in $d=7$ can be expressed in terms of 4 real functions of r (f_1,f_2,g_1,g_2) and the spinors in the **6** of SO(6) on $\mathbb{CP}^3$ in (A.23)

$$\chi_1^{\mathcal{I}}=\cos\left(\frac{\beta_1+\beta_2}{2}\right)\xi_6^{\mathcal{I}}+i\sin\left(\frac{\beta_1+\beta_2}{2}\right)\gamma_7\xi_6^{\mathcal{I}},$$
$$\chi_2^{\mathcal{I}}=\cos\left(\frac{\beta_1-\beta_2}{2}\right)\xi_6^{\mathcal{I}}+i\sin\left(\frac{\beta_1-\beta_2}{2}\right)\gamma_7\xi_6^{\mathcal{I}}, \tag{3.6}$$

where $\mathcal{I}=1,\dots,6$ and $\beta_{1,2}=\beta_{1,2}(r)$. These are only valid on round $\mathbb{CP}^3$, ie when $e^{2B}=e^{2C}$ and the fluxes depend on $\mathbb{CP}^3$ through the SO(6) invariant 2-form J_2. We will not actually make explicit use of these spinors as it turns out that general class of $\mathcal{N}=(6,0)$ is actually simply one of 2 branching classes of solution following from the $\mathcal{N}=(5,0)$ spinors below.

$\mathcal{N}=5$ spinors in $d=7$ can be decomposed in terms the spinors in the **5** of SO(5) on $\mathbb{CP}^3$ in (A.21) and 4 constraints as

$$\chi_1^{\alpha}=a_{11}(\xi_5^{\alpha}+Y_{\alpha}i\gamma_7\xi_0)+b_{11}(i\gamma_7\xi_5^{\alpha}-Y_{\alpha}\xi_0)+a_{12}Y_{\alpha}\xi_0+ib_{12}Y_{\alpha}\gamma_7\xi_0,$$
$$\chi_2^{\alpha}=a_{21}(\xi_5^{\alpha}+Y_{\alpha}i\gamma_7\xi_0)+b_{21}(i\gamma_7\xi_5^{\alpha}-Y_{\alpha}\xi_0)+a_{22}Y_{\alpha}\xi_0+ib_{22}Y_{\alpha}\gamma_7\xi_0,$$
$$a_{11}^2+b_{11}^2=a_{12}^2+b_{12}^2=a_{21}^2+b_{21}^2=a_{22}^2+b_{22}^2=1. \tag{3.7}$$

where $\alpha=1,\dots,5$, the 8 parameters $a_{11},b_{11},\dots$ are all real and have support on r alone, we have parameterised things in this fashion to make the unit norm constraints simple. These spinors are valid for squashed $\mathbb{CP}^3$.

[7] Of course $\mathbf{n}\otimes\mathbf{n}$ decomposes into singlet, symmetric traceless and anti-symmetric representations, however to see the anti-symmetric representation one would need to construct bi-linears that mix the internal spinors χ_1 and χ_2 that belong to different (1,0) sub-sectors — this is not needed for our purposes.

Finally a set of $\mathcal{N}=1$ spinors can also be defined in $d=7$, they are given by

$$\chi_1^{(0)}=\cos\left(\frac{\beta_1+\beta_2}{2}\right)\xi_0+i\sin\left(\frac{\beta_1+\beta_2}{2}\right)\gamma_7\xi_0,$$
$$\chi_2^{(0)}=\cos\left(\frac{\beta_1-\beta_2}{2}\right)\xi_0+i\sin\left(\frac{\beta_1-\beta_2}{2}\right)\gamma_7\xi_0, \tag{3.8}$$

where again $\beta_{1,2}=\beta_{1,2}(r)$ and these spinors are valid on squashed $\mathbb{CP}^3$. The 0 superscript refers to the fact that these are $\text{SO}(5)\subset\text{SO}(6)$ singlets. These are in fact nothing more than the 6th component of (3.6), however unlike the $\mathcal{N}=(6,0)$ case $e^{2C}\neq e^{2B}$ and the flux can depend on more than merely r and J_2. These spinors can be used to construct $\mathcal{N}=(1,0)$ AdS_3 solutions with SO(5) flavour symmetry, something we will report on elsewhere [64].

4 Classification of $\mathfrak{osp}(n|2)$ AdS_3 vacua on $\widehat{\mathbb{CP}}^3$ for $n=5,6$

In this section we classify AdS_3 solutions preserving $\mathcal{N}=(5,0)$ supersymmetry on squashed $\mathbb{CP}^3$. Such solutions only exist in type IIA supergravity and experience an enhancement to $\mathcal{N}=(6,0)$ when a function is appropriately fixed. We summarise our results between (4.22) and (4.25).

We take our representative $\mathcal{N}=1$ sub-sector to be

$$\chi_1=\chi_1^5,\qquad \chi_2=\chi_2^5, \tag{4.1}$$

which has the advantage that the bi-linears decompose in terms of the $\text{SO}(3)_L\times\text{SO}(3)_D$ invariant forms on the $\text{S}^2\times\text{S}^3$ fibration. We find the $d=7$ bi-linears are given by

$$\Psi_+=(\underline{\mathcal{S}}^1)_+.(\underline{\phi})_++e^k(\underline{\mathcal{S}}^2)_-.(\underline{\phi})_-\wedge dr,\qquad \Psi_-=(\underline{\mathcal{S}}^1)_-.(\underline{\phi})_-+e^k(\underline{\mathcal{S}}^2)_+.(\underline{\phi})_+\wedge dr \tag{4.2}$$

where we define

$$(\underline{\phi})_\pm=(\phi^1_\pm,\ \phi^2_\pm,\ Y_5\phi^3_\pm,\ Y_5\phi^4_\pm,\ Y_5\phi^5_\pm,\ Y_5\phi^6_\pm,\ Y_5^2\phi^7_\pm,\ Y_5^2\phi^8_\pm),\qquad Y_5=\cos\alpha, \tag{4.3}$$

for $\phi^1_\pm$ real even/odd bi-linears on $\widehat{\mathbb{CP}}^3$ decomposing in a basis of (2.3) — their explicit form is given in (B.3). We also define

$$(\underline{\mathcal{S}}^1)_+=\begin{pmatrix}a_{11}a_{21}+b_{11}b_{21}\\ a_{11}b_{21}-a_{21}b_{11}\\ a_{11}a_{22}+b_{11}b_{22}\\ a_{11}b_{22}-a_{22}b_{11}\\ a_{12}a_{21}+b_{12}b_{21}\\ a_{21}b_{12}-a_{12}b_{21}\\ a_{12}a_{22}+b_{12}b_{22}\\ a_{22}b_{12}-a_{12}b_{22}\end{pmatrix},\qquad (\underline{\mathcal{S}}^2)_-=\begin{pmatrix}a_{21}b_{11}+a_{11}b_{21}\\ b_{11}b_{22}-a_{11}a_{21}\\ a_{11}b_{22}+a_{22}b_{11}\\ b_{11}b_{22}-a_{11}a_{22}\\ a_{21}b_{12}+a_{12}b_{21}\\ b_{12}b_{21}-a_{12}a_{21}\\ a_{22}b_{12}+a_{12}b_{22}\\ b_{12}b_{22}-a_{12}a_{22}\end{pmatrix},$$

$$(\underline{\mathcal{S}}^1)_- = \begin{pmatrix} a_{21}b_{11}-a_{11}b_{21} \\ a_{11}a_{21}+b_{11}b_{21} \\ a_{22}b_{11}-a_{11}b_{22} \\ a_{11}a_{22}+b_{11}b_{22} \\ a_{21}b_{12}-a_{12}b_{21} \\ -a_{12}a_{21}-b_{12}b_{21} \\ a_{22}b_{12}-a_{12}b_{22} \\ -a_{12}a_{22}-b_{12}b_{22} \end{pmatrix}, \qquad (\underline{\mathcal{S}}^2)_+ = \begin{pmatrix} a_{11}a_{21}-b_{11}b_{21} \\ a_{21}b_{11}+a_{11}b_{21} \\ a_{11}a_{22}-b_{11}b_{22} \\ a_{22}b_{11}+a_{11}b_{22} \\ a_{12}a_{21}-b_{12}b_{21} \\ a_{21}b_{12}+a_{12}b_{21} \\ a_{12}a_{22}-b_{12}b_{22} \\ a_{22}b_{12}+a_{12}b_{22} \end{pmatrix}. \tag{4.4}$$

We begin by solving the constraints in (3.7) by parametrising the functions of the spinor ansatz as

$$\begin{aligned} a_{11}+ib_{11} &= e^{\frac{i}{2}(X_1+X_3)}, \qquad a_{12}+ib_{12} = e^{\frac{i}{2}(X_2+X_4)}, \\ a_{21}+ib_{21} &= e^{\frac{i}{2}(X_1-X_3)}, \qquad a_{22}+ib_{22} = e^{\frac{i}{2}(X_2-X_4)}, \end{aligned} \tag{4.5}$$

for $X_1 \dots X_4$ functions of r only. We shall take the magnetic component of the NS 3-form as in (2.8) and allow the RR fluxes to depend on r and all the SO(5) invariant forms, ie

$$dr, \quad J_2, \quad \tilde{J}_2, \quad \text{Re}\Omega_3, \quad \text{Im}\Omega_3, \tag{4.6}$$

and the wedge products one can form out of these. One then proceed to substitute (4.2) into the necessary conditions for supersymmetry (3.3a)–(3.3d) to fix the r dependence of the ansatz.

In IIB supergravity there are no solutions: one arrives at a set of algebraic constraints by solving for the parts of (3.3b)–(3.3c) orthogonal to dr which without loss of generality fix the phases as

$$X_1 = -\frac{\pi}{2} + 2\beta(r), \qquad X_2 = \frac{\pi}{2}, \qquad \theta = X_3 = X_4 = 0. \tag{4.7}$$

and several parts of the metric and NS 2-form as

$$e^C = 5e^A \sin\beta, \qquad e^D = 5e^A \sin\beta\cos\beta, \qquad \tilde{b} = 0. \tag{4.8}$$

Unfortunately if one then tries to solve the dr dependent terms in (3.3b) one finds the constraint

$$\cos\beta = 0, \tag{4.9}$$

which cannot be solved without setting $e^D = 0$, so no $\mathcal{N} = (5,0)$ or $(6,0)$ solutions exist on this space in type IIB.

Moving onto type IIA supergravity: some conditions one may extract from (3.3b), which simplify matters considerably going forward, are the following

$$\sin\theta = 0, \qquad \sin X_1 = -\sin X_2 = 1, \tag{4.10}$$

which we can solve without loss of generality as

$$\theta = 0 \quad \Rightarrow \quad h_0 = 0, \qquad X_1 = -X_2 = \frac{\pi}{2}. \tag{4.11}$$

We then choose to further refine the phases as

$$X_3 = \beta_1 + \beta_2, \qquad X_4 = -\beta_1 + \beta_2. \tag{4.12}$$

Plugging these into (3.3b)–(3.3d) we find the following simple definitions of various functions in the ansatz

$$\begin{aligned} e^C &= 2e^A \sin\beta_2, & e^D &= 2e^A \sin(\beta_1+\beta_2),\\ b' &= 4e^{A+k} + 2\partial_r(e^{2A}\cos\beta_1\sin(\beta_1+2\beta_2)), & \tilde{b} &= -2e^{2A}\cos(\beta_1+2\beta_2)\sin\beta_1, \end{aligned} \tag{4.13}$$

and the following ODEs that need to be actively solved

$$\begin{aligned} \partial_r(e^{3A-\Phi}\sin\beta_1\sin\beta_2) + 2me^{2A+k-\Phi}\sin\beta_1\cos\beta_2 &= 0,\\ \partial_r(e^{5A-\Phi}\sin^2\beta_2\sin(\beta_1+\beta_2)) + me^{4A+k-\Phi}\sin\beta_2\sin(\beta_1+2\beta_2) &= 0,\\ \partial_r(e^{2A}\tan\beta_2) + me^{A+k}(2\tan\beta_2\cot(\beta_1+\beta_2) - (\cos\beta_2)^{-2}) &= 0. \end{aligned} \tag{4.14}$$

We also extract expressions for the RR fluxes, though we delay presenting them explicitly until we have simplified the above. To make progress we find it useful to use diffeomorphism invariance in r to fix[8]

$$e^{A+k} = -\pi, \tag{4.15}$$

and introduce local functions of r (h, g, u) such that

$$e^{5A-\Phi}\sin^2\beta_2\sin(\beta_1+\beta_2) = \pi^2 h, \qquad e^{2A}\tan\beta_2 = g, \qquad \frac{\tan\beta_2}{\tan(\beta_1+\beta_2)} = \frac{h' + h\frac{u'}{u}}{h' - h\frac{u'}{u}}. \tag{4.16}$$

This simplifies the system of ODEs in (4.14) to

$$\begin{aligned} &u'' = 0, \quad g = 2\pi\frac{hu}{uh'-hu'}, \quad \tan\beta_1 = \frac{\text{sign}(uh)u'\sqrt{\Delta_1}}{u'(uh'-hu')+u^2h''}, \quad \tan\beta_2 = \frac{\text{sign}(uh)\sqrt{\Delta_1}}{uh'-hu'}\\ &\Delta_1 = 2hh''u^2 - (uh'-hu')^2. \end{aligned} \tag{4.17}$$

which imply supersymmetry and require $\Delta_1 > 0$. What remains is the explicit form of the magnetic components of the RR fluxes. These can be expressed most succinctly in terms of their Page flux avatars, ie $\hat{f}_+ = f_+ \wedge e^{-B_2}$, however to compute these we must first integrate b'. Combining (4.16), (4.17) and (4.13) we find

$$b = -\tilde{b} + 4\pi\left(-(r-k) + \frac{uh'-hu'}{uh''}\right), \qquad \tilde{b} = -2\pi\frac{u'}{h''}\left(\frac{h}{u} + \frac{hh''-2(h')^2}{2h'u'+uh''}\right), \tag{4.18}$$

where k is an integration constant. We then find for the magnetic Page fluxes

$$\begin{aligned} \hat{f}_0 &= F_0 = -\frac{1}{2\pi}h''',\\ \hat{f}_2 &= 2(h''-(r-k)h''')J, \end{aligned}$$

[8] The reason for the factors of π, taken here and elsewhere without loss of generality, is that they make the Page charges of the RR fluxes simple.

$$\hat{f}_4 = -4\pi\left[(2h' + (r-k)(-2h'' + (r-k)h'''))J_2\wedge J_2 + d\left(\frac{hu'}{u}\text{Im}\Omega_3\right)\right],$$

$$\hat{f}_6 = \frac{16\pi^2}{3}(6h - (r-k)(6h' + (r-k)(-3h'' + (r-k)h''')))J_2\wedge J_2\wedge J_2, \tag{4.19}$$

where we have made extensive use of the conditions derived earlier to simply these expressions. In order to have a solution we must impose that Bianchi identities of the RR flux hold (that of the NS 3-form is implied), away from sources this is equivalent to imposing that $d\hat{f}_{2n} = 0$ for $n = 0,1,2,3$, we find

$$d\hat{f}_{2n} = -\frac{1}{2\pi}(4\pi)^n\frac{1}{n!}(r-k)^n h''''dr\wedge J_2^n, \tag{4.20}$$

which tells us that the Bianchi identities in regular parts of the internal space demand

$$h'''' = 0, \tag{4.21}$$

or in other words that h is an order 3 polynomial (at least locally). This completes our local derivation of the class of solutions.

In summary the local form of solutions in this class take the following form: NS sector

$$\frac{ds^2}{2\pi} = \frac{|hu|}{\sqrt{\Delta_1}}ds^2(\text{AdS}_3) + \frac{\sqrt{\Delta_1}}{4|u|}\left[\frac{2}{|h''|}\left(ds^2(\text{S}^4) + \frac{1}{\Delta_2}(Dy_i)^2\right) + \frac{1}{|h|}dr^2\right],$$

$$e^{-\Phi} = \frac{\sqrt{|u|}|h''|^{\frac{3}{2}}\sqrt{\Delta_2}}{2\sqrt{\pi}\Delta_1^{\frac{1}{4}}},\qquad \Delta_1 = 2hh''u^2 - (uh' - hu')^2,\qquad \Delta_2 = 1 + \frac{2h'u'}{uh''},\qquad H = dB_2,$$

$$B_2 = 4\pi\left[\left(-(r-k) + \frac{uh' - hu'}{uh''}\right)J_2 + \frac{u'}{2h''}\left(\frac{h}{u} + \frac{hh'' - 2(h')^2}{2h'u' + uh''}\right)\left(J_2 - \tilde{J}_2\right)\right], \tag{4.22}$$

where (u,h) are functions of r and k is a constant. Note that positivity of the metric and dilaton holds whenever $\Delta_1 \geq 0$.[9] The $d = 10$ RR fluxes are given by

$$F_0 = -\frac{1}{2\pi}h''',\qquad F_2 = B_2F_0 + 2(h'' - (r-k)h''')J_2,$$

$$F_4 = \pi d\left(h' + \frac{hh''u(uh' + hu')}{\Delta_1}\right)\wedge\text{vol}(\text{AdS}_3) + B_2\wedge F_2 - \frac{1}{2}B_2\wedge B_2F_0$$

$$-4\pi\left[(2h' + (r-k)(-2h'' + (r-k)h'''))J_2\wedge J_2 + d\left(\frac{hu'}{u}\text{Im}\Omega_3\right)\right]. \tag{4.23}$$

Solutions within this class are defined locally by 2 ODEs: first supersymmetry demands

$$u'' = 0, \tag{4.24}$$

which must hold globally. Second the Bianchi identities of the fluxes demand that in regular regions of the internal space

$$h'''' = 0, \tag{4.25}$$

[9] Specifically reality of the metric demands $\Delta_1 \geq 0$ and that (h,u) are real. This in turn implies $hh'' \geq 0$ and it then follows that $\Delta_2 \geq 0$. Note that one needs to use that $hh'' \geq 0$ to bring the metric to this form.

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which one can integrate as

$$h = c_0 + c_1 r + \frac{1}{2} c_2 r^2 + \frac{1}{3!} c_3 r^3, \tag{4.26}$$

where c_i are integration constants here and elsewhere. However the r.h.s. of (4.25) can contain δ-function sources globally, as we shall explore in section 6. Before moving on to analyse solutions within this class it is important to stress a few things. First off that (4.24) must hold globally, and given how u, u' appear in the class means that really u is parametrising two branching possibilities — either $u' = 0$ or $u' \neq 0$.

For the first case notice that if $u' = 0$ then u actually completely drops out of the bosonic fields, so its precise value doesn't matter. Further the warping of the 4-sphere and fibered 2-sphere becomes equal, making the metric on $\mathbb{CP}^3$ the round one, and only J_2 now appears in the fluxes. There is thus an enhancement of the global symmetry of the internal space to SO(6) — indeed supersymmetry is likewise enhanced to $\mathcal{N} = (6,0)$. We shall study this limit in section 5.1.

When $u' \neq 0$ then u is an order 1 polynomial, however the class is invariant under $(r \to r + l,\ k \to k + l)$ which one can use to set the constant term in u to zero without loss of generality, the specific value of the constant u' then also drops out of the bosonic fields. Thus for the second class, preserving only $\mathcal{N} = (5,0)$ supersymmetry, one can fix $u = r$ without loss of generality. We shall study this limit in section 5.2.

Having a classes of solutions defined in terms of the ODE $h''' = -2\pi F_0$ is very reminiscent of AdS_7 vacua in massive IIA, which obey essentially the same constraint [55]. For the (6,0) case the formal similarities become more striking as both this and the AdS_7 vacua are of the form $\text{AdS}_{2p+1} \times \mathbb{CP}^{4-p}$ foliated over an interval in terms of an order 3 polynomial and it's derivatives — however we should stress these functions do not appear in the same way in each case. None the less this apparent series of local solutions does beg the question, what about $\text{AdS}_5 \times \mathbb{CP}^2$?. Establishing whether this also exists, and how much if any supersymmetry it may preserve, is beyond the scope of this work but would be interesting to pursue.

5 Local analysis of $\mathfrak{osp}(n|2)$ vacua for $n > 4$

In this section we perform a local analysis of the $\mathfrak{osp}(6|2)$ and $\mathfrak{osp}(5|2)$ AdS_3 vacua derived in the previous sections in 5.1 and 5.2. We begin with some comments about the non existence of $\mathfrak{osp}(n|2)$ AdS_3 for $n > 6$ and on the generality of classes we do find.

In the previous section we derived classes of solutions in IIA supergravity that realise the super conformal algebras $\mathfrak{osp}(n|2)$ for $n = 5, 6$ on a warped product space consisting of a foliation of $\text{AdS}_3 \times \widehat{\mathbb{CP}}^3$ foliated over an interval such that either SO(5) or SO(6) is preserved. In appendix C we prove that the case of $n = 7$ is locally $\text{AdS}_4 \times \text{S}^7$, and the same is proved for the case of $n = 8$ in [51]. Thus one only has true AdS_3 solutions for $n = 5, 6$ (or lower) and we have found them only in type IIA.

The class of $\mathfrak{osp}(6|2)$ AdS_3 vacua we find are exhaustive for type II supergravities: spinors transforming in the **6** of $\mathfrak{so}(6)$ necessitate either a round $\mathbb{CP}^3$ factor in the metric or $\mathbb{CP}^3$ with a U(1) fibered over it. This latter possibility can easily be excluded at the level of the equations of motion [51]. For the case of $\mathfrak{osp}(5|2)$ AdS_3 vacua we suspect the same is true, but are not completely certain of that.

5.1 Analysis of $\mathfrak{osp}(6|2)$ vacua

The $\mathfrak{osp}(6|2)$ AdS_3 solutions are given by the class of the previous section specialised to the case $u = 1$. We find for the NS sector

$$\begin{aligned}\frac{ds^2}{2\pi} &= \frac{|h|}{\sqrt{2hh'' - (h')^2}} ds^2(\text{AdS}_3) + \sqrt{2hh'' - (h')^2}\left[\frac{1}{4|h|}dr^2 + \frac{2}{|h''|}ds^2(\mathbb{CP}^3)\right],\\ e^{-\Phi} &= \frac{(|h''|)^{\frac{3}{2}}}{2\sqrt{\pi}(2hh'' - (h')^2)^{\frac{1}{4}}}, \qquad H = dB_2, \qquad B_2 = 4\pi\left(-(r-k) + \frac{h'}{h''}\right)J_2,\end{aligned} \tag{5.1}$$

and the RR sector

$$\begin{aligned}F_0 &= -\frac{1}{2\pi}h''', \qquad F_2 = B_2F_0 + 2(h'' - (r-k)h''')J_2,\\ F_4 &= \pi d\left(h' + \frac{hh'h''}{2hh'' - (h')^2}\right)\wedge \text{vol}(\text{AdS}_3) + B_2\wedge F_2 - \frac{1}{2}B_2\wedge B_2F_0\\ &\quad - 4\pi(2h' + (r-k)(-2h'' + (r-k)h'''))J_2\wedge J_2,\end{aligned} \tag{5.2}$$

where k is a constant and h is a function of r obeying $h'''' = 0$ in regular regions of a solution.

5.1.1 Local solutions and regularity

There are several distinct physical behaviours one can realise locally by solving $h''' = -2\pi F_0$ (for $F_0 =$ constant) in different ways, in this section we shall explore them.

The distinct local solutions the class contains can be characterised as follows. First off the domain of r should be ascertained, in principle it can be one of the following: periodic, bounded (from above and below), semi infinite or unbounded. For a well defined AdS_3 vacua dual to a $d = 2$ CFT r must be one of the first 2. In the case at hand the r dependence of h does not allow for periodic r so we seek bounded solutions. In general a solution can be bounded by either a regular zero or a physical singularity.

At a regular zero we must have that $e^{-\Phi}$ and the AdS warp factor becomes constant. The internal space should then decompose as a direct product of two sub manifolds with the first tending to the behaviour of a Ricci flat cone of radius r and the second r independent.

There are many ways to realise physical singularities that bound the space at some loci. The most simple is with D branes and O-planes: for a generic solution these objects are characterised by a metric and dilaton which decompose as

$$ds^2 = \frac{1}{\sqrt{h_p}}ds^2_{\parallel} + \sqrt{h_p}ds^2_{\perp}, \qquad e^{-\Phi} \propto h_p^{\frac{p-3}{4}} \tag{5.3}$$

where $ds^2_{\parallel}$ is the $p+1$ dimensional metric on the world volume of this object and $ds^2_{\perp}$ is the $9-p$ dimensional metric on its co-dimensions. We will consider only solutions whose

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metric is a foliation over an interval r. A Dp brane singularity (for $p<7$) is then signaled by the leading order behaviour

$$ds^2_\perp \propto dr^2 + r^2 ds^2(\text{B}^{8-p}), \quad h_p \propto \frac{1}{r^{7-p}} \tag{5.4}$$

for B^{8-p} the base of a Ricci flat cone (for instance $\text{B}^{8-p} = \text{S}^{8-p}$). The case of $p=8$ is different as while the solution is singular at such a loci, the metric neither blows up nor tends to zero so a D8 brane does not bound the solution ($p=7$ will not be relevant to our analysis). The Op plane singularity (for $p\neq 7$) on the other hand yields

$$ds^2_\perp \propto dr^2 + \alpha_0^2 ds^2(\text{B}^{8-p}), \quad h_p \propto r, \tag{5.5}$$

for α_0 some constant. Our task then is to first establish which of these behaviours can be realised by the class of solutions in this section, then whether any of these behaviours can coexist in the same local solution. Let us reiterate: D-brane and O-plane singularities do not exhaust the possible physical singularities, indeed we will find a more complicated object later in this section which we will describe when it becomes relevant.

The most obvious thing one can try is to fix $F_0 = h''' = 0$, it is not hard to see that one then has $H_3 = 0$ and F_4 becomes purely electric. One can integrate h as

$$h = c_1 + rc_2 + \frac{1}{2}\tilde{k}r^2 \tag{5.6}$$

and then upon making the redefinitions

$$c_1 = \frac{\tilde{k}L^4 + c_2^2\pi^2}{2\tilde{k}\pi^2}, \qquad -r = \frac{c_2}{\tilde{k}^2} + \frac{L^2}{\pi}\sinh x \tag{5.7}$$

one finds the solution is mapped to

$$\begin{aligned} \frac{ds^2}{L^2} &= \Big(\cosh^2 x ds^2(\text{AdS}_3) + dx^2\Big) + 4ds^2(\mathbb{CP}^3), \qquad e^{-\Phi} = \frac{\tilde{k}}{2L}, \\ F_2 &= 2\tilde{k}J_2, \qquad\qquad F_4 = \frac{3}{2}\tilde{k}L^2\cosh^3 x \text{vol}(\text{AdS}_3)\wedge dx. \end{aligned} \tag{5.8}$$

This is of course $\text{AdS}_4\times\mathbb{CP}^3$, dual to $\mathcal{N}=6$, $\text{U(N)}_{\tilde{k}}\times\text{U(N)}_{-\tilde{k}}$ Chern-Simons matter theory (where $N = 2\tilde{k}L^4$) [53]. Thus there is only one local solution when $F_0 = 0$ and it is an AdS_4 vacua preserving twice the supersymmetries of generic solutions within this class. This is the only regular solution preserving (6,0) supersymmetry.

Next we consider the sort of physical singularities the metric and dilaton in (5.1) can support for $F_0 \neq 0$. At the loci of such singularities the space terminates so the interval spanned by r becomes bounded at one end. We shall use diffeomorphism invariance to assume this bound is at $r=0$.

First off $\Delta_1 = 2hh'' - (h')^2$ appears in the metric and dilaton where one would expect the warp factor of a co-dimension 7 source to appear. Thus if Δ_1 has an order 1 zero at a loci where h, h'' have no zero one has the behaviour of O2 planes extended in AdS_3 at the tip of a G_2 cone over $\mathbb{CP}^3$. We now choose the loci of this O2 plane to be $r=0$, meaning that the constant part of Δ_1 has to vanish which forces

$$\text{O2 at } r=0:\; h=c_1+c_2 r+\frac{c_2^2}{4c_1}r^2+\frac{1}{3!}c_3 r^3, \quad c_1,c_2,c_3\neq 0, \tag{5.9}$$

where $r\in\mathbb{R}^{\pm}$ when $\text{sign}(c_1c_4)=\pm 1$.

Another type of singularity this solution is consistent with is a D8/O8 system of world volume $\text{AdS}_3\times\mathbb{CP}^3$. Such a singularity is characterised by O8 brane like behaviour in the metric and dilaton. We realise this behaviour by choosing h such that (h'',Δ_1) both have an order 1 zero at a loci where h has no zero. After using diffeomorphism invariance to place the D8/O8 at $r=0$ this is equivalent to taking h as

$$\text{D8/O8 at } r=0:\; h=c_1+\frac{1}{3!}c_2 r^3, \quad c_{1,2}\neq 0., \tag{5.10}$$

where $r\in\mathbb{R}^{\pm}$ for $\text{sign}(c_1c_2)=\pm 1$.

We are yet to find a D brane configuration, given that we have a $\mathbb{CP}^3$ factor the obvious thing to naively aim for is D2 branes at the tip of a G_2 cone similar to the O2 plane realised above. However the warp factor of a D2 brane blows up like $\lim_{r\to 0} r^{-5}$ its loci, and this is not possible to achieve for (5.1) such that h is an order 3 polynomial. It is however possible to realise a more exotic object: it is well known that if one takes $d=11$ supergravity on the orbifold $\mathbb{R}^{1,6}\times\mathbb{C}^2/\mathbb{Z}_{\tilde{k}}$ then reduces on the Hopf fibre of the Lens space (equivalently squashed 3-sphere) inside $\mathbb{C}^2/\mathbb{Z}_{\tilde{k}}$ one generates a D6 brane singularity in type IIA. One can generate the entire flat space D6 brane geometry by replacing $\mathbb{C}^2/\mathbb{Z}_{\tilde{k}}$ in the above with a Taub-Nut space and likewise reducing to IIA. One can perform an analogous procedure for $\mathbb{R}^{1,2}\times\mathbb{C}^4/\mathbb{Z}_{\tilde{k}}$, reducing this time on the Hopf fibration (over $\mathbb{CP}^3$) of a squashed 7-sphere. The resulting solution in IIA takes the from

$$ds^2=\frac{\sqrt{r}}{\tilde{k}}ds^2(\text{Mink}_3)+\frac{1}{4\tilde{k}\sqrt{r}}\Big(dr^2+4r^2ds^2(\mathbb{CP}^3)\Big), \quad e^{-\Phi}=\frac{\tilde{k}^{\frac{3}{2}}}{r^{\frac{3}{4}}}, \quad F_2=2\tilde{k}J_2, \quad r\geq 0, \tag{5.11}$$

and is singular at $r=0$. Notice that the r dependence of the dilaton and metric is the same as one gets at the loci of flat space D6 branes, but the co-dimensions no longer span a regular cone as they do in that case, indeed $dr^2+cr^2ds^2(\mathbb{CP}^3)$ is only Ricci flat for unit radius round $\mathbb{CP}^3$ when $c=\frac{2}{5}$. It is argued in [53] that the singularity in (5.11) corresponds to a coincident combination of a KK monopole and $\tilde{k}$ D6 branes (T-dual of an $(1,\tilde{k})-5$ brane) that partially intersect another KK monopole. For simplicity we shall refer to this rather complicated composite object as a $\widetilde{\text{D6}}$ brane. We can find the behaviour of this object, now extended in AdS_3, within this class — assuming it is located at $r=0$ one need only tune

$$\widetilde{\text{D6}} \text{ at } r=0:\; h=r^2(c_1+c_2 r), \quad c_{1,2}\neq 0, \tag{5.12}$$

with the caveat that as this only exists when $F_0\neq 0$, we can no longer lift to $d=11$. Again $r\in\mathbb{R}^{\pm}$ for $\text{sign}(c_1c_2)=\pm 1$.

The above exhausts the **physical** singularities we are able to identify, we do however find one final further singularity. By tuning $h=cr^3$ for $r,c>0$, the metric and dilaton then become

$$ds^2=\frac{\pi}{2\sqrt{3}r}\left(4r^2\left(ds^2(\text{AdS}_3)+ds^2(\mathbb{CP}^3)\right)+3dr^2\right),\quad e^{-\Phi}=\frac{3\sqrt{2}3^{\frac{1}{4}}c}{\sqrt{\pi}}\sqrt{r} \tag{5.13}$$

which is singular about $r=0$ in a way we do not recognise.

All the previously discussed physical singularities bound the interval spanned by r at one end. In order to have a true AdS$_3$ vacuum we need to bound a solution between 2 of them separated by a finite proper distance. However assuming that the space starts at $r=0$ with any of an O2, D8/O8 or $\widetilde{\text{D6}}$ singularity, we find that none of the warp factors appearing in metric or dilaton (5.1), (ie (h,Δ_1,h'')) either blow up or vanish until $r\to\infty$. For each case (assuming $r\geq 0$ for simplicity) the metric and dilaton as $r\to\infty$ tend to (5.13). By computing the curvature invariants it is possible to show that the metric is actually flat at this loci, hence it tends to Mink$_{10}$, however as $e^{\Phi}\to\infty$ the solution is still singular. Worse still the singularity as $r\to\infty$ is at infinity proper distance from $r=0$, so in all cases the internal space is semi infinite.

Naively one might now conclude that there are no $\mathfrak{osp}(6|2)$ AdS$_3$ vacua with bounded internal space (true vacua), however as we will show explicitly in section 6, this is not the case. The missing ingredient is the inclusion of D8 branes on the interior of r which allow one to glue local solutions of $h''''=0$, depending on different integration constants, together.

5.2 Analysis of $\mathfrak{osp}(5|2)$ vacua

The $\mathfrak{osp}(5|2)$ AdS$_3$ solutions are given by the class of section 4 for $u'\neq 0$, one can use diffemorphism invariance to fix $u=r$ for such solutions without loss of generality. The resulting NS sector takes the form

$$\begin{aligned}
\frac{ds^2}{2\pi}&=\frac{|hr|}{\sqrt{\Delta_1}}ds^2(\text{AdS}_3)+\frac{\sqrt{\Delta_1}}{4|r|}\left[\frac{2}{|h''|}\left(ds^2(\text{S}^4)+\frac{1}{\Delta_2}(Dy_i)^2\right)+\frac{1}{|h|}dr^2\right],\\
e^{-\Phi}&=\frac{|h''|^{\frac{3}{2}}\sqrt{|r|}\sqrt{\Delta_2}}{2\sqrt{\pi}\Delta_1^{\frac{1}{4}}},\qquad \Delta_1=2hh''r^2-(rh'-h)^2,\qquad \Delta_2=1+\frac{2h'}{rh''},\qquad H=dB_2,\\
B_2&=4\pi\left[\left(-(r-k)+\frac{rh'-h}{rh''}\right)J_2+\frac{1}{2h''}\left(\frac{h}{r}+\frac{hh''-2(h')^2}{2h'+rh''}\right)\left(J_2-\tilde{J}_2\right)\right],
\end{aligned} \tag{5.14}$$

while the $d=10$ RR fluxes are then given by

$$\begin{aligned}
F_0&=-\frac{1}{2\pi}h''',\qquad F_2=B_2F_0+2(h''-(r-k)h''')J_2,\\
F_4&=\pi d\left(h'+\frac{hh''r(rh'+h)}{\Delta_1}\right)\wedge\text{vol}(\text{AdS}_3)+B_2F_2-\frac{1}{2}B_2\wedge B_2F_0\\
&\quad-4\pi\left[(2h'+(r-k)(-2h''+(r-k)h'''))J_2\wedge J_2+d\left(\frac{h}{r}\text{Im}\Omega_3\right)\right],
\end{aligned} \tag{5.15}$$

where h is defined as before and k is a constant.

5.2.1 Local solutions and regularity

In this section we will explore the physically distinct local $\mathfrak{osp}(5|2)$ solution that follow from solving $h'''=-2\pi F_0$ in various ways.

Let us begin by commenting on the massless limit $F_0=h'''=0$: unlike the class of $\mathfrak{osp}(6|2)$ solutions the result is no longer locally $\text{AdS}_4\times\mathbb{CP}^3$ — it is instructive to lift the class to $d=11$. We find the metric of the solution can be written as[10]

$$\frac{ds^2_{11}}{2^{\frac{1}{3}}\pi^{\frac{2}{3}}|h''|}=\Delta_2^{\frac{1}{3}}\left[\frac{|h||r|^{\frac{4}{3}}}{\Delta_1^{\frac{2}{3}}}ds^2(\text{AdS}_3)+\frac{\Delta_1^{\frac{1}{3}}}{|r|^{\frac{2}{3}}}\left(\frac{1}{4|h|}dr^2+\frac{2}{|h''|}ds^2(\widehat{\text{S}}^7)\right)\right],$$
$$ds^2(\widehat{\text{S}}^7)=\frac{1}{4}\left[ds^2(\text{S}^4)+\frac{1}{\Delta_2}\left(L_2^i-\cos^2\left(\frac{\alpha}{2}\right)L_1^i\right)^2\right], \tag{5.16}$$

where $L^i_{1,2}$ are two sets of SU(2) left invariant forms defined as in appendix (A.2), so that the internal space is a foliation of an SP(2)×SP(1) preserving squashed 7-sphere over an interval. This enhancement of symmetry is also respected by the $d=11$ flux, to see this we need to define the SP(2)×SP(1) invariant form on this squashed 7-sphere, fortuitously these were already computed in [51], they are

$$\Lambda^0_3=\frac{1}{8}(L^1_2+\mathcal{A}^1)\wedge(L^2_2+\mathcal{A}^2)\wedge(L^3_2+\mathcal{A}^3),\quad \tilde{\Lambda}^0_3=\frac{1}{8}(L^i_2+\mathcal{A}^i)\wedge(d\mathcal{A}^i+\frac{1}{2}\epsilon^i_{jk}\mathcal{A}^j\wedge\mathcal{A}^k), \tag{5.17}$$

where $\mathcal{A}^i=-\cos^2\left(\frac{\alpha}{2}\right)L^i_1$, and their exterior derivatives. One can then show that the $d=11$ flux decompose as

$$\frac{G_4}{\pi}=d\left(h'+\frac{rhh''(rh'+h)}{\Delta_1}\right)\wedge\text{vol}(\text{AdS}_3)+4d\bigg(\frac{2(r(h')^2-h(h'+rh''))}{r(2h'+rh'')}\Lambda^0_3+\frac{h}{r}(\Lambda^0_3-\tilde{\Lambda}^0_3)\bigg). \tag{5.18}$$

For such solutions we can in general integrate $h'''=0$ in terms of an order 2 polynomial. As we shall see shortly it is possible to bound r at one end of the space in several physical ways, but when $F_0=0$ it always remains semi-infinite. Given that the massless limit of $\mathcal{N}=(6,0)$ is always locally $\text{AdS}_4\times\mathbb{CP}^3$ in IIA, it is reasonable to ask whether the massless solutions here approach this asymptotically. Such a solution preserving $\mathcal{N}=(8,0)$ was found on this type of squashing of the 7-sphere in [51] and can be interpreted as a holographic dual to a surface defect. In this case, as $r\to\infty$ the curvature invariants (5.16) all vanish, (for instance $R\sim r^{-\frac{2}{3}}$). This makes the behaviour at infinite r that of Mink_{11}, so such an interpretation is not possible here.

Let us now move back to IIA and focus on more generic solutions: by studying the zeros of $(r,\ h,\ h'',\ \Delta_1,\ \Delta_2)$ we are able to identify a plethora of boundary behaviours for $\mathfrak{osp}(5|2)$ solutions. The vast majority we are able to identify as physical and most exist for arbitrary values of F_0. We already used up translational invariance of this class to align $u=r$ so we can non longer assume that $r=0$ is a boundary of solutions in this class, rather we must consider possible boundaries at $r=0$ and $r=r_0$ for $r_0\neq0$ separately.

[10] Note that to get to this form one must rescale the canonical 11'th direction, ie if this is z and $L^i_{1,2}$ are defined as in (A.2) then $\phi_2=\frac{2}{h''}z$.

We have two physical boundary behaviours that only exist for $F_0 \neq 0$: the first of these is a regular zero for which the warp factors of AdS_3 and S^4 become constant while the (r, S^2) directions approach the origin of $\mathbb{R}^3$ in polar coordinates. This is given by tuning

$$\text{Regular zero at } r=0: \quad h = c_1 r + \frac{1}{3!} c_2 r^3, \quad c_{1,2} \neq 0, \quad \text{sign}(c_1 c_2) = 1 \tag{5.19}$$

where one can take either of $r \in \mathbb{R}^{\pm}$. This is the only regular boundary behaviour that is possible.

Next it is possible to realise a fully localised O6 plane of world volume $(\text{AdS}_3,\ \text{S}^4)$ at $r = r_0$ by tuning

$$\text{O6 plane at } r = r_0: \quad h = c_1 r_0 + c_1 (r - r_0) + \frac{1}{3!} c_2 (r - r_0)^3, \quad c_1, c_2, r_0 \neq 0. \tag{5.20}$$

The domain of r in this case depends more intimately on the tuning of c_1, c_2, r_0 than we have thus far seen: when $r_0 < 0$ one has $r \in (-\infty,\ r_0]$ for $\text{sign}(c_1 c_2) = 1$ while for $\text{sign}(c_1 c_2) = -1$ one finds that $r_0 \leq r \leq r_1 < 0$ for $r_1 = r_1(c_1, c_2)$. Conversely for $r_0 > 0$, $\text{sign}(c_1 c_2) = 1$ implies $r \in [r_0,\ \infty)$ while $\text{sign}(c_1 c_2) = -1$ implies $0 < r_1 \leq r \leq r_0$.

The remaining boundary behaviour exist whether F_0 is non trivial or not: we find the behaviour of D6 branes extended in $(\text{AdS}_3,\ \text{S}^4)$ by tuning

$$\text{D6 brane at } r=0: \quad h = c_1 r + \frac{1}{2} c_2 r^2 + \frac{1}{3!} c_3 r^3, \quad c_{1,2} \neq 0. \tag{5.21}$$

When $\text{sign}(c_1 c_2) = \pm 1$ $r = 0$ is a lower/upper bound. Given this, r is also bounded from above/below when $\text{sign}(c_1 c_3) = -1$ and is semi-infinite for $\text{sign}(c_1 c_3) = 1$ and $c_3 = 0$.

As with the $\mathfrak{osp}(6|2)$ class it is possible to realise a $\widetilde{\text{D6}}$ singularity (see the discussion below (5.10)), this time at $r = r_0$ by tuning

$$\widetilde{\text{D6}} \text{ brane at } r = r_0: \quad h = \frac{1}{2} c_1 (r - r_0)^2 + \frac{1}{3!} c_2 (r - r_0)^3, \quad r_0, c_2 \neq 0, \quad c_2 \neq -3 \frac{c_1}{r_0}. \tag{5.22}$$

For $\text{sign}(r_0 c_1 c_2) = 1$ the domain of r is semi infinite bounded from above/below when $\text{sign}(r_0) = \mp 1$. When $\text{sign}(r_0 c_1 c_2) = -1$ we find that r is bounded between r_0 and some constant $r_1 = r_1(r_0, c_1, c_2)$. Given the later behaviour, when $\text{sign}(r_0) = \pm 1$ one finds that r is strictly positive/negative with r_0 the upper/lower of the 2 bounds when $|c_2 r_0| > 3|c_1|$ and the lower/upper when $|c_2 r_0| < 3|c_1|$.

Next we find the behaviour of an O4 plane extended in $\text{AdS}_3 \times \text{S}^2$ by tuning

$$\begin{aligned} \text{O4 plane at } r = r_0: \quad & h = \frac{1}{2} c_1 \left(r_0^2 - r_0 (r - r_0) + (r - r_0)^2\right) + \frac{1}{3!} c_2 (r - r_0)^3, \\ & c_1, r_0 \neq 0, \qquad c_2 \neq -\frac{3 c_1}{r_0}. \end{aligned} \tag{5.23}$$

In this solution the domain of r has the same qualitative dependence on the signs of c_1, c_2, r_0 and whether $|c_2 r_0| > 3|c_1|$ or $|c_2 r_0| < 3|c_1|$ as the previous example, though the precise value of $r_1(c_1, c_2, r_0)$ is different.

Likewise we find the behaviour of an O2 plane extended in AdS_3 and back-reacted on a G_2 cone whose base is round $\mathbb{CP}^3$, this is achieved by tuning

$$\text{O2 plane at } r=r_0: \qquad h=2r_0c_1+\frac{1}{2}c_1(r-r_0)^2+\frac{1}{3!}c_2(r-r_0)^3, \qquad r_0,c_1\neq 0, \quad c_2\neq -\frac{3c_1}{r_0}, \tag{5.24}$$

where the domain of r is qualitatively related to the parameters as it was for the $\widetilde{\text{D6}}$.

Finally we find the behaviour of an O2' plane extended in AdS_3 and back-reacted on a G_2 cone whose base is a squashed $\mathbb{CP}^3$ (ie $4ds^2(\text{B}^6)=2ds^2(\text{S}^4)+ds^2(\text{S}^2)$) at $r=r_0$ by tuning

$$\text{O2' plane at } r=r_0: \qquad h=2c_1r_0^2+2c_1b(r-r_0)+\frac{1}{2}c_1(b-1)^2(r-r_0)^2+\frac{1}{3!}c_2(r-r_0)^3, \tag{5.25}$$

where we must additionally impose $r_0,c_1\neq 0$. This gives behaviour similar to the O2 plane, ie $e^{2A}\sim(r-r_0)^{-\frac{1}{2}}$ with the rest of the warp factors scaling as the reciprocal of this. However in general $e^{2(C-D)}=\left(\frac{b+1}{b-1}\right)^2$ at leading order about $r=r_0$ and the internal space only spans a Ricci flat cone for $e^{2(C-D)}=1,2$, with the former yielding (5.24). As such for the O2' plane we must additionally tune

$$\left(\frac{b+1}{b-1}\right)^2=2, \tag{5.26}$$

which has two solutions $b_\pm=3\pm 2\sqrt{2}$ and we must have $c_2r_0\neq -12b_\pm$. Again the domain of r has the same qualitative dependence as the $\widetilde{\text{D6}}$, though this time the relevant equalities that determine whether r_0 is an upper or lower bound are $|c_2r_0|>12b_\pm|c_1|$ or $|c_2r_0|<12b_\pm|c_1|$. This exhausts the physical singularities we have been able to identify.

As with the case of $\mathfrak{osp}(6|2)$ vacua we have been able to identify several local solution for which the domain of r is semi infinite. For these the metric as $r\to\pm\infty$ is once again flat, but at infinite distance and with a non constant dilaton. For the $\mathfrak{osp}(5|2)$ solutions however it is possible to bound the majority of the solutions for suitable tunings of the parameters on which they depend — this necessitates $F_0\neq 0$. A reasonable question to ask then is which physical singularities can reside in the same local solution? There are actually 7 distinct local solutions bounded between two physical singularities, we provide details of these in table 1. Note that the solution with regular zero is unbounded while the D6 solution can only be bounded by a singularity of the type given in (5.25), but without (5.26) being satisfied so is thus non-physical.

In this section, and the preceding one we have analysed the possible local solutions preserving $\mathcal{N}=(5,0)$ and $(6,0)$ supersymmetry that follow from various tunings of the (local) order 3 polynomial h. We found many different possibilities, many of which can give rise to a bounded interval in the $(5,0)$ case, but non of which do in the (6,0) case. This is not the end of the story, in this section we have assumed that F_0 is a constant which excludes the presence of D8 branes along the interior of the interval. In the next section we shall relax this assumption allowing for much wider classes of global solution, and in particular $(6,0)$ solutions with bounded internal space.

Bound at $r = r_0$	Bound at $r = \tilde{r}_0$	Additional tuning and comments
O6 (5.20)	O4 $\mid$ O2 $\mid$ O2’	$c_1 = \frac{(\tilde{r}_0 - r_0)^3 (r_0^2 + 4 r_0 \tilde{r}_0 - 8\tilde{r}_0^2)}{72 \tilde{r}_0^3} c_2, \quad r_0 = \frac{(\sqrt{3}\sqrt{11+8\sqrt{b_0}} - 1)\tilde{r}_0}{2}$ $b_0 = 0 \mid 1 \mid 2$
D6 (5.21)	Non-physical	$\tilde{r}_0 :$ Middle of 3 real roots of $8 c_3^2 \tilde{r}_0^3 + 36 c_2 c_2 \tilde{r}_0^2 + (27 c_2^2 + 72 c_1 c_3)\tilde{r}_0 + 72 c_1 c_2 = 0$, (5.25) like but for $e^{2C-2D} > 2$
$\widetilde{\text{D6}}$ (5.22)	O2’	$c_1 = \frac{5 - 4\sqrt{2} - \sqrt{120 + 86\sqrt{2}}}{21} c_2 r_0, \quad \tilde{r}_0 = \frac{\sqrt{4+3\sqrt{2}} r_0}{2}$
O4 (5.23)	O2	$c_1 = c_2 b_- r_0, \quad \tilde{r}_0 = b_+ r_0$, $b_\pm :$ $\pm$tive real roots of $13 b_+^3 - 29 b_+^2 + 10 b_+ = 6$, $1287 b_-^3 + 610 b_-^2 - 1287 b_- + 144 = 0$
O4 (5.23)	O2’	$c_1 = c_2 b_- r_0, \quad \tilde{r}_0 = b_+ r_0$, $b_\pm :$ $\pm$tive real roots of $2 b_+^6 - 8 b_+^5 + 40 b_+^3 - 49 b_+^2 + 6 b_+ = 9$, $81 b_-^6 - 18 b_-^5 + 37 b_-^4 + 12 b_-^3 - 93 b_-^2 + 54 b_- = 9$
O2 (5.24)	O2’	$c_1 = c_2 b_- r_0, \quad \tilde{r}_0 = b_+ r_0$, $b_\pm :$ $\pm$tive real roots of $8 b_+^6 - 16 b_+^5 - 8 b_+^4 + 224 b_+^3 - 209 b_+^2 + 26 b_+ = 169$, $711 b_-^6 - 2388 b_-^5 + 2858 b_-^4 - 972 b_-^3 - 705 b_-^2 + 648 b_- = 144$

Table 1. A list of distinct local $\mathfrak{osp}(5|2)$ solutions with bounded internal space. Note that we do not include solutions with the singularities at $r = r_0$ and $r = \tilde{r}_0$ inverted, as they are physically equivalent.

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6 Global solutions with interior D8 branes

In this section we show that it is possible to glue the local solutions of section 5 together with D8 branes placed along the interior of the interval spanned by r. This opens the way for constructing many more global solutions with bounded internal spaces.

In the previous section we studied what types of local $\mathcal{N}=(5,0)$ and $(6,0)$ it is possible to realise with various tunings of the order 3 polynomial h. The assumption we made before was that $F_0=$ constant was fixed globally, but this is not necessary, all that is actually required is that F_0 is piecewise constant. A change in F_0 gives rise to a delta function sources as

$$dF_0=\frac{\Delta F_0}{2\pi}\delta(r-r_0)dr, \tag{6.1}$$

where ΔF_0 is the difference between the values F_0 for $r>r_0$ and $r<r_0$. D8 branes give rise to a comparatively mild singularity for which the Bosonic fields neither blow up nor tend to zero so do not represent a boundary of a solution, indeed the solution continues passed them unless they appear coincident to an O8 plane. As one crosses a D8 brane the metric and dilaton and NS 3-form are continuous, but the RR sector can experience a shift. To accommodate such an object within the class of solutions of section 4 one needs to do so in terms of h, so it can lie along r. If we wish to place a D8 at $r=r_0$ one should have

$$h''''=-N_8\delta(r-r_0). \tag{6.2}$$

While the conditions that the NS sector should be continuous amounts to demanding the continuity of

$$\begin{aligned}\mathcal{N}=(6,0):&\quad (h,(h')^2,h''),\\ \mathcal{N}=(5,0):&\quad (h,h',h''),\end{aligned} \tag{6.3}$$

recall that $u''=0$ is a requirement for supersymmetry so u cannot change as one crosses the D8. The source corrected Bianchi identities in general take the form

$$(d-H\wedge)f_+=\frac{1}{2\pi}\delta(r-r_0)dr\wedge e^{\mathcal{F}}\quad\Rightarrow\quad d\hat{f}_+=\frac{1}{2\pi}\delta(r-r_0)dr\wedge e^{2\pi\tilde{f}} \tag{6.4}$$

where $\mathcal{F}=B_2+2\pi\tilde{f}$ for $\tilde{f}$ a world volume gauge field on the D8 brane and where $\hat{f}_+$ are the magnetic Page fluxes, (4.19) for the solution at hand. If $\tilde{f}$ is non zero then the D8 is actually part of a bound state involving every brane whose flux receives a source correction in $d\hat{f}_+$ Recalling the Bianchi identities (4.20), we have for the specific case at hand that

$$d\hat{f}_n=\frac{1}{2\pi}(4\pi)^n\frac{1}{n!}(r-k)^nN_8\delta(r-r_0)dr\wedge J_2^n \tag{6.5}$$

We thus see that it is consistent to set the world volume flux on the D8 brane to zero by tuning r_0, ie

$$\tilde{f}=0\quad\Rightarrow\quad r_0=k. \tag{6.6}$$

This means only F_0 receives a delta function source, the rest vanishing as $(r-r_0)^n\delta(r-r_0)\to 0$ for $n>0$. So we have shown that it is possible to place D8 branes, that do not come as a bound state, at $r = k$ and solve the Bianchi identities — but how should one interpret this? The origin of k in our classification was as an integration constant, but one can view it as the result of performing a large gauge transformation, ie shifting the NS 2-form by an exact form as $B_2 \to B_2 + \Delta B_2$ such that $b_0 = \frac{1}{(2\pi)^2}\int_{\Sigma_2} B_2$ is quantised over some 2-cycle Σ_2. Clearly squashed $\mathbb{CP}^3$ contains an S^2 which B_2 as support on, that of the fiber, one finds

$$\frac{1}{(2\pi)^2}\int_{\text{S}^2} B_2 - \frac{1}{(2\pi)^2}\int_{\text{S}^2} B_2\bigg|_{k=0} = \frac{k}{4\pi}\int_{\text{S}^2}\text{vol}(\text{S}^2) = k, \tag{6.7}$$

so provided k is quantised[11] its addition to the minimal potential giving rise to the NS 3-form is indeed the action of a large gauge transformation. The key point is that because k follows from a large gauge transformation, it does not need to be fix globally, indeed in many situations where B_2 depends on a function of the internal space it is necessary to perform such large gauge transformations as one move through the internal space to bound b_0 to with some quantised range. We conclude that the Bianchi identities are consistent with placing D8 branes at quantised $r = k$ loci provided that they are accompanied by the appropriate number of large gauge transformations of the NS 2-form.

Of course to be able claim a supersymmetric vacua the sources themselves need to have a supersymmetric embedding, further if this is the case the integrability arguments of [58, 59] imply that the remaining type II equations of motion of the Bosonic supergravity are implied by the Bianchi identities we have already established hold. The existence of a supersymmetric brane embedding can be phrased in the language of (generalised) calibrations [60]: a source extended in AdS_3 and wrapping some n-cycle Σ is supersymmetric if it obeys the following condition

$$\Psi^{\text{cal}}_n = 8\Psi_+ \wedge e^{-\mathcal{F}}\Big|_\Sigma = \sqrt{\det(g+\mathcal{F})}d\Sigma\Big|_\Sigma \tag{6.8}$$

where Ψ_+ is the bi-linear appearing in (4.2), g is the metric on the internal space and where the pull back onto Σ is understood. For a D8 brane placed along r we take $\tilde{f} = 0$, $d\Sigma = \sin^3\alpha d\alpha \wedge \text{vol}(\text{S}^3) \wedge \text{vol}(\text{S}^2)$ and B_2 defined as in (4.22) — we find

$$\begin{aligned}\Psi^{\text{cal}}_6 &= \frac{\pi^3}{2\sqrt{2}u^3 h''\sqrt{hh''}\sqrt{\Delta_2}}\Big[u\left(h(2u - r_k u') - r_k u h'\right)\left(2h\left(u + r_k u'\right) - u\left(2h' - h'' r_k\right) r_k\right)\\ &\qquad + 2\cos^2\alpha u' \Delta_1 r_k^3\Big] d\Sigma,\\ \sqrt{\det(g+\mathcal{F})}d\Sigma\Big|_\Sigma &= \frac{\pi^3}{8(uh'')^{\frac{3}{2}}\sqrt{\Delta_2}}\sqrt{2(h - h' r_k)(u - u' r_k) + u h'' r_k^2}\\ &\qquad \times\left(2h(u + u' r_k) - u r_k(2h' - h'' r_k)\right) d\Sigma\end{aligned} \tag{6.9}$$

where we use the shorthand $r_k = r - k$. It is simple to then show that (6.8) is indeed satisfied for a D8 brane at $r = k$, and so supersymmetry is preserved.

[11] Note: usually one means integer by quantised, by in the presence of fractional branes, such as in ABJ [57] which shares a $\mathbb{CP}^3$ in its internal space, it is possible for parameters such as k to merely be rational.

In summary we have shown that D8 branes can be placed along the interior of r at the loci $r=k$, for quantised k, without breaking supersymmetry provided an appropriate large gauge transformation of the NS 2-form is performed. This allows one to place a potentially arbitrary number of D8 branes along r and use them to glue the various local solutions of section 5 together provided that the continuity of (6.3) holds across each D8. In the next section we will explicitly show this in action with 2 examples.

6.1 Some simple examples with internal D8 branes

In this section we will construct two global solutions with interior D8 branes and bounded internal spaces, one preserving each of $\mathcal{N}=(6,0)$ and $(5,0)$ supersymmetry. Let us stress that this only scratches the surface of what is possible, we save a more thorough investigation for forthcoming work [63].

We shall first construct a solution preserving $\mathcal{N}=(6,0)$, meaning that we need to impose the continuity of $(h,(h')^2,h'')$ as we cross a D8. Probably the simplest thing one can do is to place a stack of D8 branes at the origin $r=0$ and bound r between D8/O8 brane singularities which are symmetric about this point. As such we can take h to be globally defined as

$$h=\begin{cases} -c_1-\frac{c_2}{3!}(r+r_0)^3 & r<0 \\ -c_1-\frac{c_2}{3!}(r_0-r)^3 & r>0 \end{cases} \tag{6.10}$$

This bounds the interval to $-r_0<r<r_0$ between D8/O8 singularities at $r=\pm r_0$ and gives rise to a source for the F_0 of charge $2c_2$, ie

$$h''''=-2c_2\delta(r) \quad\Rightarrow\quad dF_0=2c_2\frac{1}{2\pi}\delta(r)dr \tag{6.11}$$

The form that the warp factors and metric take for this solution is depicted in figure 1. Given the Page fluxes in (4.19) (with $u=1$), and that we simply have round $\mathbb{CP}^3$ for this solution for which we can take $\int_{\mathbb{CP}^n}J^n=\pi^n$, it is a simple matter to compute the Page charges of the fluxes over the $\mathbb{CP}^n$ sub-manifolds of $\mathbb{CP}^3$. By tuning

$$c_1=N_2-\frac{N_5^3N_8}{6},\qquad c_2=N_8,\qquad r_0=N_5, \tag{6.12}$$

we find that these are given globally by

$$\begin{aligned} &2\pi F_0=N_8^{\mp}=\pm N_8, && \frac{1}{2\pi}\int_{\mathbb{CP}^1}\hat{f}_2=N_5N_8, \qquad \frac{1}{(2\pi)^3}\int_{\mathbb{CP}^2}\hat{f}_4=\frac{N_5^2N_8^{\mp}}{2},\\ &-\frac{1}{(2\pi)^5}\int_{\mathbb{CP}^3}\hat{f}_6=N_2, && -\frac{1}{(2\pi)^2}\int_{(r,\mathbb{CP}^1)}H=N_5 \end{aligned} \tag{6.13}$$

where the $\mp$ superscript indicates that we are on the side of the interior D8 with $r\in\mathbb{R}^{\mp}$ and we have assumed for simplicity that $k=0$ globally in the NS 2-form.

With the expressions for the brane charges we can compute the holographic charge via the string frame analogue of the formula presented in [61], namely

$$c_{hol}=\frac{3}{2^4\pi^6}\int_{M_7}e^{A-2\Phi}\text{vol}(\text{M}_7), \tag{6.14}$$

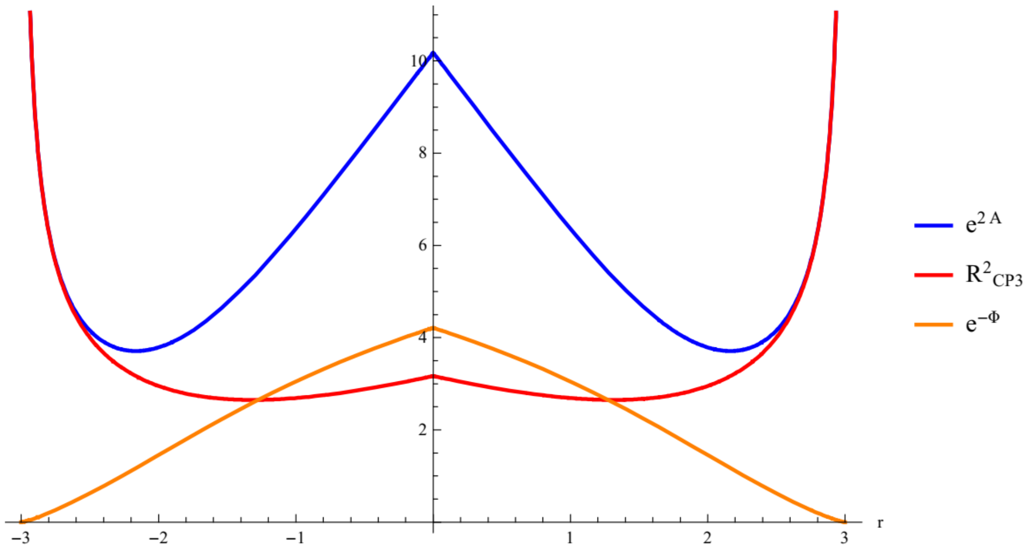

Figure 1. Plot of the warp factors in the metric and dilaton for an $\mathcal{N}=(6,0)$ solution with D8 at $r=0$ bounded between D8/O8 singularities at $r=\pm 3$ with the remaining constants in h tuned as $c_1=2$, $c_2=5$.

which gives the leading order contribution to the central charge of the putative dual CFT. Given the class of solution is section 4 we find this expression reduces to

$$c_{hol}=\frac{1}{2}\int\frac{\Delta_1}{u^2}dr. \tag{6.15}$$

For the case at hand one then finds that

$$c_{hol}=N_2N_5^2N_8-\frac{3N_5^5N_8^2}{20}. \tag{6.16}$$

The central charge of CFTs with $\mathfrak{osp}(n|2)$ superconformal symmetry takes the form of (1.1), which in the limit of large level k becomes $c=3k$. The holographic central charge is not obviously of this form, however that doesn't mean it is necessarily not the leading contribution to something that is.[12] We leave recovering this result from a CFT computation for future work.

We will now construct a globally bounded solution with interior D8 branes that preserves $\mathcal{N}=(5,0)$ — this time we will be more brief. There are many options for gluing local solutions together for this less supersymmetric case. We will choose to place a D8 brane in one of the bounded behaviour we already found in section 5.2 in the absence of interior D8 branes (see table 1), namely will insert a D8 in the solution bounded between O6 and O4 places. We remind the reader that we get local solutions containing these singularities by tuning h as

$$h_{\text{O4}}=\frac{1}{2}c_1\left(r_0^2-r_0(r-r_0)+(r-r_0)^2\right)+\frac{1}{3!}c_2(r-r_0)^3,$$

[12]Such scenarios are actually quite common, see for instance [62].

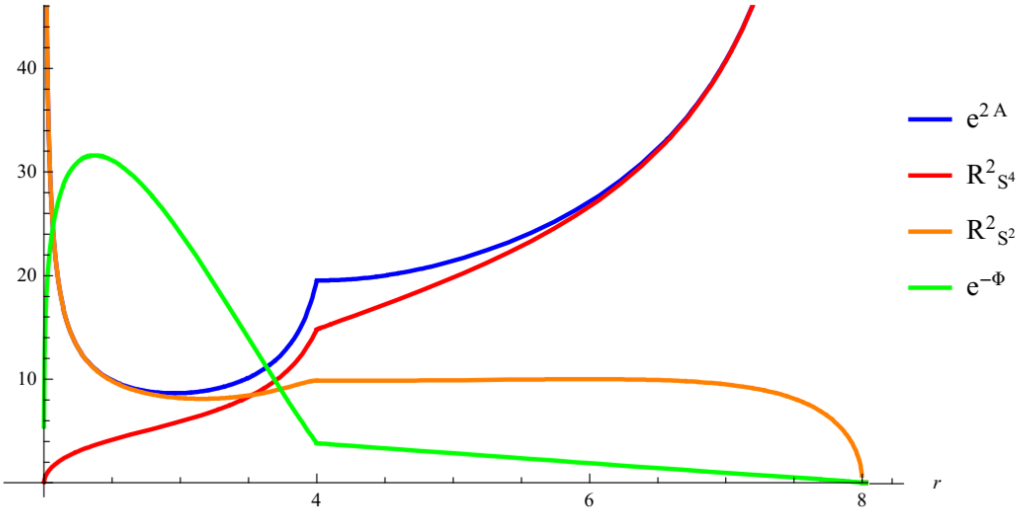

Figure 2. Plot of the warp factors in the metric and dilaton for an $\mathcal{N}=(5,0)$ solution bounded between an O4 plane at $r=2$ and an O6 plane at $r=8$ with a stack of D8 branes at $r=4$. The remaining parameter is tuned as $b_2=-6$.

$$h_{\text{O6}}=b_1\tilde{r}_0+b_1(r-\tilde{r}_0)+\frac{1}{3!}b_2(r-\tilde{r}_0)^3. \tag{6.17}$$

where the singularity are located at $(r_0,\tilde{r}_0)$ respectively. We will assume $r_0,\tilde{r}_0>0$ and place a stack of D8s at a point $r=r_s$ between the two O plane loci. The condition that the NS sector should be continuous in this case amounts to imposing that

$$(h_{\text{O4}},h'_{\text{O4}},h''_{\text{O4}})\Big|_{r=r_s}=(h_{\text{O6}},h'_{\text{O6}},h''_{\text{O6}})\Big|_{r=r_s}, \tag{6.18}$$

of course we also need the value of F_0 to change as we cross the D8. It is indeed possible to solve the continuity condition in this case, which fixes 3 parameters, (c_1,c_2,b_1) say, leaving $(r_0,\tilde{r}_0,b_2)$ as free parameters. A plot of this solution for a choice of $(r_0,\tilde{r}_0,b_2)$ is given in figure 2.

Acknowledgments

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A Derivation of spinors on $\widehat{\mathbb{CP}}^3$

In this appendix we derive all spinors transforming in the **5** and **1** of $\mathfrak{so}(5)$ on squashed $\mathbb{CP}^3$. We achieve this by reducing a set of Killing spinors on in the **3** of $\mathfrak{so}(3)$ and **5** of $\mathfrak{so}(5)$ on the 7-sphere, and then reducing them to $\mathbb{CP}^3$.

A.1 Killing spinors and vectors on S^7= SP(2)/SP(1)

The 7-sphere admits a parametrisation as an SP(2) bundle over SP(1), ie the SP(2)/SP(1) co-set. For a unit radius 7-sphere this has the metric

$$ds^2(S^7) = \frac{1}{4}\left[d\alpha^2 + \frac{1}{4}\sin^2\alpha (L_1^i)^2 + \left(L_2^i - \cos^2\left(\frac{\alpha}{2}\right)L_1^i\right)^2\right] \tag{A.1}$$

where we take the following basis of SU(2) Left invariant 1-forms

$$L^1_{1,2} + iL^2_{1,2} = e^{i\psi_{1,2}}(id\theta_{1,2} + \sin\theta_{1,2} d\phi_{1,2}), \qquad L^3_{1,2} = d\psi_{1,2} + \cos\theta_{1,2} d\phi_{1,2}. \tag{A.2}$$

The 7-sphere admits two sets of Killing spinors obeying the relations

$$\nabla_a \xi_\pm = \pm\frac{i}{2}\gamma_a \xi_\pm. \tag{A.3}$$

With respect to the vielbein and flat space gamma matrices

$$\begin{aligned}
&e^1 = \frac{1}{2}d\alpha, && e^{2,3,4} = \frac{1}{4}\sin\alpha L_1^{1,2,3}, && e^{5,6,7} = L_2^{1,2,3} - \cos^2\left(\frac{\alpha}{2}\right)L_1^{1,2,3},\\
&\gamma_1 = \sigma_1 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2, && \gamma_{2,3,4} = \sigma_2 \otimes \sigma_{1,2,3} \otimes \mathbb{I}_2 && \gamma_{5,6,7} = \sigma_3 \otimes \mathbb{I}_2 \otimes \sigma_{1,2,3},
\end{aligned} \tag{A.4}$$

where $\sigma_{1,2,3}$ are the Pauli-matrices, the Killing spinor equation (A.3) is solved by

$$\begin{aligned}
&\xi_\pm = \mathcal{M}_\pm \xi^0_\pm,\\
&\mathcal{M}_\pm = e^{\frac{\alpha}{4}(\pm i\gamma_1 + Y)} e^{\mp i\frac{\psi_1}{2}\gamma_7 P_\mp} e^{\mp i\frac{\theta_1}{2}\gamma_6 P_\mp} e^{\mp i\frac{\phi_1}{2}\gamma_7 P_\mp} e^{\frac{\psi_2}{4}(\pm i\gamma_7 + X)} e^{\frac{\theta_2}{2}(\gamma_{13}P_+ \pm \gamma_6 P_\pm)} e^{\frac{\phi_2}{2}(\gamma_{14}P_+ \pm i\gamma_7 P_\pm)},
\end{aligned} \tag{A.5}$$

where $\xi^0_\pm$ are unconstrained constant spinors and

$$P_\pm = \frac{1}{2}(\mathbb{I}_4 \pm \gamma_{1234}), \quad X = \gamma_{14} - \gamma_{23} - \gamma_{56}, \quad Y = -\gamma_{25} - \gamma_{36} - \gamma_{47}. \tag{A.6}$$

It was shown in [51] that ξ_- transform in the $(\mathbf{2},\mathbf{4})$ of $\mathfrak{sp}(1)\oplus\mathfrak{sp}(2)$ and ξ_+ in the $\mathbf{3}\oplus\mathbf{5}$ of $\mathfrak{so}(3)\oplus\mathfrak{so}(5)$ — it is the latter that will be relevant to us here. Denoting the $\mathbf{3}$ and $\mathbf{5}$ as ξ_3^i for $i=1,\dots,3$ and ξ_5^α for $\alpha=1,\dots,5$ and defining the 8 independent supercharges contained in ξ_+ as

$$\xi_+^I = \mathcal{M}_+\hat{\eta}^I, \quad I = 1,\dots,8, \tag{A.7}$$

where the I^{th} entry of $\hat{\eta}^I$ is 1 and the rest zero, these are given specifically by

$$\hat{\xi}_3^i = \frac{1}{\sqrt{2}}\begin{pmatrix} i(-\xi_+^5 + \xi_+^8)\\ \xi_+^5 + \xi_+^8\\ i(\xi_+^6 + \xi_+^7)\end{pmatrix}^i, \quad \hat{\xi}_5^\alpha = \frac{1}{\sqrt{2}}\begin{pmatrix} -i(\xi_+^1 + \xi_+^4)\\ \xi_+^1 - \xi_+^4\\ -i(\xi_+^2 - \xi_+^3)\\ \xi_+^2 + \xi_+^3\\ \xi_+^6 - \xi_+^7\end{pmatrix}^\alpha, \tag{A.8}$$

which obey

$$\xi_3^{i\dagger}\xi_3^j = \delta^{ij}, \qquad \xi_5^{\alpha\dagger}\xi_5^\beta = \delta^{\alpha\beta}, \qquad \xi_3^{i\dagger}\xi_5^\beta = 0, \tag{A.9}$$

and are Majorana with respect to the intertwiner $B=\sigma_3\otimes\sigma_2\otimes\sigma_2$. The specific Killing vectors that make up the relevant SO(3) and SO(5) in the full space are made up of the following isometries of the base and fibre metrics

$$K^{1L}_{1,2}+iK^{2L}_{1,2}=e^{i\phi_{1,2}}\left(i\partial_{\theta_{1,2}}+\frac{1}{\sin\theta_{1,2}}\partial_{\psi_{1,2}}-\frac{\cos\theta_{1,2}}{\sin\theta_{1,2}}\partial_{\phi_{1,2}}\right),\quad K^{3L}_{1,2}=-\partial_{\phi_{1,2}}, \tag{A.10a}$$

$$K^{1R}_{1,2}+iK^{2R}_{1,2}=e^{i\psi_{1,2}}\left(i\partial_{\theta_{1,2}}+\frac{1}{\sin\theta_{1,2}}\partial_{\phi_{1,2}}-\frac{\cos\theta_{1,2}}{\sin\theta_{1,2}}\partial_{\psi_{1,2}}\right),\quad K^{3R}_{1,2}=\partial_{\psi_{1,2}}, \tag{A.10b}$$

$$\hat{K}^A_{\text{SO}(5)/\text{SO}(4)}=-(\mu_A\partial_\alpha+\cot\alpha\partial_{x_i}\mu_A g_3^{ij}\partial_{x_j}),\quad A=1,\dots,4 \tag{A.10c}$$

where μ_A are embedding coordinates for the $\text{S}^3\subset\text{S}^4$, g_3^{ij} is the inverse metric of this 3-sphere and $x_i=(\theta_1,\phi_1,\psi_1)_i$, we have specifically

$$\mu_A=\left(\sin\left(\frac{\theta_1}{2}\right)\cos\left(\frac{\phi_-}{2}\right),\sin\left(\frac{\theta_1}{2}\right)\sin\left(\frac{\phi_-}{2}\right),\cos\left(\frac{\theta_1}{2}\right)\cos\left(\frac{\phi_+}{2}\right),-\cos\left(\frac{\theta_1}{2}\right)\sin\left(\frac{\phi_+}{2}\right)\right)_A,$$

for $\phi_\pm=\phi_1\pm\psi_1$. In terms of the isometries on the base and fibre we define the following Killing vectors on the 7-sphere

$$K^{iD}=K^{iR}_1+K^{iR}_2,\qquad K^A_{\text{SO}(5)/\text{SO}(4)}=\hat{K}^A_{\text{SO}(5)/\text{SO}(4)}+\cot\left(\frac{\alpha}{2}\right)\mu_B(\kappa_A)_i^B K^{iR}_2. \tag{A.11}$$

where

$$\kappa_1=\begin{pmatrix}0&0&0\\0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix},\quad \kappa_2=\begin{pmatrix}0&0&1\\0&0&0\\-1&0&0\\0&1&0\end{pmatrix},\quad \kappa_3=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\\0&0&-1\end{pmatrix},\quad \kappa_4=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&1\\0&0&0\end{pmatrix}.$$

The isometry groups in the full space are spanned by

$$\begin{aligned}&\text{SO}(3):\ K^{iL}_2\\&\text{SO}(5):\ (K^{iL}_1,\ K^{iD},\ K^A_{\text{SO}(5)/\text{SO}(4)})\end{aligned} \tag{A.12}$$

Another Killing vector on S^7 that will be relevant is

$$\tilde{K}=x_iK^{iR}_1,\qquad y_i=(\cos\psi_2\sin\theta_2,\ \sin\psi_2\sin\theta_2,\ \cos\theta_2). \tag{A.13}$$

In terms of this one can define Killing vectors that together with the SO(5) Killing vectors span SO(6), namely

$$\text{SO}(6)/\text{SO}(5):\ ([K^A_{\text{SO}(5)/\text{SO}(4)},\ \tilde{K}],\ \tilde{K}) \tag{A.14}$$

A.2 Reduction to $\mathbb{CP}^3$

It is possible to rewrite (A.1) as fibration of ∂_{ϕ_2} over $\mathbb{CP}^3$ as

$$ds^2(\text{S}^7)=ds^2(\mathbb{CP}^3)+\frac{1}{4}\left(d\phi_2+\cos\theta_2 d\psi_2-\cos^2\left(\frac{\alpha}{2}\right)x_iL^i_1\right)^2,$$

$$ds^2(\mathbb{CP}^3) = \frac{1}{4}\left[d\alpha^2 + \frac{1}{4}\sin^2\alpha (L_1^i)^2 + Dy_i^2\right], \qquad Dy_i = dy_i + \cos^2\left(\frac{\alpha}{2}\right)\epsilon_{ijk} y_j L_1^k. \tag{A.15}$$

This can be achieved by rotating the 5,6,7 components of the vielbein in (A.4) by

$$e^i \to \Lambda^i{}_j e^j, \qquad \Lambda = \begin{pmatrix} -\sin\psi_2 & \cos\psi_2 & 0 \\ -\cos\theta_2\cos\psi_2 & -\cos\theta_2\sin\psi_2 & \sin\theta_2 \\ \sin\theta_2\cos\psi_2 & \sin\psi_2\sin\theta_2 & \cos\theta_2 \end{pmatrix}. \tag{A.16}$$

The corresponding action on the spinors is defined through the matrix

$$\Omega = e^{\frac{\theta_2}{2}\gamma_{67}} e^{(\frac{\psi_2}{2}+\frac{\pi}{4})\gamma_{56}}. \tag{A.17}$$

The **3** and **5** in the rotated frame then take the form

$$\xi_3^i = \Omega\hat{\xi}_3^i, \qquad \xi_5^\alpha = \Omega\hat{\xi}_5^\alpha. \tag{A.18}$$

Any component of these spinor multiplets that is un-charged under ∂_{ϕ_2} is spinor on $\mathbb{CP}^3$, as we have rotated to a frame where translational invariance in ϕ_2 is manifest, this is equivalent to choosing the parts of (ξ_3^i, ξ_5^α) that are independent of ϕ_2. It is not hard to establish that this is all of ξ_5^α and ξ_3^3, which being a singlet under SO(5) we now label

$$\xi_0 = \xi_3^3. \tag{A.19}$$

The chirality matrix on $\mathbb{CP}^3$ is identified as $\hat{\gamma} = \gamma_7$, which is clearly an SO(5) singlet, so we can construct an additional SO(5) quintuplet and singlet by acting with this. Additionally we can define a set of embedding coordinates on S^4 via

$$i\xi_0^\dagger\gamma_7\xi_5^\alpha = Y_\alpha, \qquad Y_\alpha = (\sin\alpha\mu_A,\ \cos\alpha), \tag{A.20}$$

where μ_A are embedding coordinates on S^3 defined in (A.11) — as the left hand side of this expression is a quintuplet so too are these embedding coordinates. In summary we have the following Majorana spinors on $\mathbb{CP}^3$ respecting the $\mathbf{5}\oplus\mathbf{1}$ branching of SO(6) under its SO(5) subgroup

$$\mathbf{5}:\ (\xi_5^\alpha,\ i\gamma_7\xi_5^\alpha,\ Y_\alpha\xi_0,\ iY_\alpha\gamma_7\xi_0), \qquad \mathbf{1}:\ (\xi_0,\ i\gamma_7\xi_0), \tag{A.21}$$

which can be used to construct $\mathcal{N}=(5,0)$ and $\mathcal{N}=(1,0)$ AdS_3 solutions respectively. One might wonder if one can generate additional spinor in the **1** or **5** by acting with the SO(5) invariant forms one can define on $\mathbb{CP}^3$. These are quoted in the main text in (2.5) and (2.6), one can show that on round unit radius $\mathbb{CP}^3$

$$\begin{aligned} &\nu_2^1\xi_5^\alpha = -2Y_\alpha\xi_0 + i\gamma_7\xi_5^\alpha, \quad \nu_2^2\xi_5^\alpha = -2Y_\alpha\xi_0, \quad \text{Re}\Omega_3\xi_5^\alpha = -4iY_\alpha\xi_0, \quad \text{Im}\Omega_3\xi_5^\alpha = -4Y_\alpha\gamma_7\xi_0, \\ &\nu_2^1\xi_0 = -i\gamma_7\xi_0, \qquad\qquad \nu_2^2\xi_0 = -2i\xi_0, \qquad \text{Re}\Omega_3\xi_0 = -4\gamma_7\xi_0, \qquad \text{Im}\Omega_3\xi_0 = 4i\gamma_7\xi_0, \end{aligned} \tag{A.22}$$

where $2\nu_2^1 = \tilde{J}_2 - J_2$, $2\nu_2^2 = \tilde{J}_2 + J_2$ and the forms should be understood as acting on the spinors through the Clifford map $X_n \to \frac{1}{n!}(X_n)_{a_1\dots a_n}\gamma^{a_1\dots a_n}$, so (A.21) are in fact exhaustive. The SO(6) Killing vectors on $\mathbb{CP}^3$ are given by (A.12) and (A.14) but with the ∂_{ϕ_2}

dependence omitted, the SO(5) vectors are still Killing when one allows the base S^4 to have a different radi to the S^2 in the $\mathbb{CP}^3$ metric (ie for squashed $\mathbb{CP}^3$) this however breaks the SO(6)/SO(5) isometry. Finally note that ξ_0 are actually charged under SO(6)/SO(5), and more specifically when these isometries are not broken (ie $\mathbb{CP}^3$ is not squashed) then we have the following independent SO(6) sextuplets

$$\mathbf{6}:\ \xi_6^{\mathcal{I}} = \begin{pmatrix} \xi_5^{\alpha} \\ \xi_0 \end{pmatrix}^{\mathcal{I}}, \qquad \hat{\xi}_6^{\mathcal{I}} = \begin{pmatrix} i\gamma_7\xi_5^{\alpha} \\ i\gamma_7\xi_0 \end{pmatrix}^{\mathcal{I}}, \tag{A.23}$$

which can be used to construct $\mathcal{N}=(6,0)$ AdS_3 solutions.

B The $\text{SO}(3)_L\times\text{SO}(3)_D$ invariant $\mathcal{N}=5$ bi-linears

In the main text we will need to construct $\mathcal{N}=5$ bi-linears on the space (2.2), the non trivial part of this computation comes from the bi-linears on squashed $\mathbb{CP}^3$ — in this appendix we shall compute them.

As explained in the main text it is sufficient to solve the supersymmetry constraints for an $\mathcal{N}=1$ sub-sector of the quintuplet of SO(5) spinors defined on the internal space. A convenient component to work with is the 5th as this is a singlet under an SO(4) subgroup of SO(5). Specifically with respect to (2.2) and the discussion below it, $\chi^5_{1,2}$ are singlets with respect to $\text{SO(4)}=\text{SO}(3)_L\otimes\text{SO}(3)_D$. As such the bi-linears that follow from $\chi^5_{1,2}$ must decompose in a basis of the $\text{SO}(3)_L\otimes\text{SO}(3)_D$ invariant forms on the $S^2\times S^3$ fibration (2.3) and what one can form from these through taking wedge products. The $d=7$ spinors $\chi^5_{1,2}$ depend on $\hat{\mathbb{CP}}^3$ through

$$\eta_\pm = \xi_5^5 \pm iY_5\hat{\gamma}\xi_0, \qquad Y_5\xi_0, \qquad Y_5 i\hat{\gamma}\xi_0 \tag{B.1}$$

where these are all defined in the previous appendix — it is the bi-linears we can construct out of these that will be relevant to us. One can show that

$$\begin{aligned}
\eta_\pm\otimes\eta_\pm^\dagger &= \phi_+^1 \pm i\phi_-^1, & \eta_\pm\otimes\eta_\mp^\dagger &= \pm\phi_+^2 + i\phi_-^2,\\
\eta_+\otimes\xi_0^\dagger &= \phi_+^3 + i\phi_-^3, & \eta_-\otimes(i\hat{\gamma}\xi_0)^\dagger &= \phi_+^3 - i\phi_-^3,\\
\eta_+\otimes(i\hat{\gamma}\xi_0)^\dagger &= \phi_+^4 + i\phi_-^4, & \eta_-\otimes\xi_0^\dagger &= -\phi_+^4 + i\phi_-^4,\\
\xi_0\otimes\eta_+^\dagger &= \phi_+^5 + i\phi_-^5, & i\hat{\gamma}\xi_0\otimes\eta_-^\dagger &= \phi_+^5 - i\phi_-^5,\\
i\hat{\gamma}\xi_0\otimes\eta_+^\dagger &= \phi_+^6 + i\phi_-^6, & \xi_0\otimes\eta_-^\dagger &= -\phi_+^6 + i\phi_-^6,\\
\xi_0\otimes\xi_0^\dagger &= \phi_+^7 + i\phi_-^7, & i\hat{\gamma}\xi_0\otimes(i\hat{\gamma}\xi_0)^\dagger &= \phi_+^7 - i\phi_-^7,\\
i\hat{\gamma}\xi_0\otimes\xi_0^\dagger &= \phi_+^8 + i\phi_-^8, & \xi_0\otimes(i\hat{\gamma}\xi_0)^\dagger &= -\phi_+^8 + i\phi_-^8,
\end{aligned} \tag{B.2}$$

where $\phi_\pm^{1\dots8}$ are real bi-linears of even/odd form degree, they take the form

$$\phi_+^1 = \frac{1}{8}\sin^2\alpha\bigg(1 - \frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2 + \frac{1}{16}e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\left(e^{2D}\omega_2^1 - e^{2C}\sin^2\alpha\omega_2^4\right)\bigg),$$
$$\phi_-^1 = \frac{1}{64}e^{2C+D}\sin^4\alpha\omega_1\wedge\omega_2^2, \qquad \phi_-^2 = -\frac{1}{64}e^{2C+D}\sin^3\alpha\bigg(\sin\alpha\omega_1\wedge\omega_2^3 + d\alpha\wedge\omega_2^2\bigg)$$

$$\phi_+^2=\frac{1}{32}\sin^2\alpha\bigg(e^{2C}\sin^2\alpha\omega_2^4-e^{2D}\omega_2^1+e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\big(1-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2\big)\bigg),$$
$$\phi_+^3=\frac{1}{32}\sin^2\alpha\bigg(-e^{C+D}\omega_2^2+\frac{1}{4}e^{3C+D}\sin\alpha d\alpha\wedge\omega_1\wedge\omega_2^3\bigg),$$
$$\phi_-^3=\frac{1}{16}\sin\alpha\bigg(e^{C}\sin\alpha\omega_1\wedge\big(1-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2\big)+\frac{1}{4}d\alpha\wedge\big(e^{C+2D}\omega_2^1-e^{3C}\sin^2\alpha\omega_2^4\big)\bigg),$$
$$\phi_+^4=-\frac{1}{32}e^{C+D}\sin^2\alpha\bigg(\omega_2^3+\frac{1}{4}e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\omega_2^2\bigg),$$
$$\phi_+^5=\frac{1}{32}e^{C+D}\sin^2\alpha\bigg(\omega_2^2+\frac{1}{4}e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\omega_2^3\bigg),$$
$$\phi_-^4=\frac{1}{16}e^{C}\sin\alpha\bigg(\frac{1}{4}\sin\alpha\omega_1\wedge\big(e^{2D}\omega_2^1-e^{2C}\sin^2\alpha\omega_2^4\big)-d\alpha\wedge\big(1-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\wedge\omega_2^2\wedge\omega_2^2\big)\bigg),$$
$$\phi_-^5=\frac{1}{16}e^{C}\sin\alpha\bigg(\frac{1}{4}d\alpha\wedge\big(e^{2D}\omega_2^1-e^{2C}\sin^2\alpha\omega_2^4\big)-\sin\alpha\omega_1\wedge\big(1-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2\big)\bigg),$$
$$\phi_+^6=\frac{1}{32}e^{C+D}\sin^2\alpha\bigg(\omega_2^3-\frac{1}{4}e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\omega_2^2\bigg),$$
$$\phi_-^6=\frac{1}{16}e^{C}\sin\alpha\bigg(d\alpha\wedge\big(1-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2\big)+\frac{1}{4}\sin\alpha\big(e^{2D}\omega_2^1-e^{2C}\sin^2\alpha\omega_2^4\big)\bigg),$$
$$\phi_+^7=\frac{1}{8}\bigg(1+\frac{1}{16}e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\big(-e^{2D}\omega_2^1+e^{2C}\sin^2\alpha\omega_2^4\big)-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2\bigg),$$
$$\phi_-^7=\frac{1}{64}e^{2C+D}\sin\alpha\bigg(d\alpha\wedge\omega_2^3+\sin\alpha\omega^1\wedge\omega_2^2\bigg),\qquad \phi_-^8=\frac{1}{64}e^{2C+D}\sin\alpha\bigg(-d\alpha\wedge\omega_2^2+\sin\alpha\omega_1\wedge\omega_2^3\bigg)$$
$$\phi_+^8=-\frac{1}{32}\bigg(e^{2C}\sin\alpha d\alpha\wedge\omega_1\wedge\big(1-\frac{1}{32}e^{2(C+D)}\sin^2\alpha\omega_2^2\wedge\omega_2^2\big)+e^{2D}\omega_2^1-e^{2C}\sin^2\alpha\omega_2^4\bigg)\tag{B.3}$$

C Ruling out $\mathfrak{osp}(7|2)$ AdS$_3$ vacua

In this appendix we shall prove that all $\mathcal{N}=7$ AdS$_3$ solutions preserving the algebra $\mathfrak{osp}(7|2)$ are locally AdS$_4\times$S^7.

$\mathfrak{osp}(7|2)$ necessitates an SO(7) R-symmetry with spinor transforming in the **7**, there is only one way to achieve this. On needs a round 7-sphere in the metric with fluxes that break its SO(8) isometry to SO(7) in terms of the weak G$_2$ structure 3-forms one can define. Such an ansatz in type II can be ruled out a the level of the equations of motion [51], our focus here then will be on $d=11$ supergravity.

All AdS$_3$ solutions of 11 dimensions supergravity admit a decomposition of their bosonic fields as

$$ds^2=e^{2A}ds^2(\text{AdS}_3)+ds^2(\text{M}_8),\qquad G=e^{3A}\text{vol}(\text{AdS}_3)\wedge F_1+F_4\tag{C.1}$$

where F_1,F_4,A have support on M$_8$ only. We take AdS$_3$ to have inverse radius m. When a solutions is supersymmetric M$_8$ supports (at least one) Majorana spinor χ that one can use to define the following bi-linears

$$2e^A=|\chi|^2,\qquad 2e^Af=\chi^\dagger\hat{\gamma}^{(8)}\chi,\qquad 2e^AK=\chi_+^\dagger\gamma_a^{(8)}\chi_-e^a,$$
$$2e^A\Psi_3=\frac{1}{3!}\chi^\dagger\gamma_{abc}^{(8)}\hat{\gamma}^{(8)}\chi e^{abc},\qquad 2e^A\Psi_4=\frac{1}{4!}\chi^\dagger\gamma_{abcd}^{(8)}\chi e^{abcd}\tag{C.2}$$

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where $\gamma_a^{(8)}$ are eight-dimensional flat space gamma matrices, $\hat{\gamma}^{(8)} = \gamma^{(8)}_{12345678}$ is the chirality matrix and e^a is a vielbein on M_8. Sufficient conditions for $\mathcal{N}=1$ supersymmetry to hold can be caste as the following differential conditions the bi-linears should obey [51]

$$d(e^{2A}K)=0, \tag{C.3a}$$

$$d(e^{3A}f)-e^{3A}F_1-2me^{2A}K=0, \tag{C.3b}$$

$$d(e^{3A}\Psi_3)-e^{3A}(-\star_8 F_4+fF_4)+2me^{2A}\Psi_4=0, \tag{C.3c}$$

$$d(e^{2A}\Psi_4)-e^{2A}K\wedge F_4=0, \tag{C.3d}$$

$$6\star_8 dA-2f\star_8 F_1+\Psi_3\wedge F_4=0, \tag{C.3e}$$

$$6e^{-A}m\star_8 K-6f\star_8 dA+2\star_8 F_1+\Psi_3\wedge\star_8 F_4=0 \tag{C.3f}$$

where $\star_8$ is the hodge dual on the M_8. These conditions do not imply all of the equations of motion of 11 dimensional supergravity however. For that to follow one must additionally solve the Bianchi identity and equation of motion of the 4-form flux G. Away from the loci of sources, this amounts to imposing that

$$d(F_4)=0,\qquad d(\star_8 F_1)-\frac{1}{2}F_4\wedge F_4=0. \tag{C.4}$$

The only way to realise the SO(7) R-symmetry that $\mathfrak{osp}(7|2)$ necessitates on a 8d space is to take it to be a foliation of the $\text{SO}(7)/\text{G}_2$ co-set over an interval. As explained at greater length in section 6.2 of [51], the metric on this co-set is the round one, but the flux can depend also depend on a SO(7) invariant 3-form ϕ_3^0 such that (C.1) should be refined as

$$ds^2(\text{M}_8)=e^{2B}ds^2(\text{S}^7)+e^{2k}dr^2,\qquad e^{3A}F_1=f_1dr,\qquad F_4=4f_2\star_7\phi_3^0+f_3dr\wedge\phi_3^0. \tag{C.5}$$

where $(e^{2A},e^{2B},e^{2k},f_i)$ are functions of the interval only. The SO(7) invariants obey the following relations

$$d\phi_3^0=4\star_7\phi_3^0,\qquad \phi_3^0\wedge\star_7\phi_3^0=7\text{vol}(\text{S}^7), \tag{C.6}$$

ie they define the structure of a manifold of weak G_2 holonomy. More specifically, decomposing

$$ds^2(\text{S}^7)=d\alpha^2+\sin^2\alpha\, ds^2(\text{S}^6) \tag{C.7}$$

One has

$$\begin{aligned}\phi_3^0&=\sin^2\alpha\, d\alpha\wedge J_{\text{G}_2}+\sin^3\alpha\,\text{Re}(e^{-i\alpha}\Omega_{\text{G}_2}),\\ \star_7\phi_3^0&=-\frac{1}{2}\sin^4\alpha\, J_{\text{G}_2}\wedge J_{\text{G}_2}-\sin^3\alpha\, d\alpha\wedge\text{Im}(e^{-i\alpha}\Omega_{\text{G}_2}),\\ J_{\text{G}_2}&=\frac{1}{2}\mathcal{C}_{ijk}Y^i_{\text{S}^6}dY^j_{\text{S}^6}\wedge dY^k_{\text{S}^6},\quad \Omega_{\text{G}_2}=\frac{1}{3!}(1-i\iota_{d\alpha}\star_6)\mathcal{C}_{ijk}dY^i_{\text{S}^6}\wedge dY^j_{\text{S}^6}\wedge dY^k_{\text{S}^6},\end{aligned} \tag{C.8}$$

where $Y^i_{\text{S}^6}$ are unit norm embedding coordinates for S^6 and $\mathcal{C}_{ijk}$ are the structure constants defining the product between the octonions, ie $o^io^j=-\delta^{ij}+\mathcal{C}^{ijk}o_k$. The Killing spinors on unit radius S^7 obeying the equation

$$\nabla^{(7)}_a\xi=\frac{i}{2}\gamma^{(7)}_a\xi, \tag{C.9}$$

branch as $\mathbf{1}+\mathbf{7}$ under the SO(7) subgroup of SO(8), we denote the portions of ξ that transform in these reps as respectively ξ^0 and $\xi^I_{\mathbf{7}}$, they can be extracted from the relations

$$\mathbf{1}:\ \left(\phi^0_3+\frac{i}{7}\right)\xi=0,\quad \mathbf{7}:\ (\phi^0_3-i)\xi=0, \tag{C.10}$$

where both the $\mathbf{1}$ and $\mathbf{7}$ are Majorana. Acting with the SO(7) invariants on $\xi^I_{\mathbf{7}}$ does not generate any additional spinors in the $\mathbf{7}$, and we can without loss of generality take

$$|\xi^0|^2=|\xi^I_{\mathbf{7}}|^2=1,\quad \xi^{0\dagger}\xi^I_{\mathbf{7}}=0. \tag{C.11}$$

Thus we only have 1 spinor in the $\mathbf{7}$ and the most general Majorana spinors we can write on M_8 are $\chi=\sqrt{2}e^{\frac{A}{2}}(\chi_++\chi_-)$ where[13]

$$\chi_+=\begin{pmatrix}a_+\\0\end{pmatrix}\otimes\xi^I_{\mathbf{7}},\quad \chi_+=\begin{pmatrix}0\\ia_-\end{pmatrix}\otimes\xi^I_{\mathbf{7}} \tag{C.12}$$

where $a_\pm$ are real functions subject to $|a_+|^2+|a_-|^2=1$ — which are clearly rather constrained. The bi-linears of each component of $\xi^I_{\mathbf{7}}$ give rise to another 7 weak G_2 holonomy 3-forms as

$$\xi^{(I)}_{\mathbf{7}}\otimes\xi^{(I)\dagger}_{\mathbf{7}}=\frac{1}{8}(1+i\phi^{(I)}_3+\star_7\phi^{(I)}_3+\text{vol}(S^7)). \tag{C.13}$$

As $\phi^{(I)}_3$ are charged under SO(7) they are clearly all independent of ϕ^0_3, so there is no way to generate the invariant forms in the flux in (C.5) from (C.3c)–(C.3f), thus we must have

$$f_2=f_3=0,\quad\Rightarrow\quad F_4=0. \tag{C.14}$$

This makes the flux purely electric and it is proved in [23], that for all such solutions AdS_3 experiences an enhancement to AdS_4. As there is no longer anything breaking the isometries of the 7-sphere locally, clearly then this ansatz just leads to local $AdS_4\times S^7$. The only global possibility beyond the standard $\mathcal{N}=8$ M2 brane near horizon is an orbifolding of the 7-sphere that breaks supersymmetry to $\mathcal{N}=7$ — in any case this is certainly in no way an AdS_3 vacuum.

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