Editorial

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Numerical Analysis & No Regrets. Special Issue Dedicated to the Memory of Francisco Javier Sayas (1968–2019)

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Abstract: This is the preface of a special issue dedicated to the memory of Francisco Javier Sayas who passed away on April 2, 2019. The articles reflect Sayas' main research interests in the numerical analysis of partial differential equations, containing contributions on the scattering and propagation of acoustic and electromagnetic waves, and the analysis of discontinuous Galerkin schemes, boundary element methods, and coupled schemes. We discuss the main contributions of Sayas and give an overview of the results covered by this special issue.

Keywords: Boundary Element Method, Discontinuous Galerkin Method, Hybridizable Galerkin Method, Time Domain, Wave Equation, Scattering

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Francisco Javier Sayas was a mathematician with broad interests, reaching from abstract analysis over numerical methods in the widest sense to scientific computing and applications in physics and engineering. He was a very influential and well liked academic among colleagues and students, dedicated to research and teaching, only slightly distracted by his passion for classical music.

Javier[1] received his PhD in 1994 at the University of Zaragoza under the guidance of Michel Crouzeix from the University of Rennes. He continued to stay several years in Zaragoza, first as an Instructor and then as an Associate Professor. In 2007 he moved for three years as a Visiting Associate Professor to the University of Minnesota, collaborating there with Bernardo Cockburn. In 2010 he became Associate Professor with tenure track at the University of Delaware, being promoted to Full Professor with tenure in 2013. For several years he was the Director of Graduate Studies, enjoying to teach every course of the program at least once, and reaching to supervise fifteen PhD students during his career. After a short battle with cancer, he died on April 2, 2019 at the age of 50. Just shortly before his death, he published his third book [43] on partial differential equations, co-authored with his students Thomas Brown and Matthew Hassell, and organized the farewell meeting *Numerical Analysis & No Regrets*, whose title we dare to use for this issue.

Most of the articles in this issue stem from presentations given at a Tribute to Francisco Javier Sayas, organized as a minisymposium at the XXVI Congreso de Ecuaciones Diferenciales y Applicaciones/XVI Congreso de Matemática Aplicada, June 14–18, 2021, in Gijón, Spain. In the following we discuss some of his main contributions and give an overview of the articles in this issue.

1 He was also called Francisco and Pancho.

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Javier's initial interests were formed during his PhD years under the guidance of Michel Crouzeix and focused on the solution of boundary integral equations. In his thesis he developed error estimates for boundary element approximations through asymptotic expansions. A part of it, dealing with spline Galerkin approximations, was published some years later [13]. Together with his student Ricardo Celorrio, he also analyzed collocation schemes [4]. Throughout his career, Javier continued to work on boundary integral equations and their approximations, being influential in coupled methods and the solution of wave problems.

Together with collaborators, Javier developed the coupling of boundary elements with local discontinuous Galerkin approximations [23, 27] and with hybridizable discontinuous Galerkin (HDG) schemes [9, 11]. One of his greatest achievements is the proof of well-posedness of the Johnson–Nédélec finite element-boundary element coupling for Lipschitz domains [39], reprinted in SIAM Review [41]. This result paved the way for several non-symmetric coupling variants: with mixed finite elements [35], interior penalty discontinuous Galerkin schemes [30], HDG [22], and finite volume methods [19]. Javier's passion for the coupling of boundary elements with domain-based discretizations led to fruitful collaborations and friendships with the authors of this preface. Through these collaborations he developed strong ties with the Chilean community and made important contributions to the analysis of coupled problems and mixed finite element approximations. For instance, he co-supervised Ricardo Oyarzúa in his work on the Stokes–Darcy problem [25, 26], a research that had started with Oyarzúa's undergraduate thesis, cf. [24].

Javier was a leading expert in the solution of wave problems by boundary integral methods, specifically in the development and analysis of time domain methods. Together with students and collaborators he employed convolution quadrature [33], provided an analysis of layer potentials and boundary integral operators [16], derived energy estimates for Galerkin semi-discretizations [40], analyzed systems of time domain boundary integral operators [36], and proposed coupled schemes [2]. This research culminated with his book on retarded potentials and time domain boundary integral equations [42].

During his years at the University of Minnesota, working with Bernardo Cockburn, Javier became interested in the HDG method, making substantial contributions in the coming years. Together with Bernardo Cockburn and Jay Gopalakrishnan, he developed an error analysis framework based on projections [8] and worked on divergence conforming HDG schemes [10] (with Cockburn). Together with students and a collaborator, he created and analyzed HDG schemes for elastodynamics [18, 31], and wrote a book on the theory of HDG schemes [17] (with his student Shukai Du). For a first-hand report on the HDG contributions of Javier we refer to the article [6] by Cockburn.

Scientific computing and applications were an integral part of Javier's research activities. His publications cover, for instance, Euler–MacLaurin expansions [38], [5] (with Ricardo Celorrio), error estimates for convolution quadrature [20] (with Hasan Eruslu), and MATLAB implementations of the Argyris element [15] (with Victor Domínguez) and the HDG method in three dimensions [21] (with Zhixing Fu and Luis F. Gatica). Together with his students – Team Pancho – he made their MATLAB codes for the boundary element and HDG methods available to the community [46, 47].

The contributions to this issue reflect the research interests of Javier. They cover recent achievements in the numerical analysis of acoustic and electromagnetic wave problems, the design and analysis of boundary integral and discontinuous Galerkin (DG) methods, eigenvalue problems in linear elasticity and spectral methods for non-local operators. Let us discuss the contributions in some detail.

In [14], Domínguez, Nigam, and Ovall study eigenvalue problems appearing in linear elasticity. They give a proof of Korn's inequality for the case of vanishing normal or tangential displacements, and prove the well-posedness of the related Lamé eigenvalue problems. Shi, Antil, and Kouri present numerical techniques for the solution of fractional-order differential equations on simple domains [45]. They use the Caffarelli–Silvestre extension and a spectral method with ultraspherical and Fourier polynomials. Several numerical experiments illustrate the performance of their discretizations.

Two papers deal with discontinuous Galerkin methods. In [12], Colmenares, Oyarzúa, and Piña present a DG approximation for the stationary Boussinesq system that models a certain fluid flow with buoyancy forces. All the variables are discretized in the discontinuous fashion. Using fixed-point techniques, the authors prove well-posedness of the continuous formulation and its discrete version. The scheme provides a divergence-free approximation of the velocity and the a priori estimates do not require a small-data assumption. Cockburn, Du, and Sánchez study the time-dependent Maxwell equations [7]. They present a method where jump stabilization terms are defined via time integrals or time derivatives of the unknown functions. Different methods are analyzed, considering several combinations of time derivative and time integral, for the stabilization terms in the numerical fluxes of the electric or magnetic field. For a discretization in time with the midpoint rule, the authors prove a discrete energy conservation.

In the paper [34] by Louër and Rapún, time-harmonic acoustic scattering problems are considered. They study single-layer based approximations for penetrable, sound-soft, and hard-soft obstacles in three dimensions. Their method is a spherical harmonic-based Petrov–Galerkin scheme introduced in [29]. Additionally, they study the inverse problems of recovering the location and shape of obstacles.

Four contributions study coupled schemes. In [37], Sánchez, Sánchez–Vizuet, and Solano consider the method from [11] for the coupling of HDG with boundary elements. The underlying model problem is of second order and is posed in two space dimensions. The authors provide a proof for the convergence of the scheme, not given in [11], and an a priori error analysis. The papers [1, 28] analyze symmetric coupling schemes of finite and boundary elements for the time domain wave equation in the full space. Banjai [1] considers obstacle problems and proposes leapfrog time-stepping and convolution quadrature for the time discretization that is based on a truncated trapezoidal rule. He gives a complete stability and convergence analysis. His paper can be seen as a natural extension of his previous work with Lubich and Javier [2]. Gimperlein, Özdemir, and Stephan consider a transmission problem for the wave equation [28]. They prove the stability of the coupled scheme and derive a priori error estimates in the energy norm, employing approximations by piecewise polynomials in time and space. They also provide an a posteriori error estimator and show its reliability, and study the singular behavior of solutions at the boundary of interfaces. The fourth contribution on the analysis of coupled systems is by Schulz and Hiptmair [44]. They present a framework for domain-boundary systems with boundary integral operators of the first kind to elaborate on the presence of spurious modes for wave problems from acoustics and electromagnetism. They show that possible non-trivial kernel contributions of the domain-boundary system are eliminated by the representation formula for the exterior solution.

The contributions [3, 32] are on numerical methods for electromagnetic waves. Beni Hamad, Beck, Imperiale, and Joly study wave propagation in coaxial cables [3], and Lähivara, Monk, and Selgas consider wave scattering at multiple obstacles [32]. The method by Beni Hamad et al. employs the transverse and longitudinal components of the electric field, discretizing them by Nédélec and Lagrangian finite elements, respectively. Time discretization is of an implicit-explicit type, combining the leap-frog and Newmark centered schemes. The authors prove the stability of their method under a CFL condition, and present numerical experiments, comparing their results with the one-dimensional Telegrapher model. In [32], Lähivara, Monk, and Selgas propose a time domain linear sampling method for the inverse problem of scatterer reconstruction. They provide an analysis for impedance boundary conditions and illustrate their findings with several numerical experiments.

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