International Journal of Fatigue 163 (2022) 107069

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

A comparative review of time- and frequency-domain methods for fatigue damage assessment

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ARTICLE INFO

Keywords:
Damage fatigue
Vibration fatigue
Rainflow
Counting methods
Spectral methods

ABSTRACT

In this paper, a comparative review between time- and frequency-domain methods for fatigue damage assessment is performed. The principal steps of a fatigue study are described in detail: Material Characterization, Definition of the Reference Parameter, Treatment of Loading History, Cycle Counting Algorithm and Damage Model. Furthermore, for each of them the main differences found between the advances made in the time- and frequency-domains are highlighted. As a conclusion, this comparative literature review allows us to identify some important lights and shadows in both approaches: several efforts have been made in the development of advanced material characterization models in S-N field in the time-domain methods, either deterministic or probabilistic, but in the frequency-domain methods only the linear Basquin model is currently used. Also the ongoing discussion about the reference parameter in material characterization (stress, strain, energy, etc.) is not present in the frequency-domain methods, which are mainly based on the stress range. Contrarily, the frequency-domain methods show an advanced treatment of the rainflow histogram with different proposed statistical distributions together with theoretical and analytical relationships between the power spectral density and the expected fatigue damage, leading to a simpler and easier methodology to be applied for fatigue damage assessment than those based on time-domain.

1. Introduction and motivation

The fatigue phenomenon represents one of the most common causes of premature failure in structural and mechanical components to which several efforts have been devoted since the last two centuries. As well known, the fatigue failure occurs when a component is subject to cyclic loading that is able to initiate a crack (nucleation), which can further grow until failure (propagation) (see Suresh [1], Bolotin [2] and Schijve [3]). The fatigue analysis of real components begins with the estimation of the corresponding stress or strain distributions due to the loading history, which nowadays is usually facilitated by a finite element analysis. Then, the material characterization based on the S-N or the ε-N fields allows the fatigue lifetime to be estimated and the damage variable estimated. Accordingly, fatigue damage is mainly influenced by two factors: fatigue strength of the material and loading history applied and the quality of any fatigue damage assessment relies on a suitable and valid acquisition of information about the loading history and the fatigue strength, as the most relevant steps of the entire process. Nevertheless, the assessment of fatigue damage is a very complicated task due to the large number of models and hypotheses that is necessary to assume in order to compare these factors (see Fig. 1).

First of all, It is necessary to characterize the fatigue strength of the material, which implies the election of the reference parameter associated with the fatigue damage (i.e. $\Delta\sigma$ or $\Delta\epsilon$) and how to correlate it to the number of cycles until failure: deterministically or probabilistically. These elections will condition how the next steps of fatigue prediction procedures (Fig. 1) will be applied because the election of the reference parameter and the fatigue model will have an impact on the treatment of loading history and the interpretation of the accumulated damage.

Regarding the loading history applied to a component, it will generate complex stress (or/and strains) distributions over the material, so a methodology to transform those distributions to equivalent values comparable to the reference value defined during the fatigue characterization is necessary.

Furthermore, due to the impossibility of recording the entire loading history related to a fatigue damage scenario, a finite time interval is usually experimentally collected and taken as representative, but different approaches can be applied to characterize the entire domain based on this extraction.

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https://doi.org/10.1016/j.ijfatigue.2022.107069
Received 3 March 2022; Received in revised form 25 May 2022; Accepted 3 June 2022
Available online 9 June 2022

Nomenclature

α	scale Weibull parameter
$\alpha_{1,0.75,2}$	bandwidth parameters
β	shape Weibull parameter
$\ddot{X}(t)$	second derivative random process
$\Delta\sigma$	stress range
$\delta(.)$	delta Dirac function
$\dot{X}(t)$	first derivative random process
Γ	Gamma function
λ_m	mth spectral moment
ν_0	expected peak occurrence
ν_a	cycles count per unit time
ν_p	expected zero-crossing rate
ω	frequency
$\overline{D}_{\mathrm{NB}}$	expected fatigue damage in NB approximation
$\overline{D}_{\mathrm{RC}}$	expected fatigue damage in range-mean
$\overline{D}_{\mathrm{RFC}}$	expected fatigue damage in rainflow
$\overline{D}_{\mathrm{RFC}}^{\mathrm{AL}}$	expected damage in $\alpha_{0.75}$ method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{DK}}$	expected damage in DK method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{GM}}$	expected damage in GM method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{JM}}$	expected damage in JM method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{PZ}}$	expected damage in PZ method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{TB}}$	expected damage in TB method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{WL}}$	expected damage in WL method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{ZB}}$	expected damage in ZB method
$\overline{D}_{\mathrm{RFC}}^{\mathrm{ZN}}$	expected damage in ZN method
Φ	standard cdf Normal distribution
$\Psi(.)$	approximation function in PZ method
$\Psi_1 - \Psi_4$	approx. function coeff. in PZ method
ρ_{JM}	JM correction factor
ρ_{WL}	WL correction factor
σ_a	stress amplitude
σ_e	Goodman equivalent stress range
σ_m	stress mean
σ_X	variance of the random process
$\sigma_{\max}$	maximum stress
$\sigma_{\min}$	minimum stress
ε	strain or spectral parameter
$a(.)$	best-fitting parameters (WL method)
b	TB method parameter or weighting factor
$b(.)$	best-fitting parameters (WL method)
C	S-N curve constant
D	fatigue damage
$D_1 - D_3$	DK method parameters
$f_{\mathrm{LCC}}(\sigma_a, \sigma_m)$	joint pdf distribution from level-cross counting
$f_{\mathrm{RC}}(\sigma_a, \sigma_m))$	RC joint distribution in range-mean
$f_{\mathrm{RFC}}(\sigma_{\max}, \sigma_{\min})$	joint pdf of maximum and minimum stresses
$f_{\mathrm{RFC}}^{\mathrm{TB}}(\sigma_a, \sigma_m))$	RFC joint distribution in TB method
$G(.)$	transformation function
$h_{\mathrm{RFC}}(\sigma_a, \sigma_m)$	joint cdf of amplitude and mean stresses
$H_{\mathrm{RFC}}(\sigma_{\max}, \sigma_{\min})$	joint cdf of maximum and minimum stresses

Once the loading history is recorded, advanced methods must be implemented to estimate the number of cycles associated with each value of the reference fatigue parameter (see Dirlik [4] and Bishop [5]), such as the well-known rainflow counting method (RCM), originally developed by Matsuishi and Endo [6]. This algorithm allows the real stress-time history to be converted into a set of counted cycles with constant amplitudes.

Finally, the total damage can be estimated by computing the damage produced by each counted cycle based on a certain damage accumulation rule, such as the Palmgren–Miner (see Palmgren [7] and Miner [8]).

In this paper, the time-and frequency-domain fatigue approaches are reviewed and compared. The paper is organized following the main steps concerning a fatigue damage assessment (see Fig. 1): material characterization (Section 2), selection of the reference parameter (Section 3), loading history (Section 4), rainflow counting algorithm (Section 5) and the damage model (Section 6). In each of them, the current state-of-the-art of time- and frequency-domain methods is performed, identifying the most important contributions in literature together with major shortcomings to be dealt with in each case. It is important to remark that there are different reviews about the state-of-the-art of each of these approaches, but there is not a comparative perspective between the degree of development between them. Furthermore, some important conclusions are derived from this comparative analysis on both approaches, in order to highlight where further efforts could be focused according to current state-of-the-art.

2. Material characterization

The material characterization has to be considered the first step on a fatigue damage assessment. It consists in experimental campaigns based on standardized specimens suggested by international standards (e.g. dogbone specimens) subjected to constant amplitude dynamic loading until failure, when the reached number of cycles is collected, originally proposed by Whöler [9]. After that, a critical parameter related to the dynamic loading is defined, such as the stress range ($\Delta\sigma$) or strain range ($\Delta\epsilon$). Finally, a relation between the critical parameter and the number of cycles is defined by a fatigue life model. Although the fatigue models can be equivalently formulated in terms of the $\Delta\sigma$ - N (also known as S-N) or $\Delta\epsilon$-N fields, the proposed models in literature during the last centuries are especially focused on the former, whose can be classified into two groups depending on the approach considered (see Fig. 2 and Table 1 for consulting the detailed formulas of each model):

– Deterministic models, such as the linear (see Basquin [10]), bilinear (Palmgren [7], Stromeyer [11] and Spindel and Haibach [12]) and trilinear rules or sigmoidal curves (Stüssi [13], Weibull [14] and Kohout and Vechet [15]), among others.
– Probabilistic models, such as Bastenaire [16], Castillo and Canteli [17], Bolotin [2] and Pascual and Meeker [18], among others.

Fig. 2 illustrates schematically the typical S-N experimental results for different constant stress ranges $\Delta\sigma$ and the fitting provided by a deterministic model and a probabilistic model ($p = 0.05, 0.95$). As can be seen, the former is only focused on fitting the mean value of the experimental results whereas the later is able to consider the inherent variability of the experimental fatigue results.

It is important to remark that the development of material characterization models for the S-N field is much more prolific when fatigue time-domain methods are going to be used than when frequency-domain or spectral methods are going to be used. Although a large inherent scatter of fatigue results is extensively recognized in the literature, the vast majority of advances on frequency-domain fatigue models are still based on deterministic approaches. This could be due to the great depth of the classic models in the research and industrial environments, which mainly use the Basquin model [10] (see Benasciutti and Tovo [20,21] and Mršnik et al. [22,23]). Even though different probabilistic approaches would be also applicable to the frequency domain approach, and several efforts are being devoted

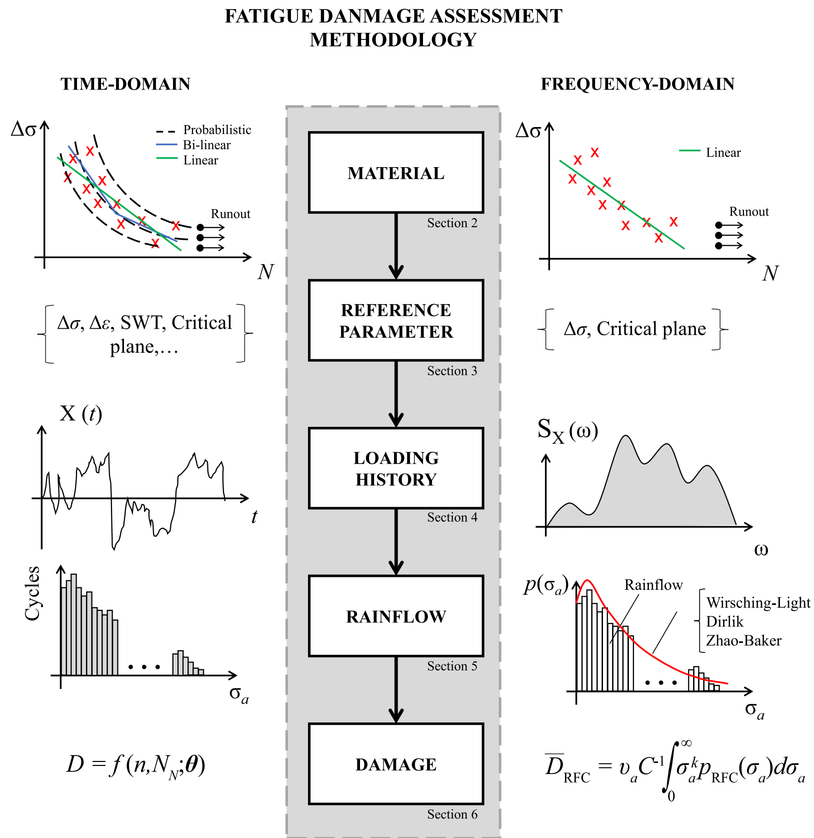

Fig. 1. General flowchart illustrating the fatigue damage assessment from both time- and frequency-domain approaches.

Table 1
Proposed deterministic and probabilistic models in literature for S-N field (see Castillo and Fernández-Canteli [19]).

Model	Expression
Basquin [10]	$\log N = A - B\log\Delta\sigma$
Stromeyer [11]	$\log N = A - B\log(\Delta\sigma - \Delta\sigma_0)$
Palmgren [7]	$\log(N+D) = A - B\log(\Delta\sigma - \Delta\sigma_0)$
Weibull [14]	$\log(N+D) = A - B\log((\Delta\sigma - \Delta\sigma_0)/(\Delta\sigma_{st} - \Delta\sigma))$
Stüssi [13]	$\log N = A - B\log((\Delta\sigma - \Delta\sigma_0)/(\Delta\sigma_{st} - \Delta\sigma))$
Bastenaire [16]	$(\log N - B)(\Delta\sigma - \Delta\sigma_0) = A\exp\left[-C(\Delta\sigma - \Delta\sigma_0)\right]$
Spindel–Haibach [12]	$\log(N/N_0) = A\log(\Delta\sigma/\Delta\sigma_0) - B\log(\Delta\sigma/\Delta\sigma_0) + B\left\{(1/\alpha)\log\left[1+(\Delta\sigma/\Delta\sigma_0)^{-2\alpha}\right]\right\}$
Castillo–Canteli [17]	$\log(N/N_0) = \dfrac{\lambda + \delta(-\log(1-p)^{\beta})}{\log(\Delta\sigma/\Delta\sigma_0)}$
Kohout–Vechet [15]	$\log\left(\dfrac{\Delta\sigma}{\Delta\sigma_\infty}\right) = \log\left(\dfrac{N+N_1}{N+N_2}\right)^b$
Pascual–Meeker [18]	$\log N = A - B\log(\Delta\sigma - \Delta\sigma_0)$

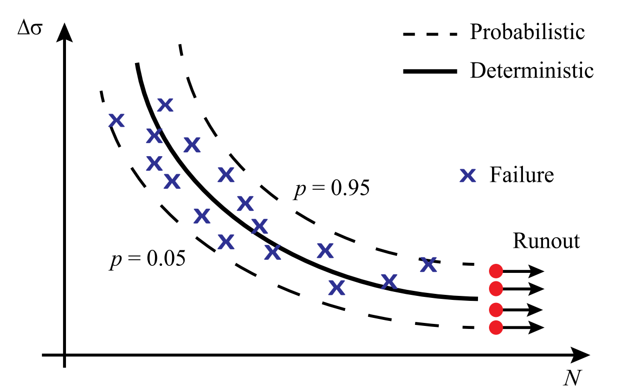

Fig. 2. Typical S-N field illustrating both probabilistic and deterministic approaches.

to improving classical deterministic models to a probabilistic version (see Paolino et al. [24] and Usabiaga et al. [25]), the Basquin Model is still the one more used under frequency-domain approach, contrary to the time-domain approach, which is associated to a large amount of deterministic and probabilistic fatigue material models (see Table 1).

3. Reference parameter

As previously mentioned, the fatigue lifetime prediction is traditionally formulated in terms of a $\Delta\sigma$-N or a $\Delta\varepsilon$ - N curve. Nevertheless, during the last decades advanced reference parameters have been suggested based on mathematical combinations of stress and strain variables, such as the Smith–Watson–Topper [26] or the Walker's proposal [27]. Additionally, energetic parameters are also of great interest as reference parameters on fatigue design, such as the plastic strain energy range (see Ellyin [28]), the total strain energy range per reversal (see Eyllin [29]) or combination between both elastic and plastic energies (see Golos and Ellyin [30,31]). Regardless these particular proposals, the discussion about the suitable reference parameter addressing the material characterization is an ongoing and a current issue (see Correia et al. [32]). As aforementioned in the previous section, the vast majority of advances related to the search for an advanced reference parameter to study fatigue life are related to studies in the time-domain, whilst the frequency-domain methods are mainly focused on stress variables ($\Delta\sigma$), and no other reference parameter is generally considered.

Furthermore, some aspects of the loading history has been researched and accounted for through the reference parameter in the time-domain approach, such as the asynchronous and non-proportionality effects due to their relevant influence on the accumulated damage.

In the first case, the stress scale factor concept is usually employed in the derivation of suitable reference parameters (see Sonsino [33] and Anes et al. [34]). In the case of the non-proportionality effect different factors are discussed in literature based on the critical-plane concept (see Kanazawa et al. [35], Bishop [36], Galer et al. [37], Susmel [38] and Bolchoun et al. [39]).

Besides, the mean stress effect could be required to be considered in the material characterization. In a simple case without non-proportional nor asynchronous effects, classical Goodmann [40], Soderberg and Gerber models or the ASME-elliptic proposal are usually applied in real experimental campaigns. Apart from this classical approach, Castillo et al. [41] have originally derived the analytical solution for considering the mean stress effect in a probabilistic approach in the S-N field based on feasible statistical and physical conditions related with the compatibility condition.

Finally, one of the main challenges when defining a reference parameter is to deal with multiaxial loading scenarios. In the time-domain, the most widespread approach is based on the critical-plane concept (see Fatemi and Socie [42,43] and Fatemi and Kurath [44]), although there are other approaches, such as the proposed by Ellyin [45], which is the modification of the energy density based on a summation of plastic strain energy divided by a multiaxial constraint ratio. On the other hand, several efforts have been made on the frequency-domain to deal with the multiaxial random loading scenarios, which represents a current and ongoing research topic where novel proposals are being devoted over the last years (see Nieslony et al. [46], Benasciutti et al. [47], Carpinteri et al. [48], Slavič et al. [23]). For example, the so-called Carpinteri et al. multiaxial fatigue criterion represents a novel computationally efficient proposal based on the critical plane concept (see Carpinteri et al. [48,49]). A comparative analysis among time- and these frequency-domain methods in multiaxial fatigue can be found in Braccesi et al. [50].

4. Loading history

The loading history represents the stress-time history $X(t)$ from a structural or mechanical component (wind turbines, ship or automobile details, off-shore platforms, bridges, etc.) under real service conditions. Due to economic and feasible reasons, only a finite time interval is experimentally registered until a certain value T, i.e. $0 \leq t \leq T$. The main difference between time- and frequency-domain studies lies in the treatment of the information collected on this finite time interval. From a frequency-domain viewpoint, the loading history can be a wide or broad-band process (BB) or a narrow-band process (NB), which mainly influences the required mathematical formulation in the following steps characterizing the fatigue damage assessment. The first case can be defined as that for which the ratio of the mean number of maxima over the mean number of up-crossings of the mean level is close to unity, that is, there is exactly one peak for every zero up crossing, while in the second case this ratio is large compared to unity, i.e. there are many peaks for each zero up crossing (see Bolotin [2] and Wirsching et al. [51]). Fig. 3 illustrates these both signal types. As can be seen, the identification of the stress cycles is more easily in the NB type process.

Hence, a narrow-band process leads to the simpler and easier formulations about the statistical properties of the random process, even with exact and theoretical solutions in some cases (see Lutes and Sarkani [52]). For this reason, the narrow-band process is usually assumed as an approximated and conservative solution to more complex loading histories. However, it is common physical phenomena exhibit spectra with several peaks, such as a mooring system response, thruster response in waves and combined wave-induced and spring motions (see Gao and Moan [53]). For example, it is very common in offshore marine structures the bimodal spectra with well-separated modes, that is, a combination of low frequency and a high frequency components (see Jiao and Moan [54]).

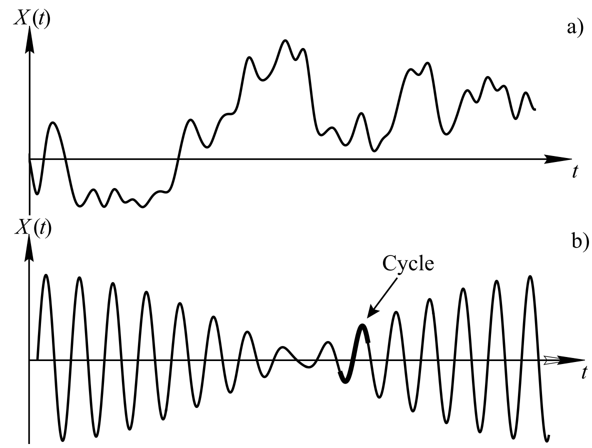

Fig. 3. Examples of random broad-band (BB) (a) and narrow-band (NB) processes (b).

A random process is univocally characterized in time-domain by the autocorrelation function $R_X(\tau)$, defined as follows (see Rice [55], Wirsching et al. [51], Sólnes [56], Newland [57], Shin and Hammond [58], Box et al. [59]):

$$R_X(\tau) = \mathrm{E}[X(t)X(t+\tau)], \tag{1}$$

where E[.] operator denotes the probabilistic expected value. Analogously, the process can be univocally characterized in the frequency-domain with the two-sided power spectral density (PSD) function, which is the Fourier transform of the autocorrelation function, that is,

$$S_X(\omega) = \int_{-\infty}^{\infty} R_X(\tau) \mathrm{e}^{-i\omega\tau} d\tau. \tag{2}$$

The statistical information in spectral density $S_X(\omega)$ can be summarized by means of the mth spectral moments λ_m:

$$\lambda_m = \int_{-\infty}^{\infty} \omega^m S_X(\omega) d\omega, \quad m = 1, 2, \ldots \tag{3}$$

with a direct relation with the time-domain characteristics of the process, since the even moments correspond with the variance σ_X^2 of the random process X and its derivatives $\dot{X}(t), \ddot{X}(t)$:

$$\lambda_0 = \sigma_X^2, \quad \lambda_2 = \sigma_{\dot{X}}^2, \quad \lambda_4 = \sigma_{\ddot{X}}^2. \tag{4}$$

From these spectral parameters, both well-known expected peak occurrence frequency ν_0 and expected positive zero-crossing rate ν_p can be directly obtained:

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}, \quad \nu_p = \frac{1}{2\pi}\sqrt{\frac{\lambda_4}{\lambda_2}}; \tag{5}$$

whose will be used in the derivation of the damage variable. Additionally, the so-called bandwidth parameters α_1, α_2, are also of interest, such that $0 \leq \alpha_1, \alpha_2 \leq 1$, being defined as follows:

$$\alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0 \lambda_2}}, \quad \alpha_2 = \frac{\lambda_2}{\sqrt{\lambda_0 \lambda_4}}. \tag{6}$$

Accordingly, the bandwidth parameters of a narrow-band process are close to unity, as previously mentioned, while for a broad-band process they tend to zero.

Finally, as one of the main differences compared to the time-domain methods, the rainflow cycle distribution can be analytically derived from the spectral parameters, particularly the distribution of the maximum stresses $p_{\sigma_{\max}}(\sigma_{\max})$ in each different cycle (see Rice [60] and Lutes [52]):

$$p_{\sigma_{\max}}(\sigma_{\max}) = \frac{\sqrt{1-\alpha_2^2}}{\sqrt{2\pi}\sigma_X} e^{\left(-\frac{\sigma_{\max}^2}{2\sigma_X^2(1-\alpha_2^2)}\right)} + \frac{\alpha_2\sigma_{\max}}{\sigma_X^2} e^{\left(-\frac{\sigma_{\max}^2}{2\sigma_X^2}\right)} \Phi\left(\frac{\alpha_2\sigma_{\max}}{\sigma_X\sqrt{1-\alpha_2^2}}\right), \tag{7}$$

where $\Phi(.)$ represents the standard normal distribution. Then, since each cycle is considered symmetric, the probability distribution of the minimum stresses $\sigma_{\min}$ will be $p(\sigma_{\min}) = p(-\sigma_{\max})$. On the other hand, if the random process is narrow-band type, the theoretical distribution of peaks is defined as a Rayleigh distribution (see Lutes [52]):

$$p_{\sigma_{\max}}(\sigma_{\max}) = \frac{\sigma_{\max}}{\sigma_X^2} \exp\left(-\frac{\sigma_{\max}^2}{2\sigma_X^2}\right). \tag{8}$$

As a matter of fact, an additional discussion was addressed about the normality assumption concerning the loading history (see Benasciutti and Tovo [20]). To this aim, a transformation function $G(.)$ relating a Gaussian process $X(t)$ and a non-Gaussian one $Z(t)$ is proposed, that is,

$$Z(t) = G[X(t)], \tag{9}$$

where $G(.)$ has to be a monotonic function for the transformation to be valid. Then, by considering the inverse transformation $G^{-1}(.)$ a non-Gaussian process can be transformed to a Gaussian one. Thus, several authors have proposed different transformation functions, as for example exponential (see Ochi and Ahn [61]), monotonic cubic Hermite polynomial (see Winterstein [62,63]), power-law (see Sarkani et al. [64] and Kihl et al. [65]) and non-parametric transformation (see Rychlik et al. [66]).

Finally, It is important to remark that, although frequency domain approaches allows us to make a more exhaustive analysis of the loading history applied, the approaches used to perform these analysis lies in the necessary assumptions concerning the random process yielding the observed random loading. For that reason, It could be concluded that both approaches, time-domain and frequency-domain, have their limitations. While time domain approaches suffer from the limited observation time and the necessary extrapolation.

5. Cycle counting approaches

Due to the inherent complexity of identifying stress cycles of constant amplitude in a real loading history, several efforts were devoted to develop algorithms, known as counting methods, to provide information that can be contrasted with the fatigue strength of the material characterized according to Section 2. The main difference between time- and frequency-domain methods relies regarding this topic lies on the approach adopted to extract the information of the loading history recorded.

On the one hand, the time-domain approaches are mainly based on the use of classical counting methods to obtain an histogram of stresses (see Fig. 4), which recopilate the number of repetitions (e.g. n_1 and n_2) of each stress amplitude registered (e.g. σ_{a1} and σ_{a2}). Fig. 4 illustrates schematically a typical histogram of different counting methods, where different stress amplitudes are identified with their corresponding number of occurrences in the time interval $0 \leq t \leq T$, .

Within these classical counting methods proposed in literature (peak count, mean crossing peak count, level-crossing count, range count, range-mean count, range-pair count, etc.) (see Dowling [67] and Dirlik [4]), the rainflow counting method (RFC) has been successfully applied since its original formulation by Matsuishi and Endo [6] as the most accuracy one (see Dowling [67] and Watson and Dabell [68]),

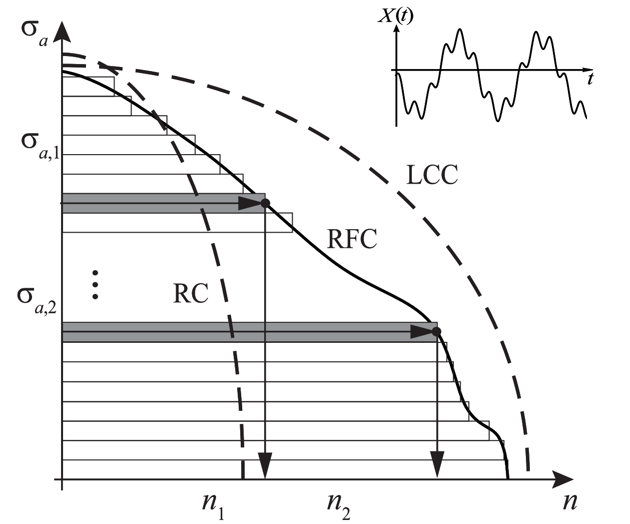

Fig. 4. Schematic illustration of stress amplitude histogram from different counting methods (RFC: rainflow, LCC:level-crossing and RC: range) for the loading history $X(t)$.

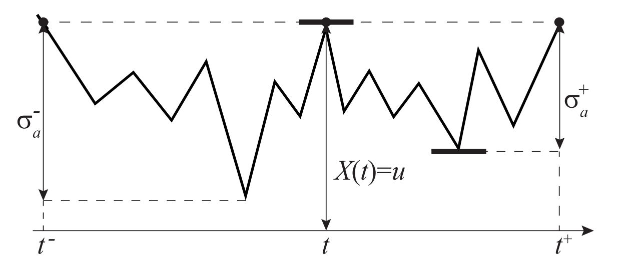

Fig. 5. Schematic illustration of the rainflow counting method (see Lindgren and Rychlik [74]).

being currently included in international standards as the reference procedure (see ASTME E1049-85 [69]). Other counting methods are known to lead to over- or underestimate counted cycles, as can be seen in Fig. 4 with the level and range-crossing counting algorithms, respectively. Due to its great relevance, several authors have proposed different algorithms seeking to improve the original version (see Downing and Socie [70], Okamura et al. [71] and Socie [72]). Rychlik attempted to derive analytical expressions in a more convenient statistical formulation (see Rychlik [73]).

In general terms, since a cycle can be defined by its highest and lowest points, the rainflow counting method allows a random process $X(t)$ in the interval $0 \leq t \leq T$ to be converted into a set of cycles of the form $\{(u(t), v(t))\}$ such that $u \geq v$, that is, each cycle from the process $X(t)$ at time t consists of a peak u and a valley v (see Lindgren and Rychlik [74] and Rychlik [75]). To this aim, consider a loading history $X(t)$ as illustrated in Fig. 5, where the rainflow is applied at time t. Firstly, the reference level $X(t) = u$ marks the level-crossing events at time t^- and t^+, that is, when the signal crosses u level. Secondly, the maximum differences between the reference level u and the loading history in the intervals $t < \tau < t^+$ and $t^- < \tau < t$ must be defined, that is,

$$\begin{aligned} \sigma_a^+ &= \max_{t<\tau<t^+} (u(t) - u(\tau)) \\ \sigma_a^- &= \min_{t^-<\tau<t} (u(t) - u(\tau)) \end{aligned} \tag{10}$$

As a result, the final rainflow cycle amplitude σ_a at time t results as follows:

$$\sigma_a = \min(\sigma_a^-, \sigma_a^+). \tag{11}$$

Nevertheless, though being widely used as the reference counting procedure, the RFC method exhibits some important disadvantages:

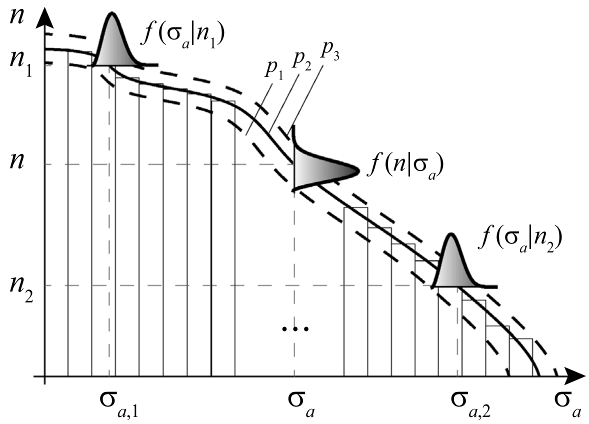

Fig. 6. Schematic illustration of the conditional densities involved in the probabilistic approach to the loading spectra (see Nagode and Fajdiga [76]).

the stress amplitude histogram depends on the particular time window analysed in the loading history, thus computation of different experimental intervals may be required to provide reliable statistical information about the acting loading, which is a highly time-consuming process (see Gao and Moan [53]). Nagode and Fajdiga [76] proposed an alternative statistical methodology, based on a previous work by Buxbaum [77], only valid for linear loading spectra, to consider as random variables both stress amplitudes and cycles in the loading spectra, thus defining the associated conditional densities in the stress amplitude-number of cycles, i.e. $f(\sigma_a|n)$ and $f(n|\sigma_a)$, and assuming them as a mixture distribution between Weibull and Normal distributions (see also Nagode et al. [78]). As a result, the loading spectra is not only a deterministic curve but also a family of percentile curves able to take into account the inherent scatter in the $p-\sigma_a-n$ field (see Fig. 6). In addition, the prediction of the spectra from a sample of measured load time histories is now available due to the statistical approach.

Contrarily to the classical discrete definitions of the loading history performed in time-domain methods (see Fig. 1), frequency-domain approaches are focused on the continuous definition of the rainflow according to its statistical distribution. From a mathematical point of view, the rainflow cycle statistical distribution can be described by the joint distribution of minimum and maximum stresses of each counted cycles $h(\sigma_{\max},\sigma_{\min})$ in which the loading history is converted by means of a certain counting method, such as rainflow, range-mean and level-crossing. Since the former is mainly preferred, the joint distribution h will be denoted as $h_{\mathrm{RFC}}(\sigma_{\max},\sigma_{\min})$. By definition of a bivariate distribution, its corresponding cumulative distribution function H_{RFC} is defined as follows:

$$H_{\mathrm{RFC}}(\sigma_{\max},\sigma_{\min}) = \int_{-\infty}^{\sigma_{\max}} \int_{-\infty}^{\sigma_{\min}} h_{\mathrm{RFC}}(x,y)\,dx\,dy, \tag{12}$$

providing the probability to count a cycle with peak lower or equal to $\sigma_{\max}$ and valley lower or equal to $\sigma_{\min}$. Note the equivalent formulation of this joint distribution if the stress amplitude σ_a and mean stress σ_m are preferred instead (see Benasciutti and Tovo [21]), that is,

$$f_{\mathrm{RFC}}(\sigma_a,\sigma_m) = 2h_{\mathrm{RFC}}(\sigma_a+\sigma_m,\sigma_a-\sigma_m), \tag{13}$$

Once the joint distribution is formulated, the practical interest lies only on one of its marginal distributions, i.e. $p^{\mathrm{RFC}}_{\sigma_a}(\sigma_m|\sigma_a)$ for the amplitude stresses and $p^{\mathrm{RFC}}_{\sigma_m}(\sigma_m|\sigma_a)$ for the mean stresses, being defined as follows (see Loeve [79] and Galambos [80]):

$$p^{\mathrm{RFC}}_{\sigma_m}(\sigma_m|\sigma_a) = \int_{-\infty}^{\infty} f_{\mathrm{RFC}}(\sigma_a,\sigma_m)\,d\sigma_m, \tag{14}$$

$$p^{\mathrm{RFC}}_{\sigma_a}(\sigma_a|\sigma_m) = \int_{-\infty}^{\infty} f_{\mathrm{RFC}}(\sigma_a,\sigma_m)\,d\sigma_a, \tag{15}$$

Commonly, the mean stress effect, described in $p^{\mathrm{RFC}}_{\sigma_m}$, is ignored and the marginal distribution of the amplitude stresses $p^{\mathrm{RFC}}_{\sigma_a}$ is selected to describe the rainflow cycle distribution. Fig. 7 illustrates a practical example due to Benasciutti and Tovo [20] of the resulting joint distribution of the rainflow together with the marginals $p^{\mathrm{RFC}}_{\sigma_a}(\sigma_m|\sigma_a)$ and $p^{\mathrm{RFC}}_{\sigma_m}(\sigma_m|\sigma_a)$.

Regretfully, due to the inherent complexity of pairing procedure peak-to-valley in the rainflow algorithm for a BB process, there is no explicit analytical solution for the distribution $f_{\mathrm{RFC}}(\sigma_a,\sigma_m)$ and, consequently, neither for the expected damage variable. For this reason, several authors have assumed different distributions as approximate solutions for the stress amplitude of the rainflow distribution $p^{\mathrm{RFC}}_{\sigma_a}$ (see Benasciutti and Tovo [21], Lalanne [82]):[1]

– Narrow-band approximation, based on the assumption of a strictly NB process, that is, every peak ad the following valley are coincident with a cycle, the amplitude stress density can be supposed to be the peak distribution in Eq. (8), thus following a Rayleigh distribution:

$$p^{\mathrm{NB}}_{\mathrm{RFC}}(\sigma_a) = \frac{\sigma_a}{\sigma_X^2}\exp\left[-\frac{1}{2}\left(\frac{\sigma_a}{\sigma_X}\right)^2\right]. \tag{16}$$

– Jiao–Moan method [54] is one of the first proposals dealing specifically with bimodal processes, that is, a power spectral density with two well-separated modes, being widely celebrated as an accurate prediction method for bimodal vibration fatigue included in international offshore engineering codes (see ISO 19901-7 [83]). Hereafter the proposed methods by Sakai-Okamura [84] and Fu and Cebon [85] were also successfully applied (see Benasciutti and Tovo [86]). The original proposal by Jiao and Moan assumes any random process $X(t)$ can be expressed as a sum of two independent narrow-band processes, that is,

$$X(t) = X_H(t) + X_L(t), \tag{17}$$

where X_H and X_L represent the high (HF) and low frequency (LF) components in which the bimodal signal can be decomposed. Fig. 8 presents an illustrative example of a bimodal signal, from which the RCF method could extract two types of cycles: large cycles with amplitude S_L, associated with the interaction between both components of the signal X_H and X_L (i.e. with the envelope of the process) and small cycles with amplitude S_H associated with HF component X_L. As a result, the amplitude stress distribution is derived as a combination of two NB process by means of the convolution integral, leading to:

$$p^{\mathrm{JM}}_{\mathrm{RFC}}(\sigma_a) = \sqrt{\frac{\lambda_1^*}{2\pi}}\exp\left(-\frac{\sigma_a^2}{2\lambda_1^*}\right) + \sqrt{\lambda_2^*}\,\sigma_a\exp\left(-\frac{\sigma_a^2}{2}\right)\Phi\left(\sqrt{\frac{\lambda_2^*}{\lambda_1^*}}\,\sigma_a\right), \tag{18}$$

where λ_1^* and λ_2^* are the bandwidth parameters of each narrow-band process.

– Gao–Moan method [53] extends previous bimodal approach to a trimodal one, such that ideally any broad-band process could be defined as a sum of three stationary, Gaussian and mutually independent narrow-band processes with well-separated frequencies, that is,

$$X(t) = X_H(t) + X_M(t) + X_L(t), \tag{19}$$

where X_H, X_M and X_L represent the components of the random process with high, intermediate and low frequencies, respectively.

[1] For the sake of simplicity in the notation, the marginal distribution of the stress amplitude $p^{\mathrm{RFC}}_{\sigma_a}$ shall be denoted simply as p_{RFC} in order to identify the authors.

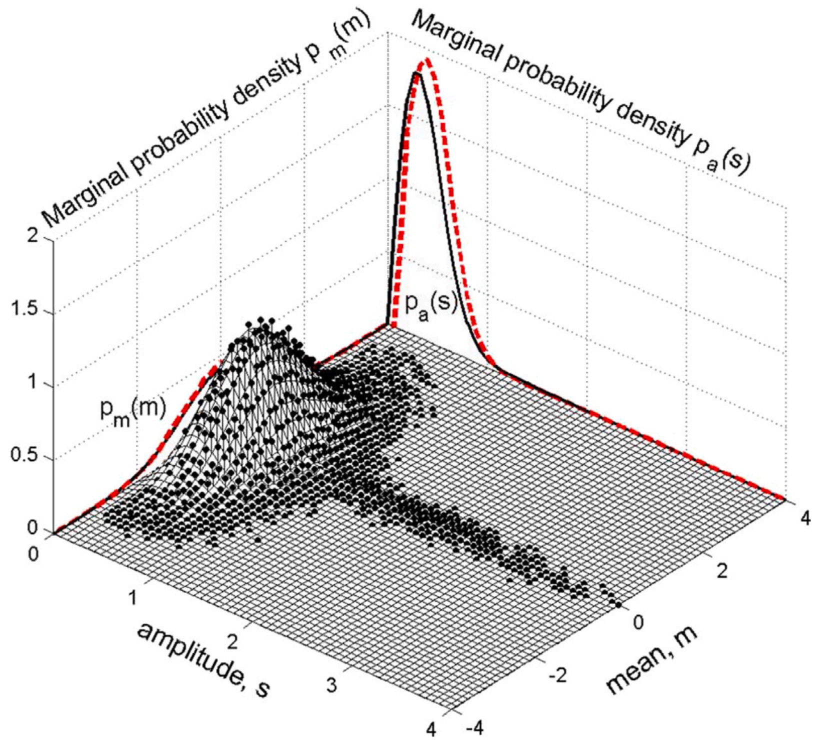

Fig. 7. Joint histogram of a simulated rainflow cycles and the theoretical joint distribution $f_{\mathrm{RFC}}^{\mathrm{TB}}(\sigma_a, \sigma_m)$ proposed by Benasciutti and Tovo [81].

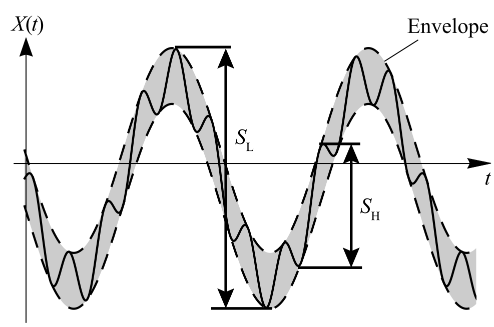

Fig. 8. Schematic illustration of a bimodal process identifying both low and high frequency components.

As in previous case, the final amplitude stress distribution is obtained by means of the convolution integral of the combination of three Rayleigh distributions. Unfortunately, an analytical formula does not exist for this distribution, which can only be solved numerically with Hermite numerical integration (see Karagiannidis and Kotsopoulos [87]).

– Dirlik [4] suggests a mixture distribution within an exponential and two Rayleigh distributions, leading to

$$p_{\mathrm{RFC}}^{\mathrm{DK}}(\sigma_a) = \frac{1}{\sigma_X}\left[\frac{D_1}{Q}\exp\left(-\frac{Z}{Q}\right) + \frac{D_2 Z}{R^2}\exp\left(-\frac{Z^2}{2R^2}\right)\right], \tag{20}$$

where $Z = \sigma_a/\sigma_X$ is the well-known normalized amplitude while other parameters are defined as follows:

$$x_m = \frac{\lambda_1}{\lambda_0}\left(\frac{\lambda_2}{\lambda_4}\right)^{1/2}; \quad D_1 = \frac{2(x_m - \alpha_2^2)}{1+\alpha_2^2};$$
$$Q = \frac{1.25(\alpha_2 - D_3 - D_2 R)}{D_1}; \quad D_2 = \frac{1-\alpha_2 - D_1 + D_1^2}{1-R}; \tag{21}$$
$$R = \frac{\alpha_2 - x_m - D_1^2}{1-\alpha_2 - D_1 + D_1^2}, \quad D_3 = 1 - D_1 - D_2;$$

As a result, the proposed rainflow cycle statistical distribution $p_{\mathrm{RFC}}^{\mathrm{DK}}$ depends only on four spectral parameters ($\lambda_0, \lambda_1, \lambda_2, \lambda_4$) with a great accuracy in practical applications (see Bouyssy et al. [88], Halfpenny [89] and Benasciutti and Tovo [21]). However, this approximated solution is not theoretically justified, besides not allowing a further extension to non-Gaussian processes.

– Zhao and Baker [90] proposes a mixture with a Rayleigh and Weibull distributions, that is,

$$p_{\mathrm{RFC}}^{\mathrm{ZB}}(Z) = w\alpha\beta Z^{\beta-1}\exp\left(-\alpha Z^{\beta}\right) + (1-w) Z \exp\left(-\frac{Z^2}{2}\right), \tag{22}$$

where w is the well-known weighting factor of the mixture ($0 \le w \le 1$), while $\alpha, \beta > 0$ are the scale and shape Weibull parameters, respectively. The weighting factor must not be arbitrarily fitted since it is intrinsically related with the spectral parameters according to the following expression:

$$w = \frac{1-\alpha_2}{1-\sqrt{\frac{2}{\pi}}\,\Gamma\left(1+\frac{1}{b}\right) a^{-1/b}}, \tag{23}$$

with parameters a and b defined as follows:

$$a = 8 - 7\alpha_2,$$
$$b = \begin{cases} 1.1 & \text{if } \alpha_2 < 0.9 \\ 1.1 + 9(\alpha_2 - 0.9) & \text{if } \alpha_2 \ge 0.9 \end{cases}$$

– Tovo and Benasciutti [81,91] proposed as joint rainflow cycle distribution $f_{\mathrm{RFC}}^{\mathrm{TB}}$ a linear combination of the range and level-crossing counting methods, based on the fact that these both values represent two limits for the rainflow cycle distribution (see Fig. 4):

$$f_{\mathrm{RFC}}^{\mathrm{TB}}(\sigma_a, \sigma_m) = b f_{\mathrm{LCC}}(\sigma_a, \sigma_m) + (1-b) f_{\mathrm{RC}}(\sigma_a, \sigma_m), \tag{24}$$

being b the weighting factor, whereas $f_{\mathrm{LCC}}(\sigma_a, \sigma_m)$ represents the joint distributions obtained from the level-crossing counting

Table 2
Proposed rainflow cycle distributions in frequency-domain methods.

Rainflow cycle distribution	Expression
Narrow-band approximation [93]	$p^{NB}_{RFC}(\sigma_a) = \frac{\sigma_a}{\sigma_X^2} \exp\left[-\frac{1}{2}\left(\frac{\sigma_a}{\sigma_X}\right)^2\right]$
Dirlik method [4]	$p^{DK}_{RFC}(\sigma_a) = \frac{1}{\sigma_X}\left[\frac{D_1}{Q}\exp\left(-\frac{Z}{Q}\right) + \frac{D_2 Z}{R^2}\exp\left(-\frac{Z^2}{2R^2}\right)\right]$
Zhao–Baker method [90]	$p^{ZB}_{RFC}(\sigma_a) = w\frac{\alpha\beta}{\sigma_X}\left(\frac{\sigma_a}{\sigma_X}\right)^{\beta-1}\exp\left[-\alpha\left(\frac{\sigma_a}{\sigma_X}\right)^{\beta}\right] + (1-w)\frac{\sigma_a}{\sigma_X^2}\exp\left[-\frac{1}{2}\left(\frac{\sigma_a}{\sigma_X}\right)^2\right]$
Tovo–Benasciutti method [81,91]	$f^{TB}_{RFC}(\sigma_a,\sigma_m) = b f_{LCC}(\sigma_a,\sigma_m) + (1-b) f_{RC}(\sigma_a,\sigma_m)$

method (see Madsen et al. [92]):

$$f_{LCC}(\sigma_a,\sigma_m) = \begin{cases} [h_{\sigma_{max}}(\sigma_a) - h_{\sigma_{min}}(\sigma_a)]\delta(\sigma_m) + \\ h_{\sigma_{min}}(\sigma_m)\delta(\sigma_a), & \text{if } \sigma_a + \sigma_m > 0, \\ h_{\sigma_{max}}(\sigma_m)\delta(\sigma_a), & \text{if } \sigma_a + \sigma_m \le 0, \end{cases} \tag{25}$$

with $h_{\sigma_{max}}(\sigma_a)$ and $h_{\sigma_{min}}(\sigma_a)$ obtained from the narrow-band approximation, and $\delta(.)$ representing the Dirac delta function, whereas $f_{RC}(\sigma_a,\sigma_m)$ is obtained from the range counting method, originally suggested by Tovo [91]:

$$f_{RC}(\sigma_a,\sigma_m) = \frac{1}{\sigma_X^2\alpha_2^2\sqrt{2\pi}} e^{\left[-\frac{\sigma_a^2+\sigma_m^2}{2\sigma_X^2(1-\alpha_2^2)}\right]} e^{\left\{-\left[\frac{\sigma_a^2}{\sigma_X^2(1-\alpha_2^2)}\right]\left[\frac{1-2\alpha_2^2}{2\alpha_2^2}\right]\right\}} \left[\frac{\sigma_a}{\sqrt{\sigma_X^2(1-\alpha_2^2)}}\right]. \tag{26}$$

The weighting factor is assumed to be dependent on the spectral parameters but any theoretical relation has been analytically derived. Nevertheless, some empirical expressions have been proposed and corroborated with experimental data (see Tovo [91]), as for example:

$$b = \min\left\{\frac{\alpha_1-\alpha_2}{1-\alpha_1}, 1\right\}; \tag{27}$$

$$b = \frac{1.112(\alpha_1-\alpha_2)(1+\alpha_1\alpha_2-(\alpha_1+\alpha_2))e^{2.11\alpha_2}}{(\alpha_2-1)^2} + \frac{(\alpha_1-\alpha_2)^2}{(\alpha_2-1)^2}. \tag{28}$$

being the last proposal the most consistent and accurate on a large group of spectra (see Slavič et al. [23]).

Table 2 summarizes the proposed stress amplitudes in literature previously detailed. Only those authors that define explicitly distribution for the stress amplitudes are here included. In the next section, additional authors will be included concerning the damage model since different proposals can be suggested based on the same stress amplitude distribution.

Finally, Fig. 9 depicts the application of these common proposed models in literature to an example of practical data due to Nieslony et al. [94].

6. Damage model

From the first Palmgren's proposal (see Palmgren [7]) for the damage accumulation in fatigue and its mathematical expression due to Miner (see Miner [8]), several efforts were devoted to develop more advanced damage models focused on modifying the mathematical laws, e.g. from linear to double-linear and non-linear, and also discussing about the suitable reference parameters. Recently, the novel continuum damage approach was also widely celebrated, especially in finite element background, but also the probabilistic approaches. In this section, a brief summary of the most relevant damage models presented in both time- and frequency-domain approaches is presented (see Table 3). After that, a critical comparison between them is presented.

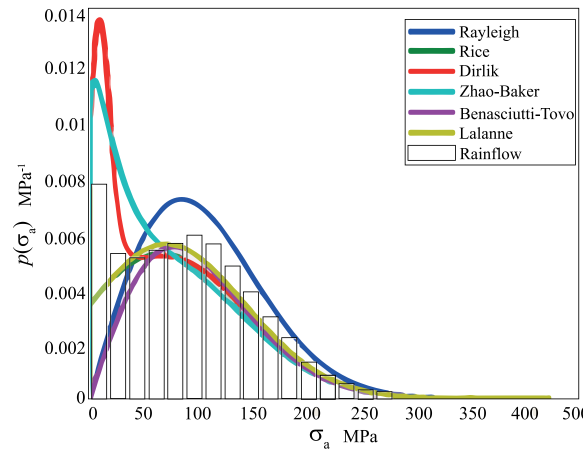

Fig. 9. Different proposed models in literature for the rainflow cycle distribution (see Nieslony et al. [94]).

Regarding time-domain approaches, the most relevant damage models are listed in the following (further details and an extensive literature review can be found in Fatemi and Yang [104]):

- Linear damage models. Palmgren [7] and Miner [8] originally proposed a linear damage model, as the first mathematical expression of the cumulative fatigue damage, based only on the ratio between the applied cycles n_i and the total cycles to failure N_{N_i} under the ith load (see Fig. 10). However, its main drawbacks are the independence with respect to both load-level and load-sequence together with a lack of load-interaction accountability. Alternatively, the so-called double-linear models, firstly proposed by Langer [105] and Grover [106], suggest to separate the stages in crack initiation N_I and crack propagation N_{II} and apply the Palmgren–Miner to each of these stages. This proposal was formulated explicitly by Manson et al. [107]
- Non-linear damage models. In order to improve aforementioned deficiencies in Palmgren–Miner model, Richart and Newmark [108] and later Marko and Starkey [95] proposed the first non-linear damage rule based on a powering cycle ratio to x_i variable for the ith loading.
- Energy damage models. As an alternative approach to previous models, Leis [96] proposed and energy-based nonlinear history-dependent damage model, related with the Smith–Watson–Topper parameter [26], including material coefficients (σ'_f and ϵ'_f as the fatigue strength and ductility coefficients) and constants b_1, c_1 as dependent on an instantaneous strain-hardening law. As an alternative, Niu et al. [97] suggested a modified damage model dependent on load-interaction, especially focused on materials with cyclic hardening.
- Continuum damage models. This novel damage approach deals with the continuum mechanical behaviour of a material in degenerating conditions, originally founded by Kachanov [109] and Rabotnov [110]. The proposal by Chaboche and Lesne [99] is one

Table 3
Proposed fatigue damage models in frequency-domain.

Fatigue damage model		Expression
Time-domain	Palmgren–Miner [7,8]	$D = \Sigma n_i / N_{N\,i}$
	Marko–Starkey [95]	$D = \Sigma (n_i / N_N)^{x_i}$
	Leis [96]	$D = \frac{4\sigma'_f}{E}(2N_f)^{2b} + 4\sigma'_f \epsilon'_f (2N_f)^{b_1+c_1}$
	Niu et al. [97]	$D = (n/N)^{1/(n'+\alpha)}$
	Fernández-Canteli [98]	$p = 1 - \exp\left[-\left(\frac{(\log N - B)(\log \Delta\sigma - C) - \lambda}{\delta}\right)^{\beta}\right]$
	Chaboche–Lesne [99]	$D = 1 - [1 - (n/N)^{1/(1-\alpha)}]^{1/(1+\beta)}$
Frequency-domain	Narrow-band approximation [93]	$\overline{D}^{\mathrm{NB}} = \nu_0 C^{-1}\left(\sqrt{2m_0}\right)^k \Gamma\left(1+\frac{k}{2}\right)$
	Range-mean approximation [92]	$\overline{D}^{\mathrm{RC}} = \nu_0 C^{-1}\left(\sqrt{2m_0}\alpha_2\right)^k \Gamma\left(1+\frac{k}{2}\right)$
	Wirsching–Light method [100]	$\overline{D}^{\mathrm{WL}}_{\mathrm{RFC}} = \left[a(k) + [1 + a(k)](1-\varepsilon)^{b(k)}\right]\overline{D}^{\mathrm{NB}}$
	$\alpha_{0.75}$ method [101]	$\overline{D}^{\mathrm{AL}}_{\mathrm{RFC}} = \alpha^2_{0.75}\overline{D}^{\mathrm{NB}}$
	Jiao–Moan method [54]	$\overline{D}^{\mathrm{JM}} = \rho_{\mathrm{JM}}\overline{D}^{\mathrm{NB}}$
	Gao–Moan method [53]	$\overline{D}^{\mathrm{GM}}_{\mathrm{RFC}} = \overline{D}_P + \overline{D}_Q + \overline{D}_H$
	Dirlik method [4]	$\overline{D}^{\mathrm{DK}}_{\mathrm{RFC}}(\sigma_a) = \frac{\nu_p}{C}\sigma_X^k\left[D_1 Q^k \Gamma(1+k) + (\sqrt{2})^k \Gamma\left(1+\frac{k}{2}\right)(D_2\lvert R\rvert^k + D_3)\right]$
	Zhao–Baker method [90]	$\overline{D}^{\mathrm{ZB}}_{\mathrm{RFC}}(\sigma_a) = \frac{\upsilon_p}{C}\sigma_X^k\left[w a^{-\frac{k}{b}}\Gamma\left(1+\frac{k}{b}\right) + (1+w)2^{\frac{k}{2}}\Gamma\left(1+\frac{k}{2}\right)\right]$
	Tovo–Benasciutti method [20,91]	$\overline{D}^{\mathrm{TB}}_{\mathrm{RFC}} = \left[b + (1-b)\alpha_2^{k-1}\right]\overline{D}^{\mathrm{NB}}$
	Petrucci–Zucarello method [102]	$\overline{D}^{\mathrm{PZ}}_{\mathrm{RFC}} = C^{-1}\nu_p\sqrt{m_0^k}e^{\Psi(\alpha_1,\alpha_2,b,\gamma)}$
	Zalaznik–Nagode method [103]	$\overline{D}^{\mathrm{ZN}}_{\mathrm{RFC}} = \nu_p Co(T)^{-1}\lambda_0^{k(T)/2}\left[D_1 Q^{k(T)}\Gamma(1+k(T)) + (\sqrt{2})^{k(T)} + \Gamma\left(1+\frac{k(T)}{2}\right)\left(D_2\lvert R\rvert^{k(T)} + D_3\right)\right]$

of the most representative one from this recent approach, which is a highly non-linear damage rule able to consider the mean stress effect, with β as a material constant and α as a function of the stress state.

– Probabilistic damage models. Fernández-Canteli [98] originally suggested a probabilistic interpretation of the Miner's linear rule, which was later formulated in mathematical terms (see Fernández-Canteli et al. [111] and Castillo et al. [112]) based on the Castillo–Canteli model for the S-N field (see Castillo and Fernández-Canteli [19]) with the normalized variable V, allowing the probability distribution of the Miner's number to be derived according to Extreme Value Theory (see also Castillo and Fernández-Canteli [113]).

Some previous models may be considered as useful and adequate, from a mathematical viewpoint, when the process exhibits a smooth behaviour in time, since there is a finite number of local extremes in the interval $[0, T]$ and a finite summation is justified. However, when considering a random loading history the number of local extremes is not obvious and the damage variable is hardly to be defined (see Rychlik [75]). For this reason, under a frequency-domain approach, and taking into account that the statistical information about the loading history is characterized by the joint distribution of the amplitudes and mean stress $f_{\mathrm{RFC}}(\sigma_a, \sigma_m)$, the fatigue damage variable can be analytically defined in the frequency-domain by assuming a S-N model in a continuous approach, i.e. as an integral, contrarily to the finite summation of Palmgren–Miner rule. Thus, by considering the Basquin law ($s^k N = C$), the total fatigue damage from the rainflow counting method results as follows (see Rychlik [75] for the mathematical derivation):

$$\overline{D}_{\mathrm{RFC}} = \nu_a C^{-1}\int_0^\infty \sigma_a^k p_{\mathrm{RFC}}(\sigma_a)d\sigma_a, \tag{29}$$

where ν_a is the number of counted cycles per unit time, assumed equal to ν_p from (5) in complete cycles, and $p_{\mathrm{RFC}}(\sigma_a)$ can be defined according to previous models. As a matter of fact, it is well-known

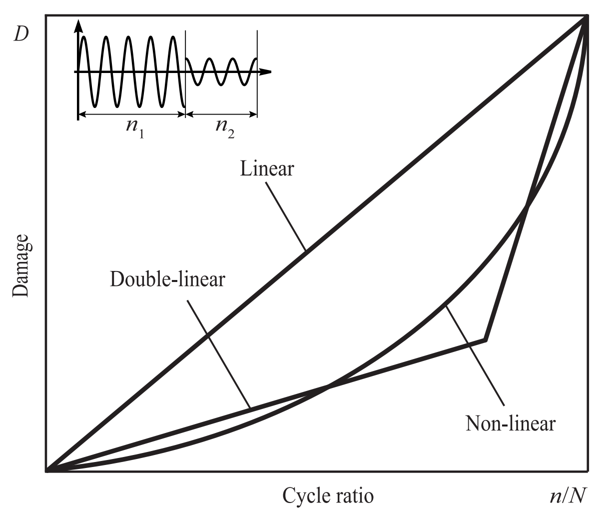

Fig. 10. Illustration of different fatigue damage rules: linear, double-linear and non-linear.

that fatigue damage obtained from rainflow counting $\overline{D}_{\mathrm{RFC}}$ is always bounded between two limit references (see Rychlik [75]), :

$$\overline{D}_{\mathrm{RC}} \le \overline{D}_{\mathrm{RFC}} \le \overline{D}_{\mathrm{NB}}, \tag{30}$$

where $\overline{D}_{\mathrm{NB}}$ corresponds with the damage by the narrow-band approximation, as the most conservative solution:

$$\overline{D}_{\mathrm{NB}} = \nu_0 C^{-1} T\left(\sqrt{2\lambda_0}\right)^m \Gamma\left(1+\frac{m}{2}\right), \tag{31}$$

and $\overline{D}_{\mathrm{RC}}$ with the range-mean counting (see Madsen [92]):

$$\overline{D}_{\mathrm{RC}} = \nu_0 C^{-1} T\left(\sqrt{2\lambda_0}\alpha_2\right)^m \Gamma\left(1+\frac{m}{2}\right), \tag{32}$$

From Eq. (31) and Eq. (32) it results that both limits are related, such that,

$$\overline{D}_{\mathrm{RC}} = \alpha_2^{m-1}\overline{D}_{\mathrm{NB}}. \tag{33}$$

Once these basic definitions has been presented, now the fatigue damage proposed models in literature in the last decades are described (see Table 3):

- Narrow-band approximation. Based on the assumption that in narrow-band processes each peak is coincident with a cycle, the amplitude cycles are thus Rayleigh distributed and the corresponding damage variable is defined as follows (see Miles [93]):

$$\overline{D}_{\mathrm{NB}} = \nu_0 C^{-1}\left(\sqrt{2m_0}\right)^k \Gamma\left(1+\frac{k}{2}\right), \tag{34}$$

  where ν_0 is the expected positive zero-crossings intensity defined in Eq. (5), C and k are material fatigue parameters from Basquin law, m_0 corresponds with the zero-th spectral moment and α_2 is the bandwidth parameter according to Eq. (6).
- Wirsching–Light method [100] proposed a modified expression based on the NB approximation in Eq. (34) by introducing a correction factor:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{WL}} = \rho_{\mathrm{WL}}\overline{D}_{\mathrm{NB}}, \tag{35}$$

  with ρ_{WL} as an empirical factor, originally investigated and simulated by Yang and coworkers [114,115], which is defined as follows:

$$\rho_{\mathrm{WL}} = \left[a(k) + [1+a(k)](1-\epsilon)^{b(k)}\right], \tag{36}$$

  where the additional parameter ϵ is related with the bandwidth parameter α_2 as follows:

$$\epsilon = \sqrt{1-\alpha_2^2}, \tag{37}$$

  while the functions $a(.)$ and $b(.)$ are dependent on the slope k of the S-N curve according to the following expressions:

$$a(k) = -0.33k + 0.926; \quad b(k) = 1.587k - 2.323. \tag{38}$$

- $\alpha_{0.75}$ method, firstly proposed by Benasciutti and Tovo [101], suggests a simple different correction of the narrow-band approximation based on the bandwidth parameters with acceptable agreement:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{AL}} = \alpha_{0.75}^2\overline{D}^{\mathrm{NB}}, \tag{39}$$

  where $\alpha_{0.75}$ is simply obtained according with (6) by calculating $\lambda_{0.75}$.
- Jiao–Moan method [54]. Especially focused on the bimodal processes, the total damage can be obtained by the combination of individual damages in both high and low components equivalently to (17), that is,

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{JM}} = \rho_{\mathrm{JM}}\overline{D}^{\mathrm{NB}}, \tag{40}$$

  where the correction factor ρ_{JM} is defined as follows:

$$\rho_{\mathrm{JM}} = \frac{\nu_{0,2}}{\nu_{0,X}} + \frac{\nu_{0,2}}{\nu_{0,X}}\left[\lambda_1^{*\left(\frac{k}{2}+2\right)}\left(1-\sqrt{\frac{\lambda_2^*}{\lambda_1^*}}\right) + \sqrt{\pi\lambda_1^*\lambda_2^*}\,\frac{k\Gamma\left(\frac{k}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{k}{2}+1\right)}\right], \tag{41}$$

  where k is the constant in the Basquin law, λ_i^* is the spectral parameters of each of the components,while $\nu_{0,2}$ and $\nu_{0,X}$ correspond with the expected zero up-crossing are of one of the components and the envelope, respectively. According to its definition in Eq. (40), this also provides information about the assumption of the narrow-band approximation, since if ρ tends to unity it corresponds with a unimodal NB case.
- Gao–Moan method [53]. As a natural extension of previous bimodal approach, the damage variable has to be calculated as the combination of each narrow-band process, each of them following a Rayleigh distribution:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{GM}} = \overline{D}_P + \overline{D}_Q + \overline{D}_H. \tag{42}$$

  where $\overline{D}_P$ is the fatigue damage due to the intermediate frequency plus the high frequency envelope process, $\overline{D}_Q$ is the fatigue damage due to the low frequency process plus the high frequency envelope process, and $\overline{D}_H$ is the fatigue damage due to high frequency process.
- Dirlik [4]. By substituting the proposed mixture distribution in (20) and integrating the fatigue damage, the Dirlik's model results as follows:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{DK}}(\sigma_a) = \frac{\nu_p}{C}\sigma_X^k\left[D_1 Q^k\Gamma(1+k) + (\sqrt{2})^k\Gamma\left(1+\frac{k}{2}\right)(D_2|R|^k + D_3)\right]. \tag{43}$$

- Zhao and Baker [90]. According to the suggested mixture between Weibull and Rayleigh distributions, the fatigue damage model proposed by Zhao and Baker is derived:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{ZB}}(\sigma_a) = \frac{\upsilon_p}{C}\sigma_X^k\left[wa^{-\frac{k}{b}}\Gamma\left(1+\frac{k}{b}\right) + (1+w)2^{\frac{k}{2}}\Gamma\left(1+\frac{k}{2}\right)\right]. \tag{44}$$

- Tovo and Benasciutti [20,91]. Based on the linear combination for the rainflow cycle distribution in Eq. (24), the fatigue damage is also obtained directly:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{TB}} = \left[b + (1-b)\alpha_2^{k-1}\right]\alpha_2\overline{D}^{\mathrm{NB}}, \tag{45}$$

  with $\overline{D}^{\mathrm{NB}}$ representing the fatigue damage under narrow-band assumption.
- Petrucci–Zucarello method [102] explored a theoretical connection between the equivalent Goodman stress for the mean stress effect and set of parameters: $\{\alpha_1, \alpha_2, k, \gamma\}$, with α_i as the bandwidth parameters, k as the constant in Basquin law and γ as the ratio between the maximum stress value $x_{\max}$ and the ultimate tensile strength of the material S_u. The final expression for estimating the fatigue damage is defined as follows:

$$\overline{D}_{\mathrm{RFC}}^{\mathrm{PZ}} = C^{-1}\nu_p\sqrt{m_0^k}e^{\Psi(\alpha_1,\alpha_2,b,\gamma)}, \tag{46}$$

  where $\Psi(\alpha_1,\alpha_2,b,\gamma)$ is called the approximation function:

$$\Psi(\alpha_1,\alpha_2,b,\gamma) = \frac{\Psi_2-\Psi_1}{6}(b-3) + \Psi_1 + \left[\frac{2}{9}\left(\Psi_4-\Psi_3-\Psi_2+\Psi_1\right)(k-3) + \frac{4}{3}(\Psi_3-\Psi_1)\right](\gamma-0.15), \tag{47}$$

  with the following constants:

$$\Psi_1 = -15.402\alpha_1^2 - 1.483\alpha_2^2 + 15.261\alpha_1\alpha_2 + 18.349\alpha_1 - 9.381\alpha_2 - 1.994; \tag{48}$$

$$\Psi_2 = -20.026\alpha_1^2 + 1.338\alpha_2^2 + 27.748\alpha_1\alpha_2 + 21.522\alpha_1 - 26.510\alpha_2 + 8.229; \tag{49}$$

$$\Psi_3 = -13.198\alpha_1^2 + 0.382\alpha_2^2 + 11.867\alpha_1\alpha_2 + 15.692\alpha_1 - 8.025\alpha_2 - 0.946; \tag{50}$$

$$\Psi_4 = -19.967\alpha_1^2 + 5.379\alpha_2^2 + 26.487\alpha_1\alpha_2 + 21.628\alpha_1 - 26.058\alpha_2 + 8.780; \quad (51)$$

After the main models of both approaches have been introduced, it is possible to conclude that the cycle counting approach (Section 5) selected to perform the treatment of the loading history (Section 4) clearly conditions the available options to develop a damage rule. The time-domain-methods are simplest on the discretization of the loading history, by using a RCF, and the histogram obtained conducts the vast majority of authors to use damage rules based on the summation of the damage performed by each $\sigma_a - n$ pair of data. On the other hand, the treatment of the loading history by the frequency-domain-methods is more advanced, by using density functions to fit it, allowing authors to define analytical and probabilistic damage models.

7. Conclusions

A comparative review of time- and frequency-domain methods for fatigue damage assessment under random loading has been performed, identifying their main drawbacks and advantages in each of the steps in a typical fatigue design process. It can be conclude that:.

- The development of material characterization models (in S-N or ε-N fields) is more prolific in time-domain methods than in the frequency case, existing several deterministic and probabilistic models in the former whereas only the Basquin rule is usually applied in the latter.
- The selection of a suitable reference parameter (stress, strain, energy, etc.) is an ongoing topic in time-domain methods, in both axial or multiaxial cases, while only in multiaxial fatigue is being discussed in the spectral methods.
- The rainflow counting method is currently used to convert the original random loading in a discrete stress amplitudes histogram, exhibiting some disadvantages: the dependency with the particular time window examined, besides not providing with reliable statistical information about the signal.
- The frequency-domain methods make use of the spectral density function of the random loading to characterize it by defining the spectral moments, supporting a statistical approach to the rainflow cycles, existing several approximated or exact proposals for the joint distribution of stress amplitudes and means.
- The development of damage models in time-domain methods deals with several reference parameters, being defined in terms of stress, strain and energy variables, while in spectral methods, though the diversity of proposals, only stresses are considered in its definition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to express their gratitude to the Spanish Ministry of Science, Innovation and Universities for the financial support of the project MCI-20-PID2019-105593GB-I00/AEI/10.13039/50110001 1033 from which this paper is inspired.

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