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EXAMPLES OF MODEL CORRELATION WITH CLOSELY SPACED MODES

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ABSTRACT

In structural dynamics, two modes with natural frequencies ω_1 and ω_2, respectively, are closely spaced if the frequency separation $\Delta\omega = \omega_2 - \omega_1$ is very small. If $\Delta\omega = 0$, the modes are repeated. On the other hand, it is well known that closely spaced modes are highly sensitive to small perturbations of mass and stiffness.

When a system with closely spaced eigenvalues is perturbed, the associated mode shapes are mainly rotating in their initial subspace. This means that we can have a good correlation in terms of mass and stiffness between the models, but low values of modal assurance criteria (MAC) can be obtained because of this rotation. In this case, the individual mode shapes should not be used for correlation using the modal assurance criteria (MAC), but the subspaces spanned by the unperturbed and the perturbed mode shapes should be correlated.

If we still want to measure the correlation using MAC, the experimental mode shapes must be previously rotated in the subspace in order to get the best correlation between the experimental and the numerical mode shapes.

In this paper, three models with closely spaced modes are studied. Firstly, an analytical model with 4 DOF's and two repeated eigenvalues is perturbed with small mass changes. The other two models are experimental models with closely spaced modes which are correlated with two numerical models assembled in ABAQUS and ANSYS. The experimental mode shapes were rotated in the subspace to get the best correlation between the models in terms of MAC.

Keywords: Model correlation, closely spaced modes, Operational Modal Analysis, Rotation matrix

1. INTRODUCTION

1.1. General information

In structural dynamics, closely spaced modes are defined as modes which are close in frequency [1-3]. A rule of thumb to define a set of mode shapes as closely spaced was proposed in [3]. If we consider two modes with close natural frequencies ω_1 and ω_2, and frequency distance $\Delta\omega = \omega_2 - \omega_1$, they can be considered closely spaced if:

$$\frac{\Delta\omega}{\omega} < \frac{1}{1000} \tag{1}$$

where $\omega = \omega_1$

The Modal Assurance Criterion (MAC) [4-5] is by far the most widely used technique to compare mode shapes. If two vectors $\boldsymbol{\phi_{FEi}}$ and $\boldsymbol{\phi_{Xj}}$, corresponding to a numerical and an experimental model, respectively, are compared, the MAC is given by:

$$MAC(\phi_{FEi}, \phi_{Xj}) = \frac{\left|\phi_{FEi}^H \cdot \phi_{Xj}\right|^2}{(\phi_{FEi}^H \cdot \phi_{FEi})(\phi_{Xj}^H \cdot \phi_{Xj})} \tag{2}$$

where the subindex 'H' indicates complex conjugate.

If a set of mode shapes are compared, a MAC matrix is obtained, which can be presented in different formats: matrix, table, 2D or 3D plot.

Closely spaced modes are highly sensitive to small mass and stiffness perturbations of the system, and they mainly rotate in their subspace [1,2,3]. Thus, we can have a good correlation in terms of mass and stiffness between the compared models, but low values of MAC can be obtained because of this rotation. This means that for closely spaced modes, correlation between different identification estimates or between a numerical model and an experimental model, should be calculated between subspaces and not between the individual mode shape vectors [2,3].

According to the structural dynamic modification (SDM) [6], the experimental mode shapes can be expressed as a linear combination of the numerical mode shapes, i.e.:

$$\boldsymbol{\phi_X} = \boldsymbol{\phi_{FE}T} \tag{3}$$

where $\boldsymbol{T}$ is a transformation matrix.

If mass normalized mode shapes are used in eq. (3) to estimate the matrix $\boldsymbol{T}$, it was demonstrated in [2] that, in case of closely spaced modes, matrix $\boldsymbol{T}$ is related to the rotation matrix **R** as:

$$\boldsymbol{T} = \boldsymbol{R^T} \tag{4}$$

In closely spaced modes the mode shapes mainly rotate in a subspace, a measure of the correlation can be obtained by means of the maximum angle θ between the subspaces defined by the experimental $\boldsymbol{\phi_X}$ and the numerical $\boldsymbol{\phi_{FE}}$ closely spaced mode shapes. This angle can be expressed as a MAC value [4,5] by:

$$MAC = \cos^2(\theta) \tag{5}$$

If the correlation is measured using MAC, the experimental mode shapes must be previously rotated in the subspace in order to get the best correlation between the experimental and the numerical mode shapes [2,3].

D'ambrogio and Fregolent [7] proposed the concept of S2MAC, similar to the MAC between two modal vectors, to correlate an experimental mode shape ϕ_X with a linear combination of two numerical closely spaced mode shapes $\boldsymbol{\phi}_{FE1}$ and $\boldsymbol{\phi}_{FE2}$, which is expressed as:

$$S2MAC = \max_{\alpha,\beta}\left(\frac{\left|\boldsymbol{\phi}_X^H(\alpha\boldsymbol{\phi}_{FE1}+\beta\boldsymbol{\phi}_{FE2})\right|^2}{\boldsymbol{\phi}_X^H\boldsymbol{\phi}_X(\alpha\boldsymbol{\phi}_{FE1}+\beta\boldsymbol{\phi}_{FE2})^H(\alpha\boldsymbol{\phi}_{FE1}+\beta\boldsymbol{\phi}_{FE2})}\right) \tag{6}$$

If case of normal modes, eq. (6) leads to:

$$S2MAC = \frac{\left(\boldsymbol{\phi}_X^T\boldsymbol{\phi}_{FE1}\right)^2 - 2\left(\boldsymbol{\phi}_X^T\boldsymbol{\phi}_{FE1}\right)\left(\boldsymbol{\phi}_{FE1}^T\boldsymbol{\phi}_{FE2}\right)\left(\boldsymbol{\phi}_X^T\boldsymbol{\phi}_{FE2}\right) + \left(\boldsymbol{\phi}_X^T\boldsymbol{\phi}_{FE2}\right)^2}{1-\left(\boldsymbol{\phi}_{FE1}^T\boldsymbol{\phi}_{FE2}\right)^2} \tag{7}$$

In this paper, three cases with closely spaced modes are studied. Firstly, a 4 DOF system with two repeated modes is perturbed with small mass changes. Then, the experimental modal parameters of a square laminated glass plate are used for correlating the results of a numerical model assembled in ANSYS [8]. Finally, the experimental modal parameters of a symmetric lab-scaled two-floor steel frame are compared with those extracted from a numerical model also assembled in ABAQUS [9].

2. A SIMULATION CASE

A 4 DOF system (system U) with two repeated eigenvalues was simulated with MATLAB [10]. The natural frequencies and the mode shapes are shown in Tables 1 and 2, respectively. The mass and the stiffness matrices were calculated from the eigenvalues and the eigenvectors. However, the solution of the eigenvalue problem gives same eigenvalues but a different set of mode shapes (see Table 3). Nevertheless, the mode shapes in Table 3 are linear combinations of those shown in Table 2.

Table 1. Natural frequencies of the two simulated systems.

Frequencies [Hz]		
System U	System P	Error [%]
0.4502	0.4408	4.12
0.4502	0.4479	1.03
0.5513	0.5409	3.74
0.6164	0.6060	3.34

Table 2. Mode shapes of system U (original)

0.3000	0.5000	1.0000	1.0000
0.8000	1.2000	0.0000	-1.0000
1.1000	0.1000	-1.0000	1.0000
1.5000	-1.0000	1.0000	-0.8000

Table 3. Mode shapes of system U (after solution of the eigenvalue problem)

-0.4507	0.3700	1.0000	-1.0000
-1.1569	0.8611	0.0000	1.0000
-1.0696	-0.2757	-1.0000	-1.0000
-1.0764	-1.4462	1.0000	0.8000

This system U was perturbed with small mass changes, the mass change matrix being presented in Table 4. The natural frequencies of the perturbed system (system P) are shown in Table 1.

Table 4. Mass change matrix (kg)

0.0170	0	0	0
0	0	0	0
0	0	0.0120	0
0	0	0	0.0100

The MAC matrix between the mode shapes of both systems (perturbed and unperturbed from Table 3) presented in Table 5. As it can be observed, the correlation between the first two modes is very poor.

Table 5. MAC

0.6979	0.3807	0.0135	0.0233
0.4990	0.4177	0.0620	0.0082
0.0660	0.0128	0.9993	0.0509
0.0272	0.0002	0.0631	0.9985

The rotation matrix presented in Table 7 was estimated from the transformation matrix $\boldsymbol{T}$ obtained with eq. (3), and which is shown in Table 6. Mass normalized numerical and experimental mode shapes were used to estimate the matrix $\boldsymbol{T}$.

Table 6. $\boldsymbol{T}$ matrix

0.7390	-0.6522	0.0152	-0.0257
0.6421	0.7506	0.0141	0.0310
-0.0138	-0.0001	0.9809	-0.0139
-0.0007	-0.0217	0.0111	0.9826

Table 7. Rotation matrix

0.7549	0.6559
-0.6559	0.7548

This rotation matrix (Table 7) was used to rotate 40.98º the perturbed closely spaced mode shapes by:

$$\boldsymbol{\phi_R} = \boldsymbol{\phi}\, \boldsymbol{R^T} \tag{8}$$

in order to obtain the best fit between the unperturbed and the perturbed mode shapes.

The new MAC obtained after the rotation of the mode shapes is shown in Table 8, where it can be observed that a very good correlation exists between both systems, which confirms that the discrepancies in terms of mass and stiffness are very low. The angle between the subspaces spanned by the closely spaced modes is 1.4727º (MAC=0.9993), which confirms the slight discrepancies between the two models. The angle between the first and the second unperturbed mode shapes is 78.4º, whereas that between the corresponding perturbed mode shapes is 79.05 º.

Table 8. MAC after rotation

0.9998	0.0383	0.0135	0.0233
0.0382	0.9995	0.0620	0.0082
0.0194	0.0764	0.9993	0.0509
0.0147	0.0186	0.0631	0.9985

3. A SQUARE LAMINATED GLASS PLATE

In this section, a square laminated glass plate with dimmensions 1400 x 1400 mm, and consisting of two glass layers with thickness 4 mm, and one polymeric interlayer with thickness 1.14 mm, is studied. The plate was fixed to a steel frame at the four corners (see Fig. 1).

A 3D finite element model was assembled in ANSYS using 20 node structural solid elements of type SOLID186 (see Fig. 2). The finite element model was meshed with 19200 elements and 97767 nodes. The numerical natural frequencies are shown in Table 9, and as it can be observed, modes 2 and 3 have repeated frequencies. The modes shapes are presented in Fig. 3.

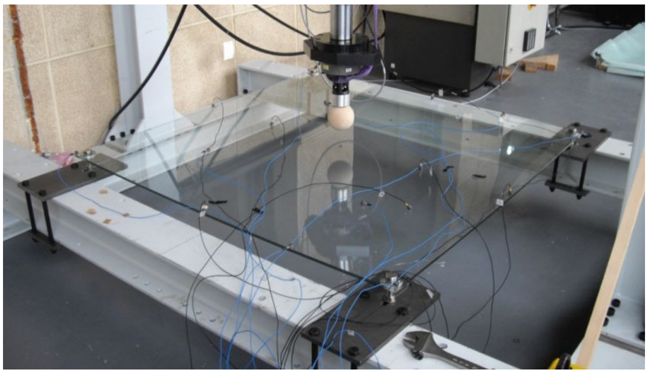

Figure 1. Data set used in the experiments.

The natural frequencies and the mode shapes were also estimated with operational modal analysis, The responses were measured with 16 accelerometers with a sensitivity of 100mV/g and registered with a TEAC LX-120 data recorder with 16 input channels. The plate was excited applying many randon small hits across the surface. The natural frequencies estimated with the EFDD (frequency domain decomposition) technique are shown in Table 9.

Table 9. Natural frequencies of the laminated glass plate.

Mode Shapes	Experimental [Hz]	Numerical [Hz]	Error [%]
Mode 1	9.35	9.72	3.80
Mode 2	19.62	21.10	7.01
Mode 3	19.83	21.12	6.10
Mode 4	22.53	24.82	9.22
Mode 5	55.76	56.11	0.62

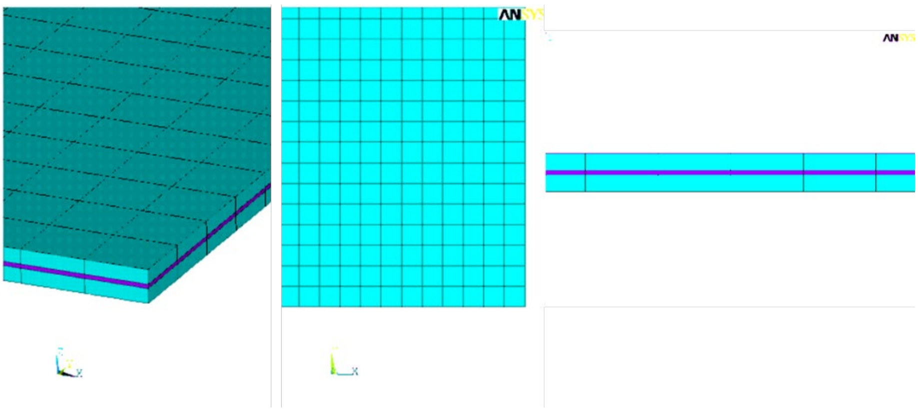

Figure 2. Numerical model.

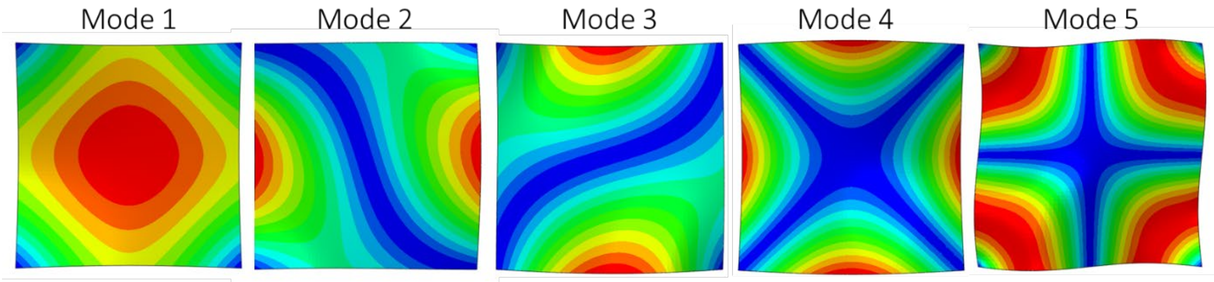

Figure 3. Numerical mode shapes

The MAC between the experimental and the numerical mode shapes is presented Table 10. As it can be observed there is no good correlation between the experimental and numerical models for the second the third modes.

Table 10. MAC.

0.9971	0.0000	0.0000	0.0000	0.0661
0.0000	0.5990	0.5099	0.0001	0.0002
0.0001	0.3965	0.4896	0.0000	0.0007
0.0000	0.0000	0.0002	0.9996	0.0000
0.0976	0.0001	0.0000	0.0000	0.9862

It can be seen Figure 4 that modes 2 and 3 are physically rotated, which explains the bad correlation between these two modes.

From the transformation matrix $\boldsymbol{T}$ (see Table 11), the rotation matrix shown in Table 12 was obtained using the same procedure presented in section 2. Mass normalized numerical mode shapes and experimental mode shapes normalized to the largest component equal to unity were used to estimate the matrix $\boldsymbol{T}$. From Table 12 it is inferred that the rotation angle is approximately 42.5º.

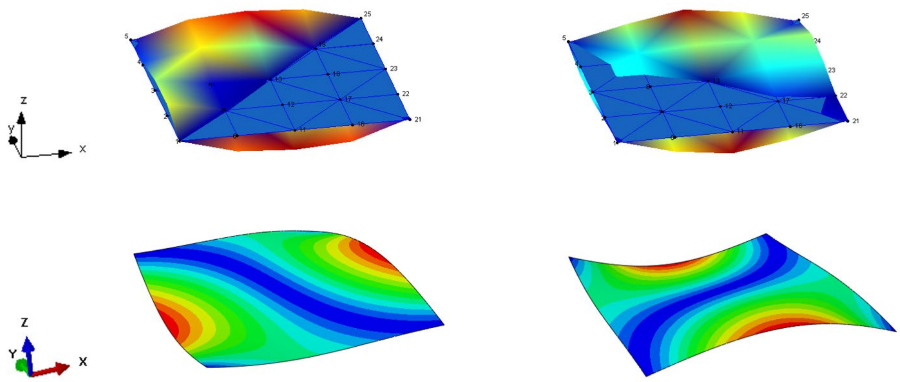

Figure 4. Numerical and experimental mode shapes 2 and 3

Table 11. ***T*** matrix.

1.4818	0.0063	-0.0065	-0.0006	0.0414
0.0018	0.9253	-0.8563	0.0111	0.0178
-0.0107	0.7523	0.8395	-0.0039	0.0313
-0.0011	0.0064	-0.0140	-0.9801	0.0005
-0.0327	0.0101	-0.0041	0.0011	1.0582

Table 12. Rotation matrix.

0.7379	0.6479
-0.6479	0.7379

Finally, the MAC between the rotated experimental mode shapes and the numerical mode shapes (Table 13) show a very good correlation between the two models.

An alternative to MAC is to calculate the angle between the subspaces spanned by the closely spaced modes. It is also interested to know if there is a relative deviation between the closely spaced eigenvectors (perfect rotation means no deviation). For this example, the angle between the subspaces spanned by the second and the third modes is 3.8203º (MAC= 0.9956). With respect to the angle between the second and the third numerical mode shapes, they are perfectly orthogonal (angle 90º), whereas in the experimental system the angle is 89.158º. These values confirm the good correlation between the two models in terms of mode shapes.

Table 13. MAC after rotation.

0.9971	0.0000	0.0000	0.0000	0.0661
0.0000	0.9965	0.0000	0.0001	0.0002
0.0001	0.0001	0.9974	0.0000	0.0007
0.0000	0.0002	0.0001	0.9996	0.0000
0.0976	0.0000	0.0000	0.0000	0.9862

4. A LAB-SCALED TWO-FLOOR STEEL FRAME

In this section a small symmetric lab scaled steel frame is studied (see Fig.5). The structure consists of four columns with square section 5 x 5 mm^2 and length 80 mm, and two square steel floors with thickness 5 mm and dimensions 30mm x 30mm.

Figure 5. Two-floor steel frame structure.

A model of the structure was assembled in ABAQUS and meshed with beam elements B33 (columns) and quadrilateral shell elements S4R (floors). The numerical natural frequencies are shown in Table 14 and the mode shapes in Fig 6.

The experimental modal parameters were estimated with operational modal analysis. The response of the structure was measured with 6 accelerometers with a sensitivity of 100mV/g and registered with a TEAC LX-120 data recorder. The natural frequencies estimated with the EFDD technique are also shown in Table 14.

Table 14. Experimental and natural frequencies

Mode Shapes	Experimental [Hz]	Numerical [Hz]	Error [%]
Mode 1- 1st bending	4.2200	4.2490	0.69
Mode 2- 1st bending	4.4280	4.2490	4.04
Mode 3-torsion	7.6735	7.8572	2.39
Mode 4-2nd bending	11.1262	11.680	4.98
Mode 5-2nd bending	11.3784	11.680	2.65
Mode 6-torsion	20.2675	21.401	5.59

Table 15. MAC.

0.9968	0.0026	0.0017	0.0004	0.0003	0.0000
0.0042	0.9773	0.3618	0.0001	0.0003	0.0066
0.0019	0.3708	0.9963	0.0001	0.0000	0.0003
0.0002	0.0000	0.0001	0.9013	0.0985	0.0346
0.0000	0.0006	0.0012	0.3132	0.6827	0.2030
0.0001	0.0022	0.0001	0.0002	0.3515	0.9992

Due to the symmetry of the structure, all the bending modes are repeated (see Fig. 6), i.e. modes 1 and 2 has repeated frequencies, and the same for modes 4 and 5. MAC between the numerical and experimental mode shapes are presented in Table 15.

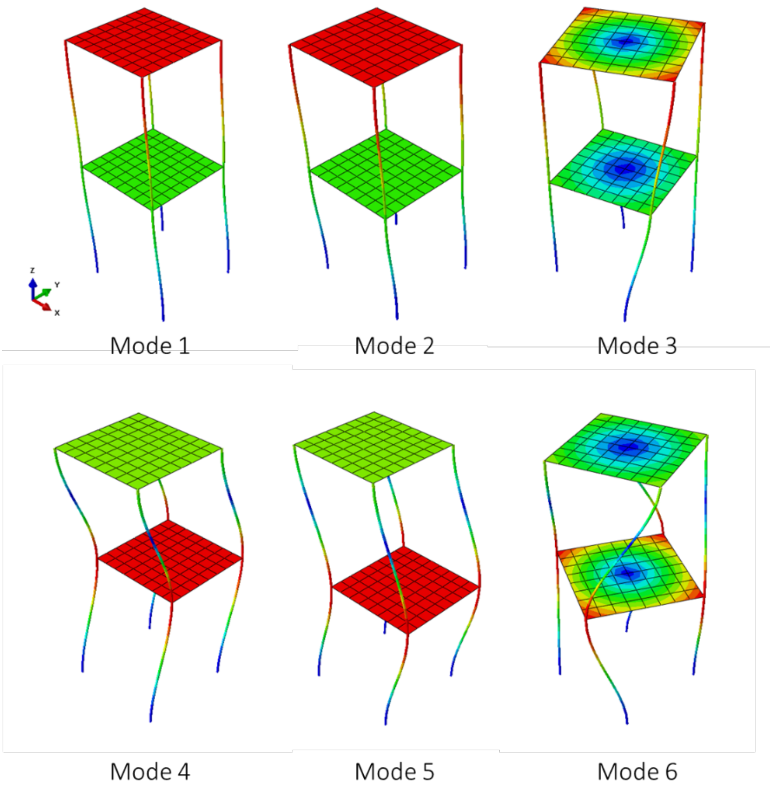

Figure 6. Numerical mode shapes.

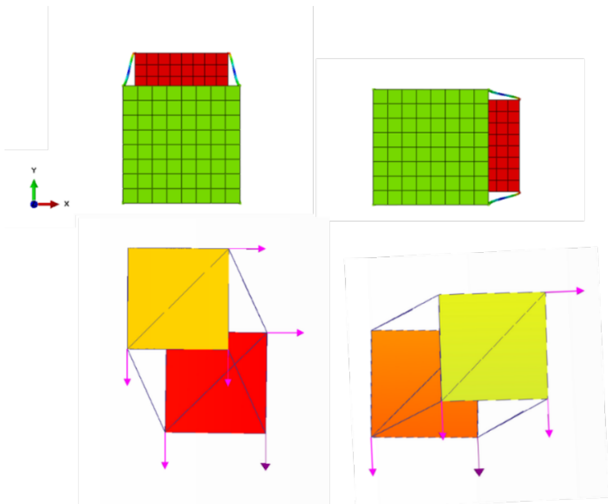

Figure 7. Numerical and experimental mode shapes 4 and 5.

Table 16. *T* matrix.

-0.0697	0.0026	-0.0011	0.0001	0.0000	-0.0001
-0.0040	-0.0610	-0.0020	-0.0001	-0.0002	0.0000
0.0005	0.0012	0.0196	-0.0001	-0.0002	0.0000
0.0002	-0.0004	0.0002	-0.0095	-0.0021	-0.0001
-0.0013	-0.0037	0.0004	0.0044	-0.0045	-0.0002
0.0002	0.0039	-0.0001	0.0001	0.0001	-0.0043

Mass normalized numerical mode shapes and experimental mode shapes normalized to the unit length were used to estimate the matrix ***T*** (see Table 16), from which the rotation matrix shown in Table 17 was obtained using the same procedure presented in section 2. From Table 17 itis inferred that the first bending modes were rotated 3.5º and the second bending modes were rotated approximately 24.7º.

Table 17. Rotation matrix

-0.9981	-0.0580	0.0000	0.0000	0.0000	0.0000
0.0580	-0.9981	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	-0.9082	0.4183	0.0000
0.0000	0.0000	0.0000	-0.4183	-0.9082	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

It can be seen in Table 18 that the MAC has improved for both the first and the second bending modes. The angles between the subspaces spanned by the closely spaced modes are 7.8436º (MAC = 0.9814). for modes 1 and 2 and 3.7228º (MAC = 0.9958) for modes 4 and 5. In the numerical model, all the bending modes are orthogonal (angle 90º), whereas in the experimental system the angle between the first bending modes is 87.65º, and 75.96º between the fourth and fifth modes.

A relatively low MAC has been obtained between the mode shapes of the fifth modes. On the other hand, the fourth and fifth experimental mode shapes are far from orthogonal, as it is the case in the numerical model. This can be attributed to discrepancies between the models or to errors in the estimation of the mode shapes.

Table 18. MAC after rotation.

0.9997	0.0001	0.0001	0.0004	0.0001	0.0000
0.0011	0.9864	0.3652	0.0001	0.0001	0.0030
0.0013	0.3710	0.9970	0.0001	0.0000	0.0003
0.0002	0.0000	0.0002	0.9737	0.0259	0.0103
0.0001	0.0006	0.0004	0.1984	0.8001	0.2510
0.0001	0.0022	0.0001	0.0002	0.3515	0.9992

5. CONCLUSIONS

According to the structural dynamic modification, an experimental system can be considered as a perturbation of a numerical model. When a numerical system with closely spaced eigenvalues is perturbed, the associated mode shapes are mainly rotating in their initial subspace [1,2,3]. This means that low MAC values can be obtained although a good correlation can exist in terms of mass and stiffness.

In order to obtain the best correlation in terms of MAC, the mode shapes have to be previously rotated. In this paper, the mode shapes of three models with closely eigenvalues have been successfully rotated to obtain the best correlation in terms of MAC. The first model was a simulated case with two repeated eigenvalues, which was perturbed with small mass changes. The other two models are experimental models with closely spaced modes which are correlated with two numerical models assembled in ANSYS and ABAQUS. The results have demonstrated that a good correlation exist between the numerical and the experimental models.

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