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New forestry tools for natural beech forests in northwestern Spain

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Abstract

Aim of study: Although beech (*Fagus sylvatic*a L.) forests in north-western Spain constitute c.a. 40% of the total area occupied by the species in the whole country, no growth or yield studies have been carried out regarding these forests. The specific objective of this study was to elaborate yield tables and stand density management diagrams for the beech forests.

Area of study: Asturias and León provinces (NW Spain).

Materials and methods: Sample plots (n=112) were established in natural beech forests, and 60 dominant trees were felled for sampling. The Asturias Government Forest Service provided data on another 351 felled trees. Yield tables and stand density management diagrams (SDMDs) were elaborated to estimate tree volume and biomass in the study area for the first time.

Main results: These forests are more productive than expected. Although they are currently not managed for forestry purposes, they could be managed again in the future and the tools are now available for this purpose.

Research highlights: The study generates new user-friendly tools to manage beech forests in northwestern Spain. These tools will also enable simulations to be conducted to determine the potential carbon storage or the capacity of the stands to sequester atmospheric carbon.

Additional key words: *Fagus sylvatica;* yield tables; SDMDs; biomass

Abbreviations used: RS (relative spacing index); SDMD (stand density management diagram); SI (site index).

Authors' contributions: JCS: data acquisition, analysis and writing. JRO: supervision and editing. MBA: conceptualization, analysis, supervision, writing & editing.

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Introduction

Forest management decisions are based on current and expected future conditions. For instance, determining standing volume is useful for research and practical purposes in forestry and contributes to sustainable forest management. However, measuring tree volume and biomass is expensive, and equations must be used to predict tree or stand values. Growth and yield models can be used to update inventories, predict future yields and to explore management alternatives, all of which help with decision-making (Vanclay, 1994).

These models can be displayed in user-friendly interfaces, such as yield tables and stand density management diagrams (SDMDs). Yield tables present values of all the main growth and yield variables for a sequence of stand ages (Matthews *et al.,* 2016), while SDMDs are average stand-level models that graphically illustrate the relationships between yield, density, and density-dependent mortality at all stages of stand development (Newton, 1997).

In Spain, beech (*Fagus sylvatica* L.) forests are distributed between the Cantabrian Range-Pyrenees (with oceanic influences) and the northern Iberian Range (with moderate continentality), with several relict areas representing southern refuges (Costa *et al.,* 1998). According to the Spanish National Forest Inventory, beech forests cover an area of 387,776 hectares in Spain. Beech stands in NW Spain constitute approximately 40%, but they only represent 23% of the total volume (DGCN, 2006). Their importance as habitats for endangered and/or key species has led to forest management being discontinued, except for occasional cutting to produce domestic firewood. However, these forests were managed in the past by means of short-duration shelterwood and could possibly be managed again in the future.

In Spain, beech yield tables have only been developed for La Rioja (Iberian Range) (Ibáñez, 1989) and Navarra (Pyrenees) (Madrigal *et al.,* 1992). SDMDs have not been developed for beech, and no similar studies have yet been conducted in the beech forests in Asturias and León. Thus, the aim of the present study was to elaborate both types of tools for these beech forests.

Material and methods

In total, 112 sampling plots were established in natural beech-dominated (≥90% of standing basal area) forests throughout the northwestern Cantabrian Range (Spain), to cover the existing range of stand structures, densities and site qualities in the area. Specifically, 97 plots were established in Asturias and 15 in León. Destructive sampling was planned in 50 of these plots to determine various metrics. However, due to prevailing forestry policy, the metrics had to be measured in a maximum sample of 30 plots. Finally, 30 plots were selected to represent all site qualities (24 plots in Asturias and 6 in León) (Fig. 1). The first two dominant trees found outside the plots to the east and west, but in the same stands, were felled. In addition, the Asturias Government Forest Service provided data on 351 trees of all sizes felled for tree volume calculations involved in elaborating former management plans for some of the same forests where sampling plots were established (Arévalo, 1954; 1956; 1960; Ramos, 1969). Summary statistics for individual tree and stand variables are shown in Table S1 [suppl].

In the first step, an individual volume equation was fitted to the data from 411 felled trees. The models used by Návar & Domínguez (1997) were used to predict volume with and without bark (Table S2 [suppl]). Subsequently, the previous volume equation was used to predict the volumes of all the trees in the 112 plots and thus obtain the total plot volume by adding the tree volumes. The stand volume equation was fitted to stand variables by using the same models proposed by Barrio (2003) for volume with and without bark (Table S2).

For elaboration of yield tables, a series of fundamental relationships that relate the predicted and measured dasometric variables must be established. The first relationship estimates the dominant height as a function of site index and age, and was already developed by Castaño-Santamaría *et al.* (2019). The second estimates the stand average height as a function of the dominant height. The third one predicts the variation in density over time. However, to avoid having to fit an equation for each site index, dominant height is used instead of age. The fourth relationship predicts the mean quadratic diameter of the stand as a function of density and dominant height. Finally, the fifth predicts timber volume as a function of basal area and dominant height. The models used by Madrigal *et al.* (1992) and Barrio (2003) were calibrated for each of these relationships. The subsequent construction of the yield tables was carried out according to Madrigal *et al.* (1992).

The forests under study have a heterogeneous silvicultural status that is reflected in the changes in tree density with different stand variables (age, dominant height, etc.), making construction of yield tables difficult. To resolve this problem, the plots were divided into two groups: plots with an above average relative spacing index (low-density plots, mean value = 28.02%), and plots with a below average relative spacing index (high-density plots, mean value = 16.86%), according to Barrio (2003).

Biomass was also included in the yield tables, because it is currently as important as volume (in relation to energy production, carbon sinks, etc.) (*e.g.* Field *et al.,* 2008; Ruiz-Peinado *et al.,* 2012). The total plot biomass was obtained by summing the total biomass of each tree

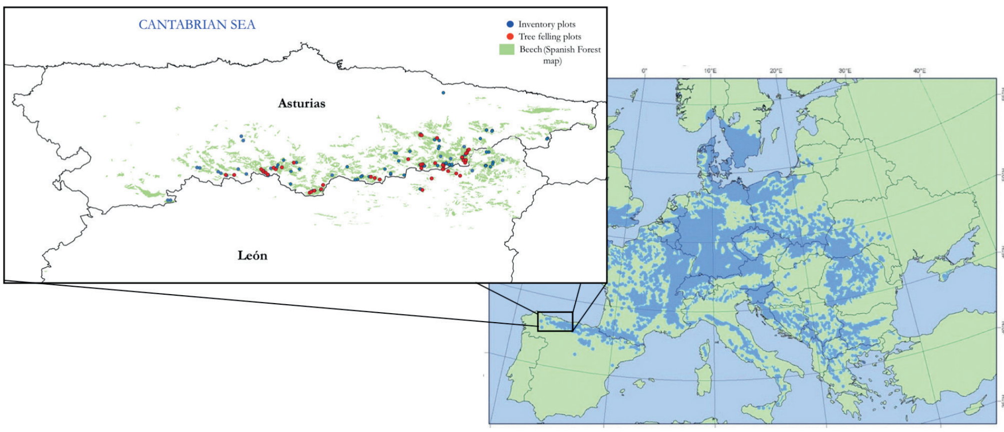

Figure 1. Location of the study area. Source of the Europe map: www.euforgen.org

calculated using the equations developed for the species by Ruiz-Peinado *et al.* (2012) as a function of diameter and height. From these data, the stand biomass equations (stem and total) were fitted using the models proposed by Castaño-Santamaría *et al.* (2013). All of the previous statistical analyses were conducted with the MODEL procedure of SAS/ETS® (SAS, 2004).

SDMDs consist of a system of two equations and the relative spacing (RS) index as basic components. The RS index is used to characterize the growing stock level and is calculated by dividing the average distance between trees by the dominant height and expressing this as a percentage. RS is a useful parameter in stand density management because it is generally independent of site quality and stand age, and because dominant height is one of the best criteria, from a biological point of view, for establishing thinning intervals (Barrio-Anta & Álvarez-González, 2005). The association between dominant height growth and forest production adds further utility to these diagrams for forest management purposes (Barrio-Anta *et al.,* 2006). As the beech forests under study are natural forests, we assumed triangular spacing between trees, so that RS can be expressed as follows:

$$RS = \frac{\sqrt{{}^{20000}/_{N\sqrt{3}}}}{H_0} \cdot 100$$

where RS is the relative spacing index (%), N is the number of stems per hectare, and H_0 is the dominant height (m).

The first step in constructing of the SDMD was to fit the non-linear system of the following two equation:

$$d_g = \beta_0 \cdot N^{\beta_1} \cdot {H_0}^{\beta_2}$$

$$V = \beta_3 \cdot {d_g}^{\beta_4} \cdot {H_0}^{\beta_5} \cdot N$$

where N is the number of stems per hectare, dg is the quadratic mean diameter (cm), H_0 is the dominant height (m), V is the stand volume (m^3 ha^{-1}) and βi (i = 0, 1, …, 5) represents the regression coefficients. These two equations define a structurally simultaneous system of equations, which is a generalization of the Seemingly Unrelated Regression method (Zellner, 1962). As the error components of the variables on the left-hand side and the right-hand side are correlated, the full information likelihood technique was used for simultaneously fitting of the equations with the MODEL procedure of SAS/ETS® (SAS, 2004). The final step in calculating the SDMDs consisted of representing the isolines for the growing stock and for the stand variables included in the stand-level model. In this study, dominant height was represented on the x-axis, and the number of trees per hectare in logarithmic scale was represented on the y-axis, following the method proposed by Barrio-Anta & Álvarez-González (2005). The isolines were obtained by solving for N in the previous equations.

Results and discussion

The expressions of the selected models are shown in Table 1. All of the parameter estimates were found to be significant at $p < 0.05$ (Table S3 [suppl]). The models yielded a good level of precision, and examination of the residuals revealed that all models were unbiased. However, several of these models differed from the original versions reported in the previous section (Návar & Domínguez, 1997; Barrio, 2003). The heteroscedasticity observed in most of the models was solved by applying a logarithmic transformation. In fitting logarithmic transformed models, a function of the variable is estimated instead of the variable itself. Therefore, the resulting systematic error must be corrected according to Meyer (1944), whose correction factor for this type of model was defined by Baskerville (1972). The models shown in Table 1 (except biomass and SDMD models) are the outcome of this process. The resulting yield tables are provided as supplementary material (Tables S4 to S11).

Tree data usually exhibit heteroscedasticity (Parresol, 1999). The logarithmic transformation is a variance-stabilizing transformation in which equation parameters can easily be estimated by the least squares procedure. In fact, it is the only transformation restricted to this procedure because it allows back transformation to the original parameters (Draper & Smith, 1998). However, there are some limitations associated with this type of transformation: i) it can only handle linear heteroscedasticity (Wilcox *et al.,* 2018) and ii) it tends to overestimate the true bias (Parresol, 1999).

Growth and volumes and biomass were higher and the rotation ages were longer in low-density plots than in high-density plots. The high density caused less growth and timber production in the studied plots. Castaño-Santamaría *et al.* (2019) distinguished four site qualities defined by heights of 6, 12, 18 and 24 m at a reference age of 80 years (Fig. S1 [suppl]). Our findings showed that rotation ages that maximize mean average increment in volume for these site qualities were more than 200, *c.a.* 170, *c.a.* 125, and *c.a.* 110 years for low-density plots, and *c.a.* 160, *c.a.* 105, *c.a.* 85 and *c.a.* 70 years for high-density plots, respectively. In the same way, the maximum mean annual increments were, respectively, 1.89, 3.94, 6.35 and 9.18 m^3 ha^{-1} $year^{-1}$ for low-density plots and 1.51, 2.89, 4.41 and 6.14 m^3 ha^{-1} $year^{-1}$ for high-density plots. Finally, total timber volumes at 150 years were respectively 264.4, 591.2, 935.2 and 1,295 m^3 ha^{-1} for low-density plots and 226.3, 410.6, 576.9 and 734.8 m^3 ha^{-1} for high-density plots.

Table 1. Best-performing models.

Model
Individual volume equation
$Vcc = 1.01222654 \cdot exp(-13.3833) \cdot d^{1.797344} \cdot h^{0.906457}$
$Vsc = 1.011298355 \cdot exp(-13.4562) \cdot d^{1.785746} \cdot h^{0.93368}$
Stand volume equation
$VCC = 1.00761799 \cdot G^{1.046832} \cdot {H_0}^{0.644167}$
$VSC = 1.008020473 \cdot G^{1.028428} \cdot {H_0}^{0.646096}$
Yield tables
$H_2 = \dfrac{23.8753 + X_0}{1 + 20526.03/X_0 \cdot {t_2}^{-1.51}}$
$X_0 = \dfrac{1}{2}\left(H_1 - 23.8753 + \sqrt{(H_1 - 23.8753)^2 + 4 \cdot 20526.03 \cdot H_1 \cdot {t_1}^{-1.51}}\right)$
$Hm = 1.01340619 \cdot {H_0}^{0.895929}$
High density: $N = 1.00630298 \cdot exp(11.77927) \cdot {H_0}^{-1.63664}$
Low density: $N = 1.0330 \cdot exp(11.89743) \cdot {H_0}^{-1.91428}$
High density: $d_g = 1.006302985 \cdot exp(5.632281) \cdot N^{-0.46971} \cdot {H_0}^{0.260024}$
Low density: $d_g = 1.00990481 \cdot exp(3.573036) \cdot N^{-0.33861} \cdot {H_0}^{0.701497}$
Biomass equations
$W_f = 0.5164583 \cdot G^{1.01162} \cdot {H_0}^{0.800449}$
$W_f = 0.5164583 \cdot G^{1.01162} \cdot {H_0}^{0.800449}$
SDMDs
$d_g = 402.6608 \cdot N^{-0.52725} \cdot {H_0}^{0.274108}$
$VCC = 0.000082 \cdot {d_g}^{1.785519} \cdot {H_0}^{0.952423} \cdot N$

Vcc = wood volume with bark (m^3 $tree^{-1}$). *Vsc* = wood volume without bark (m^3 $tree^{-1}$). *h* = tree height (m). *d* = tree diameter (mm). *VCC* = wood volume with bark (m^3 ha^{-1}). *VSC* = wood volume without bark (m^3 ha^{-1}). *G* = basal area (m^2 ha^{-1}). H_0 = dominant height (m). H_1 = predicted dominant height (m) at age t1 (years). H_2 = predicted dominant height (m) at age t_2 (years). *Hm* = average height (m). *dg* = squared mean diameter (cm). *N* = tree density (trees ha-1). *Wf* = stand stem biomass (Mg ha^{-1}). *Wt* = stand total biomass (Mg ha^{-1}).

Comparison of these results with the other yield tables developed for beech forests in Spain, high-density plots were denser than in the other studies for all rotation ages. Thus, the stands in Navarra are the most productive, followed by the present stands and finally those in the Iberian Range. However, considering the low-density plots, the yield tables for Navarra and La Rioja indicate lower densities at younger ages but higher densities at older ages than in the present stands. Rotation ages, squared mean diameters, maximum mean annual increments and total timber volumes at 150 years are lower, except for the lowest site quality. The yield tables for Navarra indicate stands with trees with smaller diameters but higher heights than the others. This phenomenon was described by Sánchez *et al.* (2008) as an imbalance that may eventually lead to structure- and stability-related problems. Yield tables for low-density stands indicate a heavier thinning regime, which results in fewer trees per hectare at the end of the rotation age, but with larger mean diameters and timber volumes, making them the most productive.

Some income could be received for beech forests regarding the amount of carbon stored or their carbon sequestration potential. Our models enable calculation of total biomass. Expansion from total biomass to the carbon

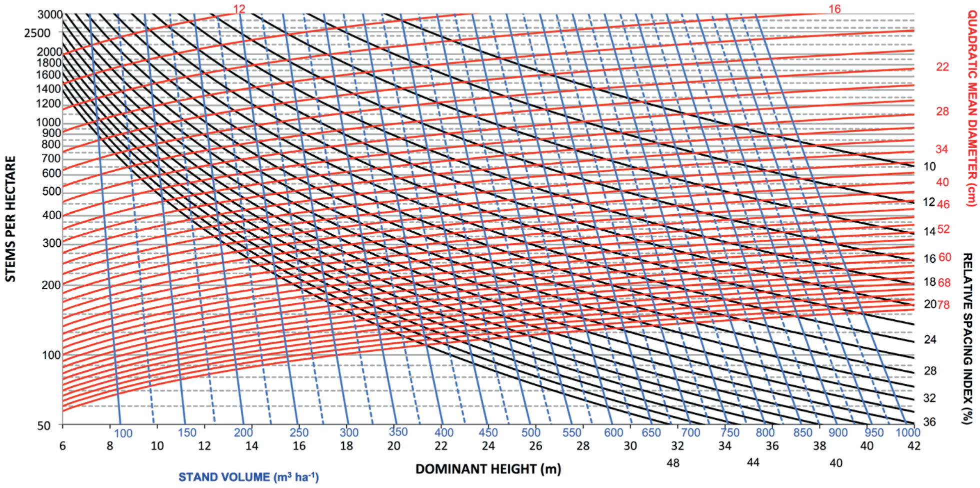

Figure 2. Stand density management diagram relating stand volume and stand density.

(factor 0.486) (Montero *et al.,* 2005) allows prediction of the effects of management prescriptions in terms of carbon. This is likely to occur in these forests in the near future. Cantabrian beech forests form part of the habitats of the Cantabrian capercaillie and the brown bear. This implies that forest planning and management should integrate forestry practices and habitat preservation. However, this challenge has been resolved by removing forestry practices from the equation. Most of these habitats are located in public forests, thus ensuring protection and maintenance of the forests and their fauna (Garcia *et al.,* 2005). These beech forests would therefore only be brought back into production if conservation policies were changed. However, the forests could be of great importance as carbon sinks for meeting emissions targets. Almost 60% of terrestrial carbon dioxide is stored in forest biomass and soils (McKinley *et al.,* 2011). Cessation of timber use resulted in an increase in carbon pools in beech forests (Mund, 2004), and it has been demonstrated that carbon continues to accumulate in long-lived forests beyond maturity (*e.g.* Keeton *et al.,* 2011).

Previous volume and biomass models should be considered estimates rather than real data. According to Cunia (1965) applying this classic methodology results in three sources of error that reduce the accuracy of the results obtained. By definition, any study that analyses a subset of the total population already has a sampling error, and measurement errors are always possible. The first source of error cannot be avoided and even if the measurements were unbiased, there will be errors in tree (Gertner, 1986) and stand (Gertner, 1990) volume estimation (also in biomass) as functions are used to estimate both attributes.

Finally, the SDMD developed is shown in Fig. 2. The range of values represented by the axes and lines were similar to the data used to construct the diagram. SDMDs can determine the initial spacing or thinning schedules required to achieve different management objectives. Selection of upper and lower growing stock limits (set with RS values) is key for this purpose, as the upper limit is chosen to obtain acceptable stand growth and vigour, and the lower limit is chosen to maintain acceptable site occupancy (Long, 1985). For example, Madrigal *et al.* (1992) proposed an RS equal to 22% during the entire rotation age as adequate growing stock, and we used this as the average value in our diagram. Different thinning regimes can be analyzed using the diagram. However, the lack of a mortality submodel limits the use of the diagram within the zone of imminent-competition mortality.

Nevertheless, the tools developed in this study represent a starting point. The lack of remeasurements prevents application of dynamic models, which will be developed in future studies.

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