Universidad de Oviedo

Programa de Doctorado en Materiales

# Estudio de la correspondencia AdS/CFT en bajas dimensiones

# Study of the AdS/CFT correspondence in low dimensions

TESIS DOCTORAL

María Anayeli Ramírez Ortiz

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TESIS DOCTORAL

Directora de tesis
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# RESUMEN DEL CONTENIDO DE TESIS DOCTORAL

| 1.- Título de la Tesis | |
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**RESUMEN (en español)**

Esta tesis está enfocada en la construcción de soluciones de supergravedad con factores AdS y sus descripciones putativas de las teorías de campo a través de la correspondencia AdS/CFT. Enfatizamos en teorías de bajas dimensiones, específicamente en geometrías con subespacios $AdS_3$ y $AdS_2$ con cuatro supersimetrías de Poincaré.

Varias técnicas son discutidas acerca de la construcción de soluciones en supergravedad, desde dualidades en teorías de cuerdas, como T-dualidad o su generalización a grupos no Abelianos, así como también en continuaciones analíticas y técnicas de *G-structures*. Con respecto a la interpretación de las soluciones obtenidas en la teoría de campo, se estudian aspectos geométricos de las soluciones, llevando a proponer teorías de campos duales que involucran productos de grupos de gauge y campos de materia, es decir, teorías de campo *quiver* cuyo contenido es determinado por Dp- y NS5-branas. Verificamos estas dualidades con diferentes observables usando holografía.

La tesis contiene nueve capítulos. El primer capítulo constituye una introducción donde ponemos en contexto este trabajo. Revisamos en detalle la construcción de soluciones en supergravedad en el Capítulo 2. El Capítulo 3 contiene el estudio de un sistema de branas D4-NS5-D6 como ejemplo de las construcciones en teoría de campo que son usadas en este trabajo. Un resumen de los principales resultados obtenidos en esta tesis es dado en el Capítulo 4. Capítulos 5, 6, 7 y 8 contienen los resultados completos de la tesis. Finalmente, las conclusiones están dadas en el Capítulo 9.

**RESUMEN (en Inglés)**

This thesis is focused on the construction of supergravity backgrounds with AdS factors and their putative field theory descriptions via the AdS/CFT correspondence. We put emphasis on lower-dimensional theories, specifically on geometries with $AdS_3$ and $AdS_2$ subspaces with four Poincaré supersymmetries.

Diverse techniques concerning the construction of supergravity solutions are discussed, from string theory dualities, such as T-duality (ATD) or its generalization to non-Abelian groups (NATD), as well as double analytical continuations and G-structure techniques. Regarding the field theory interpretation of the obtained solutions, geometrical aspects of the backgrounds are studied, leading us to propose dual field theories involving products of gauge groups and matter

fields, namely, quiver field theories whose content is determined by Dp- and NS- branes. We test such dualities through different observables using holography.

The thesis consists of nine chapters. The first chapter constitutes an introduction where the present work is set in context. We review in detail the construction of solutions in supergravity in Chapter 2. Chapter 3 contains the study of the D4-NS5-D6 brane system as an example of the field theoretical constructions used in the present work. A brief summary of the main results obtained in this work is given in Chapter 4. Chapters 5, 6, 7 and 8 contain the complete results of the thesis. Finally, the conclusions are provided in Chapter 9.

**SR. PRESIDENTE DE LA COMISIÓN ACADÉMICA DEL PROGRAMA DE DOCTORADO EN MATERIALES**

*A mis hermanos: Kathia, Tadeo y Gabriel*

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# Resumen

Esta tesis está enfocada en la construcción de soluciones de supergravedad con factores AdS y sus descripciones putativas de las teorías de campo a través de la correspondencia AdS/CFT. Enfatizamos en teorías de bajas dimensiones, específicamente en geometrías con subespacios $\mathrm{AdS}_3$ y $\mathrm{AdS}_2$ con cuatro supersimetrías de Poincaré.

Varias técnicas son discutidas acerca de la construcción de soluciones en supergravedad, desde dualidades en teorías de cuerdas, como T-dualidad o su generalización a grupos no Abelianos, así como también en continuaciones analíticas y técnicas de *G-structures*. Con respecto a la interpretación de las soluciones obtenidas en la teoría de campo, se estudian aspectos geométricos de las soluciones, llevando a proponer teorías de campos duales que involucran productos de grupos de gauge y campos de materia, es decir, teorías de campo *quiver* cuyo contenido es determinado por Dp- y NS5-branas. Verificamos estas dualidades con diferentes observables usando holografía.

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# Contents

# 1. Introduction

Since its origins in the sixties, string theory has expanded into a vast variety of topics, while providing diverse mathematical and physical toolkits. A prominent example is the AdS/CFT correspondence, which has outspread the traditional domain of string theory to areas such as condensed matter and quantum information (to say a few). Besides that, the main role or string theory has been as providing the best candidate to describe *the* theory of nature. The naive idea of replacing point particles with one dimensional objets, strings, solves the problem of reconciling general relativity with quantum mechanics, giving us a consistent joint description of gravity with the other observed forces in nature (electromagnetic, weak nuclear and strong nuclear forces).

String theory was born at the end of 1960 trying to tackle the strong nuclear force problem. However, the interest was quickly lost due to the introduction of quantum chromodynamics and the many troubles that it presented: hadrons had to live in a 26 dimensional spacetime, the spectrum of the theory contained a tachyon and predicted particles that were absent in the experiments. But the idea of string theory did not cease at all. The inclusion of fermionic degrees of freedom in the worldsheet by Ramond, and independently by Neveu and Schwarz, led to the idea of supersymmetry and superstrings [1, 2]. Later on, with the work of Gliozzi, Scherk and Olive [3], the tachyon could be removed from the spectrum and a consistent supersymmetric string theory could be formulated. Since then, extensive research has been performed around supersymmetry, as well as around its local generalisation, the so-called supergravity, also known as the supersymmetric extension of general relativity.

At that stage, string theory provided a complex framework, with three consistent superstring theories: one theory with open strings with $\mathcal{N} = 1$ supersymmetry and two theories of closed strings with extended $\mathcal{N} = 2$ spacetime supersymmetry and whose massless spectrum corresponded with two already known supergravity theories, namely, the type IIA and type IIB supergravities. Not long after, two more theories arrived: the heterotic string theories [4], leaving us with five consistent superstring theories with open and closed strings as fundamental objects. Later on it was realised that, the open strings required the existence of a second class of extended objects, the D-branes. These are non-perturbative objects on which open strings can end, which render these objects dynamical.

At the beginning of 1995, there were five superstring theories and none of them was considered the right model. At that time, Witten showed in [5] that the five theories are related among themselves by a web of dualities[1] and also, inspired by many preliminary works, with an eleven dimensional theory [6]. Such theory, introduced twenty years before, was the so-called eleven dimensional supergravity theory, the supergravity with the highest possible dimension that can be constructed. At this point, M-theory was born, and two years later so was the AdS/CFT correspondence, deeply connected to the double nature of D-branes, as end points of open strings and as solutions to the supergravities arising as low energy limit of superstrings theories [7].

In a (historical) nutshell, we have introduced the main concepts that appear in this thesis. Now, we will dig a little deeper.

### Supergravity

Let us start by describing the eleven dimensional theory previously alluded to, from this theory type IIA supergravity can be extracted via dimensional reduction. The bosonic content of the theory includes the metric and a three-form potential $C_{\mu\nu\rho}$, which appears in the action through its field strength $F_4 = dC_3$. The bosonic part of the action reads

$$S_{11} = \frac{1}{2\kappa_{11}} \int d^{11}x \sqrt{-g}(R - \frac{1}{48}F_4^2) - \frac{1}{12\kappa_{11}} \int C_3 \wedge F_4 \wedge F_4, \tag{1.1}$$

[1]The term duality is widely used by physicists to refer to the relationship between two systems with identical physics but described in a different way.

where $\kappa_{11}$ denotes the eleven dimensional gravitational coupling constant. This theory has $\mathcal{N} = 1$ supersymmetry, which in eleven dimensions corresponds to 32 supercharges. Notice the presence of a three-form gauge field, $C_{\mu\nu\rho}$. These fields couple to branes, which are sources of the potential. In this case, the three-form couples electrically to an M2-brane and magnetically to an M5-brane.

The dimensional reduction of this theory leads us to a theory with $\mathcal{N} = 2$ supersymmetries in ten dimensions. This reduction reads

$$\begin{gathered} ds_{11}^2 = e^{2\Phi/3} ds_{10}^2 + e^{4\Phi/3}(dz + C_1)^2, \\ C_{\mu\nu\rho}^{11} = C_{\mu\nu\rho}^{10}, \qquad C_{\mu\nu z}^{11} = B_{\mu\nu}. \end{gathered} \tag{1.2}$$

From the decomposition of the eleven dimensional metric, $ds_{11}^2$, one gets a ten dimensional metric $ds_{10}^2$, a gauge field $C_1$, and a scalar field known as dilaton, $\Phi$. Besides, the three-form gives rise to a NSNS two-form, $B_{\mu\nu}$, and a RR three-form, $C_{\mu\nu\rho}^{10}$. These fields, together with their Hodge duals, through

$$F_p = (-1)^{[n/2]} \star_{10} F_{10-n}, \tag{1.3}$$

to produce a five- and a seven-form, $C_5$, $C_7$, comprise the bosonic sector of the so-called, massless, type IIA supergravity.

An extension of massless type IIA supergravity can be formulated to include a mass term in the action, called Romans' mass, $F_0$. In this way, it is easy to write the action for both theories in a unique way and recover the massless theory setting $F_0 = 0$. The (bosonic sector of the) action, of both type IIA supergravities can be written in a unique way, as

$$\begin{aligned} S_{IIA} =& \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{H_3^2}{12}) - \frac{F_0^2}{2} - \frac{F_2^2}{4} - \frac{F_4^2}{48} \right) \\ & - \frac{1}{4\kappa_{10}^2} \int \left( dC_3 \wedge dC_3 \wedge B + \frac{F_0}{3} dC_3 \wedge B^3 + \frac{F_0^2}{20} B^5 \right), \end{aligned} \tag{1.4}$$

in string frame, with the field strengths of the NSNS and RR potentials given by

$$H_3 = dB, \quad F_2 = dC_1 + BF_0, \quad F_4 = dC_3 - H_3 \wedge H_3 \wedge C_1 + \frac{F_0}{2} B \wedge B. \tag{1.5}$$

The massless fermions of type IIA supergravity consist of two Majorana-Weyl gravitinos of opposite chirality and two Majorana-Weyl dilatinos of opposite chirality. Therefore, they describe a non-chiral $\mathcal{N} = 2$ supersymmetric theory in ten dimensions.

The second $\mathcal{N} = 2$ supergravity, type IIB supergravity, cannot be obtained by dimensional reduction from eleven-dimensional supergravity. They simplest way to derive it is from type IIA supergravity via a T-duality transformation, as we will see in Section 2.1. The fermionic massless spectrum of this theory consists of two left-handed Majorana-Weyl gravitinos and two right-handed Majorana-Weyl dilatinos. Since it has two spinors with the same chirality, we have a chiral theory with $\mathcal{N} = 2$ supersymmetry in ten dimensions. The NSNS bosonic sector is composed by the metric, the dilaton and a rank two antisymmetric tensor. The RR sector contains the forms fields $C_0$, $C_2$ and $C_4$, and the higher forms coming from Hodge duality of the field strengths. The action for these fields is,

$$\begin{aligned} S_{IIB} =& \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{H_3^2}{12}) - \frac{F_1^2}{2} - \frac{F_3^2}{12} - \frac{F_5^2}{480} \right) \\ &+ \frac{1}{4\kappa_{10}^2} \int \left( dC_2 \wedge H_3 \wedge (C_4 + \frac{1}{2} B \wedge C_2) \right), \end{aligned} \tag{1.6}$$

where the field strengths are,

$$H_3 = dB, \quad F_1 = dC_0, \quad F_3 = dC_2 + H_3 C_0, \quad F_5 = dC_4 + H_3 \wedge C_2. \tag{1.7}$$

The e.o.m coming from the action have to be supplemented by a self-duality condition on $F_5$,

$$F_5 = \star_{10} F_5. \tag{1.8}$$

In both theories, the fermionic fields are two gravitinos, $\psi^A$, and two dilatinos, $\lambda^A$. The condition for unbroken symmetry is that the variations of these fermionic fields vanish. That is, the bosonic part of the dilatino and gravitino is put to zero,

$$\delta \psi_M = 0, \qquad \delta \lambda = 0. \tag{1.9}$$

Explicitly, the supersymmetric variations of these fields can be written as [8],

$$\begin{aligned} \delta\lambda =& \left(\not{\partial}\phi + \frac{1}{2}\not{H}\mathcal{P}\right)\epsilon + \frac{1}{8}e^{\phi}\sum_n (-1)^n(5-n)\not{F}_n\mathcal{P}_n\epsilon, \\ \delta\psi_M =& \nabla_M\epsilon + \frac{1}{4}\not{H}_M\mathcal{P}\epsilon + \frac{1}{16}e^{\phi}\sum_n \not{F}_n\Gamma_M\mathcal{P}_n\epsilon, \end{aligned} \tag{1.10}$$

where $\epsilon$, $\lambda$ and $\psi_M$ are doublets of Majorana-Weyl spinors. The doublet $\psi_M = (\psi^1_M, \psi^2_M)$ contains the two Majorana-Weyl spinors of opposite chirality in IIA and the same chirality for IIB. The same for $\epsilon$ and $\lambda$. $\mathcal{P}$ and $\mathcal{P}_n$ are matrices and they are different in IIA and IIB. In type IIA, $\mathcal{P} = \Gamma_{11}$ and $\mathcal{P}_n = \Gamma_{11}^{n/2}\sigma^1$. In type IIB, $\mathcal{P} = -\sigma^3$ and $\mathcal{P}_n = \sigma^1$ or $\mathcal{P}_n = i\sigma^2$, for $\frac{n+1}{2}$ even or odd respectively. Besides, the RR fluxes are written as

$$\not{F}_p = \frac{1}{p!}\Gamma_{\nu_1\ldots\nu_p}F_p^{\nu_1\nu_2\ldots\nu_p}, \tag{1.11}$$

where the slash means contraction with gamma matrices.

**AdS/CFT correspondence**

At the end of the nineties, a full new line of research with far reaching consequences came up as a by-product of the so-called Maldacena conjecture [7]. Such conjecture set up a particular statement which relates a string theory in ten dimensions with a quantum field theory in a flat spacetime. In general terms, the argument considers that a set of D-branes in a given superstring (or M- ) theory admits a double description, and in a precise limit, both descriptions are equivalent. In its original formulation, Maldacena conjectured that type IIB string theory in a $\mathrm{AdS}_5\times\mathrm{S}^5$ background is dual to 4d SYM theory with $\mathcal{N} = 4$ supersymmetry. The argument in this case is based in the double description of an array of $N$ D3-branes.

In the first description, one starts with a type IIB superstring theory in a Minkowski spacetime $\mathbf{R}^{1,9}$, where an array of $N$ D3-branes is placed in the $x_0$, $x_1$, $x_2$ and $x_3$ directions, thus introducing as well open strings. In the low energy limit[1], only the massless modes of the string spectrum are meaningful, therefore, the system is described by massless states of closed strings, open strings and the massless states

[1]That is, energies $E << l_s^{-1}$, where $l_s$ is the string length scale.

coming from the interaction between closed and open strings. Thus, the total action reads,

$$S = S_{closed} + S_{open} + S_{int}. \tag{1.12}$$

The $S_{open}$ action is obtained from the excitations of the stack of $N$ coincident D3-branes, that is the massless states of the open strings. For a single D3-brane the massless fields are six scalar fields $\phi^i$, a gauge field $A_\beta$ and an spinorial field. The effective action that describes these fields is the Dirac-Born-Infeld action,

$$S_{DBI} = -\frac{1}{(2\pi)^3 g_s l_s^4} \int d^4x \, e^{-\phi} \sqrt{-\det\left(P[g]_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\right)}. \tag{1.13}$$

where we have set the Kalb-Ramond field to zero for simplicity. The worldvolume fields are $x^\mu$ with $\mu = 0, ..., 3$. $P[g]$ is the pullback of the metric to the worldvolume of the D3-brane, and $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. When one studies this system, for a single D3-brane, at low energies one finds the following action to leading order in $\alpha'$,

$$S = -\frac{1}{2\pi g_s} \int d^4x \left( \frac{F_{\alpha\beta}F^{\alpha\beta}}{4} + \mathcal{O}(\alpha') \right). \tag{1.14}$$

This is the Yang-Mills action with gauge group U(1) provided that we identify

$$g_{YM}^2 = 2\pi g_s. \tag{1.15}$$

Generalising the action to the case of $N$ coincident D3-branes, the strings attached to the stack have freedom of having their endpoints in any of the coincident branes in the stack. In this way the massless states induce a U($N$) gauge theory, i.e. the scalars and gauge fields are U($N$) valued, $\phi^i = \phi^{ia} T_a$, $A_\beta = A_\beta^a T_a$. This implies that the gauge kinetic term becomes $F_{\alpha\beta}^a F^{a\alpha\beta}$ to ensure gauge invariance. Performing the $\alpha' \to 0$ limit, one finds that $S_{open}$ is just the bosonic part of the action of $\mathcal{N} = 4$ Super Yang-Mills (SYM) theory with $g_{YM}$ as in (1.15), namely $S_{open} \to S_{\mathcal{N}=4\,\text{SYM}}$.

The U($N$) gauge group can be split as U($N$) =SU($N$)$\times$U(1), where U(1) codifies the d.o.f. associated to the center of mass of the stack, which get decoupled from the remaining d.o.f. Therefore, the U(1) can be frozen and one just considers the SU($N$) factor.

$\mathcal{N} = 4$ SYM theory has a conformal group given by SO(4,2). Besides, the theory preserves $\mathcal{N} = 4$ supersymmetries, i.e. sixteen Poincaré supercharges and sixteen superconformal supercharges. All of these supersymmetries form the supergroup PSU$(2,2|4)$ under which $\mathcal{N}=4$ SYM is invariant.

The closed string states are described by the low energy limit of the closed string theory, namely $S_{closed}$ is just the type IIB supergravity action (1.6), $S_{closed} \to S_{IIB}$. Finally, in [9], Gubser, Maldacena, et.al. showed that the interaction term is $S_{int} \sim g_s \alpha'^2$, one then can see that $S_{int} \to 0$ in the low energy limit. Thus, the total system decouples in two sectors: supergravity in 9+1 dimensions, $S_{IIB}$, and the excitations of D3-branes, where the massless states give the field content of a four dimensional $\mathcal{N}=4$ vector supermultiplet, whose dynamics is described by the $\mathcal{N}=4$ SYM theory with a gauge group SU$(N)$, i.e. the action $S_{\mathcal{N}=4\,\mathrm{SYM}}$.

In the second description, the $N$ D3-branes are described as black 3-branes, that is, as solitonic solutions of the e.o.m. of type IIB supergravity, thus providing a background where the closed strings can propagate. The black 3-branes in this case are a source of RR 5-form flux and carry charge and mass:

$$\begin{aligned} ds^2 = & \ \tfrac{1}{\sqrt{H(r)}} \eta_{\mu\nu} dx^\mu dx^\nu + \sqrt{H(r)}\big(dr^2 + r^2 ds^2_{\mathrm{S}^5}\big), \\ & \text{where} \quad H(r) = 1 + \tfrac{L^4}{r^4}, \end{aligned} \tag{1.16}$$

and $L^4 = 4\pi g_s N l_s^4$. Besides, the solution includes a constant dilaton and a RR flux, $F_5$. If one takes $L$ as a characteristic scale in the theory, one can distinguish two regions in this solution. When $r >> L$ then $H(r) \sim 1$ and the solution is a Minkowski $\mathbf{R}^{9,1}$ spacetime. When $r << L$ then $H(r) \sim \frac{L^4}{r^4}$ and (1.16) becomes

$$\begin{aligned} ds^2 &= \frac{r^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{r^2}\big(dr^2 + r^2 ds^2_{\mathrm{S}^5}\big), \\ &= \frac{L^2}{r^2}(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) + L^2 ds^2_{\mathrm{S}^5}. \end{aligned} \tag{1.17}$$

In the second line we have introduced a new coordinate $z = L^2/r$. The metric (1.17) corresponds to a product between an AdS space in five dimensions and an S$^5$ sphere, both with curvature radius $L$.

The black 3-brane excitations are closed strings in a ten dimensional asymptotically flat spacetime and also closed strings in the region close to the throat. In the low energy limit both sectors get decoupled. This may be seen as follows: consider an string excitation with energy $E_r$ measured at a fixed point close to the throat, and the energy $E_\infty$ measured at infinity, both energies are related by

$$E_\infty = H^{-1/4} E_r. \tag{1.18}$$

For fixed $E_r$, one finds that as $r \to 0$ the energy observed at infinity, $E_\infty$, goes to zero, i.e. the observer at infinity is in the low energy regime. From this point of view, the observer at infinity sees two different low energy states, supergravity modes in flat 9+1 dimensional spacetime (type IIB supergravity) and closed string excitations close to the $r = 0$ region, which corresponds to an $\mathrm{AdS}_5 \times \mathrm{S}^5$ spacetime (so-called near horizon region). Summing up, in the low energy limit both types of closed strings decouple from each other.

So far, an array of $N$ D3-branes has been described in two different ways. The two descriptions get decoupled in the low energy limit, and as an outcome of the decoupling, both systems contain the type IIB supergravity theory in a flat ten dimensional spacetime. This fact allows us to identify the remaining sectors as follows:

*An $\mathcal{N} = \mathbf{4}$ SYM theory with gauge group $\mathbf{SU(N)}$ is dual to a string theory in type IIB supergravity living in an $\mathbf{AdS_5} \times \mathbf{S^5}$ background.*

This is known as the Maldacena conjecture. Nowadays the statement has been enlarged as a duality between a conformal field theory in $p$ dimensions and a string theory living in an $\mathrm{AdS}_{p+1} \times \mathrm{M}^{9-p}$ background.

As a trivial test of the correspondence, we can analyse the symmetries in both sides of the conjecture. As we mentioned before, the $\mathcal{N} = \mathbf{4}$ SYM theory contains $\mathrm{PSU}(2,2|4)$ as symmetry group. On the other side, the $\mathrm{AdS}_5 \times \mathrm{S}^5$ theory is invariant under the isometry groups of $\mathrm{AdS}_5$ and $\mathrm{S}^5$, that is SO(4,2) and SO(6), and if we add the fermionic sector, the isometry group is extended to $\mathrm{PSU}(2,2|4)$. This is just one of the many ways to check the conjecture.

Ever since the advent of the AdS/CFT correspondence in the context of type IIB string theory, it has become increasingly important the study of supersymmetric and conformal field theories in diverse dimensions. The last two decades witnessed a large effort to extend our encyclopedic knowledge of supergravity backgrounds, Type II or M-theory, involving AdS factors.

For instance, an infinite family of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs has been discussed both from the field theoretical and holographic points of view in [10, 11, 12, 13, 14, 15]. For $d = 5$, the works [16, 17, 18, 19, 20, 21] presented backgrounds with an $\text{AdS}_6$ factor and their UV-dual SCFTs. For $\mathcal{N} = 2$ SCFTs in four dimensions, the field theories studied in [22] have holographic duals discussed in [23, 24, 25]. The case of $d = 3$ SCFTs and the dual $\text{AdS}_4$ backgrounds is studied in [26, 27, 28].

The correspondence for the case of two-dimensional and one-dimensional (half-maximal BPS) low-energy SCFTs is particularly rich and has received a lot of attention recently. For instance, two dimensional CFTs play a prominent role in string theory and in other areas of theoretical Physics (condensed matter and quantum information systems are clear examples), besides the wide landscape of two dimensional CFTs. This does of 2d superconformal algebras a perfect theoretical lab to test the AdS/CFT correspondence.

Furthermore, low dimensional AdS spaces can also be studied in the context of defect conformal field theories. The defect CFTs usually arise when a brane intersection ends on a bound state which is known to be described by an AdS vacuum in the near-horizon limit. These brane intersections break some of the isometries of the vacuum, producing lower dimensional AdS backgrounds in the near-horizon limit. These lower dimensional spaces are dual to low dimensional CFTs, that retake a defect CFTs interpretation within the higher dimensional CFTs. Some examples with these realisations can be found in [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]

Perhaps more interesting is the role that 1d and 2d CFTs are playing in the microscopic description of black holes and black strings. It is well-known that an infinitely deep $\text{AdS}_2$($\text{AdS}_3$) throat arises as the near-horizon geometry of 4d(5d) extremal black holes. Even if this limit is clear geometrically a microscopic understanding remains a demanding task [41, 42, 43]. Via the Maldacena conjecture one might presume that there is a 1d (2d) conformal field theory dual to these $\text{AdS}_2$ ($\text{AdS}_3$) geometries. This motivated various attempts at finding classifications of $\text{AdS}_2$ and $\text{AdS}_3$ backgrounds and studying their dual CFTs [35, 37, 38, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67]. In particular, the study of $AdS_2$ as a natural extension of $AdS_3$, was discussed from the field theory perspective in [68, 69, 70]. The authors prove the $CFT_1$ arises as a discrete light-cone compactification (DLC) of the 2d CFT dual to the $AdS_3$ solution.

Nevertheless, the $AdS_2$/$CFT_1$ pairs pose well-known problems related to the no-connectedness of the boundary of $AdS_2$ [71] and the interpretations of the central charge of a superconformal quantum mechanics.

It is clear that a deeper understanding of the AdS/CFT correspondence in low dimensions is needed. One of the motivations of this thesis is to find and study new AdS/CFT pairs in lower dimensions with $N = 4$ supersymmetry. As an starting point, in the next section we show the techniques used to construct the AdS solutions to type II supergravity.

# 2. Solution Generating Techniques

Since the original Maldacena conjecture was formulated there has been continuous effort in the construction and classification of type II and M-theory solutions with AdS factors. These backgrounds are conjectured to be dual to SCFT in different dimensions and with different amounts of supersymmetry. We dedicate this Section to explore a few techniques in the construction of AdS solutions.

## 2.1 Abelian T-Duality (ATD)

Dualities play a very important role in physics, especially in high energy physics. One such role is as a solution generating technique in supergravity, in particular, T-duality can be used in the construction of new AdS spaces. T-duality establishes the equivalence between a string theory propagating on a $\mathbf{R}^{q+1}\times \mathrm{S}^1$ spacetime and another, or the same, string theory propagating on $\mathbf{R}^{q+1} \times \tilde{\mathrm{S}}^1$, where $\mathrm{S}^1$ and $\tilde{\mathrm{S}}^1$ are circles of radii $R$ and $\tilde{R} \sim \alpha'/R$, respectively.

Consider a string theory propagating on a spacetime whose $x^{q+1}$ direction is compactified on a circle of radius $R$. For a closed string one takes periodic boundary conditions in the $x^{q+1}$ coordinate,

$$x^{q+1}(\tau, \sigma + 2\pi) \equiv x^{q+1}(\tau, \sigma) + 2\pi R\, W, \qquad W \in \mathbb{Z}, \tag{2.1}$$

where $W$ is an arbitrary integer called winding number, that counts the number of times that the string winds around the $\mathrm{S}^1$. Since the $x^{q+1}$ direction is compactified the momentum in this direction must be quantised as $p^{q+1} = K/R$, with $K \in \mathbb{Z}$, the Kaluza-Klein, or momentum, number. The mass formula for the spectrum of the closed string states is then

$$M^2 = \left(\frac{K}{R}\right)^2 + \left(\frac{WR}{\alpha'}\right)^2 + \dots . \tag{2.2}$$

Notice that the closed string spectrum is invariant under the following substitutions,

$$\begin{aligned} R \longleftrightarrow \tilde{R} &= \frac{\alpha'}{R}, \\ (K, W) &\longleftrightarrow (W, K), \end{aligned} \tag{2.3}$$

which suggests a remarkable feature, namely, that the compactification on a circle of radius $R$ is physically equivalent to a compactification on a circle of radius $\alpha'/R$.

This behaviour arises due to the fact that in the presence of a compact dimension the closed strings have new, so-called winding, states. A natural question to ask is what happens with T-duality in a theory containing open strings, since open strings do not have a winding sector. Indeed, for open strings the winding number does not make sense since open strings can always be contracted to a point. As the winding number is crucial in the identification of the mass spectrum one can naively think that T-duality does not apply to theories containing open strings. Nevertheless, it was shown in [72] that T-duality can be recovered including D-branes, where not only the radius of the compact dimension changes but also the dimension of the D-brane,

$$(D(q+1) - \text{brane},\ R) \longleftrightarrow (Dq - \text{brane},\ \tilde{R} = \frac{\alpha'}{R}). \tag{2.4}$$

In this case, T-duality maps an open string with Neumann boundary conditions into an open string with Dirichlet boundary conditions, that is, into an open string ending on a Dq-brane. In the first theory the open string momentum in the compact direction $x^{q+1}$ is quantised $p^{q+1} = K/R$ and we do not have a winding number. In the dual world, due to the Dirichlet boundary conditions, there are no momentum states in the compact direction and besides, the endpoints of the string must be attached to the points $x^{q+1} = x_0^{q+1} + 2\pi K\tilde{R}$, with $x_0^{q+1}$ the position of the Dq-brane in the curled up direction. The crucial point is that we get winding states with the Dirichlet boundary conditions. Therefore, the momentum states in the first theory contribute to the mass spectrum in the same way as the winding states contribute to the mass spectrum in the dual world, under the identification $\tilde{R} = \alpha'/R$.

### 2.1.1 Buscher's rules

So far we saw that $R$, the radius of the $\mathrm{S}^1$ direction, gets interchanged with $\alpha'/R$ under T-duality for strings propagating in a flat spacetime. The next question is if it is possible to generalise the T-duality transformations to strings propagating in curved spacetimes with an isometry. This idea was addressed in [73, 74] where a path integral derivation of T-duality was put forward for backgrounds with a $\mathrm{U}(1)$ isometry.

These works considered a string propagating on a background consisting on a metric, an NSNS two-form and a dilaton. The string propagation is described by the non-linear sigma model (i.e. the low energy effective field action),

$$S = \int \mathrm{d}\sigma d\tau \left( \sqrt{h} h^{\alpha\beta} g_{MN} \partial_\alpha x^M \partial_\beta x^N + \epsilon^{\alpha\beta} B_{MN} \partial_\alpha x^M \partial_\beta x^N + \alpha' \sqrt{h} R^{(2)}(h) \Phi \right). \tag{2.5}$$

Considering the case where the sigma model has at least one $\mathrm{U}(1)$ isometry and using the conformal invariance of the sigma model, the above action can be written as,

$$S = \int \mathrm{d}\sigma^2 \left( Q_{\mu\nu} \partial_+ x^\mu \partial_- x^\nu + Q_{\mu i} \partial_+ x^\mu \partial_- x^i + Q_{j\nu} \partial_+ x^j \partial_- x^\nu + Q_{ij} \partial_+ x^i \partial_- x^j \right) + S_\Phi, \tag{2.6}$$

where $Q_{MN} = g_{MN} + B_{MN}$ and the coordinares have been split into isometry directions, $i$, and spectator directions, $\mu$, that is,

$$x^M = (x^\mu, x^i) \qquad \text{and} \qquad Q_{MN} = \begin{pmatrix} Q_{\mu\nu} & Q_{\mu j} \\ Q_{i\nu} & Q_{ij} \end{pmatrix}. \tag{2.7}$$

The contribution of the dilaton to the string sigma model is of higher order in $\alpha'$ and therefore a quantum correction, for this reason we will discuss it later.

The so-called, Buscher's rules, allow to construct a T-dual background, that generalises the $R \to \alpha'/R$ transformation that defines T-duality in a flat space. The construction proceeds in three steps. The first step is to gauge the isometries introducing auxiliary $\mathrm{U}(1)$ gauge fields, $A^i_\pm$, that couple minimally to the fields $x^i$,

$$\partial_\pm x^i \to D_\pm x^i = \partial_\pm x^i + A^i_\pm \quad \text{where} \quad A^i_\pm \to A^i_\pm - \partial_\pm \lambda^i \quad \text{under} \quad x^i \to x^i + \lambda^i. \tag{2.8}$$

The second step is to introduce the Lagrange multipliers $v_i$ to enforce that the gauge fields are non-dynamical, through a term

$$-i\mathrm{Tr}(vF_\pm) \qquad \text{where} \qquad F_\pm = \partial_+ A_- - \partial_- A_+. \tag{2.9}$$

Integrating out the Lagrange multipliers imposes that $F_\pm = 0$, which forces $A_\pm$ to be pure gauge. Choosing $A_\pm = 0$ the original action (2.6) is then reproduced.

On the other hand, integrating out the gauge fields, $A_\pm$, and gauge fixing $x^i = 0$ one gets the following, dual, action,

$$S = \int \mathrm{d}\sigma^2 \left( \hat{Q}_{\mu\nu}\partial_+ x^\mu \partial_- x^\nu + \hat{Q}_{\mu i}\partial_+ x^\mu \partial_- v^i + \hat{Q}_{j\nu}\partial_+ v^j \partial_- x^\nu + \hat{Q}_{ij}\partial_+ v^i \partial_- v^j \right), \tag{2.10}$$

where the Lagrange multipliers have replaced the $x^i$ coordinates and the metric and NSNS two-form are given by

$$\begin{pmatrix} \hat{Q}_{\mu\nu} & \hat{Q}_{\mu j} \\ \hat{Q}_{i\nu} & \hat{Q}_{ij} \end{pmatrix} = \begin{pmatrix} Q_{\mu\nu} - Q_{\mu j}Q^{-1}_{ij}Q_{j\nu} & Q_{\mu i}Q^{-1}_{ij} \\ -Q^{-1}_{ij}Q_{j\nu} & Q^{-1}_{ij} \end{pmatrix}. \tag{2.11}$$

For one $\mathrm{U}(1)$ isometry direction, $\blacktriangle$, the dual metric and the NS-NS antisymmetric tensor may be obtained as,

$$\begin{gathered} \hat{g}_{\blacktriangle\blacktriangle} = \frac{1}{g_{\blacktriangle\blacktriangle}}, \quad \hat{g}_{\mu\blacktriangle} = \frac{B_{\mu\blacktriangle}}{g_{\blacktriangle\blacktriangle}}, \quad \hat{g}_{\mu\nu} = g_{\mu\nu} - \frac{g_{\mu\blacktriangle}g_{\nu\blacktriangle} - B_{\mu\blacktriangle}B_{\nu\blacktriangle}}{g_{\blacktriangle\blacktriangle}}, \\ \hat{B}_{\mu\blacktriangle} = \frac{g_{\mu\blacktriangle}}{g_{\blacktriangle\blacktriangle}}, \qquad \hat{B}_{\mu\nu} = B_{\mu\nu} - \frac{g_{\mu\blacktriangle}B_{\nu\blacktriangle} - g_{\nu\blacktriangle}B_{\mu\blacktriangle}}{g_{\blacktriangle\blacktriangle}}. \end{gathered} \tag{2.12}$$

The dilaton transformation rule requires a more careful analysis. To obtain its transformation one must take into account the first quantum correction induced by the path integral integration of the gauge fields $A_\pm$, which produces a change in the measure. Thus, we can obtain this transformation requiring invariance of the measure,

$$\sqrt{|g|}e^{-2\Phi} \quad \xrightarrow{\text{Rules (2.12)}} \quad \sqrt{|\hat{g}|}e^{-2\hat{\Phi}} = \frac{\sqrt{|g|}}{g_{\blacktriangle\blacktriangle}}e^{-2\hat{\Phi}}, \tag{2.13}$$

which implies a shift in the dilaton of the following form,

$$\hat{\Phi} = \Phi - \frac{1}{2}\log g_{\blacktriangle\blacktriangle}. \tag{2.14}$$

The expressions (2.12) and (2.14) are referred as Buscher's Rules.

### 2.1.2 Ramond-Ramond sector

Superstring theories contain as well bosonic fields in the so-called RR sector [75]. To understand the behaviour of these fields under T-duality we have to encode the RR fields in the bispinors defined by equation (1.11), and use the fact that T-duality twists the right and left movers, [76, 77, 78, 79].

The construction proceeds writing the 10 dimensional metric in terms of the vielbeins $G_{MN} = e^a_M \eta_{ab} e^b_N$ where $a, b$ are Lorentz frame indices, and realising that the T-dual solution allows for two sets of vielbeins, given by

$$(\hat{e}^M_b)_\pm = (\mathcal{Q}^M_N)_\pm e^N_a, \tag{2.15}$$

where $\mathcal{Q}^M_N$ read

$$\mathcal{Q}_\pm = \begin{pmatrix} \mp g_{\blacktriangle\blacktriangle} & \mp(G \mp B)_{\blacktriangle\nu} \\ 0 & \mathbf{I}_9 \end{pmatrix}. \tag{2.16}$$

As both vielbeins are describing the same T-dual theory they have to be related by a Lorentz transformation,

$$(\hat{e}^M_b)_+ = (\hat{e}^M_a)_- \Lambda^a_b, \tag{2.17}$$

which using (2.16) is defined by

$$\Lambda^a_b = \delta^a_b - 2\frac{e^a_\blacktriangle e_{\blacktriangle b}}{g_{\blacktriangle\blacktriangle}} \qquad \text{with} \qquad \det \Lambda = -1. \tag{2.18}$$

In [76] it was shown that using the above Lorentz transformation one can define an action on the spinors given by,

$$\Omega^{-1}\Gamma^a\Omega = \Lambda^a_b \Gamma^b, \tag{2.19}$$

with $\Omega$ the spinorial representation of $\Lambda$. The previous equation is solved with

$$\Omega = \Gamma^{11}\Gamma^\blacktriangle, \tag{2.20}$$

where $\Gamma^{11}$ is the product of all 10d gamma matrices and $(\Gamma^{11})^2 = \mathbf{I}$.

Finally, in order to obtain the transformation of the RR fields, one constructs the bispinors given by (1.11) and, from then

$$\begin{aligned} P_{odd} &= \frac{e^\Phi}{2}\sum_{n=0}^{4} \not{F}_{2n+1} \qquad \text{for type IIB supergravity}, \\ P_{even} &= \frac{e^\Phi}{2}\sum_{n=0}^{5} \not{F}_{2n} \qquad \text{for type IIA supergravity}, \end{aligned} \tag{2.21}$$

where we have used the Clifford map,

$$\alpha \equiv \sum_k \frac{1}{k!}\alpha_{i_1\dots i_k} e^{i_1} \wedge \dots \wedge e^{i_k} \quad \leftrightarrow \quad \not{\alpha} \equiv \sum_k \frac{1}{k!}\alpha_{i_1\dots i_k}\Gamma^{i_1\dots i_k} \, . \tag{2.22}$$

Using (2.20) one finds that

$$\text{type IIB} \to \text{type IIA} \qquad \hat{P}_{even} = P_{odd} \cdot \Omega^{-1}, \tag{2.23}$$

$$\text{type IIA} \to \text{type IIB} \qquad \hat{P}_{odd} = P_{even} \cdot \Omega, \tag{2.24}$$

where $\hat{P}$ are the dual RR bispinors defined from the dual fields $\hat{P}(\hat{F}_p, \hat{\Phi})$. These transformations imply that type IIA string theory is mapped under T-duality onto type IIB and viceversa.

Moreover, the presence of just one $\Gamma^{\blacktriangle}$ in (2.20) tells us that T-duality transforms a D$p$-brane into a D$(p \pm 1)$-brane, depending on whether the T-duality direction is orthogonal to the Dp-brane or contained in its worldvolume.

Since we are interested in the effect that T-duality has in the low energy limit of the superstring, the relations (2.12), (2.14) and (2.23) provide the transformation rules to go from a solution (IIA/IIB) to another solution (IIB/IIA). In this spirit, we can see T-duality as our first example of solution generating technique. We will use extensively this transformation in Section 8.

In the next section we generalise the T-duality transformation to the case of non-Abelian groups.

## 2.2 Non-Abelian T-Duality (NATD)

In this section we introduce the generalisation of Buscher's rules to the case where we have a background with a global $G$ isometry, where $G$ is a non-Abelian Lie group. In the literature, this formulation is known as non-Abelian T-duality (NATD).

The generalisation of Buscher's rules to non-Abelian isometry groups was done in [80] and then NATD was first applied as a solution generating technique in supergravity in [81], where the transformation rules of the RR fields were worked out. Since then the dualisation has been carried out in supergravity with respect to a freely acting SU$(2)$ isometry group. Here, we study the cases $G$ =SU$(2)$ and $G$ =SL$(2, \mathbf{R})$, following the review sections in [82, 83].

Consider a bosonic string sigma model that supports a $G$ isometry, such that the NSNS fields can be written as,

$$ds^2 = \frac{1}{4}g_{ij}(x)L^iL^j + G_{i\mu}(x)dx^iL^\mu + G_{\mu\nu}(x)dx^\mu dx^\nu,$$
$$B_2 = \frac{1}{8}b_{ij}(x)L^i \wedge L^j + \frac{1}{2}B_{i\mu}(x)dx^i \wedge L^\mu + B_{\mu\nu}(x)dx^\mu \wedge dx^\nu, \quad \Phi = \Phi(x), \tag{2.25}$$

for $\mu, \nu = 1, 2, ..., 7$ with $x^\mu$ being the spectator coordinates, and $L^i$ the $G$ left-invariant Maurer-Cartan forms,

$$L^i = -i\text{Tr}(t^i g^{-1}\text{d}g), \qquad \text{which obey}, \quad \text{d}L^i = \frac{1}{2}f^i{}_{jk}L^j \wedge L^k, \tag{2.26}$$

where $f^i{}_{jk}$ are the structure constants of $G$.

Our first example is the SU$(2)$ group. The generators of the SU$(2)$ algebra are $t^a = \frac{\tau_a}{\sqrt{2}}$, with $\tau_a$ the Pauli matrices,

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.27}$$

These generators satisfy,

$$\text{Tr}(t^at^b) = \delta^{ab}, \qquad [t^i, t^j] = i\sqrt{2}\epsilon_{ijk}t^k \ . \tag{2.28}$$

An arbitrary element of SU$(2)$ can be defined using Euler's parametrisation,

$$g = e^{\frac{i}{2}\phi\tau_3}e^{\frac{i}{2}\theta\tau_2}e^{\frac{i}{2}\psi\tau_3}, \quad \text{with} \quad 0 \le \theta \le \pi, \ \ 0 \le \phi < 2\pi, \ \ 0 \le \psi < 2\pi. \tag{2.29}$$

In this parametrisation, the left-invariant forms (2.26) are given by,

$$L^1 = -\sin\psi\text{d}\theta + \cos\psi\sin\theta\text{d}\phi, \qquad L^2 = \cos\psi\text{d}\theta + \sin\psi\sin\theta\text{d}\phi,$$
$$L^3 = \cos\theta\text{d}\phi + \text{d}\psi. \tag{2.30}$$

The next example is $G =$SL$(2, \mathbf{R})$. Here, the generators of the $sl(2, \mathbf{R})$ algebra are obtained by analytically continuing the $su(2)$ generators as,

$$\tilde{t}^a = \frac{\tilde{\tau}_a}{\sqrt{2}}, \qquad \text{with} \quad \tilde{\tau}_1 = i\tau_1, \qquad \tilde{\tau}_2 = \tau_2, \qquad \tilde{\tau}_3 = i\tau_3 \ . \tag{2.31}$$

These satisfy[1],

$$\mathrm{Tr}(\tilde{t}^a\tilde{t}^b) = (-1)^a\delta^{ab}, \quad [\tilde{t}^1,\tilde{t}^2] = i\sqrt{2}\tilde{t}^3, \quad [\tilde{t}^2,\tilde{t}^3] = i\sqrt{2}\tilde{t}^1, \quad [\tilde{t}^3,\tilde{t}^1] = -i\sqrt{2}\tilde{t}^2. \tag{2.32}$$

The group element $\tilde{g} \in \mathrm{SL}(2,\mathbf{R})$ is parametrised as,

$$\tilde{g} = e^{\frac{i}{2}t\tilde{\tau}_3}e^{\frac{i}{2}\theta\tilde{\tau}_2}e^{\frac{i}{2}\eta\tilde{\tau}_3} \quad \text{with} \quad 0 \le \theta \le \pi, \;\; 0 \le t < \infty, \;\; 0 \le \eta < \infty. \tag{2.33}$$

From here, the left-invariant forms (2.26) are given by,

$$\begin{gathered} L^1 = \sinh\eta \mathrm{d}\theta - \cosh\eta\sin\theta \mathrm{d}t, \qquad L^2 = \cosh\eta\mathrm{d}\theta - \sinh\eta\sin\theta\mathrm{d}t, \\ L^3 = -\cos\theta\mathrm{d}t - \mathrm{d}\eta. \end{gathered} \tag{2.34}$$

Notice that the group element $g$ depends on the target space isometry directions, realising either SU(2) or SL(2,**R**) group manifold. That is, the group manifold is an $\mathrm{S}^3$ space for SU(2), or an $\mathrm{AdS}_3$ space for SL(2,**R**).

The non-linear sigma-model that describes the propagation of a string on these backgrounds is given by

$$\begin{aligned} S = \int \mathrm{d}\sigma^2 (E_{ij}L_+^iL_-^j + &Q_{i\mu}\partial_+x^iL_-^\mu + Q_{\mu i}L_+^\mu\partial_-x^i + Q_{\mu\nu}\partial_+x^\mu\partial_-x^\nu), \\ \text{with} \qquad &E_{ij} = g_{ij} + b_{ij}, \qquad Q_{i\mu} = G_{i\mu} + B_{i\mu}, \\ &Q_{\mu i} = G_{\mu i} + B_{\mu i}, \qquad Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \end{aligned} \tag{2.35}$$

where $L^i_\pm$ are the left-invariant forms pulled back to the worldsheet. This sigma-model is invariant under $g \to \lambda^{-1}g$ for $\lambda \in \mathrm{SU}(2)$ or $\lambda \in$ SL(2,**R**).

Following [80], in order to construct the non-Abelian T-dual background, the global isometries are gauged, introducing covariant derivatives, $\partial_\pm g \to D_\pm g = \partial_\pm g - A_\pm g$ in the Maurer-Cartan forms and the condition that the gauge fields are non-dynamical is imposed through the addition of a Lagrange multiplier term, $-i\mathrm{Tr}(vF_\pm)$, where $F_\pm$ in this case is

$$F_\pm = \partial_+A_- - \partial_-A_+ - [A_+, A_-] \tag{2.36}$$

and $v = \{v_1, v_2, v_3\}$ is a Lagrange multiplier vector that takes values in the Lie algebra of the group $G$. The resulting action is invariant under,

$$g \to \lambda^{-1}g, \quad A_\pm \to \lambda^{-1}(A_\pm\lambda - \partial_\pm\lambda), \quad v \to \lambda^{-1}v\lambda, \tag{2.37}$$

[1] We take $g^{\mu\nu} = -\mathrm{Tr}(\tilde{t}^\mu\tilde{t}^\nu)$ in order to have $(+,-,+)$ signature.

with $\lambda \in \mathrm{SU}(2)$ or $\lambda \in \mathrm{SL}(2,\mathbf{R})$. In the same way as the Abelian T-Dual, after integrating out the Lagrange multipliers and fixing the gauge, the original non-linear sigma-model is recovered.

On the other hand, the dual background is obtained integrating by parts the Lagrange multiplier term and solving for the gauge fields. However, the new sigma-model has redundancies since it still relies on the spectators, Lagrange multipliers, $v_i$, and the coordinates used to parametrise the group, (2.29) or (2.33). For this reason, we need to remove the redundancy choosing a gauge fixing condition $g = \mathbf{I}$, as in the Abelian T-dual case.

The resulting action reads,

$$\hat{S} = \int \mathrm{d}\sigma^2 \left[ Q_{\mu\nu}\partial_+ x^\mu \partial_- x^\nu + (\partial_+ v_i + \partial_+ x^\mu Q_{\mu i}) M^{-1}_{ij} (\partial_- v_j - Q_{j\nu}\partial_+ x^\nu) \right], \tag{2.38}$$

$$\text{with} \qquad M_{ij} = E_{ij} + f^k{}_{ij} v_k.$$

Notice that in this action the parameters $(\phi, \theta, \psi)$ or $(t, \theta, \eta)$ have been replaced by the Lagrange multipliers $v_i$, which live in the Lie algebra of the group $G$. From (2.38) the dual NSNS sector, can be read,

$$\begin{pmatrix} \hat{Q}_{\mu\nu} & \hat{Q}_{\mu j} \\ \hat{Q}_{i\nu} & \hat{E}_{ij} \end{pmatrix} = \begin{pmatrix} Q_{\mu\nu} - Q_{\mu j} M^{-1}_{ij} Q_{j\nu} & Q_{\mu i} M^{-1}_{ij} \\ -M^{-1}_{ij} Q_{j\nu} & M^{-1}_{ij} \end{pmatrix}, \tag{2.39}$$

which generalises Buscher's rules to the non-Abelian case.

At this point we remark that the solutions generated by NATD span non-compact manifolds even in the case that the group used in the dualisation is compact (like SU(2)). The reason is that the new variables live in the Lie algebra of the dualisation group. At the level of the metric and using a suitable parametrisation for the Lagrange multipliers, the original $\mathrm{S}^3$ space is replaced by an $\mathrm{S}^2 \times \mathbf{R}^+$ space in the case of $G = \mathrm{SU}(2)$. In turn, for $G = \mathrm{SL}(2,\mathbf{R})$, the $\mathrm{AdS}_3$ subspace is supplanted by an $\mathrm{AdS}_2 \times \mathbf{R}^+$ space.

As in Abelian T-duality the dilaton receives a shift coming from the path integral analysis, that guarantees invariance of the integration measure. In this case we have

$$\hat{\Phi}(x, v) = \Phi(x) - \frac{1}{2}\log(\det M). \tag{2.40}$$

where $\det M$ is playing the role of the metric component $g_{\blacktriangle\blacktriangle}$.

### 2.2.1 Ramond-Ramond fluxes

The transformation rules for the RR fields were worked out in [81] using the spinor representation approach as in the Abelian T-dual case. That is, left and right movers transform differently under NATD, and therefore lead to two different sets of frame fields for the dual geometry,

$$\begin{aligned} \hat{e}_+ &= -\kappa M^{-T}(dv + Q^T dx) + \lambda dx, \\ \hat{e}_- &= \kappa M^{-1}(dv - Q dx) + \lambda dx, \end{aligned} \tag{2.41}$$

where $\kappa$ and $\lambda$ come from defining the frame fields in the original target space as

$$\begin{aligned} &ds^2 = \eta_{\mu\nu} e^\mu e^\nu + \delta_{ij} e^i e^j \qquad \text{for SU(2)} \\ &ds^2 = \delta_{\mu\nu} e^\mu e^\nu + \delta_{ij} e^i e^j \qquad \text{for SL}(2,\mathbf{R}) \\ \text{with} \qquad &e^\mu = e^\mu_\alpha dx^\alpha, \qquad e^i = \kappa^i_a L^a + \lambda^i_b dx^b, \qquad \kappa^i_a \kappa^j_a = g_{ij}. \end{aligned} \tag{2.42}$$

The two different sets of frame fields (2.41) define the same dual metric obtained from (2.38), and must therefore be related by a Lorentz transformation as in (2.17), where $\Lambda^\alpha{}_\beta$ is given by,

$$\Lambda = -\kappa M^{-T} M \kappa^{-1} \qquad \text{with} \qquad \det \Lambda = (-1)^{dimG}. \tag{2.43}$$

As in Section 2.1.2, the Lorentz transformation acts on spinors through the matrix $\Omega$, defined by the expression (2.19). In the NATD case the condition is solved for,

$$\Omega = \Gamma_{11} \frac{-\Gamma_{123} + \zeta_a \Gamma^a}{\sqrt{1 + \zeta_a \zeta^a}} \tag{2.44}$$

with $\zeta^a = \kappa^a_i v^i / (\sqrt{\det g})$ for $B_{ij} = 0$. Writing the RR fluxes as bispinors as in (2.21) one then extracts their transformation rules by right multiplication with the $\Omega^{-1}$ or $\Omega$ matrices as in (2.23) and (2.24). With these transformation rules it is guaranteed that a solution to type IIA (IIB) supergravity is mapped onto a solution of type IIB (IIA) supergravity [81].

We finish this subsection with some comments:

- Even if the expression for $\Omega$, given by (2.44), is more complicated than in the Abelian case, the second term in (2.44) is proportional to (2.20) for each component, which implies that a $p$-form is mapped to a $(p \pm 1)$-form. In turn, the first term consists on a product of three $\Gamma$ matrices, which means that the $p$-form is mapped to a $(p \pm 3)$-form. In other words, $F_p \to F_{p\pm1}, F_{p\pm3}$, with the sign $+$ or $-$ depending on whether the duality directions are transverse or tangential to the directions of the p-form.

- Unlike the Abelian counterpart, the NATD transformation is not an involution since the transformation destroys the dualised isometries.

- Whilst the sigma model procedure to compute the non-Abelian T-dual of a given geometry seems straightforward to follow, a subtlety arises in determining global aspects of the dual background. In the Abelian T-dual case, the extension of the transformation beyond tree level in string perturbation theory determines the global properties of the coordinate that replaces the dualised direction. The extension beyond tree level is a long-standing open problem of NATD [84, 85, 86, 87, 88, 89, 90], in such a way that a formalism that allows compactifying the new coordinates is lacking.

- One can deduce the transformation rules for the 10d Majorana Weyl Killing spinors, $\epsilon_1$ and $\epsilon_2$, from the $\Omega$ matrix

$$\hat{\epsilon}_1 = \epsilon_1, \qquad \hat{\epsilon}_2 = \Omega\epsilon_2. \tag{2.45}$$

  This transformation is well-known for $G$=SU$(2)$, [81, 82], where it reduces to a rotation of one of the Killing spinors with respect to the other. We expect the same to happen in the $G$=SL$(2, \mathbf{R})$ case, albeit this has not been shown.

## 2.3 G-structure techniques

An n-dimensional manifold $M$ admits a G-structure if the structure group on $M$ gets reduced to the subgroup $G$. This reduction can be performed due to the existence of globally defined tensors or spinors in this manifold. In recent years, G-structure techniques have been successfully used as solution generating techniques in supergravity. Prominent examples to find supersymmetric AdS solutions of supergravity with this technology are the works [8, 91, 92] for $\mathcal{N} = 1$ backgrounds.

Although in this thesis we focus in the study of $\mathrm{AdS}_3$ and $\mathrm{AdS}_2$ geometries, in this section we illustrate the procedure with a particular example: a 10 dimensional background consisting on a warped product of a 4d spacetime and a 6d compact space $M$ with $\mathcal{N}=1$ supersymmetry. That is, we take the following ansatz for the metric,

$$ds^2 = e^{2A} g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n, \tag{2.46}$$

for $\mu,\nu = 1,...,4$ and $m,n = 1,...,6$, where $e^A$ is the warp factor, that depends on the internal[1] coordinates $y^m$. In turn, 4d $\mathcal{N}=1$ supersymmetry implies that the RR and NSNS fluxes must be non-trivial on $M$. Namely,

$$F = f + e^{2A}\mathrm{vol}_4 \wedge \star_6 \lambda(f) \quad \text{where} \quad \begin{aligned} f &= f_0 + f_2 + f_4 + f_6 \quad \text{IIA} \\ f &= f_1 + f_3 + f_5 \quad \text{IIB} \end{aligned} \tag{2.47}$$

with $\lambda(X_n) = (-1)^{\frac{n(n-1)}{2}} X_n$ and $\mathrm{vol}_4$ the volume form of the 4d spacetime. Since we are interested in backgrounds preserving 4d $\mathcal{N}=1$ supersymmetry, these should have a single 4d conserved spinor $\zeta_+$, and the ten dimensional Majorana Weyl spinors, $\epsilon_1$, $\epsilon_2$, have to be decomposed accordingly, as,

$$\begin{aligned} \epsilon^1 &= \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_-, \\ \epsilon^2 &= \zeta_+ \otimes \eta^2_\mp + \zeta_- \otimes \eta^2_\pm, \end{aligned}$$

where we used + and - to indicate both four- and six-dimensional chiralities, in such a way that the upper sign in $\epsilon^2$ is for type IIA and the lower sign for type IIB. In the rest of this section we will use this notation to distinguish type IIA and IIB supergravity theories. Here, $\zeta_-$ is the Majorana conjugate of the 4d spinor $\zeta_+$ and we choose a 6d basis where complex conjugation changes the chirality, $(\eta^i_+)^* = \eta^i_-$.

In the context of solution generating techniques in supergravity, it is useful to construct two Cliff$(d,d)$ pure spinors by tensoring the internal spinors $\eta^i_\pm$,

$$\Phi_+ = \eta^1_+ \otimes \eta^{2\,\dagger}_+, \qquad \Phi_- = \eta^1_+ \otimes \eta^{2\,\dagger}_- . \tag{2.48}$$

Using then the Fierz identity and the Clifford map (2.22) the pure spinors can be seen as polyforms,

$$\eta_+ \otimes \eta^\dagger_\pm = \frac{1}{2^{[d/2]}} \sum_{k=0}^{d} \frac{1}{k!} (\eta^\dagger_\pm \gamma_{a_1...a_n} \eta_+) e^{a_1} \wedge ... \wedge e^{a_n}, \tag{2.49}$$

[1] We are interested in backgrounds with the form $\mathrm{AdS}_p \times M_{10-p}$. The coordinates covering the $M_{10-p}$ subspace are called internal coordinates and $M_{10-p}$ is known as internal space.

where in our particular case $d = 6$.

The decomposition of the spinors in four and six dimensional factors allows us to split the supersymmetry conditions (1.10) into 4d and 6d components. The differential conditions that the pure Cliff(6, 6) spinors have to obey in order to preserve $\mathcal{N} = 1$ supersymmetry were derived in [91]. They are given by

$$\begin{aligned}
&(d - H\wedge)(e^{2A-\Phi}\Phi_\pm) = 0 \\
&(d - H\wedge)(e^{2A-\Phi}\Phi_\mp) = e^{2A-\Phi} dA \wedge \bar{\Phi}_\mp + \frac{ie^{3A}}{8} \star_6 \lambda(f).
\end{aligned} \tag{2.50}$$

In order to construct the two pure spinors (2.49) we need the explicit form of the internal spinors $\eta^i$, which can be parametrised in terms of the spinors $\eta_+$ and $\chi_+$ in the following fashion,

$$\begin{aligned}
\eta^1_+ &= a\eta_+, \\
\eta^2_+ &= b(k_\| \eta_+ + k_\perp \chi_+).
\end{aligned} \tag{2.51}$$

Here $a$ and $b$ are complex numbers related to the norms of the internal spinors. The parameters $k_\|$ and $k_\perp$ satisfy $k_\|^2 + k_\perp^2 = 1$ and depending on their values one can define different G-structures on the internal manifold $M$.

### $\mathbf{k_\perp = 0}$, SU(3) structure

When $k_\perp = 0$ the internal spinor becomes parallel, i.e. we only have one global spinor on $M$, then the pure spinors define an SU(3) structure, which is characterised by one two-form and one holomorphic three-form. These forms are defined in terms of the following bilinears of the internal spinors,

$$J_{mn} = -i{\eta_+}^\dagger \gamma_{mn} \eta_+ \qquad \Omega_{mnp} = -i{\eta_-}^\dagger \gamma_{mnp} \eta_+, \tag{2.52}$$

which satisfy the conditions,

$$J \wedge \Omega = 0, \qquad J^3 = \frac{3}{4} i\Omega \wedge \bar{\Omega}. \tag{2.53}$$

Therefore, the corresponding pure spinors are,

$$\Phi_+ = \frac{ab*}{8} e^{-iJ}, \qquad \Phi_- = -i\frac{ab}{8}\Omega. \tag{2.54}$$

**$\mathrm{k}_\perp \neq 0$, SU(2) structure**

In this case the two internal spinors are independent and the $M$ manifold is said to admit an SU(2) structure. It is defined by a holomorphic one-form, $z_m = \nu_m - i w_m$, a real two-form $j$ and a holomorphic two-form $\omega$. In terms of the internal spinors,

$$\begin{aligned}&(\nu_m - i w_m) = \eta_-{}^\dagger \gamma_a \chi_+, \qquad j_{mn} = -i\eta_+{}^\dagger \gamma_{mn} \eta_+ + i\chi_+{}^\dagger \gamma_{mn} \chi_+,\\&\omega_{mn} = \eta_-{}^\dagger \gamma_{mn} \chi_- \ .\end{aligned} \tag{2.55}$$

These invariant forms satisfy the following structure contitions,

$$\begin{aligned}&j^2 = \frac{1}{2}\omega \wedge \bar{\omega}, \qquad j \wedge \omega = \omega \wedge \omega = 0,\\&(\nu - iw) \llcorner \omega = (\nu - iw) \llcorner j = 0,\end{aligned} \tag{2.56}$$

where $b \llcorner A = \frac{1}{(p-1)!} b^{\mu_1} A_{\mu_1 \dots \mu_p} \gamma^{\mu_2 \dots \mu_p}$. One can show that the pure spinors are,

$$\Phi_+ = \frac{ab*}{8} e^{-i\nu \wedge w} \wedge \omega, \qquad \Phi_- = \frac{ab}{8} e^{-ij} \wedge (\nu + iw). \tag{2.57}$$

For $k_\perp \neq 0$ we distinguish two subcases $k_\parallel = 0$ and $k_\parallel \neq 0$, the former is the case that we discussed above and it is known as orthogonal SU(2) structure. The latter, $k_\parallel \neq 0$, is called intermediate SU(2) structure.

## 2.4 Analytical continuations (AC)

In this section we study a 'trick' to construct solutions with $\mathrm{AdS}_p$ and $\mathrm{S}^q$ factors, known in the literature as double or quadruple analytic continuation. In general, if we have a $\mathrm{AdS}_p \times \mathrm{S}^q$ warped geometry over a $\mathrm{M}_{10-p-q}$ manifold one can write the $\mathrm{AdS}_p$ factor as

$$ds^2_{\mathrm{AdS}_p} = d\zeta^2 + \sinh^2\zeta \; ds^2_{p-1} = d\zeta^2 + \sinh^2\zeta \; (-d\tau^2 + \cosh^2\tau ds^2_{\mathrm{S}^{(p-2)}}), \tag{2.58}$$

and perform the following two wick rotations,

$$\zeta \equiv i\psi, \qquad \tau \equiv i\left(\frac{\pi}{2} - \alpha\right), \tag{2.59}$$

to obtain,

$$-(d\psi^2 + \sin^2\psi (d\alpha^2 + \sin^2\alpha \; ds^2_{\mathrm{S}^{(p-2)}})) = -ds^2_{\mathrm{S}^p}. \tag{2.60}$$

The transformation (2.59) is known as a double analytical continuation. Notice that, the opposite also works: starting from a $ds^2_{\mathrm{S}^q}$ metric and doing the inverse transformation of (2.59) we arrive to a $-ds^2_{\mathrm{AdS}_q}$ factor. In this way, one can carry out the four wick rotations and get the following swap,

$$\mathrm{AdS}_p \to -\mathrm{S}^p\,, \qquad \mathrm{S}^q \to -\mathrm{AdS}_q\,, \tag{2.61}$$

leading to a tentative $\mathrm{AdS}_q\times\mathrm{S}^p$ warped geometry over the same $\mathrm{M}_{10-p-q}$ manifold. We say tentative because it is necessary to complete this transformation with a careful study of the signature of the metric and the behaviour of the supergravity fields before we conclude that it gives rise to a well-defined supergravity solution. For this purpose, it is useful to consider the analytical continuations of the volume forms,

$$\mathrm{vol}_{\mathrm{AdS}_p} \to i^{(p-1)}\mathrm{vol}_{\mathrm{S}^p}\,, \qquad \mathrm{vol}_{\mathrm{S}^q} \to (-i)^{(q-1)}\,\mathrm{vol}_{\mathrm{AdS}_q}\,, \tag{2.62}$$

which must be implemented in the background fluxes. As we can see, these forms can be imaginary, so depending on the values of $p$ and $q$ the transformation given by (2.61) needs to be supplemented with an analytical continuation of the fluxes and the dilaton,

$$F_p \to iF_p, \qquad e^{\Phi} \to ie^{\Phi}. \tag{2.63}$$

Due to the lack of intuition of the resultant geometry, it is not possible to give a concrete statement about the supersymmetry of the new geometry.

As we mentioned before we will employ the techniques reviewed in this section to construct new $\mathrm{AdS}_3$ and $\mathrm{AdS}_2$ solutions with different amounts of supersymmetry. In particular, we will use the G-structure technique in Sections 5.1 and 5.2 to construct new $\mathrm{AdS}_3\times\mathrm{S}^2$ solutions, that will be the 'seed' of most of the results presented in this thesis.

# 3. Brane Pictures and Quiver Field Theories

In this Section, we review the brane picture studied by Witten in [93] (see also [94]), realising 4d $\mathcal{N} = 2$ supersymmetric gauge theories in D4-D6-NS5 brane configurations in type IIA superstring theory[1]. These ideas form the ground of our constructions of quiver field theories dual to our warped $\text{AdS}_2$ and $\text{AdS}_3$ backgrounds.

Our starting point is the brane setup shown in Table 3.1. It consists of D4-branes stretched between NS5- branes (we will explain this in a moment) and (possibly) orthogonal D6-branes. The NS5-branes are extended in the $\mathbf{R}^{1,3}, x_4, x_5$ directions, at different values in the $x_6$ coordinate, while the D4-branes have their worldvolume in the $\mathbf{R}^{1,3}, x_6$ directions. The D6-branes cover the $\mathbf{R}^{1,3}, x_7, x_8, x_9$ directions.

This brane intersection, known as a Hanany-Witten brane setup [95], is illustrated in Figure 3.1. In this brane picture the vertical lines represent the NS5-branes and the horizontal ones the D4-branes, where the field theory lives (see below). Notice that, perpendicular D6-branes have been included, represented by crosses that are introduced for free as they do not break any additional supersymmetries (see below).

The ten dimensional Lorentz group is split in the four dimensional Lorentz group and in the SO(2)×SO(3) factors, which are R-symmetries, geometrically

[1] The first example with these brane setups was described in [95] for 3d supersymmetric gauge theories with $\mathcal{N} = 4$ supersymmetry. Here we analyse the 4d example given in [93] because these brane pictures realise theories in four dimensions. In four dimensions the theory is conformal and straightforward in their study.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D4 | × | × | × | × | | | × | | | |
| D6 | × | × | × | × | | | | × | × | × |
| NS5 | × | × | × | × | × | × | | | | |

**Table 3.1:** The $\frac{1}{4}$-BPS intersection involving D4, D6 and NS5 branes. A 4d superconformal field theory lives in the common $x^0, x^1, x^2, x^3$ directions. The corresponding $\mathrm{SO}(2)_R \times \mathrm{SO}(3)_R$ R-symmetry is geometrically realised as rotations in the $(x^4, x^5)$ and $(x^7, x^8, x^9)$ coordinates, respectively.

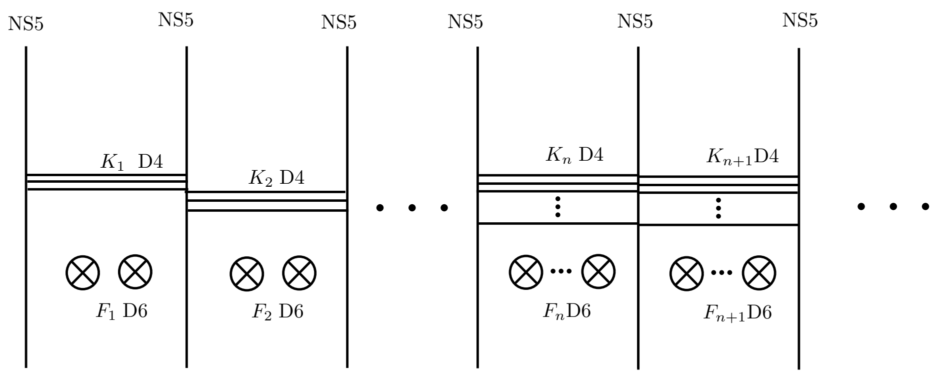


**Figure 3.1:** Brane picture of the system described in Table 3.1.

realised on $(x^4, x^5)$ and $(x^7, x^8, x^9)$, respectively. Besides, it is well-known that when we introduce Dp-branes in type II supergravities, half of the thirty two super charges are preserved, according to the condition $\epsilon_L = \Gamma^{0,\dots,p}\epsilon_R$. Likewise, introducing NS5-branes, a further half of the supersymmetries are broken, according to the type IIA conditions $\epsilon_L = \Gamma^{0,\dots,5}\epsilon_L$ and $\epsilon_R = \Gamma^{0,\dots,5}\epsilon_R$. Thus, for the system in Table 3.1 one finds eight supercharges that are undetermined. This means $\mathcal{N} = 2$ supersymmetry in four dimensions.

Strictly speaking, the field theory living on the D4-branes is 5d. However, the introduction of the NS5-branes restricts the extension of the D4-branes on the $x_6$, such that the field theory is rendered four dimensional at low energies

The gauge coupling of the theory living in the D4-branes can be extracted from their DBI action (as it was briefly explained around equation (1.15) for D3-branes).

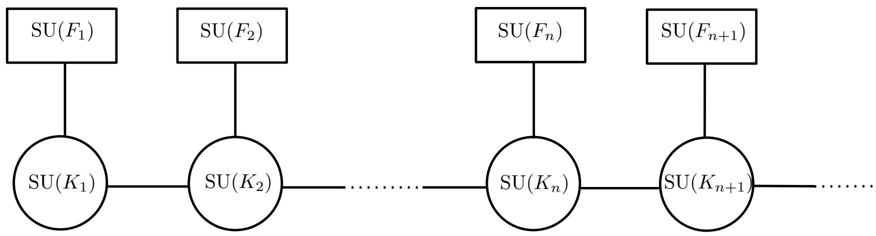


**Figure 3.2:** Four dimensional $\mathcal{N} = 2$ quiver field theory living in D4-D6-NS5 brane intersections.

It reads

$$\frac{1}{g_4^2} \sim \frac{x_{6,n+1} - x_{6,n}}{l_s}\,, \tag{3.1}$$

this shows that if we are interested in a finite coupling constant at $l_s \to 0$ (low energies) the NS5-branes have to be close between themselves.

The field content that defines the field theory living in this brane intersection arises from the quantisation of the open strings stretched between the different branes. Open strings with both ends in the same stack of coincident $K_n$ D4-branes give an $\mathcal{N} = 2$ vector multiplet. The total gauge group is then $\Pi_n$ SU$(K_n)$, where we have frozen the U(1)'s in each stack, as discussed in [93]. The strings connecting adjacent stacks of D4-branes across an NS5-brane (i.e. $K_{n-1}$, and $K_n$ D4-branes) are $\mathcal{N} = 2$ hypermultiplets in the bifundamental representation of the adjacent gauge groups. The presence of D6-branes in each interval introduces $F_n$ fundamental hypermultiplets. The gauge group living on the D6-branes becomes a flavour group because in the low energy limit these strings have their excitations decoupled, giving a global SU($F_n$) group for each SU($K_n$) gauge group.

The $\mathcal{N} = 2$ field theory can be described by the *quiver* depicted in Figure 3.2. In these representations circles represent gauge groups, boxes flavour groups and lines fields in the bifundamental or fundamental representation of the symmetry groups.

In [5], the expression for the one-loop $\beta$-function for the SU($K_n$) gauge group was calculated, and shown to be given by

$$b_{0,n} = -2K_n + K_{n-1} + K_{n+1} + F_n. \tag{3.2}$$

Thus, for the field theory to be conformal, the following condition must hold

$$F_n = 2K_n - (K_{n-1} + K_{n+1}), \tag{3.3}$$

where the term in parenthesis counts the number of bifundamental hypermultiplets that couple to the SU($K_n$) gauge group.

The 4d CFT reviewed in this section illustrates the general way in which CFTs living in Hanany-Witten brane intersections are constructed. In this thesis we will encounter more involved 2d and 1d CFTs living in $\frac{1}{8}$-BPS brane configurations, that we will study extending the construction just reviewed. In the next section, we give a brief summary of the main results obtained in this thesis.

# 4. Outline of the Thesis

In this section we summarise the content of the next Chapters, giving the reader a wide outlook and a suggested sequence, which will help organise the goals of the thesis.

In Chapter 5, we construct and study the main properties of two new classes of $\text{AdS}_3\times\text{S}^2$ solutions in type IIA and type IIB supergravities, with small $\mathcal{N}=(0,4)$ supersymmetry. These solutions are constructed using the Killing spinor techniques described in subsection 2.3, where we sketched the basic technical procedure.

The small $\mathcal{N}=(0,4)$ superconformal algebra contains the bosonic subalgebra,

$$\mathfrak{sl}(2)\oplus\mathfrak{su}(2) \tag{4.1}$$

that is realised geometrically by the ansatz,

$$\begin{aligned} ds^2 &= e^{2A}ds^2_{\text{AdS}_3}+ds^2_{\text{M}_7}, \quad \text{with} \\ ds^2_{\text{M}_7} &= e^{2C}ds^2_{\text{S}^2}+ds^2_{\text{M}_5}. \end{aligned} \tag{4.2}$$

Here the warp factors $e^{2A}, e^{2C}$ have support in $\text{M}_5$, as so has the dilaton. In turn, the fluxes depend on the $\text{AdS}_3$ and $\text{S}^2$ directions only through their volume forms.

The strategy that we follow in Chapter 5 is similar to the one explained in subsection 2.3: we construct spinors by ensuring consistency with the bosonic subalgebra (4.1), and exploit an existing $\mathcal{N}=1$ $\text{AdS}_3$ classification [96] to obtain sufficient conditions on the geometry and fluxes for a solution with small $\mathcal{N}=(0,4)$ supersymmetry to exist in type II. Finally, we study the classes of solutions that are consistent with our assumptions.

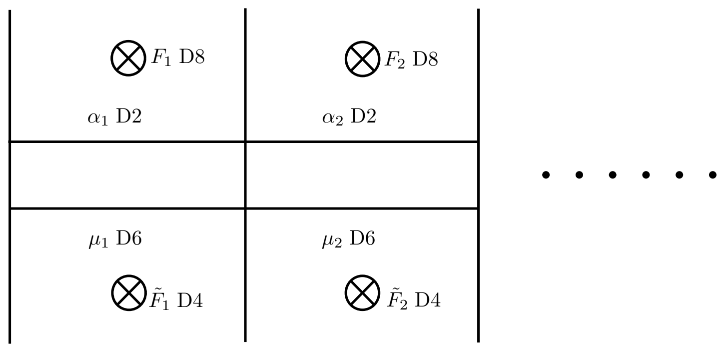


**Figure 4.1:** Hanany-Witten brane setup associated to the D2-D4-D6-D8-NS5 brane intersection and that corresponds to the ‘seed’ geometries. The horizontal lines represent D2 and D6 colour branes, vertical lines represent NS5-branes and the crosses are D4 and D8 flavour branes.

Our first assumption is small $\mathcal{N} = (0, 4)$ supersymmetry in massive type IIA supergravity with SU(2) structure on the remaining internal space. With this assumption we obtain two new classes of solutions of the form $\text{AdS}_3\times\text{S}^2\times\text{M}_4\times\text{I}$. When $\text{M}_4$=$\text{CY}_2$, that we refer as class I, the family of solutions consist on warped $\text{AdS}_3\times\text{S}^2\times\text{CY}_2\times\text{I}$ geometries with warping functions $h_8$, $h_4$ and $u$ with support on the interval and the $\text{CY}_2$. This class is a generalisation of the D4-D8 system that includes additional branes. In turn, when $\text{M}_4$ is a 4d Kähler manifold, that we refer as class II, the family of solutions consist on warped $\text{AdS}_3\times\text{S}^2\times\text{M}_4\times\text{I}$ geometries, which are a generalisation, up to T-duality, of the class of D3-branes wrapping curves in the base of an elliptically fibered $\text{CY}_3$ [49], with extra D5-branes. A subset in class I, namely when the $\text{CY}_2$ is compact, will be the main solutions discussed in this thesis. This subset is characterised by the piecewise linear functions

$$h_4 = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k), \qquad h_8 = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k), \tag{4.3}$$

where $\alpha_k$, $\beta_k$, $\mu_k$, $\nu_k$ are integration constants directly related to the quantised charges.

In Section 5.2, we extend our assumptions to consider all the $\text{AdS}_3\times\text{S}^2\times\text{M}_5$ solutions to type II supergravities preserving small $\mathcal{N} = (0, 4)$ supersymmetry. Imposing this amount of supersymmetries constraints the solutions to have between

0 and 3 a priori isometries in $M_5$. In type IIA, the case with no a priori isometries is forced to have SU(2) structure, so it gives back the class of solutions that we have just summarised. In turn, in type IIB, two general classes are obtained, with and without D7-brane sources, and with identity structure in the internal five dimensional space.

Section 5.3 is devoted to the study of the 2d CFTs dual to the 'seed' solutions. These CFTs live in D2-D4-D6-D8-NS5 brane intersections, where D2- and D6-branes play the role of colour branes and D4- and D8-branes play the role of flavour branes. The detailed study of the configuration gives rise to the Hanany-Witten brane setups, depicted in Figure 4.1. From these setups we construct a precise family of quivers depicted in Figure 4.2, that flow to $\mathcal{N} = (0, 4)$ CFTs at low energy. The field content of the proposed quiver field theories arises from the quantisation of the open strings stretched between the branes, that we have summarised in Table 4.1. As a test of the proposed duality, we pay special attention to the gauge anomaly cancelation condition, which constraints the ranks of the different groups, according to

$$\begin{aligned} F_k &= 2\mu_k - \mu_{k+1} - \mu_{k-1} = \nu_{k-1} - \nu_k, \\ \tilde{F}_k &= 2\alpha_k - \alpha_{k+1} - \alpha_{k-1} = \beta_{k-1} - \beta_k. \end{aligned} \tag{4.4}$$

These conditions are indeed satisfied by the quantised charges associated to our solutions.

We also use the relation between the R-symmetry anomaly and the right moving central charge in order to compute the latter

$$c_R = 6(n_{hyp} - n_{vec}), \tag{4.5}$$

where $n_{hyp}$ and $n_{vec}$ are the number of hypermultiplets and vector multiplets of the quiver, that we compare with the holographic central charge, computed using the Brown-Henneaux formula [97]. This gives for our solutions,

$$c_{hol} = \frac{3\pi}{2G_N}\text{Vol}_{\text{CY}_2}\int_0^{2\pi(P+1)} h_4 h_8 \text{d}\rho = \frac{3}{\pi}\int_0^{2\pi(P+1)} h_4 h_8 \text{d}\rho. \tag{4.6}$$

A non-trivial check of our proposed quivers is that both expressions agree exactly in the holographic limit, i.e. long quivers with large ranks.

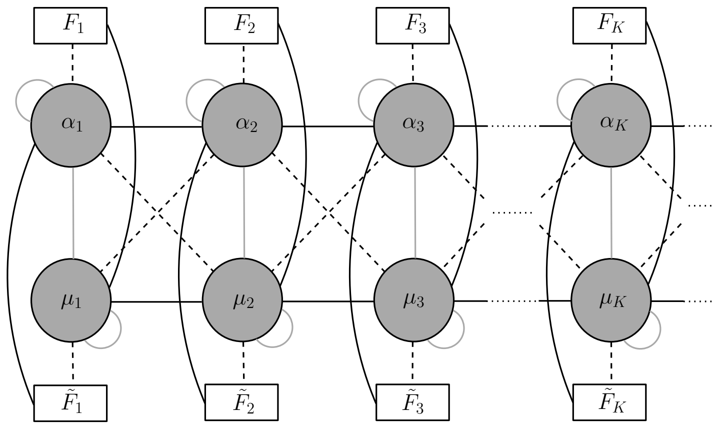


**Figure 4.2:** Quivers encoding the 2d field theories living in the D2-D4-D6-D8-NS5 brane intersections.

In Section 5.4, a concrete example in the class of the ‘seed’ solutions associated to the precise choice of warping functions,

$$u = 4L^4 M^2 \rho, \qquad h_4 = L^2 M^4 \rho, \qquad h_8 = F_0 \rho\,, \tag{4.7}$$

is studied. This solution was obtained in [81] applying NATD with respect to freely acting SU(2) group on the near horizon geometry of the D1-D5 system. Since the solution is obtained via NATD, the dual geometry is non-compact, as we mentioned at the end of Section 2.2. We exploit the fact that such background is in our class I of solutions to propose two explicit global completions of this solution, giving rise to well-defined (0,4) 2d CFTs.

A natural way to extend our work is to analyse its implications in M-theory. By taking the massless limit described in Section 5.1, we perform the uplift to M-theory –with the ansatz explained around equation (1.2). The new solutions that arise are presented in Section 6. They are solutions of the form $\mathrm{AdS}_3\times \mathrm{S}^3/\mathbf{Z}_k\times \mathrm{M}_4\times \mathrm{I}$, with $\mathrm{M}_4$=$\mathrm{CY}_2$ for class I and $\mathrm{M}_4$ a Kähler four manifold for class II, respecting small (0,4) supersymmetry. We focus on class I, for a compact Calabi-Yau two-fold, and find a M2-M5-KK-M5’ intersection underlying the geometry. This brane setup is interpreted as KK-monopoles and M2-branes stretched between M5’-branes, with extra M5-branes providing flavour groups. M2-branes suspended between parallel M5’-branes describe so-called M-strings, i.e. deformations

| String | Interval | Multiplet | Representation |
|---|---|---|---|
| D2-D2 | Same | $\mathcal{N}=(0,4)$ vector + $\mathcal{N}=(0,4)$ hyper | Adjoint |
| D2-D2 | Adjacent | $\mathcal{N}=(4,4)$ twisted hyper | bifundamental |
| D6-D6 | Same | $\mathcal{N}=(0,4)$ vector + $\mathcal{N}=(0,4)$ hyper | Adjoint |
| D6-D6 | Adjacent | $\mathcal{N}=(4,4)$ twisted hyper | bifundamental |
| D2-D6 | Same | $\mathcal{N}=(0,4)$ hyper | bifundamental |
| D2-D6 | Adjacent | $\mathcal{N}=(0,2)$ Fermi | bifundamental |
| D2-D4 | Same | $\mathcal{N}=(4,4)$ twisted hyper | bifundamental |
| D4-D6 | Same | $\mathcal{N}=(0,2)$ Fermi | bifundamental |
| D2-D8 | Same | $\mathcal{N}=(0,2)$ Fermi | bifundamental |
| D6-D8 | Same | $\mathcal{N}=(4,4)$ twisted hyper | bifundamental |

**Table 4.1:** Summary of the field content arising from the different strings stretching between branes in the D2-D4-D6-D8-NS5 brane setup.

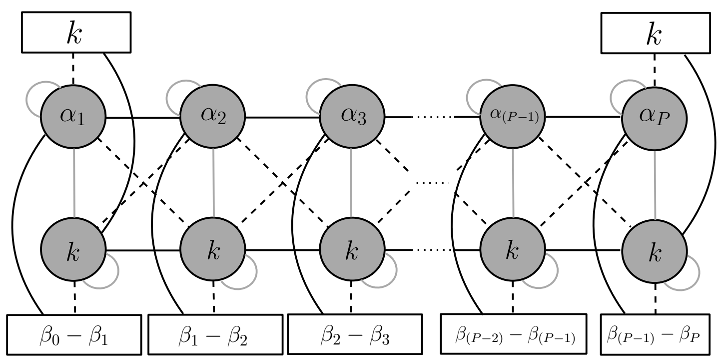


**Figure 4.3:** Quiver field theory dual to the $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbf{Z}_k\times\mathrm{CY}_2\times\mathrm{I}$ solution in M-theory.

of the self-dual strings living in M5'-branes away from criticality. Our class of solutions provides, thus, the holographic duals of these configurations. A similar analysis to the one performed in type IIA allows to construct the precise quivers depicted in Figure 4.3, that flow to CFTs in the IR. These CFTs describe self-dual strings in 6d (1,0) CFTs.

From these solutions a second set of backgrounds can be constructed via ana-

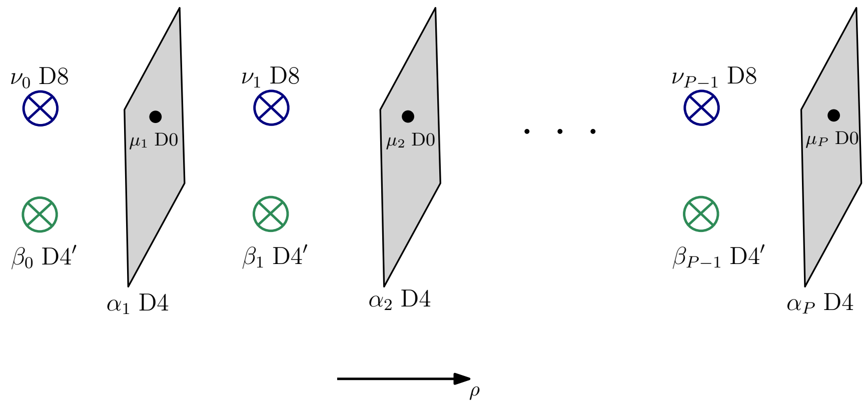


**Figure 4.4:** Hanany-Witten brane setups for the D0-F1-D4-D4'-D8 brane system.

lytical continuation,

$$\mathrm{AdS}_3 \longleftrightarrow \mathrm{S}^3\,, \qquad \mathrm{S}^3 \longleftrightarrow \mathrm{AdS}_3\,. \tag{4.8}$$

The solutions obtained are foliations of $\mathrm{AdS}_3/\mathbf{Z}_k\times\mathrm{S}^3\times\mathrm{M}_4$ on an interval preserving (0,4) supersymmetries. As these solutions come from analytical continuations, they are associated to different brane intersections, to be more precise, they are associated to M2-M5-M5' brane setups with momentum charge, where a superconformal quantum mechanics lives in the low energy limit.

In the same Section 6, we take these M-theory solutions and reduce them on the Hopf-fibre of the $\mathrm{AdS}_3/\mathbf{Z}_k$ subspace. The new class is a solution to massless type IIA supergravity of the form $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{M}_4$ warped on an interval. These solutions preserve four Poincaré supersymmetries and have SU(2) structure in the remaining internal space. We show that these solutions are also obtained through analytical continuations from the solutions constructed in Section 5.1, in the massless limit. This fact suggests that, they can be extend to massive type IIA.

The detailed study of these solutions in massive type IIA is performed in Section 7.1, again for a compact $\mathrm{CY}_2$. We show that the geometry is associated to the D0-F1-D4-D4'-D8 Hanany-Witten brane setup depicted in Figure 4.4. We explain in Section 7.1 that after a T-(S-duality)-T transformation and Hanany-Witten moves the brane setup can be interpreted as describing $\mathrm{U}(\alpha_k)$ and $\mathrm{U}(\mu_k)$ Wilson lines in their completely antisymmetric representations, in such a way that the two subsystems, D4-D4'-F1 and D0-D8-F1, can be interpreted as backreacted D0-D4 baryon

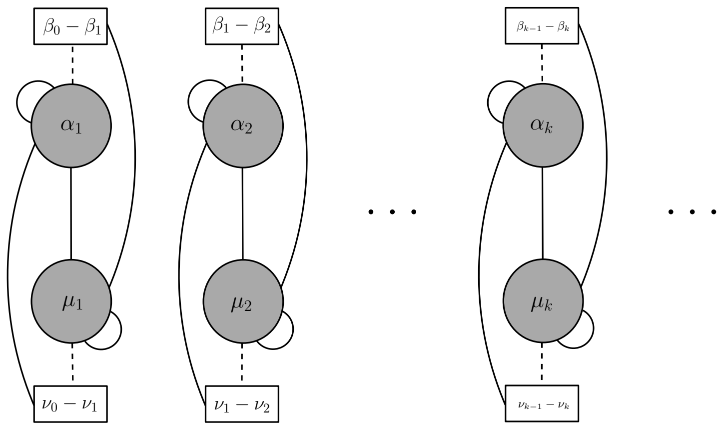


**Figure 4.5:** A generic one dimensional quiver field theory dual to the $\mathrm{AdS}_2$ solutions in massive type IIA.

vertices within the 5d $\mathcal{N} = 1$ QFT living in the worldvolume of a D4'-D8 system. A set of disconnected quivers is proposed, see Figure 4.5, giving the SCQM dual to our solutions in the IR. In these quivers the D0- and D4-branes are colour branes and the D4'- and D8-branes are flavour branes. The dynamics is described in Table 4.2.

| String | Multiplet | Representation |
|---|---|---|
| D0-D0 | $\mathcal{N} = (4,4)$ vector + $\mathcal{N} = (4,4)$ hyper | Adjoint |
| D4-D4 | $\mathcal{N} = (4,4)$ vector + $\mathcal{N} = (4,4)$ hyper | Adjoint |
| D0-D4 | $\mathcal{N} = (4,4)$ hyper | bifundamental |
| D0-D4' | $\mathcal{N} = (4,4)$ twisted hyper | bifundamental |
| D4-D8 | $\mathcal{N} = (4,4)$ twisted hyper | bifundamental |
| D0-D8 | $\mathcal{N} = (0,2)$ Fermi | bifundamental |
| D4-D4' | $\mathcal{N} = (0,2)$ Fermi | bifundamental |

**Table 4.2:** Field content of the SCQM described by the quivers in Figure 4.5.

From these $\mathrm{AdS}_2 \times \mathrm{S}^3$ geometries one can analyse explicit examples, as the one studied in Section 7.2. This example is obtained acting with non-Abelian T-duality –on a non-compact group $\mathrm{SL}(2,\mathbf{R})$– on the D1-D5 near horizon geometry, as is explained in Section 2.2. As this solution is in the class described in Section 7.1, we are able to make a concrete proposal for its dual SCQM in terms of backreacted baryon vertices described by a concrete quiver QM.

At this stage, we have one family of $\mathrm{AdS}_3\times\mathrm{S}^2$ and one family of $\mathrm{AdS}_2\times\mathrm{S}^3$ backgrounds of massive type IIA supergravity, from which we can construct new classes of solutions in type IIB acting with (Abelian or non-Abelian) T-duality. In this spirit, we construct three new classes of solutions to type IIB supergravity preserving eight supercharges of the form $\mathrm{AdS}_2\times\mathrm{S}^2\times\mathrm{CY}_2\times\Sigma_2$, where $\Sigma_2$ is a two dimensional Riemann surface. We referred to these solutions as Type A, Type B and Type C. They are analysed in detail in Chapter 8.

The solutions in Section 8.2, referred as Type A, are obtained acting with ATD on the Hopf-fibre of $\mathrm{AdS}_3$ of the solutions discussed in Section 5.1 A brane intersection is obtained consisting of colour D1- and D5-branes extended between NS5-branes and orthogonal D3 and D7 flavour branes. The quantum mechanics associated to these geometries arise by dimensional reduction along the spacetime direction of the two dimensional mother QFTs. For the coordinates that we use to T-dualise in $\mathrm{AdS}_3$, the dimensional reduction can be seen as a discrete light-cone quantisation (DLCQ) of the two dimensional CFT. In turn, we inherit the quiver field theories from those of the 'seed' geometries, depicted in Figure 4.2. In this case the gauge groups are associated to D1- and D5-branes, and the flavour groups to D3- and D7-branes, and the matter and vector fields are $\mathcal{N}=4$ multiplets.

Our second solutions, Type B, are studied in Section 8.2. They have the same warped form as the previous ones but are obtained T-dualising the solutions given in Section 7.1 on the Hopf-fibre of the $\mathrm{S}^3$. The brane intersection is again a D1-D3-D5-D7-NS5-F1 system but its quantum mechanics can be interpreted as backreacted D1-D5 baryon vertices within the four dimensional $\mathcal{N}=2$ QFT living in D3-D7 branes.

The third class of solutions that we construct in type IIB, Type C, are studied as well in Section 8.2. These are obtained acting with NATD with respect to a freely acting $\mathrm{SL}(2,\mathbb{R})$ subgroup of the $\mathrm{AdS}_3$ subspace of our original 'seed' geometries. The dual geometry is an $\mathrm{AdS}_2\times\mathrm{S}^2\times\mathrm{CY}_2\times\Sigma_2$ solution where in this case $\Sigma_2$ is an infinite strip.

In the same section, we discuss various features of the $\mathrm{AdS}_2\times\mathrm{S}^2$ geometries constructed in type IIB. We notice that, as the backgrounds Type A and Type B are

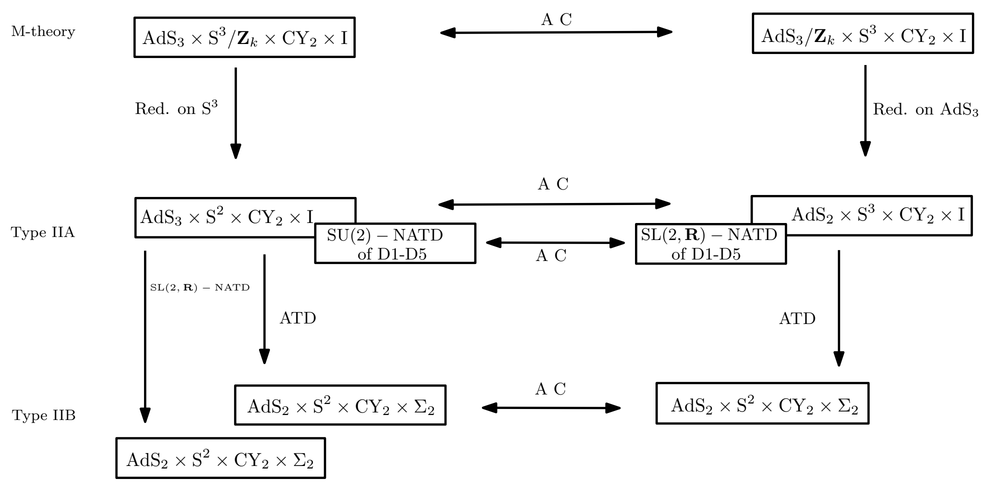


**Figure 4.6:** Connections between the infinite families of $\mathrm{AdS}_2$ and $\mathrm{AdS}_3$ solutions in M-theory, massive type IIA and type IIB supergravities studied in the thesis.

derived from type IIA solutions, related by analytical continuation, they are also connected via the analytical continuations, $\mathrm{AdS}_2 \leftrightarrow \mathrm{S}^2$ and $\mathrm{S}^2 \leftrightarrow \mathrm{AdS}_2$. Second, we show that both solutions extend the class of $\mathrm{AdS}_2\times\mathrm{S}^2\times\mathrm{CY}_2 \times \Sigma_2$ solutions constructed in [47, 48] to include D3- and D7-branes sources. We show that, the solution referred as Type C also fits in the class of [47], in the absence of sources, this time for an infinite strip. Finally, we analyse a connection between the holographic central charge and a computation carried out in the RR sector of the Type A and Type B solutions. Our computation allows to reproduce the holographic central charge with the sum of the products of the RR electric and magnetic charges of the solutions, extending previous results in $\mathrm{AdS}_2$ gravity with electric flux in [98]. Second, we use the RR field strengths to construct a functional from which the central charge can be derived from an extremisation principle in the line of [99, 100].

Figure 4.6 contains a summary of the geometries discussed in this thesis. We have completed this study with a detailed analysis of the 2d and 1d CFTs dual to these classes of solutions.

After this summary, in the following Chapters, we present the articles that compose this thesis.

# Articles (Impact Factor)

This thesis is based on the following papers:

- $\mathrm{AdS}_3$ solutions in Massive IIA with small $\mathcal{N} = (4,0)$ supersymmetry [57] .
- Two dimensional $\mathcal{N} = (4,0)$ quivers dual to $\mathrm{AdS}_3$ solutions in massive IIA [59].
- $\mathrm{AdS}_3$ solutions in massive IIA, defect CFTs and T-duality [60].
- M-strings and $\mathrm{AdS}_3$ solutions to M-theory with small $\mathcal{N} = (4,0)$ supersymmetry [62].
- New $\mathrm{AdS}_2$ backgrounds and $\mathcal{N} = 4$ Conformal Quantum Mechanics [63].
- $\mathrm{AdS}_2$ duals to ADHM quivers with Wilson lines [64] .
- $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2$ solutions in Type IIB with 8 supersymmetries [65].
- $\mathrm{AdS}_2$ geometries and non-Abelian T-duality in non-compact spaces [83].
- $\mathrm{AdS}_2 \times \mathrm{S}^2$ in IIB with small $\mathcal{N} = 4$ supersymmetry [101].

| Journal | Year | Impact Factor | Area |
|---|---|---|---|
| JHEP | 2020 | 5.810 | Physics, Particles and Fields |

**Table 4.3:** Impact factors of the scientific journal where the articles of this thesis have been published. Source: Journal Citation Reports

# 5. AdS$_3$/CFT$_2$ in Massive Type IIA

## 5.1 AdS$_3\times$S$^2\times$M$_5$ solutions in massive type IIA with SU(2) structure

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# AdS$_3$ solutions in massive IIA with small $\mathcal{N} = (4, 0)$ supersymmetry

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Abstract: We study AdS$_3 \times$ S$^2$ solutions in massive IIA that preserve small $\mathcal{N} = (4, 0)$ supersymmetry in terms of an SU(2)-structure on the remaining internal space. We find two new classes of solutions that are warped products of the form AdS$_3 \times$ S$^2 \times$ M$_4 \times \mathbb{R}$. For the first, M$_4$=CY$_2$ and we find a generalisation of a D4–D8 system involving possible additional branes. For the second, M$_4$ need only be Kahler, and we find a generalisation of the T-dual of solutions based on D3-branes wrapping curves in the base of an elliptically fibered Calabi-Yau 3-fold. Within these classes we find many new locally compact solutions that are foliations of AdS$_3 \times$ S$^2 \times$ CY$_2$ over an interval, bounded by various D brane and O plane behaviours. We comment on how these local solutions may be used as the building blocks of infinite classes of global solutions glued together with defect branes. Utilising T-duality we find two new classes of AdS$_3 \times$ S$^3 \times$ M$_4$ solutions in IIB. The first backreacts D5s and KK monopoles on the D1–D5 near horizon. The second is a generalisation of the solutions based on D3-branes wrapping curves in the base of an elliptically fibered CY$_3$ that includes non trivial 3-form flux.

Keywords: Extended Supersymmetry, Supergravity Models, AdS-CFT Correspondence, D-branes

ArXiv ePrint: 1908.09851



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JHEP01(2020)129

## Contents



# 1 Introduction

Two dimensional CFTs play a prominent role in string theory and provide the best arena to test the AdS/CFT correspondence. The conformal group in two dimensions is infinite dimensional and this makes two dimensional CFTs much more tractable than their higher dimensional counterparts, in some cases even exactly solvable [1]. In turn, certain $AdS_3$ solutions involve only NS-NS fields, and constitute exactly solvable string theory backgrounds [2, 3]. As such there is clear motivation to study the $AdS_3/CFT_2$ duality in as much detail as possible.

The canonical example of $AdS_3$ geometry is the near horizon limit of D1 and D5 branes [4] which gives rise to an $AdS_3 \times S^3 \times CY_2$ geometry realising small $\mathcal{N} = (4,4)$ superconformal symmetry. The CFT dual is believed to be the free symmetric product orbifold $\text{Sym}^N(\text{CY}_2)$ for $\text{CY}_2 = \text{T}^4$ or K3 [5]. In recent years there has been renewed interest in its study and strong support for this proposal has been provided [6, 7, 9–11].

Despite this early success, small $\mathcal{N} = (4,0)$ $AdS_3$ solutions in 10 and 11 dimensions are rare in the literature, with known cases mostly following from [4] via orbifoldings and/or string dualities. These describe the near horizon limit of D1 and D5 branes intersecting with KK-monopoles [12–15] or D9-branes [16]. These systems play a prominent role in the microscopical description of five dimensional black holes [17–22]. 2d (4,0) CFTs are also central in the description of self-dual strings in 6d (1,0) CFTs, realised in M and F theory [23–28]. The $AdS_3$ duals of D3-branes wrapped on complex curves in F-theory have been recently constructed in [28], and will play an important role in this work. More general (4,0) 2d CFTs such as the ones described by the quivers constructed in [23, 24, 26, 30], are however still lacking a holographic description. One of the motivations of this work will be to fill this gap.

This dearth of holographic duals is symptomatic of the limited classification effort aimed at $AdS_3$ in general,[1] which mostly focuses on different superconformal algebras and restrictive ansätze (see for instance [31–39]). Bucking this trend are [41, 42] and [28] which do study small $\mathcal{N} = (4,0)$ solutions in M-theory and IIB respectively, though they still take a restricted ansatz for the fluxes. In this work we shall focus on small $\mathcal{N} = (4,0)$ in massive IIA, we will make no restriction on the allowed fluxes, though we will also make some assumptions.

Our approach to finding $AdS_3$ solutions with small $\mathcal{N} = (4,0)$ superconformal symmetry is to construct Killing spinors which manifestly transform in the $(\mathbf{2}, \mathbf{2} \oplus \overline{\mathbf{2}})$ of the bosonic sub-algebra $\mathfrak{sl}(2) \oplus \mathfrak{su}(2)$ — the same as the bosonic generators of the algebra [29]. The first factor is realised by Killing spinors on $AdS_3$ which transfrom in the $\mathbf{2}$ of $\mathfrak{sl}(2)$, while the second is an SU(2) R-symmetry $SU(2)_R$ that suggests a local description of the geometry and fluxes in which this $SU(2)_R$ is realised by a 2-sphere, that we shall assume is round. We then realise the $\mathbf{2} \oplus \overline{\mathbf{2}}$ representation of $SU(2)_R$ by taking certain products of Killing spinors on $S^2$ and spinors on the internal 5-manifold. The fundamental building block in this construction, which builds on earlier work in [43–47], are the SU(2) doublets one can form from the Killing spinors on $S^2$.

A major advantage of this R-symmetry based approach to constructing spinors on the internal space is that it is possible to show that $\mathcal{N} = (4,0)$ supersymmetry is actually implied by an $\mathcal{N} = 1$ sub-sector through the action of $SU(2)_R$. As such we are able to exploit an existing geometric classification of $\mathcal{N} = 1$ $AdS_3$ solutions [38] to extract necessary conditions on the geometry and fluxes. Of course there should rather be a lot of solutions of the form $AdS_3 \times S^2 \times M_5$, so in this work we will focus on those for which $M_5$ supports an SU(2)-structure.[2] We do this in part to try and ensure that we realise small $\mathcal{N} = (4,0)$

[1]See however [40] for some systematic work addressing this from a 3d gauged supergravity perspective.
[2]A similar restriction was taken in [48] for $\mathcal{N} = 2$ $AdS_4$ in massive IIA.

rather than some larger superconformal algebra which contains this. That SU(2)-structure implies the small algebra and no more is certainly not a theorem but experience suggests to us that algebras that cointain this (such as large $\mathcal{N} = (4,0)$) will require an Identity-structure on $\mathrm{M}_5$. Another reason to focus on SU(2)-structure is to keep this work focused, and leave the more generic case with $\mathrm{M}_5$ supporting an identity structure for the future.

The layout of the paper is as follows: in section 2 (with supplementary material in appendix A) we perform the technical ground work of constructing spinors transforming in the $\mathbf{2} \oplus \overline{\mathbf{2}}$ representation of $\mathrm{SU}(2)_R$, and extracting necessary and sufficient conditions on the geometry and fluxes for a solution to realise small $\mathcal{N} = (4,0)$. We find two classes of solutions.

Class I is a warped product of the form $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \mathbb{R}$ that we study in section 3. The class is summarised and derived in sections 3.1 and 3.2 respectively. In section 3.3 (up to T-duality) we find a generalisation of the D1–D5 near horizon with source D5 branes back reacted on $\mathrm{CY}_2$. In section 3.4 we find several new compact local solutions in massive IIA that are foliations of $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ over a bounded interval.

Class II is a warped product of the form $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_4 \times \mathbb{R}$, where $\mathrm{M}_4$ is now a Kahler four-manifold. The class is summarised in section 4.1 and derived in section 4.2. Exploiting T-duality, in section 4.3, we find a generalisation of the class of D3 branes wrapping curves in the base of an elliptically fibered $\mathrm{CY}_3$ [28] — with non trival 3-form flux turned on. In section 4.4 we then find further local $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ foliations that are compact.

In section 5 we establish that the local solutions found in sections 3.4 and 4.4 may be used to construct a significantly richer variety of globally compact solutions by using defect branes to glue local solutions together.

Finally in section 6 we summarise and discuss some future directions.



## 2 $\mathrm{AdS}_3 \times \mathrm{S}^2$ solutions with small $\mathcal{N} = (4,0)$ supersymmetry and SU(2)-structure

In this section we derive geometric conditions for a class of warped $\mathrm{AdS}_3$ solutions preserving small $\mathcal{N} = (4,0)$ supersymmetry in massive IIA.

The small $\mathcal{N} = (4,0)$ super-conformal algebra contains a bosonic sub-algebra

$$\mathfrak{sl}(2) \oplus \mathfrak{su}(2) \tag{2.1}$$

that should be realised geometrically by the solutions we are interested in. The $\mathfrak{sl}(2)$ factor is simply realised by $\mathrm{AdS}_3$. The $\mathfrak{su}(2)$ factor is an R-symmetry, we shall denote $\mathrm{SU}(2)_R$, that should be realised by the 7 dimensional internal space $\mathrm{M}_7$. This indicates that $\mathrm{M}_7$ should admit a local description that contains a 2-sphere, that may be round or appear as part of an $\mathrm{SU}(2)\times\mathrm{U}(1)$ preserving squashed 3-sphere, foliated over the remaining directions. In this work we shall assume the former and seek solutions with metric decomposing as

$$ds^2 = e^{2A} ds^2(\mathrm{AdS}_3) + ds^2(\mathrm{M}_7), \qquad ds^2(\mathrm{M}_7) = e^{2C} ds^2(\mathrm{S}^2) + ds^2(\mathrm{M}_5), \tag{2.2}$$

where the warp factors $e^{2A}, e^{2C}$ and dilaton $\Phi$ have support in $\mathrm{M}_5$, and the fluxes depend on the $\mathrm{AdS}_3$ and $\mathrm{S}^2$ directions only through their respective volume forms $\mathrm{vol}(\mathrm{AdS}_3)$

and vol($S^2$). This is sufficient to ensure that we respect the isometries of $AdS_3$ and $S^2$. However to guarantee small $\mathcal{N} = (4, 0)$ supersymmetry we must solve the supersymmetry constraints. Our strategy to achieve this is as follows

1. Construct spinors on that transform in the $(\mathbf{2}, \mathbf{2} \oplus \overline{\mathbf{2}})$ representation of $SL(2) \times SU(2)$, ensuring consistency with the bosonic sub-algebra of small $\mathcal{N} = (4, 0)$ superconformal symmetry.

2. Reduce our considerations to an $\mathcal{N} = 1$ sub-sector of this spinor that manifestly implies $\mathcal{N} = (4, 0)$ through the action of the R-symmetry — this requires the bosonic supergravity fields to be $SL(2) \times SU(2)$ singlets.

3. Exploit an existing $\mathcal{N} = 1$ $AdS_3$ classification [38] to obtain sufficient conditions on the geometry and fluxes for a solution with small $\mathcal{N} = (4, 0)$ and SU(2)-structure in IIA to exist.

4. Study the classes consistent with our assumptions, and simplify them as much as possible in a coordinate patch away from the loci of possible sources.

We will deal with points 1–2 in section 2.1, which is the most technical part of the paper and can be skipped on a first reading. Section 2.2 deals with point 3. For point 4 there are 2 classes of solutions to study with SU(2)-structure, specialised to conformal Calabi-Yau and Kahler structure. We present these and study them in sections 3 and 4. Those readers merely interested in the results can find summaries of these classes in sections 3.1 and 4.1.

Following 1–4 leads to necessary and sufficient conditions for two classes of solutions to exist in the absence of localised sources. When these are present, the derivation is still completely valid away from their loci, but at these specific points we must solve some additional constraints. Namely that the source corrected Bianchi identities hold and that the sources have a supersymmetric embedding — i.e. they must be calibrated [52, 53]. We shall come back to this issue in section 5. However, from a practical perspective one should appreciate that it is often not necessary to check these conditions explicitly. In particular, if the warp factors and relevant parts of the fluxes reproduce the behaviour of known localised supersymmetric sources (i.e. branes, O-planes and their generalisations) at some point in the geometry, then one knows that these additional conditions must follow. We will exploit this fact in sections 3.4 and 4.4.

In the next section we shall construct $\mathcal{N} = (4, 0)$ spinors that manifestly transform under the action of SU(2). We shall then be able to identify an $\mathcal{N} = (1, 0)$ sub-sector, which when solved, implies the full $\mathcal{N} = (4, 0)$ under the action of $SU(2)_R$.

### 2.1 Realising an SU(2) R-symmetry

Supersymmetric solutions of type II supergravity come equipped with associated Majorana-Weyl Killing spinors $\epsilon_1, \epsilon_2$, that ensure the vanishing of the dilatino and gravitino variations. As we seek solutions with an $AdS_3$ factor that preserve $\mathcal{N} = (4, 0)$ supersymmetry we can decompose these spinors as

$$\epsilon_1 = \sum_{I=1}^{4} \zeta^I \otimes v_+ \otimes \chi_1^I, \qquad \epsilon_2 = \sum_{I=1}^{4} \zeta^I \otimes v_\mp \otimes \chi_2^I \tag{2.3}$$

where $\zeta^I$ are 4 independent Majorana Killing spinors on unit radius $\mathrm{AdS}_3$ and $\chi_{1,2}^I$ each contain 4 independent Majorana spinors on $\mathrm{M}_7$. The remaining factors $v_\pm$ are auxiliary vectors that are required to make $\epsilon_{1,2} \in \mathrm{Cliff}(1,9)$ as we decompose in terms of spinors in 3 and 7 dimensions. They also take care of 10 dimensional chirality, so the upper/lower signs are taken in IIA/B. The 10 dimensional gamma matrices undergo a similar decomposition as

$$\Gamma_M = e^A \gamma_M^{(3)} \otimes \sigma_3 \otimes \mathbb{I}, \qquad \Gamma_A = \mathbb{I} \otimes \sigma_2 \otimes \gamma_A^{(7)} \tag{2.4}$$

where $\gamma_M^{(3)}$ are real and defined on unit radius $\mathrm{AdS}_3$, and $\gamma_A^{(7)}$ are defined on $\mathrm{M}_7$. $\sigma_i$ are the Pauli matrices so the 10 dimensional chirality matrix is $\hat{\gamma} = \mathbb{I} \otimes \sigma_1 \otimes \mathbb{I}$, so that $\sigma_1 v_\pm = \pm v_\pm$. The intertwiner, defining Majorana conjugation as $\epsilon^c = B^{(10)} \epsilon^*$, is $B^{(10)} = \mathbb{I} \otimes B^{(7)}$ so that $v_\pm$ are real and $B^{(7)-1} \gamma_A^{(7)} B^{(7)} = -\gamma_A^{(7)*}$, $B^{(7)} B^{(7)*} = 1$.

There are actually several distinct types of superconformal algebras corresponding to $\mathcal{N} = (4,0)$ (see [49] for a classification). One way to ensure that we have (at least[3]) small $\mathcal{N} = (4,0)$ is to demand that the internal parts of our $\mathcal{N} = (4,0)$ spinors are charged under an SU(2) R-symmetry, specifically transforming in the $\mathbf{2} \oplus \overline{\mathbf{2}}$ representation. Then since the spinors on $\mathrm{AdS}_3$ are charged under SL(2) we manifestly realise the bosonic sub-algebra of small $\mathcal{N} = (4,0)$ superconformal symmetry (2.1). If the internal spinors are charged under $\mathrm{SU}(2)_R$ it should be possible to construct a $\chi_{1,2}^I$ realising a 4d basis of the SU(2) Lie algebra $\frac{i}{2}\Sigma_i$, when acted on by the spinoral Lie derivative — i.e.

$$\mathcal{L}_{K_i} \chi_{1,2}^I = \frac{i}{2} (\Sigma_i)^I{}_J \chi_{1,2}^J \tag{2.5}$$

where $K_i$ are the 3 Killing vectors of SU(2). Let us now construct such SU(2) spinors.

As we decompose the internal space as $\mathrm{M}_7 = \mathrm{S}^2 \times \mathrm{M}_5$, we anticipate that the Killing spinors on $\mathrm{S}^2$ will realise $\mathrm{SU}(2)_R$. For a unit norm 2-sphere, the Killing spinors $\xi$ can be taken to obey

$$\nabla_\mu^{\mathrm{S}^2} \xi = \frac{i}{2} \sigma_\mu \xi, \qquad |\xi|^2 = 1, \tag{2.6}$$

where $\mu$ are flat indices on the unit sphere and $\sigma_\mu$ are the first 2 Pauli matrices. The chirality matrix is $\sigma_3$ and Majorana conjugation is defined as $\xi^c = \sigma_2 \xi^*$. To incorporate this into $\mathrm{M}_7$ we further decompose the gamma matrices as

$$\gamma_\mu^{(7)} = e^C \sigma_\mu \otimes \mathbb{I}, \quad \gamma_a^{(7)} = \sigma_3 \otimes \gamma_a, \quad B^{(7)} = \sigma_2 \otimes B, \tag{2.7}$$

with $\gamma_a$ gamma matrices in 5d and $BB^* = -1$, $B^{-1} \gamma_a B = \gamma_a^*$. As established in [43], the $\mathrm{S}^2$ Killing spinors so defined may be used to construct two independent SU(2) doublets

$$\xi^\alpha = \begin{pmatrix} \xi \\ \xi^c \end{pmatrix}^\alpha, \qquad \hat{\xi}^\alpha = \begin{pmatrix} i\sigma_3 \xi \\ i\sigma_3 \xi^c \end{pmatrix}^\alpha, \tag{2.8}$$

[3] The small $\mathcal{N} = (4,0)$ superalgebra is a sub-algebra of several larger ones — notably the large $\mathcal{N} = (4,0)$ superalgebra $\mathfrak{D}(2,1,\alpha)$ [50]. We will not be concerned with this subtlety in this paper.



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that transform under SU(2) as

$$\mathcal{L}_{K_i}\xi^\alpha = \frac{i}{2}(\sigma_i)^\alpha{}_\beta\xi^\beta, \qquad \mathcal{L}_{K_i}\hat{\xi}^\alpha = \frac{i}{2}(\sigma_i)^\alpha{}_\beta\hat{\xi}^\beta, \tag{2.9}$$

with $\frac{i}{2}\sigma_i$ a 2d representation of the SU(2) Lie algebra and where the 1-forms dual to the Killing vectors can now be taken to be

$$K_i = \epsilon_{ijk}y_j dy_k, \tag{2.10}$$

for $y_i$ embedding coordinates on the unit 2-sphere. One can form 7 dimensional spinors giving rise to a 4 dimensional representation of SU(2) in terms of a spinor on $\mathrm{M}_5$, $\eta$, that is an SU(2) singlet, with which one defines

$$\eta^\alpha = \begin{pmatrix} \eta \\ \eta^c \end{pmatrix}^\alpha. \tag{2.11}$$

One can then contract the $\mathrm{S}^2$ and $\mathrm{M}_5$ doublets to form a 7-dimensional SU(2) spinor

$$\chi^I = \mathcal{M}^I_{\alpha\beta}\xi^\alpha\otimes\eta^\beta, \qquad \mathcal{M}^I = (\sigma_2\sigma_1, \sigma_2\sigma_2, \sigma_2\sigma_3, -i\sigma_2)^I, \tag{2.12}$$

where all components of $\chi^I$ are Majorana.[4] Using (2.9) and Pauli matrix identities it is not hard to show that $\chi^I$ transforms as (2.5) with specific 4 dimensional representation

$$\Sigma_i = (\sigma_2\otimes\sigma_1, -\sigma_2\otimes\sigma_3,\ \mathbb{I}\otimes\sigma_2)_i, \tag{2.13}$$

which is equivalent to the $\mathbf{2}\oplus\overline{\mathbf{2}}$ representation of SU(2).[5] Since $\mathrm{S}^2$ only preserves 2 supercharges, it is perhaps not obvious that (2.12) will give rise to 4. However, since (2.3) couples the 7 dimensional SU(2) spinors to 4 independent $\mathrm{AdS}_3$ spinors, this is guaranteed as long as the components of $\chi^I$ are independent — making use of appendix A it is not hard to establish that

$$\chi^{I\dagger}\chi^J = |\eta|^2\delta^{IJ} \tag{2.15}$$

which confirms this. Let us stress that although we used $\xi^\alpha$ to form $\chi^I$, we can also use $\hat{\xi}^\alpha$, which gives a further 7 dimensional SU(2) spinor independent of the first.

The most general expressions we can write for the 7 dimensional SU(2) charged factors of (2.3) are then

$$\chi^I_1 = \frac{1}{\sqrt{2}}e^{\frac{A}{2}}\mathcal{M}^I_{\alpha\beta}\big(\xi^\alpha\otimes\eta^\beta_1 + \hat{\xi}^\alpha\otimes\hat{\eta}^\beta_1\big), \qquad \chi^I_2 = \frac{1}{\sqrt{2}}e^{\frac{A}{2}}\mathcal{M}^I_{\alpha\beta}\big(\xi^\alpha\otimes\eta^\beta_2 + \hat{\xi}^\alpha\otimes\hat{\eta}^\beta_2\big), \tag{2.16}$$

[4]Demanding this actually fixes the second component of (2.11) in terms of the first. Note that the form of (2.12) is very similar to that of the SO(4) spinors constructed in [39].

[5]Specifically the similarity transformation

$$S \sim \begin{pmatrix} 0 & 0 & s & -is \\ i\overline{s} & -\overline{s} & 0 & 0 \\ 0 & 0 & s & is \\ i\overline{s} & \overline{s} & 0 & 0 \end{pmatrix}, \tag{2.14}$$

for $s = e^{i\frac{\pi}{4}}$ is such that $\frac{i}{2}S\Sigma_i S^{-1} = \frac{i}{2}\sigma_i\oplus\left(\frac{i}{2}\sigma_i\right)^*$

where we introduced 4 spinors on $\mathrm{M}_5$ $(\eta_1, \hat{\eta}_1, \eta_2, \hat{\eta}_2)$. The 10 dimensional spinors of (2.3) contain 4 independent $\mathcal{N}=1$ sub-sectors, i.e. each term in the sums. Because the solutions we seek have an $\mathrm{AdS}_3$ factor, $d=10$ supersymmetry is implied by 4 sets of reduced $d=7$ conditions — 1 for each component of $(\chi_1^I, \chi_2^I)$. As such, each component of the internal spinors is such that [38]

$$e^{\mp A}|\chi_1|^2 \pm |\chi_2|^2 = c_{\pm}, \tag{2.17}$$

for $c_{\pm}$ constant. This relates the norms of these components to the $\mathrm{AdS}_3$ warp factor, in such a way that the later can only be an SU(2) singlet if the former are. Setting the charged parts of $|\chi_{1,2}^I|$ to zero imposes the following conditions on the 5d spinors

$$\hat{\eta}_1^{c\dagger}\eta_1 = \mathrm{Im}(\hat{\eta}_1^{\dagger}\eta_1) = \hat{\eta}_2^{c\dagger}\eta_2 = \mathrm{Im}(\hat{\eta}_2^{\dagger}\eta_2) = 0, \tag{2.18}$$

for the $\mathrm{S}^2$ zero form bi-linears that give rise to these charged terms (see appendix A). In what follows we will fix $c_- = 0$ as this is requirement for non zero Romans mass. We can then take $c_+ = 2$ without loss of generality. As such the 5d spinors should also obey

$$|\eta_1|^2 + |\hat{\eta}_1|^2 = |\eta_2|^2 + |\hat{\eta}_2|^2 = 1. \tag{2.19}$$

There is one final property of the SU(2) spinors we have constructed which it is important to stress. The 4 independent $\mathcal{N}=1$ sub-sectors contained in (2.16) each couple to the same spinors in 5 dimensions, and the action of $\mathrm{SU}(2)_R$ in (2.5) provides a map between each sub-sector. Specifically, one can write $\chi_{1,2}^I$ in terms of a single component and its spinoral Lie derivative

$$\chi_{1,2}^I = \begin{pmatrix} \chi_{1,2} \\ 2\mathcal{L}_{K_3}\chi_{1,2} \\ -2\mathcal{L}_{K_2}\chi_{1,2} \\ 2\mathcal{L}_{K_1}\chi_{1,2} \end{pmatrix}^I. \tag{2.20}$$

As such, the $\mathcal{N}=1$ Killing spinor equations following from each of $\chi_{1,2}^2, \chi_{1,2}^3, \chi_{1,2}^4$ are implied by $\chi_{1,2}^1$ whenever $\mathcal{L}_{K_i}$ commutes with the dilatino and gravitino variations. This is guaranteed by imposing that all bosonic supergravity fields are singlets under $\mathrm{SU}(2)_R$.[6] Thus it is sufficient to solve for the $\mathcal{N}=1$ sub-sector involving just $\chi_{1,2}^1$ to know that $\mathcal{N}=(4,0)$ is realised by a solution.[7]

Clearly, there should be rather a lot of distinct classes of solutions consistent with $\mathrm{AdS}_3\times\mathrm{S}^2$. In particular, while (2.18)–(2.19) do constrain the 5 dimensional spinors somewhat, they will still lead to many branching possibilities, many of which will have superconformal algebras for which small $\mathcal{N}=(4,0)$ is only a subgroup. To mitigate this issue, for the rest of this paper we will constrain our focus to the particular case where $\mathrm{M}_5$ supports

[6] The proof is analogous to that in appendix B of [39].

[7] As a redundant check we also performed the analysis of section 2.2 for the other 3 $\mathcal{N}=1$ sub-sectors in $\chi_{1,2}^I$. All that changes is some signs in the components of the charged SU(2) forms on $\mathrm{S}^2$ $(y_i, K_i$ etc.) as they appear in (2.29a)–(2.29b) — no signs changes happen for the SU(2) singlet terms. After factoring out the $\mathrm{S}^2$ data one left with the same necessary and sufficient conditions in 5 dimensions irrespective of which $\mathcal{N}=1$ sub-sector you start with — as expected.

an SU(2)-structure — rather than an identity-structure as would be the case generically. We also focus on IIA, leaving IIB for future work.

In the next section we derive necessary and sufficient geometric conditions for supersymmetry when $M_5$ supports an SU(2)-structure.

### 2.2 Geometric conditions for supersymmetry

In the previous section we constructed spinors realising $\mathcal{N} = (4,0)$ and an SU(2) R-symmetry. We further argued that it is sufficient to solve for an $\mathcal{N} = 1$ sub-sector, as the rest of the $\mathcal{N} = (4,0)$ conditions are implied by this through the action of $SU(2)_R$, provided that the bosonic fields are SU(2) singlets. In this section we will derive necessary and sufficient conditions for supersymmetry in IIA under the assumption that $M_5$ supports an SU(2)-structure. We shall thus take our $\mathcal{N} = 1$ sub-sector to be

$$\begin{aligned}
\chi_1 &= \frac{e^{\frac{A}{2}}}{\sqrt{2}}(\sigma_2\sigma_1)_{\alpha\beta}\left(\sin\left(\frac{\alpha_1+\alpha_2}{2}\right)\xi^\alpha + \cos\left(\frac{\alpha_1+\alpha_2}{2}\right)\hat{\xi}^\alpha\right)\otimes\eta_1^\beta \\
\chi_2 &= \frac{e^{\frac{A}{2}}}{\sqrt{2}}(\sigma_2\sigma_1)_{\alpha\beta}\left(\sin\left(\frac{\alpha_1-\alpha_2}{2}\right)\xi^\alpha + \cos\left(\frac{\alpha_1-\alpha_2}{2}\right)\hat{\xi}^\alpha\right)\otimes\eta_2^\beta,
\end{aligned} \tag{2.21}$$

with $\alpha_{1,2}$ functions on $M_5$, and where

$$\eta_1 = \eta, \qquad \eta_2 = e^{i\beta}\eta, \qquad |\eta|^2 = 1, \tag{2.22}$$

with $\beta$ another function on $M_5$. This is the most general parametrisation solving (2.17)–(2.19) that gives rise to an SU(2)-structure.[8]

Geometric conditions that imply $\mathcal{N} = 1$ for warped $AdS_3$ solutions were recently derived in [38]. These are given in terms of a bi- spinor (that is mapped to a poly-form under the Clifford map) constructed from a pair of Majorana spinors $(\chi_1, \chi_2)$ defined on the internal $M_7$ as

$$\Psi_+ + i\Psi_- = \chi^1\otimes\chi^{2\dagger} = \frac{1}{8}\sum_{n=0}^{7}\frac{1}{n!}\chi_2^\dagger\gamma^{(7)}_{a_n,\dots,a_1}\chi_1 dx^{a_1}\wedge\dots\wedge dx^{a_n}, \tag{2.23}$$

where $\Psi_\pm$ are real poly-forms of even/odd degree. Under the assumption of equal internal spinor norm, one has $|\chi_1|^2 = |\chi_2|^2 = e^A$ and the NS 3-form has no electric component. In turn, the RR flux can be expressed as a poly-form

$$F = f + e^{2A}\text{vol}(AdS_3)\wedge\star_7\lambda(f) \tag{2.24}$$

[8]Actually one could take $\eta_2 = a\eta + b\eta^c$ with $|a|^2 + |b|^2 = 1$ and still achieve this. However when one plugs this ansatz into the supersymmetry conditions it eventually becomes clear that when $(\text{Re}b, \text{Im}b, \text{Im}a)$ are expressed in polar coordinates all the angles must be constant. They can then be set to any value with rotations of $y_i$ and the vielbein on $M_5$. One can use this freedom to fix $b = 0$ and $|a| = 1$ without loss of generality. We suppress this subtly.

with $f$ the sum of the magnetic components of the democratic fluxes. Supersymmetry for unit radius $\text{AdS}_3$ in type IIA is then implied by the following geometric conditions

$$d_H(e^{A-\Phi}\Psi_-) = 0, \tag{2.25a}$$

$$d_H(e^{2A-\Phi}\Psi_+) - 2e^{A-\Phi}\Psi_- = \frac{e^{3A}}{8}\star_7 \lambda(f), \tag{2.25b}$$

$$e^{A-\Phi}(f,\Psi_-) - \frac{1}{2}\text{vol}(\text{M}_7) = 0, \tag{2.25c}$$

where $\lambda(X_n) = (-1)^{\frac{n}{2}(n-1)}X_n$ and $(X,Y)$ is the $d=7$ Mukai pairing, defined as $(X,Y) = (\lambda(X)\wedge Y)_7$. The twisted exterior derivative is defined as $d_H = d - H\wedge$. Let us now return to the assumption of equal spinor norm made below (2.18). Had we taken 7d spinors with non equal norm instead of (2.21), the r.h.s. of (2.25a) would have become $c_- f$ [38]. This leads to the necessary condition $f_0 c_- = 0$, so a Romans mass is only possible when $c_- = 0$ — i.e. when the spinor norms are equal as in (2.21).

Plugging (2.21) into (2.23) and making use of the bi-linear on $\text{S}^2$ and $\text{M}_5$ defined in appendix A it is possible to construct $\Psi_\pm$. However, the completely general expressions for these poly-forms are rather unwieldy. Let us sketch how we simplify them to a more tractable form, by solving some necessary conditions. Upon computing the general form of $\Psi_1$, i.e. the 1-form part of $\Psi_-$, one finds that it contains the term

$$\Psi_1 = -\frac{1}{8}\cos\alpha_1 \sin\beta K_3 + \dots \tag{2.26}$$

for $K_i$ the 1-forms dual to the SU(2) Killing vectors defined in (2.10). This term is problematic for (2.25b) as there is no way to generate it under $d$ from the forms that span the $\text{S}^2$ bi-linears (A.5), and making it part of the RR flux would make them charged under SU(2) — thus one necessarily has $\cos\alpha_1 \sin\beta = 0$. To determine which factor must vanish one can examine the general form of $\Psi_2$ and $\Psi_3$. In particular the latter can only contain $K_i$ when the former does, due again to (2.25b) and the fact the NS 3-form and RR sector should be SU(2) singlets. We find

$$\begin{aligned}
\Psi_2 &= -\frac{e^C}{8}\sin\alpha_1 \sin\beta K_3 \wedge V + \dots, \\
\Psi_3 &= \frac{e^C}{8}\cos\alpha_1 (K_1\wedge j_1 + K_2 \wedge j_2 + \cos\beta K_3 \wedge j_3) + \dots,
\end{aligned} \tag{2.27}$$

where $V$ is a real vector and $(j_1, j_2, j_3)$ are real 2-forms that together span the SU(2)-structure in 5 dimensions, as in appendix A.2. As such we fix

$$\cos\alpha_1 = 0, \qquad \alpha_2 = \alpha, \tag{2.28}$$

which we achieve by setting $\alpha_1 = \frac{\pi}{2}$ without loss of generality. The 7 dimensional bi-spinors are then given by

$$8\Psi_+ = \big(\sin\alpha + \cos\alpha e^{2C}\mathrm{vol}(\mathrm{S}^2)\big)\wedge\big(y_1 j_1 + y_2 j_2 - y_3 \mathrm{Im}\psi\big)$$
$$+\big(\cos\alpha - \sin\alpha e^{2C}\mathrm{vol}(\mathrm{S}^2)\big)\wedge \mathrm{Re}\psi + -e^{C}\big(K_1\wedge j_1 + K_2\wedge j_2 - K_3\wedge\mathrm{Im}\psi\big)\wedge V, \tag{2.29a}$$

$$8\Psi_- = \big(\cos\alpha - \sin\alpha e^{2C}\mathrm{vol}(\mathrm{S}^2)\big)\wedge\big(y_1 j_1 + y_2 j_2 - y_3 \mathrm{Im}\psi\big)\wedge V$$
$$-\big(\sin\alpha + \cos\alpha e^{2C}\mathrm{vol}(\mathrm{S}^2)\big)\wedge \mathrm{Re}\psi\wedge V - e^{C}\big(dy_1\wedge j_1 + dy_2\wedge j_2 - dy_3\wedge\mathrm{Im}\psi\big), \tag{2.29b}$$

where to ease notation we introduced the exponentiated 2-form

$$\psi = e^{-i\beta}e^{-ij_3}. \tag{2.30}$$

$\Psi_\pm$ can generically be expressed in terms of an SU(3)-structure in 7 dimensions, but in this case doing so is not particularly illuminating, and (2.29a)–(2.29b) give far more compact expressions.

We now want to insert (2.29a)–(2.29b) into (2.25a)–(2.25c) and derive 5 dimensional conditions that imply these. To do so we decompose

$$H = H_3 + e^{2C}H_1\wedge\mathrm{vol}(\mathrm{S}^2), \tag{2.31}$$

and assume that the RR fluxes only depend on the $\mathrm{S}^2$ directions through $\mathrm{vol}(\mathrm{S}^2)$, and that $(e^A, e^C, e^\Phi)$ are independent of these directions. Making use of the expressions that map $(y_i, K_i, \mathrm{vol}(\mathrm{S}^2))$ under the exterior derivative and wedge-product in (A.5), and after significant massaging one arrives at necessary and sufficient conditions for supersymmetry. Those independent of the RR forms that follow from (2.25a)–(2.25b) are

$$2e^C + \sin\alpha e^A = 0, \tag{2.32a}$$

$$d(e^{3A-\Phi}\sin\alpha\sin\beta) - 2e^{2A-\Phi}\cos\alpha\sin\beta V = 0, \tag{2.32b}$$

$$e^{2C}H_1 + \frac{e^A}{2}V - \frac{1}{4}d(e^{2A}\sin\alpha\cos\alpha) = 0, \tag{2.32c}$$

$$d(e^{A-\Phi}\sin\alpha\cos\beta)\wedge V = 0, \tag{2.32d}$$

$$d(e^{3A-\Phi}\sin\alpha\Omega) - 2e^{2A-\Phi}\cos\alpha V\wedge\Omega = 0, \tag{2.32e}$$

$$d(e^{3A-\Phi}\sin\alpha\cos\beta J) - 2e^{2A-\Phi}\cos\alpha\cos\beta V\wedge J - e^{3A-\Phi}\sin\alpha\sin\beta H_3 = 0, \tag{2.32f}$$

$$(\sin\beta e^{2A}d(e^{-2A}J) + \cos\beta H_3)\wedge V = 0, \tag{2.32g}$$

$$\Omega\wedge H_3 = (\sin\beta dJ + \cos\beta H_3)\wedge J = 0, \tag{2.32h}$$

where we have repackaged $j_i$ as the more standard SU(2)-structure forms $J, \Omega$

$$J = j_3, \quad \Omega = j_1 + ij_2, \quad J\wedge\Omega = 0, \quad J\wedge J = \frac{1}{2}\Omega\wedge\overline{\Omega}. \tag{2.33}$$



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From (2.25b) we are also given the following definitions for the RR fluxes

$$
\begin{aligned}
e^{3A}\star_7 f_6 &= d(e^{3A-\Phi}\cos\alpha\cos\beta){+}2e^{2A-\Phi}\sin\alpha\cos\beta V, &\text{(2.34a)}\\
e^{3A}\star_7 f_4 &= (d(e^{3A-\Phi}\cos\alpha\sin\beta J){-}2e^{2A-\Phi}\sin\alpha\sin\beta V{\wedge}J{-}e^{3A-\Phi}\cos\alpha\cos\beta H_3) &\text{(2.34b)}\\
&\quad +\text{vol}(\text{S}^2){\wedge}\big(d(e^{3A+2C-\Phi}\sin\alpha\cos\beta){-}e^{3A+2C-\Phi}\cos\alpha\cos\beta H_1{+}2e^{2A+2C-\Phi}\cos\alpha\cos\beta V\big),\\
e^{3A}\star_7 f_2 &= -d\left(\frac{e^{3A-\Phi}}{2}\cos\alpha\cos\beta J{\wedge}J\right){-}e^{2A-\Phi}\sin\alpha\cos\beta V{\wedge}J{\wedge}J{+}e^{3A-\Phi}\cos\alpha\sin\beta J{\wedge}H_3,\\
&\quad +\text{vol}(\text{S}^2){\wedge}\Big(d(e^{3A+2C-\Phi}\sin\alpha\sin\beta J){+}e^{3A+2C-\Phi}\cos\alpha\sin\beta H_1{\wedge}J &\text{(2.34c)}\\
&\quad -2e^{2A+2C-\Phi}\cos\alpha\sin\beta V{\wedge}J{+}e^{3A+2C-\Phi}\sin\alpha\cos\beta H_3\Big),\\
e^{3A}\star_7 f_0 &= -\frac{1}{2}\text{vol}(\text{S}^2){\wedge}\Big(d(e^{3A+2C-\Phi}\sin\alpha\cos\beta J{\wedge}J){-}2e^{3A+2C-\Phi}\sin\alpha\sin\beta J{\wedge}H_3 &\text{(2.34d)}\\
&\quad +e^{3A+2C-\Phi}\cos\alpha\cos\beta H_1{\wedge}J{\wedge}J{-}2e^{2A+2C-\Phi}\cos\alpha\cos\beta V{\wedge}J{\wedge}J\Big).
\end{aligned}
$$

Finally (2.25c) gives the pairing constraints

$$
\begin{aligned}
e^{A-\Phi}(f,[(\sin\alpha+\cos\alpha e^{2C}\text{vol}(\text{S}^2))\wedge\text{Re}\psi\wedge V])+2e^{2C}\text{vol}(\text{S}^2)\wedge V\wedge J\wedge J=0,&\\
(f,[\cos\alpha-\sin\alpha e^{2C}\text{vol}(\text{S}^2))\wedge\big(y_1 j_1+y_2 j_2-y_3\text{Im}\psi\big)\wedge V]=0.& \qquad (2.35)
\end{aligned}
$$

Equations (2.32a)–(2.35) are necessary and sufficient for supersymmetry, but to ensure that we actually have a solution one must impose the Bianchi identities of the magnetic parts of the RR and NS fluxes. Away from localised sources these are

$$
dH_3=0,\qquad d(e^{2C}H_1)=0,\qquad d_H f=0. \qquad (2.36)
$$

In the presence of sources the left hand side of these expressions may be modified by $\delta$-function sources — we shall comment on this when it becomes relevant. Supersymmetry and (2.36) have been shown to imply the remaining equations of motion following from the IIA action [51].

The conditions (2.32a)–(2.32h) contain two physically distinct classes of solutions, namely for $\sin\beta=0$ and $\sin\beta\neq0$, that we explore in sections 3 and 4. To briefly illustrate the difference one can consider (2.32b)$\wedge J$ and (2.32f). These may be combined to show in general that

$$
\sin^2\beta\left(H_3-d\left(\frac{\cos\beta}{\sin\beta}J\right)\right)=0. \qquad (2.37)
$$

When $\sin\beta\neq0$ this condition gives a unique definition of $H_3$, while when $\sin\beta=0$ the condition is trivialised and (2.32g)–(2.32h) merely constrain $H_3$ such that it should give zero when wedged with each of $(J,\Omega,V)$.

Despite there being two classes, they do contain some common features. We first note that (2.32c) defines $e^{2C_1}H_1$ in such a way that its Bianchi identity can only be obeyed away from sources if

$$
d(e^AV)=0. \qquad (2.38)
$$

We solve this condition by introducing a local coordinate $\rho$ such that

$$e^{A}V = d\rho, \tag{2.39}$$

which enables us to locally decompose the internal 5-manifold as $ds^2(\mathrm{M}_5) = ds^2(\mathrm{M}_4) + e^{-2A}d\rho^2$. The second commonality is (2.32a), which fixes the warp factor of $\mathrm{S}^2$ uniquely as

$$e^{C} = -\frac{e^{A}}{2}\sin\alpha. \tag{2.40}$$

Together these conditions allow us to locally refine the metric ansatz of (2.2) as

$$ds^2 = e^{2A}\left[ds^2(\mathrm{AdS}_3) + \frac{1}{4}\sin^2\alpha ds^2(\mathrm{S}^2)\right] + ds^2(\mathrm{M}_4) + e^{-2A}d\rho^2, \tag{2.41}$$

where $\mathrm{M}_4$ supports an SU(2)-structure.[9]

Let us now summarise the main results of this section: in section 2.1 we derived general $\mathcal{N} = (4, 0)$ spinors on $\mathrm{AdS}_3 \times \mathrm{S}^2\times\ \mathrm{M}_5$ that are manifestly charged under an SU(2) R-symmetry, and compatible with type II supergravity. Any solution consistent with this spinor realises small $\mathcal{N} = (4, 0)$ superconformal symmetry. We then established that when one imposes that the physical fields of a solution (metric, dilaton and fluxes) respect $\mathrm{SU}(2)_R$, it is sufficient to solve for an $\mathcal{N} = 1$ sub-sector of this spinor to know that $\mathcal{N} = (4, 0)$ is realised, as the remaining supercharges are implied by the action of the R-symmetry. In section 2.2 we zoomed in on solutions for which $\mathrm{M}_5$ supports an SU(2)-structure in massive IIA. We exploited an existing $\mathcal{N} = 1$ $\mathrm{AdS}_3$ classification of [38] to derive necessary and sufficient conditions on the geometry and fluxes of an $\mathrm{AdS}_3$ solution preserving small $\mathcal{N} = (4, 0)$. Finally we established that there are two classes of such solutions, those for which $\sin\beta = 0$ and $\sin\beta \neq 0$.

In the next section we study the first class of solutions, where $\mathrm{M}_4$ is a conformal Calabi-Yau manifold.

## 3 Class I: conformal Calabi-Yau 2-fold case

In this section we study the first class of solutions that follows from the necessary conditions in section 2.2 with $\sin\beta = 0$. We find that they are warped products of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2\times\mathbb{R}$ with all possible massive IIA fluxes turned on.

In section 3.1 we present a summary of class I and interpret the types of solutions that lie within it. In section 3.2, we spell out precisely how class I is derived from the necessary conditions of section 2.2. Then, in section 3.3 we exploit T-duality to obtain a class of solutions in IIB with D5 branes back reacted on $\mathrm{AdS}_3 \times \mathrm{S}^3\times\mathrm{CY}_2$, with $\mathrm{S}^3$ foliated over $\mathrm{CY}_2$, that generalises the D1–D5 near horizon. We also show how to realise the sub-class with no fibration as a near horizon limit. Finally, in section 3.4 we focus on explicit local solutions in massive IIA that are foliations of $\mathrm{AdS}_3 \times \mathrm{S}^2\times\mathrm{CY}_2$ over an interval.

[9] Strictly speaking (2.41) holds in a region of space away from NS sources that do not wrap $\mathrm{S}^2$. Including such objects is in principle still possible, but they must lie at the intersection of two coordinate patches with local metrics of the form (2.41).

### 3.1 Summary of class I

The solutions of class I have the following NS sector

$$ds^2 = \frac{u}{\sqrt{h_4 h_8}}\left(ds^2(\text{AdS}_3) + \frac{h_8 h_4}{4h_8 h_4 + (u')^2}ds^2(\text{S}^2)\right) + \sqrt{\frac{h_4}{h_8}}ds^2(\text{CY}_2) + \frac{\sqrt{h_4 h_8}}{u}d\rho^2, \quad (3.1)$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2h_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8 h_4 + (u')^2}, \qquad H = \frac{1}{2}d\left(-\rho + \frac{uu'}{4h_4 h_8 + (u')^2}\right)\wedge \text{vol}(\text{S}^2) + \frac{1}{h_8}d\rho \wedge H_2.$$

Here $\Phi$ is the dilaton, $H$ the NS 3-form and $ds^2$ is in string frame. The warping $h_4$ has support on $(\rho, \text{CY}_2)$ while $u$ and $h_8$ have support on $\rho$, with $u' = \partial_\rho u$. As shall become clear below, the reason for the notation $h_4, h_8$ is that these functions may be identified with the warp factors of D4 and D8 branes when $u = 1$, the interpretation for generic $u$ is more subtle.

The 10 dimensional RR fluxes are

$$F_0 = h_8', \tag{3.2a}$$

$$F_2 = -H_2 - \frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4h_8 h_4 + (u')^2}\right)\text{vol}(\text{S}^2), \tag{3.2b}$$

$$\begin{aligned} F_4 = &\left(d\left(\frac{uu'}{2h_4}\right) + 2h_8 d\rho\right)\wedge \text{vol}(\text{AdS}_3) \\ &- \frac{h_8}{u}(\hat{\star}_4 d_4 h_4)\wedge d\rho - \partial_\rho h_4 \text{vol}(\text{CY}_2) - \frac{uu'}{2(4h_8 h_4 + (u')^2)}H_2 \wedge \text{vol}(\text{S}^2), \end{aligned} \tag{3.2c}$$

with the higher fluxes related to these as $F_6 = -\star_{10} F_4,\ F_8 = \star_{10} F_2,\ F_{10} = -\star_{10} F_0$.

Supersymmetry holds whenever

$$u'' = 0, \qquad H_2 + \hat{\star}_4 H_2 = 0, \tag{3.3}$$

which makes $u$ a linear function (i.e. an order 1 polynomial), and where $\hat{\star}_4$ is the Hodge dual on $\text{CY}_2$. In a canonical frame on $\text{CY}_2$ the associated closed-forms $\hat{J}, \hat{\Omega}$ read,

$$\hat{J} = \hat{e}^1 \wedge \hat{e}^2 + \hat{e}^3 \wedge \hat{e}^4, \qquad \hat{\Omega} = (\hat{e}^1 + i\hat{e}^2)\wedge(\hat{e}^3 + i\hat{e}^4), \tag{3.4}$$

and then $H_2$ may be express in terms of 3 arbitary functions $g_{1,2,3}$ on $\text{CY}_2$ as

$$H_2 = g_1(\hat{e}^1 \wedge \hat{e}^2 - \hat{e}^3 \wedge \hat{e}^4) + g_2(\hat{e}^1 \wedge \hat{e}^3 + \hat{e}^2 \wedge \hat{e}^4) + g_3(\hat{e}^1 \wedge \hat{e}^4 - \hat{e}^2 \wedge \hat{e}^3). \tag{3.5}$$

The Bianchi identities of the fluxes then impose

$$\begin{aligned} &h_8'' = 0, \qquad dH_2 = 0 \\ &\frac{h_8}{u}\nabla^2_{\text{CY}_2} h_4 + \partial^2_\rho h_4 + \frac{2}{h_8^3}(g_1^2 + g_2^2 + g_3^2) = 0, \end{aligned} \tag{3.6}$$

away from localised sources.

To better understand this class of solutions it is instructive to consider the case with $u=1$ and $g_1=g_2=g_3=0$, so that $H_2=0$. The metric and PDEs of (3.6) then reduce to those of a D4 brane wrapped on $\text{AdS}_3\times\text{S}^2$ and backreacted on $\text{CY}_2$, that is inside the world volume of a D8 wrapped on $\text{AdS}_3\times\text{S}^2\times\text{CY}_2$. One can compare this to the localised flat space D4–D8 system of [55] and see that indeed, the warp factors and PDEs match when $\text{CY}_2=\mathbb{R}^4$. Of course here there are additional fluxes turned on, but this should be no surprise as what was $\mathbb{R}_{1,4}$ in [55] has become $\text{AdS}_3\times\text{S}^2$. Thus one should expect additional fluxes to accommodate the fact that this space is no longer flat. More generally, turning on $H_2$ and $u\neq 1$ is essentially a deformation of this system.

In section 3.3 we establish that when one imposes that $\partial_\rho$ is an isometry, class I reduces to the T-dual of D5 branes back reacted on $\text{AdS}_3\times\text{S}^3\times\text{CY}_2$ — with $\text{S}^3$ foliated over $\text{CY}_2$. It is worth stressing that class I actually also contains the non-Abelian T-dual of this system as well. To extract this, one can fix

$$u=L\lambda\rho,\qquad h_8=c\rho,\qquad h_4=\frac{\lambda^4}{c}\rho h_5 \tag{3.7}$$

where $h_5$ depends on $\text{CY}_2$ only and should be interpreted as a D5 brane warp factor before the duality. For $h_5=1, H_2=0$ this reproduces the non-Abelian T-dual of the D1–D5 near horizon solution [56], which is of course non compact. Class I then provides a general class in which this non compact solution may be embedded. This allows to find a compact completion of this solution in the vein of [57–60], as shown in [68].

In the next section we will show how class I is obtained from the necessary supersymmetry conditions derived in section 2.1.

### 3.2 Derivation of class I

To derive class I we begin by fixing $\sin\beta=0$. We can in fact fix $\beta=0$ without loss of generality. We begin by refining (2.32a)–(2.32h) by expanding the exterior derivative in terms of the local coordinate $\rho$ introduced in (2.39), as

$$d=d_4+d\rho\wedge\partial_\rho. \tag{3.8}$$

This reduces (2.32e)–(2.32f) to

$$d_4(e^{3A-\Phi}\sin\alpha J)=d_4(e^{3A-\Phi}\sin\alpha\Omega)=0, \tag{3.9a}$$
$$\partial_\rho(e^{3A-\Phi}\sin\alpha J)-2e^{A-\Phi}\cos\alpha J=\partial_\rho(e^{3A-\Phi}\sin\alpha\Omega)-2e^{A-\Phi}\cos\alpha\Omega=0. \tag{3.9b}$$

Using both equations we establish that $d_4(e^{-2A}\cot\alpha)\wedge J=d_4(e^{-2A}\cot\alpha)\wedge\Omega=0$, from which it follows that $d_4(e^{-2A}\cot\alpha)=0$. We can solve this and (2.32d) as

$$e^{A-\Phi}\sin\alpha=h_8(\rho),\qquad e^{-2A}\cot\alpha=\frac{1}{2}\partial_\rho\log u(\rho), \tag{3.10}$$

for $h_8, u$ arbitrary functions. We can then define

$$\hat{J}=e^{-3A+\Phi}\frac{u}{\sin\alpha_2}J,\qquad \hat{\Omega}=e^{-3A+\Phi}\frac{u}{\sin\alpha_2}\Omega, \tag{3.11}$$

which are such that

$$d\hat{J} = d\hat{\Omega} = 0, \tag{3.12}$$

so that $\mathrm{M}_4$ is conformally Calabi-Yau. In turn, the conditions (2.32f)–(2.32h) constrain $H_3$ as

$$H_3 = \frac{e^A}{h_8} V \wedge H_2, \qquad J\wedge H_2 = \Omega\wedge H_2 = 0, \tag{3.13}$$

with $H_2$ otherwise free and the factor of $\frac{e^A}{h_8}$ is chosen for later convenience. A consequence of these conditions is the useful identity $\star_5 H_3 = -\frac{e^A}{h_8}H_2$ which holds because the $J\wedge H_2 = \Omega\wedge H_2 = 0$ implies that $H_2$ is anti self dual, and vice versa.

We now turn our attention to the RR fluxes. Using what has been derived thus far, and the fact that

$$\star_5\, 1 = \frac{1}{2}V\wedge J\wedge J, \qquad \star_5\, V = \frac{1}{2}J\wedge J, \qquad \star_5\, J = V\wedge J, \tag{3.14}$$

it is possible to take the Hodge dual of (2.34a)–(2.34d) and arrive at

$$f_0 = h_8' + \frac{e^{4A}u'u''}{4u^2}, \tag{3.15a}$$

$$f_2 = -H_2 - \frac{1}{2}\left(h_8 - \frac{e^{4A}u(u')^2\left(\frac{h_8}{u'}\right)'}{4u^2+e^{4A}(u')^2}\right)\mathrm{vol}(\mathrm{S}^2), \tag{3.15b}$$

$$f_4 = -\frac{e^{3A}h_8^2}{u^2}\star_5 d\left(\frac{u^2}{e^{4A}h_8}\right) - \frac{1}{2}e^{4A}\frac{uu'}{4u^2+e^{4A}(u')^2}H_2\wedge\mathrm{vol}(\mathrm{S}^2) - \frac{e^{4A}h_8u'u''}{8u^2}J\wedge J, \tag{3.15c}$$

$$f_6 = \frac{1}{2}\left[-\frac{e^{7A}h_8^2u'}{u(4u^2+e^{4A}(u')^2)}\star_5 d\left(\frac{u^2}{e^{4A}h_8}\right) + \frac{1}{2}\left(h_8 + \frac{e^{4A}h_8uu''}{u(4u^2+e^{4A}(u')^2)}\right)J\wedge J\right]\wedge\mathrm{vol}(\mathrm{S}^2). \tag{3.15d}$$

Using these definitions we can now solve (2.35), which imposes simply

$$u'' = 0. \tag{3.16}$$

At this point the supersymmetry conditions are completely solved. What remains is to solve the Bianchi identities of the fluxes. Away from localised sources these impose[10]

$$h_8'' = 0, \qquad dH_2 = 0 \tag{3.18}$$

$$\frac{h_8}{u}\nabla^2_{\mathrm{CY}_2}h_4 + \partial_\rho^2 h_4 - \frac{1}{h_8^3}\hat{\star}_4(H_2\wedge H_2) = 0.$$

[10]In deriving the last of these we make use of the identity

$$\frac{h_8^{\frac{5}{4}}}{h_4^{\frac{3}{4}}\sqrt{u}}\star_5 dh_4 = \frac{h_8}{u}(\hat{\star}_4 d_4 h_4)\wedge d\rho + \partial_\rho h_4 \mathrm{vol}(\mathrm{CY}_2), \tag{3.17}$$

and that $\nabla^2_{\mathrm{CY}_2}h_4 = \hat{\star}_4 d_4\hat{\star}_4 d_4 h_4$.

Therefore, any solution to (3.3) and (3.6) gives a solution in IIA away from localised sources. When these are included, (3.6) will have additional $\delta$-function source terms on the l.h.s. and these sources should also be calibrated. We shall return to this in section 5.

In the next section we will derive a class of solutions with D5-branes backreacted on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{CY}_2$, with $\mathrm{S}^3$ fibered over $\mathrm{CY}_2$.

### 3.3 D5 branes wrapped on $\mathbf{AdS_3 \times S^3}$ and backreacted on $\mathbf{CY_2}$

In this section we derive a class of solutions in IIB with D5 branes and formal KK monopoles that generalises the D1–D5 near horizon. We begin with the class of solutions in section 3.1 and impose that $\partial_\rho$ is an isometry. We can achieve this without loss of generality by fixing

$$u = L\lambda, \qquad h_8 = c, \qquad h_4 = \frac{\lambda^4}{c} h_5, \tag{3.19}$$

where $h_5$ depends only on the coordinates on $\mathrm{CY}_2$ and $(L, \lambda, c)$ are arbitrary constants chosen in this specific combination for convenience. The class then reduces to

$$\begin{aligned} ds^2 &= \frac{L^2}{\sqrt{h_5}}\bigg(ds^2(\mathrm{AdS}_3) + \frac{1}{4} ds^2(\mathrm{S}^2)\bigg) + \lambda^2 \sqrt{h_5} ds^2(\mathrm{CY}_2) + \frac{\sqrt{h_5}}{L^2} d\rho^2, \quad e^{\Phi} = L h_4^{-\frac{1}{4}}, \\ B &= \bigg(\frac{1}{2}\eta + \frac{1}{c}\mathcal{A}\bigg)\wedge d\rho, \quad F_2 = -H_2 - \frac{c}{2}\mathrm{vol}(\mathrm{S}^2), \quad F_4 = 2c\,\mathrm{vol}(\mathrm{AdS}_3)\wedge d\rho - \frac{c\lambda}{L^2}\hat{\star}_4 dh_5 \wedge d\rho, \end{aligned} \tag{3.20}$$

where we have introduced 1-form potentials $\eta$ and $\mathcal{A}$ such that

$$d\eta = -\mathrm{vol}(\mathrm{S}^2), \qquad d\mathcal{A} = H_2, \tag{3.21}$$

to write the 2-form NS potential $B$ in a form respecting the isometry $\partial_\rho$.

We now T-dualise on $\rho$ (see for instance [61]), which we take to have period $2\pi$, and arrive at the dual IIB solution

$$\begin{aligned} ds^2 &= \frac{L^2}{\sqrt{h_5}}\bigg[ds^2(\mathrm{AdS}_3) + \frac{1}{4}\bigg(D\psi^2 + ds^2(\mathrm{S}^2)\bigg)\bigg] + \lambda^2\sqrt{h_5} ds^2(\mathrm{CY}_2), \quad e^{\Phi} = L^2 h_5^{-\frac{1}{2}}, \\ F_3 &= 2c\bigg[\mathrm{vol}(\mathrm{AdS}_3) + \frac{1}{8} D\psi \wedge \mathrm{vol}(\mathrm{S}^2)\bigg] + \frac{1}{2} D\psi \wedge d\mathcal{A} - \frac{c\lambda}{L^2}\hat{\star}_4 dh_5, \end{aligned} \tag{3.22}$$

where $\hat{\star}_4$ is the Hodge dual on $\mathrm{CY}_2$, the NS 3-form and the remaining RR forms are now all trivial and we define

$$D\psi = d\psi + \eta + \frac{2}{c}\mathcal{A}, \tag{3.23}$$

for $\partial_\psi$ now the isometry direction. Supersymmetry requires that

$$d\mathcal{A} + \hat{\star}_4 d\mathcal{A} = 0, \tag{3.24}$$

where $\hat{J}, \hat{\Omega}$ are the 2 and 3 forms on $\mathrm{CY}_2$, and the Bianchi identity of the RR 3-form imposes

$$\nabla^2_{\mathrm{CY}_2} h_5 - \frac{L^2}{c^2\lambda^2}\hat{\star}_4(d\mathcal{A}\wedge d\mathcal{A}) = 0, \tag{3.25}$$

away from localised sources. When $\mathcal{A}=0$ and $\psi$ has period $4\pi$ this gives a class of solutions with D5 branes wrapped on $\mathrm{AdS}_3\times\mathrm{S}^3$ and backreacted in an arbitrary $\mathrm{CY}_2$. Note that one is also free to replace this $\mathrm{S}^3$ by a Lens space, by changing the period of $\psi$, as $\partial_\psi$ is an uncharged isometry generically. The effect of turning on $\mathcal{A}$ is then to formally place a Kaluza-Klein monopole into this set-up. The assumption of equal spinor norm made in section 2.1, means that this solution is in fact not the most general one of this type. However the latter can be reached by performing an $\mathrm{SL}(2,\mathbb{R})$ transformation[11] of (3.22), which will generically turn on $F_1$ and $H_3$ fluxes. In this regard (3.22) is a specific duality frame of this more general solution.

Clearly (3.22) is closely related to the near horizon limit of coincident D5 and D1 branes that respectively wrap or are smeared on $\mathrm{CY}_2$ [4, 5]. In particular if $\mathcal{A}=0$, $h_5=$ constant and $\psi\sim\psi+4\pi$, so that there is a round $\mathrm{S}^3$, we recover this class and supersymmetry is enhanced to $\mathcal{N}=(4,4)$. Replacing $\mathrm{S}^3$ with a Lens space yields the D1–D5-KK near horizon, which has also been systematically studied [12–15] and preserves only $\mathcal{N}=(4,0)$. Of course when they are non trivial, both $\mathcal{A}$ and the $\hat{\star}_4 dh_5$ term in $F_3$ break supersymmetry to $\mathcal{N}=(4,0)$ irrespective of the period of $\psi$.

Of course a viable CFT dual demands a compact internal space which restricts $\mathrm{CY}_2$ to be either $\mathrm{T}^4$ or K3 for the near horizon of D1–D5s and D1–D5-KK. This is no longer the case for generic $h_5$, as the warp factor can cause a non compact space to be restricted to a finite subregion when embedded in 10d. A simple example in this class exhibiting such behaviour was already given in [39], where a compact solution with D5s and an O5 plane backreacted on $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathbb{R}^4$ was found.

Interestingly, it turns out that the $\mathcal{A}=0$ limit of (3.22) can also be realised as a near horizon limit of intersecting branes. One begins by compactifying $\mathbb{R}^{1,9}\to\mathbb{R}^{1,5}\times\mathrm{CY}_2$, which preserves $\frac{1}{2}$ of maximal supersymmetry. For the standard D1–D5 system giving rise to $\mathrm{AdS}_3$ in the near horizon, D5 branes would then be wrapped on $\mathbb{R}^{1,1}\times\mathrm{CY}_2$ and D1 branes placed on $\mathbb{R}^{1,1}$ (smeared across $\mathrm{CY}_2$). This breaks supersymmetry to $\frac{1}{4}$ of maximal — enhanced to $\frac{1}{2}$ at the horizon. However, one can also place additional D5s on $\mathbb{R}^{1,1}$ and the common co-dimensions of the other branes at the cost of breaking supersymmetry to $\frac{1}{8}$ of maximal. This leads us to a metric of the form

$$ds^2=\frac{1}{\sqrt{H_1H_5h_5}}ds^2(\mathbb{R}^{1,1})+\sqrt{\frac{H_5H_1}{h_5}}\big(dr^2+r^2ds^2(\mathrm{S}^3)\big)+\sqrt{\frac{h_5H_1}{H_5}}ds^2(\mathrm{CY}_2) \tag{3.26}$$

where the warp factors are, respectively,

$$H_1=1+\frac{Q_1}{r^2},\qquad H_5=1+\frac{Q_5}{r^2},\qquad h_5:\quad \nabla^2_{\mathrm{CY}_2}h_5=0, \tag{3.27}$$

away from the $h_5$ sources. One can then take the near horizon limit of D1s and the D5s

[11] Equal spinor norm is in 1-to-1 correspondence with having no electric component of the NS 3 form. Such a term, when present, can always be turned off (i.e. mapped to the RR 3-form) with an $\mathrm{SL}(2,\mathbb{R})$ transformation, which maps to the case of equal spinor norm we are studying.



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corresponding to $H_5$ by expanding about $r=0$. The metric becomes

$$ds^2=\frac{1}{\sqrt{h_5}}\bigg[\frac{r^2}{\sqrt{Q_1Q_5}}ds^2(\mathbb{R}^{1,1})+\sqrt{Q_1Q_5}\frac{dr^2}{r^2}+\sqrt{Q_1Q_5}ds^2(\mathrm{S}^3)\bigg]+\sqrt{\frac{Q_1}{Q_5}}\sqrt{h_5}ds^2(\mathrm{CY}_2), \tag{3.28}$$

at leading order, which is a solution by itself with supersymmetry enhanced to $\frac{1}{4}$ of maximal. Clearly there is an $\mathrm{AdS}_3$ factor of radius $(Q_1Q_5)^{\frac{1}{4}}$, and in fact the entire metric can be easily mapped to that of (3.22) (with unit radius $\mathrm{AdS}_3$) for $\mathcal{A}=0$ by redefining $Q_1, Q_5$ and rescaling $\mathbb{R}_{1,1}, h_5$. The same is true of the fluxes, the details of which we have suppressed. Despite how seemingly obvious this near horizon is, as far as the authors are aware, it is absent from the literature. Given that a near horizon limit is known, and that the class is relatively simple it would be fruitful to study it in the future.

In the next section we study a class of local solutions in massive IIA following from class I that are a foliation of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ over an interval.

### 3.4 Local solutions with $\mathbf{AdS_3\times S^2\times CY_2}$ foliated over an interval

In this section we study the sub-class of local solutions that follow from section 3.1 by imposing that the symmetries of $\mathrm{CY}_2$ are respected by the full solution. This means that the warp factors cannot depend on the directions on $\mathrm{CY}_2$ and we must fix $H_2=0$. As such, the only way to realise a compact internal space is if $\mathrm{CY}_2$ is itself compact. This restricts our considerations to

$$\mathrm{CY}_2=\mathrm{T}^4\quad\text{ or }\quad\mathrm{CY}_2=\mathrm{K3}. \tag{3.29}$$

The supersymmetry condition (3.3) and Bianchi identities (3.6) are then all completely solved for $h_8, u, h_4$ arbitrary linear functions in $\rho$. We parametrise these in general by introducing five arbitrary constants $(c_1,\dots,c_5)$ such that

$$h_8=c_1+F_0\rho,\qquad u=c_2+c_3\rho,\qquad h_4=c_4+c_5\rho, \tag{3.30}$$

at which point the local form of a general solution in this class may be written explicitly.[12] The NS sector of the general solution is

$$\begin{aligned}
ds^2&=\frac{(c_2{+}c_3\rho)}{\sqrt{c_1{+}F_0\rho}\sqrt{c_4{+}c_5\rho}}\left[ds^2(\mathrm{AdS}_3){+}\frac{1}{4{+}\frac{c_3^2}{(c_1+F_0\rho)(c_4+c_5\rho)}}ds^2(\mathrm{S}^2)\right]{+}\sqrt{\frac{c_4{+}c_5\rho}{c_1{+}F_0\rho}}ds^2(\mathrm{CY}_2)\\
&\quad+\frac{\sqrt{c_1{+}F_0\rho}\sqrt{c_4{+}c_5\rho}}{(c_2{+}c_3\rho)}d\rho^2,\qquad e^{-\Phi}=\frac{(c_1{+}F_0\rho)^{\frac{3}{4}}\sqrt{c_3^2{+}4(c_1{+}F_0\rho)(c_4{+}c_5\rho)}}{2\sqrt{c_2{+}c_3\rho}(c_4{+}c_5\rho)^{\frac{1}{4}}},\\
H&=dB_2,\qquad B_2=\frac{1}{2}\left(2n\pi{-}\rho{+}\frac{c_3(c_2{+}c_3\rho)}{c_3^2{+}4(c_4{+}c_5\rho)(c_1{+}F_0\rho)}\right)\wedge\mathrm{vol}(\mathrm{S}^2),
\end{aligned} \tag{3.31}$$

where we have added the closed form $n\pi\mathrm{vol}(\mathrm{S}^2)$ to $B_2$ that parametrises large gauge transformations — so $n$ is an integer. The 10 dimensional RR fluxes follow from substituting (3.30) and $H_2=0$ into (3.2a)–(3.2c). However, in what follows we will find it more

[12] At least when $\mathrm{CY}_2=\mathrm{T}^4$. The metrics on K3 manifolds are not know explicitly, but they are known to exist by Yau's theorem.

useful to know the magnetic parts of the Page fluxes explicitly, i.e. $\hat{f} = f \wedge e^{-B_2}$, where $f$ encloses the magnetic components of the 10 dimensional RR fluxes. We find

$$\hat{f}_0 = F_0, \qquad \hat{f}_2 = -\frac{1}{2}(c_1 + 2n\pi F_0)\text{vol}(\text{S}^2), \tag{3.32a}$$

$$\hat{f}_4 = -c_5\text{vol}(\text{CY}_2), \quad \hat{f}_6 = \frac{1}{2}(c_4 + 2n\pi c_5)\text{vol}(\text{CY}_2) \wedge \text{vol}(\text{S}^2). \tag{3.32b}$$

Flux quantisation for Dp brane like objects requires that the Page charges, $N_p = \frac{1}{(2\pi)^{7-p}} \int_{\Sigma_{8-p}} \cdot \hat{f}_{8-p}$, are integer. This requires that one tunes

$$\begin{aligned} 2\pi F_0 &= N_8, \qquad & -\frac{c_5}{(2\pi)^3}\int_{\text{CY}_2}\text{vol}(\text{CY}_2) &= N_4, \\ -c_1 &= n_6, \qquad & \frac{c_4}{(2\pi)^4}\int_{\text{CY}_2}\text{vol}(\text{CY}_2) &= n_2, \end{aligned} \tag{3.33}$$

where $n_i \in \mathbb{Z}$, so that we have the integer Page charges $N_8, N_4$ and

$$N_6 = n_6 - nN_8, \qquad N_2 = n_2 - nN_4. \tag{3.34}$$

Of course, as they are defined in terms of arbitrary constants, not all these integers need to be non-zero in a given solution. The holographic central charge of a generic solution in this class at leading order is then given by

$$c_{\text{hol}} = \frac{3}{2^4\pi^6}\int_{\text{M}_7} e^{A-2\Phi}\text{vol}(\text{M}_7) = \frac{3}{4\pi^3}\int d\rho(2\pi n_2 - N_4\rho)(N_8\rho - 2\pi n_6), \tag{3.35}$$

where we have converted the formula of [28] to string frame. However one needs to know the range of $\rho$ to perform the final integration — which depends on how $c_i$ are tuned. For similar reasons the charge associated to $H$, which is defined on $(\rho, \text{S}^2)$ needs to be computed on a case by case basis.

A brief glance at (3.31) makes it clear that the generic solution in this section is not regular, though regularity can be achieved by tuning $c_i$. A regular solution requires the $\text{AdS}_3$ warp factor to be either constant, or constant at the boundaries of the interval spanned by $\rho$. Only the former leads to a compact space in this case, and requires tuning $h_8 \propto u \propto h_4$. We thus set

$$c_2 = L^2\lambda^2 c_1, \quad c_3 = L^2\lambda^2 F_0, \quad c_4 = \lambda^4 c_1, \quad c_5 = \lambda^4 F_0, \tag{3.36}$$

without loss of generality. The metric then reduces to

$$ds^2 = L^2 ds^2(\text{AdS}_3) + \lambda^2 ds^2(\text{CY}_2) + \frac{1}{L^2}d\rho^2 + \frac{L^2(c_1 + F_0\rho)^2}{L^4F_0^2 + 4(c_1 + F_0\rho)^2}ds^2(\text{S}^2). \tag{3.37}$$

This solution is regular: when $F_0 = 0$ the warp factors are constant so this point is trivial, $\partial_\rho$ becomes an isometry and the metric is compact if we make it parametrise a circle. For generic $F_0$, $\rho$ is bounded from below at $\rho = -\frac{c_1}{F_0}$, where the sub-manifold spanned by $(\rho, \text{S}^2)$ vanishes as $\mathbb{R}^3$ in polar coordinates. However $\rho$ is not bounded from above, with $\rho = \infty$ at infinite proper distance, so the metric is non compact. In fact when $F_0 = 0$ (3.37) is the metric of the T-dual of the D1–D5 near horizon geometry, and taking $0 < \rho < 2\pi$ in the formula for the central charge (3.35) yields $c_{\rm hol} = 6N_2N_6$, as one expects for this class. In turn, for $F_0 \neq 0$ it is the non-Abelian T-dual of this system.[13] These observations extend to the fluxes and dilaton as well. As such, the solution defined by (3.36) is somewhat novel, in that it gives a hybrid solution containing both the T and non-abelian T-duals of a known solution.

For choices of $c_i$ other than (3.36), the metric in (3.31) will necessarily contain singular loci, making the solution non regular. However non regularity is not always a reason to worry. Indeed, there are situations in which one can trust a singularity in a supergravity solution, namely when it signals the presence of a physical object in string theory and when the radius of divergent behaviour about this object, where the supergravity approximation does not hold, can be made arbitrarily small by tuning parameters. Given the form of (3.31) we anticipate D brane and O plane sources. Thus, we will allow $\rho$ to terminate at a singular point of the space when the solution reduces to the behaviour of these objects at this loci, i.e. if the leading order behaviour of the metric and dilaton are diffeomorphic to one of the following forms

$$\begin{aligned}
&\text{Dp brane}: && ds^2 \sim r^{\frac{7-p}{2}} ds^2(\mathrm{M}^{1,p}) + r^{\frac{-7+p}{2}}\Big(dr^2 + r^2 ds^2(\mathrm{B}^{8-p})\Big),\ e^{\Phi} \sim r^{\frac{(3-p)(-7+p)}{4}},\\
&\begin{matrix}\text{Dp smeared}\\ \text{on } \tilde{B}^s\end{matrix}: && ds^2 \sim r^{\frac{7-p-s}{2}} ds^2(\mathrm{M}^{1,p}) + r^{\frac{-7+p+s}{2}}\Big(dr^2 + ds^2(\tilde{\mathrm{B}}^s) + r^2 ds^2(\mathrm{B}^{8-p-s})\Big),\ e^{\Phi} \sim r^{\frac{(3-p)(-7+p+s)}{4}},\\
&\text{Op plane}: && ds^2 \sim \frac{1}{\sqrt{r}} ds^2(\mathrm{M}^{1,p}) + \sqrt{r}\Big(dr^2 + r_0^2 ds^2(\mathrm{B}^{8-p})\Big),\ e^{\Phi} \sim r^{\frac{3-p}{4}}.
\end{aligned} \tag{3.38}$$

Here $\mathrm{M}^{1,p}$ is some manifold that the object wraps, $\mathrm{B}^{8-p}$ a compact base, on which one integrates to get the associated charge of this object, and $\tilde{\mathrm{B}}^s$ is the manifold over which a brane is smeared. We have included smeared D branes but not O planes, because the former is dynamical in string theory while the latter is not. We shall also allow for coincident objects such as a Dp-brane inside the world volume of a D(p+4)-brane.[14] If $\mathrm{M}^{1,p}$ were flat, the only magnetic flux near a Dp/Op singularity would be $f_{8-p}$ — but here $\mathrm{M}^{1,p}$ will not be flat, so one should expect additional fluxes to be turned on at the singularity to accommodate this.

[13] This is perhaps made more obvious if one defines $r = c_1 + F_0\rho$ and substitutes for $\rho$ in favour of $r$.

[14] Both must depend on the same radial variable for this behaviour to occur, hence the Dp-brane is smeared over the remaining world volume directions of the D(p+4)-brane.

By tuning the constants $c_i$ we are able to find a rich variety of physical boundary behaviours, namely

| Source | Minimal tuning | $\mathrm{M}^{1,p}$ | $\tilde{\mathrm{B}}^s$ | Loci |
|---|---|---|---|---|
| D8/O8 | $c_3 = 0$ | all but $\rho$ | — | $\rho = -\frac{c_1}{F_0}$ |
| D6 | $c_3 \neq 0 \quad c_4 = bc_2, \quad c_5 = bc_3$ | $\mathrm{AdS}_3 \times \mathrm{CY}_2$ | — | $\rho = -\frac{c_2}{c_3}$ |
| O6 | generic $c_i$ | $\mathrm{AdS}_3 \times \mathrm{CY}_2$ | — | $\rho = -\frac{c_1}{F_0}$ |
| Smeared D4 | $c_3 = 0$ | $\mathrm{AdS}_3 \times \mathrm{S}^2$ | $\mathrm{CY}_2$ | $\rho = -\frac{c_4}{c_5}$ |
| Smeared $\mathrm{D2}_1$ | $c_2 = bc_1, \quad c_3 = bF_0 \neq 0$ | $\mathrm{AdS}_3$ | $\mathrm{CY}_2$ | $\rho = -\frac{c_2}{c_3}$ |
| Smeared $\mathrm{D2}_2$ | generic $c_i$ | $\mathrm{AdS}_3$ | $\mathrm{S}^2 \times \mathrm{CY}_2$ | $\rho = -\frac{c_4}{c_5}$ |
| D4 in D8 | $c_3 = 0, \quad c_4 = bc_1, \quad c_5 = bF_0$ | D8 : all but $\rho$ | D4 : $\mathrm{CY}_2$ | $\rho = -\frac{c_1}{F_0}$ |
| D2 in D6 | generic $c_i$ | D6 : $\mathrm{AdS}_3 \times \mathrm{CY}_2$ | D2 : $\mathrm{CY}_2$ | $\rho = -\frac{c_2}{c_3}$ |
| O2 in O6 | $c_4 = bc_1, \quad c_5 = bF_0$ | O6 : $\mathrm{AdS}_3 \times \mathrm{CY}_2$ | O2 : $\mathrm{S}^2 \times \mathrm{CY}_2$ | $\rho = -\frac{c_4}{c_5}$ |
| T/NATD hybrid | (3.36) | — | — | — |

Here $b$ is an arbitrary constant, and we have included the T-dual and non-Abelian T-dual hybrid solution, which is regular, as well as smeared O2 inside an O6 plane, for completeness. Note that when the D4, D2 branes are delocalised on all directions but $\rho$, one could also interpret them as smeared O4 and O2 planes respectively. To realise a compact solution from (3.31) beyond the $F_0 = 0$ limit of (3.37), we need two of these boundary behaviours to exist for the same tuning of $c_i$. There are in fact several such solutions for various tunings of $c_i$. In summary, we see that the following behaviours can exist simultaneously

| Tuning | Boundary behaviours | Loci: $\rho =$ |
|---|---|---|
| generic $c_i$ | O6 \| D2 in D6 \| $\mathrm{D2}_2$ | $-\frac{c_1}{F_0} \mid -\frac{c_2}{c_3} \mid -\frac{c_4}{c_5}$ |
| $(c_4 = bc_1, c_5 = bF_0, c_3 \neq 0)$ | O2 in D6 \| D2 in O6 | $-\frac{c_2}{c_3} \mid -\frac{c_1}{F_0}$ |
| $(c_2 = bc_4, c_3 = bc_5, c_3 \neq 0)$ | D6 \| O6 | $-\frac{c_2}{c_3} \mid -\frac{c_1}{F_0}$ |
| $(c_2 = bc_1, c_3 = bF_0, c_3 \neq 0)$ | $\mathrm{D2}_1$ \| $\mathrm{D2}_2$ | $-\frac{c_1}{F_0} \mid -\frac{c_4}{c_5}$ |
| $c_3 = 0$ | D8/O8 \| D4 | $-\frac{c_1}{F_0} \mid -\frac{c_4}{c_5},$ |

(3.39)

making for a total of 7 independent compact solutions of this type.[15] In the interest of (relative) brevity we are going to look only at the two simplest cases explicitly, those with interval bounded between D8s/O8 and D4s, and those between D6 and an O6.

**Interval bounded between D8/O8s and D4s.** To realise the first compact example with sources, one should tune $c_3 = 0$. Then one of $(c_1, c_4)$ can also be set to zero with a

[15] Really 14, as we can take $\mathrm{CY}_2 = \mathrm{T}^4$ or K3 for each.

coordinate transformation of $\rho$. Here we take $c_4 = 0$. The resulting NS sector is

$$ds^2 = \frac{c_2}{\sqrt{c_5}\sqrt{\rho}\sqrt{c_1+F_0\rho}}\left[ds^2(\text{AdS}_3)+\frac{1}{4}ds^2(\text{S}^2)\right]+\frac{\sqrt{c_5}\sqrt{\rho}}{\sqrt{c_1+F_0\rho}}ds^2(\text{CY}_2)+\frac{\sqrt{c_5}\sqrt{\rho}\sqrt{c_1+F_0\rho}}{c_2}d\rho^2,$$
$$e^{-\Phi} = \frac{c_5^{\frac{1}{4}}\rho^{\frac{1}{4}}(c_1+F_0\rho)^{\frac{5}{4}}}{\sqrt{c_2}}, \qquad B_2 = \left(n\pi-\frac{\rho}{2}\right)\wedge\text{vol}(\text{S}^2), \tag{3.40}$$

and the magnetic RR Page fluxes are given by (3.32a)–(3.32b) with $c_3 = c_4 = 0$. It should not be hard to see that close to $\rho = -\frac{c_1}{F_0} = \frac{2\pi n_6}{N_8}$ the metric and dilaton are consistent with D8 and O8 behaviour, while at $\rho = 0$ they signal D4 branes wrapped on $\text{AdS}_3 \times \text{S}^3$ and smeared on $\text{CY}_2$. This bounds the interval as $0 < \rho < \frac{2\pi n_6}{N_8}$, assuming $\frac{2\pi n_6}{N_8} > 0$. This solution is also under parametric control, with the supergravity approximation holding for $c_5 \sim N_4 \gg 1$, with the radius of divergent behaviour about the poles scaling inversely with this parameter. Given the flux quantisation conditions (3.33), the Page charges are then

$$N_8, \qquad N_4, \qquad N_6 = n_6 - nN_8, \qquad N_2 = -nN_4, \qquad N_5 = \frac{n_6}{N_8}, \tag{3.41}$$

where $N_5 = \frac{1}{(2\pi)^2}\int_{(\rho,\text{S}^2)} dB_2$ is the charge associated to NS5 branes. We now turn our attention to the large gauge transformations parameterised by $n$ in the definition of the NS 2-form. As $(\rho, \text{S}^2)$ defines a cycle at each of the singular loci, we should impose that $b = -\frac{1}{(2\pi)^2}\int_{\text{S}^2} B_2$ should be an integer at these points. This may be achieved by constraining

$$b = \frac{\rho}{2\pi} - n \quad \text{s.t.} \quad 0 \leq b < 1, \tag{3.42}$$

which implies that $\rho$ is partitioned into segments of length $2\pi$. At $\rho = 0$, $b = n$ so we can take $n = 0$ fixing $b = 0$. One should then perform a large gauge transformation sending $n \to n+1$ each time $\rho$ increases by $2\pi$. At $\rho = \frac{2\pi n_6}{N_8}$ one has $b = N_5 + m$ for $m$ the number of gauge transformations required to traverse the interval. Finally we can integrate the expression for the holographic central charge at leading order (3.35), which yields

$$c_{\text{hol}} = n_6 N_4 N_5^2. \tag{3.43}$$

At first sight this may appear confusing as the (left) central charge of small $\mathcal{N} = (4,0)$ CFTs should be related to the level of the affine SU(2) algebra as $c = 6k$. Here (3.43) contains no factor of 6, but one should recall that $c_{\text{hol}}$ is the central charge in the supergravity limit $N_5 \gg 1$, which only gives the leading order contribution to $c$, neglecting sub-leading terms in $N_5$. We believe that one will recover $c = 6k$ if one includes the 1-loop correction to $c_{\text{hol}}$ — however to our knowledge this corrections is not yet known for massive IIA, so one cannot yet check this explicitly. Adding support to this claim is [62], where in section 4 the central charges of several concrete CFTs and geometries that are locally of the form (3.32) are compared. The CFTs obey $c = 6k$, but to leading order in some parameter(s), where it makes sense to compare to supergravity, the factor of 6 is lost in many cases — non the less $c = c_{\text{hol}}$ in these limits.

**Interval bounded between D6s and an O6.** One can get the second compact solution by tuning $c_2 = bc_4, c_3 = bc_5$. One can then set $c_4 = 0$ without loss of generality with a diffeomorphism. This results in the following NS sector

$$
\begin{aligned}
ds^2 &= \frac{b\sqrt{c_5}\sqrt{\rho}}{\sqrt{c_1+F_0\rho}}\left[ds^2(\text{AdS}_3)+\frac{\rho(c_1+F_0\rho)}{b^2c_5+4\rho(c_1+F_0\rho)}ds^2(\text{S}^2)\right]+\frac{\sqrt{c_5}\sqrt{\rho}}{\sqrt{c_1+F_0\rho}}ds^2(\text{CY}_2)+\frac{\sqrt{c_1+F_0\rho}}{b\sqrt{c_5}\sqrt{\rho}}d\rho^2,\\
e^{-\Phi} &= \frac{\sqrt{c_1+F_0\rho}\sqrt{b^2c_5+4\rho(c_1+F_0\rho)}}{2\sqrt{b}c_5^{\frac{1}{4}}\rho^{\frac{3}{4}}},\ B = \left(n\pi-\frac{2\rho^2(c_1+F_0\rho)}{b^2c_5+4\rho(c_1+F_0\rho)}\right)\wedge\text{vol}(\text{S}^2),
\end{aligned}
\tag{3.44}
$$

where again $n$ is an integer parametrising large gauge transformations and the magnetic RR Page fluxes are given by (3.32a)–(3.32b) with $c_2 = c_4 = 0, c_3 = bc_5$. This time one can show that the behaviour close to $\rho = -\frac{c_1}{F_0} = \frac{2\pi n_6}{N_8}$ corresponds to an O6 plane wrapped on AdS$_3\times$ CY$_2$, while that of $\rho = 0$ is a D6 brane — bounding the interval as $\rho \in \left(0, \frac{2\pi n_6}{N_8}\right)$. The supergravity approximation is valid this time for $F_0 \gg c_1 \sim n_6 \gg 0$. The Page charges are

$$
N_8, \quad N_4, \quad N_6 = n_6 - nN_8, \quad N_2 = -nN_4, \quad N_5 = 0, \tag{3.45}
$$

with the major difference with respect to the previous example being that the NS charge $N_5 = 0$. We should again constrain

$$
b = -\frac{1}{(2\pi)^2}\int_{\text{S}^2} B_2 \quad \text{s.t.} \quad 0 < b < 1. \tag{3.46}
$$

The form of $B_2$ means that the $\rho$ dependence vanishes at the boundaries and reaches an extrema in between at $\rho = \rho_0$, which depends non trivially on the charges. As at $\rho = 0$ one sets $n = 0$, then as one traverses the interval $0 < \rho < \rho_0$ successive large gauge transformations are needed to keep $0 < b < 1$. But once one crosses $\rho = \rho_0$ one needs to start undoing the gauge transformations to keep $b$ bounded until $n = 0$, once more at $\rho = \frac{2\pi n_6}{N_8}$. From this we conclude that the charge of both the D6s and the O6 is equal to $n_6$, which gives a problem. Weak curvature requires $n_6 \gg 0$, but the charge of the O6 is fixed to be $\pm 4$. As such, the solution is strongly curved everywhere. Nonetheless, for the sake of comparison we compute the holographic central charge from (3.35), and find

$$
c_{\text{hol}} = \frac{n_6^3 N_4}{N_8^2}. \tag{3.47}
$$

**Summary of this section.** To summarise, in this section 3.4 we have studied the local solutions of class I that respect the symmetry of CY$_2$ — they are foliations of AdS$_3\times$ S$^2\times$ CY$_2$ over an interval. We have found that the general solution can be given explicitly and depends on parameters $(c_1, \ldots c_5, F_0)$, with all necessary conditions solved. Different behaviours can be achieved by tuning these parameters, and we have found an array of physical boundary behaviours for the interval. We have established that there are 8 independent compact solutions: the T-dual of the D1–D5 near horizon, which is regular, and the 7 independent combinations one can form from (3.39) with various physical singularities. We have chosen two of these solutions for a more detailed study, where the interval is bounded between either D8/O8s and D4 or D6s and an O6. We have shown that only the former has a good interpretation in supergravity, with the latter requiring higher curvature corrections.

Before moving on let us first stress that the general solution of this section is only a local one. What this really means is that every coordinate patch of a global solution can be expressed in the form of (3.31) and (3.32a)–(3.32b), but the specific values of $(F_0, c_1, \dots, c_5)$ in each of these patches may differ in principle. This fact was exploited in [54] to construct infinite classes of globally compact $\text{AdS}_7$ solutions in massive IIA, by glueing together various non compact solutions with defect branes. We shall return to this issue in section 5, where we will establish that this is also possible for the local solutions of this section. We delay a detailed analysis of such solutions until [62, 68].

In the next section we shall begin our analysis of the second class of solutions we consider in this paper, namely those containing a family of Kahler four-manifolds.

# 4 Class II: Kahler four-manifold case

In this section we study the second class of solutions following from the necessary conditions of section 2.2 for $\sin\beta \neq 0$. We find that the solutions decompose as a warped product of $\text{AdS}_3 \times \text{S}^2 \times \hat{\text{M}}_4 \times \mathbb{R}$ where $\hat{\text{M}}_4$ is a family of Kahler manifolds with metrics that depend on the interval.

In section 4.1 we summarise class II and discuss some of its general features, deferring its derivation to section 4.2. In section 4.3 we T-dualise class II along the interval and arrive at a generalisation of [28] with non trivial 3-form flux. And finally in section 4.4 we expand up section 3.4 and present further local solutions that are foliations of $\text{AdS}_3 \times \text{S}^2 {\times} \text{CY}_2$ over an interval.

## 4.1 Summary of class II

The solutions in class II have the following NS sector

$$\begin{aligned}
ds^2 &= \frac{u}{\sqrt{hw^2-v^2}}\left[ds^2(\text{AdS}_3)+\frac{hw^2-v^2}{4(hw^2-v^2)+(u')^2}ds^2(\text{S}^2)\right]+\frac{\sqrt{hw^2-v^2}}{u}\left[\frac{u}{hw}ds^2(\hat{\text{M}}_4)+d\rho^2\right],\\
H &= \frac{1}{2}d\left(-\rho+\frac{uu'}{4(hw^2-v^2)+(u')^2}\right)\wedge\text{vol}(\text{S}^2)+d\left(\frac{v}{wh}\hat{J}\right),\\
e^{-\Phi} &= \frac{wh^{\frac{1}{2}}\sqrt{4(hw^2-v^2)+(u')^2}}{2\sqrt{u}(hw^2-v^2)^{\frac{1}{4}}}.
\end{aligned}\tag{4.1}$$

Here $\hat{\text{M}}_4$ is a family of Kahler manifolds parameterised by $\rho$, with an integrable complex structure that is $\rho$ independent. $\hat{J}$ is a two-form defined on the Kahler four-manifold (the details are given below). The functions $u, v, w$ depend on $\rho$ only, while $h$ has support in $(\rho, \hat{\text{M}}_4)$. In fact $w$ is actually redundant, as it can be absorbed into $h$ and $\hat{\text{M}}_4$. We keep it for convenience as it simplifies the derivation of the classes in sections 4.3 and 4.4. Supersymmetry is ensured by the following differential conditions

$$u''=0,\qquad \partial_\rho\left(\frac{\hat{g}^{\frac{1}{2}}}{h}\right)=0,\qquad i\partial\overline{\partial}\log h=\hat{\mathfrak{R}},\tag{4.2}$$

for $\hat{\mathfrak{R}}$ the Ricci form and $\hat{g}$ the determinant of the metric on $\hat{\text{M}}_4$. $\partial, \overline{\partial}$ are Dolbeault operators expressed in terms of complex coordinates on $\hat{\text{M}}_4$ such that $d_4 = \partial + \overline{\partial}$. The 10 dimensional RR fluxes of this class take the form

$$
\begin{aligned}
&F_0 = v',\\
&F_2 = -\frac{w^2}{u} d\rho \wedge \hat{\star}_4 (d_4 h \wedge \hat{J}) - \partial_\rho (w \hat{J}) + \frac{vv'}{hw}\hat{J} - \frac{1}{2}\left(v - \frac{v'uu'}{4(hw^2 - v^2) + (u')^2}\right) \text{vol}(\text{S}^2),\\
&F_4 = \frac{1}{2}\text{vol}(\text{AdS}_3) \wedge \left(d\left(\frac{vuu'}{hw^2 - v^2}\right) + 4v d\rho\right) + \frac{v}{2h}\left(\frac{vv'}{hw^2} - \partial_\rho \log(v^{-1}hw^2)\right)\hat{J}\wedge\hat{J}\\
&\qquad - \frac{vw}{u} d\rho \wedge \hat{\star}_4 d\log h + \frac{1}{2}\left(\frac{uu'}{4(hw^2 - v^2) + (u')^2} F_2 + \frac{hw^2 - v^2}{hw}\hat{J}\right) \wedge \text{vol}(\text{S}^2),
\end{aligned}
\tag{4.3}
$$

where again $F_6 = - \star_{10} F_4$, $F_8 = \star_{10} F_2$. The Bianchi identities are then solved away from localised sources when

$$
v'' = 0, \qquad 2i\partial\overline{\partial}h = \partial_\rho^2 (w\hat{J}). \tag{4.4}
$$

The conditions (4.2) and (4.4) are necessary and sufficient for a solution to exist in the absence of sources. When these exist one should also check the source corrected Bianchi and calibration conditions at their loci.

To better understand this second class of solutions one can consider the limit $u = w = 1$ and $v = 0$ with $\partial_\rho$ an isometry. The result coincides with the Hopf fibre T-dual of the class of solutions found in [28]. These solutions are of the form $\text{AdS}_3 \times \text{S}^3 \times B$, for $B$ the base of an elliptically fibered Calabi-Yau 3-fold. They are characterised by varying axio-dilaton with D3 branes wrapped on a curve within $B$, but have no 3-form flux. If we instead consider a similar limit with $v =$ constant rather than zero, we find a generalisation of this class with non trivial 3-form flux, as we shall demonstrate in section 4.3. In addition to containing the T-dual of this IIB class, class II also contains its non-Abelian T-dual, which one can realise by fixing $w \propto u \propto v$ and taking $J$ and $\hat{\text{M}}_4$ to be $\rho$ independent. This in fact gives another hybrid solution similar to (3.37), that realises the T-dual of section 4.3 when $F_0 = 0$ and the non-Abelian T-dual for generic $F_0$.

In the next section we show how class II is derived from the necessary and sufficient conditions for supersymmetry found in section 2.1.

### 4.2 Derivation of class II

For class II we assume $\sin\beta \neq 0$, and as such we are free to divide by $\sin\beta$ which enables us to put (2.32a)–(2.32h) in the form

$$
d(e^{A-\Phi}\sin\alpha\cos\beta)\wedge V = d(e^{3A-\Phi}\sin\alpha\sin\beta) - 2\mu e^{2A-\Phi}\cos\alpha\sin\beta V = 0, \tag{4.5}
$$

$$
2e^C + e^A \sin\alpha = d\left(\frac{1}{\sin\beta}\Omega\right) = d\left(\frac{e^{-2A}}{\sin^2\beta}J\right)\wedge V = 0, \tag{4.6}
$$

$$
e^{2C}H_1 = -\frac{1}{2\mu}e^A V + \frac{1}{4}d(e^{2A}\sin\alpha\cos\alpha), \qquad H_3 = d\left(\frac{\cos\beta}{\sin\beta}J\right). \tag{4.7}
$$

We can solve (4.5) in general by introducing two functions $u(\rho), v(\rho)$ such that

$$e^{3A-\Phi}\sin\alpha\sin\beta = u, \quad 2e^{A-\Phi}\cos\alpha\sin\beta = u', \quad e^{A-\Phi}\sin\alpha\cos\beta = v. \tag{4.8}$$

In contrast to case I the conditions supersymmetry imposes on $(J,\Omega)$ do not imply that $\hat{M}_4$ is conformally Calabi-Yau in general. We will see instead that it must be a family of $\rho$ dependent Kahler four-manifolds. Taking [64] as a guide it is useful to introduce the rescaled forms and metric

$$J = \frac{\sin\beta}{\sqrt{h}}\hat{J}, \qquad \Omega = \frac{\sin\beta}{\sqrt{h}}\hat{\Omega}, \qquad ds^2(\hat{\mathrm{M}}_4) = \frac{\sin\beta}{\sqrt{h}}ds^2(\hat{\mathrm{M}}_4), \tag{4.9}$$

where we have also introduced

$$\frac{1}{h} = e^{4A}\sin^2\beta w^2 u^{-2}, \tag{4.10}$$

with $h$ a function of $\rho$ and the coordinates on $\hat{\mathrm{M}}_4$. Here $w = w(\rho)$ is an arbitrary function that is actually redundant, as it can be absorbed into the definition of $h$ and $\hat{\mathrm{M}}_4$, but extracting it now simplifies later exposition. Expanding $d = d_4 + d\rho\wedge\partial_\rho$ as before (4.6) implies the following conditions

$$d_4\hat{J} = 0, \tag{4.11a}$$

$$d_4\hat{\Omega} = \frac{1}{2}d_4\log h\wedge\hat{\Omega}, \tag{4.11b}$$

$$\partial_\rho\hat{\Omega} = \frac{1}{2}\partial_\rho\log h\hat{\Omega}. \tag{4.11c}$$

The first two conditions (4.11a)–(4.11b) imply that $ds^2(\hat{\mathrm{M}}_4)$ is a family of Kahler manifolds parameterised by $\rho$, with an associated complex structure that is $\rho$ independent. Since $\hat{\mathrm{M}}_4$ is Kahler (4.11b) can be expressed as

$$d_4\hat{\Omega} = i\hat{P}\wedge\hat{\Omega}, \qquad \hat{P} = -\frac{1}{2}d_4\log h\lrcorner\hat{J}, \qquad d_4\hat{P} = \hat{\mathfrak{R}}, \tag{4.12}$$

where $\hat{\mathfrak{R}}$ is the Ricci form on $ds^2(\hat{\mathrm{M}}_4)$, with components $\hat{\mathfrak{R}}_{ij} = \frac{1}{2}\hat{R}_{ijkl}\hat{J}^{kl}$, for $\hat{R}_{ijkl}$ the Riemann curvature tensor on $ds^2(\hat{\mathrm{M}}_4)$ computed at constant $\rho$. The condition (4.11c) then just serves to constrain the $\rho$ dependence of the Kahler metric such that its determinant $\hat{g}$ satisfies

$$\partial_\rho\left(\frac{\hat{g}^{\frac{1}{2}}}{h}\right) = 0. \tag{4.13}$$

We now turn our attention to the paring conditions (2.35). Although it is not possible to explicitly take the Hodge dual of every term in (2.34a)–(2.34d), it is still possible to solve (2.35) explicitly by making use of (4.11a)–(4.11c), (3.14), and the following identities involving an arbitrary 1-form in 5 dimensions $U = U_4 + u_0V$:

$$j_1\wedge\star_5(U\wedge j_2) = -j_2\wedge\star_5(U\wedge j_1) = U\wedge V\wedge j_3 \quad \text{and cyclic in 123} \tag{4.14}$$

$$j_1\wedge\star_5(U\wedge j_1) = j_2\wedge\star_5(U\wedge j_2) = j_3\wedge\star_5(U\wedge j_3) = \star_5U_4 + u_0j_3\wedge j_3, \tag{4.15}$$

where $J = j_3,\ \Omega = j_1 + ij_2$. After a lengthy calculation we find that (2.35) imposes simply

$$u'' = 0. \tag{4.16}$$

Having now dealt with the geometric supersymmetry constraints we turn our attention to Bianchi identities of the RR fluxes. In general the RR fluxes are rather involved, but as is often the case, the Page fluxes $\hat{f} = e^{-B} \wedge f$, where $dB = H$, are rather more simple, so let us first study these. The conditions defining $H_1, H_3$ in (4.7) can be locally integrated with ease giving rise to the NS potential

$$B = \frac{1}{2}\left(-\rho + \frac{uu'}{4(w^2h - v^2) + (u')^2}\right) \wedge \text{vol}(\text{S}^2) + \frac{v}{wh}\hat{J}, \tag{4.17}$$

in terms of which the Page fluxes take the following form[16]

$$\hat{f}_0 = f_0 = v', \tag{4.19a}$$

$$\hat{f}_2 = -\frac{w^2}{u} d\rho \wedge \hat{\star}_4(d_4 h \wedge \hat{J}) - \partial_\rho(w\hat{J}) + \frac{1}{2}(\rho v' - v)\text{vol}(\text{S}^2), \tag{4.19b}$$

$$\hat{f}_4 = \frac{v'}{2h}\hat{J} \wedge \hat{J} + \frac{1}{2}\left(\rho \hat{f}_2 + w\hat{J}\right) \wedge \text{vol}(\text{S}^2), \tag{4.19c}$$

$$\hat{f}_6 = \left(\frac{\rho}{2}\hat{f}_4 - \frac{v}{4h}\hat{J} \wedge \hat{J}\right) \wedge \text{vol}(\text{S}^2). \tag{4.19d}$$

Away from localised sources the Bianchi identities of the RR fluxes hold if and only if the Page fluxes are closed. Imposing this yields the conditions

$$v'' = 0, \tag{4.20a}$$

$$\frac{w^2}{u} d_4 \hat{\star}_4(d_4 h \wedge \hat{J}) = \partial_\rho^2(w\hat{J}), \tag{4.20b}$$

that follow from the parts of $\hat{f}_0, \hat{f}_2$ that are orthogonal to vol$(\text{S}^2)$ — closure of the rest is implied by these and supersymmetry. We can make further progress by introducing complex coordinates $z_1, z_2$ on $\hat{\text{M}}_4$ and Dolbeault operators $\partial = dz^i \partial_{z_i}$, $\overline{\partial} = d\overline{z}^i \partial_{\overline{z}_i}$ in terms of which we can expand $d_4 = \partial + \overline{\partial}$. We then have

$$\hat{\star}_4(d_4 g \wedge \hat{J}) = d_4 \log g \lrcorner \hat{J} = -i(\partial - \overline{\partial})g, \tag{4.21}$$

for $g$ an arbitrary function. This can be used to simplify some of the necessary conditions, allowing us to present the class in the form given in section 4.1.

In the next section we will derive a class of solutions in IIB that generalise the solutions in [28] to include non trivial 3-form flux.

[16] We have significantly simplified $\hat{f}_2, \hat{f}_4$ by making use of (4.14) and

$$\partial_\rho \hat{J} = \frac{1}{2}\partial_\rho \log h \hat{J} + H_2, \qquad H_2 + \hat{\star}_4 H_2 = 0, \tag{4.18}$$

where $J \wedge H_2 = \Omega \wedge H_2 = 0$, which follows from (4.11c) and the allowed torsion classes of SU(2)-structures in 5 dimensions [63].

### 4.3 Generalisation of the F-theory solutions in [28] with non trivial 3-form flux

In this section we derive a generalisation of a class of solutions in IIB found in [28]. These are characterised by varying axio-dilaton with D3-branes wrapped on complex curves within the base of an elliptically fibered $\text{CY}_3$, and vanishing 3-form flux. Our generalisation will include a non-trivial 3-form flux.

We begin with class II of section 4.1 and impose that $\partial_\rho$ is an isometry. This can be achieved without loss of generality by fixing

$$u = L^4, \qquad w = L^4\lambda^2 \qquad v = cL^2, \tag{4.22}$$

with the Kahler manifold and structure assumed to be $\rho$ independent. We also rescale $h$ for convenience as

$$h \to \frac{h}{L^4\lambda^4}, \tag{4.23}$$

for $(L, \lambda, c)$ all constant. The NS sector then becomes

$$\begin{aligned} ds^2 &= \frac{L^2}{\sqrt{h-c^2}}\left[ds^2(\text{AdS}_3) + \frac{1}{4}ds^2(\text{S}^2)\right] + \frac{\sqrt{h-c^2}}{L^2}d\rho^2 + L^2\lambda^2\frac{\sqrt{h-c^2}}{h}ds^2(\hat{\text{M}}_4), \\ B &= -\frac{1}{2}d\rho\wedge\eta + cL^2\lambda^2h^{-1}\hat{J}, \qquad e^{-\Phi} = L\sqrt{h}(h-c^2)^{\frac{1}{4}}, \end{aligned} \tag{4.24}$$

as before, $d\eta = -\text{vol}(\text{S}^2)$ and $dB = H$ — we have chosen a gauge for $B$ that makes the $\partial_\rho$ isometry explicit. The RR sector becomes

$$F_2 = id\rho\wedge(\partial - \overline{\partial})h - \frac{L^2c}{2}\text{vol}(\text{S}^2), \tag{4.25}$$

$$F_4 = 2cL^2\text{vol}(\text{AdS}_3)\wedge d\rho + cL^2\lambda^2(\hat{\star}_4 d\log h)\wedge d\rho + \frac{L^4\lambda^2}{2}\frac{h-c^2}{h}\hat{J}\wedge\text{vol}(\text{S}^2), \tag{4.26}$$

with $F_0 = 0$. The first thing we note is that the only Bianchi identity that is not solved automatically is that of the RR 2-form, due to the first term. Whenever this is satisfied away from localised sources there exists a local function $C_0$ with support on $\hat{\text{M}}_4$ such that

$$dC_0 = i(\partial - \overline{\partial})h, \tag{4.27}$$

which holds precisely when the following complex function is holomorphic

$$\sigma = C_0 + ih, \tag{4.28}$$

i.e. $\overline{\partial}\sigma = 0$ implies (4.27) and vice-versa. This is already very reminiscent of [28]. Indeed if we fix $c = 0$ we reproduce the result of T-dualising that class on the Hopf fibre of the $\text{S}^3$, with $\tau = \sigma$ the modular parameter of IIB. Generic $c \neq 0$ is a parametric deformation of this that obeys the same supersymmetry constraint, namely that

$$i\partial\overline{\partial}\log h = \frac{1}{2}d\left(\frac{dC_0}{h}\right) = \hat{\mathfrak{R}}, \tag{4.29}$$

which reproduces the geometric condition the base of the elliptically fibered $\text{CY}_3$ manifolds of [28] must obey. After performing the T-duality on $\partial_\rho$, under the assumption it has period $2\pi$, the IIB string frame solution becomes

$$\begin{aligned}
ds^2 &= \frac{L^2}{\sqrt{h-c^2}}\Big[ds^2(\text{AdS}_3)+ds^2(\text{S}^3)\Big]+L^2\lambda^2\frac{\sqrt{h-c^2}}{h}ds^2(\hat{\text{M}}_4),\\
\hat{B} &= c\lambda^2L^2h^{-1}\hat{J},\quad e^{-\hat{\Phi}}=\sqrt{h}\sqrt{h-c^2},\\
F_1 &= dC_0,\qquad F_3=-2cL^2\text{vol}(\text{AdS}_3)-cL^2\lambda^2\hat{\star}_4 d\log h+2cL^2\lambda^2\text{vol}(\text{S}^3),\\
F_5 &= -2L^4\lambda^2\frac{h-c^2}{h}(1+\star_{10})\hat{J}\wedge\text{vol}(\text{S}^3),
\end{aligned}\tag{4.30}$$

with $\hat{\Phi}$ and $\hat{B}$ the dilaton and NS 2-form potential in IIB, and where in particular $c=0$ fixes $H=F_3=0$ as in [28]. Although we choose to write this solution with an $\text{S}^3$, this is meant just locally. One could equally well replace the $\text{S}^3$ with a Lens space without breaking any further supersymmetry, as is done in [28] by sending $\text{S}^3\to\text{S}^3/\mathbb{Z}_k$.

In summary, we find a parametric deformation of the solutions of [28] with 3-form flux turned on that still preserves $\mathcal{N}=(4,0)$ supersymmetries. Converting to Einstein frame, in which the $\text{SL}(2,\mathbb{R})$ invariance of IIB is manifest, and replacing the $\text{S}^3$ with a Lens space, we arrive at the IIB solution

$$\begin{aligned}
ds^2_E &= L^2\left[\frac{h^{\frac{1}{4}}}{(h-c^2)^{\frac{1}{4}}}\left(ds^2(\text{AdS}_3)+ds^2(\text{S}^3/\mathbb{Z}_k)\right)+\lambda^2\frac{(h-c^2)^{\frac{3}{4}}}{h^{\frac{3}{4}}}ds^2(\hat{\text{M}}_4)\right],\\
\tau &= C_0+i\sqrt{h}\sqrt{h-c^2},\quad B=c\lambda^2L^2h^{-1}\hat{J},\quad F_5=-2L^4\lambda^2\frac{h-c^2}{h}(1+\star_{10})\hat{J}\wedge\text{vol}(\text{S}^3/\mathbb{Z}_k)\\
F_3 &= -2cL^2\text{vol}(\text{AdS}_3)-cL^2\lambda^2\hat{\star}_4 d\log h+2cL^2\lambda^2\text{vol}(\text{S}^3/\mathbb{Z}_k).
\end{aligned}\tag{4.31}$$

This coincides locally with the class in [28] when $c=0$, so that $\tau=C_0+ih$,[17] with AdS radius $m=1$. The complex 3-form is defined as $G=i(\text{Im}\tau)^{-1}(\tau dB-F_3)$. Supersymmetry and the Bianchi identities away from sources, simply require

$$\overline{\partial}(C_0+ih)=0,\qquad \frac{1}{2}d\left(\frac{dC_0}{h}\right)=\hat{\mathfrak{R}}.\tag{4.32}$$

Thus, as solutions were argued to exist when $c=0$, further solutions must exist for $c\neq 0$ (at least formally) as the necessary conditions for their existence are $c$ independent. A difference is that now the physical region of $\hat{\text{M}}_4$ when embedded into 10 dimensions is the portion for which $h\geq c^2$ is satisfied, with the lower bound a singular loci in the full space. The warp factors appear consistent with D5 branes wrapped on $\text{S}^3$ at this loci,[18] however

[17] The specific map is $L\to e^A,\lambda\to m_B^{-1}$.

[18] We rule out a 3-cycle in $\hat{\text{M}}_4$ because the cycle on which the D5s are wrapped should be calibrated. This ultimately means that the DBI action of the 5 brane should be equal to the pull back of some combination of the structure forms wedged with themselves, $\text{vol}(\text{AdS}_3)$ and $\hat{B}$. But since the structure group of $\hat{\text{M}}_4$ is $\text{SU}(2)$ we only have two forms at our disposal — thus any supersymmetric brane, D5 or otherwise, must wrap the $\text{S}^3$.

confirming this seems dependent on the specifics of the Kahler Manifold. Let us stress that, similar to the class of solutions in section 3.3, this is not the most general class of this type. Rather this is a specific $\mathrm{SL}(2,\mathbb{R})$ duality frame of this most general solution — see the discussion below (3.25).

It would be interesting to study the solutions in this class, but as with the $c = 0$ limit, the permissible metrics on $\hat{\mathrm{M}}_4$ are the possible bases of an elliptically fibered $\mathrm{CY}_3$, which are not explicitly known.[19] In particular, it would be interesting to find their explicit F-theory realisation [27, 28].

### 4.4 Further local $\mathbf{AdS_3 \times S^2 \times CY_2}$ foliations

In this section we shall explore the solutions contained in class II that are foliations of $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ over the interval spanned by $\rho$, similar to those found in section 3.4 — here we will be more brief. Such solutions should respect the isometries of $\mathrm{CY}_2$, which means the warp factors must be independent of the directions on $\mathrm{CY}_2$. Again $\mathrm{CY}_2$ should be compact, which reduces our considerations to $\mathrm{CY}_2 = \mathrm{T}^4$ or $\mathrm{CY}_2 = \mathrm{K3}$. The supersymmetry conditions and Bianchi identities of the fluxes (away from the loci of sources) then just impose that $(v, u, w)$ are linear functions, that we choose as

$$v = c_1 + F_0\rho, \qquad u = c_2 + c_3\rho, \qquad w = c_4 + c_5\rho, \quad h = 1 \tag{4.33}$$

where $c_i$ are all constants. The NS sector is then

$$\begin{aligned}
ds^2 &= \frac{c_2 + c_3\rho}{\sqrt{(c_2 + c_3\rho)^2 - (c_1 + F_0\rho)^2}} \left[ ds^2(\mathrm{AdS}_3) + \frac{1}{4 + \frac{c_3^2}{(c_2+c_3\rho)^2-(c_1+F_0\rho)^2}} ds^2(\mathrm{S}^2) \right] \\
&\quad + \frac{\sqrt{(c_2 + c_3\rho)^2 - (c_1 + F_0\rho)^2}}{c_2 + c_3\rho} \left[ \frac{c_2 + c_3\rho}{c_4 + c_5\rho} ds^2(\mathrm{CY}_2) + d\rho^2 \right], \\
e^{-\Phi} &= \frac{((c_4 + c_5\rho)^2 - (c_1 + F_0\rho)^2)^{\frac{3}{4}} \sqrt{4 + \frac{c_3^2}{(c_4+c_5\rho)^2-(c_1+F_0\rho)^2}} \sqrt{1 + \frac{(c_1+F_0\rho)^2}{(c_4+c_5\rho)^2-(c_1+F_0\rho)^2}}}{2\sqrt{c_2 + c_3\rho}}, \\
B &= n\pi \mathrm{vol}(\mathrm{S}^2) - \frac{c_1 + F_0\rho}{c_4 + c_5\rho} \hat{J} - \frac{1}{2} \left( \rho - \frac{c_3(c_2 + c_3\rho)}{4((c_4 + c_5\rho)^2 - (c_1 + F_0\rho)^2) + c_3^2} \right) \mathrm{vol}(\mathrm{S}^2),
\end{aligned} \tag{4.34}$$

where $n$ is an integer with which we parametrise potential large gauge transformations of the NS 2-form $B$. The magnetic Page fluxes, $\hat{f} = e^{-B} \wedge f$, for $f$ the magnetic components of the 10 dimensional RR fluxes, are

$$\hat{f}_0 = F_0, \qquad \hat{f}_2 = -c_5\hat{J} - \frac{1}{2}(c_1 + 2\pi n F_0)\mathrm{vol}(\mathrm{S}^2), \tag{4.35}$$

$$\hat{f}_4 = F_0\hat{J} \wedge \hat{J} + \frac{1}{2}(c_4 + 2\pi n c_5)\hat{J} \wedge \mathrm{vol}(\mathrm{S}^2), \qquad \hat{f}_6 = -\frac{1}{2}(c_1 + 2\pi n F_0)\hat{J} \wedge \hat{J} \wedge \mathrm{vol}(\mathrm{S}^2) \tag{4.36}$$

with $F_0$ non trivial generically, and where $\hat{J}$ is the Kahler form of $\mathrm{CY}_2$ (so $d\hat{J} = 0$).

[19] Strictly speaking this base could be $\mathrm{CY}_2$, in which case one could take $\mathrm{T}^4$ as an explicit metric. However constancy requires that $h$ is constant whenever the Kahler manifold is Ricci flat.

As was the case in section (3.4), the only way to have a regular solution is if the $\mathrm{AdS}_3$ warp factor is constant. We can achieve this by fixing

$$u = L^4, \qquad w = L^4\lambda^2 \qquad v = cL^2, \qquad h = \frac{1}{L^4\lambda^4} \tag{4.37}$$

without loss of generality. The resulting metric takes the form

$$ds^2 = \frac{L^2}{\sqrt{1-c^2}}\left[ds^2(\mathrm{AdS}_3) + \frac{1}{4}ds^2(\mathrm{S}^2)\right] + \frac{\sqrt{1-c^2}}{L^2}d\rho^2 + L^2\lambda^2\sqrt{1-c^2}ds^2(\mathrm{CY}_2). \tag{4.38}$$

When $F_0 = 0$ this reproduces the metric of (4.24), in the limit $h = 1$ and $\hat{\mathrm{M}}_4 = \mathrm{CY}_2$, which is the T-dual of the same limit of the IIB solution derived in the previous section — this solution is compact when $\mathrm{CY}_2$ and $\partial_\rho$ are assumed to be. For generic values of $F_0 \neq 0$ the solution is the non abelian T-dual of this IIB solution, where the interval spanned by $\rho$ becomes semi infinite, with a regular zero at $\rho = -\frac{c_1}{F_0}$. These statements all hold true for the fluxes also.

Allowing for D brane and O plane behaviour at the boundaries of the interval as in (3.38), as well as composite objects, we find that it is possible to realise the following physical boundary behaviours

| Source | Minimal tuning | $\mathrm{M}^{1,p}$ | $\tilde{\mathrm{B}}^s$ | Loci |
|---|---|---|---|---|
| Smeared D4 | $c_3 = 0$ | $\mathrm{AdS}_3 \times \mathrm{S}^2$ | $\mathrm{CY}_2$ | $\rho = \frac{\pm c_1 - c_4}{c_5 \mp F_0}$ |
| Smeared D2 | Generic $c_i$ | $\mathrm{AdS}_3$ | $\mathrm{CY}_2 \times \mathrm{S}^2$ | $\rho = \frac{\pm c_1 - c_4}{c_5 \mp F_0}$ |
| D2 inside D6 | Generic $c_i$ | D6: $\mathrm{AdS}_3 \times \mathrm{CY}_2$ | D2: $\mathrm{CY}_2$ | $\rho = -\frac{c_2}{c_3}$ |
| O2 inside O6 | $c_1 = bc_4,\ F_0 = bc_5$ | O6: $\mathrm{AdS}_3 \times \mathrm{CY}_2$ | O2: $\mathrm{CY}_2 \times \mathrm{S}^2$ | $\rho = -\frac{c_4}{c_5}$ |
| T/NATD hybrid | (4.37) | — | — | — |

where $b$ are arbitrary constants and we include the T-dual/non-Abelian T-dual hybrid, which is regular, for completeness. As before, one can also interpret the D branes smeared on all their compact co-dimensions as smeared O planes.

As in section (3.4), we need two of these boundary behaviours to exist for the same tuning of $c_i$ to realise a compact local solution beyond the $F_0 = 0$ limit of (4.38). We find the following possibilities

| Tuning | Boundary behaviours | Loci: $\rho =$ |
|---|---|---|
| generic $c_i$ | D2 \| D2 in D6 \| D2 | $\frac{+c_1-c_4}{c_5-F_0}$ \| $-\frac{c_2}{c_3}$ \| $\frac{-c_1-c_4}{c_5+F_0}$ |
| $(c_4 = bc_1, c_5 = bF_0, c_3 \neq 0)$ | D2 in D6 \| O2 in O6 | $-\frac{c_2}{c_3}$ \| $-\frac{c_1}{F_0}$ |
| $(c_2 = bc_1, c_3 = bF_0, c_3 \neq 0)$ | D2 \| D2 | $-\frac{c_1}{F_0}$ \| $-\frac{c_4}{c_5}$ |
| $c_3 = 0$ | D4 \| D4 | $\frac{c_1-c_2}{c_5-F_0}$ \| $\frac{-c_1-c_2}{c_5+F_0}$ |

(4.39)

Together with (3.39) this gives a total of 13 distinct foliations of $\mathrm{AdS}_3\times\mathrm{S}^3\times$ $\mathrm{CY}_2$ over intervals bounded between a rich variety of D brane and O plane behaviours. They are compact whenever $\mathrm{CY}_2 = \mathrm{T}^4$ or K3, which really doubles the number of distinct solutions to 26.

As was true of section (3.4), the general solution of this section is only local. One can actually construct more general globally compact solutions by glueing these local solutions together with defect branes. In the next section we will explore this possibility.

## 5 Glueing local solutions together with defect branes

In sections 3.4 and 4.4 we found several local compact solutions that are foliations of $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ over a finite interval bounded by various D brane an O plane behaviours. In this section we show that these compact local solutions, and more generally any local solution in these classes, may be used as the building blocks of a far larger class of globally compact solutions. This can be achieved by using defect branes to glue the various local solutions together. This follows the spirit of [54], where an infinite family of globally compact $\mathrm{AdS}_7$ solutions in massive IIA was found, that utilised D8 brane defects to glue various non compact local solutions together (see also [39] for an $\mathrm{AdS}_3$ example).

Through out most of this paper we have derived our various classes of solution under the assumption that we are in a region of the internal space away from the loci of sources. This was actually sufficient to find solutions with sources on the boundary of the internal space — as then one can explicitly see known brane/plane behaviour appearing in the physical fields. However to realise defect branes, that lie on the interior of the internal space, we will have to explicitly solve the source corrected Bianchi identities and make sure the sources are supersymmetric.

Various types of defect branes are possible in supergravity, with various signatures — the most simple is probably the D8. The singularity signalling a D8 brane defect is rather mild, giving rise only to a discontinuity in the derivatives of the metric and dilaton, with the fields themselves continuous. The NS 2-form on the other hand needs only be continuous up to a large gauge transformation. The remaining fluxes can be discontinuous across such a defect provided that this is induced by a shift in the D8 brane flux $F_0$ — which should naturally shift as one crosses a D8 brane stack. In what follows this will be one defect we use to perform glueings. The others are a D4 brane defect and a D6 defect that are both smeared over their compact co-dimensions. Such objects behave in a completely analogous way to the D8 defect, indeed for $\mathrm{CY}_2 = \mathrm{T}^4$ they are mapped into each other via T-duality, only now it is the charge of D4s/D6s rather than $F_0$ that experiences a discontinuity as we cross the defect.

Having set the scene, it will now be helpful to look at the two cases individually to show that such glueing of local solutions is possible. Let us first look at global solutions following from section 3.4.

### 5.1 Towards global solutions with defects from section 3.4

In this section we will study the possibility of gluing the local solutions of section 3.4 together with defect branes.

As explained in section 3.4, the general local form of the NS sector and RR Page fluxes are given exactly by (3.31) and (3.32) respectively. However these expressions depend on constants $(c_i, F_0)$ that can change as we cross a defect brane, so for a global solution it is more helpful to consider the form of NS sector given in (3.1), we remind the reader that here we fix $H_2 = 0$ so as to respect the symmetry of $\text{CY}_2$ and $(h_8, h_4, u)$ are all functions of $\rho$ only, the latter being linear and the behaviour of the former two determined by the Bianchi identities of the fluxes. As we shall see $(h_8, h_4)$ end up being piece-wise linear so that

$$F_0 = h_8', \qquad G_0 = h_4' \tag{5.1}$$

are not globally defined, but can change between local patches of a global solution. For the RR sector it will be most useful to know the magnetic component of the Page flux polyform

$$\hat{f} = h_8' - \frac{1}{2}(h_8 - \rho h_8')\text{vol}(\text{S}^2) - \left(h_4' - \frac{1}{2}(h_4 - \rho h_4')\text{vol}(\text{S}^2)\right) \wedge \text{vol}(\text{CY}_2). \tag{5.2}$$

Recall the Page flux is defined in terms of the NS 2-form $B$ as $\hat{F} = e^{-B} \wedge F$, for simplicity we do not consider large gauge transformations in $B$ — however we stress that their inclusion changes nothing substantive about what follows.

Let us first consider a single D8 brane defect: the Bianchi identity of the entire magnetic flux in the presence of a generic D8 brane stack takes the form

$$(d - H)f = \frac{n_8}{2\pi}\delta(\rho - \rho_0)e^{\mathcal{F}} \wedge d\rho \tag{5.3}$$

where $\rho_0$ is the loci of the stack, and $n_8$ its charge. As usual $\mathcal{F} = B + 2\pi \tilde{f}_2$ for $\tilde{f}_2$ a world-volume flux that may be turned on — this should not to be confused with the RR 2-form! As $B \sim \text{vol}(\text{S}^2)$ for the local solutions of section 3.4, we anticipate that this D8 brane is actually (at least) a D8–D6 bound state — however exactly what branes are bounded together will depend on the form of $\tilde{f}_2$ that we determine by actually solving (5.3). Following [54], we do this in terms of $\hat{f}$, for which (5.3) is equivalent to

$$d\hat{f} = \frac{n_8}{2\pi}\delta(\rho - \rho_0)e^{2\pi \tilde{f}_2} \wedge d\rho\,. \tag{5.4}$$

As we move across this defect the NS sector (3.31) should be continuous ($B$ can shift by a large gauge transformation, but for simplicity we shall assume it does not), while only $F_0$ should shift. Thus $h_8, h_4, h_4', h_4''$ should be continuous across the defect while $F_0 = h_8'$ will be discontinuous. As such integrating (5.4) across the D8 stack gives rise to

$$\Delta F_0 e^{\frac{1}{2}\rho_0 \text{vol}(\text{S}^2)} = \frac{n_8}{2\pi}e^{2\pi \tilde{f}_2} \tag{5.5}$$

for $\Delta F_0$ the difference between the values of $F_0$ for $\rho < \rho_0$ and $\rho > \rho_0$. We thus see that the Bianchi identity merely fixes

$$\Delta F_0 = \frac{n_8}{2\pi}, \qquad \tilde{f}_2 = \frac{1}{4\pi}\rho_0 \text{vol}(\text{S}^2), \qquad \Rightarrow \qquad \mathcal{F} = \frac{uu'}{8h_4h_8 + 2(u')^2}\text{vol}(\text{S}^2) \tag{5.6}$$

confirming that the defect is actually a D8–D6 bound state.

For the D4 brane defect wrapped on $\text{AdS}_3\times\text{S}^2$ and smeared over $\text{CY}_2$ things are rather similar. The Bianchi identity of such a D4 brane stack takes the from

$$(d-H)f\Big|_{\text{CY}_2}=(2\pi)^3 n_4\delta(\rho-\rho_0)e^{\mathcal{F}}\wedge d\rho\wedge\text{vol}(\text{CY}_2) \tag{5.7}$$

where $(\rho_0,n_4)$ are the loci and charge of the stack — the notation on the l.h.s. means we only consider the components parallel to $\text{vol}(\text{CY}_2)$. This time it should be only $G_0=h_4'$ which is discontinuous across the defect, so integrating the Page form avatar of (5.7) gives rise to

$$\Delta G_0 e^{\frac{1}{2}\rho_0\text{vol}(\text{S}^2)}=-(2\pi)^3 n_4 e^{2\pi\tilde{f}_2}, \tag{5.8}$$

where the volume of $\text{CY}_2$ has been factored out of both sides of this expression. We need then only fix

$$\Delta G_0=-(2\pi)^3 n_4,\qquad \tilde{f}_2=\frac{1}{4\pi}\rho_0\text{vol}(\text{S}^2),\qquad \Rightarrow\qquad \mathcal{F}=\frac{uu'}{8h_4h_8+2(u')^2}\text{vol}(\text{S}^2) \tag{5.9}$$

for the Bianchi identity to be solved — which implies that like the D8, the D4 is also a bound state, this time D4–D2.

We have shown that both D8 and smeared D4 brane defects can be placed at arbitrary points along the interval of the $\text{AdS}_3\times\text{S}^2\times\text{CY}_2$ foliations in section 3.4 and still solve the source corrected Bianchi identities — provided they come as part of a bound state (D8–D6 and D4–D2). To guarantee that we actually have a solution at these loci however the branes must have a supersymmetric embedding — then supersymmetry is preserved on the defects and the remaining EOM are implied [51]. A major advantage of the approach we took to constructing solutions in section 2.2 is that it allows us to determine this in the language of generalised calibrations [52]. This is relatively simple for us because the fundamental object of this approach is the 7d bi spinors already given in (2.29a)–(2.29b). A D brane source extended along $\text{AdS}_3$ is supersymmetric if it obeys a calibration condition — namely the DBI Lagrangian $\mathcal{L}_{\text{DBI}}=d\xi^d e^{-\Phi}\sqrt{-\det(g+\mathcal{F})}$ is equal to a calibration form. In IIA this calibration form is given by the pull back of $e^{3A-\Phi}\text{vol}(\text{AdS}_3)\wedge\Psi_+\wedge e^{\mathcal{F}}$ onto the relevant D brane world-volume. It is not hard to show that both our D8 and D4 brane defects obey this condition precisely when $\mathcal{F}$ is tuned as the Bianchi identity of each defect requires.

Thus we have established that one can place defects at arbitrary points along the interval of the $\text{AdS}_3\times\text{S}^2\times\text{CY}_2$ foliation and still have a supersymmetric solution — we need only impose that $(h_4,h_8)$ are continuous. This fact can be used to glue two local solutions of section 3.4 together provided they share a common tuning for $u$ ($u''=0$ by supersymmetry, so $u$ is globally linear). There is no limit to the number of defects one can place in a global solution, indeed in general $(h_4,h_8)$ need only be piece-wise linear with a change in slope of the former (latter) indicating the presence of a D4 (D8) brane at that loci. One can therefore construct infinite classes of global solutions for each tuning of $u$ in section 3.4. We delay a detailed exploration of these possibilities and their interpretation in terms of the $\text{AdS}_3/\text{CFT}_2$ correspondence until [62, 68].

In the next section, we explore the possibility of constructing global solutions with defects from the solutions in section 4.4.

## 5.2 Towards global solutions with defects from section 4.4

In this section we will show that it is possible to glue the local solutions of section 4.4 together with defect branes. As most of the details of this procedure are covered in the previous section, we encourage the reader to go over that first, as here we shall be brief.

As before the local solutions of section 4.4 depend on constants $(c_i, F_0)$. However, as these constants can shift between local patches in a global solution, it is more helpful to consider the NS sector in the form of (4.1), with $h = 1$ and $\hat{\mathrm{M}}_4 = \mathrm{CY}_2$ — recall $(u, v, w)$ are functions of $\rho$ only and that $u$ is such that globally $u'' = 0$ due to supersymmetry. Conversely $v, w$ are only linear functions away from localised sources — globally they need only be piecewise linear provided that the resulting $\delta$-functions appearing in their second derivatives gives rise to a source corrected Bianchi identity, and this source is calibrated. The magnetic Page flux polyform associated to these solutions is

$$\hat{f} = v' - \frac{1}{2}(v - \rho v')\mathrm{vol}(\mathrm{S}^2) - \left(w' - \frac{1}{2}(w - \rho w')\mathrm{vol}(\mathrm{S}^2)\right) \wedge \hat{J} + \left(v' - \frac{1}{2}(v - \rho v')\mathrm{vol}(\mathrm{S}^2)\right) \wedge \mathrm{vol}(\mathrm{CY}_2). \tag{5.10}$$

Given the form of this expression it might be tempting to interpret a shift in $v'$ as coincident D8–D6 and D4–D2 bound states — however such configurations fail to obey the calibration condition discussed in the previous section at generic points along the interval — so cannot be used to glue solutions together without breaking supersymmetry.[20] Shifts in $w'$ on the other hand are different and give rise to something new. To interpret it consider the following: if we take a D8–D6 brane defect wrapping $\mathrm{CY}_2 = \mathrm{T}^4$, we can express $\hat{J} = dx_1 \wedge dx_2 + dx_3 \wedge dx_4$ with $x_i$ the directions on $\mathrm{T}^4$ which are all isometries. If one T-dualises such an object on both $(x_1, x_2)$ it would generate the part of $-\big(w' - \frac{1}{2}(w - \rho w')\mathrm{vol}(\mathrm{S}^2)\big) \wedge \hat{J}$ with legs in $(x_1, x_2)$ — this is a D6–D4 brane wrapping $\mathrm{AdS}_3 \times \mathrm{S}^2$ and $(x_3, x_4)$ which is smeared on $(x_1, x_2)$. If instead one T-dualised the D8–D6 bound state on $(x_3, x_4)$, the part of the previous expression with legs in $(x_1, x_2)$ would be generated, which should be interpreted as a D6–D4 wrapping $(x_1, x_2)$ and smeared on $(x_3, x_4)$. To generate the entire $w$ dependent term in (5.10) then, one should have both of these smeared D6–D4s simultaneously. Generalising to generic $\mathrm{CY}_2$, a shift in $w'$ gives rise to a D6–D4 bound state that wraps a curve in $\mathrm{CY}_2$ and are smeared on its co-cycle and another D6–D4 that is on smeared and wraps the opposite cycles. The Bianchi identities of each bound state are essentially the same as the D4–D2 of the previous section, only this time pulled back onto the relevant curve rather than the entire of $\mathrm{CY}_2$ — they are solved as before with world volume gauge field $4\pi \tilde{f}_2 = \rho_0 \mathrm{vol}(S^2)$ and D6 brane charge proportional to $\Delta w'$ across the defect. Finally, it is not hard to establish that each of the D6–D4 bounds states are indeed calibrated at generic points in the space.

We have now established that D6–D4 defect branes can be placed at generic points along the interval of the $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ foliation of section 4.4. It would be interesting

[20] They do obey the calibration condition when $w = v$, where the metric blows up, so they still cannot be interpreted as defect branes, like in the previous section.

to explore what global solutions may be constructed by glueing the local solutions already found together with these defects. We leave that for future work.

In the next and final section we summarise this work and discuss some future directions.

## 6 Summary and future directions

In this paper we have found two classes of warped $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_5$ solutions in massive IIA that preserve small $\mathcal{N} = (4, 0)$ supersymmetry in terms of an SU(2) structure on $\mathrm{M}_5$. These classes are exhaustive for solutions of this type when one assumes that the associated spinors on $\mathrm{S}^2 \times \mathrm{M}_5$ have equal norm, a requirement for non vanishing Romans mass. For class I $\mathrm{M}_5$ decomposes as $\mathrm{CY}_2 \times \mathbb{R}$ and we are able to give explicit local expressions for the metric and fluxes up to simple Laplace like PDEs. This class contains a generalisation of the flat space system of D4s inside the world volume of D8s contained in [55], with flat space replaced by $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \mathbb{R}$. For class II we find $\mathrm{M}_5 = \mathrm{M}_4 \times \mathbb{R}$ where $\mathrm{M}_4$ is a class of warped Kahler manifolds with metrics that depend on the interval.

Performing T duality on the IIA classes, we find new classes of solutions in IIB that, modulo $\mathrm{SL}(2, \mathbb{R})$ transformations, exhaust $\mathcal{N} = (4, 0)$ solutions of the type $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{M}_4$, with $\mathrm{M}_4$ an SU(2) structure manifold. The first is a generalisation of the near horizon limit of D1–D5 branes [4], where the $\mathrm{S}^3$ becomes fibered over $\mathrm{CY}_2$ and D5 branes are backreacted on top of this. It is possible to turn off the fibre and then realise the resulting system as a near horizon limit with a modification of the D1–D5 intersection. The second class of IIB solutions is a generalisation of D3 branes wrapped on a curve inside the base of an elliptically fibered $\mathrm{CY}_3$ [28]. The generalisation depends on the same necessary geometric conditions as [28], but has an additional parameter turned on which is related to the charge of 5-branes, absent in the original construction, which tunes the 3-forms to zero.

In sections 3.4 and 4.4, we have found several new local solutions in massive IIA that are foliations of $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ over an interval, bounded between a variety of D brane and O plane behaviours. Then in section 5 we show how these may be used as the building blocks of infinite families of global solutions. These utilise defect branes to glue the various local solutions together in the vein of [54]. We will explore some possible global solutions containing defect branes and their holographic interpretation in [62, 67].

An interesting open problem that our classification of (0,4) supersymmetric solutions leaves is the identification of their 2d dual CFTs. On the other hand, as stressed in the introduction, there are large classes of 2d (0,4) linear quivers, such as the ones constructed in [23, 24, 26, 30], which lack a holographic description. In [62] we will partially fill this gap, and provide the explicit connection between $\mathrm{AdS}_3 \times \mathrm{S}^2$ solutions in class I with compact $\mathrm{CY}_2$ and 4d (0,4) quivers.

Another interesting avenue to explore as a consequence of this work is the connection between our solutions and the $\mathrm{AdS}_7$ solutions to massive IIA constructed in [54], in particular whether a generalisation to $\mathrm{AdS}_3$ solutions exists of the flows constructed in [65, 66, 69]. We will report progress in this direction in [67, 68].



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## A Spinors and bi spinors on $\mathrm{S}^2$ and $\mathrm{M}_5$

In this appendix we provide details of the spinors and bi spinors on $\mathrm{S}^2$ to supplement section 2.2. Specifically when deriving the 7 dimensional bi spinors (2.23) from (2.21), it is useful to know the 2 and 5 dimensional bi spinors on $\mathrm{S}^2$ and $\mathrm{M}_5$, which (2.23) will decompose in terms of. In fact given our decomposition of the gamma matrices (2.7), a bi spinor constructed out of tensor products of spinors in 2 and 5 dimensions ($\xi^i$ and $\eta^i$ respectively) necessarily decomposes as

$$\left[\xi^1\otimes\eta^1\right]\otimes\left[\xi^2\otimes\eta^2\right]^\dagger=(\eta^1\otimes\eta^{2\dagger})_+\wedge(\xi^1\otimes\xi^{2\dagger})+(\eta^1\otimes\eta^{2\dagger})_-\wedge(\sigma_3\xi^1\otimes\xi^{2\dagger}) \tag{A.1}$$

where $\pm$ denotes the even/odd degree components of a form only, which can be repeatedly used when computing (2.23) — and proves that it is built from bi spinors in 2 and 5 dimensions.

In the next section we present details of spinors and bi spinors on unit norm $\mathrm{S}^2$.

### A.1 Spinors and bi spinors on $\mathrm{S}^2$

There are two types of Killing spinor on unit radius $\mathrm{S}^2$, $\xi_\pm$, that are solutions to the Killing spinor equations

$$\nabla_a\xi_\pm=\pm\frac{i}{2}\sigma_a\xi_\pm,\quad a=1,2, \tag{A.2}$$

where we take the first 2 Pauli matrices as two dimensional gamma matrices. Unlike the $\xi_\pm$ equivalents on $\mathrm{S}^3$, these are not really independent and in fact one can take $\xi_-=\sigma_3\xi_+$ without loss of generality. We identify $\xi_+=\xi$ in the main text, and one has in general that both $\xi$ and $\sigma_3\xi$ transform in the same fashion under the SU(2) global symmetry on $\mathrm{S}^2$. The bi spinors that follow from $\xi$, under the assumption they have unit norm, are [43],

$$\begin{aligned}\xi\otimes\xi^\dagger&=\frac{1}{2}(1+k_3-iy_3\mathrm{vol}(\mathrm{S}^2)), & \xi\otimes\xi^{c\dagger}&=-\frac{1}{2}(k_1+ik_2-i(y_1+iy_2)\mathrm{vol}(\mathrm{S}^2)),\\ \sigma_3\xi\otimes\xi^\dagger&=\frac{1}{2}(y_3+idy_3-i\mathrm{vol}(\mathrm{S}^2)), & \sigma_3\xi\otimes\xi^{c\dagger}&=-\frac{1}{2}(y_1+iy_2-id(y_1+iy_2)),\end{aligned} \tag{A.3}$$

where $y_i$ are coordinates embedding $\mathrm{S}^2$ into $\mathbb{R}^3$ and $K_i$ are one forms dual to the Killing vectors of SU(2), which may be parameterised as

$$K_i = \epsilon_{ijk} y_j dy_k. \tag{A.4}$$

Note that (A.3) are spanned entirely by the $(y_i, dy_i, K_i, y_i \mathrm{vol}(\mathrm{S}^2))$ which transform as SU(2) triplets, and $\mathrm{vol}(\mathrm{S}^2)$, that is an SU(2) singlet. These form a closed set under the action of $d$ and wedge product, namely

$$dy_i \wedge \mathrm{vol}(\mathrm{S}^2) = K_i \wedge \mathrm{vol}(\mathrm{S}^2) = 0, \quad dK_i = 2y_i \mathrm{vol}(\mathrm{S}^2) \tag{A.5}$$

as well as the more obvious relations. We use this fact to reduce the 7d conditions that follow from inserting (2.29a)–(2.29b) into (2.25a)–(2.25c) to a set of 5d conditions no longer involving $\mathrm{S}^2$, (2.32a)–(2.35).

In the next section we give details on the bi spinors in 5d.

### A.2 Spinors and bi spinors on $M^5$

In (2.22) we decompose the independent 5d spinors appearing in (2.21) in terms of a single unit norm spinor in 5d, $\eta$. The bi-linears that follow from $\eta$ are given in [65], and read:

$$\begin{aligned} \eta \otimes \eta^\dagger &= \frac{1}{4}(1+V) \wedge e^{-ij_3}, \quad & \eta \otimes \eta^{c\dagger} &= \frac{1}{4}(1+V) \wedge \Omega, \\ \Omega &= w \wedge u, & j_3 &= \frac{i}{2}(w \wedge \overline{w} + u \wedge \overline{u}), \end{aligned} \tag{A.6}$$

where

$$v,\ w_1 = \mathrm{Re}w,\ w_2 = \mathrm{Im}w\ u_1 = \mathrm{Re}u,\ u_2 = \mathrm{Im}u \tag{A.7}$$

defines a vielbein in five dimensions. It then follows that if one decomposes

$$\Omega = j_1 + i j_2, \tag{A.8}$$

we have

$$j_a \wedge j_b = \frac{1}{2}\delta_{ab} \mathrm{vol}(\mathrm{M}_4) \tag{A.9}$$

where $V \wedge \mathrm{vol}(\mathrm{M}_4)$ is the volume form in 5d.



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## 5.2 AdS$_3\times$S$^2\times$M$_5$ solutions with identity structure in M$_5$

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# $\mathrm{AdS}_3\times\ \mathrm{S}^2$ in IIB with small $\mathcal{N}=(4,0)$ supersymmetry

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Abstract: We consider warped $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{M}_5$ backgrounds in type II supergravity preserving small $\mathcal{N}=(4,0)$ supersymmetry. We show that imposing $\mathcal{N}=(4,0)$ supersymmetry imposes between 0 and 3 a priori isometries in the internal $\mathrm{M}_5$. In this work we focus on classes of solution where $\mathrm{M}_5$ exhibits no a priori isometry which imposes additional constraints. Solving these in IIA forces $\mathrm{M}_5$ to support an SU(2)-structure, a class already studied in [10], while in IIB one arrives at two broad new classes with identity-structure that we reduce to local expressions for the physical fields and PDEs.

Keywords: Flux Compactifications, AdS-CFT Correspondence, Extended Supersymmetry

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## Contents



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## 1 Introduction and summary

Two dimensional superconformal algebras come in a wide variety of different types [1] which should be contrasted with their higher dimensional counterparts. The classification and construction of supersymmetric $\mathrm{AdS}_3$ string vacua realising these algebras[1] is a rich topic that is still mostly unknown. This is unfortunate because such solutions have rather broad applications with relevance to the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence, duals to surface defects in higher dimensional SCFTs and the near horizons of black strings. A particular case of some importance are small $\mathcal{N} = (4, 0)$ $\mathrm{AdS}_3$ vacua in 10 dimensions. The construction and classification of these is the focus of this work. See [3–20] for related small $\mathcal{N} = (4, 0)$ work[2] and [21–37] for some works realising other algebras.

[1] The algebras that can be embedded into $d = 10/11$ supergravity are classified in [2].

[2] $\mathrm{AdS}_2$ with small $\mathcal{N} = (4, 0)$ was also considered in [38–43].

The small $\mathcal{N}=(4,0)$ algebra is $\mathfrak{su}(1,1|2)/\mathfrak{u}(1)$ which has an SU(2) R-symmetry and comes equipped with a multiplet of supercurrents in the $\mathbf{2}\oplus\bar{\mathbf{2}}$ representation of this group. To construct an $\text{AdS}_3$ solution realising this algebra it is necessary that its internal space $\text{M}_7$ realises the R-symmetry. Specifically the bosonic supergravity fields should be SU(2) singlets while the internal spinors should transform in the $\mathbf{2}\oplus\bar{\mathbf{2}}$. This leads quite naturally to $\text{M}_7$ being a foliation of a 2-sphere over some $\text{M}_5$, which can be fibered over the $\text{S}^2$ provided SU(2) is preserved. In this work we shall assume $\text{M}_7=\text{S}^2\times\text{M}_5$ is warped product which enables us to use a set of general SU(2) spinors already constructed on this space in [10]. Under mild assumptions[3] IIA solution on this space with $\text{M}_5$ supporting an SU(2)-structure were completely classified in [10], leading to an interesting proposal for a particular $\text{AdS}_3/\text{CFT}_2$ correspondence in [11–13]. A main goal of this work is to move beyond SU(2)-structure and consider more generic classes of solutions where $\text{M}_5$ supports an identity-structure, with a view towards similar $\text{AdS}_3/\text{CFT}_2$ applications.

Generalising from SU(2) to Identity-structure significantly complicates matters, the reason is many of the at least $\frac{1}{4}$ BPS superconformal algebras contain an R-symmetry for which SU(2) is a subgroup — the most obvious being the large $\mathcal{N}=(4,0)$ algebra which contains two copies of the small algebra. It just so happens that these other algebras are inconsistent with the assumption of SU(2)-structure — for Identity-structure this is no longer the case. Rather than attempting to brute force ones way through the classification of all warped $\text{AdS}_3\times$ $\text{S}^2$ solutions, it would be beneficial to have some way to identify exactly what algebra a class of solution is realising before descending down a rabbit hole of computation. Another main motivation of this work is to provide precisely such a tool.

The lay out of the paper is as follows: in section 2 we spell out how we realise small $\mathcal{N}=(4,0)$ supersymmetry for warped $\text{AdS}_3$ solutions of type II supergravity. We begin in section 2.1 by reviewing the necessary geometric conditions for $\mathcal{N}=(1,0)$ supersymmetric $\text{AdS}_3$ [29]. In section 2.2 we explain how this may be used as a stepping stone to construct solutions with at least small $\mathcal{N}=(4,0)$ supersymmetry. We also give details of the ansatz we are taking, namely that the internal space decomposes as a foliation of the round $\text{S}^2$ over $\text{M}_5$, and construct general spinors on this space spinors consistent with an SU(2) R-symmetry — generically these give rise to an identity-structure on $\text{M}_5$. This section is supplemented by appendices A and B where we derive totally general geometric conditions on $\text{M}_5$ that imply $\mathcal{N}=(4,0)$ supersymmetry. In general these conditions are rather unwieldy, so in section 2.3 we introduce a new method to aid in the construction of $\text{AdS}_3$ solutions with extended supersymmetry: we introduce a matrix bilinear of Killing vectors which the spinors of an $\text{AdS}_3$ solution with at least $\mathcal{N}=(2,0)$ supersymmetry are necessarily charged under. This allows us to identify several things about a class of solutions a priori, first it makes clear under what conditions $\text{S}^2$ will experience an enhancement to $\text{S}^3$, second it tells us how many a priori isometries $(0,1,2,3)$ supersymmetry demands $\text{M}_5$ must contain, third it establishes exactly which algebra is being realised. We decide to focus specifically on the classes that realise exactly small $\mathcal{N}=(4,0)$ rather than large $\mathcal{N}=(4,0)$ or some other more supersymmetric algebra. We also focus on the cases where $\text{S}^2$ is not enhanced to $\text{S}^3$

[3] Consistency with a non trivial Romans mass and the presence of simple D brane and O plane sources.



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because all such solutions can be generated with string dualities from solutions with round 2-spheres. We prove this in section 2.4 where we also comment on the generality, modulo duality, of assuming that the $S^2$ realising the required SU(2) R-symmetry does not appear with additional U(1) isometries fibred over it as it could generically.

Finding all classes of solution on $AdS_3\times S^2\times M_5$ preserving small $\mathcal{N} = (4, 0)$ supersymmetry is a significant undertaking. We begin this process in section 3, by classifying solutions for which supersymmetry imposes no a priori isometries in $M_5$, as modulo duality these likely represent the most general classes. We leave the classification of solutions with 1–3 isometries in $M_5$ for future work. It turns out that imposing no a priori isometries in IIA restricts the ansatz to SU(2)-structure, already considered in [10], as such the focus of the rest of this work will be on type IIB where $M_5$ necessarily supports an identity-structure. The conditions for supersymmetry in appendix B truncate considerably, and we are able to establish that there are 2 classes of solution: for class I (section 3.2) the Bianchi identity of the RR 1-form is implied by supersymmetry while for class II (section 3.3) it is not, making only the latter compatible with co-dimension 2 sources (D7 branes etc).

We reduce the conditions for the existence of solutions in these classes to locally expressions for the supergravity fields and a set of PDEs which imply supersymmetry and the type IIB equations of motion. As this is an involved process, we begin by classifying a sub-class of class I in section 3.1, which is simple enough for us to explicitly explain the methods we apply more broadly. Once the Bianchi identities are also considered this sub-class branches into 2 cases i) D5 branes ending on NS5 branes both wrapping $AdS_3\times S^2$, ii) The T-dual of a IIA solution with a round 3-sphere. We then study class I in full generality in section 3.2, where the Bianchi identities no longer impose an obvious branching of solutions generically. Clearly the general class is more complicated, however the governing PDEs are still reminiscent of intersecting brane scenarios. We take the T-dual of a sub-case that yields a now squashed and fibred 3-sphere IIA class in section 3.2.1 which significantly generalises a class of solutions found in [17].

In section 3.3 we derive the second class of solutions with no a priori isometry in $M_5$. This class turns out to be significantly more involved, leading to a rather intimidating set of governing PDEs. Experience suggests to us that this indicates the class contains many physically distinct cases, and also that there are likely better local coordinates to express the system in terms of, at least once restrictions are made. We consider one such restriction in section 3.3.1 — that the metric is diagonal. In terms of a new set of coordinates we find two cases i) A deformed D5-NS5 brane intersection T-dual to a squashed and fibered 3-sphere. ii) A case with no necessary isometry governed by a system of PDEs generalising the D8-NS5-D6 $Mink_6$ system of [50], albeit this case is in IIB and with D7-NS5-D5 branes extended in $AdS_3\times S^2$.

Although we do not consider this here, the classification of solutions with no a priori isometry in $M_5$ should also be supplemented by the classification of solution with at least 1 isometry: ultimately although supersymmetry does not, the Bianchi identities of the fluxes impose an isometry in $M_5$ in several of the cases we consider. It is possible that such cases are restrictions of more general classes of solution where supersymmetry does indeed impose an isometry. Also it is entirely possible that one needs to consider such classes to capture



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the T-dual of every round 3-sphere class. We shall return to these issues in [52], where we shall also populate the classes of solution we find here.

## 2 Realising small $\mathcal{N} = (4, 0)$

In this section we illiterate how classes of $\mathrm{AdS}_3$ solutions in type II supergravity realising $\mathcal{N} = (4, 0)$ can be constructed. We begin by reviewing some features of supersymmetric $\mathrm{AdS}_3$ in general in section 2.1. In section 2.2 we find the general form of $\mathcal{N} = (4, 0)$ spinors on warped $\mathrm{AdS}_3\times$ $\mathrm{S}^2\times\mathrm{M}_5$ that transform in $2 \oplus \overline{2}$ of SU(2) and are also consistent with physical fields that are SU(2) singlets. In section 2.3 we introduce a method to analysis the isometry structure the spinors an $\mathrm{AdS}_3$ solution imply, allowing us to focus on small $\mathcal{N} = (4, 0)$ classes specifically. Finally in section 2.4 we explore the generality of the $\mathrm{AdS}_3\times$ $\mathrm{S}^2\times\mathrm{M}_5$ ansatz we make throughout this section. We comment on what happens when we allow $\mathrm{M}_5$ to be fibred over $\mathrm{S}^2$, and identify exactly what is not contained in our ansatz modulo duality.

### 2.1 Supersymmetric $\mathrm{AdS}_3$ in type II supergravity

We are interested in supersymmetric $\mathrm{AdS}_3$ solutions of type II supergravity. As such we restrict our attention to solutions for which the bosonic fields decompose as

$$ds^2 = e^{2A}ds^2(\mathrm{AdS}_3)+ds^2(\mathrm{M}_7), \quad H_{10} = c\mathrm{vol}(\mathrm{AdS}_3)+H, \quad F_{10} = f_\pm+e^{3A}(\mathrm{AdS}_3)\wedge\star_7\lambda(f_\pm), \tag{2.1}$$

where in IIA $f_+ = f_0 + f_2 + f_4 + f_6$ or IIB $f_- = f_1 + f_3 + f_5 + f_7$ is the magnetic part of the RR poly form $F_{10}$, $H_{10}$ is the NS 3-form and $\lambda X_n = (-1)^{[\frac{n}{2}]}X_n$ for any n-form. The fields $(e^A, f_\pm, H_3)$ and the dilaton $\Phi$ have support on $\mathrm{M}_7$ only and $c$ is a constant. The RR fluxes should obey $dF_{10} = H \wedge F_{10}$ away from sources, necessitating

$$d_H f_\pm = 0, \qquad d_H(e^{3A} \star_7 \lambda(f_\pm)) = cf_\pm, \tag{2.2}$$

in regular parts of a solution, where we define the twisted derivative $d_H = d - H\wedge$. An immediate consequence is that in IIA $cf_0 = 0$ in general, so either the NS 3-form is purely magnetic, or there is no Romans mass. In IIB one can always exploit SL(2,$\mathbb{R}$) duality to move to a duality frame with $c = 0$. In either IIA or IIB if we assume only space-time filling sources, the magnetic flux Bianchi identity gets modified in their presence but the electric one does not: taking $d_H$ of the later then implies that for $c \neq 0$, a RR source is only possible when an NS sources is also present at its loci — ie there can be no simple D brane or O plane sources when $c \neq 0$, only more exotic objects composite objects. For these reasons we shall fix

$$c = 0, \tag{2.3}$$

where more general solutions can be generated via duality, or, when they are in IIA, would be better studied from a d=11 perspective.

When an $\mathrm{AdS}_3$ solutions preserve at least $\mathcal{N} = (1, 0)$ supersymmetry it may be defined in terms of two real bi-spinors $\Psi_\pm$ [29], themselves defined in terms of two $d = 7$ Majorana spinors $\chi_{1,2}$ as

$$\Psi_+ + i\Psi_- = \chi_1 \otimes \chi_2^\dagger, \tag{2.4}$$

the r.h.s. of this expression is defined in (B.3). These bi-spinors are related to the supergravity fields by the geometric conditions[4]

$$d_H(e^{A-\Phi}\Psi_{\mp}) = 0, \tag{2.5a}$$

$$d_H(e^{2A-\Phi}\Psi_{\pm}) \mp 2me^{A-\Phi}\Psi_{\mp} = \frac{e^{3A}}{8} \star_7 \lambda(f_{\pm}), \tag{2.5b}$$

$$(\Psi_-, f_{\pm})_7 = \mp\frac{m}{2}e^{-\Phi}\text{vol}(\text{M}_7), \tag{2.5c}$$

where $\pm$ should be taken in IIA/IIB, we share the conventions of [33] and $|\chi_1|^2 = |\chi_2|^2 = e^A$. These condition are necessary and sufficient for supersymmetry, but not to have a solution of type II supergravity in general, for that one needs to also impose the RR and NS flux Bianchi identities and that, if sources are present, they have a supersymmetric embedding — the remaining EOM are then implied.

### 2.2 An ansatz for at least small $\mathcal{N} = (4, 0)$ supersymmetry

To have $\mathcal{N} = (4, 0)$ supersymmetry we must have 4 independent sets of Majorana spinors on the internal space $(\chi_1^I, \chi_2^I)$ for $I = 1, \dots, 4$, that each solve (2.5a)–(2.5c) for the same bosonic fields, ie the same metric, dilaton and fluxes. To realise small $\mathcal{N} = (4, 0)$ specifically it is necessary that $\chi^I_{1,2}$ transform in the $\mathbf{2} \oplus \overline{\mathbf{2}}$ representation of SU(2), the R-symmetry of the small $\mathcal{N} = (4, 0)$ — the bosonic fields should be singlets under its action. This means that for $K_i$, $i = 1, 2, 3$, the SU(2) Killing vectors we must have

$$\mathcal{L}_{K_i}\chi^I_{1,2} = \frac{i}{2}(\Sigma_i)^{IJ}\chi^J_{1,2}, \qquad \mathcal{L}_{K_i}(A, \Phi, g(\text{M}_7), f, H_3) = 0, \tag{2.6}$$

where $\frac{i}{2}\Sigma_i$ span the $\mathbf{2} \oplus \overline{\mathbf{2}}$ of $\mathfrak{su}(2)$. This provides a map between each of the 4 $\mathcal{N} = 1$ sub-sectors of $(\chi_1^I, \chi_2^I)$, and one can show that if a single one of these solves a set of sufficient conditions for $\mathcal{N} = 1$ supersymmetry the other 3 also necessarily solve these conditions [34]. The non trivial part is constructing a set of spinors such that (2.6) holds.

Given that we need an SU(2) R-symmetry it should not be hard to convince oneself that $\text{M}_7$ needs to decompose in terms of a 2-sphere and some 5 manifold $\text{M}_5$. The 2-sphere could be the round one, or $\text{M}_5$ could be fibered over it such that SU(2) is preserved. We will make the ansatz that this 2-sphere is the round one, and discuss the generality of this assumption in section 2.4. We shall thus refine (2.1) in terms of a unit radius 2-sphere as

$$ds^2(\text{M}_7) = e^{2C}ds^2(\text{S}^2) + ds^2(\text{M}_5), \quad H = e^{2C}H_1\wedge\text{vol}(\text{S}^2) + H_3, \quad f_+ = g_{1\pm} + e^{2C}g_{2\pm}\wedge\text{vol}(\text{S}^2), \tag{2.7}$$

where $(e^A, e^C, \Phi, g_1, g_2, H_1, H_3)$ have support on $\text{M}_5$ alone which does not depend on the $\text{S}^2$ coordinates. A general set of Majorana SU(2) spinors transforming in the $\mathbf{2} \oplus \overline{\mathbf{2}}$ were already derived on this geometry in [10], they are

$$\chi_1^I = \frac{e^{\frac{A}{2}}}{\sqrt{2}}(\mathcal{M}^I)_{\alpha\beta}\Big(\xi^\alpha \otimes \eta^\beta_{11} + i\sigma_3\xi^\alpha \otimes \eta^\beta_{12}\Big), \quad \chi_2^I = \frac{e^{\frac{A}{2}}}{\sqrt{2}}(\mathcal{M}^I)_{\alpha\beta}\Big(\xi^\alpha \otimes \eta^\beta_{21} + i\sigma_3\xi^\alpha \otimes \eta^\beta_{22}\Big), \tag{2.8}$$

[4] These conditions hold for $c = 0$, the general conditions were only recently derived in [36].



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where $\mathcal{M}^I = (\sigma_2\sigma_1, \sigma_2\sigma_2, \sigma_2\sigma_3, -i\sigma_2)^I$. Here $\xi^\alpha, \sigma_3\xi^\alpha$ are independent SU(2) doublets of Killing spinors on $\mathrm{S}^2$ (see appendix B.2) and $\eta^\alpha_{ij}$ are two component vectors with spinorial entries: specifically there are 4 independent spinors on $\mathrm{M}_5$ namely $\eta_{11}, \eta_{12}, \eta_{21}, \eta_{22}$ and $\eta^\alpha_{ij}$ depend on these and their Majorana conjugates $\eta^c_{ij}$ (see appendix B.1). The specific representation appearing in (2.6) for these spinors is

$$\Sigma_i = (\sigma_2 \otimes \sigma_1, \ -\sigma_2 \otimes \sigma_3, \ \mathbb{I} \otimes \sigma)_i, \tag{2.9}$$

which one can confirm is equivalent to the $\mathbf{2} \oplus \overline{\mathbf{2}}$. In what follows we shall take our $\mathcal{N} = 1$ sub-sector to be

$$\chi_1 = \chi_1^2, \quad \chi_2 = \chi_2^2 \tag{2.10}$$

Inserting (2.10) into the supersymmetry conditions (2.5a)–(2.5c) leads to a set of 5d bi-linear constraints that we derive in appendix B resulting in a sufficient but highly degenerate system of constraints (B.15a)–(B.18) for IIA and (B.20a)–(B.23) in IIB. These conditions are rather crude, and the main purpose of this and the next section is to refine them. First using (2.10) to compute $|\chi_{1,2}|^2$ it becomes apparent they generically depend on the SU(2) embedding coordinates $y_i$, which are SU(2) triplets. Fixing $|\chi_{1,2}|^2 = e^A$ as supersymmetry demands requires that we impose

$$\begin{aligned} \eta^{c\dagger}_{12}\eta_{11} = \mathrm{Im}\left(\eta^\dagger_{12}\eta_{11}\right) = \eta^{c\dagger}_{22}\eta_{21} = \mathrm{Im}\left(\eta^\dagger_{22}\eta_{21}\right) = 0,\\ |\eta_{11}|^2 + |\eta_{12}|^2 = |\eta_{21}|^2 + |\eta_{22}|^2 = 1. \end{aligned} \tag{2.11}$$

In order to solve these it is helpful to decompose the spinors in a common basis in terms of a single unit norm spinor $\eta$. Such a 5d spinor defines an SU(2)-structure in 5d as

$$\eta \otimes \eta^\dagger = \frac{1}{4}(1+V)\wedge e^{-iJ}, \quad \eta \otimes \eta^{c\dagger} = \frac{1}{4}(1+V)\wedge\Omega, \tag{2.12}$$

where $J$ is a (1,1)-form and $\Omega$ as (3,0)-form, they are defined on the sub-manifold $\mathrm{M}_4 \subset \mathrm{M}_5$ orthogonal to the real 1-form $V$, and obey

$$J^3 = \frac{3i}{4}\Omega\wedge\overline{\Omega}, \qquad J\wedge\Omega = 0. \tag{2.13}$$

This leads to a natural decomposition of the internal 5-manifold as

$$ds^2(\mathrm{M}_5) = V^2 + ds^2(\mathrm{M}_4). \tag{2.14}$$

We can decompose a generic spinor $\tilde{\eta}$ in terms of $\eta$, a holomorphic 1-form on $\mathrm{M}_4$ $Z$ and some complex functions $p_1, p_2, p_3$ as

$$\tilde{\eta} = p_1\eta + p_2\eta^c + \frac{|p_3|}{2}\overline{Z}\eta, \quad Z\eta = 0. \tag{2.15}$$

Using these facts, and after a lengthy calculation one can show that a set of general 5d spinors solving (2.11) are given by

$$\begin{aligned} \eta_{11} &= \sin\left(\frac{\alpha_1+\alpha_2}{2}\right)\eta, \quad \eta_{12} = \cos\left(\frac{\alpha_1+\alpha_2}{2}\right)\left(\cos\beta_1 + \sin\beta_1\frac{1}{2}\overline{Z}_1\right)\eta\\ \eta_{21} &= \sin\left(\frac{\alpha_1-\alpha_2}{2}\right)\eta_1, \quad \eta_{22} = \cos\left(\frac{\alpha_1-\alpha_2}{2}\right)\left(\cos\beta_2\eta_1 + \sin\beta_2\left(d_1\eta_2 + d_2\eta_2^c\right)\right) \end{aligned} \tag{2.16}$$

where

$$\eta_1 = \sqrt{1-c^2}(a\eta + b\eta^c) + \frac{c}{2}\overline{Z}_2\eta, \quad \eta_2 = c(a\eta + b\eta^c) - \sqrt{1-c^2}\frac{1}{2}\overline{Z}_2\eta. \tag{2.17}$$

Here we have to introduce two generic holomorphic 1-forms $Z_1, Z_2$, several real functions of $\mathrm{M}_5$ $(\alpha_{1,2}, \beta_{1,2}, c)$ for $|c| \leq 1$ and some complex ones constrained as

$$|a|^2 + |b|^2 = |d_1|^2 + |d_2|^2 = 1. \tag{2.18}$$

We can assume that $\eta_{11}$ never vanishes without loss of generality, but the other spinors contain some redundancy when certain parts of the other spinors are turned off. For instance when $c = 1$ we can fix $(d_1 = |d_1|, d_2 = 0)$ without loss of generality, while when $\eta_{12} = 0$ we may also fix $\beta_2 = 0$ without further cost. The presence of $(Z_1, Z_2)$ in (2.16) indicates that $\mathrm{M}_4$ generically supports an identity-structure, however when $(c = 0 = \beta_1 = \beta_2 = 0)$ the 1-forms drop out and this becomes an SU(2)-structure- in IIA this case was already completely classified in [10]. Generically $(Z_1, Z_2)$ are neither parallel nor orthogonal, rather in general they may be used to define 2 complex functions $(z, \tilde{z})$ as

$$z = \frac{1}{2}\iota_{\overline{Z}_1} Z_2, \quad \tilde{z} = \frac{1}{4}\iota_{\overline{Z}_1}\iota_{\overline{Z}_2}\Omega, \quad |z|^2 + |\tilde{z}|^2 = 1, \tag{2.19}$$

which vanish when the 1-forms are respectively orthogonal and parallel. Only when $z = 0$ do $(Z_1, Z_2)$ define a vielbein on $\mathrm{M}_4$, otherwise one can assume that $(Z_1, \frac{1}{2}\iota_{Z_1}\Omega)$ do with out loss of generality with $Z_2$ defined along each of these. At this point one can proceed to try to solve $d = 5$ supersymmetry conditions derived in the appendix B. A first import thing to note is that these contain several algebraic constraints: in IIA (B.15a)–(B.18) imply the following conditions

$$\begin{aligned}
&\eta_{21}^{c\dagger}\eta_{11} = \eta_{22}^{c\dagger}\eta_{12}, \qquad \mathrm{Im}(\eta_{21}^{\dagger}\eta_{11}) = \mathrm{Im}(\eta_{22}^{\dagger}\eta_{12}),\\
&(1 + 2me^{C-A})\eta_{22}^{c\dagger}\eta_{11} = (1 - 2me^{C-A})\eta_{21}^{c\dagger}\eta_{12},\\
&(1 + 2me^{C-A})\mathrm{Im}(\eta_{22}^{\dagger}\eta_{11}) = (1 - 2me^{C-A})\mathrm{Im}(\eta_{21}^{\dagger}\eta_{12}).
\end{aligned} \tag{2.20}$$

In IIB on the other hand (B.20a)–(B.23) imply

$$\begin{aligned}
&\eta_{22}^{c\dagger}\eta_{11} = \eta_{21}^{c\dagger}\eta_{12}, \qquad \mathrm{Im}(\eta_{22}^{\dagger}\eta_{11}) = \mathrm{Im}(\eta_{21}^{\dagger}\eta_{12}),\\
&(1 + me^{C-A})\eta_{21}^{\dagger}\eta_{11} = (1 - me^{C-A})\eta_{22}^{\dagger}\eta_{12},\\
&(1 + me^{C-A})\mathrm{Im}(\eta_{21}^{c\dagger}\eta_{11}) = (1 - me^{C-A})\mathrm{Im}(\eta_{22}^{c\dagger}\eta_{12}).
\end{aligned} \tag{2.21}$$

Unfortunately plugging our spinor ansatz into the IIA or IIB algebraic conditions leads to a lot of branching possibilities, and the constraints are rather intractable in general — though progress can be made making assumptions. Rather than attempting a brute force approach it would be beneficial to have some additional guiding principle. Here it is opportune to make one point clear: there are rather a lot of superconformal algebras consistent with $\mathrm{AdS}_3$, and many of those preserving at least 8 chiral super charges can admit solutions consistent with the ansatz taken so far. It makes sense to attempt to zoom in on those preserving the small algebra specifically, that is what we seek after all. Additionally what



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remains would be better tackled with a more specialised ansatz. For example, large $\mathcal{N}=4$ is consistent with the ansatz taken thus far, but for that we know $\mathrm{M}_5$ must contain a 2 or 3-sphere and it simplify matters to assume its presence from the start.

In the next section we shall narrow our focus to solutions that specifically realise the small $\mathcal{N}=(4,0)$ algebra rather than something larger.

### 2.3 Isolating the small algebra and a priori isometries in $\mathrm{M}_5$

In this section we introduce a method to restrict the internal spinors of an $\mathrm{AdS}_3$ solution with extended supersymmetry to those that realise a particular superconformal algebra, in this case small $\mathcal{N}=(4,0)$, though let us stress that this technique could be applied to any algebra with at least $\mathcal{N}=(2,0)$ extended supersymmetry.

In [44] generic supersymmetric solutions of type II supergravity are classified, one of their findings is that supersymmetry implies that the following $d=10$ bi-linear

$$K^{(10)}=\frac{1}{64}(\bar{\epsilon}_1\Gamma^M\epsilon_1+\bar{\epsilon}_1\Gamma^M\epsilon_1)\partial_M, \tag{2.22}$$

is a Killing vector with respect to all the bosonic fields. Additionally the $d=10$ Majorana Weyl spinors $\epsilon_{1,2}$ are singlets with respect to it. Taking an $\mathcal{N}=(2,0)$ $\mathrm{AdS}_3$ spinor ansatz for $\epsilon_{1,2}$ involving two sets of $d=7$ spinors $(\chi^1_{1,2},\chi^2_{1,2})$ it then necessarily follows that

$$K^{(\mathcal{N}=2)}=-i(\chi^1_1(\gamma^{(7)})^a\chi^2_1\mp\chi^1_2(\gamma^{(7)})^a\chi^2_2)\partial_a,\quad a=1,\dots,7 \tag{2.23}$$

defines a Killing vector on the internal space under which $(\chi^1_{1,2},\chi^2_{1,2})$ are charged and the bosonic fields are singlets. The computation is similar to that appearing in [45], and will appear for the $\mathrm{AdS}_3$ case in [46]. For a set of $\mathcal{N}=(n,0)$ spinors one can define $\frac{n}{2}(n-1)$ such $\mathcal{N}=(2,0)$ sub-sectors, so it follows that

$$K^{IJ}=-i(\chi^{I\dagger}_1(\gamma^{(7)})^a\chi^J_1\mp\chi^{I\dagger}_2(\gamma^{(7)})^a\chi^J_2)\partial_a, \tag{2.24}$$

defines an antisymmetric matrix of Killing vectors under which the bosonic fields are singlets, and $\chi^I_{1,2}$ are charged. The entries of $K^{IJ}$ (modulo antisymmetry) are not however necessarily independent.[5] It is thus natural to identify $K^{IJ}$ with the Killing vectors associated to the R-symmetry, this is almost correct, but generically $K^{IJ}$ could be a linear combination of these and a number of flavour isometries.

We shall now turn our attention to our $\mathcal{N}=(4,0)$ spinors (2.8). For these (2.24) must decompose in terms of vector bi-linears on $\mathrm{S}^2$ and $\mathrm{M}_5$, one can show that the former are

$$\begin{aligned}
\xi^{\alpha\dagger}\xi^\beta&=\delta^{\alpha\beta}, & \xi^{\alpha\dagger}\sigma_3\xi^\beta&=-y_i(\sigma_i)^{\alpha\beta},\\
\xi^{\alpha\dagger}\gamma^\mu_2\xi^\beta\partial_\mu&=(k_i)^\mu\partial_\mu(\sigma_i)^{\alpha\beta}, & \xi^{\alpha\dagger}\gamma^\mu_2\sigma_3\xi^\beta\partial_\mu&=-i(dy_i)^\mu\partial_\mu(\sigma_i)^{\alpha\beta},
\end{aligned}$$

where $\gamma^\mu_2$ are curved 2d gamma matrices, $K_i=\epsilon_{ijk}dy_jy_k$ are the 1-forms dual to the SU(2) Killing vectors on $\mathrm{S}^2$ and $y_i$ are unit norm embedding coordinates. The relevant 5d bi-linears

[5]If it is not clear this is not a weakness: small $\mathcal{N}=(4,0)$ comes with 3 independent Killing vectors, a generic antisymmetric $4\times4$ matrix 6 independent components, so we need half of them to be dependent in this case.

can be expressed in terms of the following 0 and 1-form bi-linears

$$\begin{aligned}
\mathcal{F}_1 &= 2e^{A-C}\left(\eta_{12}^\dagger\eta_{11} \mp \eta_{22}^\dagger\eta_{21}\right),\\
\mathcal{F}_2 &= e^{A-C}\left(|\eta_{11}|^2 - |\eta_{12}|^2 \mp \left(|\eta_{21}|^2 - |\eta_{22}|^2\right)\right),\\
\mathcal{V} &= e^{A}\left[\eta_{11}^\dagger\gamma_a\eta_{11} + \eta_{12}^\dagger\gamma_a\eta_{12} \mp \left(\eta_{21}^\dagger\gamma_a\eta_{21} + \eta_{22}^\dagger\gamma_a\eta_{22}\right)\right]e^a, \qquad (2.25)\\
\mathcal{U}_i &= 2e^{A}\Big(\text{Re}\left(\eta_{11}^{c\dagger}\gamma_a\eta_{12} \mp \eta_{21}^{c\dagger}\gamma_a\eta_{22}\right)e^a\,, \text{Im}\left(\eta_{11}^{c\dagger}\gamma_a\eta_{12} \mp \eta_{21}^{c\dagger}\gamma_a\eta_{22}\right)e^a\,,\\
&\qquad \text{Im}\left(\eta_{11}^{\dagger}\gamma_a\eta_{12} \mp \eta_{21}^{\dagger}\gamma_a\eta_{22}\right)e^a\Big)_i\,, \qquad (2.26)
\end{aligned}$$

where $a$ is a flat index on $\text{M}_5$. Equipped with these definitions we find for our ansatz that (2.24) becomes

$$K^{IJ} = 2\Big[-y_i\mathcal{V}^a\partial_a + \mathcal{F}_1\,(dy_i)^\mu\,\partial_\mu + \mathcal{F}_2\,(K_i)^\mu\,\partial_\mu\Big]\left(\frac{i}{2}\Sigma^i\right)^{IJ} + 2\mathcal{U}_i^a\partial_a\left(\frac{i}{2}\tilde{\Sigma}^i\right)^{IJ}, \qquad (2.27)$$

where $\tilde{\Sigma}^i = (\sigma_2\otimes\mathbb{I},\ \sigma_1\otimes\sigma_2,\ \sigma_3\otimes\sigma_2)^i$ is another SU(2) representation. Clearly that this object should be Killing imposes some constraints, as $(K_i)^\mu$ are the Killing vectors on the 2-sphere one might imagine that we should arrange for all the rest of the terms to vanish. However, the ansatz we take is consistent with $\text{S}^2$ becoming enhanced to a round 3-sphere as

$$e^{2C}ds^2(\text{S}^2) + ds^2(\text{M}_5) = e^{2C}(d\theta^2 + \sin^2\theta ds^2(\text{S}^2)) + ds^2(\tilde{\text{M}}_4). \qquad (2.28)$$

When this is the case both $(K_i)^\mu\partial_\mu$ and

$$\tilde{K}_i = y_i\partial_\theta + \cot\theta(dy_i)^\mu\partial_\mu, \qquad (2.29)$$

are SU(2) Killing vectors of respectively the anti-diagonal and diagonal SU(2) subgroups of $\text{SO}(4) = \text{SU}(2)_L\times\text{SU}(2)_R$. As such when $\text{S}^2\to\text{S}^3$, the R-symmetry Killing vectors are $(K_i)^\mu\partial_\mu + \tilde{K}_i$. None the less we shall impose

$$\mathcal{V} = \mathcal{F}_1 = 0, \qquad \mathcal{F}_2 = \text{constant} \neq 0, \qquad (2.30)$$

by doing so we are excluding the possibility of round 3-sphere solutions. The reasons for doing this are the following: first solutions with a round $\text{S}^3$ can be mapped to those with a round $\text{S}^2$ via duality, as we explain in more detail in section 2.4 — so all $\text{AdS}_3\times\text{S}^3$ solutions can be generated from a full classification of $\text{AdS}_3\times\text{S}^2$ solutions. Second a main benefit of $\text{AdS}_3\times\text{S}^3$ vacua is that they can support $\mathcal{N}=(4,4)$ supersymmetry. However we are blind to this using the chiral $\mathcal{N}=(1,0)$ conditions of (2.5a)–(2.5c). The $\mathcal{N}=(1,1)$ conditions of [36] are better suited to addressing this, as such we leave a study of $\mathcal{N}=(4,4)$ $\text{AdS}_3\times\text{S}^3$ for future work and focus on solutions satisfying (2.30).

The final term $\mathcal{U}_i^a\partial_a$ is a bit more subtle, it is not charged under the SU(2) of $\text{S}^2$ but it still generically defines a 3 Killing vectors, this time in $\text{M}_5$. Let us stress however that $\mathcal{U}_i^a\partial_a$ need not span all isometries in $\text{M}_5$ just the number of a priori isometries supersymmetry demands, more may get imposed by the Bianchi identities or one could choose to impose additional isometries in a more general class. When the $d=5$ spinors are not charged under

$\mathcal{U}_i^a \partial_a$, the solution still preserves the small $\mathcal{N} = (4,0)$ algebra. If the 5d spinors are charged under $\mathcal{U}_i^a \partial_a$ then the algebra experiences some enhancement: for example, if these Killing vectors span a 3-sphere then the ansatz becomes consistent with large $\mathcal{N} = (4,0)$ on $\mathrm{S}^2 \times \mathrm{S}^3$. For small $(4,0)$ specifically then we should impose that $\mathcal{U}_i^a \partial_a$ are flavour isometries, but not that they should be zero in general. Instead one should be able to construct several different types of solution classes, distinguished by the number of a priori isometries $\mathrm{M}_5$ necessarily comes equipped with (ie 0,1,2,3). That these should be uncharged isometries means that one is free to T-dualise on them: this suggests that one can generate much of what is contained in the small $\mathcal{N} = (4,0)$ classes with a priori isometries from the classes with no such isometries via duality.

### 2.4 On the generality of the round $\mathrm{S}^2$ ansatz modulo duality

In this section we shall comment on the generality of the ansatz we have made thus far. In particular we shall illustrate how much of what is omitted by the round 2-sphere ansatz we have made can actually be generated with duality starting from a round $\mathrm{S}^2$ classification of both IIA and IIB.

In (2.7) we have made the assumption that the SU(2) R-symmetry is realised by a round $\mathrm{S}^2$. Generically one could look for solutions where $\mathrm{M}_5$ is fibered over the $\mathrm{S}^2$ such that SU(2) is preserved. However the SU(2) of the 2-sphere is only lifted to an isometry of this full space if the connection 1-forms that mediate this transform as gauge fields with respect to SU(2) [34]. As we also need spinors transforming in the $\mathbf{2} \oplus \overline{\mathbf{2}}$ this restricts the additional possibilities to $\mathrm{M}_7$ containing a torus fibration over $\mathrm{S}^2$ with spinors that are singlets under of the U(1)s of the torus,[6] ie the metric must decompose as

$$\begin{aligned} ds^2(\mathrm{M}_7) &= \frac{1}{4}\left[\sum_{q=1}^{n} e^{2C_q} D\psi_q^2 + e^{2C} ds^2\left(\mathrm{S}^2\right)\right] + ds^2(\mathrm{M}_{5-n}), \\ D\psi_q &= (d\psi_q - \eta + \mathcal{A}_q), \quad d\eta = \mathrm{vol}\left(\mathrm{S}^2\right), \end{aligned} \tag{2.31}$$

where $(e^A, e^{C_q}, e^C, \Phi, \mathcal{A}_q)$ depend on $\mathrm{M}_{5-n}$ only and the fluxes may only depend on the $\mathbb{T}^n$ and $\mathrm{S}^2$ directions via $(D\psi_q, \mathrm{vol}(\mathrm{S}^2))$ only. We would now like to establish when a solution of this type can be generated from a round 2-sphere solution by duality. For simplicity let us just assume that $n = 1$ so that we have simply an $\mathrm{SU}(2)\times\mathrm{U}(1)$ squashed 3-sphere. The case of generic $n$ works analogously, one simply needs to apply T-duality more times. Such solutions may be distinguished by the form the NS 3-form takes, in general this can depend on the $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariants as

$$H_3 = h_0 D\psi \wedge \mathrm{vol}(\mathrm{S}^2) + D\psi \wedge H_2 + e^{2C_2} H_1 \wedge \mathrm{vol}(\mathrm{S}^2) + \tilde{H}_3 \tag{2.32}$$

where $h_0$ is a constant. Since $\partial_\psi$ should be a flavour isometry one is free to T-dualise on it without breaking the $(4,0)$ supersymmetry. When $h_0 = 0$ this maps a solution in IIA/IIB

[6] To be more precise we mean the $\mathcal{N} = (4,0)$ spinors, a geometry might also support additional spinors charged under one or more of the U(1)s, but they would need to be $(0,n)$ spinors that are singlets of SU(2). Thus they are auxiliary to this argument.



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to IIB/IIA with a round 2-sphere. When $h_0 \neq 0$ T-duality maps one between solutions with squashed 3-spheres and non trivial $h_0$. However, in the IIA duality frame of such a solution, we must have that the magnetic RR 2-form decomposes as

$$f_2 = \tilde{g}_0 \text{vol}(\text{S}^2) + \tilde{g}_1 \wedge D\psi + \tilde{g}_2, \tag{2.33}$$

none of which give rise to $D\psi \wedge \text{vol}(\text{S}^2)$ under $d$. Thus similar to our discussion below (2.2), the Bianchi identity of the RR 2-form imposes

$$f_0 h_0 = 0. \tag{2.34}$$

If we take $f_0 = 0$ we can lift this solution to $d = 11$. Generically such a solution would have a $\mathbb{T}^2$ (spanned by $\psi$ and the M-theory circle) fibered over $\text{S}^2$, but if $(\tilde{g}_0 = 0, \tilde{g}_1 = 0)$ the M-theory circle is not fibered over the $\text{S}^2$, so if we reduce back to IIA now on $\psi$ we arrive at a round $\text{S}^2$ in IIA. Note in particular that this means that all solutions with a round $\text{S}^3$ can be mapped by duality to solutions in type II with a round $\text{S}^2$ justifying our assumption (2.30). More generally one can cover almost all small $\mathcal{N} = (4, 0)$ solutions of type II supergravity modulo duality, by considering a round $\text{S}^2$ in each of IIA and IIB, what remains is rather constrained and could be studied modulo duality from an 11d perspective.

In the next section we shall classify $\text{AdS}_3{\times}\text{S}^2$ solutions containing no a priori isometry in $\text{M}_5$.

## 3 Classes of solution with no a priori isometry

In this section we classify solutions on $\text{AdS}_3{\times}\text{S}^2{\times}\ \text{M}_5$ that preserve small $\mathcal{N} = (4, 0)$ supersymmetry and contain no a priori isometry in $\text{M}_5$. As we shall see, only in IIB does this lead to new classes of solution, specifically 2 distinguished by their compatibility with D7 brane (like) sources. Class I, for which $dF_1 = 0$ globally, can be found in section 3.2 while class II, for which $dF_1 = 0$ need only hold away from the loci of sources, can be found in section 3.3. For both classes we reduce the conditions that define the existence of supersymmetric solutions to local expressions for the supergravity fields and a number of PDEs. We illiterate the methods we employ to achieve this by deriving a sub-class of class I in explicit detail in section 3.1. The general classes are rather broad, so we also consider some restricted cases of interest in sections 3.2.1 and 3.3.1.

In the previous sections we give criteria to classify solutions on $\text{AdS}_3{\times}\text{S}^2 \times \text{M}_5$, namely we define a set of Killing vector bi-linears (2.27) which can be used to establish when

1. $\text{S}^2$ does not experience an enhancement to $\text{S}^3$: $\mathcal{V} = \mathcal{F}_1 = d\mathcal{F}_2 = 0$
2. $\text{M}_5$ contains additional a priori isometries: spanned by independent components of $\mathcal{U}_i$

We reiterate that we discard 3-sphere solutions as they can all be mapped to round 2-sphere solutions with duality. A class of solutions preserves small $\mathcal{N} = (4, 0)$ rather than large $\mathcal{N} = (4, 0)$ or some more supersymmetric algebra when $\mathcal{U}_i$ is spanned by flavour isometries in $\text{M}_5$. This is a little tricky to impose on our spinor ansatz a priori, so a practical approach is to classify solutions with 0,1,2,3 a priori isometries one at a time. When we simply fix $\mathcal{U}_i = 0$, there are no a priori isometries in $\mathrm{M}_5$ to worry about so solutions certainly preserve just small $\mathcal{N} = (4,0)$. The focus of the rest of this work will be to classify all such solutions, leaving $\mathcal{U}_i \neq 0$ for future work. We expect that much of what can be derived for the cases of $1,2,3$ flavour isometries can be generated from classes with no a priori isometry via duality, though we are not claiming that it all can: for instance the round $\mathrm{S}^2$ element of the duality orbit of some 3-sphere classes may necessitate 1 a priori isometry. None the less classifying $\mathcal{U}_i = 0$ is a sensible place to start as, mod duality, these are likely the most general classes with exactly[7] small $\mathcal{N} = (4,0)$.

In both IIA and IIB it is possible to solve both the 2-sphere and no a priori isometry constraints in general. For IIA, we already know that the SU(2)-structure classes of [10], contain no a prior isometry in $\mathrm{M}_5$. Under the assumption that the functions appearing in (2.16) are not tuned so as to necessarily reduce the ansatz to SU(2)-structure, it is possible to show that (2.20), (2.30) and $\mathcal{U}_i = 0$ can be solved as

$$\begin{aligned} &a = d_1 = 1, \quad Z_1 = Z_2, \quad b = d_2 = 0, \quad \alpha_2 = \frac{1}{2}(\beta_1 + \beta_2) = \frac{\pi}{2}, \\ &c = \frac{\sin\alpha_1 \sin\beta_2}{\sqrt{\cos\beta_2^2 + \sin^2\alpha_1 \sin^2\beta_2}}, \quad d(e^{A-C}\sin\alpha_1) = 0 \end{aligned} \tag{3.1}$$

and $\sin\alpha_1 \neq 0$ without loss of generality. Unfortunately this ansatz dies off very quickly once the remaining supersymmetry conditions are considered. This can easily be seen from the 1-form part of (B.15c) which demands that

$$c\cos\alpha_1 = 0 \tag{3.2}$$

for this tuning of the functions of the spinor ansatz. Setting either factor in the above to zero reduces the spinors to an SU(2)-structure ansatz anyway. Thus under our assumptions we can conclude that

$$\text{IIA} + \text{no apriori isometry in } \mathrm{M}_5 \quad \Rightarrow \quad \text{SU(2)-structure} \tag{3.3}$$

as these solutions are already classified we shall not comment on them further here.

For our purposes the status of IIB is more promising: it is possible to show that (2.21), (2.30) and $\mathcal{U}_i = 0$ can be solved without loss of generality as

$$a = d_1 = c = 1, \quad Z_1 = Z_2 = Z, \quad b = d_2 = 0, \quad \alpha_1 = \frac{\pi}{2} \quad \beta_2 = -\beta_1 = \beta, \quad d(e^{A-C}\cos\alpha) = 0 \tag{3.4}$$

where $\cos\alpha_2 = \cos\alpha \neq 0$ — note we are dropping the indices on $(\alpha_2, \beta_1)$. Unlike IIA this tuning of the spinor ansatz necessarily yields an identity-structure, and does not collapse once the rest of the supersymmetry constraints are considered. Plugging the spinors (2.16) for the tuning (3.4) into (B.20a)–(B.23) one finds that these conditions truncate rather a lot, to show this we find it convenient to redefine the vielbein on $\mathrm{M}_5$ in terms of $\{U, e^1, e^2, e^3, K\}$ as

$$\begin{aligned} &\mathrm{Re}Z = -\cos\beta K + \sin\beta U, \quad V = \cos\beta U + \sin\beta K, \\ &e^1 = -\frac{1}{2}\mathrm{Im}\iota_{\overline{Z}}\Omega, \quad e^2 = \frac{1}{2}\mathrm{Re}\iota_{\overline{Z}}\Omega, \quad e^3 = \mathrm{Im}Z, \end{aligned} \tag{3.5}$$

[7] More correctly at most 8 left chiral supercharges, there could be additional right chiral supercharges.

with orientation $\text{vol}(\text{M}_7) = e^{2C}\text{vol}(\text{S}^2)\wedge\text{vol}(\text{M}_5)$ for $\text{vol}(\text{M}_5) = U\wedge e^{123}\wedge K$. One then finds that the IIB avatars of (2.5a)–(2.5b) are implied by the following conditions independent of the RR fluxes

$$e^C = \frac{1}{2m}e^A\cos\alpha,\quad 2me^{2C}H_1 = -d\left(e^{A+C}\cos\beta\sin\alpha\right) - e^A\left(\cos\beta U - \cos\alpha\sin\beta K\right), \tag{3.6a}$$

$$d(e^A\left(\cos\beta U - \cos\alpha\sin\beta K\right) = d\left(e^{2A-\Phi}\cos\alpha U\wedge K\right) = 0, \tag{3.6b}$$

$$d\left(e^{-A-C+\Phi}\sin\beta\sin\alpha\right) - e^{-A-2C+\Phi}\sin^2\alpha\sin\beta U = d\left(e^{2A+C-\Phi}e^i\right) + e^{2A-\Phi}\sin\alpha U\wedge e^i = 0, \tag{3.6c}$$

$$\begin{aligned}&\frac{1}{2}\epsilon_{ijk}\left[d\left(e^{2A+C-\Phi}e^j\wedge e^k\wedge(\cos\alpha\sin\beta U + \cos\beta V)\right) + e^{2A-\Phi}\cos\beta\sin\alpha e^j\wedge e^k\wedge U\wedge K\right]\\&\qquad = e^{2A+C-\Phi}H_3\wedge e^i,\\&d\left(e^{2A-\Phi}e^{123}\wedge(\cos\alpha\cos\beta U - \sin\beta K)\right) = e^{2A-\Phi}\cos\alpha U\wedge K\wedge H_3,\end{aligned} \tag{3.6d}$$

and define the components of the magnetic portions of the RR fluxes that are respectively orthogonal and parallel to $\text{vol}(\text{S}^2)$ as

$$\begin{aligned}e^{3A+2C}\star_5\lambda g_{1+} &= d(e^{3A+2C-\Phi}(\sin\beta U + \cos\alpha\cos\beta K))\\&\quad - e^{3A+2C-\Phi}\sin\alpha H_1\wedge K + 2me^{2A+2C-\Phi}\cos\beta\sin\alpha U\wedge K\\&\quad - d(e^{3A+2C-\Phi}\cos\alpha e^{123}) + e^{3A+2C-\Phi}(\sin\beta U + \cos\alpha\cos\beta K)\wedge H_3\\&\quad + e^{3A+2C-\Phi}\cos\beta\sin\alpha H_1\wedge e^{123} + 2me^{2A+2C-\Phi}\sin\alpha e^{123}\wedge U,\end{aligned} \tag{3.7a}$$

$$\begin{aligned}e^{3A}\star_5\lambda g_{2+} &= -d(e^{3A-\Phi}\sin\alpha K) + 2me^{2A-\Phi}\cos\alpha U\wedge K - e^{3A-\Phi}\sin\alpha K\wedge H_3\\&\quad + d(e^{3A-\Phi}\cos\beta\sin\alpha e^{123}) + 2me^{2A-\Phi}e^{123}\wedge(\cos\alpha\cos\beta U - \sin\beta K).\end{aligned} \tag{3.7b}$$

Given these, one can eliminate the flux terms from (2.5c) arriving at a single condition[8]

$$\begin{aligned}&\Big(d\left(e^{2A}\cos\beta\cos\alpha\sin\alpha\right)\wedge(\sin\beta U + \cos\beta\cos\alpha K)\\&\qquad - \sin(2\alpha)\left(d\left(e^{2A}\cos\alpha\right) + me^A\sin\alpha U\right)\wedge K\Big)\wedge e^{123} = 0.\end{aligned} \tag{3.8}$$

Of this system of sufficient conditions for supersymmetry, (3.6a) and (3.7a)–(3.7b) just act as definitions for certain physical fields — it is (3.6b)–(3.6d) and (3.8) that we must actively solve. To this end one first needs to define local coordinates and a vielbein from the conditions in (3.6b)–(3.6c), then plug this into the remaining conditions, extracting $H_3$ and PDEs that imply the remaining conditions. Of course this only fixes the local form of the physical fields up to some PDEs that imply supersymmetry, in addition to solving these, to actually have a solution we must also solve the Bianchi identities of the NS and magnetic RR fluxes which imply additional PDEs. The conditions (3.6a)–(3.8) actually give rise to 2 physically distinct classes; $\sin\beta = 0$ or not. To establish this one must solve the supersymmetry conditions under the assumptions that $\sin\beta \neq 0$ and $\cos\beta \neq 0$, one

[8]Generically (2.5c) decomposes in terms of SU(2) singlet and triplet contributions, but in this case the latter are implied.

finds that the Bianchi identity of $F_1$ is implied by supersymmetry in the former case but not the latter — ie source D7 branes are only possible when $\sin\beta = 0$. A special limit is when $\cos\beta = 0$, though this is part of the more general $\sin\beta \neq 0$ class, in this limit the PDEs governing the system give rise to a harmonic function constraint common to partially localised brane intersections [48].

In the next section we give the local form of the class of solution with $\cos\beta = 0$ turning our attention to the general case in section 3.2.

### 3.1 $\cos\beta = 0$: A sub-class of class I with harmonic function rule

In this section we derive the class of solutions that follows from fixing $\cos\beta = 0$, we can set $\beta = \frac{\pi}{2}$ without loss of generality. This is actually a limit of the more general class I with $\sin\beta \neq 0$ in the next section but classifying that in this limit has value as it gives is relatively simple class, which the general classes are not. First it serves as a warm up illustrating how we reduce the supersymmetry constraints and Bianchi identities to physical fields and PDEs — in the more general classes we use the same methods but give a less detailed derivation. Second it gives some understanding of the types of solutions that class I contains.

We begin by noting that the second of (3.6c) implies $d(e^{-C}\sin\alpha U) = 0$. One might be tempted to use this to define a local coordinate, however this would not be valid when $\sin\alpha = 0$. In general it implies that we can introduce a function $f$ on $\text{M}_5$ and then integrate the second of (3.6c) as follows

$$e^{-C}\sin\alpha U = d\log f \qquad \Rightarrow e^{2A+C-\Phi} f e^i = dz_i \tag{3.9}$$

where $z_i$ for $i = 1, 2, 3$ are local coordinates on $\text{M}_5$, which fixes 3 components of the vielbein. We define the final 2 components with (3.6b): we integrate the first of these in terms of a local coordinate $y$ as

$$e^A \cos\alpha K = -dy \tag{3.10}$$

so that the second condition becomes $d(e^{A-\Phi}U) \wedge dy$. One can show that this can be integrated in general in terms of a final local coordinate $x$ and $\lambda = \lambda(x, y, z_i)$ as

$$e^{A-\Phi}U = dx + \lambda dy, \tag{3.11}$$

at which point a set of local coordinates and vielbein on $\text{M}_5$ are determined without loss of generality. Plugging this back into (3.6c) we find first that $P = P(x, y)$, and then

$$2m\tan\alpha = e^{2A-\Phi}\partial_x \log f, \qquad d(f^{-2}\partial_x f) = 0, \qquad \partial_y f = \lambda \partial_x f. \tag{3.12}$$

In general these imply $cf^{-1} = x + g(y)$ for $c$ a constant, and $\lambda = \partial_y g$ — however we then have that $U \sim d(x + g)$, so up to a change in coordinates we can simply fix

$$\lambda = 0, \qquad f^{-1} = u(x), \qquad u'' = 0, \tag{3.13}$$

making the metric diagonal. Next we note that the first of (3.6d) wedged with $v_i$ is independent of $H_3$, so gives rise to constraints on the local functions derived thus far: this and (3.8) respectively imply

$$\partial_y\left(e^{2A-\Phi}\right) = 0, \qquad \partial_x\left(e^{4A}u^{-1}\cos^2\alpha\right) = 0, \tag{3.14}$$

which together with (3.13) are actually all the PDEs supersymmetry demands that we solve as shall be come clear momentarily. Supersymmetry also demands that we solve (3.6d) so to proceed we take a general ansatz for $H_3$ in terms of our local coordinates and 10 functions with support on $\mathrm{M}_5$

$$H_3 = H^0 dz_1\wedge dz_2\wedge dz_3 + H^0_i dx\wedge dy\wedge dz_i + \frac{1}{2}H^x_i\epsilon_{ijk}dx\wedge dz_j\wedge dz_k + \frac{1}{2}H^y_i\epsilon_{ijk}dy\wedge dz_j\wedge dz_k. \tag{3.15}$$

Inserting this into (3.6d), making use of what has been derived thus far, we find that it just implies

$$\begin{aligned}&H^0 = 8m^3\partial_x\left(\frac{1}{e^{8A-2\Phi}\cos^4\alpha}\right),\\ &H^0_i = H^y_i = 0,\qquad H^x_i = -2m\partial_{z_i}(e^{-4A+2\Phi}u).\end{aligned} \tag{3.16}$$

What remains to be dealt with is (3.6a) and (3.7a)–(3.7b), which are all just definitions of physical fields. We do however still need to take the hodge dual of the latter to construct the magnetic RR fluxes — this is not difficult as we have an explicit vielbein to work with, we find

$$\begin{aligned}
&f_1 = f_7 = 0,\qquad e^{2C}H_1 = -\frac{1}{2m}dy,\\
&f_3 = -\frac{1}{4m^2}\left(e^{2A-\Phi}\cos\alpha\sin\alpha - 2mx\right)\wedge\mathrm{vol}(\mathrm{S}^2) + m\epsilon_{ijk}\partial_{z_i}\left(\frac{u}{e^{4A}\cos^2\alpha}\right)dz_j\wedge dz_k\wedge dy\\
&\qquad -8m^3e^{-4A+2\Phi}(u')^2\partial_y\left(\frac{u}{e^{4A}\cos^2\alpha}\right)dz_1\wedge dz_2\wedge dz_3,\\
&f_5 = \frac{e^{4A-2\Phi}}{4m^2\cos^2\alpha u^2}\Bigg[e^{2A-\Phi}u\tan\alpha\epsilon_{ijk}\partial_{z_i}\left(e^{-4A+2\Phi}u^2\right)dz_j\wedge dz_k\wedge dx\\
&\qquad \frac{u}{e^{4A}\cos^2\alpha}\left(16m^4e^{-4A+2\Phi}u^4 - 8m^2e^{-2A+\Phi}u^3\tan\alpha\partial_x\left(e^{-4A+2\Phi}u\right)\right)dz_1\wedge dz_2\wedge dz_3\Bigg]\wedge\mathrm{vol}(\mathrm{S}^2).
\end{aligned} \tag{3.17}$$

We have now reduced the supersymmetry conditions to definitions of the physical fields and 3 PDEs (3.13)–(3.14), but we still need to solve the Bianchi identities of the fluxes to have a solution. If we assume only (at least partially) localised sources, away from the loci of these this amounts to solving $dH_3 = 0$ and $d_H f_- = 0$ which must hold in all regular regions of a solution with or without sources. Our approach to deal with the Bianchi identities will be to assume we are in a local regular region and reduce the Bianchi identities to a set of PDES that define the class. For specific solutions one then needs to check whether this local region can be extended to potential singular loci, ie one must additionally check that the PDEs give appropriate $\delta$-function sources and that these have a supersymmetric embedding. Away from the loci of sources the Bianchi identities of $H_3$ and $f_3$ impose

$$\begin{aligned}
&\partial_y\left(\frac{u}{e^{4A}\cos^2\alpha}\right)\partial_x\left(e^{-4A+2\Phi}u^2\right) = 0,\\
&\partial^2_{z_i}\left(e^{-4A+2\Phi}u^2\right) + 4m^2\frac{u^2}{e^{4A}\cos^2\alpha}\partial^2_x\left(e^{-4A+2\Phi}u^2\right) = 0,\\
&\partial^2_{z_i}\left(\frac{u}{e^{4A}\cos^2\alpha}\right) + 4m^2e^{-4A+2\Phi}u^2\partial^2_y\left(\frac{u}{e^{4A}\cos^2\alpha}\right) = 0
\end{aligned} \tag{3.18}$$



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The first of these is a harmonic function rule that induces a splitting of the class into 2 cases, depending on which factor vanishes. The Bianchi identity for $f_5$ is implied by these and the supersymmetry conditions. At this point we have reduced the conditions to have a supersymmetric solution to some PDEs — but we can express the class in a more concise fashion in terms of some new functions as

$$P = \frac{4m^2 u}{e^{4A}\cos^2\alpha}, \qquad G = 4m^2 e^{-4A+2\Phi} u^2, \tag{3.19}$$

in terms of which (3.14) become simply $G = G(x, z_i)$, $P = P(y, z_i)$. At this point the class is defined by just the Bianchi identities. This completes our derivation of the PDEs governing this class.

In summary the class of solutions in this section has an NS sector of the form

$$\begin{aligned}
ds^2 &= 2m\Bigg[\sqrt{1+\frac{(u')^2}{G}}\left(\sqrt{\frac{u}{P}}ds^2\left(\mathrm{AdS}_3\right)+\frac{G}{4m^2\sqrt{u}}\left(\frac{1}{\sqrt{Pu}}dx^2+\sqrt{P}dz_i^2\right)+\frac{1}{4m^2}\sqrt{\frac{P}{u}}dy^2\right)\\
&\qquad +\frac{1}{4m^2\sqrt{1+\frac{(u')^2}{G}}}\sqrt{\frac{u}{P}}ds^2\left(\mathrm{S}^2\right)\Bigg],\\
e^{-\Phi} &= \sqrt{\frac{Pu}{G\left(1+\frac{(u')^2}{G}\right)}},\\
2mH &= -dy\wedge\mathrm{vol}\left(\mathrm{S}^2\right)-\frac{1}{2u}\epsilon_{ijk}\partial_{z_i}G dx\wedge dz_j\wedge dz_k+P\partial_x G dz_{123},
\end{aligned} \tag{3.20}$$

where $G = G(x, z_i)$, $P = P(y, z_i)$, $u$ is a linear function of $x$ and we use the short hand notation $dz_{123} = dz_1 \wedge dz_2 \wedge dz_3$. The non trivial ten dimensional RR fluxes are

$$\begin{aligned}
F_3 &= \frac{1}{2m}d\left(\frac{uu'}{G+(u')^2}-x\right)\wedge\mathrm{vol}(\mathrm{S}^2)+\frac{1}{4m}\epsilon_{ijk}\partial_{z_i}P dy\wedge dz_j\wedge dz_k-\frac{1}{2m}G\partial_y P dz_{123}, \qquad (3.21)\\
F_5 &= (1+\star_{10})f_5,\\
f_5 &= \frac{1}{8m^3(G+(u')^2)}\left[mu'\epsilon_{ijk}\partial_{z_i}G dx\wedge dz_j\wedge dz_k+2mP\left(G^2-u'u^2\partial_x(Gu^{-1})\right)dz_{123}\right]\wedge\mathrm{vol}(\mathrm{S}^2).
\end{aligned}$$

One has a solution whenever the Bianchi identities of the fluxes are satisfied, away from possible sources these impose

$$\partial_x G\partial_y P = 0, \qquad \partial_{z_i}^2 P + G\partial_y^2 P = 0, \qquad \partial_{z_i}^2 G + uP\partial_x^2 G = 0. \tag{3.22}$$

More generically the latter two of these could have $\delta$-function sources, when this is the case they should have a supersymmetric embedding for the remaining equations of motion of a solution to necessarily follow. The first condition is a harmonic function rule: it states that either $\partial_x G = 0$, or $\partial_y P = 0$ yielding two cases.

**Case 1.** For the first case, when $\partial_x G = 0$, the Bianchi identities reduce to those of D5 branes ending on NS5 branes smeared over 1 direction in flat space [48] (see section 4.5). Comparing to this suggests that this case formally describes localised D5 branes of world volume $(\mathrm{AdS}_3, \mathrm{S}^2, x)$ ending at NS5 branes on $(\mathrm{AdS}_3, \mathrm{S}^2, y)$ that are delocalised in $x$. Unlike the flat space case, $\partial_x$ is not an isometry of the solution in general, the warping in $G$ is more complicated and additional fluxes and flux components are turned on. The latter difference is to be expected as $\mathrm{Mink}_5$ is replaced by $\mathrm{AdS}_3 \times \mathrm{S}^2$. Though the warping in $G$ is more complicated than the flat space case, notice that as $G$ becomes large it does tend to what one would expect in the flat space case. The dependence of the metric on $u(x)$ is essentially a deformation of this system: when $u = 1$, the warp factors becomes precisely what one would expect for this D5-NS5 system and $x$ becomes an isometry, so we can take it to be a compact direction. When $u \neq 1$, as nothing else depends on $x$ this direction is unbounded.

**Case 2.** For the second case when $\partial_y P = 0$ the Bianchi identities do not reduce to those of a simple flat space brane intersection, though they are not far removed — the issue is the function $u$. As $P$ is the only object with $y$ dependence generically, this now becomes an isometry for this case. Examining the metric and NS and RR fluxes, it should not be hard to see that performing T-duality on $\partial_y$ maps to a class of solutions in IIA with a round 3-sphere (locally) and non trivial fluxes $(H, F_2, F_4)$. In IIB when $u = 1$ the Bianchi identities reduce to what one would expect for localised NS5 branes of world volume $(\mathrm{AdS}_3, \mathrm{S}^2, y)$ ending on D5 brane of world volume ($\mathrm{AdS}_3$, $\mathrm{S}^2$, $x$) that are smeared over $y$, when $u \neq 1$ we have a deformation of this system. Even when $u \neq 1$ for large $G$ the functions $P, G$ appear where one would expect for D5 and NS5 brane warp factors respectively, but the additional $u$ dependence in the metric further deforms this picture. As we shall explain at greater length in section 3.2.1 this case represents a of a class derived in [17], which is actually related to an SU(2)-structure class in IIA via duality.

Having derived and interpreted the class with $\cos\beta = 0$, we shall now move onto its generalisation with $\sin\beta \neq 0$ in the next section.

### 3.2 $\sin\beta \neq 0$: Class I

The class of the previous section is actually a sub-case of the more general class we consider in this section, this is consistent with fixing $\cos\beta = 0$ but does not require it. The method of reducing this class to PDEs is analogous to that of the previous section, so we will be more brief.

We again use (3.6b)–(3.6c) to define the vielbein and dilaton in terms of local coordinates $(x, y, z_i)$, this time as

$$\begin{aligned} e^A(\cos\beta U - \cos\alpha\sin\beta K) = dy, \qquad \frac{e^{A-\Phi}\cos\alpha}{\cos^2\beta + \cos^2\alpha\sin^2\beta}(\cos\alpha\sin\beta U + \cos\beta K) = dx + \lambda dy,\\ e^{2A+C-\Phi} f e^i = dz_i, \end{aligned} \tag{3.23}$$

without loss of generality — note that this makes the metric on $\mathrm{M}_5$ non diagonal generically becoming diagonal for $\cos\beta = 0$. Plugging this ansatz into what remains non trivial in (3.6b)–(3.6c), fixes $\lambda$ to a value we shall quote momentarily and up to diffeomorphisms imposes

$$2m\tan\alpha\sin\beta = -e^{2A-\Phi}\partial_x \log u(x), \qquad u'' = 0, \tag{3.24}$$

which we can take to define $\alpha$. Now as it simplifies the final result we find it convenient to introduce functions $(G,P,Q)$ with support on $\mathrm{M}_5$ as

$$GP\Delta_2\cot^2\beta = Q^2u,\qquad e^{4A}P\Delta_1 = 4m^2u\left(1+\frac{(u')^2}{G}\right),\quad e^{4A-2\Phi}G\Delta_1\left(1+\frac{(u')^2}{G}\right) = 4m^2u\Delta_2,$$
$$\Delta_1 = 1-\frac{uQ^2}{GP},\qquad \Delta_2 = \Delta_1+\frac{(u')^2}{G}. \tag{3.25}$$

With these definitions the expression for $\lambda$ that follows from (3.6b)–(3.6c) is simply

$$\lambda = \frac{Qu}{G}. \tag{3.26}$$

Thus when $Q=0$ the class reduces to the previous section. We then essentially follow the same steps as in the previous section to reduce (3.6a)–(3.8) to physical fields and PDEs. The computation is of course more demanding but in essence the only real difference is that supersymmetry demands that the following PDEs are satisfied

$$\partial_y Q = \partial_x P,\qquad \partial_y G = u\partial_x Q, \tag{3.27}$$

which are less trivial than those of the previous section. There the PDEs could simply be integrated while now the possible geometries depend on how (3.27) are solved. Note that (3.27) gives a definition for $\partial_y\partial_x Q$ and $\partial_x\partial_y Q$ whose consistency implies

$$\partial_y^2 G = u\partial_x^2 P, \tag{3.28}$$

at least for a sufficiently smooth $Q$. In addition to this we find that the NS sector must take the form

$$\begin{aligned}
ds^2 &= 2m\sqrt{1+\frac{(u')^2}{G}}\Bigg[\sqrt{\frac{u}{P\Delta_1}}\left(ds^2(\mathrm{AdS}_3)+\frac{1}{4m^2}\frac{\Delta_1}{\Delta_2}ds^2(\mathrm{S}^2)\right)\\
&\qquad\qquad + \frac{1}{4m^2}\left(\sqrt{\frac{P}{u\Delta_1}}Dy^2 + G\sqrt{\frac{\Delta_1}{u}}\left(\frac{1}{\sqrt{P}u}dx^2+\sqrt{P}dz_i^2\right)\right)\Bigg],\\
e^{-\Phi} &= \frac{\sqrt{\frac{Pu}{G}}\sqrt{\Delta_2}}{1+\frac{(u')^2}{G}}\\
2mH &= -d\left(\frac{Quu'}{GP\Delta_2}+y\right)\wedge \mathrm{vol}(\mathrm{S}^2)\\
&\qquad -\frac{1}{2}\epsilon_{ijk}\left(\frac{1}{u}\partial_{z_i}Gdx+\partial_{z_i}Qdy\right)\wedge dz_j\wedge dz_k + \partial_x(GP\Delta_1)dz_{123},
\end{aligned} \tag{3.29}$$

where we define the following to make the expressions more compact

$$\Delta_1 = 1-\frac{uQ^2}{GP},\qquad \Delta_2=\Delta_1+\frac{(u')^2}{G},\qquad Dy = dy+\frac{Q}{P}dx,\qquad dz_{123}=dz_1\wedge dz_2\wedge dz_3 \tag{3.30}$$

clearly the metric on $\mathrm{M}_5$ contains cross terms in $(x,y)$ generically — we remind the reader that $u$ is a linear function of $x$. In addition to this, the class has all possible ten dimensional

RR fluxes non trivial, they take the form

$$F_1=dC_0,\qquad C_0=-\frac{Qu}{(G+(u')^2)},\qquad F_5=(1+\star_{10})f_5 \tag{3.31}$$

$$F_3=-C_0H+\frac{1}{2m}\bigg[d\left(\frac{uu'}{G\Delta_2}-x\right)\wedge\text{vol}(\text{S}^2)$$

$$+\frac{1}{2}\epsilon_{ijk}\left(\partial_{z_i}Qdx+\partial_{z_i}Pdy\right)\wedge dz_j\wedge dz_k-\partial_y(GP\Delta_1)dz_{123}\bigg],$$

$$f_5=\frac{P}{4m^2G\Delta_2}\bigg[\frac{u'}{2}\epsilon_{ijk}\bigg(u\partial_{z_i}(QP^{-1})dy+\bigg(P^{-1}\partial_{z_i}G-\frac{u}{2P^2}\partial_{z_i}(Q^2)\bigg)dx\bigg)\wedge dz_j\wedge dz_k$$

$$+P^{-2}\bigg(G^2P^2\Delta_1^2+uu'\left(G^{-2}\partial_y(G^3PQ)-P^2\partial_xG-2uQ^2\partial_xP-uG\partial_x(u^{-1}P^2)\right)\bigg)dz_{123}\bigg]$$

$$\wedge\,\text{vol}(\text{S}^2)$$

The definitions of the physically fields in (3.29)–(3.31) along with the PDEs of (3.27) imply supersymmetry. Again to have a solution we need to solve the Bianchi identities of the fluxes, although the fluxes are a little complicated their Bianchi identities are not especially as many components are implied by (3.27). First off clearly $dF_1=0$ is implied, so we can have no D7 brane sources in this class. As with the previous case the Bianchi identities of $(H,F_3)$ imply that of $F_5$, in regular regions of a solution these impose the following

$$\partial_{z_i}^2Q+\partial_x\partial_y(GP\Delta_1)=0,\qquad \partial_{z_i}^2G+u\partial_x^2(GP\Delta_1)=0,\qquad \partial_{z_i}^2P+\partial_y^2(GP\Delta_1)=0, \tag{3.32}$$

which are clearly more exotic generically than the PDEs one would expect of a simple brane intersection, but are still reminiscent of this. Solutions in this class are in 1 to 1 correspondence with the solutions to the combined systems of (3.27) and (3.32).

In the next section we shall derive a class of solutions in type IIA with a squashed and fibred 3-sphere that follows from imposing that $\partial_y$ is an isometry direction and then T-dualising on it.

### 3.2.1 A IIA class with fibered 3-sphere

In this section we derive a new class of solutions in IIA via T-duality from class I. Clearly there is generically no isometry to perform this duality on, so we must impose one on the class — we shall take $\partial_y$ to be an isometry. This means that the metric (3.29) should be $y$ independent — examining the various metric components and given (3.27) this reduces the conditions for a solution to

$$P=P(z_i),\qquad Q=Q(z_i),\qquad G=G(x,z_i),\qquad \partial_{z_i}^2P=0,\qquad \partial_{z_i}^2Q=0,\qquad \partial_{z_i}^2G+uP\partial_x^2G=0 \tag{3.33}$$

Performing T-duality on the $\partial_y$ direction in (3.29)–(3.31) then results in the following NS sector

$$\frac{ds^2}{2m}=\sqrt{1+\frac{(u')^2}{G}}\left[\sqrt{\frac{u}{P\Delta_1}}ds^2\left(\text{AdS}_3\right)+\frac{1}{4m^2}G\sqrt{\frac{\Delta_1}{u}}\left(\frac{1}{\sqrt{P}u}dx^2+\sqrt{P}dz_i^2\right)\right]$$

$$+\frac{1}{4m^2}\frac{1}{\sqrt{1+\frac{(u')^2}{G}}}\sqrt{\frac{u\Delta_1}{P}}\left(D\phi^2+\frac{G+(u')^2}{G\Delta_2}ds^2\left(\text{S}^2\right)\right)$$

$$
\begin{aligned}
e^{2\Phi} &= \frac{2mG}{\Delta_2}\sqrt{\frac{\Delta_1}{uP^3}\left(1+\frac{(u')^2}{G}\right)^3},\\
B_2 &= B - \frac{1}{2m}\eta\wedge d\left(\frac{Quu'}{GP\Delta_2}\right) + \frac{Q}{2mP}dx\wedge D\phi,
\end{aligned}
\tag{3.34}
$$

where we introduce 1-forms $(D\phi, \eta, \mathcal{A})$ and 2-form $B$ such that

$$
\begin{aligned}
D\phi &= d\phi + \mathcal{A} + \eta, \qquad \text{with} \qquad d\mathcal{A} = -\frac{1}{2}\epsilon_{ijk}\partial_{z_i}Q dz_j\wedge dz_k, \qquad d\eta = \text{vol}(\text{S}^2),\\
dB &= \frac{1}{2m}\Big(\partial_x(PG\Delta_1)dz_{123} - \frac{1}{2}\epsilon_{ijk}\frac{\partial_{z_i}G}{u}dx\wedge dz_j\wedge dz_k\Big),
\end{aligned}
\tag{3.35}
$$

here $\phi$ is the dual coordinate to $y$ after a rescaling. Notice that $(D\phi, \text{S}^2)$ now span an SU(2)$\times$U(1) preserving squashed and fibered 3-sphere. In addition, the background is supported by the RR fluxes

$$
\begin{aligned}
&F_0=0, \qquad F_2=\frac{1}{4m}\epsilon_{ijk}\left(\partial_{z_i}P+C_0\partial_{z_i}Q\right)dz_j\wedge dz_k+\frac{C_0}{2m}\text{vol}(\text{S}^2)+\frac{1}{2m}D\phi\wedge dC_0,\\
&F_4=2m\frac{Quu'}{GP\Delta_1}(P\partial_{z_i}Q-Q\partial_{z_i}P)dz_i\wedge\text{vol}(\text{AdS}_3)+\frac{Puu'}{8m^2G\Delta_2}\epsilon_{ijk}\,\partial_{z_i}(QP^{-1})dz_j\wedge dz_k\wedge\text{vol}(\text{S}^2)\\
&\quad+\frac{1}{4m^2}\bigg[C_0\,\partial_x(PG\Delta_1)dz_{123}+\frac{1}{2}\epsilon_{ijk}\left(C_0\left(\frac{Q}{P}\partial_{z_i}Q-\frac{\partial_{z_i}G}{u}\right)-(\partial_{z_i}Q+\frac{Q}{P}\partial_{z_i}P)\right)dx\wedge dz_j\wedge dz_k\\
&\qquad+\left(dx-d\left(\frac{uu'}{G\Delta_2}\right)-C_0\,d\left(\frac{Quu'}{QP\Delta_2}\right)\right)\wedge\text{vol}(\text{S}^2)\bigg]\wedge D\phi,
\end{aligned}
\tag{3.36}
$$

where $C_0, \Delta_1$ and $\Delta_2$ are defined as in (3.30)–(3.31).

To interpret this class it is instructive to first fix $Q=0$ and $u=1$ so that $\Delta_1 = 1$, $G\Delta_2 = G+(u')^2$ and the 3-sphere spanned by $(D\phi, \text{S}^2)$ becomes the round one. We then find that the governing PDE of the system reduces to $\partial_{z_i}^2 G + P\partial_x^2 G = 0$, the same PDEs as the system of fully localised NS5 branes inside the world volume of D6 branes derived in [49]. Here however, rather than Mink$_6$, the branes share the world volume directions AdS$_3\times$S$^3$ with the D6 further extended in $x$. In this limit $P$ and $G$ appear where one would expect for the warp factors of such D6 and NS5 branes, this is also true when $u\neq 1$ but $G$ is large. The effect of turning on $Q$ appears to be to place formal KK monopoles into this D6-NS5 brane system which squashes the 3-sphere. The effect of turning on $u$ is then a deformation.

This case actually generalises an known class of solutions: in [10], there is a class of solution with D4 branes extended on AdS$_3\times$S$^2$ and localised in CY$_2$ times an interval, that lie inside the world volume of D8 branes. This set up can actually be realised as a near horizon limit of intersecting branes, however the class in [10] is a generalisation of this which depends on a linear function of the interval $\tilde{u}$ that cannot be so realised.[9] Upon setting the Romans mass to zero this class may be lifted to an AdS$_3\times$S$^3\times$CY$_2$ class in d=11 with source M5 branes and this additional $\tilde{u}$ deformation (see [14]). If one then imposes

[9] Or at least has not yet been so realised. Additionally this class depends in on a primitive $(1,1)$ on CY$_2$ that we assume is set to zero here.

that $\mathrm{CY}_2 = \mathbb{R}^4$ expressed in polar coordinates, and assumes SO(4) rotational symmetry for the M5 brane warp factors, one can reduce back to IIA, but this time on the Hopf fibre of a second 3-sphere $\tilde{\mathrm{S}}^3 \subset \mathbb{R}^4$. Upon fixing $\tilde{u} = 1$ the resulting class is a system of localised D6 branes with localised NS5 branes inside them both extended in $\mathrm{AdS}_3\times\mathrm{S}^3$ under the assumption that their common co-dimensions preserve an SO(3) isometry (see appendix B of [17]). Generically the presence of $\tilde{u}$ deforms this system. This sounds quite similar to the $Q = 0$ limit of what we have here if we impose an SO(3) isometry in the directions $z_i$. Indeed the SO(3) preserving $Q = 0$ limit of the case we present here can be precisely mapped to the class of [17].

We have thus constructed a broad generalisation of a class of $\mathrm{AdS}_3\times\mathrm{S}^3$ solutions found in [17], such that formal KK monopoles are included and the internal space has no SO(3) isometry generically.

### 3.3 $\sin\beta = 0$: Class II

In this section we derive the class of solutions that follows from fixing $\sin\beta = 0$ in the supersymmetry conditions, specifically we will fix $\beta = 0$ without loss of generality.

Fixing $\beta = 0$ in (3.6b)–(3.6c) changes the character of the solutions some what as we shall see. We can still take the same terms in (3.6b)–(3.6c) to define the vielbein, this time as

$$e^{A}U = dx, \qquad e^{A-\Phi}\cos\alpha K = dy + \tilde{\lambda}dx, \qquad e^{2A+C-\Phi}fe^{i} = \frac{1}{2m}dz_i, \tag{3.37}$$

for $\tilde{\lambda}$ and $f$ functions of all the coordinates on $\mathrm{M}_5$. Again (3.6b)–(3.6c) contain additional constraints which allow us to fix

$$f = u^{-1}, \qquad \tan\alpha = -\frac{1}{2m}e^{2A}\partial_x \log u(x), \qquad u'' = 0, \tag{3.38}$$

without loss of generality. We now find it helpful to introduce functions $h, g, \lambda$ with support on $(x, y, z_i)$ such that

$$e^{2A-\Phi} = 2m\sqrt{\frac{u\Xi}{g}}, \qquad e^{4A} = \frac{gu^2}{h}, \qquad \tilde{\lambda} = \frac{\lambda}{g}, \qquad \Xi = 1 + \frac{g(u')^2}{4m^2h}. \tag{3.39}$$

This reduces (3.6d) to a single PDE and definition of $H_3$, namely

$$\partial_y\lambda = \partial_x g, \qquad 4m^2H_3 = -\frac{1}{2}\epsilon_{ijk}(\partial_{z_i}g dy + \partial_{z_i}\lambda dx)\wedge dz_j\wedge dz_k + \partial_y h dz_{123}. \tag{3.40}$$

What remains of the supersymmetry conditions just defines $H_1$ and the Hodge dual of the magnetic parts of the RR fluxes — the Hodge dual can be taken with respect to (3.37) without difficulty. Let us just explain how we extract the Bianchi identities before then summarising our results for this class. First (3.40) clearly implies that away from sources, where $dH_3 = 0$ should hold we must have

$$\partial^2_{z_i}g + \partial^2_y h = 0, \qquad \partial^2_{z_i}\lambda + \partial_x\partial_y h = 0. \tag{3.41}$$

Moving onto the RR sector, we find that the 1-form is

$$f_1 = -d\left(\frac{\lambda}{g}\right) + dx\bigg(\partial_x\left(\frac{\lambda}{g}\right) - \frac{1}{2}\partial_y\left(\frac{\lambda^2}{g^2}\right) - \frac{4m^2u}{g}\partial_y\left(\frac{h}{g^2u^2}\right)\bigg). \tag{3.42}$$

As such, away from the loci of sources we must have that the coefficient of $dx$ in this expression is a function of $x$ only for $df_1 = 0$ to hold. We can thus introduce a function $w = w(x)$ and identify $f_1 = dC_0$ for

$$C_0 = w - \frac{\lambda}{g}. \tag{3.43}$$

The remaining magnetic RR fluxes can then be more compactly expressed in terms of the following auxiliary functions

$$\mathcal{S} = w\partial_y h - \partial_x h, \qquad \mathcal{T} = \lambda - wg, \qquad \mathcal{X} = \frac{4m^2hu^{-1} + \lambda\mathcal{T}}{g}. \tag{3.44}$$

We find that only the magnetic part of the RR 3-form orthogonal to $\mathrm{S}^2$ gives a Bianchi identity not implied by what has been derived thus far — this takes the form

$$g_{31} + C_0H = \frac{1}{4m^2}\left(\frac{1}{2}\epsilon_{ijk}(\partial_{z_i}\mathcal{T}dy + \partial_{z_i}\mathcal{X}dx)\wedge dz_j\wedge dz_k + \mathcal{S}dz_{123}\right). \tag{3.45}$$

The Bianchi identity of this flux components is implied when the r.h.s. of this expression is closed, which requires that

$$\partial_y\mathcal{X} = \partial_x\mathcal{T}, \qquad \partial^2_{z_i}\mathcal{T} = \partial_y\mathcal{S}, \qquad \partial^2_{z_i}\mathcal{X} = \partial_x\mathcal{S}, \tag{3.46}$$

however only the last of these is not implied by the supersymmetry and Bianchi identity PDEs derived thus far.

In summary the class of solutions we derive in this section has an NS sector of the form

$$\begin{aligned}
ds^2 &= \sqrt{\frac{g}{h}}u\bigg(ds^2(\mathrm{AdS}_3) + \frac{1}{4m^2\Xi}ds^2(\mathrm{S}^2)\bigg)\\
&\quad + \sqrt{\frac{h}{g}}\frac{dx^2}{u} + \frac{g^{\frac{3}{2}}}{4m^2\sqrt{h}}\left(dy + \frac{\lambda}{g}dx\right)^2 + \frac{1}{4m^2}\sqrt{gh}(dz_i)^2,\\
e^{-\Phi} &= \frac{2m}{g}\sqrt{\frac{h\Xi}{u}},\\
2mH &= d\left(\frac{guu'}{4m^2h\Xi} - x\right)\wedge\mathrm{vol}(\mathrm{S}^2) - \frac{1}{2m}\bigg(\frac{1}{2}\epsilon_{ijk}(\partial_{z_i}g dy + \partial_{z_i}\lambda dx)\wedge dz_j\wedge dz_k - \partial_y h dz_{123}\bigg).
\end{aligned}$$

It also has the following non trivial ten dimensional RR fluxes

$$\begin{aligned}
F_1 &= dC_0, \qquad C_0 = w - \frac{\lambda}{g}, \qquad F_5 = (1 + \star_{10})f_5 \\
F_3 &= -C_0H + \frac{1}{4m^2}\left(\frac{1}{2}\epsilon_{ijk}(\partial_{z_i}\mathcal{T}dy + \partial_{z_i}\mathcal{X}dx)\wedge dz_j\wedge dz_k + \mathcal{S}dz_{123}\right)\\
&\quad - \frac{1}{2m}\left(d\left(\frac{\mathcal{T}uu'}{4m^2h\Xi} + y\right) + wdx\right)\wedge\mathrm{vol}(\mathrm{S}^2),
\end{aligned} \tag{3.47}$$

$$f_5 = \frac{g^2}{2m^5 h\Xi}\Bigg[\epsilon_{ijk}\left(\frac{uu'}{2}\partial_{z_i}\left(\frac{\lambda}{g}\right)dy + \left(\frac{2m^2u'}{g}\partial_{z_i}\left(\frac{h}{g}\right) + \frac{uu'}{4}\partial_{z_i}\left(\frac{\lambda^2}{g^2}\right)\right)dx\right)dz_j\wedge dz_k$$
$$+\left(\frac{4m^2h^2}{g^2} - \frac{u^2u'}{g}\partial_x(u^{-1}h) + \frac{uu'\lambda}{g}\partial_y h\right)dz_{123}\Bigg]\wedge \text{vol}(\text{S}^2).$$

Solutions in this class are in 1-to-1 correspondence with the solutions to the following PDEs: first supersymmetry demands that we impose

$$\partial_y\lambda = \partial_x g. \tag{3.48}$$

Second, away form the loci of sources, the Bianchi identities impose

$$w' = \partial_x\left(\frac{\lambda}{g}\right) - \frac{1}{2}\partial_y\left(\frac{\lambda^2}{g^2}\right) - \frac{4m^2u}{g}\partial_y\left(\frac{h}{g^2u^2}\right),$$
$$\partial^2_{z_i}g + \partial_y^2 h = 0, \qquad \partial^2_{z_i}\lambda + \partial_x\partial_y h = 0, \qquad \partial^2_{z_i}\mathcal{X} = \partial_x\mathcal{S}. \tag{3.49}$$

Clearly the system of constraints that needs to be solved for this class is in general very complicated. This indicates two things: first that the class is likely quite broad, containing sub-classes with qualitatively different physics;[10] Second it suggests that there probably exists a better set of local coordinates on $\text{M}_5$ that simplifies these conditions some what. We have not made progress on the second point in general, but we evidence the first point by deriving 2 simplified cases in the next section, assuming a diagonal metric ansatz.

#### 3.3.1 2 interesting cases with diagonal metric

The general $\sin\beta = 0$ class is rather complicated, so here we shall derive some simplified sub-classes. It is a generic feature of local classifications of supergravity solutions that those with the simplest PDES governing them come with a diagonal metric, so this is what we shall pursue. The most obvious way to achieve this is to set $\lambda = 0$, however it is possible to be more general than this: while it is true that the definition of (3.37) integrates $d(e^{A-\Phi}\cot\alpha K)\wedge dx$ without loss of generality, one could equally well take

$$e^{A-\Phi}\cos\alpha K = a(x,y)(b(y)dy + \hat{\lambda}dx), \tag{3.50}$$

the difference is a diffeomorphism in $(x,y)$ that turns on $(a,\ b)$, setting $\hat{\lambda}=0$ here or $\tilde{\lambda}=0$ in (3.37) both give a diagonal metric, but the former is more general. We shall then take $(U, e_i)$ as defined in (3.37), $\alpha, f, u$ as in (3.38) and redefine the physical fields in terms of auxiliary functions as

$$e^{2A-\Phi} = 2m\sqrt{\frac{au\tilde{\Xi}}{g}}, \qquad e^{4A+3u}a = \frac{gu^2}{h}, \qquad \tilde{\Xi} = 1 + \frac{g(u')^2}{4m^2ah}, \quad \hat{\lambda}=0, \tag{3.51}$$

the last of these being the diagonal ansatz. Repeating the same steps as the previous section we find that supersymmetry requires $\partial_x g = 0$ and

$$4m^2H_3 = -\frac{b}{2}\epsilon_{ijk}\partial_{z_i}g dz_j\wedge dz_k\wedge dy + \frac{1}{ab}\partial_y h dz_{123}, \quad f_1 = b\partial_x a dy - \frac{4m^2u}{bg}\partial_y\left(\frac{ag}{gu^2}\right)dx. \tag{3.52}$$

[10] A similarly complicated system derived for $\text{Mink}_4\times\text{S}^2$ in [47] was shown to contain all (known) half BPS $\text{AdS}_{5,6,7}$ solutions modulo duality, as well as compact Minkowski vacua.

In order for these flux components to satisfy their respective Bianchi identities we must have that

$$\partial_x\left(\frac{1}{a}\partial_y h\right)=0,\qquad \partial_{z_i}\left(\frac{1}{bg}\partial_y\left(\frac{ag}{gu^2}\right)\right)=0. \tag{3.53}$$

These conditions do not have a unique general solution instead they represent a branching of possible classes of solution — ie the solutions that follow from fixing $\partial_y\left(\frac{ag}{gu^2}\right)=0$ are distinct from those not obeying this constraint and so on. We have not found every branch that follows from (3.53), but shall provide 2 that result in interesting physical systems. These follow from fixing the functions of our local ansatz as

$$\begin{aligned}
&\text{case 1}: \quad h=4m^2RT,\quad g=4m^2T,\quad a=k,\quad b=1 \\
&\text{case 2}: \quad h=12m^2T^2S,\quad g=12\sqrt{3}m^2T,\quad a=\sqrt{3}k^{\frac{5}{3}}\partial_y S,\quad b=\frac{1}{3}k^{\frac{4}{3}},
\end{aligned} \tag{3.54}$$

where the assumption comes with the choice of $(h,g,a)$, we are free to fix $b$ as we choose with diffeomorphisms. In all cases $v=v(y)$ and $k=k(x)$, $R=R(x,z_i)$, $S=S(x,y)$ and $T=T(z_i)$. We will now present the cases that follow from these tunings

**Case 1: A deformed D7-D5-NS5 brane intersection.** For case 1 $\partial_y$ is actually an isometry direction and the local form of solutions reduces to

$$\begin{aligned}
ds^2&=\frac{u}{\sqrt{kR}}\left[ds^2\left(\text{AdS}_3\right)+\frac{1}{4m^2}\frac{1}{1+\frac{(u')^2}{4m^2kR}}ds^2\left(\text{S}^2\right)\right]+\frac{\sqrt{kR}}{u}dx^2+T\left(\sqrt{\frac{k}{R}}dy^2+\sqrt{\frac{R}{k}}\left(dz_i\right)^2\right)\\
e^{-\Phi}&=\frac{k\sqrt{R}}{\sqrt{Tu}}\sqrt{1+\frac{(u')^2}{4m^2kR}},\\
2mH&=d\left(\frac{uu'}{4m^2kR\left(1+\frac{(u')^2}{4m^2kR}\right)}-x\right)\wedge\text{vol}\left(\text{S}^2\right)-m\epsilon_{ijk}\partial_{z_i}Tdz_j\wedge dz_k\wedge dy,\\
F_1&=k'dy,\qquad F_5=(1+\star_{10})f_5\\
F_3&=\frac{1}{2m}\left(-k+\frac{uu'k'}{4m^2kR+(u')^2}\right)dy\wedge\text{vol}\left(\text{S}^2\right)+\frac{k}{2u}\epsilon_{ijk}\partial_{z_i}Rdz_j\wedge dz_k\wedge dx-\partial_xRTdz_{123},\\
f_5&=\frac{1}{4mkR+(u')^2}\left[\frac{ku'}{4m}\epsilon_{ijk}\partial_{z_i}Rdz_j\wedge dz_k\wedge dx\right.\\
&\qquad\left.-\frac{1}{2m}TR\left(4m^2k+u'\partial_x\left(uR^{-1}\right)\right)dz_{123}\right]\wedge\text{vol}(\text{S}^2).
\end{aligned} \tag{3.55}$$

Solutions in this case are defined entirely in terms of the Bianchi identities of the fluxes which impose the following system of PDEs away from the loci of sources

$$k''=0,\qquad \partial_{z_i}^2T=0,\qquad k\partial_{z_i}^2R+uT\partial_x^2R=0. \tag{3.56}$$

If we fix $u=k=1$ this is the system of PDEs of a flat space intersection of D5 branes ending on NS5 branes that are smeared over $y$ [48]. The warp factors $T,R$ appear where one would expect if they are to be identified NS5 and D5 branes extended in $\text{AdS}_3\times\text{S}^2$ and

$x$ or $y$ respectively. The effect of turning on $k$ is clearly to add D7 branes smeared on $y$ to this system. When $u$ is non trivial, this cannot simply be interpreted as the warp factor of a brane, rather $u$ represents a deformation of this system, where the roles that $(R,T,k)$ play depend on the details of how (3.56) is solved. Finally we note T-dualising of $\partial_y$ maps us to a solution in massive IIA with an SU(2)$\times$ U(1) preserving squashed and fibred 3-sphere.

**Case 2: An Imamura-like case with D7-NS5-D5.** For case 2 not only is $\partial_y$ not generically an isometry, it is manifestly never an isometry. The solutions in this case all take the local form

$$\begin{aligned}
ds^2 &= \frac{u}{\sqrt{S\partial_y ST}v^{\frac{5}{6}}}\left[ds^2(\text{AdS}_3)+\frac{1}{4m^2\tilde{\Xi}}ds^2\left(\text{S}^2\right)\right]+\frac{\sqrt{S\partial_y ST}}{v^{\frac{5}{6}}u}dx^2\\
&\quad+\frac{T}{v^{\frac{5}{6}}}\left(\sqrt{\frac{\partial_y S}{ST}}\frac{dy^2}{v}+3\sqrt{\frac{ST}{\partial_y S}}(dz_i)^2\right),\\
e^{-\Phi} &= \frac{\sqrt{S}\partial_y S v^{\frac{5}{3}}}{\sqrt{3}\sqrt{u}}\sqrt{\tilde{\Xi}},\qquad \tilde{\Xi}=1+\frac{(u')^2}{4m^2 S\partial_y STv^{\frac{5}{3}}}\\
2mH &= d\left(\frac{uu'}{4m^2Sv^{\frac{5}{3}}T\tilde{\Xi}}-x\right)\wedge\text{vol}\left(\text{S}^2\right)-\frac{\sqrt{3}m}{v^{\frac{4}{3}}}\left(\epsilon_{ijk}\partial_{z_i}Tdz_j\wedge dz_k-8vT^2dz_{123}\right),\\
F_1 &= \frac{1}{\sqrt{3}}d\left(v^{\frac{1}{3}}\partial_x S\right)+\frac{v'}{3\sqrt{3}}\left(\frac{v^2S\partial_y S}{u}dx-\frac{\partial_x S}{v^{\frac{2}{3}}}dy\right),\qquad F_5=(1+\star_{10})\tilde{f}_3\wedge\text{vol}\left(\text{S}^2\right),\\
F_3 &= \frac{S\partial_y Sv^{\frac{5}{3}}}{2u}\epsilon_{ijk}\partial_{z_i}Tdz_j\wedge dz_k\wedge dx-3T^2\partial_x Sdz_{123}-\frac{v^{\frac{1}{3}}}{2\sqrt{3}m}\partial_y Sdy\wedge\text{vol}\left(\text{S}^2\right)\\
&\quad+\frac{v^{\frac{1}{3}}}{8\sqrt{3}m^3S\partial_y STv^{\frac{5}{3}}\Xi}\left(uu'\partial_x\partial_y Sdy+\frac{1}{3}u'\left(S\partial_y Sv^{\frac{5}{3}}v'+3u\partial_x^2S\right)\right)dx\wedge\text{vol}(\text{S}^2),\\
\tilde{f}_3 &= \frac{1}{8m^3S\partial_y STv^{\frac{5}{3}}\tilde{\Xi}}\\
&\quad\times\left[\frac{1}{2}S\partial_y Sv^{\frac{5}{3}}u'\epsilon_{ijk}\partial_{z_i}Tdz_j\wedge dz_k\wedge dx+3T^2\left(4m^2S\partial_y STv^{\frac{5}{3}}+S\partial_x(uu'S^{-1})\right)dz_{123}\right],
\end{aligned}\tag{3.57}$$

where $v$ is a non zero but otherwise arbitrary linear function of $y$. The Bianchi identities of the fluxes impose

$$\partial_{z_i}^2T=v'T^2,\qquad u\partial_x^2(\sqrt{v}S)+v^{\frac{13}{6}}\partial_y^2((\sqrt{v}S)^2)=0\tag{3.58}$$

away from the loci of sources, there are no other conditions to be solved — we note that $v''=0$ is implied by the first of these, even in the presence of sources. When $T=v=u=1$ the system of PDEs becomes very much like those of the D8-D6-NS5 system of [50], a $\frac{1}{4}$ BPS Mink$_6$ class in IIA (see also [51] for a version without an SO(3) isometry). Of course here we are in IIB, so the interpretation must be a little different. Examining the warp factors in the metric in this limit $\partial_y S$ appears to play the role of a localised D7 brane warp factor while $S$ that of D5 branes smeared on $z_i$. Turning on $T$ and defining $h_{D7}=\partial_y S$, $h_{\text{NS5}}=T$ and $h_{\text{D5}}=ST$ we see that these quantities appear in the metric where one would formally expect the warp factors of a D7-D5-NS5 brane intersection to appear, however $S$ would need to be independent of $x$ for this interpretation. Additionally all these putative branes would have $\text{AdS}_3\times$ $\text{S}^2$ in their world volume and $z_i$ in their co dimensions, making this more closely resemble the generalisation of [50] presented in [51], T-dualised on a spacial direction in $\text{Mink}_6$ to get $\text{Mink}_5$ in IIB then compactified to $\text{AdS}_3\times\text{S}^2$. The effect of further turning on $(u, v)$ is to deform this system.

We have found that class II is really rather broad and that, even when rather draconian constraints are imposed on it, it is possible to extract interesting physical cases. We plan to explore the solutions contained here, in class II more broadly and in class I in forthcoming work [52].


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## A Gamma matrices on $\mathbf{M_7}$

To find the supersymmetric solutions given in the main text, we use the bispinor approach, thus, we start by taking a basis of 7d gamma matrices, $\gamma_A^{(7)}$, that respect a 2+5 split (on $\text{S}^2\times\text{M}_5$) which decompose as,

$$\gamma_i^{(7)} = e^C\sigma_i\otimes\mathbb{I}, \qquad \gamma_a^{(7)} = \sigma_3\otimes\gamma_a, \qquad \text{where} \qquad i\gamma^{(7)}_{1234567} = 1. \tag{A.1}$$

Here, $\sigma_i$ (with $i = 1, 2$) are the gamma matrices on $\text{S}^2$ with chirality matrix $\sigma_3$, i.e. $\sigma_{1,2,3}$ are the Pauli matrices. In turn, $\gamma_a$ are the gamma matrices in 5d, namely $a = 1, \ldots, 5$, such that the 5d intertwiner, $B_5$, satisfies

$$B_5B_5^* = -\mathbb{I}, \qquad B_5\gamma_aB_5^{-1} = \gamma_a^*. \tag{A.2}$$

Finally, the intertwiner in 7d is defined as and satisfies

$$B^{(7)} = \sigma_2\otimes B_5 \qquad \text{with} \qquad B^{(7)}B^{(7)^*} = \mathbb{I}, \qquad B^{(7)}\gamma_A^{(7)}(B^{(7)})^{-1} = -\gamma_A^{(7)^*} \tag{A.3}$$

and $A = 1, \ldots, 7$.

## B Geometric 5d conditions for supersymmetry

In this appendix we derive a set of geometric constraints for the five-dimensional manifold which are sufficient for supersymmetry. To be precise, we express these supersymmetric conditions in terms of five-dimensional bi-linears.

### B.1 Matrix bi-linears on $\mathrm{M}_5$

We introduce four matrix bi-linears expressed in terms of the four 5d spinors, $\eta$, and their Majorana conjugates, namely

$$(\psi_{\sigma\rho})^{\alpha\beta} = \eta_{1\sigma}^{\alpha} \otimes \eta_{2\rho}^{\beta\dagger}, \qquad \eta^{\alpha} = \begin{pmatrix} \eta \\ \eta^c \end{pmatrix}^{\alpha}. \tag{B.1}$$

Note that,

$$\begin{aligned} \eta_1^{\alpha} \otimes \eta_2^{\beta\dagger} &= \begin{pmatrix} \eta_1 \otimes \eta_2^{\dagger} & \eta_1 \otimes \eta_2^{c\dagger} \\ (-\eta_1 \otimes \eta_2^{c\dagger})^* & (\eta_1 \otimes \eta_2^{\dagger})^* \end{pmatrix} \\ &= \mathrm{Re}\eta_1 \otimes \eta_2^{\dagger}\delta^{ab} + i\mathrm{Im}\eta_1 \otimes \eta_2^{c\dagger}(\sigma_1)^{ab} + i\mathrm{Re}\eta_1 \otimes \eta_2^{c\dagger}(\sigma_2)^{\alpha\beta} + i\mathrm{Im}\eta_1 \otimes \eta_2^{\dagger}(\sigma_3)^{\alpha\beta} \end{aligned} \tag{B.2}$$

here the fundamental object is the 5 dimensional bi-spinor, which can be computed with following definition,

$$\epsilon_1 \otimes \epsilon_2^{\dagger} = \frac{1}{2^{[d/2]}} \sum_{n=0}^{d} \frac{1}{n!} \epsilon_2^{\dagger} \gamma_{a_1 \dots a_n} \epsilon_1 e^{a_1} \wedge \dots \wedge e^{a_n}, \tag{B.3}$$

in this case $\epsilon_1$ and $\epsilon_2$ are two d dimensional spinors, $\gamma_a$ a basis of the flat space gamma matrices in d dimensions and $e^a$ are the vielbein on the d dimensional space.

In the next sub-section we present details of the bi-spinors on $\mathrm{S}^2$.

### B.2 Matrix bi-linears on $\mathrm{S}^2$

On $\mathrm{S}^2$, there exist Killing spinors, $\xi$, which obey

$$\nabla_{\mu}\xi = \frac{i}{2}\sigma_{\mu}\xi \tag{B.4}$$

where we used the Pauli matrices to represent the Clifford algebra on $\mathrm{S}^2$, $\mu = 1, 2$ is a flat index on the unit sphere and the 2d intertwiner defining Majorana conjugation is $\sigma_2$, such that $\xi^c = \sigma_2 \xi^*$. As shown in [47], the SU(2) doublets take the following form,

$$\xi^{\alpha} = \begin{pmatrix} \xi \\ \xi^c \end{pmatrix}^{\alpha}, \quad \sigma_3\xi^{\alpha} = \begin{pmatrix} \sigma_3\xi \\ \sigma_3\xi^c \end{pmatrix}^{\alpha} \tag{B.5}$$

where $\alpha$ and other Greek indices run over 1,2. We can form matrix bi-linears out of these doublets,

$$\begin{aligned} \Xi^{\alpha\beta} &= \xi^{\alpha} \otimes \xi^{\beta\dagger}, & \hat{\Xi}^{\alpha\beta} &= \sigma_3\xi^{\alpha} \otimes \xi^{\beta\dagger}, \\ \sigma_3\xi^{\alpha} \otimes (\sigma_3\xi^{\beta})^{\dagger} &= \Xi_+^{\alpha\beta} - \Xi_-^{\alpha\beta}, & \xi^{a} \otimes (\sigma_3\xi^{\beta})^{\dagger} &= \hat{\Xi}_+^{\alpha\beta} - \hat{\Xi}_-^{\alpha\beta}, \end{aligned} \tag{B.6}$$

here $\Xi_{\pm}^{\alpha\beta}$, $\hat{\Xi}_{\pm}^{\alpha\beta}$ are poly-forms containing only even/odd forms, which arise from the decomposition of $\Xi^{\alpha\beta}$ and $\hat{\Xi}^{\alpha\beta}$ via (B.3). In addition, $\Xi^{\alpha\beta}$ is linearly independent of $\hat{\Xi}^{\alpha\beta}$, component by component and at every form degree. Further one can show that, the only mixings of the $\mathrm{S}^2$ bi-linears, which appear in the $\mathcal{N}=1$ supersymmetry constraints and are different to zero under d and wedge product are

$$\begin{aligned} &d\mathrm{Re}\Xi_1^{\alpha\beta} = -2\mathrm{Im}\Xi_2^{\alpha\beta}, \qquad d\mathrm{Im}\Xi_1^{\alpha\beta} = 2\mathrm{Re}\Xi_2^{\alpha\beta}, \qquad d\mathrm{Re}\hat{\Xi}_0^{\alpha\beta} = \mathrm{Im}\hat{\Xi}_1^{\alpha\beta},\\ &d\mathrm{Im}\hat{\Xi}_0^{\alpha\beta} = -\mathrm{Re}\hat{\Xi}_1^{\alpha\beta}, \quad \Xi_0^{\alpha\beta}\mathrm{vol}(\mathrm{S}^2) = -\mathrm{Im}\hat{\Xi}_2^{\alpha\beta}, \quad \mathrm{Re}\hat{\Xi}_0^{\alpha\beta}\mathrm{vol}(\mathrm{S}^2) = -\mathrm{Im}\Xi_2^{\alpha\beta},\\ &\mathrm{Im}\hat{\Xi}_0^{\alpha\beta}\mathrm{vol}(\mathrm{S}^2) = \mathrm{Re}\Xi_2^{\alpha\beta}, \end{aligned} \tag{B.7}$$

and the only singlets are contained in the terms,

$$\Xi_0^{\alpha\beta} = \frac{1}{2}\delta^{\alpha\beta}, \qquad \mathrm{Im}\hat{\Xi}_2^{\alpha\beta} = -\frac{1}{2}\mathrm{vol}(\mathrm{S}^2)\delta^{\alpha\beta}, \tag{B.8}$$

which give rise to contributions to the RR fluxes. Notice that $\mathrm{Im}\Xi_0^{\alpha\beta} = \mathrm{Re}\hat{\Xi}_2^{\alpha\beta} = 0$.

In the next sub-section we will use the previous expressions to factor out the $\mathrm{S}^2$ matrix bi-linears from the 7d bi-linears constraints leaving us with 5d conditions.

### B.3 7d bi-linears as matrix bi-linear contractions

In this section, we express the 7d bi-linears, as contractions of the $\mathrm{S}^2$ and $\mathrm{M}_5$ data. We take the representative $\mathcal{N}=1$ sub-sector of the $\mathcal{N}=4$ Majorana spinors as follows,

$$\chi_1 = \frac{e^{\frac{A}{2}}}{\sqrt{2}}\Big(\xi^\alpha\otimes\eta_{11}^\alpha + i\sigma_3\xi^\alpha\otimes\eta_{12}^\alpha\Big), \quad \chi_2 = \frac{e^{\frac{A}{2}}}{\sqrt{2}}\Big(\xi^\alpha\otimes\eta_{21}^\alpha + i\sigma_3\xi^\alpha\otimes\eta_{22}^\alpha\Big). \tag{B.9}$$

Using the general feature that the bi-spinor can be decomposed as

$$\chi_1\otimes\chi_2^\dagger = \Psi_+ + i\Psi_- \tag{B.10}$$

and using the results established in the previous sections of this appendix, we can compute the following 7d bi-linears,

$$\begin{aligned} \Psi_+ = \frac{e^A}{2}\Big(&\Xi_0^{\alpha\beta}\mathrm{Re}\left(\psi_{11}^{(\alpha\beta)}+\psi_{22}^{(\alpha\beta)}\right)_+ + e^{2C}\mathrm{Im}\hat{\Xi}_2^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{12}^{(\alpha\beta)}-\psi_{21}^{(\alpha\beta)}\right)_+\\ &+\mathrm{Re}\hat{\Xi}_0^{\alpha\beta}\mathrm{Im}\left(\psi_{12}^{(\alpha\beta)}-\psi_{21}^{(\alpha\beta)}\right)_+ + \mathrm{Im}\hat{\Xi}_0^{\alpha\beta}\mathrm{Re}\left(\psi_{12}^{[\alpha\beta]}-\psi_{21}^{[\alpha\beta]}\right)_+\\ &+e^C\mathrm{Re}\Xi_1^{\alpha\beta}\wedge\mathrm{Im}\left(\psi_{12}^{(\alpha\beta)}+\psi_{21}^{(\alpha\beta)}\right)_- + e^C\mathrm{Im}\Xi_1^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{12}^{[\alpha\beta]}+\psi_{21}^{[\alpha\beta]}\right)_-\\ &-e^C\mathrm{Re}\hat{\Xi}_1^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{11}^{[\alpha\beta]}-\psi_{22}^{[\alpha\beta]}\right)_- + e^C\mathrm{Im}\hat{\Xi}_1^{\alpha\beta}\wedge\mathrm{Im}\left(\psi_{11}^{(\alpha\beta)}-\psi_{22}^{(\alpha\beta)}\right)_-\\ &+e^{2C}\mathrm{Re}\Xi_2^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{11}^{[\alpha\beta]}+\psi_{22}^{[\alpha\beta]}\right)_+ - e^{2C}\mathrm{Im}\Xi_2^{\alpha\beta}\wedge\mathrm{Im}\left(\psi_{11}^{(\alpha\beta)}+\psi_{22}^{(\alpha\beta)}\right)_+\Big), \end{aligned} \tag{B.11a}$$

$$\begin{aligned} \Psi_- = \frac{e^A}{2}\Big(&-\Xi_0^{\alpha\beta}\mathrm{Re}\left(\psi_{12}^{(\alpha\beta)}-\psi_{21}^{(\alpha\beta)}\right)_- + e^{2C}\mathrm{Im}\hat{\Xi}_2^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{11}^{(\alpha\beta)}+\psi_{22}^{(\alpha\beta)}\right)_-\\ &+\mathrm{Re}\hat{\Xi}_0^{\alpha\beta}\mathrm{Im}\left(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)}\right)_- + \mathrm{Im}\hat{\Xi}_0^{\alpha\beta}\mathrm{Re}\left(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]}\right)_-\\ &+e^C\mathrm{Re}\Xi_1^{\alpha\beta}\wedge\mathrm{Im}\left(\psi_{11}^{(\alpha\beta)}-\psi_{22}^{(\alpha\beta)}\right)_+ + e^C\mathrm{Im}\Xi_1^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{11}^{[\alpha\beta]}-\psi_{22}^{[\alpha\beta]}\right)_+\\ &+e^C\mathrm{Re}\hat{\Xi}_1^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{12}^{[\alpha\beta]}+\psi_{21}^{[\alpha\beta]}\right)_+ - e^C\mathrm{Im}\hat{\Xi}_1^{\alpha\beta}\wedge\mathrm{Im}\left(\psi_{12}^{(\alpha\beta)}+\psi_{21}^{(\alpha\beta)}\right)_+\\ &-e^{2C}\mathrm{Re}\Xi_2^{\alpha\beta}\wedge\mathrm{Re}\left(\psi_{12}^{[\alpha\beta]}-\psi_{21}^{[\alpha\beta]}\right)_- + e^{2C}\mathrm{Im}\Xi_2^{\alpha\beta}\wedge\mathrm{Im}\left(\psi_{12}^{(\alpha\beta)}-\psi_{21}^{(\alpha\beta)}\right)_-\Big) \end{aligned} \tag{B.11b}$$

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in terms of the symmetric and antisymmetric parts of $\Psi^{\alpha\beta}_{ij}$. These expressions, along with (B.7)–(B.8), will be useful in the next section, in the computations of the supersymmetric constraints.

### B.4 From 7d to 5d supersymmetry constraints

An $\mathcal{N}$=1 supersymmetric AdS$_3$ solution in type II supergravity, with purely magnetic NS flux, must obey the conditions [33]

$$d_H(e^{A-\Phi}\Psi_{\mp}) = 0, \tag{B.12a}$$

$$d_H(e^{2A-\Phi}\Psi_{\pm}) \mp 2me^{A-\Phi}\Psi_{\mp} = \frac{e^{3A}}{8} \star_7 \lambda(f), \tag{B.12b}$$

$$(\Psi_{\mp}, f)_7 = \mp\frac{m}{2}e^{-\Phi}\text{vol}(\text{M}_7), \tag{B.12c}$$

where $(\Psi_{\mp}, f_{\pm})_7 \equiv (\Psi_{\mp} \wedge \lambda(f_{\pm}))_7$ with $(X,Y)_7$ denoting the projection to the seven-form component. In (B.12a)–(B.12c), an upper sign applies to type IIA and a lower one to type IIB. The twisted exterior derivative is defined as $d_H = d - H\wedge$, and in turn we are assuming a purely magnetic NS flux

$$H = e^{2C}H_1 \wedge \text{vol}(\text{S}^2) + H_3. \tag{B.13}$$

Thus, one can write the RR flux poly-form term appearing in (B.12b) as following

$$\star_7\lambda f_{\pm} = -\star_5\lambda g_{2\pm} + e^{2C}\text{vol}(\text{S}^2)\wedge\star_5\lambda g_{1\pm} = \delta^{\alpha\beta}(-\Xi^{\alpha\beta}\wedge\star_5\lambda g_{2\pm} + ie^{2C}\hat{\Xi}^{\alpha\beta}\wedge\star_5\lambda g_{1\pm}). \tag{B.14}$$

#### B.4.1 IIA 5d conditions

One can show that given the 7d bi-linears in (B.11a)–(B.11b) and making use of the expressions (B.7) the supersymmetric constraints (B.12a)–(B.12b), independent of the RR forms, are equivalent to the following conditions in 5d,

$$d_{H_3}(e^{2A-\Phi}\text{Re}(\psi^{(\alpha\beta)}_{12} - \psi^{(\alpha\beta)}_{21})_-) = 0, \tag{B.15a}$$

$$d_{H_3}(e^{2A+C-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{12} + \psi^{[\alpha\beta]}_{21})_+) + e^{2A-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{11} + \psi^{[\alpha\beta]}_{22})_- = 0, \tag{B.15b}$$

$$d_{H_3}(e^{2A+C-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{12} + \psi^{(\alpha\beta)}_{21})_+) + e^{2A-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{11} + \psi^{(\alpha\beta)}_{22})_- = 0, \tag{B.15c}$$

$$d_{H_3}(e^{2A+2C-\Phi}\text{Re}(\psi^{(\alpha\beta)}_{11} + \psi^{(\alpha\beta)}_{22})_-) - e^{2A+2C-\Phi}H_1 \wedge \text{Re}(\psi^{(\alpha\beta)}_{12} - \psi^{(\alpha\beta)}_{21})_- = 0, \tag{B.15d}$$

$$\begin{aligned} &d_{H_3}(e^{2A+2C-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{12} - \psi^{(\alpha\beta)}_{21})_-) + e^{2A+2C-\Phi}H_1 \wedge \text{Im}(\psi^{(\alpha\beta)}_{11} + \psi^{(\alpha\beta)}_{22})_- \\ &\quad - 2e^{2A+C-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{11} - \psi^{(\alpha\beta)}_{22})_+ = 0, \end{aligned} \tag{B.15e}$$

$$\begin{aligned} &d_{H_3}(e^{2A+2C-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{12} - \psi^{[\alpha\beta]}_{21})_-) + e^{2A+2C-\Phi}H_1 \wedge \text{Re}(\psi^{[\alpha\beta]}_{11} + \psi^{[\alpha\beta]}_{22})_- \\ &\quad - 2e^{2A+C-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{11} - \psi^{[\alpha\beta]}_{22})_+ = 0, \end{aligned} \tag{B.15f}$$

$$\begin{aligned} &d_{H_3}(e^{3A+C-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{11} - \psi^{[\alpha\beta]}_{22})_-) - e^{3A-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{12} - \psi^{[\alpha\beta]}_{21})_+ \\ &\quad - 2me^{2A+C-\Phi}\text{Re}(\psi^{[\alpha\beta]}_{12} + \psi^{[\alpha\beta]}_{21})_+ = 0, \end{aligned} \tag{B.15g}$$

$$\begin{aligned} &d_{H_3}(e^{3A+C-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{11} - \psi^{(\alpha\beta)}_{22})_-) - e^{3A-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{12} - \psi^{(\alpha\beta)}_{21})_+ \\ &\quad - 2me^{2A+C-\Phi}\text{Im}(\psi^{(\alpha\beta)}_{12} + \psi^{(\alpha\beta)}_{21})_+ = 0, \end{aligned} \tag{B.15h}$$



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$$d_{H_3}\big(e^{3A+2C-\Phi}\mathrm{Re}(\psi^{[\alpha\beta]}_{11}+\psi^{[\alpha\beta]}_{22})_+\big)+2e^{3A+C-\Phi}\mathrm{Re}(\psi^{[\alpha\beta]}_{12}+\psi^{[\alpha\beta]}_{21})_-$$
$$+2me^{2A+2C-\Phi}\mathrm{Re}(\psi^{[\alpha\beta]}_{12}-\psi^{[\alpha\beta]}_{21})_- - e^{3A+2C-\Phi}H_1\wedge\mathrm{Re}(\psi^{[\alpha\beta]}_{12}-\psi^{[\alpha\beta]}_{21})_+=0, \qquad \text{(B.15i)}$$
$$d_{H_3}\big(e^{3A+2C-\Phi}\mathrm{Im}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_+\big)+2e^{3A+C-\Phi}\mathrm{Im}(\psi^{(\alpha\beta)}_{12}+\psi^{(\alpha\beta)}_{21})_-$$
$$+2me^{2A+2C-\Phi}\mathrm{Im}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_- - e^{3A+2C-\Phi}H_1\wedge\mathrm{Im}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_+=0, \qquad \text{(B.15j)}$$

and some more that are equivalent to,

$$\mathrm{Re}(\psi^{[\alpha\beta]}_{12}+\psi^{[\alpha\beta]}_{21})_+\wedge dH_3=\mathrm{Im}(\psi^{(\alpha\beta)}_{12}+\psi^{(\alpha\beta)}_{21})_+\wedge dH_3=0, \qquad \text{(B.16a)}$$
$$e^{2C}\mathrm{Im}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_-\wedge dH_3-d(e^{2C}H_1)\wedge\mathrm{Im}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_-=0, \qquad \text{(B.16b)}$$
$$e^{2C}\mathrm{Re}(\psi^{[\alpha\beta]}_{12}-\psi^{[\alpha\beta]}_{21})_-\wedge dH_3-d(e^{2C}H_1)\wedge\mathrm{Re}(\psi^{[\alpha\beta]}_{11}+\psi^{[\alpha\beta]}_{22})_-=0 \qquad \text{(B.16c)}$$
$$\mathrm{Re}(\psi^{[\alpha\beta]}_{11}-\psi^{[\alpha\beta]}_{22})_-\wedge dH_3=\mathrm{Im}(\psi^{(\alpha\beta)}_{11}-\psi^{(\alpha\beta)}_{22})_-\wedge dH_3=0, \qquad \text{(B.16d)}$$
$$e^{2C}\mathrm{Re}(\psi^{[\alpha\beta]}_{11}+\psi^{[\alpha\beta]}_{22})_+\wedge dH_3+d(e^{2C}H_1)\wedge\mathrm{Re}(\psi^{[\alpha\beta]}_{12}-\psi^{[\alpha\beta]}_{21})_+=0, \qquad \text{(B.16e)}$$
$$e^{2C}\mathrm{Im}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_+\wedge dH_3+d(e^{2C}H_1)\wedge\mathrm{Im}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_+=0. \qquad \text{(B.16f)}$$

These previous expressions are implied when the sourceless Bianchi identity of the NS flux is obeyed and more broadly constrain exactly what source terms $dH_3$ and $\mathrm{d}e^{2C}H_1$ we can have.

Using (B.14), the RR field-strengths are derived from (B.11b),

$$e^{3A}\delta^{\alpha\beta}\star_5\lambda g_2=-4d_{H_3}\big(e^{3A-\Phi}\mathrm{Re}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_+\big)-8me^{2A-\Phi}\mathrm{Re}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_-, \qquad \text{(B.17a)}$$
$$e^{3A+2C}\delta^{\alpha\beta}\star_5\lambda g_1=-4d_{H_3}\big(e^{3A+2C-\Phi}\mathrm{Re}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_+\big)+8me^{2A+2C-\Phi}\mathrm{Re}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_-$$
$$-4e^{3A+2C-\Phi}H_1\wedge\mathrm{Re}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_+. \qquad \text{(B.17b)}$$

Finally, considering $\lambda(f_+)=-\lambda g_{1+}-e^{2C}\mathrm{vol}(\mathrm{S}^2)\wedge\lambda g_{2+}$ in the pairing equation (B.12c) yields the additional restrictions,

$$\delta^{\alpha\beta}\big(\mathrm{Re}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_-\wedge\lambda g_{1+}+\mathrm{Re}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_-\wedge\lambda g_{2+}\big)=-2\mu e^{-(\Phi+A)}\mathrm{vol}(\mathrm{M}_5), \qquad \text{(B.18a)}$$
$$\mathrm{Im}(\psi^{(\alpha\beta)}_{11}+\psi^{(\alpha\beta)}_{22})_-\wedge\lambda g_{2+}=\mathrm{Im}(\psi^{(\alpha\beta)}_{12}-\psi^{(\alpha\beta)}_{21})_-\wedge\lambda g_{1+}, \qquad \text{(B.18b)}$$
$$\mathrm{Re}(\psi^{[\alpha\beta]}_{11}+\psi^{[\alpha\beta]}_{22})_-\wedge\lambda g_{2+}=\mathrm{Re}(\psi^{[\alpha\beta]}_{12}-\psi^{[\alpha\beta]}_{21})_-\wedge\lambda g_{1+}. \qquad \text{(B.18c)}$$

The equations (B.15), (B.16), (B.17), (B.18) are sufficient constraints for the preservation of supersymmetry in type IIA.

Some particularly important conditions are the matrix 0-form constraints coming from (B.15), as these are purely algebraic namely

$$\begin{aligned}&\eta^{c\dagger}_{21}\eta_{11}=\eta^{c\dagger}_{22}\eta_{12},\qquad \mathrm{Im}(\eta^\dagger_{21}\eta_{11})=\mathrm{Im}(\eta^\dagger_{22}\eta_{12}),\\&(1+2me^{C-A})\eta^{c\dagger}_{22}\eta_{11}=(1-2me^{C-A})\eta^{c\dagger}_{21}\eta_{12},\\&(1+2me^{C-A})\mathrm{Im}(\eta^\dagger_{22}\eta_{11})=(1-2me^{C-A})\mathrm{Im}(\eta^\dagger_{21}\eta_{12}).\end{aligned} \qquad \text{(B.19)}$$

We will use these to constrain the spinors in the main text.

### B.4.2 IIB 5d conditions

In IIB, the supersymmetric constraints (B.12a)–(B.12b), independent of the RR forms, are equivalent to the following conditions in 5d,

$$d_{H_3}\big(e^{2A-\Phi}\mathrm{Re}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_+\big)=0, \tag{B.20a}$$

$$d_{H_3}\big(e^{2A+C-\Phi}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}-\Psi_{22}^{[\alpha\beta]})_-\big)-e^{2A-\Phi}\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}-\Psi_{21}^{[\alpha\beta]})_+=0, \tag{B.20b}$$

$$d_{H_3}\big(e^{2A+C-\Phi}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}-\Psi_{22}^{(\alpha\beta)})_-\big)-e^{2A-\Phi}\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_+=0, \tag{B.20c}$$

$$\begin{aligned}&d_{H_3}\big(e^{2A+2C-\Phi}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]})_+\big)-e^{2A+2C-\Phi}H_1\wedge\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}-\Psi_{21}^{[\alpha\beta]})_+\\&\quad+2e^{2A+C-\Phi}\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}+\Psi_{21}^{[\alpha\beta]})_-=0,\end{aligned} \tag{B.20d}$$

$$\begin{aligned}&d_{H_3}\big(e^{2A+2C-\Phi}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_+\big)-e^{2A+2C-\Phi}H_1\wedge\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_+\\&\quad+2e^{2A+C-\Phi}\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}+\Psi_{21}^{(\alpha\beta)})_-=0,\end{aligned} \tag{B.20e}$$

$$d_{H_3}\big(e^{2A+2C-\Phi}\mathrm{Re}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_+\big)+e^{2A+2C-\Phi}H_1\wedge\mathrm{Re}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_+=0, \tag{B.20f}$$

$$\begin{aligned}&d_{H_3}\big(e^{3A+C-\Phi}\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}+\Psi_{21}^{[\alpha\beta]})_+\big)+e^{3A-\Phi}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]})_-\\&\quad+2me^{2A+C-\Phi}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}-\Psi_{22}^{[\alpha\beta]})_-=0,\end{aligned} \tag{B.20g}$$

$$\begin{aligned}&d_{H_3}\big(e^{3A+C-\Phi}\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}+\Psi_{21}^{(\alpha\beta)})_+\big)+e^{3A-\Phi}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_-\\&\quad+2me^{2A+C-\Phi}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}-\Psi_{22}^{(\alpha\beta)})_-=0,\end{aligned} \tag{B.20h}$$

$$\begin{aligned}&d_{H_3}\big(e^{3A+2C-\Phi}\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}-\Psi_{21}^{[\alpha\beta]})_-\big)-2e^{3A+C-\Phi}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}-\Psi_{22}^{[\alpha\beta]})_+\\&\quad-2me^{2A+2C-\Phi}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]})_++e^{3A+2C-\Phi}H_1\wedge\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]})_-=0,\end{aligned} \tag{B.20i}$$

$$\begin{aligned}&d_{H_3}\big(e^{3A+2C-\Phi}\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_-\big)-2e^{3A+C-\Phi}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}-\Psi_{22}^{(\alpha\beta)})_+\\&\quad-2me^{2A+2C-\Phi}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_++e^{3A+2C-\Phi}H_1\wedge\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_-=0\end{aligned} \tag{B.20j}$$

and some more expressions, which are implied when the sourceless Bianchi identity of the NS flux is obeyed, that are equivalent to,

$$\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}-\Psi_{22}^{[\alpha\beta]})_-\wedge dH_3=\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}-\Psi_{22}^{(\alpha\beta)})_-\wedge dH_3=0, \tag{B.21a}$$

$$e^{2C}\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_+\wedge dH_3+d(e^{2C}H_1)\wedge\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_+=0, \tag{B.21b}$$

$$e^{2C}\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]})_+\wedge dH_3+d(e^{2C}H_1)\wedge\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}-\Psi_{21}^{[\alpha\beta]})_+=0, \tag{B.21c}$$

$$\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}+\Psi_{21}^{[\alpha\beta]})_+\wedge dH_3=\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}+\Psi_{21}^{(\alpha\beta)})_+\wedge dH_3=0, \tag{B.21d}$$

$$e^{2C}\mathrm{Re}(\Psi_{12}^{[\alpha\beta]}-\Psi_{21}^{[\alpha\beta]})_-\wedge dH_3-d(e^{2C}H_1)\wedge\mathrm{Re}(\Psi_{11}^{[\alpha\beta]}+\Psi_{22}^{[\alpha\beta]})_-=0, \tag{B.21e}$$

$$e^{2C}\mathrm{Im}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_-\wedge dH_3-d(e^{2C}H_1)\wedge\mathrm{Im}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_-=0. \tag{B.21f}$$

In turn, the flux equations derive from (B.11a) and (B.14) are,

$$e^{3A}\delta^{\alpha\beta}\star_5\lambda g_2=4d_{H_3}\big(e^{3A-\Phi}\mathrm{Re}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_-\big)-8me^{2A-\Phi}\mathrm{Re}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_+, \tag{B.22a}$$

$$\begin{aligned}e^{3A+2C}\delta^{\alpha\beta}\star_5\lambda g_1=&-4d_{H_3}\big(e^{3A+2C-\Phi}\mathrm{Re}(\Psi_{11}^{(\alpha\beta)}+\Psi_{22}^{(\alpha\beta)})_-\big)-8me^{2A+2C-\Phi}\mathrm{Re}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_+\\&+4e^{3A+2C-\Phi}H_1\wedge\mathrm{Re}(\Psi_{12}^{(\alpha\beta)}-\Psi_{21}^{(\alpha\beta)})_-\end{aligned} \tag{B.22b}$$



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and considering $\lambda(f_-) = -\lambda g_{1-} - e^{2C}\mathrm{vol}(\mathrm{S}^2)\wedge\lambda g_{2-}$ in the pairing equation (B.12c) we obtain the following restrictions,

$$\delta^{\alpha\beta}\big(\mathrm{Re}(\psi_{11}^{(\alpha\beta)}+\psi_{22}^{(\alpha\beta)})_+\wedge\lambda g_{2-} - \mathrm{Re}(\psi_{12}^{(\alpha\beta)}-\psi_{21}^{(\alpha\beta)})_+\wedge\lambda g_{1-}\big) = -2\mu e^{-(\Phi+A)}\mathrm{vol}(\mathrm{M}_5), \tag{B.23a}$$

$$\mathrm{Im}(\psi_{12}^{(\alpha\beta)}-\psi_{21}^{(\alpha\beta)})_+\wedge\lambda g_{2-} = -\mathrm{Im}(\psi_{11}^{(\alpha\beta)}+\psi_{22}^{(\alpha\beta)})_+\wedge\lambda g_{1-}, \tag{B.23b}$$

$$\mathrm{Re}(\psi_{12}^{[\alpha\beta]}-\psi_{21}^{[\alpha\beta]})_+\wedge\lambda g_{2-} = -\mathrm{Re}(\psi_{11}^{[\alpha\beta]}+\psi_{22}^{[\alpha\beta]})_+\wedge\lambda g_{1-}. \tag{B.23c}$$

The matrix 0-form constraints coming from (B.20) are,

$$\begin{aligned}
&\eta_{22}^{c\dagger}\eta_{11} = \eta_{21}^{c\dagger}\eta_{12}, \qquad \mathrm{Im}(\eta_{22}^{\dagger}\eta_{11}) = \mathrm{Im}(\eta_{21}^{\dagger}\eta_{12}),\\
&(1+me^{C-A})\eta_{21}^{\dagger}\eta_{11} = (1-me^{C-A})\eta_{22}^{\dagger}\eta_{12},\\
&(1+me^{C-A})\mathrm{Im}(\eta_{21}^{c\dagger}\eta_{11}) = (1-me^{C-A})\mathrm{Im}(\eta_{22}^{c\dagger}\eta_{12}).
\end{aligned} \tag{B.24}$$

which will be used in the main text in order to find restrictions in the spinors.



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## 5.3 2d Conformal field theories for class I

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# Two dimensional $\mathcal{N} = (0, 4)$ quivers dual to AdS$_3$ solutions in massive IIA

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Abstract: In this paper we discuss an infinite family of new solutions in massive Type IIA supergravity with AdS$_3\times$S$^2$ factors, preserving $\mathcal{N} = (0, 4)$ SUSY. After studying geometrical aspects of the backgrounds we propose a duality with a precise family of quivers that flow to (0,4) fixed points at low energies. These quivers consist on two families of (4,4) linear quivers coupled by matter fields. We present various tests of our proposal.

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## Contents



## 1 Introduction

The study of generic quantum field theories (QFTs) is one of the main topics of interest in present-day theoretical Physics. Perturbative and non-perturbative investigations in the recent decades have shown that remarkable progress can be achieved when the system under study is symmetric enough.

One major line of research that came as a by-product of the Maldacena conjecture [1], is the study of supersymmetric and conformal field theories in diverse dimensions. Superconformal Field Theories (SCFTs) exist in space-time dimensions $d < 7$ [2]. The last two decades witnessed a large effort in the classification of Type II or M-theory backgrounds with $\mathrm{AdS}_{d+1}$ factors, see for example [3, 4]. The solutions are conjectured to be dual to SCFTs in $d$ dimensions with different amounts of SUSY. In the case in which we have eight Poincaré supercharges major progress has been achieved (the number of real supercharges doubles by the presence of the conformal partner supercharges).

For the case of $\mathcal{N} = 2$ SCFTs in four dimensions, the field theories studied in [5] have holographic duals first discussed in [6], and further elaborated (among other works) in [7]–[12]. The case of five dimensional SCFTs was analysed from the field theoretical and holographic viewpoints in [13]–[18], among many other interesting works. An infinite family of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs was discussed from both the field theoretical and holographic points of view in [19]–[27]. For three-dimensional $\mathcal{N} = 4$ SCFTs, the field theories presented in [28] were discussed holographically in [29]–[32], among other works.

The case of two-dimensional SCFTs and their AdS duals is particularly attractive. The interest that CFTs in two dimensions and $\mathrm{AdS}_3$ solutions present in other areas of theoretical Physics (condensed matter systems, black holes, etc), and the power of the 2-d super conformal algebra present us with a perfect *theoretical lab* to test various ideas explicitly. This motivated various attempts at finding classifications of $\mathrm{AdS}_3$ backgrounds and studying their dual CFTs — for a sample of papers see [33]–[47].

In this work we add a new entry to the dictionary between SCFTs and string backgrounds with an AdS-factor described above. We deal with $\mathcal{N} = (0, 4)$ (small algebra) SCFTs. We define our SCFTs as the IR fixed points of $\mathcal{N} = (0, 4)$ UV finite QFTs. These QFTs are described by quivers, consisting of two long rows of gauge groups connected by hypermultiplets and Fermi multiplets. There are also global (flavour) symmetry groups, joined with the gauge groups by Fermi multiplets. Quantum theories of this kind (with some differences regarding the field content and R-symmetry charges) have been proposed in the study of solitonic strings in six-dimensional $\mathcal{N} = (1, 0)$ SCFTs, see for example [41].[1] We show that the new background solutions to massive IIA supergravity constructed recently in [47] contain the needed isometries to be dual to our SCFTs. These backgrounds may be trusted when the number of nodes of the quiver is large and so are the ranks of each gauge group.[2] We show that they reproduce the central charge of our SCFTs in the holographic limit.

The contents of this paper are distributed as follows. In section 2 we summarise the general massive Type IIA backgrounds that we constructed recently in [47], and find new solutions, also presented in [48]. These backgrounds have the structure

$$\mathrm{AdS}_3 \times \mathrm{CY}_2 \times \mathrm{S}^2 \times \mathrm{I}_\rho. \tag{1.1}$$

By $\mathrm{I}_\rho$ we denote an interval parametrised by a coordinate that we label $\rho$. There are warp factors in front of each metric component (also for each of the RR and NS fluxes compatible

[1] See also [49] for realisations in terms of D3-brane boxes.

[2] See the recent paper [50] for long 5d quivers.

with the isometries of the background). We discuss various observable quantities of these backgrounds, like the Page charges, the explicit presence of branes (we map these data into Hanany-Witten brane set-ups) and the holographic central charges. All these quantities are described in terms of the functions that define the warp factors.

In section 3 we define the QFTs of our interest. In order to do this we take a small detour through 2-d $\mathcal{N} = (0, 2)$ multiplets. In terms of them we write the field content of our $\mathcal{N} = (0, 4)$ QFTs. We pay special attention to the cancellation of gauge anomalies. We propose that these QFTs flow in the IR to strongly coupled $\mathcal{N} = (0, 4)$ SCFTs with small superconformal algebra. We use this to link the R-symmetry anomaly (the level of the Kac-Moody algebra) with the central charge (the leading coefficient in the OPE of energy-momentum tensors). We finally propose a generic duality between our SCFTs and the backgrounds discussed in section 2.

In section 4 (of pedagogical character), we present a detailed set of examples that serve as tests of our proposed duality. In those examples we show how the supergravity backgrounds (with the predicted number of *colour* and *flavour* branes) have the precise *combinatorics* to be dual to long quivers with non-anomalous gauge symmetries and flavour symmetries. We calculate the central charge in the SCFT and the holographic central charge in the gravity background showing a clean matching between both descriptions.

We close the paper with a brief summary and some ideas for further research in section 5. The presentation is complemented by appendices of technical nature.

## 2 The holographic backgrounds

In this section we start by discussing the solutions to massive IIA supergravity (with localised sources) obtained in the recent work [47]. We propose that these backgrounds are holographic duals to two dimensional CFTs preserving $\mathcal{N} = (0, 4)$ SUSY. The particular CFTs will be discussed in section 3. The Neveu-Schwarz (NS) sector of these bosonic solutions reads,

$$\begin{aligned}
ds^2 &= \frac{u}{\sqrt{\hat{h}_4 h_8}}\left(ds^2(\mathrm{AdS}_3) + \frac{h_8\hat{h}_4}{4h_8\hat{h}_4 + (u')^2}ds^2(\mathrm{S}^2)\right) + \sqrt{\frac{\hat{h}_4}{h_8}}ds^2(\mathrm{CY}_2) + \frac{\sqrt{\hat{h}_4 h_8}}{u}d\rho^2,\\
e^{-\Phi} &= \frac{h_8^{\frac{3}{4}}}{2\hat{h}_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8\hat{h}_4 + (u')^2}, \quad H = \frac{1}{2}d\left(-\rho + \frac{uu'}{4\hat{h}_4 h_8 + (u')^2}\right) \wedge \mathrm{vol}(\mathrm{S}^2) + \frac{1}{h_8}d\rho \wedge H_2,
\end{aligned} \tag{2.1}$$

here $\Phi$ is the dilaton, $H = dB_2$ is the NS 3-form and $ds^2$ is written in string frame. The warping function $\hat{h}_4$ has support on $(\rho, \mathrm{CY}_2)$. On the other hand, $u$ and $h_8$ only depend of $\rho$. We denote $u' = \partial_\rho u$ and similarly for $h_8'$. The RR fluxes are

$$F_0 = h_8', \quad F_2 = -H_2 - \frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4h_8\hat{h}_4 + (u')^2}\right)\mathrm{vol}(\mathrm{S}^2), \tag{2.2a}$$

$$\begin{aligned}
F_4 &= \left(d\left(\frac{uu'}{2\hat{h}_4}\right) + 2h_8 d\rho\right) \wedge \mathrm{vol}(\mathrm{AdS}_3)\\
&\quad - \frac{h_8}{u}(\star_4 d_4\hat{h}_4) \wedge d\rho - \partial_\rho\hat{h}_4\mathrm{vol}(\mathrm{CY}_2) - \frac{uu'}{2(4h_8\hat{h}_4 + (u')^2)}H_2 \wedge \mathrm{vol}(\mathrm{S}^2),
\end{aligned} \tag{2.2b}$$

with the higher fluxes related to them as $F_6 = -\star_{10} F_4$, $F_8 = \star_{10} F_2$, $F_{10} = -\star_{10} F_0$. It was shown in [47] that supersymmetry holds whenever

$$u'' = 0, \qquad H_2 + \hat{\star}_4 H_2 = 0, \tag{2.3}$$

where $\hat{\star}_4$ is the Hodge dual on $\mathrm{CY}_2$. In what follows we will concentrate on the set of solutions for which $H_2 = 0$. The Bianchi identities of the fluxes then impose (away from localised sources)

$$h_8'' = 0, \qquad \frac{h_8}{u}\nabla^2_{\mathrm{CY}_2}\hat{h}_4 + \partial_\rho^2 \hat{h}_4 = 0.$$

A further restriction consists in assuming that $\hat{h}_4 = \hat{h}_4(\rho)$. After this, the string frame background reads,

$$\begin{aligned}
ds^2_{st} &= \frac{u}{\sqrt{\hat{h}_4 h_8}}\left(ds^2(\mathrm{AdS}_3) + \frac{h_8\hat{h}_4}{4h_8\hat{h}_4 + (u')^2}ds^2(\mathrm{S}^2)\right) + \sqrt{\frac{\hat{h}_4}{h_8}}ds^2(\mathrm{CY}_2) + \frac{\sqrt{\hat{h}_4 h_8}}{u}d\rho^2,\\
e^{-\Phi} &= \frac{h_8^{\frac{3}{4}}}{2\hat{h}_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8\hat{h}_4 + (u')^2}, \qquad B_2 = \frac{1}{2}\left(-\rho + 2\pi k + \frac{uu'}{4\hat{h}_4 h_8 + (u')^2}\right)\mathrm{vol}(\mathrm{S}^2),\\
\hat{F}_0 &= h_8', \qquad \hat{F}_2 = -\frac{1}{2}\Big(h_8 - h_8'(\rho - 2\pi k)\Big)\mathrm{vol}(\mathrm{S}^2),\\
\hat{F}_4 &= \left(\partial_\rho\left(\frac{uu'}{2\hat{h}_4}\right) + 2h_8\right)d\rho\wedge\mathrm{vol}(\mathrm{AdS}_3) - \partial_\rho\hat{h}_4\mathrm{vol}(\mathrm{CY}_2).
\end{aligned}\tag{2.4}$$

We have written the Page fluxes $\hat{F} = e^{-B_2}\wedge F$ that are more useful for our purposes. Notice that we have also allowed for large gauge transformations $B_2 \to B_2 + \pi k \mathrm{vol}(\mathrm{S}^2)$, for $k = 0, 1, \ldots, P$. The transformations are performed every time we cross an interval $[2\pi k, 2\pi(k+1)]$. To motivate this consider the following: in the limit where $\hat{h}_4(\rho)$ and/or $h_8(\rho)$ become large compared with $u(\rho)$ the NS 2-form in the presence of $k$ large gauge transformations is approximately

$$B_2 \sim \frac{1}{2}(-\rho + 2\pi k)\mathrm{vol}(\mathrm{S}^2) \implies \hat{b}_0 = -\frac{1}{(2\pi)^2}\int_{\mathrm{S}^2} B_2 \sim \frac{1}{2\pi}(\rho - 2\pi k). \tag{2.5}$$

This can be archived by tuning certain integration constants in the solutions presented below, and in fact coincides with the limit of weak curvature where the supergravity approximation can be trusted. Demanding that $\hat{b}_0$ lies in the fundamental region $\hat{b}_0 \in [0, 1)$ partitions the real line spanned by $\rho$ into segments of length $2\pi$. A large gauge transformation ($B_2 \to B_2 + \pi\mathrm{vol}(\mathrm{S}^2)$) is required as one crosses between these segments, such that the NS 2-form quoted in (2.4) is valid in the segment $2k\pi \le \rho < 2\pi(k+1)$ with $k = 0, 1, 2 \ldots$.

The background in (2.4) is a SUSY solution of the massive IIA equations of motion if the functions $\hat{h}_4, h_8, u$ satisfy (away from localised sources),

$$\hat{h}_4''(\rho) = 0, \quad h_8''(\rho) = 0, \quad u''(\rho) = 0. \tag{2.6}$$

The three functions are thus linear. Various particular solutions were analysed in [47]. Here we will present an infinite family of solutions for which the functions are piecewise continuous.

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### 2.1 The local solutions

We shall be interested in solutions that in the interval $2\pi k \leq \rho \leq 2\pi(k+1)$ (for $k = 0, 1, \ldots, P$) are of the form,

$$\hat{h}_4^{(k)} = \Upsilon\left(\alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)\right), \quad h_8^{(k)} = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k), \quad u^{(k)} = a_k + \frac{b_k}{2\pi}(\rho - 2\pi k).$$

Here $(\Upsilon, \alpha_k, \beta_k, \mu_k, \nu_k, a_k, b_k)$ are arbitrary constants whose physical meaning we shall discuss below. In particular, we impose that these three functions vanish at $\rho = 0$ (where the space begins) and that the space ends at $\rho = 2\pi(P+1)$, by considering the situation for which $\hat{h}_4$ and/or $h_8$ vanish at this point. These conditions leave us with functions of the form,

$$\hat{h}_4(\rho) = \Upsilon h_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \leq \rho \leq 2\pi \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi(k+1), \quad k := 1, \ldots, P-1 \\ \alpha_P + \frac{\beta_P}{2\pi}(\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1). \end{cases} \tag{2.7}$$

$$h_8(\rho) = \begin{cases} \frac{\nu_0}{2\pi}\rho & 0 \leq \rho \leq 2\pi \\ \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi(k+1), \quad k := 1, \ldots, P-1 \\ \mu_P + \frac{\nu_P}{2\pi}(\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1). \end{cases} \tag{2.8}$$

$$u(\rho) = \begin{cases} \frac{b_0}{2\pi}\rho & 0 \leq \rho \leq 2\pi \\ a_k + \frac{b_k}{2\pi}(\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi(k+1), \quad k := 1, \ldots, P-1 \\ a_P + \frac{b_P}{2\pi}(\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1). \end{cases} \tag{2.9}$$

If the function $\hat{h}_4(\rho)$ vanishes at $\rho = 2\pi(P+1)$, ending the space there, we need that $\alpha_P = -\beta_P$. Similarly if $h_8\big(2\pi(P+1)\big) = 0$, we must impose that $\nu_P = -\mu_P$.

Demanding that the metric, dilaton and $B_2$ field are continuous across the different intervals imposes additional conditions on the various constants.[3] The details are discussed in appendix A. Here we quote one simple solution to these continuity equations,

$$\mu_k = \sum_{j=0}^{k-1} \nu_j, \quad \alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad b_k = b_0, \quad a_k = k b_0. \tag{2.10}$$

These conditions imply the continuity of the functions $\hat{h}_4, h_8$. Their derivatives can, however, present jumps. This will imply discontinuities in the RR sector, that we will interpret as generated by the presence of branes in the background, that modify the Bianchi identities. In turn, notice that (2.10) implies that $u(\rho) = \frac{b_0}{2\pi}\rho$ in all intervals, which is consistent with the supersymmetry requirement (2.3) that $u'' = 0$ globally.

These supergravity backgrounds can be trusted (with localised singularities) if the numbers $P, \alpha_k, \mu_k$ are large. Indeed, the Ricci scalar only diverges at the points where the sources are localised. Choosing the numbers $\nu_k, \beta_k$ to be large controls this divergence. On the other hand $P$ is taken to be large to have these singularities separated enough that we can trust the geometric description given here.

[3] We do not impose the continuity of $H = dB_2$ since $H = F(\rho) d\rho \wedge \mathrm{vol}(\mathrm{S}^2)$. This implies that $dH = 0$ and the continuity of $H$ is not needed to avoid the presence of NS brane sources.



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## 2.2 The $\rho$-interval

Let us analyse more closely these solutions. The background functions defined in the first interval $[0, 2\pi]$ show that the space begins at $\rho = 0$ in a smooth fashion. On the other hand, the $\rho$-interval ends at a generic point $\rho = 2\pi(P+1)$ if any of the functions $\hat{h}_4$ and/or $h_8$ vanish at that point. Let us analyse the behaviour of the metric and dilaton close to the end of the space for the three possible cases:

- The space ends by virtue of the function $\hat{h}_4$ whilst $h_8$ is generically non-vanishing at $\rho = 2\pi(P+1)$. In the last interval the functions defining the background are then

$$\hat{h}_4 = \Upsilon\left(\alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P)\right), \quad h_8 = \mu_P + \frac{\nu_P}{2\pi}(\rho - 2\pi P), \quad u = \frac{b_0}{2\pi}\rho.$$

  In this case, expanding the metric and the dilaton close to $\rho = 2\pi(P+1)$ we find, for small values of $x = 2\pi(P+1) - \rho$,

$$ds^2 \sim \frac{m_1}{\sqrt{x}} ds^2(\mathrm{AdS}_3) + \frac{\sqrt{x}}{m_1}\Big[dx^2 + m_1 m_2 ds^2(\mathrm{S}^2) + m_3 m_1 ds^2(\mathrm{CY}_2)\Big], \quad e^{-4\Phi} = \frac{m_4}{x}. \tag{2.11}$$

  The numbers $(m_1, \ldots, m_4)$ are written in terms of $\mu_P, \alpha_P, \nu_P, b_0, \Upsilon$. This asymptotic behaviour indicates that close to the end of the space we have a D2 brane that extends on $\mathrm{AdS}_3$ and is delocalised (or smeared) on $\mathrm{CY}_2\times\mathrm{S}^2$ — see [47] for a generic analysis of singularities. Note that one could also view this as an O2 plane smeared on $\mathrm{CY}_2\times\mathrm{S}^2$ or a superposition of both D2s and O2s.

- The space ends by virtue of the function $h_8$ while $\hat{h}_4$ is generically non-vanishing at $\rho = 2\pi(P+1)$. In the last interval the functions are then

$$h_8 = \mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P), \quad \hat{h}_4 = \Upsilon\left(\alpha_P + \frac{\beta_P}{2\pi}(\rho - 2\pi P)\right), \quad u = \frac{b_0}{2\pi}\rho.$$

  For small $x = 2\pi(P+1) - \rho$, the metric and dilaton scale as,

$$ds^2 \sim \frac{1}{\sqrt{x}}\Big[n_1 ds^2(\mathrm{AdS}_3) + n_3 ds^2(\mathrm{CY}_2)\Big] + \frac{\sqrt{x}}{n_1}\Big[dx^2 + n_1 n_2 ds^2(\mathrm{S}^2)\Big], \quad e^{-4\Phi} = n_4 x^3. \tag{2.12}$$

  The numbers $(n_1, \ldots, n_4)$ are written in terms of $\mu_P, \alpha_P, \beta_P, b_0, \Upsilon$. This asymptotic behaviour indicates that at $\rho = 2\pi(P+1)$ we have an O6 plane that extends on $\mathrm{AdS}_3\times\mathrm{CY}_2$.

- Finally, consider the more symmetric case for which the space is closed by the simultaneous vanishing of $\hat{h}_4$ and $h_8$ at $\rho = 2\pi(P+1)$. In this case the functions in the last interval read,

$$h_8 = \mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P), \quad \hat{h}_4 = \Upsilon\left(\alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P)\right), \quad u = \frac{b_0}{2\pi}\rho. \tag{2.13}$$

  For small values of $x = 2\pi(P+1) - \rho$, the metric and dilaton scale as,

$$ds^2 \sim \frac{s_1}{x} ds^2(\mathrm{AdS}_3) + s_3 ds^2(\mathrm{CY}_2) + \frac{x}{s_1}\Big[dx^2 + s_1 s_2 ds^2(\mathrm{S}^2)\Big], \quad e^{-4\Phi} = s_4 x^2. \tag{2.14}$$

The numbers $(s_1, \dots, s_4)$ are written in terms of $\mu_P, \alpha_P, b_0, \Upsilon$. Notice that each quantity above is the product of those in (2.11)–(2.12). This indicates the superposition of O2-O6 planes.

This more symmetric way of ending the space is the one on which we will concentrate our forthcoming analysis. An important observation is that, from the gravity perspective, the behaviour we are finding close to the end of the interval is the least healthy of the three analysed, as the O2s need to be smeared. We believe that the presence of smeared O-planes is an artifact of the supergravity approximation.

To be used below, let us quote the explicit expressions for the different numerical values of $(s_1, s_2, s_3, s_4)$,

$$\begin{aligned} s_1 &= \frac{4\pi^2 b_0 (P+1)}{\sqrt{\alpha_P \mu_P \Upsilon}}, \qquad s_2 = 2\pi(P+1)\frac{\sqrt{\alpha_P \mu_P \Upsilon}}{b_0}, \\ s_3 &= \sqrt{\frac{\Upsilon \alpha_P}{\mu_P}}, \qquad\qquad s_4 = \frac{b_0^2 \mu_P^3}{2^{10}\pi^6 \alpha_P (P+1)^2 \Upsilon}. \end{aligned} \tag{2.15}$$

Notice that in order for the $\mathrm{CY}_2$ space to be large compared with the string size, we need that $\Upsilon \alpha_P \sim \mu_P$. Otherwise the gravity background is not trustable.

In the following section we study the Page charges and discuss the presence of branes in our solutions. These are of the form given by eq. (2.4), with the functions $(\hat{h}_4, h_8, u)$ satisfying eq. (2.6), away from localised sources, and piecewise continuous, as in (2.7)–(2.9). The condition for continuity of the defining functions $\hat{h}_4, h_8$ is given by (2.10). This implies the continuity of the NS-sector of the solution. From all the possibilities to end the space we focus on solutions whose last interval's functions are given by (2.13). The non-compact solution with $\hat{h}_4 \sim h_8 \sim u \sim \rho$ all over the space will be discussed in detail in [51].

### 2.3 Page charges

The Page charges are important observable quantities characterising a supergravity solution. Since they are quantised they imply the quantisation of some of the constants defining the solution in (2.7)–(2.9). The Page charge of Dp-branes is given by the integral of the magnetic part of the Page $\hat{F}_{8-p}$ form. This is,

$$(2\pi)^{7-p} g_s \alpha'^{(7-p)/2} Q_{Dp} = \int_{\Sigma_{8-p}} \hat{F}_{8-p}. \tag{2.16}$$

In what follows, we choose units consistent with $\alpha' = g_s = 1$. Also, we will use that $\hat{h}_4 = \Upsilon h_4$, as seen in (2.7).

We find the following Page charges for our solutions in the interval $[2\pi k, 2\pi(k+1)]$,

$$\begin{aligned} Q_{D8} &= 2\pi F_0 = 2\pi h_8' = \nu_k. \\ Q_{D6} &= \frac{1}{2\pi}\int_{\mathrm{S}^2} \hat{F}_2 = h_8 - h_8'(\rho - 2\pi k) = \mu_k. \\ Q_{D4} &= \frac{1}{8\pi^3}\int_{\mathrm{CY}_2} \hat{F}_4 = \Upsilon \frac{\mathrm{Vol}(\mathrm{CY}_2)}{16\pi^4}\beta_k, \\ Q_{D2} &= \frac{1}{32\pi^5}\int_{\mathrm{CY}_2\times \mathrm{S}^2} \hat{F}_6 = \Upsilon \frac{\mathrm{Vol}(\mathrm{CY}_2)}{16\pi^4}(h_4 - h_4'(\rho - 2\pi k)) = \Upsilon \frac{\mathrm{Vol}(\mathrm{CY}_2)}{16\pi^4}\alpha_k. \end{aligned} \tag{2.17}$$

We have used that the magnetic part of $\hat{F}_6$ is

$$\hat{F}_{6,\text{mag}} = \hat{f}_6 = \frac{\Upsilon}{2}\left(h_4 - h_4'(\rho - 2\pi k)\right) \text{vol}(\text{S}^2) \wedge \text{vol}(\text{CY}_2). \tag{2.18}$$

We also have one NS-five brane every time we cross the value $\rho = 2\pi k$ (for $k = 1, \ldots, P$). The total number of NS-five branes is $Q_{NS} = \frac{1}{4\pi^2}\int_{\rho\times S^2} H_3 = (P+1)$.

In what follows, we choose the constant $\Upsilon$ to satisfy $\Upsilon\text{Vol}(\text{CY}_2) = 16\pi^4$. This implies that the constants $\alpha_k, \beta_k$ are integer numbers (like $\nu_k, \mu_k$ are). They are directly related with the number of branes in the associated Hanany-Witten brane set-up.

To understand which branes are present in our backgrounds, let us study the Bianchi identities for the Page fluxes.

#### 2.3.1 Hanany-Witten brane set-up

We now calculate the Bianchi identities for the Page fluxes. The goal is to determine which branes are actually present in our background solutions, either as sources or dissolved into fluxes.

Let us start with the flux $F_0 = h_8'(\rho)$. We calculate $dF_0 = h_8''(\rho)d\rho$. Now, at a generic point of the $\rho$-coordinate we will have $h_8'' = 0$, according to (2.6). However, due to our definition of the functions $\hat{h}_4$ and $h_8$ — see (2.7)–(2.8), something special occurs at the points where the functions change slope. In fact, for both $\hat{h}_4$ and $h_8$ we find,

$$h_8'' = \sum_{k=1}^{P}\left(\frac{\nu_{k-1} - \nu_k}{2\pi}\right)\delta(\rho - 2\pi k), \quad \hat{h}_4'' = \Upsilon\sum_{k=1}^{P}\left(\frac{\beta_{k-1} - \beta_k}{2\pi}\right)\delta(\rho - 2\pi k). \tag{2.19}$$

As a consequence of this we have,

$$\begin{aligned} dF_0 &= \sum_{k=1}^{P}\left(\frac{\nu_{k-1} - \nu_k}{2\pi}\right)\delta(\rho - 2\pi k)d\rho, \\ d\hat{F}_4 &= \Upsilon\sum_{k=1}^{P}\left(\frac{\beta_{k-1} - \beta_k}{2\pi}\right)\delta(\rho - 2\pi k)d\rho \wedge \text{vol}(\text{CY}_2), \end{aligned} \tag{2.20}$$

indicating that at the points $\rho = 2\pi k$ there may be localised D8 and semi-localised D4 branes. In fact, explicit D8 and D4 branes are present at $\rho = 2\pi k$ when the slopes of $h_8, \hat{h}_4$ are different at both sides.

Let us investigate the same about D2 and D6 branes. For the magnetic part of the Page fluxes, we compute in the interval $[2\pi k, 2\pi(k+1)]$

$$\begin{aligned} d\hat{F}_2 &= \frac{1}{2}h_8'' \times (\rho - 2\pi k)d\rho \wedge \text{vol}(\text{S}^2), \\ d\hat{F}_6 &= d\hat{f}_6 = \frac{1}{2}\hat{h}_4'' \times (\rho - 2\pi k)d\rho \wedge \text{vol}(\text{S}^2) \wedge \text{vol}(\text{CY}_2). \end{aligned} \tag{2.21}$$

Using (2.19) and that $x\delta(x) = 0$, we then find that there are no sources for D2 or D6 branes present. This is precisely because a large gauge transformation of the NS two-form

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D2 | x | x |  |  |  |  | x |  |  |  |
| D4 | x | x |  |  |  |  |  | x | x | x |
| D6 | x | x | x | x | x | x | x |  |  |  |
| D8 | x | x | x | x | x | x |  | x | x | x |
| NS5 | x | x | x | x | x | x |  |  |  |  |

**Table 1**. $\frac{1}{8}$-BPS brane intersection underlying our geometry. The directions $(x^0, x^1)$ are the directions where the 2d CFT lives (dual to our $\mathrm{AdS}_3$). The directions $(x^2, \dots, x^5)$ span the $\mathrm{CY}_2$, on which the D6 and the D8-branes are wrapped. The coordinate $x^6$ is the direction associated with $\rho$. Finally $(x^7, x^8, x^9)$ are the transverse directions realising an SO(3)-symmetry associated with the isometries of $\mathrm{S}^2$.



is performed at the loci of the D8 and D4s, were this not the case a source term for D6 and D2 would be induced as in section 5.1 of [47].[4]

This study suggests that the D2 and D6 branes will play the role of *colour* branes, while the D4 and D8 branes that of *flavour* branes. The global symmetry in the dual CFT is gravitationally realised by the gauge fields that fluctuate on the D4 or D8 branes.

Studying the associated Hanany-Witten [52] set-up, we find that in flat space the branes are distributed as indicated in table 1. Our proposal is that the geometries described by (2.4), capture the near horizon, or decoupling limit, of the brane configuration, once a suitable large number of NS and D-branes is considered.

Using our result for the Page charges in (2.17) and the modified Bianchi identities in (2.20), we find that the number of D-branes in the interval $[2\pi(k-1), 2\pi k]$ (in between two NS-five branes) is,

$$N_{D8}^{[k-1,k]} = \nu_{k-1} - \nu_k, \qquad N_{D4}^{[k-1,k]} = \beta_{k-1} - \beta_k, \tag{2.22}$$

$$N_{D6}^{[k-1,k]} = \mu_k = \sum_{i=0}^{k-1} \nu_i, \qquad N_{D2}^{[k-1,k]} = \alpha_k = \sum_{i=0}^{k-1} \beta_i. \tag{2.23}$$

We then have a Hanany-Witten brane set-up, that in the interval $[2\pi(k-1), 2\pi k]$ (bounded by NS-five branes), has $N_{D6}^{[k-1,k]}$, $N_{D2}^{[k-1,k]}$ colour branes and $N_{D8}^{[k-1,k]}$, $N_{D4}^{[k-1,k]}$ flavour branes. See figure 1.

### 2.4 Holographic central charge

To close our study of the background in (2.4) we will calculate the holographic central charge associated with these solutions. The idea is to compare with the central charge of the proposed dual conformal field theory, that we study in the coming sections.

[4]The D8 and D4 can also be shown to be supersymmetric by a small modification of the argument in [47]. There, it was assumed that no gauge transformations are performed on the brane, which lead to D8 and D4 world volume gauge fields being required by supersymmetry and the source corrected Bianchi identities. Here these gauge fields have been absorbed by the large gauge transformation of the NS two-form. The branes now restricted to lie at $\rho = 2\pi(k+1)$, $k = 0, 1, 2 \dots$ We give some details in appendix B.

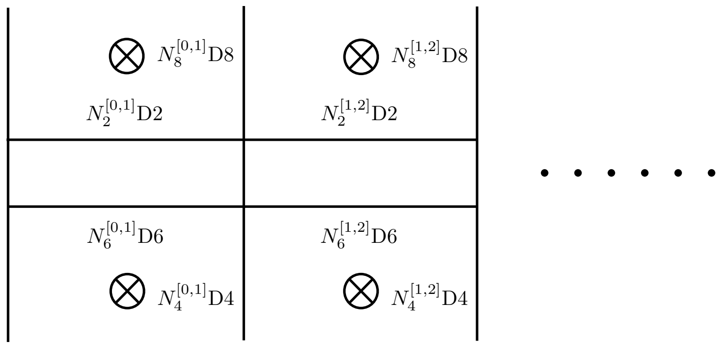


**Figure 1**. The generic Hanany-Witten set-up associated with our backgrounds. The vertical lines are NS-five branes. The horizontal lines represent D2 and D6 branes. The crosses indicate D4 and D8 branes.

The central charge is one of the important observables for conformal field theories. It appears when calculating the trace of the energy-momentum tensor, for a theory defined on a curved space. In the case of two dimensional conformal field theories, there is only one relevant quantity — denoted by "$c$" — that appears when computing $< T_\mu^\mu >= -\frac{c}{24\pi} R$. Here $R$ is the Ricci scalar of the manifold on which the CFT is defined and $c$ is the central charge.

The holographic calculation of this quantity has a very interesting history. It was first obtained in [53] (before the Maldacena conjecture was formulated), then calculated in [54]. In the context of AdS-supergravity, it was holographically computed in [55] and [56]. In [57] generic supergravity solutions were considered that were later generalised in [58]. This is the formalism we will use. It basically boils down to computing the volume of the internal space (excluding $\mathrm{AdS}_3$).

In a putative compactification to an effective 3-d supergravity this volume is the inverse of the 3-d Newton constant. However, in general, it needs to be weighted by factors of the dilaton and other warp factors. In fact, for a generic dilaton and background of the form,

$$ds^2 = a(r,\vec{\theta})(dx_{1,d}^2 + b(r)dr^2) + g_{ij}(r,\vec{\theta})d\theta^i d\theta^j, \quad \Phi(r,\vec{\theta}), \tag{2.24}$$

one should calculate the auxiliary quantity [58]

$$\hat{H} = \left(\int d\vec{\theta}\sqrt{e^{-4\Phi}\det[g_{ij}]a(r,\vec{\theta})^d}\right)^2.$$

With this, one computes the holographic central charge (see [58, 59] for the derivation) to be,

$$c_{\text{hol}} = 3 \times \frac{d^d}{G_N}\frac{b(r)^{d/2}(\hat{H})^{\frac{2d+1}{2}}}{(\hat{H}')^d}. \tag{2.25}$$

The factor of "3" in (2.25) is introduced as a normalisation, to coincide with the standard result of [53].

For the case at hand, comparing with the solutions in (2.4) and using Poincaré coordinates for $\mathrm{AdS}_3$, we have

$$\begin{aligned}
& a(r,\vec{\theta}) = \frac{u}{\sqrt{\hat{h}_4 h_8}} r^2, && b(r) = \frac{1}{r^4}, \qquad d = 1, \\
& \det[g_{ij}] = u\sqrt{\frac{\hat{h}_4^7}{h_8}} \frac{\sin^2\chi}{\left(4\hat{h}_4 h_8) + (u')^2\right)^2}, && \sqrt{e^{-4\Phi}\det[g_{ij}]a} = \frac{r}{4}\hat{h}_4 h_8 \sin\chi, \\
& \hat{H} = \mathcal{N}^2 r^2, && \mathcal{N} = \pi \mathrm{Vol}(\mathrm{CY}_2)\int_0^{2\pi(P+1)} \hat{h}_4 h_8 d\rho.
\end{aligned} \tag{2.26}$$

We then obtain,

$$c_{\mathrm{hol}} = \frac{3}{2G_N}\mathcal{N} = \frac{3\pi}{2G_N}\mathrm{Vol}(\mathrm{CY}_2)\int_0^{2\pi(P+1)} \hat{h}_4 h_8 d\rho = \frac{3}{\pi}\int_0^{2\pi(P+1)} h_4 h_8 d\rho, \tag{2.27}$$

where in the last equality we have used — see below (2.18),

$$\Upsilon \mathrm{Vol}(\mathrm{CY}_2) = 16\pi^4, \quad \hat{h}_4 = \Upsilon h_4, \quad G_N = 8\pi^6.$$

It is useful to express the holographic central charge in terms of the constants $\alpha_k$, $\beta_k$, $\mu_k$, $\nu_k$ defining the solution,

$$c_{\mathrm{hol}} = \sum_{j=0}^{P}\Big(6\alpha_j\mu_j + 3\alpha_j\nu_j + 3\beta_j\mu_j + 2\beta_j\nu_j\Big). \tag{2.28}$$

We shall come back to these expressions in section 4 when we discuss the matching between the holographic quantities studied in this section and the field theory observables discussed below.

# 3 The $\mathcal{N} = (0,4)$ SCFTs

As we advanced in the Introduction, the idea of this work is to propose a duality between the new background solutions in massive IIA found in [47] (summarised in section 2) and a set of CFTs. These CFTs are thought to be arising as low energy fixed points in the RG flows of well defined $\mathcal{N} = (0,4)$ two dimensional quantum field theories.

In this section we discuss the weakly coupled UV description of such quantum field theories.

## 3.1 The UV description

Let us start with a brief discussion of the fields involved in the weakly coupled description. It is usual to describe $\mathcal{N} = (0,4)$ SUSY in terms of $\mathcal{N} = (0,2)$ superfields. In this paper we will not use the detailed structure of each $(0,2)$ multiplet. We shall content ourselves with listing the degrees of freedom together with the R-charges for the fermions involved. As we explain below, these are the details we need to discuss cancellation of gauge anomalies, the R-charge anomaly and the central charge of the IR CFT.

The superfields of $\mathcal{N} = (0,2)$ two-dimensional SUSY are well described in various references. We found particularly clear and enlightening the papers [60]–[64]. They contain some of the results we summarise in this section.

As we advanced, instead of going into the details of the $(0,2)$ supermultiplets we describe the degrees of freedom involved in each of them:

- *Vector multiplet, $U$.* It contains a gauge field $A_\mu$ and one left moving fermion $\lambda_-$.
- *Chiral multiplet, $\Phi$.* It consists of a complex scalar $\varphi$ and a right moving fermion $\psi_+$. By the context, we hope the reader will be able to distinguish between the chiral multiplet and the dilaton in massive IIA, that we denote with the same character $\Phi$.
- *Fermi multiplet, $\Theta$.* This is a constrained superfield for which only a left handed fermion $\psi_-$ propagates. The constraint defining the Fermi superfield generates interactions between the Fermi and the chiral multiplets. The field strength multiplet is an example of a Fermi multiplet. It being constrained agrees with the fact that in two dimensions, a gauge field has no propagating degrees of freedom.

We are interested in theories for which the amount of SUSY is $\mathcal{N} = (0,4)$. In this case the quantum field theories are formulated in terms of combinations of $(0,2)$ superfields. For $(0,4)$ SUSY we have:

- $(0,4)$ *vector multiplet.* It is expressed as a combination of a $(0,2)$ vector multiplet and a $(0,2)$ Fermi multiplet. There are two left handed fermions $\lambda_-^a$ with $a = 1,2$ and a gauge field $A_\mu$.
- $(0,4)$ *hypermultiplet.* Defined as the combination of two chiral multiplets. The degrees of freedom are two complex scalars and two right handed fermions $\psi_+^a$.
- $(0,4)$ *twisted hypermultiplet.* Also written as a superposition of two chiral multiplets. The degrees of freedom are two right handed fermions $\tilde{\psi}_+^a$ and two complex scalars. The difference with the (non-twisted) hypermultiplet discussed above is in the R-charge assignment. This is reflected in the interactions with other multiplets.
- $(0,4)$ *Fermi multiplet.* It is the superposition of two $(0,2)$ Fermi multiplets. As such, it contains two left handed fermionic degrees of freedom, $\psi_-^a$.
- $(0,2)$ *Fermi multiplet.* As explained in [61], it is compatible with $(0,4)$ SUSY to have the single left handed fermion of the $(0,2)$ Fermi multiplet.

The couplings between these multiplets and the constraints on some of them determine the interactions. These can be derived from a superpotential. See [60]–[62] for the details.

In a similar vein one can write the $\mathcal{N} = (4,4)$ SUSY field content in terms of $\mathcal{N} = (0,4)$ fields. Notice that in both $(0,4)$ hypers, we have right handed fermions and in the $(0,4)$ vector multiplet left handed ones. In fact, a $(4,4)$ vector multiplet contains a $(0,4)$ vector multiplet and a $(0,4)$ twisted-hypermultiplet (this is: a vector, a Fermi and two chirals of

(0,2) SUSY). A $\mathcal{N} = (4,4)$ hypermultiplet contains a $(0,4)$ hypermultiplet and a $(0,4)$ Fermi multiplet, hence containing two Fermi and two chiral multiplets of $(0,2)$ SUSY.

The R-symmetry of $\mathcal{N} = (0,4)$ field theories is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. We single out a $\mathrm{U}(1)_R$ inside $\mathrm{SU}(2)_R$ and quote the $\mathrm{U}(1)_R$ charge of each fermion in the above multiplets. This will be used below to calculate the anomaly of the global R-symmetry. See equation (3.13) in the paper [63] for the same charge assignment.

For the $(0,4)$ vector multiplet we have that the left handed fermion inside the vector has $R[\lambda_-^v] = 0$ while the left handed fermion inside the Fermi multiplet has $R[\lambda_-^f] = 1$. Similarly, for the $(0,4)$ twisted hypermultiplet we have that for both right handed fermions $R[\tilde{\psi}_+^a] = 0$. For both right handed fermions inside the $(0,4)$ hypermultiplet we have $R[\psi_+^a] = -1$. Finally, the fermion inside the $(0,2)$ Fermi multiplet (allowed in theories with $(0,4)$ SUSY) is such that $R[\lambda_-^f] = 0$.

Now, we explore the condition for cancellation of gauge anomalies.

### 3.2 Anomaly cancellation

We are dealing with chiral theories. Their consistency requires one to be careful with the field content, so that gauge anomalies are vanishing. In this work we only need to use that the anomaly of a (gauged or global) non-Abelian symmetry is given by the correlator of the symmetry currents, $< J_\mu^A(x) J_\nu^B(x) >\sim k\delta^{A,B}\delta_{\mu\nu}$. Notice that there is no mixing between non-Abelian currents. On the other hand, Abelian currents can mix. The coefficient $k$ is calculated by computing $\mathrm{Tr}[\gamma_3 J_{\mathrm{SU}(N)} J_{\mathrm{SU}(N)}]$. This should be read as the difference between the right handed fermions times their charge squared and the left handed fermions times their charge squared. Let us study in detail the contribution to the $\mathrm{SU}(N)$ anomaly coming from the various $\mathcal{N} = (0,2)$ multiplets mentioned above:

- Chiral multiplets. If they are in the adjoint representation of the symmetry group $\mathrm{SU}(N)$, they contribute with a factor $N$. If they transform in the (anti) fundamental, they contribute with a factor $\frac{1}{2}$.
- Fermi multiplets. If they are in the adjoint representation of the symmetry group $\mathrm{SU}(N)$, they contribute with a factor $-N$. If they transform in the (anti) fundamental, they contribute with a factor $-\frac{1}{2}$.
- Vector multiplets. They are in the adjoint representation of the symmetry group $\mathrm{SU}(N)$. They contribute with a factor $-N$.

### 3.3 Building block of our theories

Let us discuss now what will be the 'building block' of our quantum field theories. See figure 2. We have an $\mathrm{SU}(N)$ gauge group. In the gauge group the matter content is that of a $(4,4)$ vector multiplet, namely — in $(0,2)$ notation, a vector, two twisted chirals and a Fermi multiplet in the adjoint representation of $\mathrm{SU}(N)$. This gauge group is joined with other (gauged of global) symmetry groups $\mathrm{SU}(\hat{P})$, $\mathrm{SU}(R)$ and $\mathrm{SU}(Q)$. The connection with the $\mathrm{SU}(\hat{P})$ symmetry group is mediated by $(4,4)$ hypers. In $(0,2)$ notation, $2 \times N \times \hat{P}$ Fermi multiplets and $2 \times N \times \hat{P}$ chiral multiplets run over the black solid line. The connection

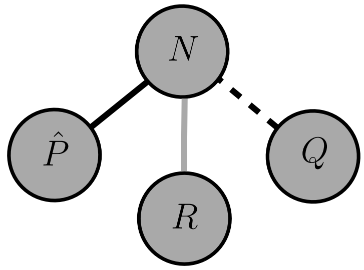


**Figure 2**. The building block of our theories. The solid black line represents a $(4,4)$ hypermultiplet. The grey line represents a $(0,4)$ hypermultiplet. The dashed line represents a $(0,2)$ Fermi multiplet. Inside the gauge group $\mathrm{SU}(N)$ run $(4,4)$ SUSY vector multiplets. The groups $\mathrm{SU}(\hat{P})$, $\mathrm{SU}(Q)$ and $\mathrm{SU}(R)$ can be gauge or global.



with the $\mathrm{SU}(R)$ symmetry group is via $(0,4)$ hypermultiplets. In $(0,2)$ notation $2 \times N \times R$ chiral multiplets propagate over the grey lines. Finally, over the dashed line run $N \times Q$ Fermi multiplets in $(0,2)$ notation. Notice that a similar (but not the same!) field content to this was proposed in [41], in the study of the field theories associated with tensionless strings in $\mathcal{N} = (0,1)$ six-dimensional SCFTs.

Let us now calculate the anomaly of the gauged $\mathrm{SU}(N)$ symmetry group and impose that it vanish. We focus only on the gauged $\mathrm{SU}(N)$ group, but a similar job should be done for all other gauged symmetry groups. Let us spell the various contributions:

- The contribution of the adjoint fields is $2N - N - N = 0$. This is expected, as the field content is that of a $(4,4)$ vector multiplet.
- The contribution of the bifundamentals connecting with $\mathrm{SU}(\hat{P})$ is $(\frac{1}{2} - \frac{1}{2})2\hat{P}N = 0$. Again, this vanishing contribution is expected as we are dealing with $(4,4)$ hypers.
- The link with the symmetry $\mathrm{SU}(R)$ contributes a factor $2 \times N \times R \times \frac{1}{2} = NR$.
- Finally the bifundamentals running on the link with the $\mathrm{SU}(Q)$ symmetry group contribute $-\frac{1}{2}NQ$.

Thus, in order to have a non anomalous gauged symmetry we need to impose that the four contributions above add to zero, that is

$$2R = Q\,. \tag{3.1}$$

This mechanism should apply to all other gauged symmetry groups. When we construct our gauge theories, they will be represented by quivers obtained by 'assembling' the building blocks of figure 2.

### 3.4 U(1) R-symmetry anomaly

It is instructive to compute the R-symmetry anomaly for our 'building block'. Once again, we focus the attention on the $\mathrm{SU}(N)$ gauge group. We use the values for the $\mathrm{U}(1)_R$

charges quoted near the end of section 3.1. We find that the $U(1)_R$ anomaly, following from $\mathrm{Tr}[\gamma_3 Q_i^2]$ is given by the sum of various contributions. In detail, we have,

- For the fields in the adjoint of the $SU(N)$ gauge group, the only contribution is from the fermions inside the Fermi multiplet (all the other fermions have zero $U(1)_R$ charge). The contribution of these particular left handed fermions is $-(N^2-1)$. This coincides with (minus) the number of $(0,4)$ vector multiples in $SU(N)$.

- The contribution from the bifundamentals joining $SU(N)$ with $SU(\hat{P})$ is $N \times \hat{P}$. This is the number of $(0,4)$ hypermultiplets in that link.

- The contribution coming from the fields running over the grey line, joining $SU(N)$ with $SU(R)$, is $N \times R$, once again, counting the number of $(0,4)$ hypers running on the link.

- Finally, the fields running over the dashed line do not contribute as the R-charge of the left handed fermion is zero, as we discussed above.

In summary, we find that

$$\mathrm{Tr}[\gamma_3 Q_i^2] \sim (n_{\mathrm{hyp}} - n_{\mathrm{vec}}). \tag{3.2}$$

Thus, the R-symmetry anomaly is proportional to the number of $(0,4)$ hypers minus the number of $(0,4)$ vectors.

### 3.5 Central charge, R-anomaly and the superconformal algebra

Up to this point, we have found the condition for our building block to be non-anomalous, see (3.1), and the contribution of the matter charged under $SU(N)$ to the $U(1)_R$ anomaly, see (3.2). If the theory becomes conformal and strongly coupled — as we shall propose our quivers do when flowing to low energies — the coefficients for the anomalies cannot be computed by summing over fermions at the conformal point (as we do not have a particle-like description of the CFT). But since these coefficients are 't Hooft anomalies, they are invariants under RG-flow. Hence UV-QFT calculations are good for the same IR-CFT quantity (we are assuming that the proposed R-symmetry does not mix in the IR with other Abelian symmetries). We propose that our quivers become conformal in the IR and then the central charge of the quiver and the R-symmetry anomaly get related by the superconformal algebra.

In our case the relevant superconformal algebra is the small $\mathcal{N} = (0,4)$ algebra. This consists of eight operators: the energy momentum tensor $T(z)$, four fermionic superpartners $G^a(z)$ and three Kac-Moody currents $J^i(z)$. The dimensions of these operators are $(2, \frac{3}{2}, 1)$ respectively. The modes of these operators satisfy an algebra that can be derived from the OPE's of the small $\mathcal{N} = (0,4)$ algebra. In particular among the various relations we have,

$$T(z)T(0) \sim \frac{c}{z^4} + 2\frac{T(0)}{z^2} + \frac{\partial T}{z} + \text{regular}, \quad J^i(z)J^l(0) \sim \frac{k^{il}}{z^2} + \text{regular}.$$

A relation between $c$ and $k^{il} = k\delta^{il}$ appears by virtue of the algebra of (anti) commutators. The relation is that $c = 6 \times k$. In other words, for our building block

$$c = 6(n_{\text{hyp}} - n_{\text{vec}}). \tag{3.3}$$

This relation — also derived in [63], is of importance to us. Let us briefly discuss it, as well as the proposed duality and its implications.

### 3.6 The proposed duality

In what follows we shall define $\mathcal{N} = (0,4)$ SUSY quiver field theories. These quivers will consist of colour and flavour groups joined by hypermultiplets or $\mathcal{N} = (0,2)$ Fermi multiplets as indicated in our building block. We must be careful to have all anomalies of gauged groups vanishing. We will also calculate the R-symmetry anomaly and the 'central charge' via the relation in (3.3).[5] The calculation will be performed in the weakly coupled description of the field theory, in the UV before the conformal point is attained. But as we mentioned, these are 't Hooft coefficients, hence invariants of the RG flow. Importantly, we assume that there is no mixing between the R-symmetry and other global symmetries. If such mixing were to exist, an extremisation procedure like the one devised in [65, 66] would be needed. It would be nice to prove that for our quivers there is no mixing between the R-symmetry and other global symmetries. As a plausible argument for the non-mixing, notice that the non-Abelian R-symmetry SU(2) cannot mix with U(1) global symmetries in two dimensions. There is no other non-Abelian global R-symmetry to mix with. Let us then focus on the end of the RG flow to low energies.

As advanced, we propose that our quivers flow to a strongly coupled CFT with $\mathcal{N} = (0,4)$ SUSY and central charge given by (3.3), as enforced by the superconformal algebra. The second part of our proposal is that the holographic backgrounds are dual to these CFTs. The holographic central charge calculated in (2.27) should coincide with the result of (3.3), in the case of long quivers with large ranks (as this is the regime in which we can trust the supergravity solutions).

Another check of our proposal will be the matching of global symmetries on both sides of the duality. In fact the SCFTs have SO(2, 2) space-time and SU(2) R-symmetries. The backgrounds in (2.4) match these with the isometries of $\text{AdS}_3$ and $\text{S}^2$ respectively. Also eight supercharges are preserved both by the CFT and the background. Indeed, there are four space-time (Q's) and four conformal (S's) supercharges. More interestingly, the flavour symmetries of the SCFT are matched by the presence of 'flavour branes' in the background (giving place to Bianchi identities modified by the presence of sources). The counting of Page charges also coincides with the ranks of the colour and flavour groups, or, analogously, with the numbers of (D2,D6) colour branes and (D4,D8) flavour branes in the associated Hanany-Witten brane set-ups.

Let us be more concrete. A generic background of the form in (2.4) is defined by the functions $\hat{h}_4, h_8, u$. In the type of solutions we consider in this paper (those where the

[5] Strictly speaking, we should not call this quantity central charge as (in the UV) we are not at a fixed point of the RG flow. The relation in (3.3) is only valid at the fixed point.

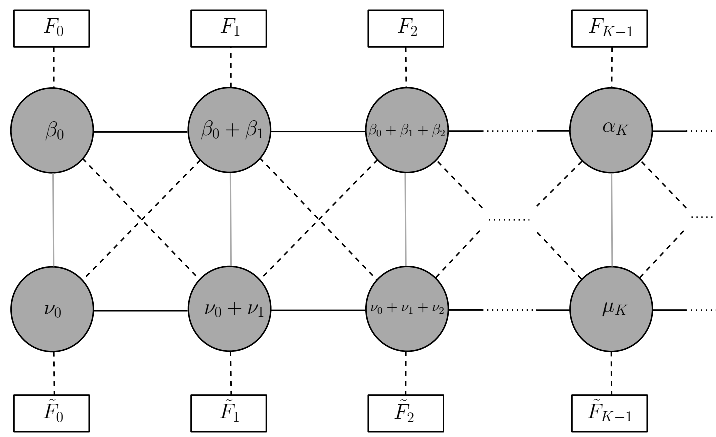


**Figure 3**. A generic quiver field theory whose IR is dual to the holographic background defined by the functions in (3.4)–(3.5). As before, the solid black line represents a $(4,4)$ hypermultiplet. The grey line represents a $(0,4)$ hypermultiplet and the dashed line represents a $(0,2)$ Fermi multiplet. $\mathcal{N}=(4,4)$ vector multiplets are the degrees of freedom in each gauged node.

space ends at $\rho_*=2\pi(P+1)$, where we have $\hat{h}_4(\rho_*)=h_8(\rho_*)=0$), we generically have — see (2.7)–(2.8) and (2.13),

$$\hat{h}_4(\rho)=\Upsilon h_4(\rho)=\Upsilon\begin{cases}\frac{\beta_0}{2\pi}\rho & 0\leq\rho\leq 2\pi\\ \beta_0+\frac{\beta_1}{2\pi}(\rho-2\pi) & 2\pi\leq\rho\leq 4\pi\\ (\beta_0+\beta_1)+\frac{\beta_2}{2\pi}(\rho-4\pi) & 4\pi\leq\rho\leq 6\pi\\ (\beta_0+\beta_1+\ldots+\beta_{k-1})+\frac{\beta_k}{2\pi}(\rho-2\pi k) & 2\pi k\leq\rho\leq 2\pi(k+1),\quad k:=3,\ldots,P-1\\ \alpha_P-\frac{\alpha_P}{2\pi}(\rho-2\pi P) & 2\pi P\leq\rho\leq 2\pi(P+1).\end{cases} \tag{3.4}$$

$$h_8(\rho)=\begin{cases}\frac{\nu_0}{2\pi}\rho & 0\leq\rho\leq 2\pi\\ \nu_0+\frac{\nu_1}{2\pi}(\rho-2\pi) & 2\pi\leq\rho\leq 4\pi\\ (\nu_0+\nu_1)+\frac{\nu_2}{2\pi}(\rho-4\pi) & 4\pi\leq\rho\leq 6\pi\\ (\nu_0+\nu_1+\ldots+\nu_{k-1})+\frac{\nu_k}{2\pi}(\rho-2\pi k) & 2\pi k\leq\rho\leq 2\pi(k+1),\quad k:=3,\ldots,P-1\\ \mu_P-\frac{\mu_P}{2\pi}(\rho-2\pi P) & 2\pi P\leq\rho\leq 2\pi(P+1).\end{cases} \tag{3.5}$$

and

$$u=\frac{b_0}{2\pi}\rho.$$

The background in (2.4) for the functions $\hat{h}_4, h_8, u$ above is dual to the CFT describing the low energy dynamics of a two dimensional quantum field theory encoded by the quiver in figure 3 and the Hanany-Witten set-up of figure 4.

Let us see how the correspondence works. For the first two gauge groups SU$(\nu_0)$ and SU$(\beta_0)$, the cancellation of gauge anomalies in (3.1) implies that,

$$F_0+\nu_0+\nu_1=2\nu_0\rightarrow F_0=\nu_0-\nu_1,\quad \tilde{F}_0+\beta_0+\beta_1=2\beta_0\rightarrow\tilde{F}_0=\beta_0-\beta_1. \tag{3.6}$$

This is precisely the number of flavour D8 and D4 branes predicted by the Bianchi identities in the interval $[0,2\pi]$ — see (2.22) for $k=1$. Similarly, the ranks of the first two gauge

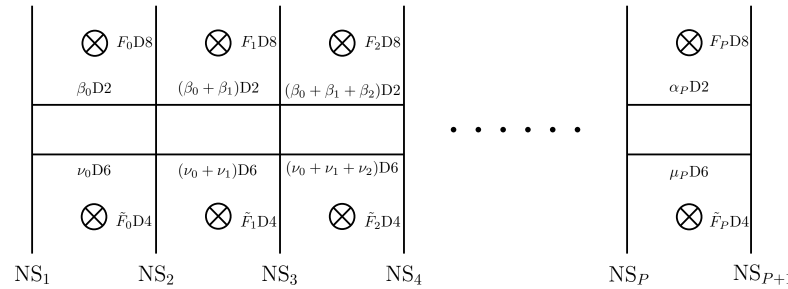


**Figure 4**. Hanany-Witten set-up associated with our generic quiver in figure 3. The vertical lines denote NS five branes, horizontal lines D2 and D6 colour branes. The crosses, D4 and D8 flavour branes.

groups, namely $\beta_0$ and $\nu_0$, are precisely the numbers of D2 and D6 colour branes predicted by eq. (2.23) in the first interval (for $k=1$).

This works similarly for all other entries in the quiver. For example, for the SU$(\alpha_k)$ colour group, we obtain that in the interval $[2\pi(k-1), 2\pi k]$ of the associated Hanany-Witten set up in figure 4, there are $\alpha_k$ D2 branes, with

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j.$$

The cancellation of gauge anomalies for the SU$(\alpha_k)$ gauge group imposes that,

$$F_{k-1} + \mu_{k+1} + \mu_{k-1} = 2\mu_k \to F_{k-1} = \nu_{k-1} - \nu_k, \tag{3.7}$$

which, according to (2.22), is precisely the number of flavour D8 branes in the $[2\pi(k–1), 2\pi k]$ interval of the brane set-up. Things work analogously if we replace D2 for D6 (or $\alpha_k \to \mu_k$) and D8 for D4 ($\nu_k \to \beta_k$) and deal with the lower-row gauge group SU$(\mu_k)$.

We can calculate the field theory central charge by counting the number of $(0,4)$ hypermultiplets, the number of $(0,4)$ vector multiplets and using (3.3). We find,

$$n_{\rm vec} = \sum_{j=1}^{P}\Big(\alpha_j^2 + \mu_j^2 - 2\Big), \quad n_{\rm hyp} = \sum_{j=1}^{P}\alpha_j\mu_j + \sum_{j=1}^{P-1}\Big(\alpha_j\alpha_{j+1} + \mu_j\mu_{j+1}\Big),$$
$$c = 6 \times \left(\sum_{j=1}^{P}\Big(\alpha_j\mu_j - \alpha_j^2 - \mu_j^2 + 2\Big) + \sum_{j=1}^{P-1}\Big(\alpha_j\alpha_{j+1} + \mu_j\mu_{j+1}\Big)\right). \tag{3.8}$$

When the number of nodes is large $P \gg 1$, and the ranks of each gauge group $\alpha_i, \mu_i$ are large numbers, the supergravity backgrounds are trustable and the holographic central charge calculated according to (2.27) should coincide at leading order in these large parameters with (3.8).

For pedagogical purposes, in the next section we present some explicit examples (in increasing level of complexity) of quiver-supergravity dual pairs. We shall check the cancellation of gauge anomalies and the leading order matching of (2.27) and (3.8).

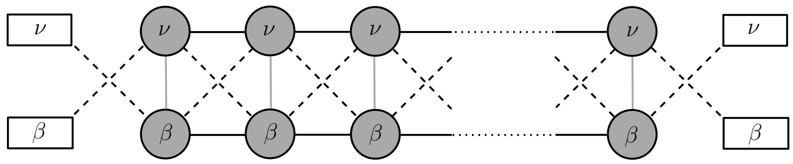

**Figure 5**. The quiver encoding our first example of quantum field theory. The conventions for the fields running along the different lines are the same as those in section 3.

## 4 Various checks of our proposed duality

In this section we discuss various examples of dual holographic pairs. We check anomaly cancellation and the leading order matching of the CFT and holographic central charges. We start from the simplest possible example of a quiver field theory flowing to a superconformal $\mathcal{N} = (0, 4)$ SCFT that admits a viable supergravity dual, and move on to examples of increasing complexity. These will provide stringent checks of our proposal.[6]

### 4.1 Example I

Consider the quiver of figure 5, where we depict $P$ gauge groups $\mathrm{SU}(\nu)$ and $P$ gauge groups $\mathrm{SU}(\beta)$. They are joined by bifundamentals, all complemented by flavour groups (rectangular boxes). This quiver encodes the kinematical content of our first field theory. We propose that this QFT flows in the IR to a CFT. Let us focus on the first gauge group of the top row, $\mathrm{SU}(\nu)$. We compare with our building block in figure 2 to find that,

$$\hat{P} = \nu, \quad Q = 2\beta, \quad R = \beta. \tag{4.1}$$

This is precisely what our formula (3.1) requires for the cancellation of the $\mathrm{SU}(\nu)$ gauge anomaly. For the first $\mathrm{SU}(\beta)$ gauge group in the lower row, we have $\hat{P} = \beta, Q = 2\nu, R = \nu$ and (3.1) is also satisfied.

Similarly, one can calculate for the top and bottom gauge groups at the right end of the figure and check that all of them satisfy (3.1). Finally, for any intermediate $\mathrm{SU}(\nu)$-node, we have $\hat{P} = \nu$, $Q = 2\beta$, $R = \beta$. Analogous statements hold true for the lower row groups. Hence all of the gauge symmetries are non-anomalous.

We can now calculate the number of $(0, 4)$ hypermultiplets and vector multiplets with a view on computing the central charge of the IR CFT. We find,

$$\begin{aligned} n_{\text{vec}} &= P(\nu^2 + \beta^2 - 2), \quad n_{\text{hyp}} = (P-1)(\nu^2 + \beta^2) + P\nu\beta. \\ c &= 6(n_{\text{hyp}} - n_{\text{vec}}) = 6\nu\beta P\left(1 + \frac{2}{\beta\nu} - \frac{\beta}{\nu P} - \frac{\nu}{\beta P}\right) \sim 6\nu\beta P. \end{aligned} \tag{4.2}$$

In the last approximation we used that the ranks are large numbers $(\nu, \beta) \to \infty$ and that the quiver is long enough, hence $P \gg 1$, to meaningfully compare with the dual massive IIA solution.

[6] In the examples that follow we write the function $h_4(\rho)$. As discussed above, the function that appears in the background is $\hat{h}_4 = \Upsilon h_4$. The value $\Upsilon \mathrm{Vol}(\mathrm{CY}_2) = 16\pi^4$ is used to have well quantised charges in terms of the integer numbers $(\alpha_k, \beta_k, \mu_k, \nu_k)$.

The holographic background dual to this CFT is given in terms of the functions $u = \frac{b_0}{2\pi}\rho$ and

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \nu & 2\pi \le \rho \le 2\pi P \\ \frac{\nu}{2\pi}(2\pi(P+1)-\rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.3}$$

$$h_4(\rho) = \begin{cases} \frac{\beta}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \beta & 2\pi \le \rho \le 2\pi P \\ \frac{\beta}{2\pi}(2\pi(P+1)-\rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.4}$$

The holographic central charge is found by the simple calculation in (2.27),

$$c_{\text{hol}} = \frac{3}{\pi}\left(\int_0^{2\pi} \frac{\beta\nu}{4\pi^2}\rho^2 d\rho + \int_{2\pi}^{2\pi P} \beta\nu d\rho + \int_{2\pi P}^{2\pi(P+1)} \frac{\beta\nu}{4\pi^2}(2\pi(P+1)-\rho)^2 d\rho\right)$$

$$c_{\text{hol}} = 6\beta\nu P\left(1 - \frac{1}{3P}\right) \sim 6P\beta\nu. \tag{4.5}$$

This coincides with the field theoretical result in (4.2). Finally, notice that the number of D4 and D8 flavour branes, dictated by (2.22), precisely provide the flavour symmetries at the beginning and end of the quiver. One finds the same by inspecting (2.23) for the number of colour branes, coinciding with the ranks of the gauge groups of our quiver.

### 4.2 Example II

Let us slightly complicate our previous example. We consider now a quiver with two rows of linearly increasing colour groups. These two rows are finished after $P$ nodes by the addition of a flavour group for each row. See figure 6. This type of quivers can be used as a completion of the background obtained via the application of non-Abelian T-duality on $\text{AdS}_3 \times \text{S}^3 \times \text{CY}_2$, inspired by the treatments in [67]–[70]. See [51] for a careful discussion of this. The anomalies of each of the gauge groups can be easily seen to vanish. In fact, for any of the intermediate gauge nodes, say $\text{SU}(k\nu)$ and referring to our building block in figure 2, we have $Q = 2k\beta, R = k\beta$. This implies that (3.1) is satisfied and a generic intermediate gauge group is not anomalous. If we refer to the last gauge group in the upper-row $\text{SU}(P\nu)$ we have that $Q = (P+1)\beta + (P-1)\beta = 2P\beta$ and $R = P\beta$. As a consequence (3.1) is satisfied and the gauged group $\text{SU}(P\nu)$ is not anomalous. The same occurs for the lower-row gauge groups.

We can easily count the number of $(0,4)$ hypers and the number of $(0,4)$ vector multiplets,

$$n_{\text{vec}} = \sum_{j=1}^{P}\Big(j^2(\nu^2+\beta^2) - 2\Big), \quad n_{\text{hyp}} = \sum_{j=1}^{P-1} j(j+1)(\nu^2+\beta^2) + \sum_{j=1}^{P} j^2\nu\beta. \tag{4.6}$$

The central charge of the IR CFT is,

$$\begin{aligned} c &= 6(n_{\text{hyp}} - n_{\text{vec}}) \\ &= 6\nu\beta\left(\frac{P^3}{3} + \frac{P^2}{2} + \frac{P}{6}\right) - 3(\nu^2+\beta^2)(P^2+P) + 12P \sim 2\nu\beta P^3. \end{aligned} \tag{4.7}$$

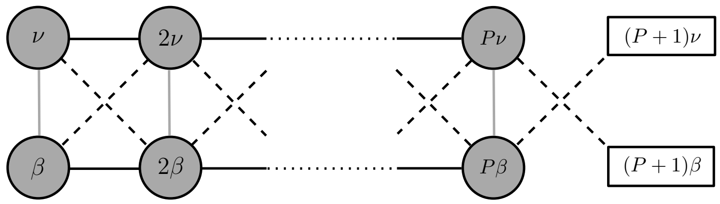


**Figure 6**. The quiver encoding our second example. There are $P$ gauged nodes with increasing rank in each row. The conventions for the fields running along the different lines are the same as those in section 3.



The holographic description of this system is in terms of the functions,

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\nu P}{2\pi}(2\pi(P+1)-\rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.8}$$

$$h_4(\rho) = \begin{cases} \frac{\beta}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\beta P}{2\pi}(2\pi(P+1)-\rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.9}$$

Using (2.27), we calculate the holographic central charge,

$$c_{\text{hol}} = \frac{3}{\pi}\left(\frac{\beta\nu}{4\pi^2}\right)\left(\int_0^{2\pi P}\rho^2 d\rho + \int_{2\pi P}^{2\pi(P+1)} P^2(2\pi(P+1)-\rho)^2 d\rho\right) = 2\nu\beta P^3\left(1+\frac{1}{P}\right) \sim 2\nu\beta P^3. \tag{4.10}$$

Again, we observe that in the limit of a long quiver, there is matching for the central charge in the CFT — see (4.7), with that of the dual description — see (4.10).

Let us now discuss a more involved example, providing us with a much stringent check of our proposed duality.

### 4.3 Example III

In this case we consider the more involved field theory encoded by the quiver in figure 7.

In this quiver we have a line of linearly increasing nodes $\mathrm{SU}(\nu)\times\mathrm{SU}(2\nu)\times\ldots\times\mathrm{SU}(K\nu)$ followed by $q \times \mathrm{SU}(K\nu)$ nodes. The gauge groups $\mathrm{SU}(G_l)$ have ranks

$$G_l = K\nu\left(1 - \frac{l}{P+1-K-q}\right), \qquad l = 1, \ldots, P-K-q. \tag{4.11}$$

For the lower row we have analogous kinematics: Linearly increasing ranks $\mathrm{SU}(\beta) \times \mathrm{SU}(2\beta) \times \ldots \times \mathrm{SU}(K\beta)$, followed by $q \times \mathrm{SU}(K\beta)$ nodes. The gauge groups $\mathrm{SU}(\tilde{G}_l)$ have ranks,

$$\tilde{G}_l = K\beta\left(1 - \frac{l}{P+1-K-q}\right), \qquad l = 1, \ldots, P-K-q. \tag{4.12}$$

Let us analyse anomalies for the upper row groups (the lower row ones work analogously). The linearly increasing chain is non-anomalous like our previous example in section 4.2 was. Namely, for a generic $\mathrm{SU}(j\nu)$ node, we have $Q = 2j\beta$ and $R = j\beta$.

The chain of $q$ $\mathrm{SU}(K\nu)$ groups works exactly as any intermediate group in section 4.1, namely for any generic (intermediate) node we have $Q = 2K\beta$ and $R = K\beta$, satisfying (3.1).

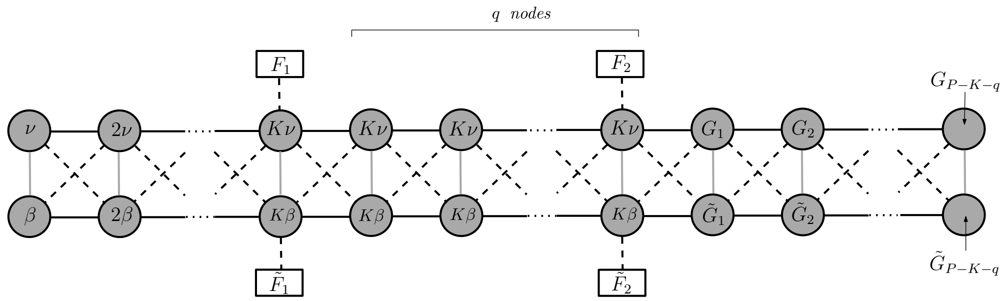


**Figure 7**. The quiver encoding our third example. There are $K$ gauged nodes with linearly increasing ranks in each row. These are followed by $q-\mathrm{SU}(K\nu)$ (top row) and $q-\mathrm{SU}(K\beta)$ nodes (lower row). The ranks of the next $\mathrm{SU}(G_i)$ and $\mathrm{SU}(\tilde{G}_i)$ nodes is given in the text. The conventions for the fields running along the different lines are the same as those in section 3.

More interesting are the first and last of these $q$-nodes. For the first node we have $Q = F_1 + (K-1)\beta + K\beta$ and $R = K\beta$. Observe that (3.1) forces

$$F_1 = \beta.$$

For the last of these $q$-nodes we have $Q = K\beta + \tilde{G}_1 + F_2$ and $R = K\beta$. Then the vanishing of the gauge anomaly forces

$$F_2 = \frac{K\beta}{P+1-K-q}.$$

For any generic group $\mathrm{SU}(G_i)$ we have $Q = \tilde{G}_{i-1} + \tilde{G}_{i+1}$ and $R = \tilde{G}_i$. Using (4.12) we find that $Q = 2R$ as imposed in (3.1) for the vanishing of the gauge anomalies.

Analogously, for the lower row groups, we find that the vanishing of the gauge anomalies imposes

$$\tilde{F}_1 = \nu, \qquad \tilde{F}_2 = \frac{K\nu}{P+1-K-q}. \tag{4.13}$$

To calculate the CFT central charge we need to compute the number of $(0,4)$ hypers and vectors. We find

$$\begin{aligned}
n_{\mathrm{vec}} &= \sum_{j=1}^{K}\left(j^2(\nu^2+\beta^2)-2\right) + q(K^2(\nu^2+\beta^2)-2) \\
&\quad + \sum_{j=1}^{P-K-q}\left(K^2(\nu^2+\beta^2)\left(1-\frac{j}{P+1-K-q}\right)^2 - 2\right), \\
n_{\mathrm{hyp}} &= \sum_{j=1}^{K-1} j(j+1)(\nu^2+\beta^2) + \sum_{j=1}^{K} j^2\beta\nu + K^2 q(\nu^2+\beta^2+\beta\nu) \\
&\quad + \sum_{j=0}^{P-K-q-1} K^2(\beta^2+\nu^2)\left(1-\frac{j}{P+1-K-q}\right)\left(1-\frac{j+1}{P+1-K-q}\right) \\
&\quad + \sum_{j=1}^{P-K-q} K^2\beta\nu\left(1-\frac{j}{P+1-K-q}\right)^2 .
\end{aligned} \tag{4.14}$$

The field theory central charge is after a lengthy calculation,

$$
\begin{aligned}
c &= 6(n_{\text{hyp}} - n_{\text{vec}}) \\
&\sim \begin{cases} 2\beta\nu K^2 P + 12P + O(1, 1/P), & \text{if} \quad P \gg 1, \\ 4\beta\nu K^2 q + O(1, 1/q), & \text{if} \quad q \gg 1, \\ 2\beta\nu K^2 (1 + 2q + P) + O(1, 1/K), & \text{if} \quad K \gg 1. \end{cases}
\end{aligned}
\tag{4.15}
$$

We have expanded the exact result for the three possible ways in which the quiver may be considered to be 'long'. We also need to take $(\nu, \beta)$ to be large numbers.

Now, let us compare with the holographic description. The functions $h_4$ and $h_8$ for this case read,

$$
h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \leq \rho \leq 2\pi K \\ \nu K & 2\pi K \leq \rho \leq 2\pi(K + q) \\ \frac{\nu K}{2\pi(P+1-K-q)}(2\pi(P + 1) - \rho) & 2\pi(K + q) \leq \rho \leq 2\pi(P + 1). \end{cases} \tag{4.16}
$$

$$
h_4(\rho) = \begin{cases} \frac{\beta}{2\pi}\rho & 0 \leq \rho \leq 2\pi k \\ \beta K & 2\pi K \leq \rho \leq 2\pi(K + q) \\ \frac{\beta K}{2\pi(P+1-K-q)}(2\pi(P + 1) - \rho) & 2\pi(K + q) \leq \rho \leq 2\pi(P + 1). \end{cases} \tag{4.17}
$$

The holographic central charge is given by (2.27), that after some algebra yields

$$
c_{\text{hol}} = 2\beta\nu K^2(P + 2q + 1) = \begin{cases} 2\beta\nu K^2 P & \text{if} \quad P \gg 1 \\ 4\nu\beta K^2 q & \text{if} \quad q \gg 1 \\ 2\beta\nu K^2(P + 2q + 1) & \text{if} \quad K \gg 1. \end{cases} \tag{4.18}
$$

The comparison with (4.15) shows that this is a very stringent check of our proposal.

Finally, the reader can check, using (2.22), that the numbers of flavour D8 and D4 branes coincide with the numbers $F_1, F_2$ and $\tilde{F}_1, \tilde{F}_2$ quoted above — see (4.13). The same happens with the gauge groups and the numbers of D2 and D6 branes in the associated brane set-up calculated using (2.23), and comparing with (4.11)), (4.12).

Let us now study a qualitatively different example. It will raise a puzzle with an instructive resolution.

### 4.4 Example IV: a puzzle and its resolution

Qualitatively, the QFTs discussed above share the fact that the lower row gauge groups 'mirror' the behaviour of the upper row ones. The groups both grow, stabilise and decrease at the same points. It is interesting to consider an example for which this is not the case. Let us consider the quiver in figure 8.

We can easily calculate the number of $(0, 4)$ hypermultiplets, vector multiplets and the central charge,

$$
\begin{aligned}
n_{\text{vec}} &= P(\beta^2 - 1) + \sum_{j=1}^{P}\Big(j^2\nu^2 - 1\Big), \quad n_{\text{hyp}} = \sum_{j=1}^{P} j\beta\nu + \sum_{j=1}^{P-1} j(j + 1)\nu^2 + \beta^2(P - 1), \\
c &= 3P^2(\beta\nu - \nu^2) + (12 + 3\beta\nu - 3\nu^2)P - 6\beta^2.
\end{aligned}
\tag{4.19}
$$

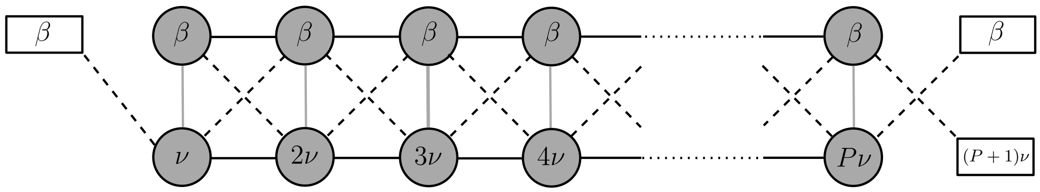


**Figure 8**. The quiver encoding our fourth example. The conventions for the fields running along the different lines are the same as those in section 3.

We can anticipate troubles with the holographic description. Indeed, if we were to take $\nu > \beta$ and large $P$, we could get a negative central charge.

Let us write the functions $h_4, h_8$ describing holographically the IR dynamics of this quiver (as usual $u = \frac{b_0}{2\pi}\rho$),

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\nu P}{2\pi}(2\pi(P+1) - \rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.20}$$

$$h_4(\rho) = \begin{cases} \frac{\beta}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \beta & 2\pi \le \rho \le 2\pi P \\ \frac{\beta}{2\pi}(2\pi(P+1) - \rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.21}$$

The holographic central charge is calculated using (2.27). After some algebra this results in,

$$c_{\text{hol}} = 3P^2\beta\nu\left(1 + \frac{2}{3P} - \frac{1}{3P^2}\right) \sim 3P^2\beta\nu. \tag{4.22}$$

Comparing the expressions for the field theoretical and holographic central charges in (4.19), (4.22), we see a mismatch if we keep the leading order in $P, \nu, \beta$. This raises a puzzle. The resolution to this puzzle is given by (2.15). The last interval of the functions $h_4, h_8$ in this example is written as

$$\begin{aligned} h_4^{P,P+1} &= \frac{\alpha_P}{2\pi}(2\pi(P+1) - \rho), \quad \alpha_P = \beta, \\ h_8^{P,P+1} &= \frac{\mu_P}{2\pi}(2\pi(P+1) - \rho), \quad \mu_P = P\nu. \end{aligned}$$

Using (2.15), this implies that the $\text{CY}_2$ space is of sub-stringy size, for large $P$. This invalidates the supergravity solution which does not include the dynamics of massless states due to strings or branes wrapping the $\text{CY}_2$ — see the comment below (2.15). The way out of this puzzle is to decouple these light states (by making them heavy and hence the supergravity solution valid). To do this, one must scale $\beta \sim \hat{\beta} \times P$. Then, both the field theoretical and the holographic central charges in (4.19), (4.22) coincide to $c \sim 3\hat{\beta}\nu P^3$.

We close this section here. A more involved example is discussed in appendix C.

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## 5 Conclusions

This paper presents a new entry in the mapping between SCFTs and AdS-supergravity backgrounds, for the particular case of two-dimensional small $\mathcal{N} = (0,4)$ SCFTs and backgrounds with $\mathrm{AdS}_3 \times \mathrm{S}^2$ factors. The most general solutions of this type that support an SU(2)-structure on the internal space were recently classified in [47].

We have constructed new solutions of the type $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$, belonging to class I in the classification in [47], with compact $\mathrm{CY}_2$, whose defining functions are piecewise continuous. We elaborated on their regime of validity and on various general aspects of their mapping with SCFTs. In particular, we matched the background isometries and the global symmetries (both space-time and flavour) of the SCFTs. We computed Page charges and put them in correspondence with the putative colour and flavour branes in the Hanany-Witten set-ups associated to our SCFTs. The CFTs are defined as the IR limit of UV well-behaved long quivers with $(0,4)$ SUSY, that generalise 2-d (0,4) quivers previously discussed in the literature — see [41, 49]. Our $(0,4)$ quivers consist of two families of $(4,4)$ quivers coupled by $(0,4)$ and $(0,2)$ matter fields. The $(4,4)$ quivers are associated to D2-NS5-D4 and D6-NS5-D8 brane systems, the latter wrapped on the $\mathrm{CY}_2$, which by themselves do not give rise to 2d CFTs in the IR. Our work shows that the coupling between the two families of quivers through matter fields that reduce the supersymmetry to $(0,4)$ renders a 2d CFT in the IR, which admits an $\mathrm{AdS}_3$ dual. After presenting our proposed duality we discussed a number of examples of increasing complexity that together constitute a stringent test of our proposal. These examples exhibit perfect agreement between the holographic and field theoretical central charges (in the regime where both descriptions are valid), gauge-anomaly cancellation and matching between isometries and 'flavour' symmetries on both sides of the duality.

It is clear that this paper just scratches the surface of a rich line of work. In the forthcoming paper [51] we will apply the developments in this paper to (among other things) construct a symmetric solution that can be thought of as a completion of the background obtained via non-Abelian T-duality on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{CY}_2$. Indeed, non-Abelian T-duality has been one of the inspirations of the exhaustive classification presented in [47], and further discussed in this work. This classification provides one more example that shows the huge impact of non-Abelian T-duality as a solution generating technique in supergravity — see for example [71]–[77]. One can speculate that an approach similar to the one in [47] can be used to classify generic backgrounds in different dimensions and with different amounts of SUSY from particular solutions generated through this technique.

More related to the present paper a number of interesting problems can be tackled. For example, operators of spin two have been studied in correspondence with certain fluctuations of the background metric [78, 79]. It would be interesting to study the analog operators in our CFTs. Similarly, long operators like those in [18] should exist in our CFTs and their associated backgrounds. An obvious open problem is to discuss the CFTs dual to the solutions terminated by the two types of boundary conditions discussed in section 2, not tackled in this paper. In the same vein, it would be interesting to explore the CFT duals of the solutions referred as class II in [47], where the $\mathrm{CY}_2$ is replaced by a

4-d Kahler manifold. It would be nice to explore other tests and (more interestingly) find predictions of our proposed duality. The richness of the 2-d SCFTs suggests that stringy tests and mappings along the lines of [80]–[85] should be possible. We hope to report on these projects soon.


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## A Continuity of the NS sector of our solutions

In this section we study the conditions imposed by the continuity of the NS-sector, on the constants $(a_k, b_k, \alpha_k, \beta_k, \mu_k, \nu_k)$ defining our solutions in section 2.1. In particular, we consider solutions that in the interval $[2\pi k, 2\pi(k+1)]$ are given by,

$$\hat{h}_4^{(k)} = \Upsilon\left(\alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)\right), \quad h_8^{(k)} = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k), \quad u^{(k)} = a_k + \frac{b_k}{2\pi}(\rho - 2\pi k). \tag{A.1}$$

Below, we quote the value of each component of the metric, $e^{-4\Phi}$ and $B_2$-field when calculated at the point $\rho = 2\pi(k+1)$ in terms of the general decomposition

$$ds^2 = e^{2A}ds^2(\mathrm{AdS}_3) + e^{2C}ds^2(\mathrm{S}^2) + e^{2D}ds^2(\mathrm{CY}_2) + e^{-2A}d\rho^2, \quad B = B_0\mathrm{vol}(\mathrm{S}^2). \tag{A.2}$$

If using the solution in (A.1) we denote them with a superscript $-$. Then, we calculate the NS quantities at the same point $\rho = 2\pi(k+1)$, but using the solution in the next interval (with $\alpha_k \to \alpha_{k+1}$, etc), we denote this with a supra-index $+$. Imposing the continuity of each element of the metric and other NS fields, we find conditions for the numbers $(a_k, b_k, \alpha_k, \beta_k, \mu_k, \nu_k)$.

In more detail, we find,

$$
\begin{aligned}
e^{2A^-} &= \frac{(a_k+b_k)}{\sqrt{\Upsilon(\alpha_k+\beta_k)(\mu_k+\nu_k)}}, & e^{2A^+} &= \frac{a_{k+1}}{\sqrt{\Upsilon\alpha_{k+1}\mu_{k+1}}}. \\
e^{2D^-} &= \sqrt{\frac{\Upsilon(\alpha_k+\beta_k)}{(\mu_k+\nu_k)}}, & e^{2D^+} &= \sqrt{\frac{\Upsilon\alpha_{k+1}}{\mu_{k+1}}}. \\
e^{2C^-} &= 4\pi^2\frac{(a_k+b_k)\sqrt{\Upsilon(\alpha_k+\beta_k)(\mu_k+\nu_k)}}{b_k^2+16\pi^2\Upsilon(\alpha_k+\beta_k)(\mu_k+\nu_k)}, & e^{2C^+} &= 4\pi^2\frac{a_{k+1}\sqrt{\Upsilon\alpha_{k+1}\mu_{k+1}}}{b_{k+1}^2+16\pi^2\Upsilon\alpha_{k+1}\mu_{k+1}}. \\
e^{-4\Phi^-} &= \frac{(\mu_k+\nu_k)^3\left[b_k^2+16\pi^2\Upsilon(\alpha_k+\beta_k)(\mu_k+\nu_k)\right]^2}{256\pi^4\Upsilon(\alpha_k+\beta_k)(a_k+b_k)^2}, \\
e^{-4\Phi^+} &= \frac{(\mu_{k+1})^3\left[b_{k+1}^2+16\pi^2\Upsilon\alpha_{k+1}\mu_{k+1}\right]^2}{256\pi^4\Upsilon\alpha_{k+1}(a_{k+1})^2}. \\
\frac{B_0^-}{\pi} &= \frac{a_kb_k-16\pi^2\Upsilon(\alpha_k+\beta_k)(\mu_k+\nu_k)}{b_k^2+16\pi^2\Upsilon(\alpha_k+\beta_k)(\mu_k+\nu_k)}, & \frac{B_0^+}{\pi} &= \frac{a_{k+1}b_{k+1}-b_{k+1}^2-16\pi^2\Upsilon\alpha_{k+1}\mu_{k+1}}{b_{k+1}^2+16\pi^2\Upsilon\alpha_{k+1}\mu_{k+1}}.
\end{aligned}
\tag{A.3}
$$

Continuity across $\rho = 2\pi(k+1)$ imposes the matching of the analog quantities above. One possible solution is,

$$
a_{k+1} = a_k + b_k, \quad b_k = b_{k+1} = b_0, \quad \alpha_{k+1} = \alpha_k + \beta_k, \quad \mu_{k+1} = \mu_k + \nu_k. \tag{A.4}
$$

These are precisely the same conditions that result from imposing the continuity of $\hat{h}_4, h_8, u$ across each interval. Notice that (A.4) is equivalent to (2.10).

## B A general analysis of Bianchi identities and counting branes in our Hanany-Witten set-ups

In this appendix we study the charges of D2 and D6 branes induced on D8 and D4 flavour branes. We finish by presenting expressions to calculate the total number of D8, D6, D4 and D2 branes in a generic Hanany-Witten set-up.

As in the main body of the paper, we denote by $f_p$ the magnetic part of the form $F_p$ and with $\hat{f}_p$ the magnetic part of the Page field strength $\hat{F}_p = F \wedge e^{-B_2}$. In the presence of $N_4$ D4 and $N_8$ D8 branes on which we switch a gauge field strength $\tilde{f}_2$ and form the combination $\mathcal{F}_2 = B_2 + 2\pi\tilde{f}_2$. The Bianchi identities read,

$$
\begin{aligned}
dF_0 &= \frac{N_8}{2\pi}\delta(\rho-\rho_0)d\rho, \\
df_2 - H_3F_0 &= \frac{N_8}{2\pi}\delta(\rho-\rho_0)\mathcal{F}_2\wedge d\rho, \\
df_4 - H_3\wedge f_2 &= (2\pi)^3N_4\delta(\rho-\rho_0)\delta^4(\vec{y}-\vec{y}_0)d\rho\wedge d^4\vec{y} + \frac{1}{2}\frac{N_8}{2\pi}\delta(\rho-\rho_0)\mathcal{F}_2\wedge\mathcal{F}_2\wedge d\rho, \\
df_6 - H_3\wedge f_4 &= (2\pi)^3N_4\delta(\rho-\rho_0)\delta^4(\vec{y}-\vec{y}_0)\mathcal{F}_2\wedge d\rho\wedge d^4\vec{y} + \frac{1}{6}\frac{N_8}{2\pi}\delta(\rho-\rho_0)\mathcal{F}_2\wedge\mathcal{F}_2\wedge\mathcal{F}_2\wedge d\rho.
\end{aligned}
\tag{B.1}
$$

The D8 branes are localised in the $\rho$-direction at the point $\rho_0$, as indicated in the first line of (B.1). The D4 branes are localised at $\rho = \rho_0$ and at a point $\vec{y}_0$ inside the $\mathrm{CY}_2$ space (we denote by $d^4\vec{y} = \mathrm{vol}(\mathrm{CY}_2)$ its volume form).

The explicit definition of the Page field strengths (we only quote the magnetic part here) is,

$$\hat{f}_2 = f_2 - B_2 F_0, \quad \hat{f}_4 = f_4 - B_2 \wedge f_2 + \frac{1}{2} B_2 \wedge B_2 F_0,$$
$$\hat{f}_6 = f_6 - B_2 \wedge f_4 + \frac{1}{2} B_2 \wedge B_2 \wedge f_2 - \frac{1}{6} B_2 \wedge B_2 \wedge B_2 F_0. \tag{B.2}$$

Combining (B.1) with (B.2), we find

$$d\hat{f}_2 = N_8 \delta(\rho - \rho_0) \tilde{f}_2 \wedge d\rho. \tag{B.3}$$

In the case in which there is no gauge field switched on in the D8 branes, there is no induced D6-brane charge, as implied by the first line in (2.21). Otherwise D6-*flavour* charge is induced, as indicated by (B.3). A similar analysis shows that,

$$d\hat{f}_4 = (2\pi)^3 N_4 \delta(\rho - \rho_0) \delta^4(\vec{y} - \vec{y}_0) d\rho \wedge d^4\vec{y} + 2\pi N_8 \delta(\rho - \rho_0) \tilde{f}_2 \wedge \tilde{f}_2 \wedge d\rho. \tag{B.4}$$

This indicates that D4 brane charge might originate from either localised D4 branes, or on localised D8 branes with a gauge field strength $\tilde{f}_2$ switched on, such that $\tilde{f}_2 \wedge \tilde{f}_2 \wedge d\rho$ is non-zero. For our background, we have, consistently with (2.20)

$$d\hat{f}_4 = (2\pi)^3 N_4 \delta(\rho - \rho_0) \delta^4(\vec{y} - \vec{y}_0) d\rho \wedge d^4\vec{y}. \tag{B.5}$$

The analogous expression for $\hat{f}_6$ is obtained combining the expressions in (B.1)–(B.2),

$$d\hat{f}_6 = (2\pi)^4 N_4 \delta(\rho - \rho_0) \delta^4(\vec{y} - \vec{y}_0) \tilde{f}_2 \wedge d\rho \wedge d^4\vec{y} + \frac{1}{6}(2\pi)^2 N_8 \delta(\rho - \rho_0) \tilde{f}_2 \wedge \tilde{f}_2 \wedge \tilde{f}_2 \wedge d\rho. \tag{B.6}$$

We thus have $d\hat{f}_6 = 0$, in agreement with (2.21).

To close this appendix, let us present simple expressions counting the *total* number of D branes in the Hanany-Witten set-ups associated with our gauge theories and holographic backgrounds. These formulas are similar to those derived in [11, 24] for CFTs in four and six dimensions. They read,

$$N_{D8}^{\text{total}} = 2\pi \left[ h_8'(0) - h_8'(2\pi(P+1)) \right], \qquad N_{D4}^{\text{total}} = 2\pi \left[ h_4'(0) - h_4'(2\pi(P+1)) \right], \tag{B.7}$$
$$N_{D6}^{\text{total}} = \frac{1}{2\pi} \int_0^{2\pi(P+1)} h_8 d\rho, \qquad N_{D2}^{\text{total}} = \frac{1}{2\pi} \int_0^{2\pi(P+1)} h_4 d\rho.$$

These can be successfully checked in all the examples in section 4 and in appendix C.

## C A more stringent check of the duality

In this appendix we work out the details of a more complicated, generic and demanding example, shown in figure 9. Extending the examples studied in the body of the paper, we consider a quiver that starts with linearly increasing nodes. This is followed by $q$-nodes with $\mathrm{SU}(G_l)$, $\mathrm{SU}(\tilde{G}_l)$ gauge groups in the top and lower row respectively, where

$$G_l = \frac{\hat{G}_0}{q} l + \nu K \left(1 - \frac{l}{q}\right), \quad \tilde{G}_l = \frac{\hat{G}_0}{q} l + \beta K \left(1 - \frac{l}{q}\right), \qquad l = 1, \ldots, q \tag{C.1}$$

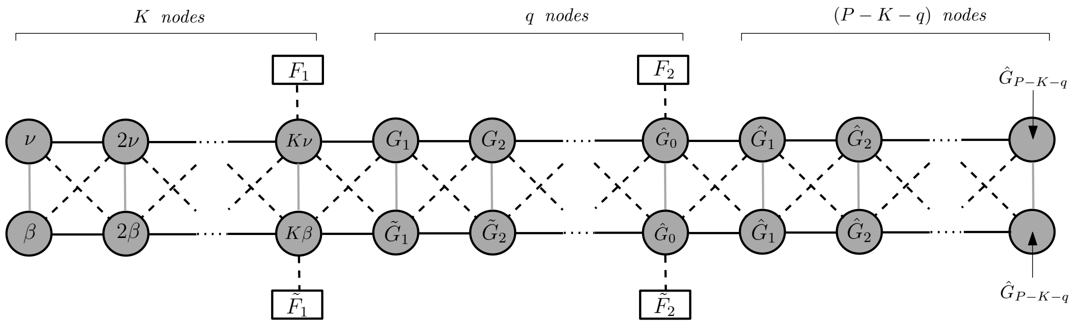


**Figure 9**. A more complicated quiver with $K$ linearly increasing rank nodes in each row, followed by $q$ nodes with $\mathrm{SU}(G_l)$ and $\mathrm{SU}(\tilde{G}_l)$ gauge groups in the top and lower rows, respectively, and ending with $(P-K-q)$ nodes with $\mathrm{SU}(\hat{G}_i)$ gauge groups in both rows.

Following them, there are $(P-K-q)$ $\mathrm{SU}(\hat{G}_l)$ gauge groups with ranks

$$\hat{G}_i = \frac{\hat{G}_0}{P-K-q+1}(P-K-q+1-i), \qquad i=1,\ldots,(P-K-q) \tag{C.2}$$

in both rows.

As in the examples studied in the main body of the paper, the gauge anomaly vanishes in the linearly increasing rows. Following the logic of section 3, for the $\mathrm{SU}(K\nu)$ node we have $Q = F_1 + \tilde{G}_1 + (K-1)\beta$ and $R = K\beta$. A vanishing gauge anomaly for the $\mathrm{SU}(K\nu)$ node, see (3.1), forces

$$F_1 = K\beta + \beta - \tilde{G}_1 = \frac{\beta}{q}(K+q) - \frac{\hat{G}_0}{q}. \tag{C.3}$$

Similarly, for the $\mathrm{SU}(K\beta)$ node the condition is $\tilde{F}_1 = K\nu + \nu - G_1 = \frac{\nu}{q}(K+q) - \frac{\hat{G}_0}{q}$.

For the next gauge group, $\mathrm{SU}(G_1)$, we have the contibutions $R = \tilde{G}_1$ and $Q = K\beta + \tilde{G}_2$. The gauge anomaly then implies $2\tilde{G}_1 - \tilde{G}_2 - K\beta = 0$, which is true in virtue of (C.1). For all $\mathrm{SU}(G_l)$ and $\mathrm{SU}(\tilde{G}_l)$ gauge groups we also have a vanishing gauge anomaly. In the $\mathrm{SU}(\hat{G}_0)$ gauge group — $q$-steps forward in the top row — the contributions are $R = \hat{G}_0$ and $Q = F_2 + \tilde{G}_{q-1} + \hat{G}_1$, where (3.1) is satisfied whenever

$$F_2 = 2\hat{G}_0 - \hat{G}_1 - \tilde{G}_{q-1} = \hat{G}_0\left(\frac{1}{q} + \frac{1}{P+1-K-q}\right) - \frac{\beta K}{q}. \tag{C.4}$$

The same is true for the $\mathrm{SU}(\hat{G}_0)$ lower gauge group, in this case $\tilde{F}_2 = 2\hat{G}_0 - \hat{G}_1 - G_{q-1} = \hat{G}_0(\frac{1}{q} + \frac{1}{P+1-K-q}) - \frac{\nu K}{q}$. Considering (C.2), the gauge anomaly vanishes similarly for the rest of the gauge groups.

To calculate the central charge we compute the number of $(0,4)$ hypers and vectors

$$\begin{aligned} n_{\text{vec}} &= \sum_{j=1}^{K}\left(j^2(\nu^2+\beta^2)-2\right) + \sum_{j=1}^{q}(G_j^2 + \tilde{G}_j^2 - 2) + \sum_{j=1}^{P-K-q} 2(\hat{G}_j^2 - 1), \\ n_{\text{hyp}} &= \sum_{j=1}^{K} j^2\beta\nu + \sum_{j=1}^{K-1} j(j+1)(\nu^2+\beta^2) + \sum_{j=1}^{q} G_j\tilde{G}_j + \sum_{j=0}^{q-1}(G_jG_{j+1} + \tilde{G}_j\tilde{G}_{j+1}) + \\ &\quad + \sum_{j=1}^{P-K-q}\hat{G}_j^2 + \sum_{j=0}^{P-K-q-1} 2\hat{G}_j\hat{G}_{j+1}, \end{aligned} \tag{C.5}$$

where for the number of hypers we are considering $G_0 = \nu K$, $\tilde{G}_0 = \beta K$ and $\hat{G}_0 = G_q = \tilde{G}_q$. As in the previous examples, we are interested in the case of a long quiver. To leading order the central charge, in the three possible limits, is

$$\begin{aligned} c &= 6(n_{\text{hyp}} - n_{\text{vec}}) \\ &= \begin{cases} 2\hat{G}_0^2 P + 12P + O(1, 1/P), & \text{if} \quad P \gg 1, \\ (2\nu\beta K + (\beta+\nu)\hat{G}_0)Kq + O(1, 1/q), & \text{if} \quad q \gg 1, \\ 2\beta\nu K^3 + O(1, 1/K), & \text{if} \quad K \gg 1. \end{cases} \end{aligned} \tag{C.6}$$

Now, we can compare the result in (C.6) with the holographic central charge. The $h_8$ and $h_4$-profiles are given by

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \le \rho \le 2\pi K \\ \nu K + \frac{\hat{G}_0 - \nu K}{2\pi q}(\rho - 2\pi K) & 2\pi K \le \rho \le 2\pi(K+q) \\ \hat{G}_0 - \frac{\hat{G}_0}{2\pi(P-K-q+1)}(\rho - 2\pi(K+q)) & 2\pi(K+q) \le \rho \le 2\pi(P+1). \end{cases} \tag{C.7}$$

$$h_4(\rho) = \begin{cases} \frac{\beta}{2\pi}\rho & 0 \le \rho \le 2\pi K \\ \beta K + \frac{\hat{G}_0 - \beta K}{2\pi q}(\rho - 2\pi K) & 2\pi K \le \rho \le 2\pi(K+q) \\ \hat{G}_0 - \frac{\hat{G}_0}{2\pi(P-K-q+1)}(\rho - 2\pi(K+q)) & 2\pi(K+q) \le \rho \le 2\pi(P+1). \end{cases} \tag{C.8}$$

The holographic central charge, using (2.27), results into

$$\begin{aligned} c_{\text{hol}} &= 2\beta\nu K^2(K+q) + (\beta+\nu)Kq\hat{G}_0 - 2\hat{G}_0^2(K-P-1) \\ &= \begin{cases} 2\hat{G}_0^2 P & \text{if} \quad P \gg 1, \\ (2\nu\beta K + (\beta+\nu)\hat{G}_0)Kq & \text{if} \quad q \gg 1 \\ 2\beta\nu K^3 & \text{if} \quad K \gg 1. \end{cases} \end{aligned} \tag{C.9}$$

We can then easily see that (C.9) is in complete agreement with the output of (C.6).



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## 5.4 SU(2)-NATD solution as an example in class I

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# AdS$_3$ solutions in massive IIA, defect CFTs and T-duality

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Abstract: We establish a map between AdS$_3\times$S$^2$ and AdS$_7$ solutions to massive IIA supergravity that allows one to interpret the former as holographic duals to D2-D4 defects inside 6d (1,0) CFTs. This relation singles out in a particular manner the AdS$_3\times$S$^2$ solution constructed from AdS$_3\times$S$^3\times$CY$_2$ through non-Abelian T-duality, with respect to a freely acting SU(2). We find explicit global completions to this solution and provide well-defined (0,4) 2d dual CFTs associated to them. These completions consist of linear quivers with colour groups coming from D2 and D6 branes and flavour groups coming from D8 and D4 branes. Finally, we discuss the relation with flows interpolating between AdS$_3\times$S$^2\times$T$^4$ geometries and AdS$_7$ solutions found in the literature.

Keywords: AdS-CFT Correspondence, Extended Supersymmetry, String Duality

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## Contents



## 1 Introduction

Defect QFTs play an important role in our current understanding of Quantum Field Theories. Of particular interest is the situation when the ambient QFT is a CFT with a holographic dual. In this case, introducing appropriate branes in the dual geometry it is possible to construct the gravity dual of the defect QFT, that can then be studied holographically [1–3]. When the defect QFT is a CFT, the explicit AdS dual geometry can be constructed in terms of the fully backreacted geometry [4, 5], if the number of defect branes is sufficiently large.

2d defect CFTs breaking half of the supersymmetries of the ambient CFT have been studied in [6–8], and their corresponding $\mathrm{AdS}_3$ gravity duals have been constructed.[1] The ambient CFT is either a 6d (1,0) CFT [6, 7] or a 5d fixed point theory [8].[2] In the first case the 2d CFT lives in D2-D4 branes introduced in the D6-NS5-D8 brane intersections that underlie 6d (1,0) CFTs. In the second case it lives in D2-NS5-D6 branes in the D4-D8 brane set-ups that give rise to 5d $\mathrm{Sp}(N)$ fixed point theories.

In this work we will be interested in an extension of the first realisation. We will show that a sub-class of the local solutions constructed recently in [11], preserving small $\mathcal{N} = (0, 4)$ supersymmetry on a foliation of $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ over an interval, can be used to construct globally compact solutions dual to 2d (0,4) SCFTs that have an interpretation in terms of D2-D4 defects in 6d (1,0) CFTs. More precisely, we will be using the word defect to indicate the presence of extra branes in Hanany-Witten brane set-ups that would otherwise arise from compactifying higher dimensional branes. This provides a new scenario in which 2d (0,4) CFTs appear in string theory.

2d (0,4) CFTs play a key role in the microscopical description of 5d black holes with $\mathrm{AdS}_3 \times \mathrm{S}^2$ near horizon geometries [12–17]. In string theory they can be realised in D1-D5-KK systems [18–21] and D1-D5-D9 systems [22]. They also play a prominent role in the description of self-dual strings in 6d (1,0) CFTs realised in M- and F-theory [23–28]. Their extensions to 2d (0,4) CFTs with large superconformal algebra have also received a good deal of attention [29–33]. Very recently we have also shown that they can be realised in larger D2-D4-D6-NS5-D8 brane systems [34, 35].

In [11] $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_4$ solutions in massive IIA supergravity preserving $\mathcal{N} = (0, 4)$ supersymmetry with SU(2)-structure were classified. These solutions are warped products of $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_4$ over an interval, with $\mathrm{M}_4$ either a $\mathrm{CY}_2$ or a Kahler manifold. The CFT duals of the first class were studied in [34, 35]. They are described by (0,4) quiver gauge theories with gauge groups $\prod_{i=1}^{n} \mathrm{SU}(k_i) \times \mathrm{SU}(\tilde{k}_i)$. $\mathrm{SU}(k_i)$ is the gauge group associated to $k_i$ D2 branes stretched between NS5 branes and $\mathrm{SU}(\tilde{k}_i)$ is the gauge group associated to $\tilde{k}_i$ D6-branes, wrapped on the $\mathrm{CY}_2$, also stretched between the NS5 branes. On top of these there are D4 and D8 branes that provide flavour groups to both types of nodes of the quiver. These quivers are a generalisation of the linear quivers studied in [26], where the D6 branes are unwrapped and are thus non-dynamical. In this paper we give an interpretation to our brane systems as D2-D4 brane defects in the D6-NS5-D8 branes associated to 6d (1,0) CFTs.

The organisation of the paper is as follows. In section 2 we review the main properties of the $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2$ solutions constructed in [11], and summarise the key features of their 2d dual CFTs, following [35]. In section 3 we construct a mapping that relates a sub-class of these solutions with the $\mathrm{AdS}_7$ solutions in massive IIA supergravity constructed in [36]. Using this map we can interpret the 2d dual CFTs as associated to D2-D4 defects in the D6-NS5-D8 brane set-ups dual to the $\mathrm{AdS}_7$ solutions, wrapped on the $\mathrm{CY}_2$. This suggests that it should be possible to construct RG flows that interpolate between these two classes

[1] 1d CFTs and their $\mathrm{AdS}_2$ duals have been addressed in [9].

[2] SUSY-preserving defects in 5d CFTs have been studied recently in [10].



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of solutions. In section 4 we discuss the $\mathrm{AdS}_7$ solution that describes the 6d linear quiver with gauge groups of increasing ranks terminated by D6 branes, in relation to the map constructed in section 3. By means of this study we *rediscover* the non-Abelian T-dual (NATD) of the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ geometry, constructed in [37] (see also [30]), as the *leading order* in an expansion on the number of gauge groups, of this solution. Then in section 5 we start a detailed study of the non-Abelian T-dual solution. We show that it provides a simple explicit example in the general classification in [11], that describes a 2d (0,4) CFT with two families of gauge groups [35] with increasing ranks. As in other AdS solutions generated through non-Abelian T-duality, the solution is non-compact, and this renders and infinitely long dual quiver CFT. Remarkably, we are able to provide explicit global completions of the solution that have associated well-defined 2d (0,4) dual CFTs, that we describe. This solution thus provides a useful example where it is possible to use holography in a very explicit way to determine global properties of non-compact solutions generated through non-Abelian T-duality, following the ideas in [38–42]. In section 6 we attempt to make connection with RG flows in the literature that connect $\mathrm{AdS}_3$ geometries in the IR, with an interpretation as 2d defect CFTs, with $\mathrm{AdS}_7$ solutions in the UV [6, 7]. Our results are negative, and thus exclude the RG flows constructed in these references as interpolating between the $\mathrm{AdS}_3$ solutions in [11] and the $\mathrm{AdS}_7$ solutions in [36]. Section 7 contains our conclusions and future directions. Appendix A contains some explicit derivations useful in section 5. Appendix B contains details of the BPS flow constructed in [6], upon which section 6 is built.

## 2 $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ solutions in massive IIA and their CFT duals

In [11] $\mathrm{AdS}_3\times\mathrm{S}^2$ solutions in massive IIA with small (0,4) supersymmetry and SU(2) structure were classified. Two classes of solutions that are warped products of the form $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{M}_4\times\mathrm{I}$ were found, for $\mathrm{M}_4$ either a $\mathrm{CY}_2$ manifold, class I, or a family of Kahler 4 manifolds depending on the interval, class II. The solutions in the first class provide a generalisation of D4-D8 systems involving additional branes, while those in the second class are a generalisation of the (T-duals of the) solutions in [28], based on D3-branes wrapping curves in F-theory. In this paper we will be interested in the first class of solutions, that we now summarise.

The explicit form of the NS sector of the solutions referred as class I in [11] is given by:

$$ds^2 = \frac{u}{\sqrt{h_4 h_8}}\left(ds^2(\mathrm{AdS}_3) + \frac{h_8 h_4}{4h_8 h_4 + (u')^2}ds^2(\mathrm{S}^2)\right) + \sqrt{\frac{h_4}{h_8}}ds^2(\mathrm{CY}_2) + \frac{\sqrt{h_4 h_8}}{u}d\rho^2, \quad (2.1)$$
$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2h_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8 h_4 + (u')^2}, \qquad H = \frac{1}{2}d\left(-\rho + \frac{uu'}{4h_4 h_8 + (u')^2}\right)\wedge \mathrm{vol}(\mathrm{S}^2) + \frac{1}{h_8}d\rho\wedge H_2.$$

Here $\Phi$ is the dilaton, $H$ the NS 3-form and $ds^2$ is the metric in string frame. The warpings are determined from three independent functions $h_4, u, h_8$. $h_4$ has support on $(\rho, \mathrm{CY}_2)$ while $u$ and $h_8$ have support on $\rho$, with $u' = \partial_\rho u$. The reason for the notation $h_4, h_8$ is that these functions may be identified with the warp factors of intersecting D4 and D8 branes when $u = 1$.[3]

The 10 dimensional RR fluxes are

$$
\begin{aligned}
F_0 &= h_8', && \text{(2.2a)}\\
F_2 &= -H_2 - \frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4h_8h_4 + (u')^2}\right)\mathrm{vol}(\mathrm{S}^2), && \text{(2.2b)}\\
F_4 &= \left(d\left(\frac{uu'}{2h_4}\right) + 2h_8 d\rho\right)\wedge \mathrm{vol}(\mathrm{AdS}_3)\\
&\quad - \frac{h_8}{u}(\hat{\star}_4 d_4 h_4)\wedge d\rho - \partial_\rho h_4 \mathrm{vol}(\mathrm{CY}_2) - \frac{uu'}{2(4h_8h_4 + (u')^2)} H_2 \wedge \mathrm{vol}(\mathrm{S}^2), && \text{(2.2c)}
\end{aligned}
$$

with the higher fluxes related to these as $F_6 = -\star_{10} F_4,\ F_8 = \star_{10} F_2,\ F_{10} = -\star_{10} F_0$.

Supersymmetry holds whenever

$$u'' = 0, \qquad H_2 + \hat{\star}_4 H_2 = 0, \tag{2.3}$$

which makes $u$ a linear function. Here $\hat{\star}_4$ is the Hodge dual on $\mathrm{CY}_2$. In turn, the Bianchi identities of the fluxes impose

$$
\begin{gathered}
h_8'' = 0, \qquad dH_2 = 0 \\
\frac{h_8}{u}\nabla^2_{\mathrm{CY}_2} h_4 + \partial_\rho^2 h_4 - \frac{2}{h_8^3}\hat{\star}_4(H_2\wedge H_2) = 0,
\end{gathered} \tag{2.4}
$$

away from localised sources.

In this paper we will be interested in the subclass of solutions for which the symmetries of the $\mathrm{CY}_2$ are respected by the full solution. This enforces $H_2 = 0$ and a compact $\mathrm{CY}_2$. Thus, we will be dealing with $\mathrm{T}^4$ or K3. The supersymmetry and Bianchi identities are then all solved for $h_8, u, h_4$ arbitrary linear functions in $\rho$.

The magnetic components of the Page fluxes $\hat{F} = F \wedge e^{-B_2}$, are given by

$$
\begin{aligned}
\hat{f}_0 &= h_8', && \text{(2.5)}\\
\hat{f}_2 &= -\frac{1}{2}\Big(h_8 - (\rho - 2n\pi)h_8'\Big)\mathrm{vol}(\mathrm{S}^2) && \text{(2.6)}\\
\hat{f}_4 &= -h_4'\mathrm{vol}(\mathrm{CY}_2), && \text{(2.7)}\\
\hat{f}_6 &= \frac{1}{2}\Big(h_4 - (\rho - 2n\pi)h_4'\Big)\mathrm{vol}(\mathrm{CY}_2)\wedge \mathrm{vol}(\mathrm{S}^2), && \text{(2.8)}
\end{aligned}
$$

where we have included large gauge transformations of $B_2$ of parameter $n$, such that

$$B_2 = \frac{1}{2}\left(2n\pi - \rho + \frac{uu'}{4h_4h_8 + (u')^2}\right)\wedge \mathrm{vol}(\mathrm{S}^2). \tag{2.9}$$

The 2d CFTs dual to this class of solutions were constructed in [35]. They are described by (0,4) supersymmetric quivers with gauge groups associated to D2 and D6 branes, the

[3] The interpretation for generic $u$ is more subtle.

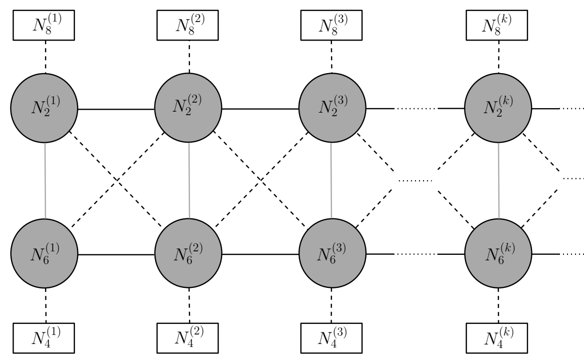


**Figure 1**. Generic quiver field theory whose IR is holographic dual to the solutions discussed in this section. The solid black line represents a (4,4) hypermultiplet, the grey line a (0,4) hypermultiplet and the dashed line a (0,2) Fermi multiplet. (4,4) vector multiplets are the degrees of freedom at each node.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D2 | x | x | | | | | x | | | |
| D4 | x | x | | | | | | x | x | x |
| D6 | x | x | x | x | x | x | x | | | |
| D8 | x | x | x | x | x | x | | x | x | x |
| NS5 | x | x | x | x | x | x | | | | |

**Table 1**. $\frac{1}{8}$-BPS brane intersection underlying the quiver depicted in figure 1. $(x^0, x^1)$ are the directions where the 2d CFT lives, $(x^2, \ldots, x^5)$ span the $\mathrm{CY}_2$, on which the D6 and the D8-branes are wrapped, $x^6$ is the direction along the linear quiver, and $(x^7, x^8, x^9)$ are the transverse directions on which the $\mathrm{SO}(3)_R$ symmetry is realised.

latter wrapped on the $\mathrm{CY}_2$ manifold, stretched between NS5 branes. Having finite extension in this direction, the field theory living in both the D2 and D6 branes is two dimensional at low energies compared to the inverse separation between the NS5-branes. It was shown in [35] that these quivers are rendered non-anomalous with adequate flavour groups at each node, coming from D4 and D8 branes. Remarkably, the flavour groups associated to gauge groups originating from D2 branes arise from D8 branes (wrapped on the $\mathrm{CY}_2$) while those associated to the gauge groups originating from wrapped D6-branes arise from D4-branes. The corresponding quiver is depicted in figure 1. The underlying brane set-up is summarised in table 1.

The 2d CFTs dual to the solutions in class I thus generalise the (0,4) quivers studied in [26] from D2, NS5 and D6 branes, in two ways. First, the D6 branes are compact, and therefore give rise to gauge, as opposed to global, symmetries. Second, there are D8 branes between the NS5 branes that can give rise to different flavour groups to each gauge group coming from D2 branes [44, 45]. Non-compact D4 branes provide the necessary flavour groups that render the nodes associated to the new, colour, D6 branes non-anomalous. Our quivers also generalise the (0,4) quivers constructed in [32] from D3-brane box configurations to gauge nodes with different gauge groups.



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$\frac{1}{8}$-BPS brane set-ups such as the one depicted in table 1 were discussed in [7] in the context of 2d defect CFTs originating from D2-D4 branes living in 6d (1,0) CFTs. In the next section we find that it is indeed possible to give an interpretation to some of the CFTs studied in [35] in these terms. We will discuss the connection with the solutions constructed in [7] in section 6.

## 3 A map between $\mathbf{AdS_3{\times}S^2}$ and $\mathbf{AdS_7}$ solutions in massive IIA

In [36] an infinite class of $\mathrm{AdS}_7$ solutions in massive IIA was constructed,[4] preserving 16 supersymmetries (eight Poincare and eight conformal) on a foliation of $\mathrm{AdS}_7\times\mathrm{S}^2$ over an interval. In this section we show that they can be related to our solutions in [11], preserving (0,4) supersymmetries on a foliation of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ over an interval, through a map that reduces supersymmetry by half. As opposed to the mappings in [47] between $\mathrm{AdS}_7$ solutions and the $\mathrm{AdS}_5$ and $\mathrm{AdS}_4$ solutions in [48, 49], this mapping is not one-to-one, due to the presence of D2-D4 defects, whose backreaction introduces new 4-form and 6-form fluxes.

We start by briefly summarising the solutions constructed in [36]. Using the parametrisation in [50], these solutions can be completely determined by a function $\alpha(z)$ that satisfies the differential equation

$$\dddot{\alpha} = -162\pi^3 F_0. \tag{3.1}$$

Where $F_0$ is the Ramond zero-form. Explicitly, the metric and fluxes are given by

$$\begin{aligned}
ds^2_{10} &= \pi\sqrt{2}\left(8\sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds^2(\mathrm{AdS}_7) + \sqrt{-\frac{\ddot{\alpha}}{\alpha}}dz^2 + \frac{\alpha^{3/2}(-\ddot{\alpha})^{1/2}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}ds^2(\mathrm{S}^2)\right) &(3.2)\\
e^{2\Phi} &= 2^{5/2}\pi^5 3^8 \frac{(-\alpha/\ddot{\alpha})^{3/2}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}} &(3.3)\\
B_2 &= \pi\left(-z + \frac{\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}\right)\mathrm{vol}(\mathrm{S}^2) &(3.4)\\
F_2 &= \left(\frac{\ddot{\alpha}}{162\pi^2} + \frac{\pi F_0 \alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}\right)\mathrm{vol}(\mathrm{S}^2). &(3.5)
\end{aligned}$$

These backgrounds were shown to arise as near horizon geometries of D6-NS5-D8 brane intersections [51, 52] (see also [50, 53] for previous hints), from which 6d linear quivers with 8 supercharges can be constructed [44, 45]. In these quivers anomaly cancelation implies that for every gauge group the number of flavours must double the number of gauge multiplets, $N_f = 2N_c$ [53]. In reference [50] a prescription was given to calculate the function $\alpha(z)$ that encodes the explicit $\mathrm{AdS}_7$ solution dual to a given 6d quiver diagram. In this quiver diagram the NS5 branes are located at different values of $z$, the D6-branes are stretched between them along this direction and the D8 branes are perpendicular. The corresponding brane set-up is depicted in table 2.

[4] See [46] for orientifold constructions thereof.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D6 | x | x | x | x | x | x | x | | | |
| D8 | x | x | x | x | x | x | | x | x | x |
| NS5 | x | x | x | x | x | x | | | | |

**Table 2**. $\frac{1}{4}$-BPS brane intersection underlying the 6d (1,0) CFTs dual to the $\mathrm{AdS}_7$ solutions in [36]. $(x^0, \ldots, x^5)$ are the directions where the 6d CFT lives, $x^6$ is the direction along which the NS5-branes are located, and $(x^7, x^8, x^9)$ realise the SU(2) R-symmetry of the internal space.

After this brief summary we can introduce the mapping that relates these solutions to the solutions in class I in [11], summarised in the previous section. The mapping reads

$$\rho \leftrightarrow 2\pi z \tag{3.6}$$

$$u \leftrightarrow \alpha \tag{3.7}$$

$$h_8 \leftrightarrow -\frac{\ddot{\alpha}}{81\pi^2} \tag{3.8}$$

$$h_4 \leftrightarrow \frac{81}{8}\alpha\,. \tag{3.9}$$

Using these relations one can match the $B_2$ field, dilaton, $F_0$ and $F_2$ fluxes of the two solutions, as well as the $S^2 \times I$ components of the metric. For the rest of the metric one must consider the mapping

$$ds^2(\mathrm{AdS}_3) + \frac{3^4}{2^3} ds^2(\mathrm{CY}_2) \leftrightarrow 4\, ds^2(\mathrm{AdS}_7)\,. \tag{3.10}$$

Besides, the $F_4$ and $F_6$ fluxes, which would violate the symmetries of the $\mathrm{AdS}_7$ solution, must be disregarded when using the mapping from $\mathrm{AdS}_3$ to $\mathrm{AdS}_7$. These fluxes clearly sign the presence of a D2-D4 defect in the $\mathrm{AdS}_3$ solution. As we discuss below, its backreaction has also the effect of modifying the dependence of the different functions on both sides of (3.7)–(3.9) on the respective *field theory* directions (related through (3.6)).

Indeed, (3.7) and (3.9) relate linear functions in $\rho$ with a cubic function of $z$. This mapping is therefore essentially different from the mappings found in [47], where other than the replacements of $\mathrm{AdS}_5 \times \Sigma_2$ or $\mathrm{AdS}_4 \times \Sigma_3$ with $\mathrm{AdS}_7$, the internal space is just distorted by some numerical factors. This difference is due to the presence of the D2-D4 defect in the $\mathrm{AdS}_3$ solution, which is also responsible for the reduction of the supersymmetry from 1/2 BPS to 1/4 BPS.

Using (3.8) and (3.6) it is possible to obtain the $\mathrm{AdS}_7$ solution related to a particular $\mathrm{AdS}_3 \times \mathrm{CY}_2$ solution. One finds

$$h_8 = F_0\rho + c \quad \leftrightarrow \quad \ddot{\alpha} = -162\pi^3 F_0 z + \tilde{c}\,, \tag{3.11}$$

from which $\alpha(z)$, and thus, the explicit $\mathrm{AdS}_7$ solution in [36], can be determined. This mapping does not however give the expressions for the $u$ and $h_4$ functions that define the $\mathrm{AdS}_3$ solution. Still, one can exploit (3.11) to show that the D8-brane charges of the $\mathrm{AdS}_7$ and $\mathrm{AdS}_3$ solutions, determined, respectively, from $h_8'$ and $-\dddot{\alpha}/(162\pi^3)$, agree, and that the same holds for the D6-brane charges, given that the corresponding $\hat{f}_2$ Page fluxes satisfy

$$\hat{f}_{2(\mathrm{AdS}_3)} = -\frac{1}{2}\Big(h_8 - (\rho - 2n\pi)h_8'\Big)\mathrm{vol}(S^2) \leftrightarrow \left(\frac{\ddot{\alpha}}{162\pi^2} + F_0(z - n\pi)\right)\mathrm{vol}(S^2) = \hat{f}_{2(\mathrm{AdS}_7)}. \tag{3.12}$$

This implies that the D6-NS5-D8 *sector* of the $\mathrm{AdS}_3$ solution is simply obtained by compactifying on the $\mathrm{CY}_2$ the D6-NS5-D8 branes that underlie the $\mathrm{AdS}_7$ solution.

However, as we have mentioned, the $u$ and $h_4$ linear functions needed to fully specify the $\mathrm{AdS}_3$ solution, cannot be determined from the $\mathrm{AdS}_7$ solution using this mapping, other than the fact that they have to be proportional to each other.[5] This was to be expected, since, as we showed in [11], these functions encode the information of the additional D2-D4 branes present in the $\mathrm{AdS}_3$ solution. This is, once more, essentially different from the mappings between $\mathrm{AdS}_5$ and $\mathrm{AdS}_4$ and $\mathrm{AdS}_7$ solutions found in [47], where it is not possible to identify 6 and 4-cycles on which additional D2 or D4 brane charges can be defined. In this case this is possible due to the non-trivial $\mathrm{CY}_2$ 4-cycle in the internal space of the $\mathrm{AdS}_3$ solutions.

The symmetry between the D6-NS5-D8 and D2-NS5-D4 *sectors*, manifest in the expressions of the RR Page fluxes of the $\mathrm{AdS}_3$ solutions,

$$\hat{f}_0 = h_8', \qquad \hat{f}_2 = -\frac{1}{2}\Big(h_8 - (\rho - 2n\pi)h_8'\Big)\mathrm{vol}(\mathrm{S}^2) \tag{3.13}$$

and

$$\hat{f}_4 = -h_4'\mathrm{vol}(\mathrm{CY}_2), \qquad \hat{f}_6 = \frac{1}{2}\Big(h_4 - (\rho - 2n\pi)h_4'\Big)\mathrm{vol}(\mathrm{CY}_2) \wedge \mathrm{vol}(\mathrm{S}^2), \tag{3.14}$$

stress the role of both D2 and D6 branes as colour branes in the 2d CFT dual to the $\mathrm{AdS}_3$ solution, and of D4 and D8 branes as flavour branes [35]. The resulting 2d (0,4) CFT thus contains two types of nodes, associated to the gauge groups of D2 and compact, wrapped on the $\mathrm{CY}_2$, D6 branes. This is the generalisation of the (0,4) quivers discussed in [26] that we found in [35]. Note that compactification on the $\mathrm{CY}_2$ of the 6d CFT living in D6-NS5-D8 branes preserves (4,4) supersymmetries.[6] The D2-D4 branes further reduce the supersymmetries by one half [7] (see also [59]). Alternatively, one could start with the D2-NS5-D4 Hanany-Witten brane set-ups discussed in [60, 61], realising 2d (4,4) field theories, and intersect them with wrapped D6 and D8 branes, which would also reduce the supersymmetries by a half. The resulting $\frac{1}{8}$ BPS configuration (increasing to $\frac{1}{4}$ at the near horizon) is the one that we depicted in table 1.

[5] We will see below that this guarantees that the two solutions share the same singularity structure, or, in other words, that the $\mathrm{S}^2$ shrinks in the same way to produce topologically an $\mathrm{S}^3$.

[6] Gauge theories with (4, 4) supersymmetry in two dimensions may be viewed as the dimensional reduction of 6d (1, 0) gauge theories. The six dimensional gauge theories have an $\mathrm{SU}(2)_R$ R-symmetry. Upon dimensional reduction to two dimensions there is an additional $\mathrm{SO}(4) = \mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ symmetry acting on the four reduced dimensions. This is also an R-symmetry since the supercharges are a spinor of this SO(4) group; the left-moving (positive chirality) supercharges are in the (2,1,2) representation of $\mathrm{SU}(2)_l \times \mathrm{SU}(2)_r \times \mathrm{SU}(2)_R$ while the right-moving (negative chirality) supercharges are in the (1, 2, 2) representation [54, 55].



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Let us now discuss the physical reason for the condition $h_4 \sim u$, implied by (3.9) and (3.7), in the $\mathrm{AdS}_3$ solutions. As we have mentioned, the functions $u$ and $h_4$, needed to completely determine the $\mathrm{AdS}_3\times\mathrm{CY}_2$ solution, cannot be computed from (3.7) and (3.9), due to the different dependence on $\rho$ and $z$ of these functions and $\alpha(z)$, respectively. Rather, the relation $h_4 = 81u/8$ has to be seen as a restriction on the class of $\mathrm{AdS}_3\times\mathrm{CY}_2$ solutions that can be interpreted as defects in the CFTs dual to $\mathrm{AdS}_7$ solutions. This restriction comes from the condition that both solutions share the same singularity structure. In order to see this we note that in both solutions the range of the interval is determined by the points at which the $\mathrm{S}^2$ shrinks, such that the $\mathrm{S}^2\times\mathrm{I}$ space is topologically an $\mathrm{S}^3$. In $\mathrm{AdS}_7$ there is a D6 brane when $\alpha = 0$, $\ddot{\alpha} \neq 0$, and a O6 when $\ddot{\alpha} = 0$, $\alpha \neq 0$. In turn, when $\alpha = 0$, $\ddot{\alpha} = 0$ the $\mathrm{S}^2$ shrinks smoothly [48]. Similarly, for $\mathrm{AdS}_3$ solutions satisfying $h_4 \sim u$ there is a D6 brane when $u \sim h_4 = 0$, $h_8 \neq 0$ and a O6 when $h_8 = 0$, $u \sim h_4 \neq 0$. In turn, the $S^2$ shrinks smoothly for $u \sim h_4 = 0$, $h_8 = 0$ [11]. The role played by the D6 branes terminating the space as flavour branes is discussed in section 4.

Let us summarise our findings so far in this section. We have shown that a subclass of the solutions in [11][7] can be interpreted as arising from D2-D4 *defect branes* inside the D6-NS5-D8 brane intersections underlying the $\mathrm{AdS}_7 \times \mathrm{S}^2 \times I$ solutions in [36], wrapped on the $\mathrm{CY}_2$ of the internal manifold. 6d (1,0) CFTs compactified in $\mathrm{CY}_2$ manifolds give rise to 2d (4,4) field theories that are not conformal [54, 55]. Therefore, $\mathrm{AdS}_3$ solutions cannot be obtained from the $\mathrm{AdS}_7$ solutions in [36] simply by extending the construction of $\mathrm{AdS}_5$ and $\mathrm{AdS}_4$ solutions in [47] to 4d manifolds. As we showed in [11] extra D2 and D4 branes are needed, that further reduce the supersymmetries down to 1/8 BPS and the $\mathrm{AdS}_3$ solutions to 1/4-BPS. These branes backreact in the compactified geometry, and modify the simple mappings found in [47] such that the dependence of the functions defining the $\mathrm{AdS}_3$ and $\mathrm{AdS}_7$ solutions change, due to the backreaction. One can thus think of the 2d CFT associated to the $\mathrm{AdS}_3$ solutions as comprised of two sectors, one coming from D6-NS5-D8 branes wrapped on the $\mathrm{CY}_2$, which by itself does not give rise to a 2d CFT, and one coming from extra, D2-D4 branes, which would not give rise either to 2d CFTs together with the NS5-branes [60]. One can in this sense interpret the D2-D4 branes as defects inside D6-NS5-D8 brane systems. We would like to stress that this defect interpretation is essentially different from the defect interpretation in terms of punctures that can be given to the Gaiotto theories in 4d [56], dual to the Gaiotto-Maldacena geometries [57]. In this last case both the field theory in the absence of punctures (dual to the Maldacena-Nunez solution [58]) and the ones with punctures are well- defined 4d CFTs, in contrast with the 2d CFTs dual to our $\mathrm{AdS}_3$ solutions.

Further light on the relation between the 2d (0,4) CFTs dual to the $\mathrm{AdS}_3$ solutions and compactifications on $\mathrm{CY}_2$ of the 6d (1,0) CFTs dual to the $\mathrm{AdS}_7$ solutions comes from comparing their respective central charges, following [62]. The holographic central charge of the 6d CFTs dual to the $\mathrm{AdS}_7$ solutions was computed in [63]:

$$c_{\mathrm{AdS}_7} = \frac{1}{G_N}\frac{2^4}{3^8}\int dz(-\alpha\ddot{\alpha}). \tag{3.15}$$

[7] Those that share the same singularity structure of the solutions in [36], in the sense that we have just explained.

In turn, the holographic central charge of the 2d CFTs dual to the $\mathrm{AdS}_3\times\mathrm{CY}_2$ solutions is [35]

$$c_{\mathrm{AdS}_3} = \frac{3\pi}{2G_N}\mathrm{Vol}(\mathrm{CY}_2)\int d\rho(h_8 h_4)\,. \tag{3.16}$$

Using the mapping given by (3.6)–(3.9) this becomes

$$c_{\mathrm{AdS}_3} \leftrightarrow \frac{3}{2^3 G_N}\mathrm{Vol}(\mathrm{CY}_2)\int dz(-\alpha\ddot{\alpha}) = \frac{3^9}{2^7}\mathrm{Vol}(\mathrm{CY}_2)\, c_{\mathrm{AdS}_7}\,. \tag{3.17}$$

Thus, there exists a universal relation between the central charges associated to both types of solutions. Similarly, in [64] (see also [65]) $\mathrm{AdS}_3\times\Sigma_4$ solutions of massive IIA were constructed whose 2d (0,1) and (0,2) CFT duals arise as compactifications of the 6d (1,0) theories dual to the $\mathrm{AdS}_7$ solutions. Their respective free energies were shown to satisfy the relation

$$\frac{\mathcal{F}_2}{\mathcal{F}_6} = \frac{1}{(2X_{IR})^5}\mathrm{Vol}(\Sigma_4), \tag{3.18}$$

where $\Sigma_4$ is the compactification manifold and $X_{IR}$ is a constant that characterises the $\mathrm{AdS}_3$ solution.[8] Our result is thus in agreement with an interpretation of the 2d CFTs dual to our solutions as compactified 6d (1,0) theories in $\mathrm{CY}_2$ manifolds, with extra degrees of freedom coming from the 2d defects. It would be very interesting to obtain explicit flows connecting the $\mathrm{AdS}_3\times\mathrm{CY}_2$ solutions in the IR with the $\mathrm{AdS}_7$ solutions in the UV. In particular, it would be interesting to clarify whether these involve $\mathbb{R}_{1,1}\times\mathrm{CY}_2$ warped product geometries, which would be the natural extension of the flows constructed in [62, 64, 65], or wrapped $\mathrm{AdS}_3$ subspaces, more directly related to defects, as in [6–8]. In [7] different limits of the D2-D4-D6-NS5-D8 intersections depicted in table 1 were studied, giving rise to either $\mathrm{AdS}_7$ or $\mathrm{AdS}_3\times\mathrm{S}^3\times$I' geometries, associated to the UV or IR limits of the intersection, respectively. In particular, $\mathrm{AdS}_3\times\mathrm{T}^4$ geometries should arise when the branes are smeared on the $\mathrm{T}^4$. In section 6 we explore the connection between the BPS flows constructed in [6, 7] and the subclass of $\mathrm{AdS}_3\times\mathrm{T}^4$ solutions defined by the mapping discussed in this section.

## 4 The linear quiver with infinite number of nodes

As we have mentioned, the mapping found in the previous section is formal, in the sense that it relates $\alpha$, a cubic function in $z$, to $h_4 \sim u$, which are linear in $\rho$ (with $z$ and $\rho$ related as in (3.6)). In this section we discuss a particular instance in which $\alpha$ and $h_4 \sim u$ can be explicitly related.

Consider an $\mathrm{AdS}_7$ solution in which the $\mathrm{S}^2\times$I geometry is smooth at $z=0$ and terminates at $z=P+1$, such that

$$F_0 = -\frac{\alpha'''(z)}{162\pi^3} = \frac{N}{2\pi}\begin{cases} 1\,, & 0\le z\le P \\ -P\,, & P\le z\le P+1. \end{cases} \tag{4.1}$$

[8] $X_{IR}$ is the value in the IR of the $X$ scalar field of 7d minimal supergravity (see section 6).

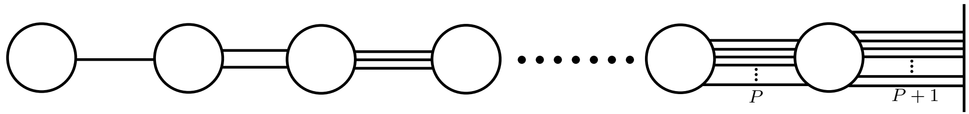


**Figure 2**. D6-NS5-D8 brane set-up associated to a linear quiver with increasing ranks terminated by a flavour group. NS5 branes are denoted by circles, D6 branes by horizontal lines and D8 branes by vertical lines.

For this we need $N(P+1)$ D8-branes at $z=P$, given that

$$dF_0 = \frac{N(P+1)}{2\pi}\delta(z-P)dz. \tag{4.2}$$

As shown in [63], for a particular choice of the integration constants such that $\alpha(0) = \alpha(P+1) = 0$, and $\alpha$ and $\alpha'$ are continuous functions, we have

$$\alpha(z) = \frac{27\pi^2 N}{2}\begin{cases} P(P+2)z - z^3\,, & 0 \le z \le P \\ Pz^3 - 3P(P+1)z^2 + P(3P^2+4P+2)z - P^3(P+1)\,, & P \le z \le P+1, \end{cases} \tag{4.3}$$

and the dual CFT is a linear quiver with gauge group

$$\mathrm{SU}(N)\times\mathrm{SU}(2N)\times\mathrm{SU}(3N)\times\mathrm{SU}(4N)\times\ldots\times\mathrm{SU}(PN), \tag{4.4}$$

finished with a $\mathrm{SU}((P+1)N)$ flavour group, represented by the D8 branes. The brane set-up associated to this quiver is depicted in figure 2.

Now, consider the situation in which $P$ is very large, so that the region of interest reduces to $0\le z\le P$ and we can take $\alpha(z) = \frac{27\pi^2 N}{2}(P(P+2)z - z^3)$ for all $P$.[9] Redefining $z = \sqrt{P(P+2)}x$, we can write the solution in this region as

$$\begin{aligned} \frac{ds^2}{\sqrt{P(P+2)}} &= \frac{8\pi}{\sqrt{3}}\sqrt{1-x^2}\,ds^2(\mathrm{AdS}_7) + \frac{2\sqrt{3}\pi}{\sqrt{1-x^2}}\Big[dx^2 + \frac{x^2(1-x^2)^2}{1+6x^2-3x^4}\,ds^2(\mathrm{S}^2)\Big], \\ e^{4\Phi} &= \frac{12}{F_0^4\pi^2 P(P+2)}\frac{(1-x^2)^3}{(1+6x^2-3x^4)^2}, \qquad F_0 = \frac{N}{2\pi} \\ B_2 &= -2\pi\sqrt{(P(P+2)}\frac{x^3(5-3x^2)}{1+6x^2-3x^4}\,\mathrm{vol}(\mathrm{S}^2), \quad F_2 = F_0 B_2, \quad \hat{f}_2 = n\mathrm{vol}(\mathrm{S}^2). \end{aligned} \tag{4.5}$$

This solution can be expanded close to $x=1$ (the end of the space) by defining $x = 1-v$. We then have a metric and dilaton that for small values of $v$ read,

$$\begin{aligned} ds^2 &\sim 8\pi\sqrt{\frac{2}{3}}\sqrt{v}\,ds^2(\mathrm{AdS}_7) + \frac{\sqrt{6}\pi}{\sqrt{v}}(dv^2 + v^2 ds^2(\mathrm{S}^2)), \\ e^{4\Phi} &\sim v^3. \end{aligned} \tag{4.6}$$

It is thus clear that close to $v\sim 0$ or $x\sim 1$, in the end of the space, we have D6 branes that extend along $\mathrm{AdS}_7$. As discussed in [48], these D6 branes can play the role of flavour

[9] Note that strictly speaking this would extend the region of interest to $0\le z\le\sqrt{P(P+2)}$, but this is equivalent to $0\le z\le P$ when $P$ is large.

branes even when their dimensionality is the same as that of the colour branes. They differ in that the colour branes are extended along the six Minkowski directions of $\mathrm{AdS}_7$ plus a bounded interval, while the flavour D6-branes are extended on the whole $\mathrm{AdS}_7$. Being non-compact they can act as flavour branes, as happens in many other (qualitatively different) examples, like [66, 67].

Now, we would like to use the mapping between $\mathrm{AdS}_3$ and $\mathrm{AdS}_7$ solutions described by (3.6)–(3.9). This tells us that we should identify,

$$h_8 = \frac{N}{2\pi}\rho, \quad u = \frac{27\pi N}{4}\left(P(P+2)\rho - \frac{\rho^3}{4\pi^2}\right), \quad h_4 = \frac{81}{8}u. \tag{4.7}$$

This is not a solution of the equations of motion of the $\mathrm{AdS}_3$ system. Nevertheless, if we take $P \to \infty$ or, equivalently, $\rho \to 0$, we have

$$h_8 = \frac{N}{2\pi}\rho, \quad u = \frac{27\pi N}{4}P(P+2)\rho, \quad h_4 = \frac{3^7\pi N}{2^5}P(P+2)\rho, \tag{4.8}$$

which defines a non-compact $\mathrm{AdS}_3$ solution. This is the solution constructed in [37] acting with non-Abelian T-duality on the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ solution dual to the D1-D5 system [54, 55, 68–70].

As we discuss in the next section, the non-compact nature of the non-Abelian T-dual solution is reflected in the dual CFT in the existence of an infinite number of gauge groups of increasing ranks. In this section we have *rediscovered* it as the *leading order* of the solution defined by (4.5), dual to a well-defined six dimensional CFT.[10] Since we are working at very small values of $z$ (equivalently, very small values of $\rho$), we do not see the flavour D6 branes, and the space is rendered non-compact. Conversely, taking $P \to \infty$ we see no sign of these branes closing the space.

We discuss the non-Abelian T-dual solution in detail in the next section, and describe other possible ways to define it globally using $\mathrm{AdS}_3/\mathrm{CFT}_2$ holography.

## 5 The non-Abelian T-dual of $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$

In this section we discuss in detail one of the simplest solutions in the classification of $\mathrm{AdS}_3\times\mathrm{S}^2$ geometries in [11], with a focus on the description of its 2d dual CFT, following [35]. This solution arises acting with non-Abelian T-duality on the near horizon of the D1-D5 system, and was originally constructed in [37]. In reference [30] it was shown that the (4,4) supersymmetry of the D1-D5 system is reduced to (0,4) upon dualisation, and that the solution can be further T-dualised and uplifted to M-theory such that it fits in the class of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{S}^2\times\mathrm{CY}_2$ solutions in [71].[11] This solution is particularly interesting in the study of the interplay between non-Abelian T-duality and holography, since it allows for simple explicit global completions of the geometry using field theory arguments.

[10] To be more precise, (4.8) selects a particular non-Abelian T-dual solution, with a given relation between the D2 and D6 brane charges. We give more details in the next section.

[11] Actually, it provides the only known example in this class with SU(2) structure.



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In this section we also discuss another solution in the class in [11] that arises from the D1-D5 system, and that can be obtained as a limit of the non-Abelian T-dual solution [38, 39, 72]. This is the Abelian T-dual (ATD) of $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ along the Hopf-fibre of the $\mathrm{S}^3$, and orbifolds thereof, that also preserve (0,4) of the supersymmetries of the original D1-D5 system. The orbifold solutions describe the D1-D5-KK system, and are dual to (0,4) CFTs that have been discussed in the literature [18–21, 25, 32].

### 5.1 The NATD solution

The non-Abelian T-dual (NATD) of $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{T}^4$ with respect to a freely acting SU(2) subgroup of its SO(4) R-symmetry group was constructed in [37]. As in other NATD examples, the space dual to $\mathrm{S}^3$ becomes, locally, $\mathbb{R}\times\mathrm{S}^2$. The SO(4) R-symmetry is reduced to an SU(2) R-symmetry, and the solution is rendered (0,4) supersymmetric [30]. Due to our lack of knowledge of how non-Abelian T-duality extends beyond spherical worldsheets [73], the space is globally unknown. In this section we will resort to holography in order to construct a compact internal space for which a well-defined 2d dual CFT exists, following the strategy in [38–42].

We start generalising the solution constructed in [37] to arbitrary D1 and D5 brane charges and a compact $\mathrm{CY}_2$ four dimensional internal space. The most general solution reads

$$
\begin{aligned}
ds^2_{10} &= 4L^2 ds^2(\mathrm{AdS}_3) + M^2 ds^2(\mathrm{CY}_2) + 4L^2 ds^2(\mathrm{S}^3) && (5.1)\\
e^{2\Phi} &= 1 && (5.2)\\
F_3 &= 8L^2 \mathrm{vol}(\mathrm{S}^3) && (5.3)\\
F_7 &= -8L^2 M^4 \mathrm{vol}(\mathrm{S}^3)\wedge \mathrm{vol}(\mathrm{CY}_2)\,. && (5.4)
\end{aligned}
$$

The corresponding D1 and D5 brane charges are given by

$$
\begin{aligned}
N_1 &= \frac{1}{(2\pi)^6}\int_{\mathrm{S}^3\times\mathrm{CY}_2} F_7 = \frac{4L^2M^4}{(2\pi)^4}\mathrm{Vol}(\mathrm{CY}_2) && (5.5)\\
N_5 &= \frac{1}{(2\pi)^2}\int_{\mathrm{S}^3} F_3 = 4L^2 && (5.6)
\end{aligned}
$$

The NATD with respect to a freely acting SU(2) group on the $\mathrm{S}^3$ reads

$$
\begin{aligned}
ds^2_{10} &= 4L^2 ds^2(\mathrm{AdS}_3) + M^2 ds^2(\mathrm{CY}_2) + \frac{d\rho^2}{4L^2} + \frac{L^2\rho^2}{4L^4+\rho^2} ds^2(\mathrm{S}^2) && (5.7)\\
e^{2\Phi} &= \frac{4}{4L^6+L^2\rho^2} && (5.8)\\
B_2 &= -\frac{\rho^3}{2(4L^4+\rho^2)}\mathrm{vol}(\mathrm{S}^2) && (5.9)\\
F_0 &= L^2 && (5.10)\\
F_2 &= -\frac{L^2\rho^3}{2(4L^4+\rho^2)}\mathrm{vol}(\mathrm{S}^2) && (5.11)\\
F_4 &= -L^2M^4\mathrm{vol}(\mathrm{CY}_2) && (5.12)\\
F_6 &= \frac{L^2M^4\rho^3}{2(4L^4+\rho^2)}\mathrm{vol}(\mathrm{CY}_2)\wedge\mathrm{vol}(\mathrm{S}^2) && (5.13)
\end{aligned}
$$

It is easy to see that this solution fits locally in the class of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ solutions constructed in [11], with the simple choices

$$u = 4L^4M^2\rho \tag{5.14}$$
$$h_4 = L^2M^4\rho \tag{5.15}$$
$$h_8 = F_0\rho\,. \tag{5.16}$$

These functions define a regular, albeit non-compact, solution. We will shortly be discussing various possibilities that define it globally. For now let us analyse the associated quantised charges.

We start discussing the relevance of large gauge transformations. Close to $\rho = 0$ the 3d transverse space is $\mathbb{R}^3$, while for large $\rho$ it is $\mathbb{R}\times\mathrm{S}^2$. This implies that for finite $\rho$ there is a non-trivial $\mathrm{S}^2$ on which we can compute $\int_{\mathrm{S}^2} B_2$, which needs to satisfy

$$\frac{1}{4\pi^2}\Big|\int_{\mathrm{S}^2} B_2\Big| \in [0,1). \tag{5.17}$$

For $B_2$ as in (5.9) this implies that a large gauge transformation needs to be performed as we move in $\rho$, such that $B_2 \to B_2 + n\pi\mathrm{vol}_{S^2}$ for $\rho\in[\rho_n,\rho_{n+1}]$, with

$$\frac{\rho_n^3}{4L^4+\rho_n^2} = 2n\pi. \tag{5.18}$$

The non-compactness of $\rho$ is then reflected in the existence of large gauge transformations of infinite gauge parameter $n$. Moreover, taking into account large gauge transformations, we see that even if the 2-form and 6-form Page fluxes vanish identically,

$$\hat{f}_2 = F_2 - F_0\wedge B_2 = 0, \qquad \hat{f}_6 = F_6 - B_2\wedge F_4 = 0, \tag{5.19}$$

implying the absence of D6 and D2 brane quantised charges, there is a non-zero contribution when $n\neq 0$, such that

$$N_8 = 2\pi F_0 = 2\pi L^2 \tag{5.20}$$
$$N_6 = \frac{F_0}{2\pi}n\pi\mathrm{Vol}(\mathrm{S}^2) = nN_8 \tag{5.21}$$
$$N_4 = \frac{1}{(2\pi)^3}\int_{\mathrm{CY}_2} F_4 = \frac{L^2M^4}{(2\pi)^3}\mathrm{Vol}(\mathrm{CY}_2) \tag{5.22}$$
$$N_2 = \frac{1}{(2\pi)^5}\int_{\mathrm{CY}_2} F_4 n\pi\mathrm{Vol}(S^2) = nN_4 \tag{5.23}$$
$$N_5 = \frac{1}{(2\pi)^2}\int_{\rho_n}^{\rho_{n+1}}\int_{S^2} H_3 = 1. \tag{5.24}$$

These conserved charges suggest that the D1-D5 system that underlies the Type IIB $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ solution has been mapped under the NATD transformation onto a brane system consisting on $n$ D2-D6 branes at each $[\rho_n,\rho_{n+1})$ interval, dissolved in a D4-D8 bound state, due to the non-vanishing $B_2$-charge. The corresponding brane distribution is depicted in table 3.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D2 | x | x | | | | | x | | | |
| D4 | x | x | | | | | x | | x | x |
| D6 | x | x | x | x | x | x | x | | | |
| D8 | x | x | x | x | x | x | x | | x | x |
| NS5 | x | x | x | x | x | x | | | | |

**Table 3**. Distribution of branes compatible with the quantised charges of the NATD solution. $(y^0, y^1)$ are the directions where the 2d CFT lives, $(y^2, \ldots, y^5)$ parameterise the $\mathrm{CY}_2$, $y^6 = \rho$, $y^7$ is the radius of $\mathrm{AdS}_3$ and $(y^8, y^9)$ span the $\mathrm{S}^2$.

This configuration is the same as the one underlying the solutions constructed in [7], and, as in that case, it can be related to the $\frac{1}{8}$-BPS brane set-up depicted in table 1, where the $\mathrm{SU}(2)_R$ symmetry is manifest, through a rotation in the $(x^6, x^7)$ subspace. Due to the non-compactness of $\rho$ the brane system is however infinite. This suggests a relation with the linear quiver with infinite gauge groups discussed in section 4, that we can now make more explicit.

Indeed, given that $h_4$ and $u$, as given by (5.15) and (5.14), satisfy the condition $h_4 \sim u$, the NATD solution fits in the class of solutions that can be related to $\mathrm{AdS}_7$ solutions, discussed in section 3. Both solutions are related explicitly through the mapping

$$u = 162 F_0 L^4 \rho\,, \qquad P = \frac{2\sqrt{3}}{\pi} L^2, \tag{5.25}$$

with $P$ as introduced in (4.3). This selects the NATD solution with $M^2 = \frac{3^4}{2} L^2$,[12] as the one related to the 6d (1,0) linear quiver discussed in section 4. These relations show that in the supergravity limit $L \gg 1$ the D6-branes are sent off to infinity. In this way we can think of the NATD solution as the *leading order* in an expansion in $P$, of the $\mathrm{AdS}_7$ solution dual to the 6d linear quiver with gauge groups of increasing ranks, terminated with flavour D6-branes.

In the next subsections we define other ways of completing the NATD solution with compact $\mathrm{AdS}_3$ solutions. This will be valid for arbitrary values of the charges.

### 5.2 2d (0,4) dual CFT

As we have seen, the quantised charges of the NATD solution are compatible with an infinite brane system consisting on D2 and D6 branes stretched between NS5 branes. The D6 branes are wrapped on the $\mathrm{CY}_2$, and thus share the same number of non-compact directions of the D2 branes.

General 2d (0,4) quiver theories associated to the 1/8-BPS D2-D4-D6-D8-NS5 brane configurations depicted in table 1 were constructed in [35]. For the particular configuration corresponding to the NATD solution the quiver contains two infinite families of nodes, associated to D2 and wrapped D6 branes, with gauge groups of increasing ranks, and no

[12] This restriction is imposed because the $\mathrm{AdS}_7$ solution depends on one single parameter, $P$, while a generic NATD solution depends on two parameters, $L$ and $M$.

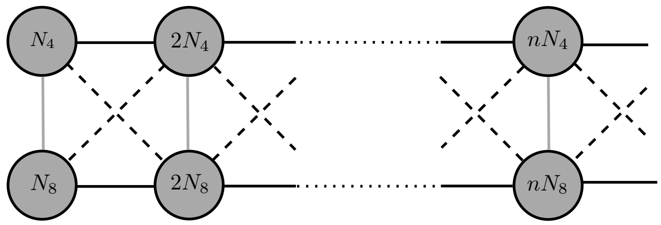


**Figure 3**. Infinite quiver associated to the NATD solution.

flavours. This quiver is depicted in figure 3. We next summarise its main ingredients (the reader can find more details in reference [35]):

- To each gauge node corresponds a (0,4) vector multiplet plus a (0,4) twisted hypermultiplet in the adjoint representation of the gauge group. In terms of (0,2) multiplets, the first consists on a vector multiplet and a Fermi multiplet in the adjoint, and the second to two chiral multiplets forming a (0,4) twisted hypermultiplet, also in the adjoint. The (0,4) vector and the (0,4) twisted hypermultiplet combine to form a (4,4) vector multiplet. They are represented by circles.

- Between each pair of horizontal nodes there are two (0,2) Fermi multiplets, forming a (0,4) Fermi multiplet, and two (0,2) chiral multiplets, forming a (0,4) hypermultiplet, each in the bifundamental representation of the gauge groups. The (0,4) Fermi multiplet and the (0,4) hypermultiplet combine into a (4,4) hypermultiplet. They are represented by black solid lines.

- Between each pair of vertical nodes there are two (0,2) chiral multiplets forming a (0,4) hypermultiplet, in the bifundamental representation of the gauge groups. They are represented by grey solid lines.

- Between each gauge node and any successive or preceding node there is one (0,2) Fermi multiplet in the bifundamental representation. They are represented by dashed lines.

- Between each gauge node and a global symmetry node there is one (0,2) Fermi multiplet in the fundamental representation of the gauge group. They are again represented by dashed lines.

Note that the resulting quiver, depicted in figure 3, can be divided into two, horizontal, (4,4) linear quivers consisting on (4,4) gauge groups with increasing ranks connected by (4,4) bifundamental hypermultiplets. They correspond to the two (4,4) D6-NS5-D8 and D2-NS5-D4 *subsectors* of the brane configuration. The coupling between these two linear quivers through (0,4) hypermultiplets and (0,2) Fermi multiplets renders however the complete quiver (0,4) supersymmetric (see [35] for more details).

The previous fields contribute to the gauge anomaly of a generic SU($N_i$) gauge group as:

- A (0,2) vector multiplet contributes with a factor of $-N_i$.

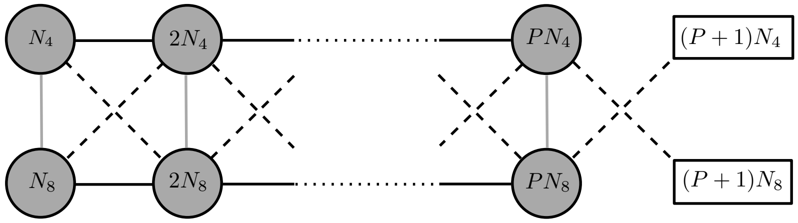


**Figure 4**. Completed quiver with a finite number of gauge groups.

- A (0,2) chiral multiplet in the adjoint representation contributes with a factor of $N_i$.
- A (0,2) chiral multiplet in the bifundamental representation contributes with a factor of $\frac{1}{2}$.
- A (0,2) Fermi multiplet in the adjoint representation contributes with a factor of $-N_i$.
- A (0,2) Fermi multiplet in the fundamental or bifundamental representation contributes with a factor of $-\frac{1}{2}$.

Following these rules it is easy to see that the coefficient of the anomalous correlator of the symmetry currents $< J^A_\mu(x) J^B_\nu(x) >\sim k\delta_{\mu\nu}\delta^{AB}$ vanishes for each gauge group (see [35] for more details) - hence the gauge anomalies vanish. By assigning R-charges to the different multiplets (see [35] for the precise assignation), we can calculate the $\mathrm{U}(1)_R$ anomaly (for $\mathrm{U}(1)_R$ inside $\mathrm{SU}(2)_R$). The correlation function $< j_\mu(x) j_\nu(y) >$ for two $\mathrm{U}(1)_R$ currents is proportional to the number of $\mathcal{N} = (0,4)$ hypermultiplets minus the number of $\mathcal{N} = (0,4)$ vector multiplets. This result is conserved when flowing to lower energies. In the far IR, when the theory is proposed to become conformal the R-symmetry anomaly is related to the central charge as indicated below.

### 5.2.1 Central charge

Let us now discuss the central charge associated to this quiver. We compute it using the formula (see [35, 43])

$$c = 6(n_{\text{hyp}} - n_{\text{vec}}), \tag{5.26}$$

where $n_{\text{hyp}}$ counts the number of fundamental and bifundamental hypermultiplets and $n_{\text{vec}}$ of vector multiplets. Clearly, these numbers are infinite for our quiver in figure 3. However, since they are subtracted in the computation of the central charge, they could still render a finite value. Terminating the space at a given $n = P$ and analysing the behaviour when $P$ goes to infinity we show however that this is not the case. Anomaly cancellation enforces that flavour groups must be added to both gauge groups at the end of the quiver. The resulting quiver is the one shown in figure 4. This quiver was discussed in [35], as one of the anomaly free examples analysed therein. For completeness we reproduce here the computation of its central charge.

The hypermultiplets that contribute to the counting of $n_{\text{hyp}}$ are the two chiral multiplets in each solid horizontal line, plus the two chiral multiplets in each vertical line.

They give

$$n_{\rm hyp}=\sum_{j=1}^{P-1} j(j+1)(N_4^2+N_8^2)+\sum_{j=1}^{P} j^2 N_4 N_8 = (N_4^2+N_8^2)\left(\frac{P^3}{3}-\frac{P}{3}\right)+N_4N_8\left(\frac{P^3}{3}+\frac{P^2}{2}+\frac{P}{6}\right) \tag{5.27}$$

Vector multiplets come from each node in the quiver, such that:

$$n_{\rm vec} = \sum_{j=1}^{P}(j^2N_4^2-1+j^2N_8^2-1) = (N_4^2+N_8^2)\left(\frac{P^3}{3}+\frac{P^2}{2}+\frac{P}{6}\right)-2P \tag{5.28}$$

This gives for the central charge

$$c = 6\left[-(N_4^2+N_8^2)\left(\frac{P^2}{2}+\frac{P}{2}\right)+N_4N_8\left(\frac{P^3}{3}+\frac{P^2}{2}+\frac{P}{6}\right)+2P\right]. \tag{5.29}$$

To leading order in $P$ we have,

$$c \sim 2N_4N_8P^3. \tag{5.30}$$

The central charge thus diverges with $P^3$ for the infinite quiver dual to the NATD solution. Still, it is useful to show that (5.30) coincides with the holographic central charge for $\rho\in[0,\rho_P]$, with $\rho_P$ satisfying (5.18). Note that for large $P$ we can simply take $\rho_P = 2\pi P$. Using (3.16) we find for $\rho\in[0,2\pi P]$,

$$c_{\rm hol} = \frac{\pi}{2G_N}(2\pi)^5 N_4N_8P^3 = 2N_4N_8P^3, \tag{5.31}$$

in agreement with the field theory result.

Our calculation shows the precise way in which the central charge diverges due to the non-compact field theory direction. It also gives us a possible way to regularise the infinite CFT dual to the NATD solution. Indeed, the quiver depicted in figure 4 describes a well-defined 2d (0,4) CFT, that can be used to find a global completion of the non-Abelian T-dual solution. This completion is obtained glueing the non-Abelian T-dual solution at $\rho_P = 2\pi P$ to another solution in [11] that terminates the space at $\rho = 2\pi(P+1)$. We present the details of this completion in the next subsection. In section 5.3.2 we present a different completion, which makes manifest that this procedure is not unique and that one can device different global completions of the NATD solution, as stressed in [38].

### 5.3 Completions

In this section we present two possible completions of the NATD solution. The AdS$_3$ example is particularly useful in this respect, because the completed solution is not only explicit but also extremely simple, as opposed to other examples in higher dimensions [38, 39, 41].

#### 5.3.1 Completion with O-planes

The simplest way to complete the NATD solution is by terminating the infinite linear quiver at a certain value of $\rho$, as we have done in the previous subsection. We take this to



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be $\rho = 2\pi(P+1)$, with $P \in \mathbb{Z}$, and choose the $u$, $h_8$ and $h_4$ functions such that:

$$u = 4L^4M^2\rho, \qquad 0 \le \rho \le 2\pi(P+1) \tag{5.32}$$

$$h_8(\rho) = F_0.\begin{cases} \rho & 0 \le \rho \le 2\pi P \\ P(2\pi(P+1)-\rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{5.33}$$

$$h_4(\rho) = L^2M^4.\begin{cases} \rho & 0 \le \rho \le 2\pi P \\ P(2\pi(P+1)-\rho) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{5.34}$$

The explicit form of the metric, dilaton and fluxes in the $2\pi P \le \rho \le 2\pi(P+1)$ region can be found in appendix A. One can check that the NS sector is continuous at $\rho = 2\pi P$. The 2-form and 6-form Page fluxes are also continuous once large gauge transformations are taken into account. They are given by

$$\hat{f}_2 = -F_0\,\mathrm{vol}(\mathrm{S}^2).\begin{cases} n\pi & 0 \le n \le P \\ P\pi(P+1-n) & P \le n \le P+1 \end{cases} \tag{5.35}$$

$$\hat{f}_6 = L^2M^4\,\mathrm{vol}(\mathrm{S}^2)\wedge\mathrm{vol}(\mathrm{CY}_2).\begin{cases} n\pi & 0 \le n \le P \\ P\pi(P+1-n) & P \le n \le P+1, \end{cases} \tag{5.36}$$

so they vanish at $n = P+1$, where the geometry terminates. We show below that at this point the background has a singularity associated to O6-O2 planes. In turn there is a discontinuity in $F_0$ and $F_4$ at $n = P$ that is translated into $(P+1)N_8$ and $(P+1)N_4$ additional flavours connected to the nodes corresponding to $PN_4$ D2 and $PN_8$ D6 branes, respectively. This is exactly as in the quiver depicted in figure 4.

The expressions of the metric and dilaton in the $2\pi P \le \rho \le 2\pi(P+1)$ region, given by equations (A.1), (A.2) in appendix A, show that close to $\rho = 2\pi(P+1)$ they behave as

$$ds^2 \sim x^{-1}ds^2(\mathrm{AdS}_3) + M^2ds^2(\mathrm{CY}_2) + x\big(dx^2 + ds^2(\mathrm{S}^2)\big), \qquad e^{2\phi} \sim x^{-1} \tag{5.37}$$

where $x = \rho - 2(P+1)$. This singular behaviour corresponds to the intersection of an O6 fixed plane lying on $\mathrm{AdS}_3\times\mathrm{CY}_2$ with O2-planes lying on $\mathrm{AdS}_3$ and smeared on $\mathrm{CY}_2\times\mathrm{S}^2$. Even if it is not clear what this object is in string theory, the fact that the solution has a well-defined dual CFT suggests that it should be possible to give it a meaning.

#### 5.3.2 Glueing the NATD to itself

Another interesting way of defining globally the NATD solution is by glueing it to itself. In this case we take:

$$u(\rho) = 4L^4M^2, \qquad 0 \le \rho \le 4\pi P. \tag{5.38}$$

$$h_8(\rho) = F_0.\begin{cases} \rho & 0 \le \rho \le 2\pi P \\ 4\pi P - \rho & 2\pi P \le \rho \le 4\pi P. \end{cases} \tag{5.39}$$

$$h_4(\rho) = L^2M^4.\begin{cases} \rho & 0 \le \rho \le 2\pi P \\ 4\pi P - \rho & 2\pi P \le \rho \le 4\pi P. \end{cases} \tag{5.40}$$

The explicit form of the metric, dilaton and fluxes in the $2\pi P \leq \rho \leq 4\pi P$ region can be found in appendix A. One can check that the NS sector is continuous at $\rho = 2\pi P$. The 2-form and 6-form Page fluxes are also continuous once large gauge transformations are taken into account. They are given by

$$\hat{f}_2 = -F_0 \,\mathrm{vol}(\mathrm{S}^2). \begin{cases} n\pi & 0 \leq n \leq P \\ (2P-n)\pi & P \leq n \leq 2P \end{cases} \tag{5.41}$$

and

$$\hat{f}_6 = L^2 M^4 \,\mathrm{vol}(\mathrm{S}^2) \wedge \mathrm{vol}(\mathrm{CY}_2). \begin{cases} n\pi & 0 \leq n \leq P \\ (2P-n)\pi & P \leq n \leq 2P \end{cases} \tag{5.42}$$

Therefore, they are both continuous at $n = P$ and vanish at $n = 2P$. The corresponding quantised charges are:

$$N_6 = \begin{cases} nN_8 & 0 \leq n \leq P \\ (2P-n)N_8 & P \leq n \leq 2P \end{cases} \tag{5.43}$$

and

$$N_2 = \begin{cases} nN_4 & 0 \leq n \leq P \\ (2P-n)N_4 & P \leq n \leq 2P \end{cases} \tag{5.44}$$

where $N_6$ denotes anti-D6 brane charge, $N_2$ D2-brane charge and $N_8 = \pm 2\pi F_0$ in the two regions. For $N_4$ we have

$$N_4 = \frac{1}{(2\pi)^3} \int \hat{f}_4 = \frac{1}{(2\pi)^3} \int F_4 = \mp \frac{L^2 M^4}{(2\pi)^3} \mathrm{Vol}(\mathrm{CY}_2) \tag{5.45}$$

in the two regions. Thus, the D2 and D6 brane charges increase linearly in the $0 \leq n \leq P$ region, corresponding to the NATD solution, and decrease linearly in the $P \leq n \leq 2P$ region, till they vanish at $n = 2P$, where the geometry terminates. At this point the $\mathrm{S}^2$ shrinks smoothly. The discontinuity of $N_8$ and $N_4$ at $n = P$ is translated into $2N_8$ and $2N_4$ additional flavours at the nodes with flavour groups $PN_4$ and $PN_8$, respectively. The associated quiver is the one depicted in figure 5. The $2N_8$ and $2N_4$ flavour groups contribute each with one (0,2) Fermi multiplet in the fundamental representation of the corresponding gauge group. As for the quivers constructed in [35], the flavour group introduced at the node associated to D2-branes arises from D8-branes while that introduced at the node associated to D6-branes arises from D4-branes.

The central charge of this quiver is given by

$$c = 6\left[(N_4^2 + N_8^2)(-P) + N_4 N_8 \left(\frac{2}{3}P^3 + \frac{P}{3}\right) + 4P - 2\right]. \tag{5.46}$$

To leading order in $P$ this gives

$$c = 4N_4 N_8 P^3, \tag{5.47}$$

which one can check is in agreement with the holographic central charge.

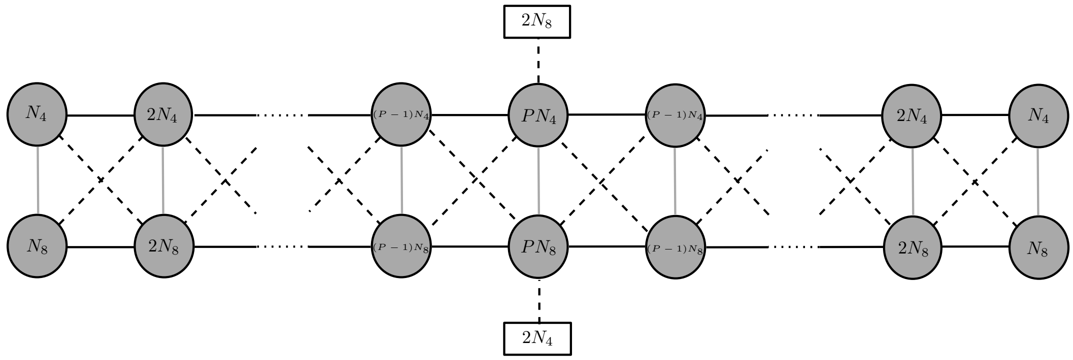


**Figure 5**. Symmetric quiver associated to the NATD solution glued to itself.

### 5.4 The Abelian T-dual limit

The non-Abelian T-dual solution defined in $\rho \in [\rho_n, \rho_{n+1}]$ gives rise to the Abelian T-dual, along the Hopf-fibre of the $\mathrm{S}^3$, of the original $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ background, in the limit in which $n$ goes to infinity [38, 39, 72]. In this subsection we will be interested in the ATD solution, and orbifolds thereof, in its own right, as another explicit example in the class in [11].

The ATD solution is given by

$$ds^2_{10} = 4L^2 ds^2(\mathrm{AdS}_3) + M^2 ds^2(\mathrm{CY}_2) + \frac{d\psi^2}{4L^2} + L^2 ds^2(\mathrm{S}^2) \tag{5.48}$$

$$e^{2\Phi} = \frac{4}{L^2} \tag{5.49}$$

$$B_2 = -\frac{\psi}{2}\mathrm{vol}(\mathrm{S}^2) \tag{5.50}$$

$$F_2 = -\frac{L^2}{2}\mathrm{vol}(\mathrm{S}^2) \tag{5.51}$$

$$F_6 = \frac{1}{2}M^4 L^2 \mathrm{vol}(\mathrm{CY}_2)\wedge\mathrm{vol}(\mathrm{S}^2), \tag{5.52}$$

where $\psi$ is the ATD of the Hopf-fibre direction, normalised such that $\psi \in [0, 2\pi]$. Upon dualisation, the (4,4) supersymmetries of the original solution are reduced to (0,4) [30], and the solution fits in the classification in [11]. The corresponding $u$, $h_4$ and $h_8$ functions are given by

$$u = 4L^4 M^2 \tag{5.53}$$

$$h_4 = L^2 M^4 \tag{5.54}$$

$$h_8 = L^2. \tag{5.55}$$

The quantised charges are,

$$N_2 = \frac{L^2 M^4}{(2\pi)^4}\mathrm{Vol}(\mathrm{CY}_2)\,, \qquad N_6 = L^2\,, \qquad N_5 = 1 \tag{5.56}$$

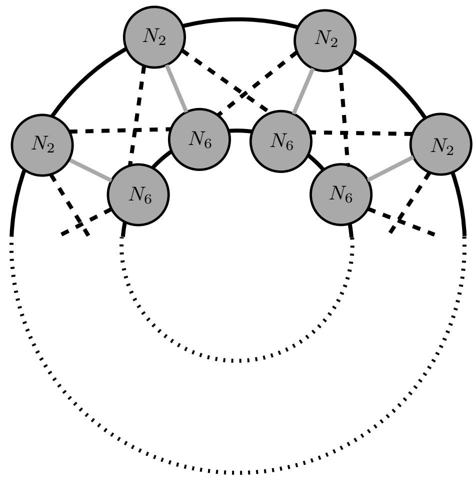


**Figure 6**. Circular quiver associated to the (orbifolded) ATD solution.

so using (3.16) the holographic central charge gives

$$c_{\rm hol} = 6N_2N_6. \tag{5.57}$$

One can check that this is reproduced from the NATD solution for $\rho \in [\rho_n, \rho_{n+1}]$ and $n$ large, using that $N_2 = nN_4$ and $N_6 = nN_8$ in this interval. The brane set-up describing the ATD solution consists on $N_2$ D2-branes and $N_6$ D6-branes, wrapped on the $\mathrm{CY}_2$, stretched along the $\psi$ circular direction between two NS5-branes that are identified.

Orbifolds of this solution can be constructed taking $\psi \in [0, 2\pi N]$. They are T-dual to the $\mathrm{AdS}_3\times \mathrm{S}^3/\mathbb{Z}_N\times \mathrm{CY}_2$ solution in Type IIB that describes the D1-D5-KK system [18–21, 25]. The Type IIA brane realisation of this system is depicted in figure 6. From this quiver we have that

$$n_{\rm vec} = (N_2^2 - 1 + N_6^2 - 1)N, \qquad n_{\rm hyp} = (N_2^2 + N_6^2 + N_2N_6)N. \tag{5.58}$$

One then obtains a central charge

$$c = 6(n_{\rm hyp} - n_{\rm vec}) = 6N_2N_6N + 12N. \tag{5.59}$$

This gives in the large $N_2, N_6$ limit,

$$c \sim 6N_2N_6N, \tag{5.60}$$

in agreement with the central charge of the D1-D5-KK system [18].[13]

For $N = 1$ the quiver in figure 6 reduces to the quiver depicted in figure 7. The (4,4) hypermultiplets connecting $N_2$ nodes and $N_6$ nodes among themselves become (4,4) hypermultiplets in the adjoint representation. In turn, the (0,2) Fermi multiplets connecting each

[13]This central charge was computed using the Brown-Henneaux formula [74]. One can also use (3.16), which generalises the central charge therein to non-trivial warping and dilaton.

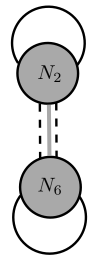


**Figure 7**. Quiver associated to the ATD solution.

$N_2$ ($N_6$) node with adjacent $N_6$ ($N_2$) nodes combine into (0,4) Fermi multiplets connecting each $N_2$ node with its respective $N_6$ node, which together with the (0,4) hypermultiplets between them give (4,4) hypermultiplets in the bifundamental. In this way supersymmetry is enhanced to (4,4), and the quiver describes the D1-D5 system in terms of the D2 and D6-brane charges of the Abelian T-dual solution.[14]

## 6 Relation with the $\mathbf{AdS_3 \times S^2}$ flows of Dibitetto-Petri

In [6, 7] Dibitetto and Petri (DP) constructed various BPS flows within minimal $\mathcal{N} = 1$ 7d supergravity that are asymptotically locally $\mathrm{AdS}_7$. These flows are described by warped $\mathrm{AdS}_3$ solutions triggered by a non-trivial dyonic 3-form potential. A particularly interesting solution was constructed in [6], which was shown to interpolate between asymptotically locally $\mathrm{AdS}_7$ and $\mathrm{AdS}_3\times\mathrm{T}^4$ geometries. The UV $\mathrm{AdS}_7$ limit is (asymptotically locally) the reduction to 7d of the $\mathrm{AdS}_7$ solutions of massive IIA constructed in [36]. In this subsection we would like to explore the 10d uplift of the IR $\mathrm{AdS}_3\times\mathrm{T}^4$ limit, in connection with the subclass of solutions discussed in section 2, in the case in which $\mathrm{CY}_2 = \mathrm{T}^4$.

The $\mathrm{AdS}_3$ solution constructed in [6] reads (see appendix B for the details),

$$\begin{aligned} ds_7^2 &= e^{2\mathrm{U}(r)} ds(\mathrm{AdS}_3)^2 + e^{2V(r)} dr^2 + e^{2W(r)} ds(\mathrm{S}^3)^2, \\ X &= X(r), \\ B_{(3)} &= k(r)\mathrm{vol}(\mathrm{AdS}_3) + l(r)\mathrm{vol}(\mathrm{S}^3), \end{aligned} \tag{6.1}$$

where $X$, $U$, $V$, $W$, $k$ and $l$ are functions of $r$ discussed in the appendix B. This solution is asymptotically locally $\mathrm{AdS}_7$ when $r \to \infty$ , while when $r \to 0$ it flows to an $\mathrm{AdS}_3\times\mathrm{T}^4$ non-singular limit, given by,[15]

$$ds_7^2 = \frac{2^{31/5}}{g^2}\left(\frac{3^{2/5}}{5^2} ds^2(\mathrm{AdS}_3) + \frac{4}{3^{8/5}} ds^2(\mathrm{T}^4)\right), \tag{6.2}$$

and

$$B_3 = -\frac{1}{2}\mathrm{vol}(\mathrm{AdS}_3) - 4r^4\mathrm{vol}(\mathrm{S}^3). \tag{6.3}$$

[14]See [75], section 4, for this analysis in Type IIB.
[15]As compared to [6], we write the 7d metric in terms of an $\mathrm{AdS}_3$ space of radius one.

As the $\mathrm{AdS}_7$ asymptotic limit, this geometry is not a solution of 7d $\mathcal{N}=1$ minimal supergravity by itself, but rather the IR leading asymptotics of the flow. In the discussion that follows it will be useful to recall from appendix B that the values of the 7d scalar $X$ in the $r\to\infty$ and $r\to 0$ limits are $X=1$ and $X^5=2^2/3$, respectively.

7d $\mathcal{N}=1$ minimal supergravity can be consistently uplifted to massive IIA on a squashed $\mathrm{S}^3$ [64]. Using the uplift formulae provided in appendix B, a family of $\mathrm{AdS}_3$ solutions to massive IIA can thus be constructed from the DP flow. This gives rise in the $r\to\infty$ limit to 10d geometries that asymptote to the $\mathrm{AdS}_7\times\mathrm{S}^2\times\mathrm{I}$ family of solutions in [36]. In turn, the geometry that is obtained in the $\mathrm{AdS}_3\times\mathrm{T}^4$ limit reads[16]

$$\begin{aligned} ds^2_{10} &= 8\sqrt{2}\pi\sqrt{-\frac{\alpha}{\ddot{\alpha}}}\left(\frac{2^3\sqrt{3}}{5^2}ds^2(\mathrm{AdS}_3)+\frac{2^5}{3\sqrt{3}}ds^2(\mathrm{T}^4)\right)\\ &\quad+\frac{2\sqrt{2}}{\sqrt{3}}\pi\sqrt{-\frac{\ddot{\alpha}}{\alpha}}dz^2+2\sqrt{6}\pi\frac{\alpha^{3/2}(-\ddot{\alpha})^{1/2}}{3\dot{\alpha}^2-8\alpha\ddot{\alpha}}ds^2(\mathrm{S}^2) \end{aligned} \tag{6.4}$$

$$e^{2\Phi} = 2^3 3^8\sqrt{6}\pi^5\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{3/2}\frac{1}{3\dot{\alpha}^2-8\alpha\ddot{\alpha}} \tag{6.5}$$

$$B_2 = \pi\left(-z+\frac{3\alpha\dot{\alpha}}{3\dot{\alpha}^2-8\alpha\ddot{\alpha}}\right)\mathrm{vol}(\mathrm{S}^2) \tag{6.6}$$

$$F_2 = \left(\frac{\ddot{\alpha}}{162\pi^2}+\frac{3\pi F_0\alpha\dot{\alpha}}{3\dot{\alpha}^2-8\alpha\ddot{\alpha}}\right)\mathrm{vol}(\mathrm{S}^2) \tag{6.7}$$

$$F_4 = \frac{2^9}{3^4\pi}\left(\frac{\ddot{\alpha}}{5^3}\mathrm{d}z\wedge\mathrm{vol}(\mathrm{AdS}_3)-\frac{2^5}{3^3}\dot{\alpha}\mathrm{vol}(\mathrm{T}^4)\right) \tag{6.8}$$

$$F_6 = -\frac{2^9}{5^3 3^7}\frac{\alpha\ddot{\alpha}}{3\dot{\alpha}^2-8\alpha\ddot{\alpha}}\left(2^8 5^3\alpha\mathrm{vol}(\mathrm{T}^4)+3^4\dot{\alpha}\mathrm{vol}(\mathrm{AdS}_3)\wedge\mathrm{d}z\right)\wedge\mathrm{vol}(\mathrm{S}^2). \tag{6.9}$$

As in 7d, the uplift of the $r\to 0$ limit of the DP flow is not a solution to massive IIA by itself, but rather its IR leading asymptotics. We would like to see whether it can be completed by an $\mathrm{AdS}_3\times\mathrm{T}^4$ solution in the class of [11], with the same asymptotics. For that it is easy to realise that one can absorb the constant $X$ that causes the distortion of the internal space (we are referring to (B.25)–(B.29) in appendix B) by simply modifying the mapping for the $h_4$ function in (3.9) as $h_4=\frac{81}{8}X^5u\leftrightarrow\frac{81}{8}X^5\alpha$. We then have for the IR geometry given by (6.4)–(6.9),

$$\rho \leftrightarrow 2\pi z \tag{6.10}$$

$$u \leftrightarrow \alpha \tag{6.11}$$

$$h_8 \leftrightarrow -\frac{\ddot{\alpha}}{81\pi^2} \tag{6.12}$$

$$h_4 = \frac{27}{2}u\leftrightarrow\frac{27}{2}\alpha\,. \tag{6.13}$$

This gives for the $\mathrm{AdS}_3\times\mathrm{T}^4$ subspace

$$\frac{u}{\sqrt{h_4h_8}}ds^2(\mathrm{AdS}_3)+\sqrt{\frac{h_4}{h_8}}ds^2(\mathrm{T}^4)\leftrightarrow\sqrt{6}\pi\sqrt{-\frac{\alpha}{\ddot{\alpha}}}\left(ds^2(\mathrm{AdS}_3)+\frac{3^3}{2}ds^2(\mathrm{T}^4)\right). \tag{6.14}$$

[16] Here we have taken $g^3=8\sqrt{2}$, which is the value for which the internal space and fluxes of the $\mathrm{AdS}_7$ solutions in [36] are recovered.

The result is a bonna fide $\mathrm{AdS}_3\times\mathrm{T}^4$ solution to massive IIA, supplemented with $F_4$ and $F_6$ fluxes satisfying (2.7) and (2.8). The resulting 7d metric does not share however the asymptotics of the 7d metric arising from (6.4). Thus, the IR limit of the DP flow cannot be completed into an $\mathrm{AdS}_3\times\mathrm{T}^4$ solution in the class of [11], that shares its same asymptotics. This result excludes the RG flows constructed in [6] as solutions interpolating between $\mathrm{AdS}_3\times\mathrm{T}^4$ geometries (in the subclass defined in section 3) and the $\mathrm{AdS}_7$ solutions constructed in [36]. Still, it should be possible to construct these flows, perhaps as $\mathbb{R}_{1,1}\times\mathrm{CY}_2$ warped product geometries, as the ones discussed in [64].

## 7 Conclusions

In this paper we have discussed some aspects of the class of $\mathrm{AdS}_3\times\mathrm{S}^2$ solutions with small $\mathcal{N}=(0,4)$ supersymmetry and SU(2)-structure constructed in [11]. We have focused our analysis on a sub-set of solutions contained in "class I" of [11], which are warped products of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ over an interval with warpings respecting the symmetries of $\mathrm{CY}_2$. 2d (0,4) CFTs dual to these solutions have been proposed recently in [34, 35].

We have established a map between the previous solutions and the $\mathrm{AdS}_7$ solutions in [36], that allows one to interpret the former as duals to defects in 6d (1,0) CFTs. More precisely, the 2d dual CFT arises from wrapping on the $\mathrm{CY}_2$ the D6-NS5-D8 branes that underlie the $\mathrm{AdS}_7$ solutions, and intersecting them with D2 and D4 branes. In this sense it combines *wrapped branes* and *defect branes*. The D2-branes are stretched between the NS5-branes, as the D6-branes, and the D4-branes are perpendicular, as the D8-branes. They give rise to (0,4) quivers with two families of gauge groups connected by matter fields [35]. Each family is described by a (4,4) linear quiver and is connected with the other family by (0,4) and (0,2) multiplets, rendering the final quiver (0,4) supersymmetric.

The previous mapping suggests that it should be possible to construct flows connecting the $\mathrm{AdS}_3\times\mathrm{CY}_2$ solutions in the IR with the $\mathrm{AdS}_7$ solutions in the UV. The presence of D2-D4 defects suggests that one should look at warped $\mathrm{AdS}_3$ flows, as the ones discussed in [6], which interpolate between asymptotically locally $\mathrm{AdS}_3\times\mathrm{T}^4$ geometries, with an interpretation as 2d defect CFTs, and $\mathrm{AdS}_7$ solutions. We have found however that our solutions have different asymptotics than the IR $\mathrm{AdS}_3$ geometries considered in [6]. This discrepancy could originate on the wrapped branes present in our solutions, more suggestive of an $\mathbb{R}^{1,1}\times\mathrm{CY}_2$ flow [62], as the one constructed in [64]. It would be very interesting to find the explicit flow that interpolates between these two classes of solutions.

We have provided a thorough analysis of the $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ solution that arises from the Type IIB solution dual to the D1-D5 system through non-Abelian T-duality. Using the map between $\mathrm{AdS}_3$ and $\mathrm{AdS}_7$ solutions derived in the first part of the paper, we have *rediscovered* this solution as the leading order of the $\mathrm{AdS}_7$ solution in the class in [36] dual to a 6d linear quiver with gauge groups of increasing ranks, terminated by D6-branes. Secondly, we have provided two explicit global completions with $\mathrm{AdS}_3$ solutions in the class in [11]. One of these completions is obtained glueing the non-Abelian T-dual solution to itself, in a sort of orbifold projection around the point where the space terminates. This solution has a well-defined 2d dual CFT that we have studied. Orbifolds have previously played a role in the completion of NATD solutions, remarkably in the example discussed in [41], but this is the first time the explicit completed geometry has been constructed. The $\mathrm{AdS}_3$ example provides indeed a very useful set-up where to test the role played by holography in extracting global information of NATD in string theory, following the ideas in [38–42].


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## A Completions of the NATD solution

**Completion with O-planes.** The metric, dilaton and fluxes of the NATD solution completed as indicated in section 5.3.1 read, in the $2\pi P \leq \rho \leq 2\pi(P+1)$ region,

$$
\begin{aligned}
ds^2 &= \frac{4L^2\rho}{P(2\pi(P+1)-\rho)} ds^2(\mathrm{AdS}_3) + M^2 ds^2(\mathrm{CY}_2) + \frac{P(2\pi(P+1)-\rho)}{4L^2\rho} d\rho^2 \\
&\quad + \frac{L^2 P\rho\,(2\pi(P+1)-\rho)}{4L^4 + P^2(2\pi(P+1)-\rho)^2} ds^2(\mathrm{S}^2) \qquad \text{(A.1)} \\
e^{2\Phi} &= \frac{4\rho}{L^2 P(2\pi(P+1)-\rho)\Big(4L^4 + P^2(2\pi(P+1)-\rho)^2\Big)} \qquad \text{(A.2)} \\
B_2 &= -\frac{\rho P^2(2\pi(P+1)-\rho)^2}{2\Big(4L^4 + P^2(2\pi(P+1)-\rho)^2\Big)} \mathrm{vol}(\mathrm{S}^2) \qquad \text{(A.3)} \\
F_0 &= -PL^2 \qquad \text{(A.4)} \\
F_2 &= -\frac{L^2\Big(P^3\,(2\pi\,(P+1)-\rho)^3 + 8L^4\pi P\,(P+1)\Big)}{2\,\Big(4L^4 + P^2\,(2\pi\,(P+1)-\rho)^2\Big)} \mathrm{vol}(\mathrm{S}^2) \qquad \text{(A.5)} \\
F_4 &= L^2 M^4 P\,\mathrm{vol}(\mathrm{CY}_2) \qquad \text{(A.6)}
\end{aligned}
$$

**NATD solution glued to itself.** The metric, dilaton and fluxes of the NATD solution glued to itself read, in the $2\pi P \leq \rho \leq 4\pi P$ region,

$$ds^2 = \frac{4L^2\rho}{4\pi P-\rho}ds^2(\mathrm{AdS}_3) + M^2 ds^2(\mathrm{CY}_2) + \frac{4\pi P-\rho}{4L^2\rho}d\rho^2 + \frac{L^2\rho(4\pi P-\rho)}{4L^4+(4\pi P-\rho)^2}ds^2(\mathrm{S}^2) \tag{A.7}$$

$$e^{2\Phi} = \frac{4\rho}{L^2(4\pi P-\rho)\Big(4L^4+(4\pi P-\rho)^2\Big)} \tag{A.8}$$

$$B_2 = -\frac{\rho(4\pi P-\rho)^2}{2\Big(4L^4+(4\pi P-\rho)^2\Big)}\mathrm{vol}(\mathrm{S}^2) \tag{A.9}$$

$$F_0 = -L^2 \tag{A.10}$$

$$F_2 = -\frac{L^2\Big((4\pi P-\rho)^3+16\pi PL^4\Big)}{2\Big(4L^4+(4\pi P-\rho)^2\Big)}\mathrm{vol}(\mathrm{S}^2) \tag{A.11}$$

$$F_4 = L^2M^4\,\mathrm{vol}(\mathrm{CY}_2) \tag{A.12}$$

# B The Dibitetto-Petri flow in minimal $\mathcal{N}=1$ 7d supergravity

The solution discussed in section 6 was obtained in [6] taking the following ansatz:

$$\begin{aligned} ds_7^2 &= e^{2\mathrm{U}(r)}ds^2(\mathrm{AdS}_3) + e^{2V(r)}dr^2 + e^{2W(r)}ds^2(\mathrm{S}^3), \\ X &= X(r), \\ B_{(3)} &= k(r)\mathrm{vol}(\mathrm{AdS}_3) + l(r)\mathrm{vol}(\mathrm{S}^3), \end{aligned} \tag{B.1}$$

and vanishing vector fields. Here $ds^2(\mathrm{S}^3)$ is the metric of an $\mathrm{S}^3$ with radius $\frac{2}{\kappa}$, parameterised as:

$$\begin{aligned} e^1 &= \frac{1}{\kappa}d\theta_2, \\ e^2 &= \frac{1}{\kappa}\cos\theta_2 d\theta_3, \\ e^3 &= \frac{1}{\kappa}(d\theta_1 + \sin\theta_2 d\theta_3), \end{aligned} \tag{B.2}$$

and $ds^2(\mathrm{AdS}_3)$ is the metric of an $\mathrm{AdS}_3$ with radius $\frac{2}{L}$, parameterised as:

$$\begin{aligned} e^1 &= \frac{1}{L}(dt - \sinh x_1 dx^2), \\ e^2 &= \frac{1}{L}dx^1, \\ e^3 &= \frac{1}{L}\cosh x_1 dx^2. \end{aligned} \tag{B.3}$$

vol($S^3$) and vol($AdS_3$) represent their corresponding volume forms. DP showed that (B.1) is a solution to minimal 7d sugra with $X$, $U$, $V$, $W$, $k$ and $l$ given by,

$$X(r) = \frac{2^{2/5}h^{1/5}\left(-1+\rho^8\right)^{2/5}}{\left(-8L\rho^4\left(1+\rho^8\right)+\sqrt{2}g\left(1+4\rho^4+4\rho^{12}+\rho^{16}\right)\right)^{1/5}}, \tag{B.4}$$

$$e^{2\mathrm{U}(r)} = \frac{(\rho^4+1)^2}{4\rho^4X^2}, \tag{B.5}$$

$$e^{2V(r)} = \frac{4X^8}{h^2}, \tag{B.6}$$

$$e^{2W(r)} = \frac{(\rho^4-1)^2}{4\rho^4X^2}, \tag{B.7}$$

$$\begin{aligned} l(r) &= \frac{1}{16h\rho^4\left(\rho^4+1\right)^2}[\sqrt{2}g\left(-1+4\rho^4+4\rho^8+4\rho^{12}-\rho^{16}\right) \\ &\quad +2L\left(1-4\rho^4-2\rho^8-4\rho^{12}+\rho^{16}\right)], \end{aligned} \tag{B.8}$$

$$\begin{aligned} k(r) &= \frac{1}{16h\rho^4\left(\rho^4-1\right)^2}\Big[\sqrt{2}g\left(1+4\rho^4+4\rho^{12}+\rho^{16}\right) \\ &\quad -2L\left(1+4\rho^4-2\rho^8+4\rho^{12}+\rho^{16}\right)\Big], \end{aligned} \tag{B.9}$$

where $r = \log\rho$ and $\kappa$ and $L$ satisfy,

$$\kappa + L = \sqrt{2}g. \tag{B.10}$$

In these expressions $g$ and $h$ are the gauge coupling of the vector fields[17] and the topological mass of the 3-form potential, respectively, of minimal $\mathcal{N}=1$ 7d supergravity.

### B.1 The $r\to\infty$, $AdS_7$ limit

When $r\to\infty$ the previous solution is asymptotically locally $AdS_7$, for any values of $\kappa$ and $L$ respecting the constraint given by their equation (4.27). The explicit way in which $AdS_7$ arises is as follows.

The $r\to\infty$ limit of the previous functions gives, for $g = 2\sqrt{2}h$,[18]

$$X \simeq 1,$$

$$e^{2U} \simeq \frac{\rho^4}{4} = \frac{e^{4r}}{4}, \tag{B.11}$$

$$e^{2V} \simeq \frac{4}{h^2}, \tag{B.12}$$

$$e^{2W} \simeq \frac{\rho^4}{4} = \frac{e^{4r}}{4}, \tag{B.13}$$

$$k \simeq -\frac{\rho^4}{16} = -\frac{e^{4r}}{16}, \tag{B.14}$$

$$l \simeq \frac{\rho^4}{16} = \frac{e^{4r}}{16}. \tag{B.15}$$

[17] This constant enters in the superpotential even for vanishing profile for the vector fields.
[18] This value is fixed such that $X=1$ asymptotically.

This gives for the 7d metric,

$$ds_7^2 = \frac{e^{4r}}{L^2} ds^2(\mathrm{AdS}_3) + \frac{4}{h^2} dr^2 + \frac{e^{4r}}{\kappa^2} ds^2(\mathrm{S}^3), \tag{B.16}$$

in terms of unit radius $\mathrm{S}^3$ and $\mathrm{AdS}_3$ spaces. In turn, the 3-form potential is given by,

$$B_3 = \frac{\sqrt{2}g - 2L}{16h} e^{4r} \Big( \mathrm{vol}(\mathrm{AdS}_3) - \mathrm{vol}(\mathrm{S}^3) \Big). \tag{B.17}$$

For arbitrary $L$ and $\kappa$, the scalar curvature is

$$R = -\frac{3}{2} e^{-4r} \Big( 28 e^{4r} h^2 + L^2 - \kappa^2 \Big), \tag{B.18}$$

and thus asymptotes to that of an $\mathrm{AdS}_7$ space of radius $1/h$. The geometry in the UV can thus be *completed* by an $\mathrm{AdS}_7$ space with vanishing 3-form potential, that solves the equations of motion and gives rise to an $\mathrm{AdS}_7$ solution to massive IIA supergravity upon uplift to ten dimensions [64].

### B.2 The $r \to 0$, $\mathbf{AdS}_3 \times \mathbf{T}^4$ limit

In turn, the $r \to 0$ limit of the expressions (B.4)–(B.9) is non-singular for the special value

$$L = \frac{5g}{4\sqrt{2}}, \tag{B.19}$$

which is also the value for which the leading order behaviour of the scalar potential $\nu(X)$,

$$\nu(r) = \frac{h^{2/5} (5\sqrt{2}g - 8L)^{8/5}}{2^{3/10} r^{16/5}} + \dots \tag{B.20}$$

is non-singular. Note that from (B.10),

$$\kappa = \frac{3g}{4\sqrt{2}}. \tag{B.21}$$

Substituting these values in (B.4)–(B.9) and taking the $r \to 0$ limit, one finds

$$\begin{aligned} X &\simeq \frac{2^{2/5}}{3^{1/5}}, \\ e^{2U} &\simeq \frac{3^{2/5}}{2^{4/5}}, \\ e^{2V} &\simeq \frac{2^8}{3g^2} \left( \frac{2}{3^3} \right)^{1/5}, \\ e^{2W} &\simeq 3^{2/5} 2^{6/5} r^2. \end{aligned} \tag{B.22}$$

This gives, for the metric in (B.1)

$$ds_7^2 = \frac{2^{31/5}}{g^2} \left( \frac{3^{2/5}}{5^2} ds^2(\mathrm{AdS}_3) + \frac{4}{3^{8/5}} ds^2(\mathrm{T}^4) \right), \tag{B.23}$$

and for the 3-form potential

$$B_3 = -\frac{1}{2} \mathrm{vol}(\mathrm{AdS}_3) - 4r^4 \mathrm{vol}(\mathrm{S}^3). \tag{B.24}$$

As discussed in [6], this geometry is not a solution of 7d $\mathcal{N} = 1$ minimal supergravity by itself, but rather the IR leading profile of the flow for $L$ and $\kappa$ given by (B.19), (B.21).

### B.3 Uplift to massive IIA

7d $\mathcal{N}=1$ minimal supergravity can be consistently uplifted to massive IIA on a squashed $S^3$ [64]. The uplift formulae were provided in that reference. They read, in the parameterisation used in [50] and for vanishing vector fields:

$$\begin{aligned} ds_{10}^2 &= \frac{16}{g}\pi\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{1/2} X^{-1/2} ds_7^2 \\ &\quad + \frac{16}{g^3}\pi X^{5/2}\left[\left(-\frac{\ddot{\alpha}}{\alpha}\right)^{1/2} dz^2 - \left(-\frac{\alpha}{\ddot{\alpha}}\right)^{1/2} \frac{\alpha\ddot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}X^5} ds^2(S^2)\right] \end{aligned} \tag{B.25}$$

$$e^{2\Phi} = \frac{X^{5/2}}{g^3}\frac{3^8 2^6 \pi^5}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}X^5}\left(-\frac{\alpha}{\ddot{\alpha}}\right)^{3/2} \tag{B.26}$$

$$B_2 = \frac{2^3\sqrt{2}}{g^3}\left(\frac{\pi\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}X^5} - \pi z\right)\mathrm{vol}(S^2) \tag{B.27}$$

$$F_2 = \left(\frac{2^3\sqrt{2}}{g^3}F_0\frac{\pi\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}X^5} + \frac{\ddot{\alpha}}{3^4 2\pi^2}\right)\mathrm{vol}(S^2) \tag{B.28}$$

$$F_4 = \frac{2^3}{3^4\pi}\left[-\ddot{\alpha}dz\wedge B_{(3)} - \dot{\alpha}\mathrm{d}B_{(3)}\right], \tag{B.29}$$

where in the last expression we have used the odd-dimensional self-duality condition [76]

$$X^4 *_7 \mathcal{F}_4 = -2hB_3. \tag{B.30}$$



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# 6. AdS$_3$/CFT$_2$ in M-theory

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# $M$-strings and AdS$_3$ solutions to M-theory with small $\mathcal{N} = (0, 4)$ supersymmetry

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Abstract: We construct a general class of (small) $\mathcal{N} = (0, 4)$ superconformal solutions in M-theory of the form AdS$_3 \times S^3/\mathbb{Z}_k \times$ CY$_2$, foliated over an interval. These solutions describe M-strings in M5-brane intersections. The $M$-strings support $(0, 4)$ quiver CFTs that are in correspondence with our backgrounds. We compute the central charge and show that it scales linearly with the total number of $M$-strings. We introduce momentum charge, thus allowing for a description in terms of M(atrix) theory. Upon reduction to Type IIA, we find a new class of solutions with four Poincaré supercharges of the form AdS$_2 \times S^3 \times$ CY$_2 \times \mathcal{I}$, that we extend to the massive IIA case. We generalise our constructions to provide a complete class of AdS$_3$ solutions to M-theory with (0,4) supersymmetry and SU(2) structure. We also construct new AdS$_2 \times S^3 \times$ M$_4 \times \mathcal{I}$ solutions to massive IIA, with M$_4$ a 4d Kähler manifold and four Poincaré supercharges.

Keywords: AdS-CFT Correspondence, M-Theory

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## Contents



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## 1 Introduction

Two-dimensional $\mathcal{N} = (0, 4)$ CFTs play a prominent role in the microscopic description of 5d black holes [1–6]. They are also central in the description of 6d (1,0) CFTs deformed away from the conformal point. In fact, when the M5-branes are separated in an extra transverse direction one gets a theory of interacting strings. These strings support a $(0, 4)$ supersymmetric quiver gauge theory, whose elliptic genus has been shown to capture the full supersymmetric partition function of the 6d theory [7, 8].

M2-branes suspended between parallel M5-branes lead to strings on their boundaries. We refer to them as $M$-strings [9]. For M5-branes probing $A$-type singularities, the case that will concern us in this paper, these strings are referred as $M_A$-strings [7]. They support 2d $(0, 4)$ quiver gauge theories with unitary gauge groups. Other general configurations of $M$-strings can be obtained for M5-branes probing D-type singularities, or "end of the space" M9-branes. These support quiver gauge theories involving symplectic and orthogonal gauge groups, and exceptional gauge groups, respectively [8]. More general configurations can be obtained beyond the realm of M-theory, using F-theory [10, 11]. In all cases, once the quiver gauge theory is specified the elliptic genus can be computed using localisation.

Explicit $\mathrm{AdS}_3$ holographic duals to 2d (0,4) quiver gauge theories were however quite rare in the literature, with known examples reducing to intersections of D1-D5 branes [12] with KK-monopoles [13–16] or D9-branes [17]. The recent results in [18] significantly contributed to fill this gap.[1]

The geometries constructed in [18] are $\mathrm{AdS}_3\times\, \mathrm{S}^2\times\, \mathrm{M}_4$ foliations over an interval, with $\mathrm{M}_4$ either a $\mathrm{CY}_2$ (class I) or a 4d Kähler (class II) manifold. They are solutions to massive Type IIA supergravity involving D2-D4-D6-NS5-D8 brane configurations. They preserve small (i.e. with only one SU(2) R-symmetry) $\mathcal{N} = (0,4)$ supersymmetries and posses an SU(2) structure. The dual CFTs of the first class were studied in [34, 35]. They arise in the infrared limit of (0,4) quiver gauge theories containing two families of unitary gauge groups, $\prod_{i=1}^{n} \mathrm{SU}(k_i)\times \mathrm{SU}(\tilde{k}_i)$. The gauge group $\mathrm{SU}(k_i)$ is associated to $k_i$ D2-branes while the gauge group $\mathrm{SU}(\tilde{k}_i)$ is associated to $\tilde{k}_i$ D6-branes, wrapped on the $\mathrm{CY}_2$. Both D2 and D6 branes are stretched between NS5-branes. On top of this, there are D4 and D8 branes that provide flavour groups to both types of gauge groups, and render the field theory anomaly-free.

The uplift of these solutions to M-theory provides explicit holographic duals to the 2d $(0,4)$ quiver gauge theories with unitary gauge groups supported by $M_A$-strings. We will see that they are $\mathrm{AdS}_3\times \mathrm{S}^3/\mathbb{Z}_k\times\, \mathrm{CY}_2$ foliations over an interval that still realise small $(0,4)$ superconformal symmetry. This will be one of the main results in this paper. Our class of solutions extends previous results in the literature, which took more restricted ansatze for the fluxes [36]. Furthermore, we are able to show that they are in one to one correspondence with 2d quiver CFTs describing $M_A$-strings. The CFTs arise as infrared fixed points of 2d field theories living on M2-branes and M-theory Kaluza-Klein monopoles (wrapped on the $\mathrm{CY}_2$, and thus behaving effectively as 2-branes) stretched between parallel M5-branes. This set-up realises two families of unitary gauge groups, supported by flavour groups coming from extra M5-branes that render the quivers non-anomalous. Our field theories generalise quivers constructed in the literature [8]. The key ingredient is that we are able to obtain them within controlled string theory set-ups with known holographic duals. They provide examples for 2d $(0,4)$ quiver gauge theories for which the elliptic genus can be computed.

The contents of the paper are distributed as follows. In section 2 we summarise the main properties of the backgrounds of the form $\mathrm{AdS}_3\times\, \mathrm{S}^2\times\, \mathrm{CY}_2$ foliated over an interval constructed in [18]. We focus our attention on compact Calabi-Yau 2-folds. We give a brief account of the 2d $(0,4)$ quiver CFTs that are dual to these solutions [34, 35]. In section 3 we perform the uplift of the sub-class of solutions with vanishing Romans' mass to eleven dimensions. We construct the explicit 2d quivers dual to these backgrounds and compute the central charge, both holographically and field theoretically, finding agreement in the holographic limit. Furthermore, we show that the central charge scales linearly with the total number of $M_A$-strings of the configuration. This identifies the latter with the defining degrees of freedom of our theories, and allows us to reinterpret with generality previous results obtained in more restricted scenarios [13]. In section 4 we construct

[1]See the papers [19–33] for more general $\mathrm{AdS}_3$ solutions with different amounts of supersymmetries.

new $\mathrm{AdS}_3/\mathbb{Z}_k\times$ $\mathrm{S}^3\times$ $\mathrm{CY}_2$ solutions in M-theory, foliated over an interval, preserving four Poincaré supersymmetries. We achieve this through a double analytical continuation. The new solutions are associated to M2-M5-M5' brane intersections with momentum charge, and provide a holographic description for the superconformal quantum mechanics (SCQM) that arises in the low energy limit. These SCQMs generalise quantum mechanical descriptions of M-branes in the context of M(atrix) theory studied in the literature [37–42]. In section 5 we construct a new family of $\mathrm{AdS}_2\times$ $\mathrm{S}^3\times$ $\mathrm{CY}_2\times$ I solutions to Type IIA with four Poincaré supercharges, upon reduction from M-theory. These solutions are associated to D0-F1-D4-D4' brane intersections. We naturally extend them to backgrounds of massive IIA supergravity, upon double analytical continuation from the solutions summarised in section 2. In section 6 we present our conclusions and future lines of research motivated by our results. Appendix A summarises the class I and class II solutions presented in [18]. In appendix B we present the most general class of $\mathrm{AdS}_3\times\mathrm{S}^2\times$ $\mathrm{M}_4$ solutions to M-theory with (0,4) supersymmetries and SU(2) structure. Appendix C contains more general $\mathrm{AdS}_2\times$ $\mathrm{S}^3\times\mathrm{M}_4$ solutions to massive IIA where $\mathrm{M}_4$ is a 4d Kähler manifold. The geometries studied in the main body of the paper are special cases of those in the appendices. It would be interesting to understand the holographic dual to these more general backgrounds.

## 2 Review of the $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2$ solutions to massive Type IIA

In [18] the most general class of $\mathrm{AdS}_3\times$ $\mathrm{S}^2$ solutions to massive IIA supergravity with small (0,4) supersymmetry and SU(2) structure was presented. These solutions are foliations of $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{M}_4$ over an interval, with $\mathrm{M}_4$ either a $\mathrm{CY}_2$ or a 4d Kähler manifold. The first type of solutions were referred to as class I. The second, which contain as a particular case the T-duals of the solutions found in [11], were referred to as class II. The backgrounds in class I for which the symmetries of the $\mathrm{CY}_2$ are respected by the solution constitute a particularly interesting subclass for which the full family of 2d (0,4) dual CFTs can be identified [34, 35]. This subclass of solutions is the focus of our main interest in this work. From them we will construct a general class of $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbb{Z}_k\times$ $\mathrm{CY}_2$ solutions to M-theory, to which we will associate 2d (0,4) quiver CFTs supported by $M_A$-strings. The uplifts of the most general solutions in class I and class II will be presented in appendix B. Our solutions provide altogether a complete classification of $\mathrm{AdS}_3$ solutions to M-theory with (0,4) supersymmetries and SU(2) structure.

We begin our analysis by reviewing the class I geometries constructed in [18], with the further restriction that the symmetries of the Calabi-Yau 2-fold are respected by the full solution. This requires the Calabi-Yau to be compact, and therefore we will take it to be either $T^4$ or $K3$. The NS sector of this subclass of solutions reads,

$$\begin{aligned} \mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 + (u')^2}\mathrm{d}s^2_{S^2}\right) + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\,, \\ e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4}\sqrt{u}}\sqrt{4\widehat{h}_4 h_8 + (u')^2}\,, \quad H_{(3)} = \frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2}\right)\wedge \widehat{\mathrm{vol}}_{\mathrm{S}^2}\,. \end{aligned} \tag{2.1}$$

Here $\Phi$ is the dilaton, $H_{(3)}$ the NS three-form and the metric is given in string frame. A prime denotes a derivative with respect to $\rho$.

The RR sector reads

$$\begin{aligned} F_{(0)} &= h_8' , \quad F_{(2)} = -\frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4h_8\widehat{h}_4 + (u')^2}\right)\widehat{\mathrm{vol}}_{\mathrm{S}^2} , \\ F_{(4)} &= -\left(\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right) + 2h_8\mathrm{d}\rho\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_3} - \partial_\rho\widehat{h}_4\widehat{\mathrm{vol}}_{\mathrm{CY}_2} . \end{aligned} \tag{2.2}$$

Higher RR fluxes are related to these as $F_{(6)} = -\star F_{(4)}$, $F_{(8)} = \star F_{(2)}$, $F_{(10)} = -\star F_{(0)}$, where $\star$ is the ten-dimensional Hodge-dual operator. Supersymmetry holds when

$$u'' = 0 , \tag{2.3}$$

which makes $u$ a linear function of $\rho$. In turn, the Bianchi identities of the fluxes impose

$$h_8'' = 0 , \quad \widehat{h}_4'' = 0, \tag{2.4}$$

which make $h_8$ and $\widehat{h}_4$ also linear functions. The particular configurations reviewed above are independent of the $\mathrm{CY}_2$-fold coordinates and $\widehat{h}_4$ has support on the $\rho$ coordinate only. The supersymmetry and Bianchi identities are satisfied for $u$, $h_8$, $\widehat{h}_4$ arbitrary linear functions in $\rho$. This is the above mentioned restriction we adopt with respect to [18]. We shall keep this restriction hereafter, with the exception of the material in the appendices.

The magnetic components of the Page fluxes $\widehat{F} = F \wedge e^{-B_{(2)}}$ are given by

$$\widehat{F}_{(0)} = h_8' \tag{2.5}$$

$$\widehat{F}_{(2)} = -\frac{1}{2}\Big(h_8 - (\rho - 2\pi j)h_8'\Big)\widehat{\mathrm{vol}}_{\mathrm{S}^2} \tag{2.6}$$

$$\widehat{F}_{(4)} = -\widehat{h}_4'\widehat{\mathrm{vol}}_{\mathrm{CY}_2} \tag{2.7}$$

$$\widehat{F}_{(6)} = \frac{1}{2}\Big(\widehat{h}_4 - (\rho - 2\pi j)\widehat{h}_4'\Big)\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\widehat{\mathrm{vol}}_{\mathrm{S}^2}, \tag{2.8}$$

where we have included large gauge transformations in $B_{(2)}$ of parameter $j$, such that

$$B_{(2)} = \frac{1}{2}\left(2\pi j - \rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{S}^2} . \tag{2.9}$$

### 2.1 Brief description of the 2d dual CFTs

Associated to the Page fluxes there is a D2-D4-D6-D8-NS5 brane system, depicted in table 1. The 2d CFTs that live on these brane set-ups were analysed in [34, 35], to which the reader is referred for more details. They are described by $(0,4)$ superconformal quivers with gauge groups associated to stacks of D2 and D6 branes (the latter wrapped on the $\mathrm{CY}_2$ manifold), both stretched between NS5 branes. Being the extension of the D2 and D6 branes finite in the $\rho$ direction, the field theory living on their intersection is effectively two dimensional at low energies. These quivers are rendered non-anomalous with adequate flavour groups at each node, coming from D4 and D8 branes. Figure 1 illustrates their



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| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D2 | x | x | | | | | x | | | |
| D4 | x | x | | | | | | x | x | x |
| D6 | x | x | x | x | x | x | x | | | |
| D8 | x | x | x | x | x | x | | x | x | x |
| NS5 | x | x | x | x | x | x | | | | |

**Table 1**. $\frac{1}{8}$-BPS brane intersection underlying the $\mathrm{AdS}_3$ solutions. $(x^0, x^1)$ are the directions where the 2d CFT lives, $(x^2, \ldots, x^5)$ span the $\mathrm{CY}_2$, on which the D6, the D8 and the NS5 branes are wrapped, $x^6$ is the direction along the $\rho$-interval, and $(x^7, x^8, x^9)$ are the transverse directions on which the $\mathrm{SO}(3)_R$ symmetry is realised.

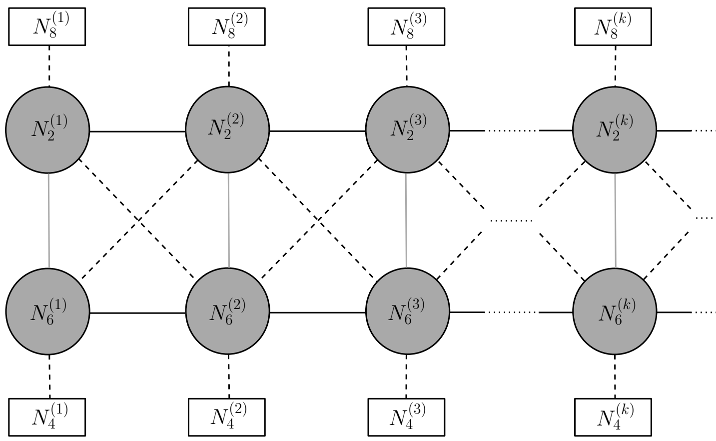


**Figure 1**. Generic quiver field theory whose IR is holographic dual to the solutions reviewed in this section. The solid black line represents a (4,4) hypermultiplet, the grey line a (0,4) hypermultiplet and the dashed line a (0,2) Fermi multiplet. The degrees of freedom at each node are (4,4) vector multiplets.



general structure. The quivers can be divided into two long linear quivers consisting on $(4,4)$ gauge groups connected horizontally by $(4,4)$ bifundamental hypermultiplets, coupled through $(0,4)$ hypermultiplets (vertically) and $(0,2)$ Fermi multiplets (in the diagonals). The flavour degrees of freedom couple through $(0,2)$ Fermi multiplets with its corresponding gauge node. These couplings render the quiver $(0,4)$ supersymmetric.

Let us see how the cancellation of gauge anomalies works. For a given $\mathrm{SU}(N_2^{(i)})$ gauge group we are concerned with the contributions to the anomaly coming from the $(0,4)$ hypermultiplets that connect it to the $\mathrm{SU}(N_6^{(i)})$ gauge node and with the various $(0,2)$ Fermi multiplets. The $(0,4)$ hypermultiplets in the bifundamental representation are composed of two $(0,2)$ chiral multiplets, which contribute to the gauge anomaly a factor of 1. In turn, the $(0,2)$ Fermi multiplets in the fundamental or bifundamental representations contribute a factor of $-\frac{1}{2}$. Putting these together, we have that for a $\mathrm{SU}(N_2^{(i)})$ gauge group the gauge anomaly cancellation condition is

$$2N_6^{(i)} = N_6^{(i-1)} + N_6^{(i+1)} + N_8^{(i)}\,. \tag{2.10}$$

This becomes

$$2N_2^{(i)} = N_2^{(i-1)} + N_2^{(i+1)} + N_4^{(i)}\,, \tag{2.11}$$

for $\mathrm{SU}(N_6^{(i)})$ gauge groups. The reader is referred to [35] for more details.

In the next section we study the M-theory lift of the backgrounds in (2.1)–(2.2).

## 3 New $\mathrm{AdS}_3 \times \mathrm{S}^3/\mathbb{Z}_k \times \mathrm{CY}_2$ solutions in M-theory

Let us consider the uplift to eleven dimensions of the solutions discussed in the previous section. To perform this lift we need $F_{(0)} = 0$, which according to (2.2) imposes the function $h_8$ to be a constant. Thus, the IIA brane configuration that we lift consists on intersecting D2-D4-D6-NS5 branes. The restriction to vanishing Romans' mass implies that the number of D6-branes ($N_6$) must remain constant between all pairs of NS5-branes. In the lift to eleven dimensions this number becomes a modding parameter of the geometry, associated with KK-monopole charge.

Once this lift is performed, we obtain a class of $\mathrm{AdS}_3{\times}\mathrm{S}^3/\mathbb{Z}_k{\times}$ $\mathrm{CY}_2$ solutions to M-theory foliated over an interval. They preserve $\mathcal{N} = (0, 4)$ supersymmetry. These solutions read

$$\mathrm{d}s_{11}^2 = \Delta\left(\frac{u}{\sqrt{\widehat{h}_4 h_8}}\mathrm{d}s^2_{\mathrm{AdS}_3} + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\right) + \frac{h_8^2}{\Delta^2}\mathrm{d}s^2_{\mathrm{S}^3/\mathbb{Z}_k}\,, \tag{3.1}$$

$$\begin{aligned} G_{(4)} &= -\mathrm{d}\left(\frac{uu'}{2\widehat{h}_4} + 2\rho h_8\right)\wedge \widehat{\mathrm{vol}}_{\mathrm{AdS}_3} + 2h_8\,\mathrm{d}\left(-\rho + \frac{uu'}{4\widehat{h}_4 h_8 + u'^2}\right)\wedge \widehat{\mathrm{vol}}_{\mathrm{S}^3/\mathbb{Z}_k} \\ &\quad - \partial_\rho \widehat{h}_4\,\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\,, \qquad\qquad (3.2) \\ \Delta &= \frac{h_8^{1/2}(4\widehat{h}_4 h_8 + u'^2)^{1/3}}{2^{2/3}\widehat{h}_4^{1/6}u^{1/3}}\,, \end{aligned}$$

where $k = h_8 = N_6$. The quotiented 3-sphere is written as an $S^1_z$ Hopf fibration over an $\mathrm{S}^2$,

$$\mathrm{d}s^2_{\mathrm{S}^3/\mathbb{Z}_k} = \frac{1}{4}\left[\left(\frac{\mathrm{d}z}{k} + \eta\right)^2 + \mathrm{d}s^2_{\mathrm{S}^2}\right] \qquad \text{with} \qquad \mathrm{d}\eta = \widehat{\mathrm{vol}}_{\mathrm{S}^2}. \tag{3.3}$$

In the previous solutions the symmetries $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ and $\mathrm{SU}(2)$ are realised geometrically on the $\mathrm{AdS}_3$ and the quotiented 3-sphere, respectively.

The dual quivers associated to this class of solutions are depicted in figure 2. The gauge anomaly is automatically cancelled for the $\mathrm{SU}(N_2^{(i)})$ gauge groups, once an extra $\mathrm{SU}(N_6)$ flavour group is added to the first node, while for the $\mathrm{SU}(N_6)$ gauge groups the condition (2.11) has been enforced. In what follows, we concentrate on the backgrounds in (3.1)–(3.2). In appendix B we discuss the lift to eleven dimensions of the more general backgrounds constructed in [18].

### 3.1 Brane set-up

In the new class of solutions given by (3.1)–(3.2), the number of Type IIA D6-branes became the orbifold parameter in $\mathrm{S}^3/\mathbb{Z}_k$, $k = N_6 = h_8$, and thus corresponds to KK-monopole

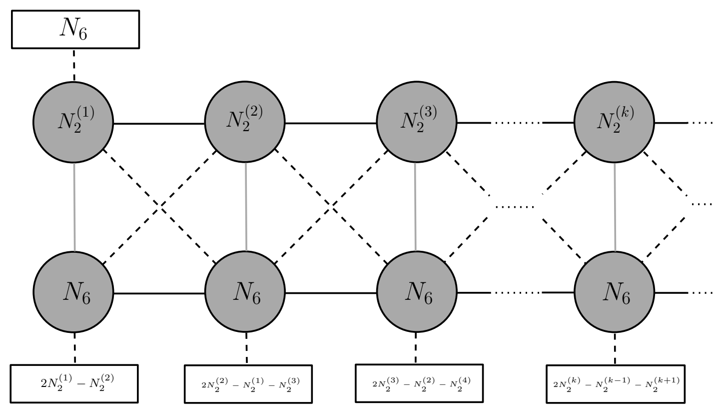


**Figure 2**. Generic quiver field theories dual to the $\mathrm{AdS}_3$ solutions with vanishing Romans' mass.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| M2 | x | x | | | | | x | | | | |
| M5 | x | x | | | | | | x | x | x | x |
| KK | x | x | x | x | x | x | x | | | | z |
| M5' | x | x | x | x | x | x | | | | | |

**Table 2**. $\frac{1}{8}$-BPS brane intersection underlying the $\mathrm{AdS}_3\times$ $\mathrm{S}^3/\mathbb{Z}_k$ solutions in M-theory. The directions $(x^0, x^1)$ are those where the 2d dual CFT lives, $(x^2, \dots, x^5)$ span the $\mathrm{CY}_2$, $x^6$ is the 'field space' direction and $(x^7, x^8, x^9)$ are the transverse directions on which the $\mathrm{SO}(3)_R$ symmetry is realised. Finally, $x^{10}$ is the extra eleventh direction, which spans the $S^1/\mathbb{Z}_k \subset S^3/\mathbb{Z}_k$ and plays the role of Taub-NUT direction of the KK-monopole.

charge. The D2-branes became M2-branes. Their charge in the interval $\rho \in [2\pi j, 2\pi(j+1)]$ is obtained by integrating the Page flux $\widehat{G}_{(7)} = G_{(7)} - G_{(4)} \wedge C_{(3)}$. The component of $\widehat{G}_7$ relevant to calculate the number of M2 branes is given by

$$\widehat{G}_{(7)} = 2h_8\Big(\widehat{h}_4 - (\rho - 2\pi j)\widehat{h}_4'\Big)\widehat{\mathrm{vol}}_{\mathrm{S}^3/\mathbb{Z}_k} \wedge \widehat{\mathrm{vol}}_{\mathrm{CY}_2}\,. \tag{3.4}$$

The D4-branes of the Type IIA solution became M5-branes. Their presence is captured by non-trivial flux of $G_4$ through the $\mathrm{CY}_2$. Finally, the NS5 branes became M5'-branes, whose charge is given by a non-trivial flux of $G_4$ through the $(\rho, S^3/\mathbb{Z}_k)$ cycle. Therefore, the D2-D4-D6-NS5 branes underlying the Type IIA solutions become M2-M5-KK-M5' branes, intersecting as shown in table 2. The KK-monopoles (wrapped on the $\mathrm{CY}_2$) and the M2 branes are stretched between parallel M5'-branes and there are extra M5-branes providing for flavour groups. This describes $M_A$-strings, supplemented with extra M5-branes. The corresponding dual quivers are the ones depicted in figure 2, with upper row nodes associated to M2-branes and lower row nodes to KK-monopoles. The M5-branes provide for extra flavour groups that render the quivers non-anomalous (and the supergravity equations of motion satisfied).

Our new solutions in M-theory (3.1)–(3.2), provide for explicit $\mathrm{AdS}_3$ geometries that can be used to study these quivers holographically. It would be interesting to see these

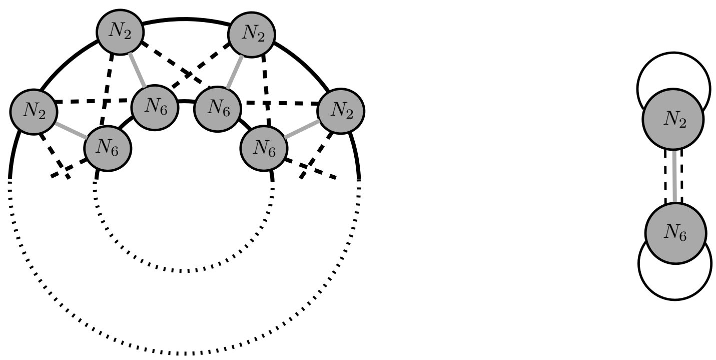


**Figure 3**. Left: Generic quiver field theory whose IR limit is holographic dual to the $AdS_3$ solutions with $I = S^1$. Right: Quiver field theory for $M = 1$. In the right quiver, the (0,4) hypermultiplets combine with two (0,2) Fermi multiplets to produce (4,4) hypermultiplets. Supersymmetry is thus enhanced to (4,4).

geometries emerging in the near horizon limit of intersecting M2-M5-MKK-M5' brane systems. This is currently under investigation [43].

Note that when $u' = 0$ the M5-branes wrapped on $AdS_3 \times S^3/\mathbb{Z}_k$ support self-dual strings on their worldvolumes, coupled to the (self-dual) 3-form field

$$C_{(3)} = -2\rho\, h_8(\widehat{\text{vol}}_{AdS_3} + \widehat{\text{vol}}_{S^3/\mathbb{Z}_k})\,. \tag{3.5}$$

They arise from M2-branes ending on the M5-branes. These solutions provide then for fully backreacted near horizon geometries for OM theory [44], the theory conjectured to be the UV completion of the (2,0) theory with constant background 3-form field living on the M5-branes [45]. In our explicit set-up the 3-form depends on the positions of the M5-branes in the $\rho$-direction. Extra intersecting M5'-branes and KK-monopoles further reduce the supersymmetries by a half.

An interesting particular case contained in our class of solutions is when $\mathcal{I} = S^1$, for which $\widehat{h}_4' = u' = 0$. This case was discussed in [7] (see also [46]). In this case the background [47] is the uplift of the T-dual of the $AdS_3 \times S^3/\mathbb{Z}_M \times \text{CY}_2$ geometry that describes D1-D5 branes in a $A_{M-1}$ singularity, introduced by $M$ KK-monopoles. The IIB KK-monopoles become the M5'-branes in M-theory, with their Taub-NUT charge provided by the Type IIB D5-branes. The associated 2d quivers are those on the left of figure 3. When $M = 1$ supersymmetry is enhanced to (4,4), the brane system becomes a M2-M5' brane intersection and the associated quiver becomes the one on the right.

Another interesting case is when $k = 1$ and there is just one KK-monopole stretched between the M5'-branes. The resulting quivers are depicted in figure 4 (left). In M-theory one KK-monopole is equivalent to no modding, and therefore the brane system reduces to the M2-M5-M5' brane intersection included in table 2. This intersection is still 1/8-BPS. These brane intersections might play a role as brane set-ups where 2d defect CFTs could be realised, in connection with the phenomenon of deconstruction [48]. Indeed, our quivers generalise (by the inclusion of flavours) the 2d defect CFTs living in D3-D3'-KK intersections studied in [49], which deconstruct 4d $\mathcal{N} = 2$ CFTs living in M5-brane

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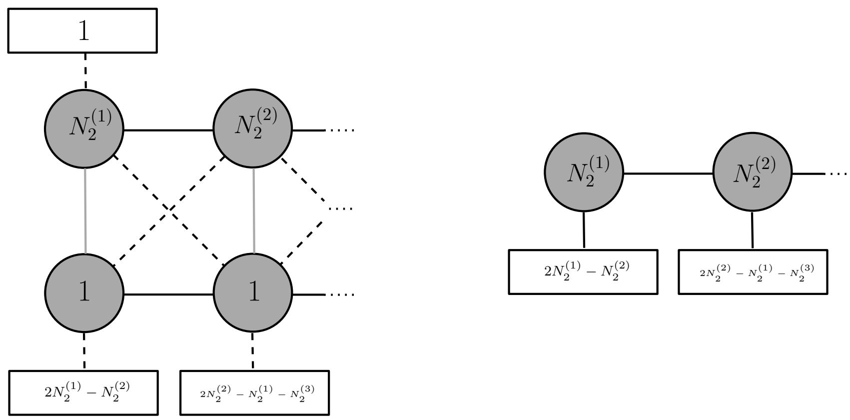


**Figure 4**. Left: 2d (0,4) quiver CFT dual to the AdS$_3\times$ S$^3\times$CY$_2\times$ I solution. Right: 4d $\mathcal{N} = 2$ quiver CFT with flavours.

intersections. One might expect that the 2d quivers depicted on the left of figure 4 could emerge through a similar mechanism as the one described in [49], by coupling the 4d $\mathcal{N} = 2$ CFT depicted on the right with an Abelian field theory. It would be interesting to explore this possibility.

### 3.2 Central charge

In this section we compute the (right moving) central charge of the CFTs dual to our solutions. We consider generic quivers such as the ones depicted in figure 2, that we terminate by adding adequate flavour groups, rendering the quiver non-anomalous, with large but finite length (see [35] for more details). One possibility is the completed quiver depicted in figure 5. The corresponding functions $h_8$ and $\widehat{h}_4(\rho) = \Upsilon h_4(\rho)$ are given by,

$$h_8 = N_6, \quad 0 \leq \rho \leq 2\pi(P+1).$$
$$\widehat{h}_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \leq \rho \leq 2\pi \\ \alpha_j + \frac{\beta_j}{2\pi}(\rho - 2\pi j) & 2\pi j \leq \rho \leq 2\pi(j+1), \\ \alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1), \end{cases} \tag{3.6}$$

with $\alpha_j = \sum_{r=0}^{j-1}\beta_r$ and $j = 1, \ldots, P-1$. We have $\widehat{h}_4(0) = \widehat{h}_4(2\pi(P+1)) = 0$. At these values of $\rho$, the asymptotic analysis indicates the presence of M5-branes extended on AdS$_3\times$ S$^3/\mathbb{Z}_k$ and the space terminates. In what follows, we choose $\Upsilon$ such that $\Upsilon \mathrm{vol}_{\mathrm{CY}_2} = 16\pi^4$.

The numbers of M2 and M5 branes at each $2\pi j \leq \rho \leq 2\pi(j+1)$ interval, with $j = 1, \ldots, P$, are given by

$$N_2^{(j)} = \frac{1}{(2\pi)^6}\int_{\mathrm{CY}_2\times S^3/\mathbb{Z}_k} \widehat{G}_{(7)} = \frac{2}{(2\pi)^6}\Big(\widehat{h}_4 - (\rho - 2\pi j)\widehat{h}_4'\Big)\mathrm{vol}_{\mathrm{CY}_2}\mathrm{vol}_{S^3} = \alpha_j \tag{3.7}$$

and

$$N_5^{(j)} = \frac{1}{(2\pi)^3}\int_{\mathrm{CY}_2} G_{(4)} = \begin{cases} \beta_j & 2\pi j \leq \rho \leq 2\pi(j+1); \;\; j = 0, \ldots, P-1 \\ -\alpha_P & 2\pi P \leq \rho \leq 2\pi(P+1). \end{cases} \tag{3.8}$$

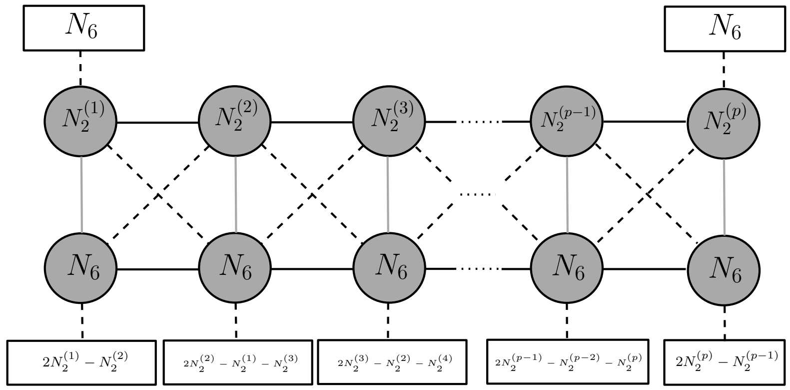


**Figure 5**. Completed quiver field theories whose IR limits are holographic duals to the $\mathrm{AdS}_3\times \mathrm{S}^3/\mathbb{Z}_k$ solutions in M-theory. $N_2^{(j)}$ refer to M2-brane charges and $N_6 = k$ to the constant, KK-monopole charge. M5-branes provide for the $2N_2^{(i)} - N_2^{(i-1)} - N_2^{i+1)}$ flavour groups that render the quiver non-anomalous.

Notice that $\beta_{j-1}-\beta_j = 2N_2^{(j)} - N_2^{(j-1)} - N_2^{(j+1)}$ is the number of flavours at each $2\pi j \leq \rho \leq 2\pi(j+1)$ interval, with $j = 1, \ldots, P-1$, and there are extra $\alpha_P + \beta_{P-1} = 2N_2^{(P)} - N_2^{(P-1)}$ flavours at the $2\pi P \leq \rho \leq 2\pi(P+1)$ interval, as depicted in figure 5.

We proceed now with the computation of the holographic central charge. The interesting result we shall obtain is that the central charge is proportional to the total number of $M_A$-strings and is also related to the action of the $M_5'$-branes.

The central charge being directly proportional to the number of $M_A$-strings indicates that the fundamental degrees of freedom of this theory should be understood as $M_A$-strings. On the other hand, the relation between the action of $M_5'$-branes and the central charge indicates that these branes (that provide a boundary condition for the membranes to end) capture on their world-volumes the dynamics of the lower dimensional branes. This is a non-trivial fact, already encountered in [50]. It would be of interest to reproduce it in other holographic systems to fully understand its origin.

The right-moving central charge of the $\mathrm{AdS}_3\times\mathrm{S}^2\times$ $\mathrm{CY}_2$ solutions constructed in [18] was computed in [35]. The expression found there remains valid upon uplift to eleven dimensions. In terms of the ten dimensional Newton's constant we have,

$$c = \frac{3\pi}{2G_N}\mathrm{vol}_{\mathrm{CY}_2}\int \mathrm{d}\rho\, h_8 \widehat{h}_4. \tag{3.9}$$

This gives, for the functions $\widehat{h}_4$ and $h_8$ displayed above

$$c = \frac{3}{\pi} h_8 \int_0^{2\pi(P+1)} \mathrm{d}\rho\, h_4 = 6 h_8 \sum_{j=1}^{P} \alpha_j = 6 h_8 \sum_{j=1}^{P} N_2^{(j)} = 6kN_2 = 6\, N_{M_A}\,, \tag{3.10}$$

where $N_{M_A}$ stands for the total number of $M_A$-strings in the configuration, taking into account the orbifolding by $\mathbb{Z}_k$. This result emphasises the fact that the $M_A$-strings holographically capture the degrees of freedom of the conformal field theory. This suggests that the $M_A$-strings actually are the degrees of freedom of the strongly coupled conformal field theory. Notice the factor of "6" in eq. (3.10). This factor is fixed on purely algebraic grounds. See for example [51].

We show next that this result can be reproduced from the action describing the M5'-branes of the configuration, where the M2-branes end, realising the $M_A$-strings introduced in [7, 9].

The M5'-branes on which the M2-branes end, span the $(t, x^1, \mathrm{CY}_2)$ directions of the geometry, and are positioned along the $\rho$-interval at $\rho = 2\pi j$. Their worldvolume effective action is given by

$$S_{M5'(j)} = T_{M5'} \int \mathrm{d}^6\xi \sqrt{\det g} = \frac{1}{4} T_{M5'} \mathrm{vol}_{\mathrm{CY}_2} \int \left( \widehat{h}_4 h_8 + \frac{1}{4} u'^2 \right) \cosh r \, \mathrm{d}t \mathrm{d}x^1 \,. \tag{3.11}$$

For an M5'-brane located at $\rho = 2\pi j$ and $r = 0$ this becomes

$$S_{M5'(j)} = \frac{1}{4\,(2\pi)^4} \mathrm{vol}_{\mathrm{CY}_2} \mathrm{vol}_{\mathbb{R}} \left( \widehat{h}_4\,(2\pi j)\, h_8 + \frac{1}{4} u'^2 \right) = \frac{1}{4} \mathrm{vol}_{\mathbb{R}} \left( \alpha_j h_8 + \frac{u'^2}{4\Upsilon} \right). \tag{3.12}$$

Summing the contributions of all M5'-branes we have, to leading order in $P$,

$$S_{M5'} = \sum_{j=1}^{P} S_{M5'(j)} \sim h_8 \sum_{j=1}^{P} \alpha_j \; = N_{M_A} \,. \tag{3.13}$$

This reproduces the scaling of the central charge to leading order within the context of the M5'-branes effective action.

Our discussion in the previous subsection about the interpretation of the $M_A$-strings as self-dual strings when $u' = 0$ suggests that we should also be able to reproduce the scaling of the central charge from the M5-branes effective action, where the M2-branes realise self-dual strings. However, to check this we would need an action for non-Abelian M5-branes.

### 3.2.1 Field theory calculation

Finally, we check for consistency that the previous central charge coincides with the field theory result for long quivers with large ranks — the regime in which we can trust the supergravity solutions (see [35]). At the conformal point the central charge is related to the two point $\mathrm{U}(1)_R$ current correlation function (see for example [52]), such that

$$c = 6(n_{hyp} - n_{vec}) \,, \tag{3.14}$$

where $n_{hyp}$ is the number of $\mathcal{N} = (0,4)$ hypermultiplets and $n_{vec}$ the number of $\mathcal{N} = (0,4)$ vector multiplets of the quiver in the UV description.

For the quivers considered in figure 5, we find

$$n_{hyp} = \sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} + (P-1) N_6^2 + N_6 \sum_{j=1}^{P} N_2^{(j)} \,, \tag{3.15}$$

and

$$n_{vec} = \sum_{j=1}^{P} \Big( (N_2^{(j)})^2 - 1 \Big) + P(N_6^2 - 1) \,. \tag{3.16}$$

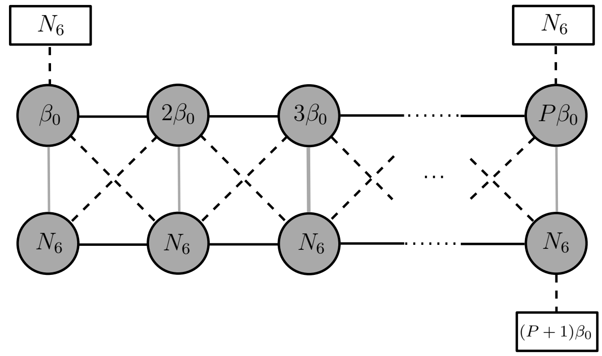


**Figure 6**. 2d (0,4) quiver CFT with gauge groups of linearly increasing ranks.

After defining $N_2 = \sum_{j=1}^{P} N_2^{(j)}$, this gives for the central charge

$$c = 6\,(n_{hyp} - n_{vec}) = 6\left(N_6 N_2 + \sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} - \sum_{j=1}^{P} (N_2^{(j)})^2 - N_6^2 + 2P\right). \tag{3.17}$$

Now, the net contribution to this expression of the term $\left(\sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} - \sum_{j=1}^{P} (N_2^{(j)})^2\right)$ is subleading when compared with the contribution of $N_6 N_2$. This hierarchy occurs when the number of gauge nodes is large (for long quivers). As a consequence, to leading order in the number of nodes $P$, we find

$$c = 6N_6N_2 + \mathcal{O}(P). \tag{3.18}$$

The only situation in which the two competing terms above scale similarly in $P$, is when there are no intermediate flavour groups, i.e. when $N_2^{(j)} = j\beta_0$ for $i = 1, \dots, P$. This particular situation corresponds to the quiver with gauge groups of linearly increasing ranks, depicted in figure 6.Next we show that the contribution of $\left(\sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} - \sum_{j=1}^{P} (N_2^{(j)})^2\right)$ is indeed subleading with respect to that of $N_6 \sum_{j=1}^{P} N_2^{(j)}$. To do this, we should impose that $N_6$ is much bigger than $\beta_0$. This is required to have a trustable supergravity background — see the analysis in section 4.4 of [35]. The central charge then reads

$$c = 6\left[N_6 N_2 + \beta_0^2\Big(\sum_{j=1}^{P-1} j(j+1) - \sum_{j=1}^{P} j^2\Big) - N_6^2 + 2P\right]. \tag{3.19}$$

Keeping in mind that $N_2 = \beta_0 \sum_{j=1}^{P} j = \beta_0 \frac{P(P+1)}{2}$ we get

$$c = 3\beta_0(N_6 - \beta_0)P^2 + \mathcal{O}(P) \sim 3\beta_0 N_6 P^2\,. \tag{3.20}$$

We used that $N_6$ must be much larger than $\beta_0$ for the supergravity background to be trustable.

One can now easily check that this is in agreement with the holographic calculation. In fact, the function $\widehat{h}_4$ representing the quiver with linearly increasing ranks is

$$\widehat{h}_4(\rho) = \Upsilon h_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\beta_0 P}{2\pi}(2\pi(P+1) - \rho) & 2\pi P \le \rho \le 2\pi(P+1), \end{cases} \tag{3.21}$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| M0 | x | z | | | | | | | | | |
| M2 | x | x | | | | | x | | | | |
| M5 | x | x | | | | | | x | x | x | x |
| M5' | x | x | x | x | x | x | | | | | |

**Table 3**. $\frac{1}{8}$-BPS brane intersection underlying the $\mathrm{AdS}_3/\mathbb{Z}_k\times$ $\mathrm{S}^3$ solutions in M-theory. $x^1$ is the direction of propagation of the wave. $(x^2,\ldots,x^5)$ span the $\mathrm{CY}_2$, $x^6$ is the direction along the $\rho$-interval, $(x^7,x^8,x^9,x^{10})$ are the transverse directions on which the SO(4) symmetry is realised. The presence of the wave renders the dual CFT one-dimensional.

with $h_8=N_6$. Using our expression for the central charge (3.9) we find,

$$c=\frac{3}{\pi}N_6\left(\int_0^{2\pi P}\frac{\beta_0}{2\pi}\rho d\rho+\int_{2\pi P}^{2\pi(P+1)}\frac{\beta_0}{2\pi}(2\pi(P+1)-\rho)d\rho\right)=3N_6\beta_0P^2\left(1+\frac{1}{P}\right), \quad (3.22)$$

in coincidence with eq. (3.20) when long quivers are considered.

Any other situation with intermediate (many, but sparse enough) flavour groups will work along similar lines, showing the validity of eq. (3.18). This shows that the holographic calculation and the field theoretical one coincide for long quivers with large enough ranks.

This closes our analysis of the backgrounds in equations (3.1)–(3.2). In the next section we present a new branch of $\mathrm{AdS}_3$ solutions in M-theory.

## 4 Double analytic continuation

A double analytic continuation in the $\mathrm{AdS}_3\times$ $\mathrm{S}^3/\mathbb{Z}_k\times$ $\mathrm{CY}_2\times$ I solutions presented in (3.1)–(3.2), gives rise to a second class of solutions in which the $\mathrm{AdS}_3$ subspace is quotiented instead of the $\mathrm{S}^3$. These solutions preserve the same amount of supersymmetries. The KK-monopoles turn into M0-branes, or waves, with the Taub-NUT direction of the KK-monopoles turning into the direction of propagation of the waves. The solutions are then associated to the M0-M2-M5-M5' brane intersections depicted in table 3. The double analytic continuation of the background given in (3.1) works as follows. The $\mathrm{AdS}_3$ and $\mathrm{S}^3$ factors can be swapped as

$$AdS_3\to -S^3\,,\qquad S^3\to -AdS_3\,. \quad (4.1)$$

In order to get a spacetime with the correct signature the $u$, $\widehat{h}_4$, $h_8$ functions need to be also analytically continued, as follows

$$u\to -iu\,,\qquad \widehat{h}_4\to i\widehat{h}_4\,,\qquad h_8\to ih_8\,, \quad (4.2)$$

together with $\rho\to i\rho$.

Applying this set of transformations to the solutions to M-theory discussed in (3.1)–(3.2) gives rise to the following new solutions

$$\mathrm{d}s_{11}^2=\frac{h_8^2}{\Delta^2}\mathrm{d}s^2_{AdS_3/\mathbb{Z}_k}+\Delta\left(\frac{u}{\sqrt{\widehat{h}_4h_8}}\mathrm{d}s^2_{S^3}+\sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2}+\frac{\sqrt{\widehat{h}_4h_8}}{u}\mathrm{d}\rho^2\right) \quad (4.3)$$



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| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D0 | x | | | | | | | | | |
| F1 | x | | | | | x | | | | |
| D4 | x | x | x | x | x | | | | | |

**Table 4**. $\frac{1}{8}$-BPS intersection involving D0, D4 and F1 branes. A $\mathcal{N}=4$ supersymmetric quantum mechanics lives in the common $x^0$ direction. $(x^1,\ldots,x^4)$ span the directions on the $\mathrm{CY}_2$. $x^5$ is the direction along the $\rho$-interval. The $(x^6,\ldots,x^9)$ directions enjoy an SO(4) rotational symmetry, of which an SU(2) is the R-symmetry of the SU(1,1|2) supergroup and another SU(2) a global symmetry.

$$G_{(4)} = -\mathrm{d}\left(-\frac{uu'}{2\widehat{h}_4}+2\rho h_8\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{S}_3} - 2h_8\,\mathrm{d}\left(\rho+\frac{uu'}{4\widehat{h}_4h_8-u'^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_3/\mathbb{Z}_k} - \partial_\rho\widehat{h}_4\,\widehat{\mathrm{vol}}_{\mathrm{CY}_2} \tag{4.4}$$

$$\Delta = \frac{h_8^{1/2}(4\widehat{h}_4h_8-u'^2)^{1/3}}{2^{2/3}\widehat{h}_4^{1/6}u^{1/3}} \tag{4.5}$$

where $k=h_8$ and the quotiented $\mathrm{AdS}_3$ subspace is written as a Hopf fibration over $\mathrm{AdS}_2$,

$$\mathrm{d}s^2_{\mathrm{AdS}_3/\mathbb{Z}_k} = \frac{1}{4}\left[\left(\frac{\mathrm{d}z}{k}+\eta\right)^2+\mathrm{d}s^2_{\mathrm{AdS}_2}\right] \qquad \text{with} \qquad \mathrm{d}\eta = \widehat{\mathrm{vol}}_{\mathrm{AdS}_2}. \tag{4.6}$$

### 4.1 Dual quantum mechanics

Due to the momentum charge, the previous class of solutions is dual to a 1d superconformal quantum mechanics (SCQM). Holographically, they are thus essentially different from the $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbb{Z}_k$ solutions on which the double analytic continuation was performed. From the isometries of the background, we see that the $\mathcal{N}=4$ SCQM must preserve $\mathfrak{su}(1,1|2)$ superconformal algebra, whose bosonic sub-algebra is $\mathfrak{sl}(2)\oplus\mathfrak{su}(2)$ [53].[2]

A particular solution that can be used to provide some hint on the nature of the dual quantum mechanics is the one for which $\mathcal{I}=S^1$. This is the background that follows from setting $\widehat{h}_4'=u'=0$ in (4.3)–(4.5). This solution is associated to a M0-M2-M5' brane intersection, and is the uplift to M-theory of the T-dual of the $\mathrm{AdS}_3/\mathbb{Z}_M\times\mathrm{S}^3\times\mathrm{CY}_2$ geometry that describes D1-D5 branes with $M$ units of momentum along the Hopf-fibre direction of $\mathrm{AdS}_3$. T-dualising on the Hopf-fibre gives rise to D0-branes, D4-branes and F1-strings, as shown in table 4, which upon uplift give the M0-M2-M5' brane intersection included in table 3. When $M=1$ supersymmetry is enhanced to 2d (4,4) and the brane intersection becomes the M2-M5' brane set-up discussed in section 3.1. The associated quiver is the one depicted on the right of figure 3. Switching on momentum charge allows for a quantum mechanical description of this system within M(atrix) theory, upon taking the Sen-Seiberg limit [54, 55]. The $\mathrm{AdS}_3/\mathbb{Z}_M\times\mathrm{S}^3\times\mathrm{CY}_2\times\mathrm{S}^1$ solution (or its $\mathrm{AdS}_2$ reduction to Type IIA) provides for an alternative holographic description of this quantum mechanics.

[2]Note that one of the SU(2) isometry groups of the 3-sphere is a global symmetry.



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More general $\mathrm{AdS}_3/\mathbb{Z}_k\times \mathrm{S}^3\times \mathrm{CY}_2\times \mathrm{I}$ solutions in our class should be dual to M(atrix) quantum mechanics describing M2-M5-M5' brane intersections. M5-M5' brane intersections were discussed in this context in [42], but these give rise to 4d SCFTs in their common worldvolume, and are therefore different from the intersections considered in this paper. The M(atrix) theory description of the 2d SCFTs living in our M-brane intersections is currently under investigation [56].

Connections between $\mathrm{AdS}_2$ solutions and M(atrix) theory have been discussed in the literature in various contexts (see for instance [50, 57–60]). The results in [50] are particularly interesting regarding our system. Indeed, the system depicted in table 4 can be thought of as the result of adding F1-branes to the 1/4-BPS D0-D4 brane system discussed in [50], dual to a $\mathrm{AdS}_2\times\mathrm{S}^3\times\ \mathrm{S}^4$ geometry foliated over an interval. The D0-D4 brane system describes in M(atrix) theory longitudinal M5-branes, in terms of a $\mathrm{U}(k)$ gauge theory with hypermultiplets in the adjoint representation and $N$ fundamentals [39]. This quantum mechanics is the reduction on a circle of the quiver CFT dual to the D1-D5 system (depicted on the right of figure 3).[3] Alternatively, one could think of our system in terms of a 1/4-BPS D0-F1 brane system with extra D4-branes. 1/4-BPS D0-F1 branes are dual to a $\mathrm{AdS}_2\times\mathrm{S}^7$ geometry foliated over an interval [50, 61]. Our solutions can be interpreted in these set-ups as the fully backreacted supergravity solutions that arise when F1-strings are placed in the $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{S}^4$ geometry dual to the D0-D4 brane system, or D4-branes are placed in the $\mathrm{AdS}_2\times\mathrm{S}^7\times\mathrm{I}$ solutions dual to the D0-F1 brane system, uplifted to eleven dimensions.

## 5 New $\mathbf{AdS_2\times S^3\times CY_2}$ solutions to massive Type IIA

The $\mathrm{AdS}_3/\mathbb{Z}_k\times \mathrm{S}^3\times\mathrm{CY}_2\times \mathrm{I}$ solutions to M-theory presented in (4.3)–(4.4), can be reduced on the Hopf-fibre of $\mathrm{AdS}_3$. This produces a new class of $\mathrm{AdS}_2\times \mathrm{S}^3\times \mathrm{CY}_2\times \mathrm{I}$ solutions to massless Type IIA supergravity. These solutions are associated to D0-F1-D4-D4' brane systems, preserve four Poincaré supersymmetries and have an SU(2) structure. In fact, one can check that they are just the double analytic continuation of the $\mathrm{AdS}_3\times \mathrm{S}^2\times \mathrm{CY}_2\times \mathrm{I}$ solutions reviewed in section 2, when restricted to the massless case. Therefore, these backgrounds can be extended straightforwardly to the massive case. In this section we present this new class of solutions. We leave their detailed study to our forthcoming publication [56].

Performing the analytic continuation explained in the previous section on the class of solutions given by (2.1)–(2.2) we find a NS sector,

$$\begin{aligned} \mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8-(u')^2}\mathrm{d}s^2_{\mathrm{AdS}_2}+\mathrm{d}s^2_{\mathrm{S}^3}\right)+\sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2}+\frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\,,\\ e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4}\sqrt{u}}\sqrt{4\widehat{h}_4 h_8-(u')^2}\,,\qquad H_{(3)}=-\frac{1}{2}\mathrm{d}\left(\rho+\frac{uu'}{4\widehat{h}_4 h_8-(u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}. \end{aligned} \tag{5.1}$$

[3] In the limit in which the $\mathrm{CY}_2$ is taken to be very large, such that the group associated to the D5-branes becomes global.



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The RR sector reads

$$F_{(0)} = h_8' \,, \quad F_{(2)} = -\frac{1}{2}\left(h_8 + \frac{h_8' u' u}{4h_8\widehat{h}_4 - (u')^2}\right)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \,,$$
$$F_{(4)} = \left(-\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right) + 2h_8 \mathrm{d}\rho\right) \wedge \widehat{\mathrm{vol}}_{\mathrm{S}^3} - \partial_\rho \widehat{h}_4 \widehat{\mathrm{vol}}_{\mathrm{CY}_2}. \tag{5.2}$$

The background in equations (5.1)–(5.2) solves the equations of motion provided that $u'' = \widehat{h}_4'' = h_8'' = 0$. The last two conditions come from the Bianchi identities for the RR sector. Note that we must have $4\widehat{h}_4 h_8 - (u')^2 > 0$, in order for the metric to be of the correct signature and the dilaton to be real.

The Page fluxes are given by

$$\widehat{F}_{(0)} = F_{(0)} = h_8' \,, \tag{5.3}$$

$$\widehat{F}_{(2)} = F_{(2)} - F_{(0)} B_{(2)} = -\frac{1}{2}(h_8 - \rho h_8')\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \,, \tag{5.4}$$

$$\widehat{F}_{(4)} = F_{(4)} = -\widehat{h}_4' \widehat{\mathrm{vol}}_{\mathrm{CY}_2} - \left(2h_8 - \left(\frac{uu'}{2h_4}\right)'\right)\widehat{\mathrm{vol}}_{\mathrm{S}^3} \wedge \mathrm{d}\rho \,, \tag{5.5}$$

$$\widehat{F}_{(6)} = \frac{1}{2}(\widehat{h}_4 - \rho \widehat{h}_4')\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \wedge \widehat{\mathrm{vol}}_{\mathrm{CY}_2} + \left(\left(\frac{u(\rho u' - u)}{4h_4}\right)' - \rho h_8\right)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \wedge \widehat{\mathrm{vol}}_{\mathrm{S}^3} \wedge \mathrm{d}\rho \,, \tag{5.6}$$

$$\widehat{F}_{(8)} = \left(2\widehat{h}_4 - \left(\frac{uu'}{2h_8}\right)'\right)\widehat{\mathrm{vol}}_{\mathrm{CY}_2} \wedge \widehat{\mathrm{vol}}_{\mathrm{S}^3} \wedge \mathrm{d}\rho \,. \tag{5.7}$$

The class of solutions given by (5.1) and (5.2) provide a new class of backgrounds to Type IIA with four Poincaré supersymmetries and SU(2)-structure, which are warped products of $\mathrm{AdS}_2 \times \mathrm{S}^3 \times$ $\mathrm{M}_4$ over an interval, with $\mathrm{M}_4$ a Calabi-Yau 2-fold. The $\mathrm{AdS}_2 \times$ $\mathrm{S}^3$ subspace realises an $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SO}(4)$ isometry group. As mentioned above, one of the SU(2)'s in $\mathrm{SO}(4) \cong \mathrm{SU}(2) \times \mathrm{SU}(2)$ is a global symmetry, so the R-symmetry is that of the $\mathrm{SU}(1,1|2)$ supergroup. This identifies the superconformal group of the associated dual quantum mechanics. As in section 2, we have restricted ourselves to the case in which the symmetries of the $\mathrm{CY}_2$ are respected by the full solution. We will construct the most general class of solutions with SU(2) structure in appendix C, where we will relax this condition on the class I solutions in [18] and analytically continue the solutions in class II. Note that a more general class of solutions with the same supersymmetry can in principle be obtained taking an identity structure instead of the SU(2)-structure considered here.

### 5.1 Brane set-up

Associated to the Page fluxes we find the following quantised charges,

$$N_8 = 2\pi \int_{\mathcal{I}_\rho} \mathrm{d}\widehat{F}_{(0)} \,, \qquad \mathrm{d}\widehat{F}_{(0)} = h_8'' \mathrm{d}\rho \,. \tag{5.8}$$

According to (5.8), we have a natural definition of D8-branes as objects localised in the $\rho$ direction. This, in turn, leads to the fact that D8-branes are not dissolved into fluxes, and effectively behave as flavour branes. They span the $\mathrm{AdS}_2 \times$ $\mathrm{S}^3 \times$ $\mathrm{CY}_2$ sub-manifold.



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| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D0 | x | | | | | | | | | |
| F1 | x | | | | | x | | | | |
| D4 | x | | | | | | x | x | x | x |
| D4' | x | x | x | x | x | | | | | |
| D8 | x | x | x | x | x | | x | x | x | x |

**Table 5**. $\frac{1}{8}$-BPS brane intersection underlying the $\mathrm{AdS}_2\times\mathrm{S}^3$ solutions in Type IIA. $(x^1,\dots,x^4)$ span the $\mathrm{CY}_2$ and $(x^6,x^7,x^8,x^9)$ are the transverse directions on which the SO(4) symmetry group is realised. A $\mathcal{N}=4$ supersymmetric quantum mechanics lives in the common $x^0$ direction.

From the expression for $\widehat{F}_{(4)}$ in (5.5) we identify two manifolds supporting $\widehat{F}_{(4)}$-fluxes. These are $\tilde{\mathcal{M}}_4 = \mathrm{CY}_2$ and $\mathcal{M}_4 = S^3\times\mathcal{I}$. There are therefore two quantised charges, associated to D4 and D4' branes

$$\begin{aligned} N_4 &= \frac{\mathrm{vol}_{\mathrm{CY}_2}}{(2\pi)^3}\int_{\mathcal{I}}\mathrm{d}\rho\,\widehat{h}_4''\,, \\ N_{4'} &= \frac{\mathrm{vol}_{S^3}}{(2\pi)^3}\int_{\mathcal{I}}\mathrm{d}\rho\Big(2h_8-\Big(\frac{uu'}{2h_4}\Big)'\Big)\,. \end{aligned} \tag{5.9}$$

Given that

$$\mathrm{d}\widehat{F}_{(4)} = \widehat{h}_4''\,\mathrm{d}\rho\wedge\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\,, \tag{5.10}$$

the D4-branes provide localised sources, and are therefore flavour branes. They are extended on $\mathrm{AdS}_2\times\mathrm{S}^3$. In turn, the D4'-branes are dissolved into fluxes and therefore do not provide additional physical sources. They are colour branes and extend on $(t,\mathrm{CY}_2)$.

Finally, there is D0 brane charge,

$$N_0 = \frac{1}{(2\pi)^7}\mathrm{vol}_{\mathrm{CY}_2}\mathrm{vol}_{S^3}\int_{\mathcal{I}}\mathrm{d}\rho\,\Big(2\widehat{h}_4-\Big(\frac{uu'}{2h_8}\Big)'\Big)\,. \tag{5.11}$$

Given that $\mathrm{d}\widehat{F}_8$ vanishes identically the D0-branes are colour branes. On top of this there are F1-strings, associated to the electric components of $H_{(3)}$, in (5.1). These F1 extend on $\mathrm{AdS}_2$.

The brane set-up associated to the quantised charges is summarised in table 5. Note that this is exactly what is obtained reducing the M-brane configuration in the previous section, with the addition of extra D8-branes, not present in M-theory. Similar brane intersections have been discussed in [62], in connection with $\mathrm{AdS}_2\times$ $\mathrm{S}^3\times\mathrm{S}^3$ geometries warped over a strip. In this reference the dual SCQM was interpreted in terms of D0-F1-D4 brane defects inside the 5d Sp($N$) fixed point theory dual to the $\mathrm{AdS}_6$ Brandhuber-Oz background [63]. It is likely that a similar interpretation is at place for our brane system [56].

## 6 Conclusions

In this paper we have presented and studied new families of solutions preserving four Poincaré supersymmetries.

The first is a new class of solutions to M-theory preserving $\mathcal{N} = (0,4)$ (small) supersymmetry, of the type $\mathrm{AdS}_3\times$ $\mathrm{S}^3/\mathbb{Z}_k\times$ $\mathrm{CY}_2$ foliated over an interval. These solutions are holographic duals to 2d (small) $\mathcal{N} = (0,4)$ SCFTs supported by $M_A$-strings in M5-brane intersections. We identified the precise quivers in correspondence with the functions defining the backgrounds. Through the computation of the central charge we have checked the compliance with the holographic dictionary and the identification of $M_A$-strings as the defining degrees of freedom of our theories. We calculated the central charge of a two dimensional $\mathcal{N} = (0,4)$ QFT that flows to a conformal sigma model. To perform our calculation we used a brane intersection of M2 and M5 branes and the geometry they generate. If we were to think of our supergravity solutions as near horizons of higher dimensional black holes, our central charge would be identified with the entropy of these black holes. Notice that these ideas are similar to those in [1]. In fact, the entropy of black holes was calculated in [1] using a 2d $\mathcal{N} = (0,4)$ sigma model. Even when the brane configuration considered in [1] is different from the one we discussed here, the ideology is resemblant.

Through analytic continuation, we have constructed a second family of new solutions for which the modding is performed on the Hopf fibre of $\mathrm{AdS}_3$. These solutions are holographically dual to SCQM, which are the M(atrix) theory descriptions of M2-M5-M5' brane intersections, upon Sen-Seiberg limit. We have postponed a more detailed analysis of these supersymmetric quantum mechanics to our forthcoming publication [56]. We have shown that the subclasses of $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbb{Z}_k$ and $\mathrm{AdS}_3/\mathbb{Z}_k\times$ $\mathrm{S}^3$ M-theory solutions with $u' = 0$, contain self-dual strings. Therefore, they provide with explicit fully backreacted $\mathrm{AdS}_3\times$ $\mathrm{S}^3$ OM-theory [45] backgrounds [44].

Upon reduction, we have constructed a third new class of solutions to Type IIA supergravity with four Poincaré supercharges, of the type $\mathrm{AdS}_2\times$ $\mathrm{S}^3\times$ $\mathrm{CY}_2$ foliated over an interval. These solutions should be holographic duals to the quantum mechanical systems described above, in the regime of validity of the Type IIA description. We have extended these solutions to the massive case noticing that they are related through analytic continuation to the $\mathrm{AdS}_3\times\mathrm{S}^2\times$ $\mathrm{CY}_2$ solutions to massive Type IIA supergravity constructed in [18]. The dual quantum mechanics is under investigation in [56]. Three appendices extend our solutions to the most general class of $\mathrm{AdS}_3\times\mathrm{S}^2$ solutions to M-theory with (0,4) supersymmetries and SU(2) structure, and to new $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{M}_4$ solutions to massive IIA where $\mathrm{M}_4$ is a Kähler manifold. It would be interesting to understand the holographic duals for the general backgrounds presented there.

There is an interesting connection between our work and holographic duals of defect CFTs constructed in the literature. It was shown in [64] that a subclass of the solutions in [18] allowed for an interpretation in terms of 2d D2-D4 defects in the 6d $(1,0)$ CFTs living in D6-NS5-D8 brane intersections. Key to this realisation was the identification of a mapping between these solutions and the $\mathrm{AdS}_7$ solutions to massive Type IIA supergravity constructed in [65]. In the same vein, one would expect that a similar interpretation should be possible for our $\mathrm{AdS}_3$ M-theory solutions, this time in terms of 2d M2-M5 defects in the 6d (1,0) CFTs living in M5'-branes probing $A$-type singularities. In this direction, it would be interesting to show whether our solutions bear any relation to the flows constructed in [66, 67], which are asymptotically locally $\mathrm{AdS}_7$ in the UV. Along closely related lines, $\mathrm{AdS}_2\times\mathrm{S}^3\times$ R flows to asymptotically locally $\mathrm{AdS}_6$ in the UV have been constructed in [68, 69], and interpreted as 1d defects in the 5d Sp($N$) fixed point theory dual to the $\mathrm{AdS}_6$ Brandhuber-Oz solution [63]. We would expect that our $\mathrm{AdS}_2\times$ $\mathrm{S}^3$ solutions to massive Type IIA bear a relation to these, thus allowing for an interpretation as D0-F1-D4 brane defects within the Sp($N$) fixed point theory living in D4'-D8 branes. These issues are currently under investigation [43].


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## A Review of the general $\mathbf{AdS_3\times S^2\times M_4}$ solutions in [18]

In this appendix we summarise the generic backgrounds found in [18]. These backgrounds were divided in two classes: class I, for which the $\mathrm{M}_4$ is a Calabi-Yau 2-fold, and class II, for which $\mathrm{M}_4$ is a general 4d Kähler manifold. The particular case in class I in which the full solution respects the symmetries of the Calabi-Yau manifold, and therefore the Calabi-Yau manifold needs to be compact, was discussed in section 2. This is the case that concerned us in the main body of the paper. In these appendices we complete the analysis by providing the most general solutions in M-theory with (0,4) supersymmetries and SU(2) structure. In appendix B we present the uplift to M-theory of the most general solutions in class I and of the solutions in class II. In appendix C we construct $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{M}_4$ solutions to massive Type IIA supergravity through double analytical continuation of the class I and class II solutions.

We start reviewing the most general class I backgrounds in [18].

**Class I: $\mathbf{M_4=CY_2}$.** The explicit form of the NS sector of the solutions referred to as class I in [18] is given by,

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\bigg(\mathrm{d}s^2_{\mathrm{AdS}_3}+\frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8+(u')^2}\mathrm{d}s^2_{S^2}\bigg)+\sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2}+\frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\,,\\
e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4}\sqrt{u}}\sqrt{4\widehat{h}_4 h_8+(u')^2}\,,\\
H_{(3)} &= \frac{1}{2}\mathrm{d}\bigg(-\rho+\frac{uu'}{4\widehat{h}_4 h_8+(u')^2}\bigg)\wedge\widehat{\mathrm{vol}}_{\mathrm{S}^2}+\frac{1}{h_8}\mathrm{d}\rho\wedge H_2\,.
\end{aligned}\tag{A.1}$$

Here $\Phi$ is the dilaton, $H_{(3)}$ the NS three-form and the metric is given in string frame. A prime denotes a derivative with respect to $\rho$. The two-form $H_2$ is defined on the $\mathrm{CY}_2$ as we specify below. The RR sector reads

$$
\begin{aligned}
&F_{(0)} = h_8' , \quad F_{(2)} = -H_2 - \frac{1}{2}\Big(h_8 - \frac{h_8' u' u}{4h_8\widehat{h}_4 + (u')^2}\Big)\widehat{\mathrm{vol}}_{\mathrm{S}^2} , \\
&F_{(4)} = -\left(\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right) + 2h_8\mathrm{d}\rho\right) \wedge \widehat{\mathrm{vol}}_{\mathrm{AdS}_3} - \partial_\rho \widehat{h}_4 \widehat{\mathrm{vol}}_{\mathrm{CY}_2} - \frac{h_8}{u}(\widehat{\star}_4 \mathrm{d}_4 \widehat{h}_4) \wedge d\rho \\
&\qquad - \frac{u'u}{2(4\widehat{h}_4 h_8 + (u')^2)} H_2 \wedge \widehat{\mathrm{vol}}_{\mathrm{S}^2} ,
\end{aligned} \tag{A.2}
$$

where $\widehat{\star}_4$ is the Hodge dual on the $\mathrm{CY}_2$. Higher RR fluxes are related to these as $F_{(6)} = -\star F_{(4)}$, $F_{(8)} = \star F_{(2)}$, $F_{(10)} = -\star F_{(0)}$, where $\star$ is the ten-dimensional Hodge-dual operator. Supersymmetry holds when

$$
u'' = 0 , \qquad H_2 + \widehat{\star}_4 H_2 = 0 , \tag{A.3}
$$

which makes $u$ a linear function of $\rho$. $H_2$ is defined in terms of three functions $g_{1,2,3}$ on the $\mathrm{CY}_2$ and the vielbein on $\mathrm{M}_4$, $\widehat{e}^i$,

$$
H_2 = g_1(\widehat{e}^1 \wedge \widehat{e}^2 - \widehat{e}^3 \wedge \widehat{e}^4) + g_2(\widehat{e}^1 \wedge \widehat{e}^3 + \widehat{e}^2 \wedge \widehat{e}^4) + g_3(\widehat{e}^1 \wedge \widehat{e}^4 - \widehat{e}^2 \wedge \widehat{e}^3). \tag{A.4}
$$

Hence, the Bianchi identities of the fluxes impose

$$
\begin{aligned}
&h_8'' = 0 , \qquad dH_2 = 0 \\
&\frac{h_8}{u}\nabla^2_{\mathrm{CY}_2}\widehat{h}_4 + \partial^2_\rho \widehat{h}_4 + \frac{2}{h_8^3}(g_1^2 + g_2^2 + g_3^2) = 0.
\end{aligned} \tag{A.5}
$$

In the particular case when $H_2$ vanishes and $\widehat{h}_4$ has support on the $\rho$ coordinate we find that the supersymmetry and Bianchi identities are satisfied for $u$, $h_8$, $\widehat{h}_4$ arbitrary linear functions in $\rho$. We are then in the case reviewed in section 2 and lifted to eleven dimensions in section 3.

**Class II: $\mathbf{M_4}$ = Kähler.** We now summarise the details of the class II backgrounds in [18]. These are warped products of the form $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_4 \times \mathrm{I}$, where $\mathrm{M}_4$ is a family of Kähler four-manifolds with metrics that depend on the interval coordinate $\rho$, and with an integrable complex structure that is $\rho$-independent. These solutions have the following NS sector

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{hw^2 - v^2}}\left[\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{hw^2 - v^2}{4(hw^2 - v^2) + (u')^2}\mathrm{d}s^2_{\mathrm{S}^2}\right] + \frac{\sqrt{hw^2 - v^2}}{u}\left[\frac{u}{hw}\mathrm{d}s^2_{\mathrm{M}_4} + \mathrm{d}\rho^2\right], \\
H_{(3)} &= \frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4(hw^2 - v^2) + (u')^2}\right) \wedge \mathrm{vol}_{\mathrm{S}^2} + \mathrm{d}\left(\frac{v}{wh}\widehat{J}\right) , \\
e^{-\Phi} &= \frac{wh^{\frac{1}{2}}\sqrt{4(hw^2 - v^2) + (u')^2}}{2\sqrt{u}(hw^2 - v^2)^{\frac{1}{4}}}.
\end{aligned} \tag{A.6}
$$



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The functions $u, v$ and $w$ depend on $\rho$, while $h$ has support in $\rho$ and $\mathrm{M}_4$. $\widehat{J}$ is a two-form defined on the Kähler manifold.[4] The RR fluxes are given by,

$$
\begin{aligned}
F_{(0)} &= v', \\
F_{(2)} &= -\frac{w^2}{u}\mathrm{d}\rho\wedge\widehat{\star}_4(\mathrm{d}_4 h\wedge\widehat{J}) - \partial_\rho(w\widehat{J}) + \frac{vv'}{hw}\widehat{J} - \frac{1}{2}\left(v - \frac{v'uu'}{4(hw^2-v^2)+(u')^2}\right)\mathrm{vol}_{\mathrm{S}^2}, \\
F_{(4)} &= \frac{1}{2}\mathrm{vol}_{\mathrm{AdS}_3}\wedge\left(\mathrm{d}\left(\frac{vuu'}{hw^2-v^2}\right) + 4v\mathrm{d}\rho\right) + \frac{v}{2h}\left(\frac{vv'}{hw^2} - \partial_\rho\log(v^{-1}hw^2)\right)\widehat{J}\wedge\widehat{J} \\
&\quad - \frac{vw}{u}\mathrm{d}\rho\wedge\widehat{\star}_4\mathrm{d}\log h + \frac{1}{2}\left(\frac{uu'}{4(hw^2-v^2)+(u')^2}F_2 + \frac{hw^2-v^2}{hw}\widehat{J}\right)\wedge\mathrm{vol}_{\mathrm{S}^2}.
\end{aligned} \tag{A.7}
$$

Here $\mathrm{d}_4 = \partial + \overline{\partial}$, with $\partial, \overline{\partial}$ defined as the Dolbeault operators, expressed in terms of complex coordinates on $\mathrm{M}_4$.

Supersymmetry and the Bianchi identities (away from localised sources) hold by the following conditions,

$$
\begin{aligned}
&& u'' = 0, \qquad && \partial_\rho\left(\frac{\widehat{g}^{\frac{1}{2}}}{h}\right) = 0, \qquad && i\partial\overline{\partial}\log h = \widehat{\mathfrak{R}} \\
&\text{and} \qquad && v'' = 0, \qquad && 2i\partial\overline{\partial}h = \partial_\rho^2(w\widehat{J})\,. &&
\end{aligned} \tag{A.8}
$$

The quantity $\widehat{g}$ is the determinant of the metric and $\widehat{\mathfrak{R}}$ the Ricci form on $\mathrm{M}_4$.

## B New $\mathbf{AdS_3\times S^2\times M_4}$ solutions in M-theory

In this appendix we consider the uplift to eleven dimensions of the most general solutions in class I and the solutions in class II reviewed in the previous appendix. Our backgrounds provide the most general class of $\mathrm{AdS}_3\times\mathrm{S}^2$ solutions in M-theory with (0,4) supersymmetries and SU(2) structure. Note that a more general class of solutions with (0,4) SUSY can in principle be obtained taking an identity structure instead of the SU(2)-structure considered here. We will focus separately on the class I and class II backgrounds. In both cases, conditions must be imposed to allow the lift to eleven dimensions.

**Lift of the class I backgrounds.** We consider the class I geometries first. Imposing that $F_{(0)} = 0$ to allow the lift of the solutions described by equations (A.1)–(A.2), we find the eleven dimensional configurations,

$$
\begin{aligned}
\mathrm{d}s_{11}^2 &= \Delta\left(\frac{u}{\sqrt{\widehat{h}_4 h_8}}\mathrm{d}s^2_{\mathrm{AdS}_3} + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\right) + \frac{h_8^2}{4\Delta^2}\left(\mathrm{d}s^2_{\mathrm{S}^2} + (\mathrm{D}\tilde{\psi})^2\right), \\
G_{(4)} &= -\left(\mathrm{d}\left(\frac{uu'}{2\widehat{h}_4}\right) + 2h_8\mathrm{d}\rho\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_3} - \partial_\rho\widehat{h}_4\widehat{\mathrm{vol}}_{\mathrm{CY}_2} - \frac{uu'}{2(\widehat{h}_4 h_8 + (u')^2)}H_2\wedge\widehat{\mathrm{vol}}_{\mathrm{S}^2} \\
&\quad - \frac{h_8}{u}\star_4\mathrm{d}_4\widehat{h}_4\wedge\mathrm{d}\rho + \frac{h_8}{2}\left[\frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{S}^2} + \frac{1}{h_8}\mathrm{d}\rho\wedge H_2\right]\wedge\mathrm{D}\tilde{\psi}\,,
\end{aligned} \tag{B.1}
$$

[4] The interested reader can find a detailed explanation in [18].

where we have defined the following expressions,

$$
\begin{aligned}
H_2 &= -\mathrm{d}\mathcal{A}\,, \\
\mathrm{D}\tilde{\psi} &= \mathrm{d}\tilde{\psi} + \tilde{\mathcal{A}} + \omega \qquad \text{with} \qquad \mathrm{d}\omega = \widehat{\mathrm{vol}}_{S^2}\,, \\
\Delta &= \frac{h_8^{1/2}(4\widehat{h}_4 h_8 + u'^2)^{1/3}}{2^{2/3}\widehat{h}_4^{1/6} u^{1/3}}\,,
\end{aligned}
\tag{B.2}
$$

with $\tilde{\psi} = \frac{2}{h_8}\psi$ and $\tilde{\mathcal{A}} = \frac{2}{h_8}\mathcal{A}$. In (B.2) we have assumed that $dH_2 = 0$ holds globally, allowing us to globally define $H_2 = -d\mathcal{A}$. Notice that the connection $\tilde{\mathcal{A}} + \omega$ makes the fibre over the $\mathrm{S}^2$ and the $\mathrm{CY}_2$ non trivial. The uplift to eleven dimensions preserves the $\mathcal{N} = (0, 4)$ supersymmetry of the Type IIA solutions, as well as their SU(2)-structure.

Considering a sub-class of solutions –with $H_2 = 0$ and $\widehat{h}_4 = \widehat{h}_4(\rho)$ –we obtain $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbb{Z}_k\times\mathrm{CY}_2$ solutions to M-theory with $(0,4)$ (small) supersymmetry, warped over an interval. These were the solutions written in equations (3.1)–(3.2).

**Lift of the class II backgrounds.** To allow for a lift to M-theory, we impose that $F_{(0)} = 0$. Considering $v' = 0$ and uplifting the solution described by equations (A.6)–(A.7) we find,

$$
\begin{aligned}
ds_{11}^2 &= \Delta\left[\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{hw^2 - v^2}{hwu}\left(\mathrm{d}s^2_{M_4} + \frac{hw}{u}\mathrm{d}\rho^2\right)\right] + \frac{u^2w^2h}{4(hw^2 - v^2)\Delta^2}\left[\mathrm{d}s^2_{\mathrm{S}^2} + \frac{v^2}{w^2h}(\mathrm{D}\tilde{\psi})^2\right] \\
G_{(4)} &= \frac{1}{2}\mathrm{vol}_{\mathrm{AdS}_3}\wedge\left[\mathrm{d}\left(\frac{vuu'}{hw^2 - v^2}\right) + 4v\,\mathrm{d}\rho\right] - \frac{v}{2h}\left(\partial_\rho \log(v^{-1}hw^2)\right)\widehat{J}\wedge\widehat{J} \\
&\quad - \frac{vw}{u}\mathrm{d}\rho\wedge\widehat{\star}_4\mathrm{d}\log h + \frac{1}{2}\left(\frac{uu'}{4(hw^2 - v^2) + (u')^2}J_2 + \frac{hw^2 - v^2}{hw}\widehat{J}\right)\wedge\mathrm{vol}_{\mathrm{S}^2} \\
&\quad + \frac{v}{2}\left[\frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4(hw^2 - v^2) + (u')^2}\right)\wedge\mathrm{vol}_{\mathrm{S}^2} + \mathrm{d}\left(\frac{v}{wh}\widehat{J}\right)\right]\wedge\mathrm{D}\tilde{\psi},
\end{aligned}
\tag{B.3}
$$

where we have defined the following expressions,

$$
\begin{aligned}
\mathrm{D}\tilde{\psi} &= \mathrm{d}\tilde{\psi} + \tilde{\mathcal{J}} + \eta \qquad \text{with} \qquad \mathrm{d}\eta = \widehat{\mathrm{vol}}_{S^2}\,, \\
J_2 &= \mathrm{d}\tilde{\mathcal{J}} = -\frac{w^2}{u}\mathrm{d}\rho\wedge\widehat{\star}_4(\mathrm{d}_4 h\wedge\widehat{J}) - \partial_\rho(w\widehat{J}), \\
\Delta &= \left(\frac{uw\sqrt{h}\sqrt{4(hw^2 - v^2) + u'^2}}{2(hw^2 - v^2)}\right)^{2/3}.
\end{aligned}
\tag{B.4}
$$

Here $\tilde{\psi} = \frac{2}{v}\psi$ and $\tilde{\mathcal{J}} = \frac{2}{v}\mathcal{J}$. As before, the connection $\tilde{\mathcal{J}} + \eta$ makes the fibre over the $\mathrm{S}^2$ and the $\mathrm{M}_4$ non trivial. In order to find this uplift we have assumed that $\mathrm{d}J_2 = 0$ holds globally, allowing us to globally define $J_2 = \mathrm{d}\tilde{\mathcal{J}}$.

The uplift to eleven dimensions preserves the $\mathcal{N} = (0, 4)$ supersymmetry of the Type IIA solutions, as well as their SU(2)-structure.

## C New $\mathrm{AdS}_2 \times \mathrm{S}^3 \times \mathrm{M}_4$ solutions in massive Type IIA

Applying the set of transformations discussed around equation (4.2) to the previous M-theory solutions gives rise to $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{M}_4$ solutions with 4 Poincaré supercharges and

SU(2) structure. In these solutions the $\mathrm{AdS}_2$ is non-trivially fibrered. These solutions give upon reduction to Type IIA the double analytical continuation of the class I and class II solutions reviewed in appendix A. Thus, by acting with these rules directly on these sets of solutions we can generalise the backgrounds to the massive case. We present these backgrounds in this appendix. The $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{M}_4$ M-theory solutions arise upon uplift when $F_{(0)} = 0$.

**Class I backgrounds.** A sub-class of these solutions was presented in section 5. Here we generalise this class to the case in which there is a dependence of the fluxes on the $\mathrm{CY}_2$. Performing the double analytical continuation

$$u \to -iu\,, \qquad \widehat{h}_4 \to i\widehat{h}_4\,, \qquad h_8 \to ih_8\,, \qquad \rho \to i\rho\,, \tag{C.1}$$

together with

$$AdS_3 \to -S^3\,, \qquad S^2 \to -AdS_2\,, \tag{C.2}$$

in the most general solutions in class I, given by equations (A.1), (A.2), we arrive at

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8-(u')^2}\mathrm{d}s^2_{\mathrm{AdS}_2}+\mathrm{d}s^2_{\mathrm{S}^3}\right)+\sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2}+\frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\,,\\
e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4}\sqrt{u}}\sqrt{4\widehat{h}_4 h_8-(u')^2}\,,\\
H_{(3)} &= -\frac{1}{2}\mathrm{d}\left(\rho+\frac{uu'}{4\widehat{h}_4 h_8-(u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}+\frac{1}{h_8}\mathrm{d}\rho\wedge H_2\,.
\end{aligned}\tag{C.3}$$

The RR sector reads

$$\begin{aligned}
F_{(0)} &= h_8'\,, \quad F_{(2)} = -H_2-\frac{1}{2}\left(h_8+\frac{h_8' u' u}{4h_8\widehat{h}_4-(u')^2}\right)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}\,,\\
F_{(4)} &= \left(-\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right)+2h_8\mathrm{d}\rho\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{S}^3}-\frac{h_8}{u}\widehat{\star}_4\mathrm{d}_4 h_4\wedge d\rho-\partial_\rho\widehat{h}_4\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\\
&\quad+\frac{u'u}{2(4\widehat{h}_4 h_8-(u')^2)}H_2\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}\,.
\end{aligned}\tag{C.4}$$

These backgrounds generalise the solutions in section 5 to the case in which $H_2 \neq 0$ and $\nabla_{\mathrm{CY}_2}\widehat{h}_4 \neq 0$.

Supersymmetry holds when

$$u'' = 0\,, \qquad H_2+\widehat{\star}_4 H_2 = 0\,. \tag{C.5}$$

$H_2$ is defined in terms of three functions $g_{1,2,3}$ on the $\mathrm{CY}_2$ and the vielbein on $\mathrm{M}_4$, $\widehat{e}^i$, as in eq. (A.4). The Bianchi identities of the fluxes impose

$$\begin{aligned}
&h_8'' = 0\,, \qquad dH_2 = 0\\
&\frac{h_8}{u}\nabla^2_{\mathrm{CY}_2}\widehat{h}_4+\partial^2_\rho\widehat{h}_4+\frac{2}{h_8^3}(g_1^2+g_2^2+g_3^2) = 0.
\end{aligned}\tag{C.6}$$

Note that it must be that $4\widehat{h}_4 h_8-(u')^2 > 0$, in order for the metric to be of the correct signature and the dilaton to be real.



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**Class II backgrounds.** In this case we consider the following analytical continuation

$$u \to -iu\,, \qquad v \to iv\,, \quad w \to iw\,, \qquad \rho \to i\rho \tag{C.7}$$

together with

$$AdS_3 \to -S^3\,, \qquad S^2 \to -AdS_2\,, \tag{C.8}$$

of the class II solutions reviewed in appendix A. The NS sector of the background we get reads

$$\begin{aligned} \mathrm{d}s^2 &= \frac{u}{\sqrt{hw^2-v^2}}\left[\frac{hw^2-v^2}{4(hw^2-v^2)-(u')^2}\mathrm{d}s^2_{\mathrm{AdS_2}}+\mathrm{d}s^2_{\mathrm{S^3}}\right]+\frac{\sqrt{hw^2-v^2}}{u}\left[\frac{u}{hw}\mathrm{d}s^2_{\mathrm{M_4}}+\mathrm{d}\rho^2\right]\,,\\ e^{-\Phi} &= \frac{wh^{1/2}\sqrt{4(hw^2-v^2)-(u')^2}}{2u^{1/2}(hw^2-v^2)^{1/4}}\,, \quad H_{(3)}=\frac{1}{2}\mathrm{d}\left(-\rho-\frac{uu'}{4(hw^2-v^2)-(u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS_2}}\,. \end{aligned} \tag{C.9}$$

The RR sector is given by

$$\begin{aligned} F_{(0)} &= v'\,,\\ F_{(2)} &= -\frac{w^2}{u}\mathrm{d}\rho\wedge\widehat{\star}_4(\mathrm{d}_4h\wedge\widehat{J})-\partial_\rho w\widehat{J}+\frac{vv'}{hw}\widehat{J}-\frac{1}{2}\left(v+\frac{v'uu'}{4(hw^2-v^2)-(u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS_2}}\,,\\ F_{(4)} &= -\frac{1}{2}\widehat{\mathrm{vol}}_{\mathrm{S^3}}\wedge\left(\mathrm{d}\left(\frac{vuu'}{hw^2-v^2}\right)-4v\mathrm{d}\rho\right)+\frac{v}{2h}\left(\frac{vv'}{hw^2}-\partial_\rho\log(v^{-1}hw^2)\right)\widehat{J}\wedge\widehat{J}\\ &\quad -\frac{vw}{u}\mathrm{d}\rho\wedge\widehat{\star}_4\mathrm{d}\log h+\frac{1}{2}\left(-\frac{uu'}{4(hw^2-v^2)-(u')^2}F_{(2)}+\frac{hw^2-v^2}{hw}\widehat{J}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS_2}}\,. \end{aligned} \tag{C.10}$$

Here $\mathrm{d}_4=\partial+\overline{\partial}$, with $\partial,\overline{\partial}$ defined as the Dolbeault operators, expressed in terms of complex coordinates on $\mathrm{M_4}$. Supersymmetry and the Bianchi identities hold by the conditions,

$$\begin{aligned} &u''=0, \qquad \partial_\rho\left(\frac{\widehat{g}^{\frac{1}{2}}}{h}\right)=0, \quad i\partial\overline{\partial}\log h=\widehat{\mathfrak{R}}\\ \text{and} \qquad &v''=0, \qquad\quad 2i\partial\overline{\partial}h=\partial^2_\rho(w\widehat{J})\,. \end{aligned} \tag{C.11}$$

The backgrounds presented in this appendix provide the most general class of $AdS_2\times S^3$ solutions to massive Type IIA supergravity with 4 Poincaré supercharges and SU(2) structure.



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# 7. AdS$_2$/SCQM in Massive Type IIA

## 7.1 AdS$_2\times$S$^3\times$M$_5$ solutions in massive type IIA

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# $\mathrm{AdS}_2$ duals to ADHM quivers with Wilson lines

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Abstract: We discuss $\mathrm{AdS}_2 \times S^3 \times \mathrm{CY}_2 \times I_\rho$ solutions to massive Type IIA supergravity with 4 Poincaré supersymmetries. We propose explicit dual quiver quantum mechanics built out of D0 and D4 colour branes coupled to D4' and D8 flavour branes. We propose that these quivers describe the interactions of instantons and Wilson lines in 5d gauge theories with 8 Poincaré supersymmetries. Using the RR Maxwell fluxes of the solutions, conveniently put off-shell, we construct a functional from which the holographic central charge can be derived through a geometrical extremisation principle.

Keywords: AdS-CFT Correspondence, Brane Dynamics in Gauge Theories, String Duality

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## Contents





## 1 Introduction

Recent progress in Holography deepened our understanding of lower dimensional realisations of the AdS/CFT correspondence [1]. These scenarios are particularly relevant for the development of the black hole microstate counting programme. Indeed, 4d and 5d extremal black holes exhibit $\mathrm{AdS}_2$ and $\mathrm{AdS}_3$ geometries close to their horizons.

Efforts in classifying $\mathrm{AdS}_2$ and $\mathrm{AdS}_3$ geometries in M-theory and Type II supergravity have revealed that a plethora of these solutions exists, exhibiting different geometrical structures and preserving different amounts of supersymmetries, see for example [2–39].

Indeed, as the dimensionality of the internal space increases, the richer the structure of all possible geometries and topologies thereof becomes. Exhausting all possibilities through classifications becomes then increasingly complex.

Major recent progress has been achieved in the classification of $\mathrm{AdS}_3$ geometries with $\mathcal{N} = (0, 4)$ supersymmetries and SU(2) structure [15, 28]. Remarkably, the 2d CFTs dual to subsets of these solutions have been identified [40–42] (see also [43–45]), thus providing for explicit $\mathrm{AdS}_3/\mathrm{CFT}_2$ pairs where the microscopical description of 5d black holes can be addressed. Generalising recent developments in five dimensional BPS black holes (see for instance [46–48]) to these new set-ups constitutes a promising new avenue that is still awaiting for further development.

$\mathrm{AdS}_2$ geometries arise as near horizon geometries of 4d extremal black holes, and are thus ubiquitous in their microscopical studies. The precise realisation of the $\mathrm{AdS}_2/\mathrm{CFT}_1$ holographic correspondence presents however important technical and conceptual problems [49–52]. These mainly originate from the fact that the boundary of $\mathrm{AdS}_2$ is non-connected [53]. As a result this correspondence is much less understood than its higher dimensional counterparts.

A promising approach to the study of the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence is to exploit its connection with the better understood $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence. This approach has been explored recently in [38]. At the geometrical level the $\mathrm{AdS}_3$ and $\mathrm{AdS}_2$ spaces are related by Abelian T-duality. At the level of the dual CFTs the Superconformal Quantum Mechanics (SCQM) dual to the $\mathrm{AdS}_2$ solutions arise from the 2d CFTs dual to the $\mathrm{AdS}_3$ backgrounds upon dimensional reduction. In this manner new families of $\mathrm{AdS}_2$ solutions to Type IIB supergravity with $\mathcal{N} = 4$ supercharges and their dual SCQMs were constructed in [38], using as seed solutions the $\mathcal{N} = (0, 4)$ $\mathrm{AdS}_3$ solutions to massive Type IIA supergravity recently found in [35]. These constructions provide explicit string theory set-ups in which one of the chiral sectors of the 2d CFT is decoupled in the SCQM, as explained in [54–56]. Remarkably, the corresponding quivers inherit, by construction, many of the properties of the parent 2d quiver CFTs, like the matter content that guarantees gauge anomaly cancellation in 2d.

Given that SCQMs do not have these constraints, one would expect that more general quivers than those arising upon reduction could be constructed. With this goal in mind we will follow in this paper an alternative road to the study of $\mathrm{AdS}_2/\mathrm{CFT}_1$ pairs. Our starting point will be the new class of $\mathrm{AdS}_2$ solutions with $\mathcal{N} = 4$ supercharges recently constructed in [35], using the technique of double analytical continuation. Therefore, a priori one does not expect any relation between these solutions and 2d CFTs.[1]

The technique of double analytical continuation has been extensively used in the construction of $\mathrm{AdS}_2$ solutions. Indeed, the only requirement to produce an $\mathrm{AdS}_2$ solution is that an $S^2$ exists in the internal space of an already known supergravity background, that can be analytically continued to $\mathrm{AdS}_2$. Fortunately, many $\mathrm{AdS}_p$ supergravity solutions contain $S^2$'s in their transverse spaces. The $\mathrm{AdS}_p$ subspace itself gives rise upon the

[1]Besides the fact that, as we will see, the 1d dual CFTs will be formulated in terms of (0,4) 2d matter fields.



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analytical continuation to an $S^p$ that is part of the internal space of the new solutions, and realises some of its isometries. Some examples of $\mathrm{AdS}_2$ solutions constructed in this way are the $\mathrm{AdS}_2 \times S^6$ backgrounds constructed in [19, 25] from the class of $\mathrm{AdS}_6 \times S^2$ solutions to Type IIB in [57–60], the $\mathrm{AdS}_2 \times S^7$ solutions to massive Type IIA constructed in [21] from the $\mathrm{AdS}_7 \times S^2$ solutions of [61], or the $\mathrm{AdS}_2 \times S^4 \times S^2$ solutions constructed in [32], from the $\mathrm{AdS}_4 \times S^2 \times S^2$ backgrounds in [62, 63]. These solutions preserve different amounts of supersymmetries, and allow, in some cases, for interesting interpretations as line defect CFTs [24, 36, 37] or deconstructed higher dimensional CFTs [32].

In this paper we focus our study on the $\mathrm{AdS}_2 \times S^3 \times \mathrm{CY}_2 \times I_\rho$ solutions to massive Type IIA supergravity recently constructed in [35]. These solutions can be thought of as the dual description of the quantum mechanics of a point like defect in a five-dimensional CFT. The contents of this work are distributed as follows. In section 2 we review the main properties of these solutions, we compute the quantised charges and propose the underlying brane set-up. This is a 1/8 BPS D0-D4-D4'-D8-F1 brane intersection, previously studied in [24, 35, 37]. We show that the D0, D4 and F1 branes are most naturally counted with their (regularised) electric charges, rather than their magnetic ones. This is in agreement with their interpretation in terms of instantons in the worldvolumes of the D4' and D8 branes, interacting with Wilson lines. In section 3 we set to discuss the superconformal mechanics dual to these solutions. We start reviewing the CFT dual to the D0-D4-F1 system, described in the near horizon limit by one of our solutions. This example is used to introduce the low-energy fields that will enter in the quantum mechanics dual to more general solutions. We construct explicit quiver quantum mechanics that we interpret as describing D0 and D4 brane instantons in the worldvolumes of D4' and D8 branes, interacting with Wilson lines in the antisymmetric representations of their gauge groups. We use the notion of central charge for superconformal quantum mechanics put forward in [38], defined from the number of vacuum states of the theory, and compare it to the holographic calculation. In section 4 we further elaborate on the relation between the holographic central charge and the product of electric and magnetic RR charges of $\mathrm{AdS}_2$ solutions pointed out in [38]. Furthermore, we propose an extremising functional constructed from the RR Maxwell fluxes from which we derive the holographic central charge through an extremisation principle. Section 5 contains our conclusions and future directions. Appendix A contains a summary of the $\mathrm{AdS}_3 \times S^2 \times \mathrm{CY}_2$ solutions and their 2d dual CFTs studied in [28, 40–42]. These motivate the families of backgrounds for which we construct dual SCQMs in this paper. Appendix B contains a detailed derivation of the low-energy field content of the D0-D4-D4'-D8 brane web. Appendix C studies different probe branes of interest in our backgrounds. Finally, appendices D and E contain specific geometrical properties of our solutions.

## 2 The $\mathrm{AdS}_2 \times S^3 \times$ $\mathrm{CY}_2$ solutions to massive IIA

In this section we present and further discuss the $\mathrm{AdS}_2 \times S^3 \times$ $\mathrm{CY}_2$ solutions to massive IIA supergravity constructed in [35]. These solutions were obtained through a double analytic continuation of the $\mathrm{AdS}_3 \times S^2 \times$ $\mathrm{CY}_2$ backgrounds to massive IIA constructed in [28], and summarised in appendix A (see eqs. (A.1), (A.3a) and (A.3b)). We give a thorough study of the underlying geometry that will allow us to propose concrete superconformal quantum mechanics dual to the solutions.

The class of solutions referred as class I in [35] contain a transverse space $\mathrm{M}_4 = \mathrm{CY}_2$.[2] They are given by

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8-(u')^2}\mathrm{d}s^2_{\mathrm{AdS}_2}+\mathrm{d}s^2_{S^3}\right)+\sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2}+\frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\,,\\
e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4}\sqrt{u}}\sqrt{4\widehat{h}_4 h_8-(u')^2}\,,\\
H_{(3)} &= -\frac{1}{2}\mathrm{d}\left(\rho+\frac{uu'}{4\widehat{h}_4 h_8-(u')^2}\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}+\frac{1}{h_8^2}\mathrm{d}\rho\wedge H_2\,.
\end{aligned} \tag{2.1}$$

Here $\Phi$ is the dilaton, $H_{(3)} = \mathrm{d}B_{(2)}$ is the NS 3-form and the metric is written in string frame. The warping function $\widehat{h}_4$ has support on $(\rho, \mathrm{CY}_2)$. On the other hand, $u$ and $h_8$ only depend of $\rho$. We denote $u' = \partial_\rho u$ and similarly for $h_8'$. Note that it must be that $4\widehat{h}_4 h_8 - (u')^2 > 0$, in order for the metric to be of the correct signature and the dilaton to be real.

The RR sector reads

$$\begin{aligned}
F_{(0)} &= h_8'\,,\\
F_{(2)} &= -\frac{1}{h_8}H_2-\frac{1}{2}\left(h_8+\frac{h_8' u' u}{4h_8\widehat{h}_4-(u')^2}\right)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}\,,\\
F_{(4)} &= \left(-\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right)+2h_8\mathrm{d}\rho\right)\wedge\widehat{\mathrm{vol}}_{S^3}-\frac{h_8}{u}\widehat{\star}_4\mathrm{d}_4 h_4\wedge d\rho-\partial_\rho\widehat{h}_4\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\\
&\quad+\frac{u'u}{2h_8(4\widehat{h}_4 h_8-(u')^2)}H_2\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}\,,
\end{aligned} \tag{2.2}$$

with the higher dimensional fluxes related to these as $F_{(6)} = -\star_{10} F_{(4)}$, $F_{(8)} = \star_{10} F_{(2)}$, $F_{(10)} = -\star_{10} F_{(0)}$. Supersymmetry holds whenever

$$u'' = 0, \qquad H_2 + \widehat{\star}_4 H_2 = 0, \tag{2.3}$$

where $\widehat{\star}_4$ is the Hodge dual on $\mathrm{CY}_2$. In what follows we will restrict ourselves to the set of solutions for which $H_2 = 0$ and $\widehat{h}_4 = \widehat{h}_4(\rho)$. In that case the background is a solution of the massive IIA equations of motion if the functions $\widehat{h}_4, h_8$ satisfy the conditions (away from localised sources),

$$\widehat{h}_4''(\rho) = 0, \qquad h_8''(\rho) = 0, \tag{2.4}$$

which make them linear functions of $\rho$.

[2]A second class of solutions, referred as class II, contain an $\mathrm{M}_4$ Kähler manifold.

|  | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D0 | – |  |  |  |  |  |  |  |  |  |
| D4 | – | – | – | – | – |  |  |  |  |  |
| D4' | – |  |  |  |  |  | – | – | – | – |
| D8 | – | – | – | – | – |  | – | – | – | – |
| F1 | – |  |  |  |  | – |  |  |  |  |

**Table 1**. Brane set-up, where $-$ marks the spacetime directions spanned by the various branes. $x^0$ corresponds to the time direction of the ten dimensional spacetime, $x^1, \ldots, x^4$ are the coordinates spanned by the $\mathrm{CY}_2$, $x^5$ is the direction where the F1-strings are stretched, and $x^6, x^7, x^8, x^9$ are the coordinates where the SO(4) symmetry is realised.

### 2.1 Brane set-up

The brane set-up associated to the previous solutions was identified in [37] (see also [24]), for the case of $u =$ constant.[3] It consists on a D0-F1-D4-D4'-D8 brane intersection, depicted in table 1, preserving four supersymmetries. This is compatible with the quantised charges obtained from the Page fluxes, as we show below.

The Page fluxes associated to the background given in (2.1), (2.2), $\widehat{F} = F \wedge e^{-B_{(2)}}$, are given by[4],

$$\begin{aligned}
\widehat{F}_{(0)} &= h_8' \,, \\
\widehat{F}_{(2)} &= -\frac{1}{2}\Big(h_8 - (\rho - 2\pi k)h_8'\Big)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \,, \\
\widehat{F}_{(4)} &= -\widehat{h}_4'\widehat{\mathrm{vol}}_{\mathrm{CY}_2} - \left(2h_8 + \frac{u'(u\widehat{h}_4' - \widehat{h}_4 u')}{2\widehat{h}_4^2}\right)\widehat{\mathrm{vol}}_{S^3} \wedge \mathrm{d}\rho \,, \\
\widehat{F}_{(6)} &= \frac{1}{2}\Big(\widehat{h}_4 - (\rho - 2\pi k)\widehat{h}_4'\Big)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \wedge \widehat{\mathrm{vol}}_{\mathrm{CY}_2} \\
&\quad - \left((\rho - 2\pi k)h_8 - \frac{(u - (\rho - 2\pi k)u')(\widehat{h}_4 u' - u\widehat{h}_4')}{4\widehat{h}_4^2}\right)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \wedge \widehat{\mathrm{vol}}_{S^3} \wedge \mathrm{d}\rho \,, \\
\widehat{F}_{(8)} &= \left(2\widehat{h}_4 + \frac{u'(uh_8' - h_8 u')}{2h_8^2}\right)\widehat{\mathrm{vol}}_{\mathrm{CY}_2} \wedge \widehat{\mathrm{vol}}_{S^3} \wedge \mathrm{d}\rho \,, \\
\widehat{F}_{(10)} &= \left((\rho - 2\pi k)\widehat{h}_4 - \frac{(u - (\rho - 2\pi k)u')(uh_8' - h_8 u')}{4h_8^2}\right)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \wedge \widehat{\mathrm{vol}}_{S^3} \wedge \widehat{\mathrm{vol}}_{\mathrm{CY}_2} \wedge \mathrm{d}\rho \,.
\end{aligned} \tag{2.5}$$

The F1-branes are electrically charged with respect to the NS-NS 3-form. We compute their charges according to

$$Q^e_{\mathrm{F1}} = \frac{1}{(2\pi)^2}\int_{\mathrm{AdS}_2 \times \mathrm{I}_\rho} H_{(3)} \,, \tag{2.6}$$

[3]The $u$ non-constant case was recently analysed in [64] for the $\mathrm{AdS}_3 \times S^2 \times \mathrm{CY}_2$ backgrounds from which our solutions are constructed by analytical continuation, in the massless case. The brane intersection involves in this case dyonic branes placed at conical singularities.

[4]These fluxes take into account the effect of the large gauge transformations $B_2 \to B_2 + \pi k \widehat{\mathrm{vol}}_{\mathrm{AdS}_2}$, for $k = 0, 1, \ldots, P$. These transformations are performed every time a $\rho$-interval $[2\pi k, 2\pi(k+1)]$ is crossed, as explained below.

in units of $\alpha' = g_s = 1$. We regularise the volume of the $\mathrm{AdS}_2$ space such that[5]

$$\mathrm{Vol}_{\mathrm{AdS}_2} = 4\pi \,. \tag{2.7}$$

This gives for $H_{(3)}$ as in (2.1) (and $H_2 = 0$),

$$Q^e_{\mathrm{F1}} = \frac{1}{2\pi}\left(\rho - \frac{uu'}{4h_4h_8 - u'^2}\right)\Big|^{\rho_f}_{\rho_i}, \tag{2.8}$$

where this must be computed at both ends of the $I_\rho$-interval. From now and for the rest of the paper we will focus our analysis on the $u =$ constant case. In such case

$$Q^e_{\mathrm{F1}} = \frac{\rho_f - \rho_i}{2\pi}. \tag{2.9}$$

Therefore, there are $k$ F1-strings for $\rho \in [0, 2\pi k]$, with one F1-string being created as we move in $\rho$-intervals of length $2\pi$. Enforcing that with our regularisation prescription (2.7) $B_2$ lies in the fundamental region,

$$\frac{1}{(2\pi)^2}\Big|\int_{\mathrm{AdS}_2} B_{(2)}\Big| \in [0, 1]\,, \tag{2.10}$$

then implies that a large gauge transformation of parameter $k$ needs to be performed for $\rho \in [2\pi k, 2\pi(k + 1)]$, such that

$$B_{(2)} = -\frac{1}{2}(\rho - 2k\pi)\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}. \tag{2.11}$$

This affects the RR Page fluxes, as

$$\widehat{F}_{(p)} \to \widehat{F}_{(p)} - k\pi\widehat{F}_{(p-2)} \wedge \widehat{\mathrm{vol}}_{\mathrm{AdS}_2}, \tag{2.12}$$

which has already been taken into account in the expressions in (2.5).

As we will see, the D0 and D4 branes will have an interpretation in the dual field theory as instantons. Therefore, we will characterise them by their electric charges. We use that the electric charge of a Dp-brane is given by

$$Q^e_{\mathrm{Dp}} = \frac{1}{(2\pi)^{p+1}}\int_{\mathrm{AdS}_2\times\Sigma_p} \widehat{F}_{(p+2)}, \tag{2.13}$$

in units of $\alpha' = g_s = 1$. Substituting the electric components of the $\widehat{F}_{(2)}$ and $\widehat{F}_{(6)}$ fluxes in (2.5) and regularising the volume of $\mathrm{AdS}_2$ as indicated by equation (2.7), we find for the electric charges of the D0 and D4 branes,

$$\begin{aligned} Q^e_{\mathrm{D0}} &= h_8 - (\rho - 2\pi k)h_8' \\ Q^e_{\mathrm{D4}} &= h_4 - (\rho - 2\pi k)h_4'\,, \end{aligned} \tag{2.14}$$

[5]This regularisation prescription is based on the analytical continuation that relates the $\mathrm{AdS}_2$ space with an $S^2$.

where in the last equation we have used that $\widehat{h}_4 \equiv \Upsilon h_4$ and $\Upsilon \mathrm{Vol}_{\mathrm{CY}_2} = 16\pi^4$. Both the D0 and the D4 branes play the role of colour branes in the brane set-up, as implied by the fact that both $\mathrm{d}\widehat{F}_{(8)}$ and the second component of $\mathrm{d}\widehat{F}_{(4)}$ vanish identically.

The D4' and D8 branes will find an interpretation in the dual field theory as branes where the D0 and D4 brane instantons live. We will characterise them by their magnetic charges. As usual these are computed from

$$Q^m_{\mathrm{Dp}} = \frac{1}{(2\pi)^{7-p}} \int_{\Sigma_{8-p}} \widehat{F}_{(8-p)}\,, \tag{2.15}$$

in units of $\alpha' = g_s = 1$. The second component of $\widehat{F}_{(4)}$ in (2.5) is the Hodge-dual of the $\widehat{F}_{(6)}$ used in (2.14) to compute the electric charge of the D4-branes. In turn, the first component gives rise to a magnetic charge associated to a second type of D4'-branes. This charge reads

$$Q^m_{\mathrm{D4}'} = \frac{\mathrm{Vol}_{\mathrm{CY}_2}}{(2\pi)^3}\, \widehat{h}_4' = 2\pi h_4'\,. \tag{2.16}$$

Given that

$$\mathrm{d}\widehat{F}_{(4)} = \widehat{h}_4''\, \mathrm{d}\rho \wedge \widehat{\mathrm{vol}}_{\mathrm{CY}_2}\,, \tag{2.17}$$

these branes provide sources localised in the $\rho$ direction. They are thus flavour branes. Being localised in $\rho$ and transverse to the $\mathrm{CY}_2$, they are naturally seen to wrap $\mathrm{AdS}_2 \times S^3$. In turn, given that $\mathrm{d}\widehat{F}_{(0)} \neq 0$ if $h_8'' \neq 0$, according to

$$\mathrm{d}\widehat{F}_{(0)} = h_8'' \mathrm{d}\rho\,, \tag{2.18}$$

there are also D8 brane sources localised in the $\rho$ direction, also behaving as flavour branes. They are wrapped on $\mathrm{AdS}_2 \times S^3 \times \mathrm{CY}_2$. Their magnetic charge is given by

$$Q^m_{\mathrm{D8}} = 2\pi h_8'. \tag{2.19}$$

### 2.2 The local solutions

For $u =$ constant a generic background in our class is defined by the functions $\widehat{h}_4$, $h_8$. We will be interested in solutions that in the $\rho \in [2\pi k, 2\pi(k+1)]$ interval are of the form

$$\widehat{h}_4^{(k)} = \Upsilon\left(\alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)\right), \qquad h_8^{(k)} = \left(\mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k)\right), \tag{2.20}$$

with the space starting and ending at $\rho = 0$ and $\rho = 2\pi(P+1)$, respectively, where we take both $\widehat{h}_4$ and $h_8$ to vanish. We thus have that

$$\begin{aligned} \widehat{h}_4(\rho) =& \Upsilon\, h_4(\rho) \\ =& \Upsilon \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1), \quad k = 1, \dots, P-1 \\ \alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1), \end{cases} \end{aligned} \tag{2.21}$$

$$h_8(\rho) = \begin{cases} \frac{\nu_0}{2\pi}\rho & 0 \le \rho \le 2\pi \\ \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1), \quad k = 1, \dots, P-1 \\ \mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{2.22}$$

By imposing continuity the $\alpha_k$, $\mu_k$ integration constants are determined from $\beta_k$, $\nu_k$ as (see appendix B),

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad \mu_k = \sum_{j=0}^{k-1} \nu_j. \tag{2.23}$$

These profiles for the $\widehat{h}_4$ and $h_8$ functions were the ones taken in [40–42] for the construction of the 2d CFTs dual to the $\mathrm{AdS}_3 \times S^2 \times \mathrm{CY}_2$ solutions in [28]. These results are summarised in appendix A. The supergravity backgrounds can be trusted when $\beta_k, \nu_k, P$ are large. Taking $\beta_k, \nu_k$ to be large controls the divergence of the Ricci scalar at the points where the sources are localised. In turn, these singularities are spread out for $P$ large.

The behaviour of the solutions defined by (2.21) and (2.22) (and $u =$ constant) at both ends of the $\rho$-interval is that of a superposition of D4 branes smeared on the $\mathrm{CY}_2$ and D8 branes. Note that the same behaviour is obtained from a superposition of O4 and O8 orientifold fixed planes, with the O4 smeared on the $\mathrm{CY}_2$. The smearing of these objects is an effect of the solutions being only $\rho$-dependent. A more elaborated configuration, for which $\widehat{h}_4$ depends both on $\rho$ and the coordinates of the $\mathrm{CY}_2$, would have these object localised. We may think as our solutions as the zero mode in a putative Fourier decomposition on the $\mathrm{CY}_2$. We will find an interesting interpretation for this behaviour in the following sections. Indeed, for very small values of $\rho$, the metric and dilaton read

$$\mathrm{d}s^2 \simeq \rho^{-1}\left(\mathrm{d}s^2_{\mathrm{AdS}_2} + 4\mathrm{d}s^2_{S^3}\right) + \mathrm{d}s^2_{\mathrm{CY}_2} + \rho\, \mathrm{d}\rho^2\,, \qquad e^{-2\Phi} \simeq \rho^3\,, \tag{2.24}$$

while for $\rho \to 2\pi(P+1)$ we have

$$\mathrm{d}s^2 \simeq x^{-1}\left(\mathrm{d}s^2_{\mathrm{AdS}_2} + 4\mathrm{d}s^2_{S^3}\right) + \mathrm{d}s^2_{\mathrm{CY}_2} + x\, \mathrm{d}x^2\,, e^{-2\Phi} \simeq x^3\,, \tag{2.25}$$

with $x = 2\pi(P+1) - \rho$. We recognise these behaviours as those of a superposition of (smeared) D4 and D8 branes and/or (smeared) O4 and O8 orientifold fixed planes.

Using the electric and magnetic charges discussed in the previous subsection to count, respectively, the colour and flavour brane charges, we find for $\rho$ in the $[2\pi k, 2\pi(k+1)]$ interval,

$$Q^{e\,(k)}_{\mathrm{D0}} = \mu_k\,, \qquad\qquad Q^{e\,(k)}_{\mathrm{D4}} = \alpha_k \tag{2.26}$$

$$Q^{m\,(k)}_{\mathrm{D4}'} = \beta_k\,, \qquad\qquad Q^{m\,(k)}_{\mathrm{D8}} = \nu_k\,. \tag{2.27}$$

These equations show that the constants $\alpha_k$, $\mu_k$, $\beta_k$, $\nu_k$ must be integer numbers. Moreover, they show that the D0 and D4 brane charges in each $[2\pi k, 2\pi(k+1)]$ interval are equal to the total D8 and D4' brane charges in the $[0, 2\pi k]$ previous intervals. Namely, $\alpha_k = \sum_{j=0}^{k-1} \beta_j$, $\mu_k = \sum_{j=0}^{k-1} \nu_j$. We will find an interesting interpretation for this result when we discuss the dual field theory in section 3.2.

### 2.3 Holographic central charge

We compute the holographic central charge following the prescription in [38]. In this section we consider general solutions with $u$ a linear function of $\rho$. We recall that as discussed in that reference the holographic central charge for an $\mathrm{AdS}_2$ solution is to be interpreted as the number of vacuum states of the dual SCQM. The prescription in [38] reads,

$$c_{\text{holo}} = \frac{3V_{\text{int}}}{4\pi G_N}\,, \tag{2.28}$$

where $G_N = 8\pi^6$ and $V_{\text{int}}$ is the volume of the internal space, which must be corrected by a dilaton term, following [65, 66]. For the backgrounds described by (2.1) and (2.2), $V_{\text{int}}$ reads

$$V_{\text{int}} = \int \mathrm{d}^8x\sqrt{e^{-4\Phi}\det g_{8,ind}} = \frac{\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{S^3}}{4}\int_0^{2\pi(P+1)} \mathrm{d}\rho\left(4\widehat{h}_4 h_8 - u'^2\right), \tag{2.29}$$

from where, using that $\Upsilon \mathrm{Vol}_{\mathrm{CY}_2} = 16\pi^4$, we find

$$c_{\text{holo}} = \frac{3}{4\pi}\int_0^{2\pi(P+1)} \mathrm{d}\rho\left(4h_4h_8 - u'^2\right). \tag{2.30}$$

We will refer to this expression in the next sections for our particular solutions with $u =$ constant.

## 3 The dual superconformal quantum mechanics

In this section we discuss the $\mathcal{N} = 4$ super-conformal quantum mechanical theories that we propose as duals to the $\mathrm{AdS}_2$ solutions with the defining functions given by equations (2.21), (2.22) (and $u =$ constant). We provide a UV $\mathcal{N} = 4$ quantum mechanics, that conjecturally flows to a super conformal quantum mechanics dual to these backgrounds.

We start analysing a well-known particular solution in this class. This is the Abelian T-dual of the $\mathrm{AdS}_3 \times S^3 \times$ $\mathrm{CY}_2$ solution to Type IIB, along the $S^1$ fibre direction of the $\mathrm{AdS}_3$ space. Thus, the dual SCQM to this solution arises as the IR fixed point of the quantum mechanical quiver that is obtained dimensionally reducing the 2d QFT living in the D1-D5 system. Though many of the subtleties in our generic case do not appear in this simple setting, it is useful to study it first. In fact, this example illustrates the field content that will appear in our quivers and sets up the discussion for the extension to more general ones. Appendix B contains a detailed account of the low-energy field content emerging from the brane web associated to our solutions.

### 3.1 Warm up: quantum mechanics of the D0-D4 system

In order to motivate the construction of the quantum mechanics dual to our $\mathrm{AdS}_2$ backgrounds we focus first on the Abelian T-dual of the D1-D5 system, whose near horizon geometry is the $\mathrm{AdS}_3 \times S^3 \times$ $\mathrm{CY}_2$ solution of Type IIB. In this case the T-duality is performed on the $S^1$ Hopf fibre contained in $\mathrm{AdS}_3$, leaving an $\mathrm{AdS}_2$ solution in our class where the $\rho$-direction lives in the T-dual circle. In this case $u$, $h_4$ and $h_8$ are the constant functions,

$$u = 16L^4M^2\,, \qquad h_4 = 4L^2M^4\,, \qquad h_8 = 4L^2\,, \tag{3.1}$$

| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D0 | – | | | | | | | | | |
| D4 | – | – | – | – | – | | | | | |
| F1 | – | | | | | – | | | | |

**Table 2**. Brane set-up associated to the D0-D4-F1 brane system T-dual to the D1-D5 system.

and the solution reads

$$\mathrm{d}s^2 = L^2\Big(\mathrm{d}s^2_{\mathrm{AdS}_2} + 4\mathrm{d}s^2_{S^3}\Big) + M^2\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{1}{4L^2}\mathrm{d}\rho^2 \tag{3.2}$$

$$e^{-\Phi} = 2L \tag{3.3}$$

$$H_{(3)} = -\frac{1}{2}\mathrm{d}\rho\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2} \tag{3.4}$$

$$F_{(4)} = -8L^2\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho \tag{3.5}$$

$$F_{(8)} = 8L^2M^4\widehat{\mathrm{vol}}_{S^3}\wedge\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho\,, \tag{3.6}$$

with $\rho\in[0,2\pi]$. As in many other examples where T-duality is performed along a Hopf-fibre direction, the T-dual background preserves half of the supersymmetries of the original solution. In our case the $\mathrm{SO}(4)_R$ symmetry of the $\mathrm{AdS}_3\times S^3\times\mathrm{CY}_2$ solution is broken to $\mathrm{SU}(2)_R$, the other SU(2) becoming a global symmetry. Together with the $\mathrm{SL}(2,\mathbb{R})$ isometry group of $\mathrm{AdS}_2$, $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SU}(2)$ span the bosonic subgroup of $\mathrm{SU}(1,1|2)$. This is one of the two possible superconformal groups with 4 supercharges in one dimension, the other being $D(2,1;\alpha)$, $\alpha\neq-1,0$, for which the R-symmetry is two SU(2)'s, and thus corresponds to large superconformal symmetry.

The D1-D5 system gives rise upon T-duality to D0-D4 branes plus an extra F1-brane. The corresponding 1/4-BPS brane set-up is depicted in table 2. The numbers of D0 and D4 branes are computed using equations (2.14). We obtain that $Q^e_{\mathrm{D0}}=h_8$, $Q^e_{\mathrm{D4}}=h_4$. There is also one F1-string that extends in the $\rho$-direction. If before the T-duality the $\mathrm{AdS}_3$ subspace is orbifolded by $\mathbb{Z}_N$, which is equivalent to introducing $N$ units of momentum in the D1-D5 system, breaking the supersymmetries to (0,4), then $N$ F1-strings are generated after the T-duality. These strings stretch in $\rho$ between $2\pi k$ and $2\pi(k+1)$, with $k=0,\ldots,N-1$. The central charge of the D1-D5-wave system [67],

$$c = 6\,Q_w Q_{\mathrm{D1}}Q_{\mathrm{D5}} \tag{3.7}$$

is reproduced after the T-duality as

$$c = 6\,Q_{\mathrm{F1}}Q_{\mathrm{D0}}Q_{\mathrm{D4}}\,. \tag{3.8}$$

As a consistency check for expression (2.30) we can show that it reproduces this field theory result.

We next discuss in some detail the quiver depicted in figure 1. This quiver is the dimensional reduction of the 2d (4,4) quiver CFT that lives in the D1-D5 system. The

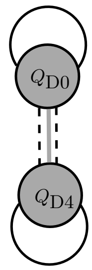


**Figure 1**. Quiver quantum mechanics associated to the D0-D4-F1 brane system. The solid black lines represent (4,4) adjoint hypermultiplets, the grey line a (0,4) hypermultiplet and the dashed lines two $(0,2)$ Fermi multiplets. Circles represent $\mathcal{N} = (4,4)$ vector multiplets.



usage of 2d (0,4) notation to describe 1d super quantum mechanics is widespread in the literature. The reader is referred to [68, 69] for a detailed presentation of this description. In our example this is further motivated by its explicit relation to the 2d (4,4) CFT living in the D1-D5 system. The detailed field content of the quiver depicted in figure 1 is as follows (see appendix B):

- Circles represent (4,4) vector multiplets. They are associated to gauge nodes. They come from open strings with both ends on the D0 or the D4 branes.

- Black lines connecting one gauge node to itself represent (4,4) hypermultiplets in the adjoint representation of the gauge group. They also originate from open strings with both ends on the D0 or the D4 branes.

- The grey line connecting the two gauge nodes represents a (0,4) bifundamental hypermultiplet.

- The two dashed lines connecting the two gauge nodes represent (0,2) bifundamental Fermi multiplets. These combine into a (0,4) Fermi multiplet. Together with the (0,4) bifundamental hypermultiplet they form a (4,4) hypermultiplet in the bifundamental representation of the two gauge groups. This originates from open strings stretched between the D0 and the D4 branes. The resulting quiver is thus (4,4) supersymmetric.

Having introduced our notation we now set out to describe the more general quiver quantum mechanics dual to the solutions defined by equations (2.21), (2.22). We will make use of the low-energy field content detailed in appendix B.

### 3.2 ADHM quantum mechanics with Wilson loops

The previous D0-D4-F1 brane system can be extended to include D4' and D8 branes while keeping the same number of supersymmetries, giving rise to the brane set-up depicted in table 1. We show in this and the next subsection that this brane set-up suggests an interpretation in terms of D0 and D4-brane self-dual instantons in the 5d $\mathcal{N} = 1$ theory living in the D4'-D8 branes, with extra BPS Wilson loops.

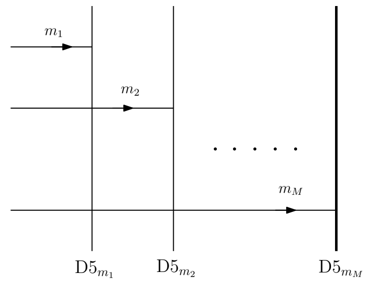


**Figure 2**. Array of $M$ D5-branes with $(m_1, m_2, \ldots m_M)$ F1-strings stretched between them and the $N$ D3-branes. Note that even if the branes are separated for illustration purposes they are actually coincident.

| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D3 | – | – | – | – | | | | | | |
| D5 | – | | | | | – | – | – | – | – |
| F1 | – | | | | – | | | | | |

**Table 3**. Brane set-up associated to the D3-D5-F1 brane configuration that describes Wilson loops in the antisymmetric representation of U($N$) in 4d $\mathcal{N} = 4$ SYM.

We start recalling the brane realisation of Wilson loops in arbitrary U($N$) representations. This was worked out in [70–72] for 4d $\mathcal{N} = 4$ SYM. In [70, 71] it was shown that a half-BPS Wilson loop in a U($N$) antisymmetric representation is described by an array of $M$ D5-branes with fundamental string charges dissolved in their worldvolumes. This is the realisation in the near horizon limit of a configuration of $M$ stacks of D5-branes separated by a distance $L$ from the $N$ D3-branes, with $(m_1, m_2, \ldots m_M)$ F1-strings stretched between the stacks, as depicted in figure 2, in the limit $L \to \infty$. The brane set-up is depicted in table 3. In turn, in [71, 72] it was shown that a half-BPS Wilson loop in a symmetric representation of U($N$) is described by an array of $P$ D3-branes with fundamental string charges dissolved in their worldvolumes. This is the realisation in the near horizon limit of a configuration of $P$ D3-branes, separated by a distance $L$ from a stack of $N$ coincident D3-branes, with $(n_1, n_2, \ldots n_P)$ F1-strings stretched between the stacks, in the limit $L \to \infty$. The F1-string charges dissolved in the different D5-branes (D3-branes) of the array $m_j$, $j = 1, \ldots, M$ ($n_i$, $i = 1, \ldots, P$), then realise a Wilson loop operator in the antisymmetric (symmetric) U($N$) representation labeled by the Young tableau depicted in figure 3. This generalises the description of a Wilson loop in the fundamental representation in [73, 74] to all other representations.

Let us now see how this is realised in our brane system. We start by considering the D4-D4'-F1 brane subsystem in table 1. In this brane set-up the D4-D4'-F1 branes are distributed exactly as the D3-D5-F1 brane configuration that describes Wilson loops in antisymmetric representations in 4d $\mathcal{N} = 4$ SYM, depicted in table 3. In other words, the strings extending between the D4' and the D4 branes have as their lowest energy excitation a fermionic field. Integrating out this massive field leads to a Wilson loop

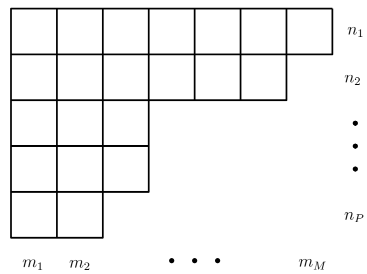


**Figure 3**. Young tableau labelling the irreducible representations of $\mathrm{U}(N)$.

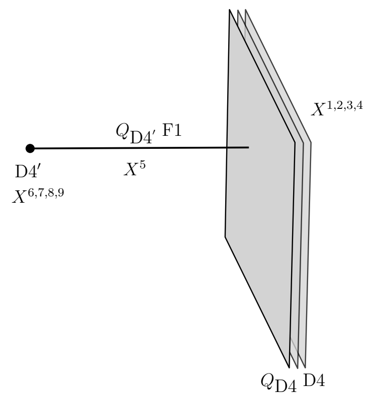


**Figure 4**. Wilson loop in the $Q_{D4'}$-th antisymmetric representation of $\mathrm{U}(Q_{\mathrm{D4}})$.

in the antisymmetric representation. Indeed, a BPS Wilson line can be introduced into the low-energy 5d SYM theory living on the D4-branes by probing the D4-branes with fundamental strings [73, 74]. These can in turn be taken to originate on additional D4'-branes, orthogonal to the D4-branes. This can be described through the coupling

$$S_{\mathrm{D4}} = T_4 \int \widehat{F}_{(4)} \wedge A_t \tag{3.9}$$

in the worldvolume effective action of the D4-branes. If the D4-branes are wrapped on the $\mathrm{CY}_2$, as in our brane set-up, the D4'-branes must be orthogonal to them and must carry a magnetic charge

$$Q_{\mathrm{D4}'} = \frac{1}{(2\pi)^3} \int_{\mathrm{CY}_2} \widehat{F}_{(4)} \,. \tag{3.10}$$

This charge is then equal to the number of F1-strings dissolved in the worldvolume of the D4-branes. For $Q_{\mathrm{D4}'}$ D4'-branes this configuration, depicted in figure 4, describes then a Wilson loop in the $Q_{\mathrm{D4}'}$-th antisymmetric representation of $\mathrm{U}(Q_{\mathrm{D4}})$, where $Q_{\mathrm{D4}}$ is the number of D4-branes. BPS Wilson loops in arbitrary representations of $\mathrm{U}(Q_{\mathrm{D4}})$ can then be obtained adding arrays of D4'-branes with fundamental string charges dissolved in their worldvolumes, as in figure 2.

Let us consider now the D0-D8-F1 brane subsystem in table 1. In this brane set-up the D0-D8-F1 branes are again distributed exactly as the D3-D5-F1 and D4-D4'-F1 brane configurations described above. In this case a BPS Wilson line can be introduced into the low energy quantum mechanics living on the D0-branes by probing the D0-branes with fundamental strings, originating on D8-branes [75]. As discussed above, the fermionic string stretched between the D8 and the D0 branes implies that the Wilson loop will be in the antisymmetric representation. This is indeed what is inferred from the coupling

$$S_{\mathrm{D0}} = T_0 \int F_{(0)} \wedge A_t \tag{3.11}$$

in the worldvolume effective action of the D0-branes. D8-branes with charge

$$Q_{\mathrm{D8}} = 2\pi F_{(0)} \tag{3.12}$$

induce $Q_{\mathrm{D8}}$ F1-strings dissolved in the worldline of the D0-branes. This describes then a Wilson loop in the $Q_{\mathrm{D8}}$'th antisymmetric representation of $\mathrm{U}(Q_{\mathrm{D0}})$, for $Q_{\mathrm{D0}}$ D0-branes. As for the D4-D4'-F1 subsystem described above, BPS Wilson loops in arbitrary representations of $\mathrm{U}(Q_{\mathrm{D0}})$ can then be obtained adding arrays of D8-branes with fundamental string charges dissolved in their worldvolumes, as depicted in figure 2.

Therefore, the complete D0-D4-D4'-D8-F1 brane system depicted in table 1 can be interpreted as describing Wilson loops in the $Q_{\mathrm{D4'}} \times Q_{\mathrm{D8}}$ antisymmetric representation of $\mathrm{U}(Q_{\mathrm{D4}}) \times \mathrm{U}(Q_{\mathrm{D0}})$.

The complete brane system has, however, a richer dynamics, as one must also consider the interactions between the D4-D4'-F1 and D0-D8-F1 subsystems. Indeed, the D0 and D4 branes can be seen as instantons in the worldvolumes of the D4' and D8 branes, respectively. This is inferred from the couplings [76]

$$S_{\mathrm{D4'}} = T_4 \int \mathrm{Tr}[C_{(1)} \wedge F \wedge F]\,, \qquad S_{\mathrm{D8}} = T_8 \int \mathrm{Tr}[C_{(5)} \wedge F \wedge F]\,, \tag{3.13}$$

in the D4' and D8 branes worldvolume effective actions. These show that a D0-brane can be absorbed by a D4'-brane and converted into an instanton, while a D4-brane wrapped on the $\mathrm{CY}_2$ can be absorbed by a D8-brane and converted as well into an instanton. The one dimensional $\mathcal{N} = 4$ gauged quantum mechanics living on the complete brane system would describe then the interactions between the two types of instantons and the two types of Wilson lines previously described. This generalises the ADHM quantum mechanics discussed in [69, 77].

Indeed, in the previous references gauged quantum mechanics describing the interactions between D0-brane instantons and Wilson lines in the 5d $\mathcal{N} = 2$ SYM theory living in D4-branes were constructed. In our brane set-up we have extra D8-branes. These allow for extra D4-brane instantons wrapped on the $\mathrm{CY}_2$. Moreover, the D8-branes introduce additional F1-strings ending on the D0-branes. Given this, we propose that the gauged quantum mechanics living on the complete D0-D4-D4'-D8-F1 brane set-up describes the interactions between instantons and Wilson lines in the 5d $\mathcal{N} = 1$ SYM theory living in D4'-D8 branes.

D4-D8 brane set-ups must include O8 orientifold fixed planes in order to flow to 5d fixed point theories in the UV [78, 79]. We have indeed seen in section 2.1 that the

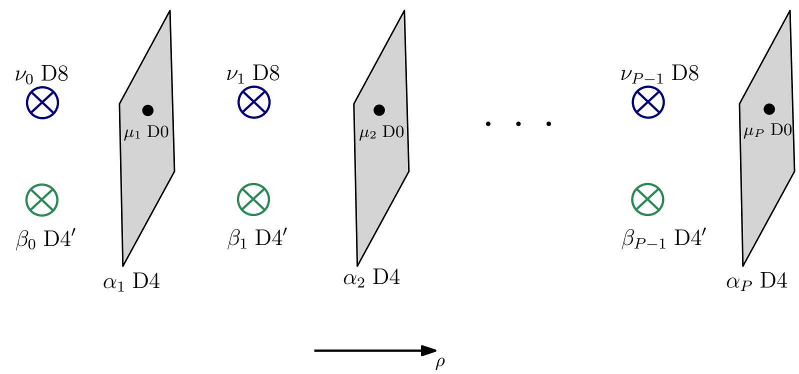


**Figure 5**. Hanany-Witten like brane set-up associated to the quantised charges of the solutions.

behaviour of the solutions at both ends of the space is compatible with the presence of O8 orientifold fixed points. These could provide for a fully consistent brane picture. In the next subsection, we construct quiver quantum mechanics that we propose describe instanton and Wilson line defects within the 5d Sp(N) D4-D8/O8 brane system of [78, 79].

### 3.3 The dual quiver quantum mechanics

In this section we propose quiver quantum mechanics supported by the D0-D4-D4'-D8-F1 brane system. The full dynamics is described in terms of the matter fields that enter in the description of the D0-D4-F1 system, introduced in section 3.1, plus additional fields that connect these branes with the D4' and the D8 branes. The extra fields are (twisted) (4,4) bifundamental hypermultiplets, coming from the open strings that connect the D4'-branes with the D0-branes and the D8-branes with the D4-branes, and (0,2) bifundamental Fermi multiplets, coming from the open strings that connect the D4'-branes with the D4-branes and the D8-branes with the D0-branes [68, 77]. This field content is detailed in appendix B.

Let us analyse in more detail the brane picture introduced in table 1. In subsection 2.1 we found the following quantised charges at each $[2\pi k, 2\pi(k+1)]$ $\rho$-interval,

$$Q^{e\,(k)}_{\rm D4} = \alpha_k = \sum_{j=0}^{k-1} \beta_j\,, \qquad Q^{e\,(k)}_{\rm D0} = \mu_k = \sum_{j=0}^{k-1} \nu_j\,, \tag{3.14}$$

$$Q^{m\,(k)}_{\rm D4'} = \beta_k\,, \qquad Q^{m\,(k)}_{\rm D8} = \nu_k\,, \qquad Q^{e\,(k)}_{\rm F1} = 1\,. \tag{3.15}$$

We can associate a Hanany-Witten like brane set-up to these charges, as depicted in figure 5. In this figure there are $\mu_k$ D0-branes and $\alpha_k$ D4-branes, and orthogonal $\nu_k$ D8-branes and $\beta_k$ D4'-branes, in the $[2\pi k, 2\pi(k+1)]$, $k = 1, \dots P$, $\rho$-intervals. As discussed in section 2.1 the D0 and D4 branes are interpreted as colour branes while the D4' and D8 branes play the role of flavour branes. In the last $[2\pi P, 2\pi(P+1)]$ interval there are extra $-\alpha_P$, $-\mu_P$, D4' and D8 brane charges that cancel the total D4' and D8 brane charges of the compact space. Note that these charges may originate from D4'/O4 and D8/O8 superpositions, consistently with the singularity structure at the end of the space.

The brane set-up depicted in figure 5 can be related to a Hanany-Witten brane set-up in Type IIB upon a T+S duality transformation. This is depicted in figure 6. The D0 and

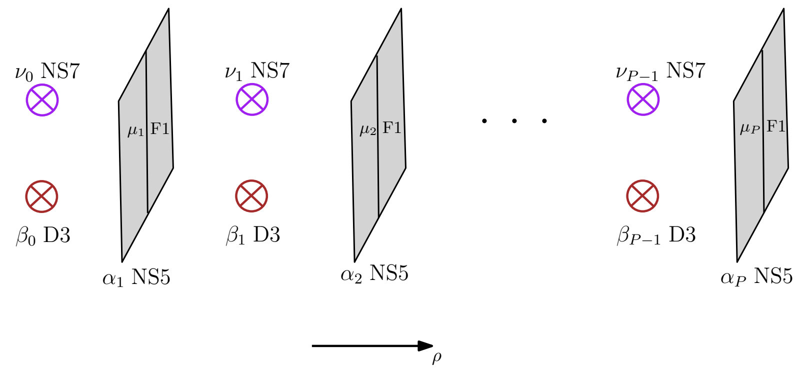


**Figure 6**. Hanany-Witten brane set-up associated to the T+S duals of our solutions.

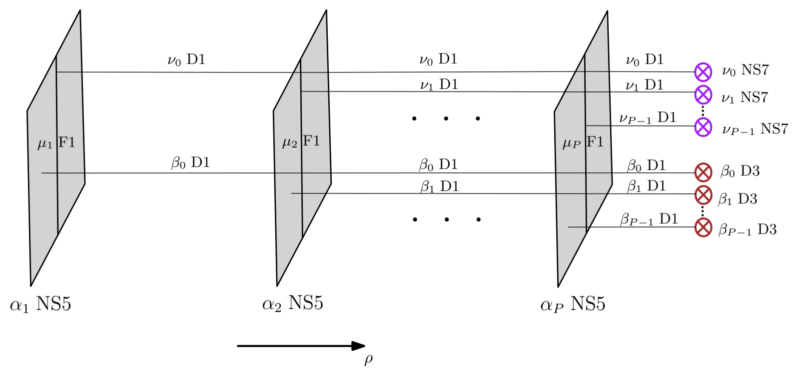


**Figure 7**. Hanany-Witten brane set-up of figure 6 after Hanany-Witten moves.

D4 branes are now F1 and NS5 branes, while the D8 and D4' branes become NS7 and D3 branes, respectively. In this brane set-up one can make Hanany-Witten moves that turn the configuration onto the one depicted in figure 7. Here all the $\beta_k$ stacks of D3-branes and $\nu_k$ stacks of NS7-branes are now coincident, and there are $\alpha_k$ D1-branes and $\mu_k$ D1-branes ending on each stack of $\alpha_k$ NS5-branes and $\mu_k$ F1-strings, originating, respectively, from the $(\beta_0, \dots, \beta_{k-1})$ and $(\nu_0, \dots, \nu_{k-1})$ stacks of D3 and NS7-branes.

Back to Type IIA, the previous configuration is mapped onto the one depicted in figure 8, where $\alpha_k$ F1-strings originating in $(\beta_0, \beta_1, \dots, \beta_{k-1})$ stacks of D4'-branes end on a given stack of $\alpha_k$ D4-branes, and $\mu_k$ F1-strings originating in $(\nu_0, \nu_1, \dots, \nu_{k-1})$ stacks of D8-branes end on a given stack of $\mu_k$ D0-branes. This is exactly the description of $\mathrm{U}(\alpha_k)$ and $\mathrm{U}(\mu_k)$ Wilson loops in the antisymmetric representations $(\beta_0, \dots, \beta_{k-1})$ of $\mathrm{U}(\alpha_k)$ and $(\nu_0, \dots, \nu_{k-1})$ of $\mathrm{U}(\mu_k)$, that we discussed in the previous subsection. The respective Young tableaux are depicted in figure 9.

Therefore, our proposal is that the quantum mechanics dual to our $\mathrm{AdS}_2$ solutions describes the interactions between Wilson loops in the $(\beta_0, \dots, \beta_{k-1})$ and $(\nu_0, \dots, \nu_{k-1})$ antisymmetric representations of the gauge groups $\mathrm{U}(\alpha_k) \times \mathrm{U}(\mu_k)$, and $\mu_k$ D0 and $\alpha_k$ D4 brane instantons, with $k = 1, \dots, P$.[6]

[6]Note that if O4-O8 orientifold fixed planes are present at both ends of the $\rho$-interval the gauge groups would actually be $\mathrm{Sp}(\alpha_1)$, $\mathrm{Sp}(\mu_1)$ and $\mathrm{Sp}(\alpha_P)$, $\mathrm{Sp}(\mu_P)$ in the first and last $\rho$-intervals [82].



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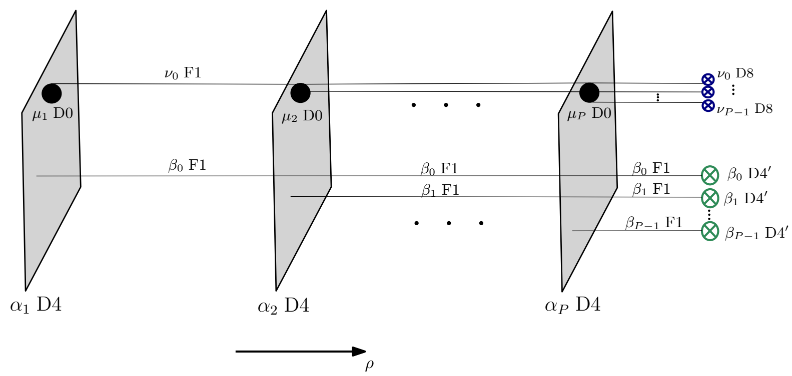


**Figure 8**. Hanany-Witten like brane set-up equivalent to the brane configuration in figure 5.

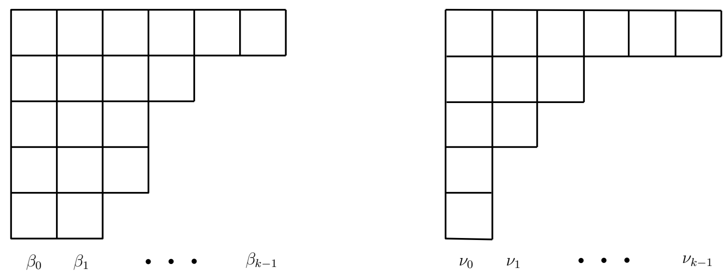


**Figure 9**. Young tableaux labelling the irreducible representations of $\mathrm{U}(\alpha_k)$ and $\mathrm{U}(\mu_k)$.

Our proposed quantum mechanics can be given a quiver-like description in terms of a set of disconnected quivers showing the interactions between the D0-D4-D4'-D8 branes in each $\rho$-interval. These quivers can be read off directly from the brane set-up depicted in figure 5, before the Hanany-Witten moves that connect the different intervals are made. Keeping in mind that D0 and D4 branes, and D4 and D8 branes, are connected by (4,4) hypermultiplets, and D4' and D4 branes, and D8 and D0 branes, through (0,2) Fermi multiplets, all of them in the bifundamental representations of the respective groups, and that there are (4,4) vectors and (4,4) adjoint hypermultiplets at each gauge node (see appendix B), we can depict the quivers shown in figure 10. These quivers can be interpreted as partitions of the total number of D4' and D8 branes due to the insertions of the D0 and D4 brane instanton defects. This results in independent quantum mechanics living in the different D0-D4 brane instantons. In turn, the Wilson line defects do not show in the quivers, since the associated fermionic strings are massive. The integration of these Fermi fields leads to the insertion of the Wilson loop. Note that in these quivers we have taken into account that the number of D4' and D8 *source* branes in each interval is given by the difference of the quantised charge in the given interval and that in the preceding one. This is implied by the source terms (2.17), (2.18), together with the derivatives

$$\widehat{h}_4'' = \frac{1}{2\pi}\sum_{k=1}^{P}(\beta_{k-1}-\beta_k)\delta(\rho-2\pi k)\,, \qquad h_8'' = \frac{1}{2\pi}\sum_{k=1}^{P}(\nu_{k-1}-\nu_k)\delta(\rho-2\pi k)\,, \tag{3.16}$$

derived from the $\widehat{h}_4$ and $h_8$ functions defined by (2.21) and (2.22). Flavour groups in the

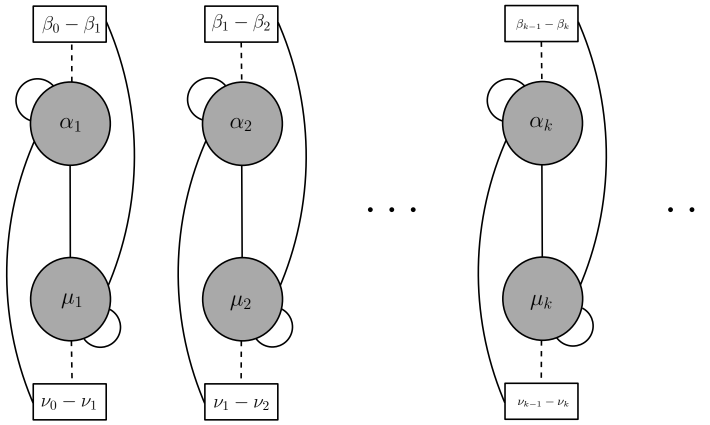


**Figure 10**. Disconnected quivers describing the SCQMs dual to our solutions.

last $\rho$-interval should be present associated to the D4' and D8 flavour branes at the end of the space.

Our suggested interpretation of the $AdS_2$ solutions in [35] as duals to line defect quantum mechanics living in the D4'-D8 brane system of [78, 79] is in agreement with the findings in [37] (see also [24]). In these references it was shown that the solutions with $CY_2 = T^4$ flow in the UV to the $AdS_6$ solution dual to the 5d CFT living in the D4'-D8 brane system. Accordingly, a defect interpretation in terms of D0-D4-F1 branes was given. Our analysis suggests that the D0 and D4 branes would find an interpretation in terms of instantons inside the D4' and D8 branes and that the F1-strings would arise in the near horizon limit from F1-strings stretched between the D0 and the D8 branes and the D4 and the D4' branes, realising Wilson loops in antisymmetric representations. Quiver quantum mechanics describing line defects in 5d gauge theories realised in $(p, q)$ 5-brane webs have been proposed in [80, 81] (see also [68, 75]). It would be interesting to clarify the relation between these SCQM and the ones proposed in this paper.

### 3.4 Quantum mechanical central charge

One possible check of our proposed quivers would require that a notion of central charge existed for the superconformal quantum mechanics, that could be matched with the holographic central charge constructed in section 2.3. Such a notion indeed exists for 1d quiver CFTs that originate from 2d $\mathcal{N} = (0, 4)$ CFTs upon dimensional reduction [38]. In that case one can measure the number of vacua of the 1d CFT using the same expression that defines the central charge of the 2d CFT, in terms of the two-point $U(1)_R$ current correlation function (see for example [83]),

$$c = 6(n_{\text{hyp}} - n_{\text{vec}})\,, \tag{3.17}$$

where $n_{\text{hyp}}$ is the number of $\mathcal{N} = (0, 4)$ hypermultiplets and $n_{\text{vec}}$ the number of $\mathcal{N} = (0, 4)$ vector multiplets in the UV description (of either the 1d or the 2d CFT). It was shown in [38] that for the class of $AdS_2$ solutions considered therein the result matches the

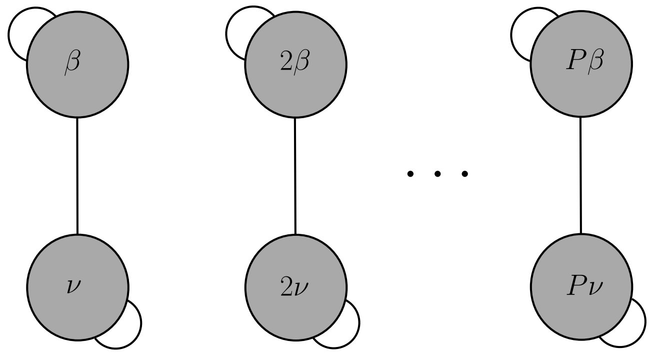


**Figure 11**. Quiver mechanics associated to the backgrounds obtained from eqs. (3.19)–(3.20).

holographic calculation for long quivers with large ranks. We show next that equation (3.17) matches as well the holographic result for the quiver quantum mechanics defined in the previous subsection, not originating from 2d CFTs. For these quivers $n_{\rm hyp}$ and $n_{\rm vec}$ are the numbers of $\mathcal{N} = (0,4)$ hypermultiplets and $\mathcal{N} = (0,4)$ vector multiplets of the quantum mechanics.

As a first check we show that equation (3.17) reproduces the right central charge of the D0-D4-F1 system, T-dual to the D1-D5-wave system. In this case, with $N$-waves,

$$\begin{aligned} n_{\rm hyp} &= NQ_{\rm D0}Q_{\rm D4} + N(Q_{\rm D0}^2 + Q_{\rm D4}^2)\,, \qquad n_{\rm vec} = N(Q_{\rm D0}^2 + Q_{\rm D4}^2) \\ c &= 6(n_{\rm hyp} - n_{\rm vec}) = 6NQ_{\rm D0}Q_{\rm D4}\,. \end{aligned} \tag{3.18}$$

This is in exact agreement with equation (3.8) and (2.30).

As a second example we consider the following profiles for the $\widehat{h}_4$ and $h_8$ linear functions:

$$\widehat{h}_4(\rho) = \Upsilon h_4(\rho) = \Upsilon \begin{cases} \frac{\beta}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\beta P}{2\pi}(2\pi(P+1) - \rho)\,, & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{3.19}$$

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\nu P}{2\pi}(2\pi(P+1) - \rho)\,, & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{3.20}$$

The corresponding $\mathcal{N} = 4$ quantum mechanical quiver is the one depicted in figure 11.[7]

Substituting in (3.17) we find

$$\begin{aligned} n_{\rm hyp} &\sim \sum_{j=1}^{P} j^2\Big(\beta^2 + \nu^2 + \beta\nu\Big), & n_{\rm vec} &\sim \sum_{j=1}^{P} j^2(\beta^2 + \nu^2), \\ c &\sim \beta\nu P(P+1)(2P+1) \sim 2\beta\nu P^3\,, & c_{\rm hol,1d} &= 2\beta\nu P^2(P+1) \sim 2\beta\nu P^3. \end{aligned}$$

[7] Flavour groups coupled to the last gauge nodes should be present associated to the D4' and D8 flavour branes at the end of the space. Their contribution to the central charge is subleading and has not been incorporated into the (therefore approximate) expressions for the number of hypermultiplets and vector multiplets.

We then see that in the holographic limit (large $P$, $\nu$, $\beta$) the field theory and holographic central charges coincide. One can elaborate many other examples in which the two calculations are also shown to agree.

In order to justify this agreement one can draw a comparison between the central charge calculation proposed in (3.17) and the dimension of the Higgs branch of the quantum mechanics calculated in [84–87]. In these references the following formula is used to compute the dimension of the Higgs branch for $\mathcal{N}=4$ quantum mechanics with gauge group $\Pi_v \mathrm{U}(N_v)$

$$\mathcal{M}=\sum_{v,w} N_v N_w - \sum_v N_v^2 + 1\,. \tag{3.21}$$

In this formula $N_w$ stands for the ranks of the colour groups adjacent to a given colour group of rank $N_v$. Our expression in (3.17) extends this fomula for counting the degrees of freedom of the quantum mechanics to more general quivers including flavours. Moreover, our quivers need not be related by dimensional reduction to 2d CFTs, as the quivers discussed in [38], where this formula was shown to reproduce the central charge of 1d CFTs obtained from 2d CFTs upon dimensional reduction.

In the next section we further elaborate on the holographic central charge discussed in section 2.3, and relate it to an extremisation principle.

## 4 Holographic central charge, electric-magnetic charges and a minimisation principle

In this section we show that the holographic central charge of the $\mathrm{AdS}_2$ solutions discussed in this paper can be related to the product of the RR electric and magnetic charges. The second result of this section is to show that it can also be obtained through a minimisation principle. The first relation was already encountered for the $\mathcal{N}=4$ $\mathrm{AdS}_2$ solutions studied in [38], and, as in that case, it generalises an argument for $\mathrm{AdS}_2$ gravity coupled to a gauge field put forward in [88]. Our second result is a minimisation principle that allows to obtain the holographic central charge in the spirit of [89–93], by minimising a functional defined as the integral of various geometrical forms. In analogy with the findings in [38], we show that these geometric forms can be directly related to the RR electric and magnetic fluxes of the background. In contrast with [38], we use the Maxwell (instead of Page) fluxes to establish the connection.

### 4.1 Relation with the electric-magnetic charges

As it happened for the $\mathrm{AdS}_2$ solutions in [38], there exists an interesting relation between the holographic central charge of our $\mathrm{AdS}_2$ solutions, given by equation (2.30), and the electric and magnetic charges of the underlying Dp-branes. Using our definitions (2.13) and (2.15) for the electric and magnetic charges of Dp-branes and taking the absolute value of the charges, we find that the quantity

$$\mathcal{Q}=\sum_p Q^e_{\mathrm{Dp}} Q^m_{\mathrm{Dp}} \tag{4.1}$$

is proportional to the holographic central charge, up to a boundary term. More concretely, we find

$$\mathcal{Q}=\frac{\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{S^3}\mathrm{Vol}_{\mathrm{AdS}_2}}{(2\pi)^8}\int \mathrm{d}\rho\Bigg[2\widehat{h}_4h_8-\frac{(u')^2}{2}-\frac{(\rho-2\pi k)}{2}(4\widehat{h}_4h_8-(u')^2)\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right)$$
$$+\frac{u^2-2(\rho-2\pi k)uu'}{4}\left(\frac{\widehat{h}_4'^2}{\widehat{h}_4^2}+\frac{h_8'^2}{h_8^2}\right)\Bigg] \tag{4.2}$$

where the sum has been taken over the D0-D4-D4'-D8 branes of our backgrounds. This expression needs to be regularised in order to obtain a finite result, given the integral over the infinite $\mathrm{AdS}_2$ space present in the computation of the electric charges. We can regularise it for instance as in (2.7).
Let us now use the BPS equation $u''=0$ on the expression (4.2). We may think about the application of the BPS equation as an extremisation of the quantity $\mathcal{Q}$. We find

$$\mathcal{Q}=\frac{\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{S^3}\mathrm{Vol}_{\mathrm{AdS}_2}}{(2\pi)^8}\int \mathrm{d}\rho\Bigg[4\widehat{h}_4h_8-(u')^2+\frac{u^2-2(\rho-2\pi k)uu'}{4}\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right)$$
$$-\partial_\rho\left(2(\rho-2\pi k)\widehat{h}_4h_8-\frac{uu'}{2}+\frac{u^2-2(\rho-2\pi k)uu'}{4}\left(\frac{(\widehat{h}_4h_8)'}{\widehat{h}_4h_8}\right)\right)\Bigg]. \tag{4.3}$$

Finally, we use the expressions (3.16) to obtain,

$$\mathcal{Q}=\frac{\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{S^3}\mathrm{Vol}_{\mathrm{AdS}_2}}{(2\pi)^8}\int \mathrm{d}\rho\Bigg[4\widehat{h}_4h_8-(u')^2 \tag{4.4}$$
$$+\frac{u^2-2(\rho-2\pi k)uu'}{8\pi}\sum_{k=1}^{P}\left(\frac{(\beta_{k-1}-\beta_k)}{h_4}+\frac{(\nu_{k-1}-\nu_k)}{h_8}\right)\delta(\rho-2\pi k)$$
$$-\partial_\rho\left(2(\rho-2\pi k)\widehat{h}_4h_8-\frac{uu'}{2}+\frac{u^2-2(\rho-2\pi k)uu'}{4}\left(\frac{(\widehat{h}_4h_8)'}{\widehat{h}_4h_8}\right)\right)\Bigg].$$

In the absence of sources, the second line in (4.4) vanishes and the quantity $\mathcal{Q}$ in (4.2) coincides, up to a finite boundary term with the holographic central charge in (2.30).

Consider now solutions with $u'=0$ as in the rest of this paper. In the presence of sources, the second line in eq. (4.4) is,

$$\frac{u_0^2}{8\pi}\sum_{k=1}^{P}\left(\frac{(\beta_{k-1}-\beta_k)}{\alpha_k}+\frac{(\nu_{k-1}-\nu_k)}{\mu_k}\right).$$

This is subleading in the regime of large parameters, with respect to the term in the first line of (4.4). Interestingly, the boundary term gives a divergent contribution similar to that found in [38]. In fact, for $\widehat{h}_4$ and $h_8$ given by equations (2.21) and (2.22), we find

$$\int_0^{2\pi(P+1)}\partial_\rho\left(2(\rho-2\pi k)\widehat{h}_4h_8+u^2\frac{(\widehat{h}_4h_8)'}{4\widehat{h}_4h_8}\right)=-\lim_{\epsilon\to 0}\frac{u_0^2}{8\pi\epsilon}(\alpha_P+\beta_0+\mu_P+\nu_0)+O(\epsilon^2)$$
$$=\lim_{\epsilon\to 0}\frac{u_0^2}{8\pi\epsilon}(Q_{\mathrm{D4}'}^{\mathrm{sources}}+Q_{\mathrm{D8}}^{\mathrm{sources}}), \tag{4.5}$$

where $\widehat{h}_4(\rho=0)=h_8(\rho=0)=\epsilon$, and the same at $\rho=2\pi(P+1)$, with $\epsilon\to 0$. The boundary term thus yields a divergence directly related to the presence of the D4' and D8 brane *sources* of the background. In summary, once regularised, equation (4.2) is proportional, up to a boundary term, to the holographic central charge in (2.30).

As in [38], we find an explicit realisation of the proposal in [88], which relates the central charge in the algebra of symmetry generators of $\mathrm{AdS}_2$ with an electric field to the square of the electric field. This proposal is realised in our fully consistent string theory set-up, where various branes with electric and magnetic charges enter the calculation. We show next that we can also derive the central charge by extremising an action functional constructed out of the RR Maxwell fluxes.

### 4.2 An action functional for the central charge

We showed in [38] that a natural way to define an action functional from which the central charge can be derived through an extremisation principle is to put "off-shell" the electric and magnetic RR Page fluxes of the associated background. We implement next this idea in our current backgrounds following closely the derivation in [38], with the difference that in the present case we use the Maxwell fluxes.

In fact, we use the fluxes $F_0, F_2, F_4$ defined in (2.2) — as in the rest of the paper we consider the case $H_2=0$ and $\widehat{h}_4(\rho)$. We also use the dual fluxes $F_6, F_8, F_{10}$ defined as Hodge duals below (2.2).

On these fluxes we impose the "restriction" procedure explained in [38]. Basically, we define forms constructed from the Maxwell fluxes, after excising from them the $\mathrm{AdS}_2$ part. In this way we define the forms $[\mathcal{J}_0, \tilde{\mathcal{J}}_0, \mathcal{J}_4, \tilde{\mathcal{J}}_4, \mathcal{F}_4, \tilde{\mathcal{F}}_4, \mathcal{F}_8, \tilde{\mathcal{F}}_8]$. To be concrete, the Maxwell fluxes of our backgrounds and the "restricted" forms read,

$$\begin{aligned}
&F_{(0)}=\mathcal{J}_0, && F_{(2)}=-\tilde{\mathcal{J}}_0\,\widehat{\mathrm{vol}}_{\mathrm{AdS}_2},\\
&F_{(4)}=\mathcal{J}_4-\tilde{\mathcal{J}}_4, && F_{(6)}=\mathcal{F}_4\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2}+\tilde{\mathcal{F}}_4\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2},\\
&F_{(8)}=\mathcal{F}_8, && F_{(10)}=\tilde{\mathcal{F}}_8\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_2},
\end{aligned}\tag{4.6}$$

$$\begin{aligned}
&\mathcal{J}_0=h_8', \qquad \tilde{\mathcal{J}}_0=\frac{1}{2}\Big(h_8+\frac{u'uh_8'}{4\widehat{h}_4h_8-u'^2}\Big), \qquad \mathcal{J}_4=-\widehat{h}_4'\,\widehat{\mathrm{vol}}_{\mathrm{CY}_2},\\
&\tilde{\mathcal{J}}_4=\left(2h_8+\frac{uu'\widehat{h}_4'-\widehat{h}_4u'^2}{2\widehat{h}_4^2}\right)\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho,\\
&\mathcal{F}_4=\frac{h_8\widehat{h}_4'u^2}{\widehat{h}_4(4\widehat{h}_4h_8-u'^2)}\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho,\\
&\tilde{\mathcal{F}}_4=\frac{1}{2}\Big(\widehat{h}_4+\frac{u'u\widehat{h}_4'}{4\widehat{h}_4h_8-u'^2}\Big)\,\widehat{\mathrm{vol}}_{\mathrm{CY}_2},\\
&\mathcal{F}_8=\left(2\widehat{h}_4+\frac{uu'h_8'-h_8u'^2}{2h_8^2}\right)\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho,\\
&\tilde{\mathcal{F}}_8=-\frac{h_8'\widehat{h}_4u^2}{h_8(4\widehat{h}_4h_8-u'^2)}\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho.
\end{aligned}\tag{4.7}$$



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With the forms as in (4.6), we define the functional,

$$\begin{aligned}\mathcal{C} &= \frac{1}{2}\int_{X_8}[\tilde{\mathcal{J}}_4\wedge\tilde{\mathcal{F}}_4+\mathcal{J}_0\wedge\tilde{\mathcal{F}}_8+\mathcal{F}_4\wedge\mathcal{J}_4+\tilde{\mathcal{J}}_0\wedge\mathcal{F}_8)]\\ &= \int_{X_8}\left(\widehat{h}_4h_8-\frac{1}{8}\left[u^2\left(\frac{\widehat{h}_4'^2}{\widehat{h}_4^2}+\frac{h_8'^2}{h_8^2}\right)-2uu'\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right)+2u'^2\right]\right)\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho.\end{aligned}\tag{4.8}$$

We now extremise this functional by imposing the Euler-Lagrange equation for $u(\rho)$,

$$2u''=u\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right).\tag{4.9}$$

In the absence of sources, this equation implies

$$h_8''=0,\qquad \widehat{h}_4''=0,\qquad u''=0.\tag{4.10}$$

In this case the functional $\mathcal{C}$ is proportional to the central charge in (2.30). The functional in (4.8) can be written as

$$4\mathcal{C}=\int_{X_8}\left(4\widehat{h}_4h_8-(u')^2-\frac{u^2}{2}\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right)+\partial_\rho\left[\frac{u^2}{2}\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right)\right]\right)\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho.$$

Using the expressions in (3.16) and following the procedure described below (4.3), we find for constant $u=u_0$

$$\begin{aligned}4\mathcal{C}=&\int_{X_8}\left(4\widehat{h}_4h_8-(u')^2-\frac{u_0^2}{4\pi}\sum_{k=1}^{P}\left(\frac{\beta_{k-1}-\beta_k}{h_4}+\frac{\nu_{k-1}-\nu_k}{h_8}\right)\delta(\rho-2\pi k)\right)\widehat{\mathrm{vol}}_{\mathrm{CY}_2}\wedge\widehat{\mathrm{vol}}_{S^3}\wedge\mathrm{d}\rho\\ &-\lim_{\epsilon\to 0}\frac{u_0^2}{2\epsilon}(\mu_P+\nu_0+\alpha_P+\beta_0)\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{S^3}.\end{aligned}\tag{4.11}$$

As in section 4.1, we find that up-to a boundary term and the subleading source-term, the extremisation of the functional $\mathcal{C}$ in (4.8) is proportional to the holographic central charge (2.30). The functional $\mathcal{C}$ is defined in terms of the Maxwell fields of the background. This development extends the ideas of [89–93] to the present case, for manifolds with boundary in the presence of sources.

## 5 Conclusions

In this paper we have studied the $\mathrm{AdS}_2\times S^3\times\mathrm{CY}_2\times I_\rho$ solutions to massive Type IIA supergravity recently constructed in [35] and proposed SCQM dual to them. Our solutions can be thought of as describing the background near a string like defect inside $\mathrm{AdS}_6$. Conversely, they can be thought of as dual to the Quantum Mechanics describing the excitations of $(0+1)$-dimensional defects in a five dimensional SCFT.

We have started by identifying the 1/8-BPS brane set-up that underlies the solutions. This is the D0-D4-D4'-D8-F1 brane intersection studied in [24, 37]. From the study of this brane set-up we have revealed that the D0 and D4 branes have an interpretation as instantons in the worldvolumes of the D4' and D8 branes. Accordingly, these branes are counted by their electric charges. In turn, the F1-strings provide for Wilson lines in the antisymmetric representations of the D0 and D4 branes gauge groups. This generalises the constructions in [70, 71] for 4d $\mathcal{N} = 4$ SYM to our more complicated brane configurations.

We have proposed explicit quiver quantum mechanics dual to our solutions, that generalise previous constructions dual to $\mathrm{AdS}_2$ solutions in [38]. In [38] $\mathrm{AdS}_2/\mathrm{CFT}_1$ duals with 4 supercharges were constructed using T-duality from the $\mathrm{AdS}_3/\mathrm{CFT}_2$ pairs in [28, 40–42].

These quivers inherited some of the properties of the 2d "mother" CFTs, like the matter content that guarantees gauge anomaly cancelation in 2d. This condition does not have to be satisfied in one dimension, and indeed the quivers that we have constructed in this paper consist on a set of disconnected matrix models that do not satisfy it. It would be interesting to relate the SCQMs constructed in this paper to the quiver quantum mechanics studied in [80, 81], living in D1-F1-D3-D5-NS5-D7 brane systems. These brane systems are the T-dual realisations of our D0-D4-D4'-D8-F1 brane configurations, so one would expect the dual SCQMs to be related.

It was shown in [37] (see also [24]) that the $\mathrm{AdS}_2 \times S^3 \times T_4 \times I_\rho$ solutions with $h_4$ a particular function of the $T^4$ and $\rho$ asymptote locally to $\mathrm{AdS}_6 \times S^3 \times I$ in the UV. This allowed to interpret these solutions as D0-D4-F1 line defect CFTs within the 5d $\mathcal{N} = 1$ gauge theory living in the D4'-D8 brane system. It is plausible that we have found concrete realisations of these CFTs in the form of D0-D4 brane instantons interacting with F1 Wilson lines, connecting with the results in [68, 69, 75, 77, 80, 81].

We have seen that the holographic central charge can be related to the product of the electric and magnetic charges of the D-branes present in the solutions. This realises in a controlled string theory set-up the proposal in [88], where the central charge in the algebra of symmetry generators of $\mathrm{AdS}_2$ with a gauge field was related to the square of the electric field, and adds to the controlled string theory set-ups realising this proposal already found in [32, 38]. It would be interesting to see if further set-ups realising this proposal can be found in more general situations, especially in higher dimensions.

Moreover, we have provided one more example where the holographic central charge can be derived from an action functional, following the ideas of geometric extremisation [89–93]. Our results extend the results in [89–93] by the presence of sources and boundaries. We should stress that a physical reason that underlies the need for extremisation in a field theory with a non-Abelian R-symmetry, such as ours, remains as an interesting open problem that deserves further investigation.

An interesting open avenue that also deserves further investigation is the application of exact calculational techniques to our new backgrounds. This would allow to understand the properties of our theories in the IR and would set the stage for their applications to the study of black holes, along the lines of [94]. It would be interesting to study our constructions with a more algebraic point of view, along the lines of [95, 96]. Similarly, the connection or similarities with the dynamics uncovered in papers like [97] should be nice to clarify.



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## A The seed backgrounds and their dual SCFTs

In this appendix we recall the main properties of the solutions to massive IIA supergravity (with localised sources) obtained in the recent work [28]. It was proposed in [40–42] that these backgrounds are holographic duals to two dimensional CFTs preserving $\mathcal{N}=(0,4)$ SUSY that we also summarise below.

The Neveu-Schwarz (NS) sector of these bosonic solutions reads,

$$\mathrm{d}s^2 = \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{h_8\widehat{h}_4}{4h_8\widehat{h}_4+(u')^2}\mathrm{d}s^2_{S^2}\right) + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2, \tag{A.1}$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2\widehat{h}_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8\widehat{h}_4+(u')^2}, \tag{A.2}$$

$$H_{(3)} = \frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4\widehat{h}_4 h_8+(u')^2}\right)\wedge\widehat{\mathrm{vol}}_{S^2} + \frac{1}{h_8^2}\mathrm{d}\rho\wedge H_2.$$

Generically, the warping function $\widehat{h}_4$ has support on $(\rho,\mathrm{CY}_2)$. On the other hand, $u$ and $h_8$ only depend of $\rho$. We denote $u'=\partial_\rho u$ and similarly for $h_8'$. The RR fluxes are

$$F_{(0)} = h_8', \quad F_{(2)} = -\frac{1}{h_8}H_2 - \frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4h_8\widehat{h}_4+(u')^2}\right)\widehat{\mathrm{vol}}_{S^2}, \tag{A.3a}$$

$$\begin{aligned} F_{(4)} = &-\left(\mathrm{d}\left(\frac{uu'}{2\widehat{h}_4}\right) + 2h_8\mathrm{d}\rho\right)\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_3} \\ &- \frac{h_8}{u}(\widehat{\star}_4\mathrm{d}_4\widehat{h}_4)\wedge\mathrm{d}\rho - \partial_\rho\widehat{h}_4\widehat{\mathrm{vol}}_{\mathrm{CY}_2} - \frac{uu'}{2h_8(4h_8\widehat{h}_4+(u')^2)}H_2\wedge\widehat{\mathrm{vol}}_{S^2}, \end{aligned} \tag{A.3b}$$

with the higher fluxes related to them as $F_{(6)}=-\star_{10}F_{(4)}$, $F_{(8)}=\star_{10}F_{(2)}$, $F_{(10)}=-\star_{10}F_{(0)}$. It was shown in [28] that supersymmetry holds whenever

$$u''=0, \qquad H_2+\widehat{\star}_4 H_2 = 0, \tag{A.4}$$

where $\widehat{\star}_4$ is the Hodge dual on $\mathrm{CY}_2$. In what follows we will restrict ourselves to the set of solutions for which $H_2=0$ and $\widehat{h}_4=\widehat{h}_4(\rho)$. After this, the background reads,

$$\mathrm{d}s^2_{st} = \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{h_8\widehat{h}_4}{4h_8\widehat{h}_4+(u')^2}\mathrm{d}s^2_{S^2}\right) + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2,$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2\widehat{h}_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8\widehat{h}_4+(u')^2},$$

$$
\begin{aligned}
B_2 &= \frac{1}{2}\left(-\rho + 2\pi k + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2}\right)\widehat{\mathrm{vol}}_{S^2},\\
\widehat{F}_{(0)} &= h_8',\\
\widehat{F}_{(2)} &= -\frac{1}{2}\Big(h_8 - h_8'(\rho - 2\pi k)\Big)\widehat{\mathrm{vol}}_{S^2},\\
\widehat{F}_{(4)} &= -\left(\partial_\rho\left(\frac{uu'}{2\widehat{h}_4}\right) + 2h_8\right)\mathrm{d}\rho\wedge\widehat{\mathrm{vol}}_{\mathrm{AdS}_3} - \partial_\rho\widehat{h}_4\widehat{\mathrm{vol}}_{\mathrm{CY}_2}.
\end{aligned} \tag{A.5}
$$

We have written the Page fluxes, $\widehat{F} = e^{-B_{(2)}} \wedge F$, more useful for our purposes. Notice that we have also allowed for large gauge transformations $B_{(2)} \to B_{(2)} + \pi k\ \widehat{\mathrm{vol}}_{S^2}$, for $k = 0, 1, \ldots, P$. The transformations are performed every time we cross an interval $[2\pi k, 2\pi(k+1)]$. The space begins at $\rho = 0$ and ends at $\rho = 2\pi(P+1)$. This will become more apparent once the functions $\widehat{h}_4, h_8, u$ are specified below.

The background in (A.5) is a SUSY solution of the massive IIA equations of motion if the functions $\widehat{h}_4, h_8, u$ satisfy (away from localised sources),

$$
\widehat{h}_4''(\rho) = 0, \quad h_8''(\rho) = 0, \quad u''(\rho) = 0. \tag{A.6}
$$

Various particular solutions were analysed in [28]. Here we will consider an infinite family of solutions for which the functions are piecewise continuous. These were carefully studied in [40–42] and a precise dual field theory was proposed. Let us briefly summarise aspects of these SCFTs.

**The associated dual SCFTs.** A generic background of the form in (A.5) is defined by the functions $\widehat{h}_4, h_8, u$. For the type of solutions that were considered in [40–42] the $\rho$-interval was bounded in $[0, 2\pi(P+1)]$. This range was determined by the vanishing of the functions $\widehat{h}_4, h_8$. Generically these functions are piecewise linear in $\rho$, and can be taken as,

$$
\begin{aligned}
\widehat{h}_4(\rho) &= \Upsilon\, h_4(\rho)\\
&= \Upsilon \begin{cases}
\frac{\beta_0}{2\pi}\rho & 0 \le \rho \le 2\pi\\
\alpha_1 + \frac{\beta_1}{2\pi}(\rho - 2\pi) & 2\pi \le \rho \le 4\pi\\
\alpha_2 + \frac{\beta_2}{2\pi}(\rho - 4\pi) & 4\pi \le \rho \le 6\pi\\
\alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1), \quad k := 3, \ldots, P-1\\
\alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1).
\end{cases}
\end{aligned} \tag{A.7}
$$

$$
h_8(\rho) = \begin{cases}
\frac{\nu_0}{2\pi}\rho & 0 \le \rho \le 2\pi\\
\mu_1 + \frac{\nu_1}{2\pi}(\rho - 2\pi) & 2\pi \le \rho \le 4\pi\\
\mu_2 + \frac{\nu_2}{2\pi}(\rho - 4\pi) & 4\pi \le \rho \le 6\pi\\
\mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1), \quad k := 3, \ldots, P-1\\
\mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1),
\end{cases} \tag{A.8}
$$

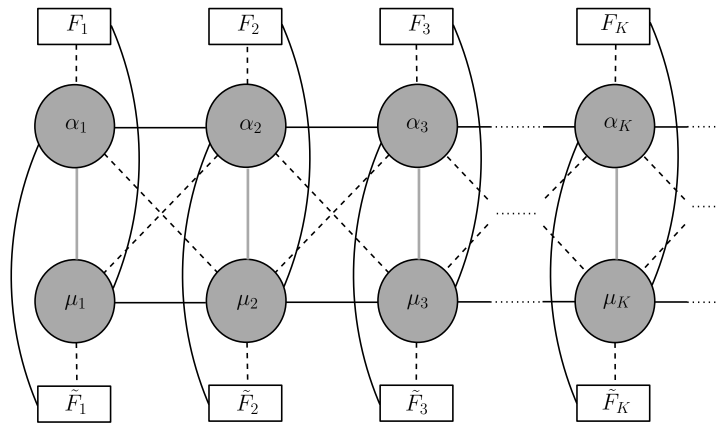


**Figure 12**. A generic quiver field theory whose IR is dual to the holographic background defined by the functions in (A.7)–(A.8). The solid black line represents a $(4,4)$ hypermultiplet. The grey line represents a $(0,4)$ hypermultiplet and the dashed line represents a $(0,2)$ Fermi multiplet. $\mathcal{N} = (4,4)$ vector multiplets are the degrees of freedom in each gauged node.

where

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j \,, \qquad \mu_k = \sum_{j=0}^{k-1} \nu_j \,. \tag{A.9}$$

The choice of constants is imposed by continuity of the metric and dilaton, while the fluxes can present discontinuities associated to the presence of branes (see [28] for more details). In turn, the $u$ function, also linear in $\rho$, does not enter in the magnetic Page fluxes associated to the solutions, and thus, does not affect the type of quivers that can be constructed from the brane set-up. Here we will restrict to the simplest case, $u' = 0$.

The background in eq. (A.5) for the functions $\widehat{h}_4, h_8$ specified above is dual to the CFT describing the low energy dynamics of a two dimensional quantum field theory encoded by the quiver in figure 12. In this quiver the ranks of the flavour groups are determined by the absence of gauge anomalies, to be [40–42]

$$F_k = \nu_{k-1} - \nu_k, \quad \tilde{F}_k = \beta_{k-1} - \beta_k. \tag{A.10}$$

The most stringent check in [40–42] for the validity of this proposal was the matching between the field theory and holographic central charges. The $\mathrm{U}(1)_R$ current two-point function can be identified when flowing to the far IR (a conformal point is reached) with the right moving central charge of the $\mathcal{N} = (0,4)$ conformal field theory. Following [83, 99] it was found for a generic quiver with $n_{\text{hyp}}$ hypermultiplets and $n_{\text{vec}}$ vector multiplets, that the central charge (the anomaly of the $\mathrm{U}(1)_R$ current) is

$$c_{\text{CFT}} = 6(n_{\text{hyp}} - n_{\text{vec}}). \tag{A.11}$$

In [40–42] a variety of examples of long linear quivers for which the ranks of each of the nodes is a large number were discussed. In each of these qualitatively different examples, it was found that the field theoretical central charge of eq. (A.11) coincides (in the limit of long quivers with large ranks) with the holographic central charge, computed as

$$c_{hol} = \frac{3\pi}{2G_N}\mathrm{Vol}_{\mathrm{CY}_2}\int_0^{2\pi(P+1)} \widehat{h}_4 h_8 \,\mathrm{d}\rho = \frac{3}{\pi}\int_0^{2\pi(P+1)} h_4 h_8 \,\mathrm{d}\rho, \tag{A.12}$$

where $G_N = 8\pi^6$ (with $g_s = \alpha' = 1$) and $\Upsilon \mathrm{Vol}_{\mathrm{CY}_2} = 16\pi^4$.

## B D0-D4-D4'-D8 quantum mechanics

In this appendix we include a study of the low-energy field content emerging from the brane web given in table 1. The four coordinates $x^{1,2,3,4}$ parametrise a Calabi-Yau, whose nature we need not specify in general. Just to be concrete we can take it to be a four-torus. The compactification to a four-torus breaks the SO(4) symmetry associated to rotations on the four-dimensional subspace spanned by $x^{1,2,3,4}$. However, in what follows, we will continue to use the SO(4) algebra to organise fields.[8]

As all the branes are localised in the $x^5 = \rho$ direction, strings stretched between branes in adjacent intervals $\mathcal{I}_k$ and $\mathcal{I}_{k+1}$, with $\mathcal{I}_k = [2\pi k, 2\pi(k+1)]$, are necessarily long, hence describing massive states. Therefore, the Hilbert space of the full system is given by the direct sum of individual Hilbert spaces associated with D0-D4-D4'-D8 systems, that we now describe.

In the following, we will use that the rank of the gauge and flavour groups associated with D0, D4, D4' and D8 branes in a given interval $\mathcal{I}_k$ is given by $\mu_k$, $\alpha_k$, $\tilde{F}_k$ and $F_k$, respectively, with $\tilde{F}_k = \beta_{k-1} - \beta_k$ and $F_k = \nu_{k-1} - \nu_k$, in agreement with the notation used in the main text. The elementary excitations on the branes are given by strings with ends attached to the branes. We will often use 2d language, exploiting the fact that our brane web is T-dual to a D1-D5-D5'-D9 system, and we need only to dimensionally reduce to 0 + 1 dimensions to get back to D0-D4-D4'-D8. Much of the conventions in denoting fields is borrowed from [99], while quantisation of open strings can be easily done following [100, 101], to which we refer for more details.

- D0-D0 strings: for simplicity, let us start by considering a sytem of $\mu_k$ parallel D1$-$D1 branes stretched along $x^{0,5}$ (eventually we will reduce down to 1d). The system consists of a U$(\mu_k)$ gauge theory and can be obtained by dimensional reduction of a 10d $\mathcal{N} = 1$ U$(\mu_k)$ gauge theory, where the field content is that of a 10d gauge field and a 10d Majorana-Weyl spinor.

  In order to see what this entails, consider the decomposition of the 10d Lorentz group as $\mathrm{SO}(1,9) \to \mathrm{SO}(1,1) \times \mathrm{SO}(8)$. A Majorana-Weyl spinor in the **16** of[9] SO(1,9) can be decomposed as $\mathbf{16} = (+\frac{1}{2}, \mathbf{8}_s) \oplus (-\frac{1}{2}, \mathbf{8}_c)$. We should then further split the SO(8)

[8] See [98], Ch. 4, where subtleties and special limits concerning quantisation on a compact four-torus are discussed.

[9] We should really say Spin(1, 9). In this appendix, we will not care much about global structure of groups.

as SO(8) $\to G$ with $G = \mathrm{SO}(4)^- \times \mathrm{SO}(4)^+$, where the two SO(4)'s are realised on the D4 and D4' branes. We may also rewrite $G$ as

$$G = \mathrm{SU}(2)^-_L \times \mathrm{SU}(2)^-_R \times \mathrm{SU}(2)^+_L \times \mathrm{SU}(2)^+_R \,, \tag{B.1}$$

where $\mathrm{SU}(2)^-_R \times \mathrm{SU}(2)^+_R$ can be understood as the R-symmetry of the quantum mechanical system.

A ten-dimensional gauge field decomposes as a two-dimensional gauge field $u_\mu$ plus 8 scalars. The scalars, that we denote collectively as $Y$ and $Z$, transform in the $(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2})$ and $(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})$ of $G$, respectively. Decomposing also the eight-dimensional spinors under representations of $G$, and using $\mathcal{N}=(0,4)$ language, we get[10]

$$\begin{array}{rll}
\text{Vector multiplet:} & u_\mu & (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1}) \\
 & \zeta & (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2}) \\
\text{Twisted hypermultiplet:} & Y & (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2}) \\
 & \tilde{\zeta} & (\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1}) \\
\text{Hypermultiplet:} & Z & (\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1}) \\
 & \lambda & (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2}) \\
\text{Fermi multiplet:} & \tilde{\lambda} & (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2})
\end{array} \tag{B.2}$$

i.e. the field content of a $(4,4)$ vector and a $(4,4)$ hypermultiplet. The latter was denoted in the main text as a black solid line starting and ending on the same gauge groups, see figure 10. All the multiplets above transform in the adjoint of the gauge group $\mathrm{U}(\mu_k)$ on the D1 branes.

Let us now dimensionally reduce to one-dimensional field theory. The gauge field $u_\mu$, upon dimensional reduction, decomposes as $u_\mu \to (u_t, \sigma)$, with $u_t$ the 1d vector field and $\sigma$ a real scalar of the 1d theory. The fermions and scalars are inert, leading indeed to the field content (B.2). Of course, going to down to one dimension, chirality for the fermions is lost.

Quantisation of D4-D4 strings, upon reduction to 0+1 dimensions, leads to the same conclusions for the gauge group $\mathrm{U}(\alpha_k)$.

- D0-D4$'$ strings: the system of D0-D4' branes is a brane web with 4 ND relative boundary conditions, T-dual of the well-known D1-D5 system. The NS and R sectors give rise, upon imposing the GSO projection, to a scalar in the $(\mathbf{1},\mathbf{2})$ of the internal $\mathrm{SO}(4)^+$ and a 6d Weyl spinor which is a singlet under the internal $\mathrm{SO}(4)^+$. We can dimensionally reduce to 1d to get an $\mathcal{N}=(4,4)$ twisted hypermultiplet. In $\mathcal{N}=(0,4)$

[10] We are considering the following decomposition

$$\mathbf{8}_s = (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2}) \oplus (\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})\,, \quad \mathbf{8}_c = (\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2})\,,$$

from which (B.2) follows. The eight complex fermions obtained by decomposing a 10d Majorana-Weyl spinor are denoted, in some compact notation, as $\zeta$, $\tilde{\zeta}$, $\lambda$ and $\tilde{\lambda}$.

language we have

$$\begin{aligned} \text{Twisted Hypermultiplet:} \quad & \phi' \quad (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2}) \\ & \psi' \quad (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1}) \\ \text{Fermi multiplet:} \quad & \tilde{\psi}' \quad (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1}) \end{aligned} \tag{B.3}$$

Each of these multiplets transform in the $\boldsymbol{\mu_k}$ of the gauge group $\mathrm{U}(\mu_k)$ and in the anti-fundamental of the global $\mathrm{SU}(\tilde{F}_k)$.

- D0-D4 strings: the field content is the same as for the D0-D4' system. The only difference is that $\mathrm{SO}(4)^- \times \mathrm{SO}(4)^+$ are exchanged. From our discussion about D0-D4' strings we find

$$\begin{aligned} \text{Hypermultiplet:} \quad & \tilde{\phi} \quad (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1}) \\ & \psi \quad (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2}) \\ \text{Fermi multiplet:} \quad & \tilde{\psi} \quad (\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1}) \end{aligned} \tag{B.4}$$

  i.e. a $(4,4)$ hypermultiplet. Each of these multiplets transform in the $\boldsymbol{\mu_k}$ of the gauge group $\mathrm{U}(\mu_k)$ and in the $\overline{\boldsymbol{\alpha}}_{\boldsymbol{k}}$ of the other gauge group $\mathrm{U}(\alpha_k)$.

- D4-D4' strings: this system is an example of a brane web with 8 relative ND boundary conditions. Strings in the NS sector are automatically massive. Indeed, such a brane web can be T-dualised to a $\mathrm{D}0-\mathrm{D}8$ system, where the R sector gives rise to an $\mathcal{N}=(0,2)$ Fermi multiplet, singlet of all internal symmetries,

$$\mathcal{N}=(0,2) \quad \text{Fermi multiplet:} \quad \chi \quad (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})\,. \tag{B.5}$$

  Such a Fermi multiplet transforms in the $\boldsymbol{\alpha_k}$ of the gauge group $\mathrm{U}(\alpha_k)$ and in the antifundamental of the global group $\mathrm{SU}(\tilde{F}_k)$.

- D0-D8 strings: again, this is a system with 8 relative ND boundary conditions. Therefore, also in this case we find a $\mathcal{N}=(0,2)$ Fermi multiplet, which is a singlet of all internal symmetries

$$\mathcal{N}=(0,2) \quad \text{Fermi multiplet:} \quad \xi \quad (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})\,. \tag{B.6}$$

  This $\mathcal{N}=(0,2)$ Fermi multiplet transforms in the $\boldsymbol{\mu_k}$ of the gauge group $\mathrm{U}(\mu_k)$ and in the $\overline{\boldsymbol{F}}_{\boldsymbol{k}}$ of the global group $\mathrm{SU}(F_k)$.

- D4-D8 strings: finally, we have a system of D4-D8 branes with 4 relative ND boundary conditions, with degenerate NS and R sectors. The quantisation of D4-D8 strings leads again to an $\mathcal{N}=(4,4)$ twisted hypermultiplet. Its field content is given by

$$\begin{aligned} \text{Twisted hypermultiplet:} \quad & \varphi \quad (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2}) \\ & \eta \quad (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1}) \\ \text{Fermi multiplet:} \quad & \tilde{\eta} \quad (\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})\,, \end{aligned} \tag{B.7}$$

  where each multiplet transforms in the $\boldsymbol{\alpha_k}$ of the gauge group $\mathrm{U}(\alpha_k)$ and in the $\overline{\boldsymbol{F}}_{\boldsymbol{k}}$ of the global group $\mathrm{SU}(F_k)$.



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Putting altogether, we get the field content of figure 10, where black solid lines represent $(4,4)$ hypermultiplets and dashed black lines $(0,2)$ Fermi multiplets. Interactions between the various fields can be constructed following the rules of e.g. [77, 99]. In particular, interactions should not spoil supersymmetry. See [99] where this is done consistently.

## C Holographic calculation of SQM observables

In this appendix we probe the background in (2.1) and (2.2) by introducing probe D branes. The action describing the coupling of a generic Dp brane to the NS-NS and RR closed string fields contains the usual DBI + WZ terms,

$$\begin{aligned} S_{\mathrm{Dp}} &= S_{\mathrm{DBI}} + S_{\mathrm{WZ}}\,, \\ S_{\mathrm{DBI}} &= -T_{\mathrm{p}} \int \mathrm{d}^{p+1}\xi \left\{ e^{-\phi} \left[ -\det(g_{ab} + B_{ab} + 2\pi\alpha' \mathcal{F}_{ab}) \right]^{\frac{1}{2}} \right\} \\ S_{\mathrm{WZ}} &= T_{\mathrm{p}} \int_{p+1} \exp(2\pi\alpha' \mathcal{F}_{(2)} + B_{(2)}) \wedge \sum_q C_{(q)}\,, \end{aligned} \tag{C.1}$$

where $\mathcal{F}$ is the gauge field living on the brane.

- Let us begin by considering probe D0 branes. The field theory living on a D0 brane is $(0+1)-$dimensional. We can define a metric for such a $(0+1)-$dimensional field theory from the pullback of (2.1),

$$\mathrm{d}s^2_{\mathrm{ind}} = -\frac{u\sqrt{\widehat{h}_4 h_8}}{4\widehat{h}_4 h_8 - (u')^2}\mathrm{d}t^2\,. \tag{C.2}$$

  Note that the pullback of $B_{(2)}$ and the field strength $\mathcal{F}_{ab}$ on the D0 brane are automatically zero, due to their (anti)symmetric properties. Given that

$$e^{-\Phi}\sqrt{\det g_{\mathrm{ind}}} = \frac{h_8(\rho_*)}{2}\cosh(r_*)\,, \tag{C.3}$$

  where the "$*$" simply refers to the fact that we are keeping $r$ and $\rho$ fixed at some values, for a probe D0 brane we find the following action

$$S_{\mathrm{D0}} = -T_0 \int_{\mathbb{R}} \mathrm{d}t\, e^{-\Phi}\sqrt{\det g_{\mathrm{ind}}} + T_0 \int C_{(1)}\,, \tag{C.4}$$

  which leads to

$$S_{\mathrm{D0}} = -T_0 \frac{h_8(\rho_*)}{2}\cosh(r_\star) \int_{\mathbb{R}} \mathrm{d}t + T_0 \frac{\mu_k}{2}\sinh(r_\star) \int_{\mathbb{R}} \mathrm{d}t\,. \tag{C.5}$$

  If we choose $\rho_* = 2\pi k$, we find

$$S_{\mathrm{D0}} = T_0 \frac{\mu_k}{2}(\sinh r_* - \cosh r_*) \int_{\mathbb{R}} \mathrm{d}t\,. \tag{C.6}$$

  We then find that the brane is calibrated only when $r_* = \infty$, for which, however, we have a vanishing action.

Consider instead the coupling of a D0 brane to an external one form, call it $\mathcal{A}_{(1)}$. The action describing such a coupling is

$$S = 2\pi T_0 \int F_{(0)} \mathcal{A}_{(1)} \,. \tag{C.7}$$

If, as in (2.2), $F_{(0)} = h_8'$, we get

$$S = 2\pi T_{\mathrm{D}0} \frac{\nu_k}{2\pi} \int_{\mathbb{R}} \mathcal{A}_{(t)} \mathrm{d}t \,. \tag{C.8}$$

Using that $T_0 = 1$, we find that the action (C.8) describes a Chern-Simons term with $k_{CS} = \nu_k$. If we were to make sense of the action (C.8) also on a topologically non-trivial one-dimensional manifold, say on a circle $S^1$, we would find that it describes a gauge invariant action if and only if $\nu_k$ is an integer. Moreover, without loss of generality, we can take $\nu_k$ to be positive as well, as a Chern-Simons term is odd under parity.

- Let us move on to the case of a probe D4 brane stretched along $(t, \mathrm{CY}_2)$. The induced metric on the worldvolume of such a D4 brane, from the pullback of (2.1), reads

$$\mathrm{d}s^2_{\mathrm{ind}} = -\frac{u\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 - (u')^2} \mathrm{d}t^2 + \sqrt{\frac{\widehat{h}_4}{h_8}} \mathrm{d}s^2_{\mathrm{CY}_2} \,. \tag{C.9}$$

The pullback of $B_{(2)}$ vanishes, whereas we will ignore for now the field strength $\mathcal{F}_{ab}$ along the brane.[11] Given that

$$e^{-\Phi} \sqrt{\det g_{\mathrm{ind}}} = \frac{\widehat{h}_4(\rho_*)}{2} \cosh(r_*) \,, \tag{C.10}$$

for a probe colour D4 brane the DBI action reads

$$S_{\mathrm{DBI}} = -T_4 \mathrm{Vol}_{\mathrm{CY}_2} \frac{\widehat{h}_4(\rho_*)}{2} \cosh(r_*) \int_{\mathbb{R}} \mathrm{d}t \,. \tag{C.11}$$

Using that $T_4 = 1/(2\pi)^4$ and $\widehat{h}_4 = \Upsilon h_4$ we find

$$S_{\mathrm{DBI}} = -\frac{\Upsilon \mathrm{Vol}_{\mathrm{CY}_2}}{(2\pi)^4} \frac{h_4(\rho_*)}{2} \cosh(r_*) \int_{\mathbb{R}} \mathrm{d}t \,. \tag{C.12}$$

Let us consider now the WZ part of the D4 brane action

$$S_{\mathrm{WZ}} = T_4 \int C_{(5)} + 2\pi C_{(3)} \wedge \mathcal{F}_{(2)} + 4\pi^2 C_{(1)} \wedge \mathcal{F}_{(2)} \wedge \mathcal{F}_{(2)} \,. \tag{C.13}$$

If we place the D4 brane at $\rho = 2\pi k$, its action reads

$$\begin{aligned} S &= S_{\mathrm{DBI}} + T_4 \int C_{(5)} + 2\pi T_4 \int \widehat{F}_{(4)} \wedge \mathcal{A}_{(1)} \,, \\ &= T_4 \Upsilon \mathrm{Vol}_{\mathrm{CY}_2} \frac{\alpha_k}{2} (\sinh r_* - \cosh r_*) \int_{\mathbb{R}} \mathrm{d}t + 2\pi T_4 \int \widehat{F}_{(4)} \wedge \mathcal{A}_{(1)} \,. \end{aligned} \tag{C.14}$$

[11] An $\alpha'$ expansion of the DBI action would produce a Maxwell kinetic term for $\mathcal{F}$. Eventually we will be interested in the dimensional reduction to 1 dimension where such a kinetic term would be absent.



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Here we have used that $C_{(5)}|_{\text{D4}} = -\frac{1}{2}(\widehat{h}_4 - \widehat{h}_4'(\rho - 2\pi k))\sinh(r)\text{d}t \wedge \widehat{\text{vol}}_{\text{CY}_2}$, where $\text{d}C_{(5)} = \widehat{F}_{(6)}$ on the brane. In turn, the term $2\pi C_{(3)} \wedge F_{(2)}$ contributes as a Chern-Simons term, $2\pi \widehat{F}_{(4)} \wedge \mathcal{A}_{(1)}$, with $\widehat{F}_{(4)} = \widehat{h}_4' \widehat{\text{vol}}_{\text{CY}_2}$. This Chern-Simons term reads explicitly,

$$2\pi T_4 \int \widehat{F}_{(4)} \wedge \mathcal{A}_{(1)} = 2\pi T_4 \widehat{h}_4' \text{Vol}_{\text{CY}_2} \int \mathcal{A}_{(1)} \,. \tag{C.15}$$

Using that $\widehat{h}_4 = \Upsilon h_4$ and choosing $\Upsilon$ as usual such that $\Upsilon T_4 \text{Vol}_{\text{CY}_2} = 1$ we find a CS action with level $k_{CS} = \beta_k$. Again, invariance under large gauge transformations implies that $\beta_k$ has to be an integer. Parity allows us to take it to be positive.

# D Continuity of the NS sector

Let us comment briefly on the continuity of the NS sector of (2.1). Being $u$, $\widehat{h}_4$ and $h_8$ linear functions of $\rho$, we consider the following expressions for them in the interval $[2\pi k, 2\pi(k+1)]$,

$$\widehat{h}_4^{(k)} = \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k)\,, \quad h_8^{(k)} = \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k)\,, \quad u^{(k)} = a_k + \frac{b_k}{2\pi}(\rho - 2\pi k)\,. \tag{D.1}$$

Let us now see what conditions should be imposed on the constants $\{\alpha_k, \beta_k, \mu_k, \nu_k, a_k, b_k\}$ in order for the NS sector in (2.1) to be continuous. We rewrite (2.1) as

$$\text{d}s^2 = f_1 \text{d}s^2_{\text{AdS}_2} + f_2 \text{d}s^2_{S^3} + f_3 \text{d}s^2_{\text{CY}_2} + f_4 \text{d}\rho^2\,, \qquad B_{(2)} = f_5 \widehat{\text{vol}}_{\text{AdS}_2}\,, \qquad e^{-2\Phi} = f_6\,. \tag{D.2}$$

Then, we should impose the continuity of the $f_i$'s at all points $\rho_k = 2\pi k$, where the functions $u, \widehat{h}_4$ and $h_8$ change defining laws. This is achieved by demanding

$$\lim_{\rho \to \rho_k^-} f_i = \lim_{\rho \to \rho_k^+} f_i\,. \tag{D.3}$$

We will refer to the left and right hand side of (D.3) as $f_i^-$ and $f_i^+$, respectively, and therefore continuity corresponds to requiring $f_i^- = f_i^+$. A straightforward computation shows that

$$f_1^- = f_1^+ \Rightarrow \quad \frac{(a_{k-1} + b_{k-1})\sqrt{(\alpha_{k-1} + \beta_{k-1})(\mu_{k-1} + \nu_{k-1})}}{16\pi^2(\alpha_{k-1} + \beta_{k-1})(\mu_{k-1} + \nu_{k-1}) - b_{k-1}^2} = \frac{a_k\sqrt{\alpha_k \mu_k}}{16\pi^2 \alpha_k \mu_k - b_k^2}\,, \tag{D.4}$$

$$f_2^- = f_2^+ \Rightarrow \quad \frac{(a_{k-1} + b_{k-1})}{\sqrt{(\alpha_{k-1} + \beta_{k-1})(\mu_{k-1} + \nu_{k-1})}} = \frac{a_k}{\sqrt{\alpha_k \mu_k}}\,, \tag{D.5}$$

$$f_3^- = f_3^+ \Rightarrow \quad \sqrt{\frac{\alpha_{k-1} + \beta_{k-1}}{\mu_{k-1} + \nu_{k-1}}} = \sqrt{\frac{\alpha_k}{\mu_k}}\,, \tag{D.6}$$

$$f_4^- = f_4^+ \Rightarrow \quad \frac{\sqrt{(\alpha_{k-1} + \beta_{k-1})(\mu_{k-1} + \nu_{k-1})}}{a_{k-1} + b_{k-1}} = \frac{\sqrt{\alpha_k \mu_k}}{a_k}\,, \tag{D.7}$$

$$f_5^- = f_5^+ \Rightarrow \quad \frac{a_{k-1}b_{k-1} + 16\pi^2(\alpha_{k-1} + \beta_{k-1})(\mu_{k-1} + \nu_{k-1})}{16\pi^2(\alpha_{k-1} + \beta_{k-1})(\mu_{k-1} + \nu_{k-1}) - b_{k-1}^2} = -1 + \frac{a_k b_k}{b_k^2 - 16\pi^2 \alpha_k \mu_k}\,, \tag{D.8}$$

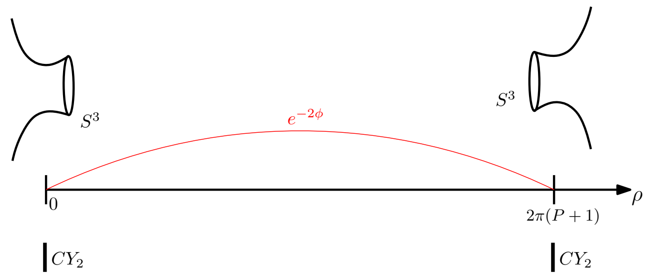


**Figure 13**. Behaviour of the solutions at both ends of the $\rho$-interval for $u$ = constant. We find that the $\mathrm{CY}_2$ is of finite size, while the $S^3$ diverges at both ends of the interval.

$$f_6^- = f_6^+ \Rightarrow \quad \frac{(\mu_{k-1}+\nu_{k-1})^{3/2}\left[16\pi^2(\alpha_{k-1}+\beta_{k-1})(\mu_{k-1}+\nu_{k-1})-b_{k-1}^2\right]}{\sqrt{\alpha_{k-1}\beta_{k-1}}(a_{k-1}+b_{k-1})} = \frac{\mu_k^{3/2}\left[16\pi^2(\alpha_k+\beta_k)-b_k^2\right]}{a_k\sqrt{\alpha_k}}\,. \tag{D.9}$$

A possible solution for such a system is

$$a_k = a_{k-1}+b_{k-1}\,,\quad b_k = b_{k-1} = b_0\,,\quad \alpha_k = \alpha_{k-1}-\beta_{k-1}\,,\quad \mu_k = \mu_{k-1}-\nu_{k-1}\,, \tag{D.10}$$

which, in turn, implies

$$a_k = a_0 + kb_0\,,\quad \alpha_k = \alpha_0 + \sum_{j=0}^{k-1}\beta_j\,,\quad \mu_k = \mu_0 + \sum_{j=0}^{k-1}\nu_j\,. \tag{D.11}$$

These are the very same conditions assuring continuity of $u$, $\widehat{h}_4$ and $h_8$. Therefore, we conclude that imposing continuity of the functions $u$, $\widehat{h}_4$ and $h_8$ is sufficient to get continuity for the NS sector.

## E Volumes and stringy volumes

Analysing the volume of the compact submanifolds of the solutions in eqs. (2.21)–(2.22) we run into the possibility that some of these submanifolds have infinite size. However, in spite of a divergent warp factor, the "stringy size" of the submanifold is actually finite or vanishing at the ends of the space. The finite stringy-volume case does not pose any problem in interpreting a D-brane wrapping such cycle. The case in which the cycle shrinks may suggest an interpretation of the singularity in terms of new massless degrees of freedom (branes wrapping the shrinking cycles) that the supergravity solution is not encoding.

In the case with $u' = 0$ we see in eqs. (2.24), (2.25) that the $S^3$ diverges at both ends of the $\rho$-interval. A representation of the various compact submanifolds is given in figure 13. We can nevertheless calculate the stringy volume of the $S^3$, at any value of the $\rho$-coordinate,

$$V_s[S^3] = \int \widehat{\mathrm{vol}}_{S^3}\, e^{-\Phi}\sqrt{\det[g+B]} = 2\pi^2 u\sqrt{\frac{h_8}{\widehat{h}_4}}. \tag{E.1}$$

This is finite at the ends of the $\rho$-interval. Hence, branes wrapping this $S^3$ will have finite action.

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## 7.2 SL$(2, \mathrm{R})$-NATD solution as an example in the class of AdS$_2\times$S$^3$ solutions.

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# $\mathrm{AdS}_2$ geometries and non-Abelian T-duality in non-compact spaces

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Abstract: We obtain an $\mathrm{AdS}_2$ solution to Type IIA supergravity with 4 Poincaré supersymmetries, via non-Abelian T-duality with respect to a freely acting $\mathrm{SL}(2,\mathbf{R})$ isometry group, operating on the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ solution to Type IIB. That is, non-Abelian T-duality on $\mathrm{AdS}_3$. The dual background obtained fits in the class of $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2$ solutions to massive Type IIA constructed in [1]. We propose and study a quiver quantum mechanics dual to this solution that we interpret as describing the backreaction of the baryon vertex of a D4-D8 brane intersection.

Keywords: D-branes, String Duality, AdS-CFT Correspondence

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## Contents





## 1 Introduction

Our understanding of four- and five-dimensional extremal black holes has extended our knowledge of supergravity backgrounds involving $\mathrm{AdS}_2$ and $\mathrm{AdS}_3$ geometries. For instance, an infinitely deep $\mathrm{AdS}_2$ throat arises as the near horizon geometry of 4d extremal black holes that have associated an $\mathrm{SL}(2,\mathbf{R})\times \mathrm{U}(1)$ isometry, which includes the conformal group in 1d. Even if this limit is clear geometrically a microscopic understanding remains a demanding task [2–4]. Via the AdS/CFT correspondence [5] one might presume that there is a conformal quantum mechanics dual to these $\mathrm{AdS}_2$ geometries. Nevertheless, $\mathrm{AdS}_2/\mathrm{CFT}_1$ pairs pose important conceptual puzzles [6–9] originated from the boundary of $\mathrm{AdS}_2$ being non-connected [10].

Partial attempts at studying $\mathrm{AdS}_2$ and $\mathrm{AdS}_3$ solutions in 10 and 11 dimensions, with vast and rich structures coming from the high dimensionality of the internal space, admitting many possible geometries, topologies and amounts of supersymmetry, have been carried out, [1, 11–43]. In particular, recent progress has been reported on the construction of new $\mathrm{AdS}_3$ solutions with four Poincaré supersymmetries [18, 24, 26, 34, 37] as well as on the identification of their 2d (half-maximal BPS) dual CFTs [27–29, 34, 37, 44]. In the same vein, $\mathrm{AdS}_2/\mathrm{CFT}_1$ pairs have been explored as a natural extension of $\mathrm{AdS}_3/\mathrm{CFT}_2$ pairs through T-duality [38] and double analytical continuation, [1, 41], in each case, providing different families of quiver quantum mechanics with four Poincaré supersymmetries.

Part of the motivation for this work is to construct $\mathrm{AdS}_2$ solutions through non-Abelian T-duality acting on $\mathrm{AdS}_3$ spaces. Non-Abelian T-duality (NATD) was introduced in the 90's [45] as a transformation of the string $\sigma$-model, generalising to non-Abelian isometry groups the path integral approach to Abelian T-duality put forward in [46]. From these studies other important groundwork arose, see for example [47–51]. In spite of this initial progress and unlike its Abelian counterpart, the NATD transformation did not reach the status of a string theory symmetry [48, 50–55], due to two main difficulties. Firstly, NATD has only been worked out as a transformation in the worldsheet for spherical topologies (namely, at tree level in string perturbation theory) and second, the conformal symmetry of the string $\sigma$-model is only known to survive the NATD transformation at first order in $\alpha'$.

Sfetsos and Thompson [56] reignited the interest in NATD by showing that it can be successfully used as a solution generating technique in supergravity, with the derivation of the transformation rules of the RR sector. This study was initiated with the dualisation of the $\mathrm{AdS}_5\times\mathrm{S}^5$ and $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ backgrounds with respect to a freely acting SU(2) isometry group (SU(2)-NATD). This work was of particular interest to tackle the rôle NATD might have in the context of AdS/CFT correspondence. In this vein, interesting examples of AdS spacetimes generated through NATD in different contexts have been constructed to date [56–73]. Holographically, the field theoretical interpretation of NATD was first addressed in [29, 74–77], where the main conclusion is that NATD changes the field theory dual to the original theory. Remarkably, in all examples so far of NATD in supergravity — in the context of holography — the dualisation took place with respect to a freely acting SU(2) subgroup of the entire symmetry group of the solutions.

The main purpose of this work is to construct an $\mathrm{AdS}_2$ solution to massive Type IIA supergravity acting with NATD on the well-known D1-D5 near horizon system. Here the dualisation is performed with respect to a freely acting SL(2,$\mathbf{R}$) group (SL(2,$\mathbf{R}$)-NATD). Second, we give a proposal for its dual superconformal quantum mechanics, in terms of D0 and D4 colour branes coupled to D4′ and D8 flavour branes, inspired by the results in [1].

The organisation of the paper is as follows. In section 2, we develop the technology necessary to construct solutions through SL(2,$\mathbf{R}$)-NATD. In the same section we apply these results to the D1-D5 near horizon system, generating a new $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2$ geometry foliated over an interval. The brane set-up, charges and holographic central charge are carefully studied. Section 3 contains a summary of the infinite family of $\mathrm{AdS}_2$ solutions to massive Type IIA supergravity with four Poincaré supersymmetries constructed in [1], as well as of the quiver quantum mechanics proposed there as duals to these geometries. In section 4, we show that our SL(2,$\mathbf{R}$)-NATD solution provides an explicit example in the classification in [1]. At the end of this section we study an explicit completion of this solution and propose a quiver quantum mechanics that admits a description in terms of interactions between Wilson lines and D0 and D4 instantons in the world-volumes of the D4′ and D8 branes. Our conclusions are contained in section 5.

## 2 NATD of $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ with respect to a freely acting $\mathrm{SL}(2,\mathbf{R})$

In this section we review the dualisation procedure and apply it to the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ solution of Type IIB supergravity. We address the construction of the brane set-up, Page charges and holographic central charge of the resulting background and propose a quiver quantum mechanics that flows in the IR to the superconformal quantum mechanics dual to our solution.



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### 2.1 NATD with respect to $\mathbf{SL}(2,\mathbf{R})_L$

The study of NATD as a solution generating technique in supergravity was initiated in [56], where the dualisation was carried out with respect to a freely acting SU(2) isometry group. Since then, several works have taken advantage of this technology to generate new AdS solutions, some of which avoiding previously existing classifications (see for instance [58, 69, 78, 79]). Most of these examples possess rich isometry groups containing at least an SU(2) factor that can be used to dualise. Instead, in this work we will use a non-compact, freely acting, SL(2,$\mathbf{R}$) group to dualise. This is the first time that NATD with respect to a non-compact isometry group has been applied as a solution generating technique in supergravity.[1] Following [52] we perform the NATD transformation with respect to one of the freely acting SL$(2,\mathbf{R})$ isometry groups of the AdS$_3$ subspace of the AdS$_3\times$S$^3\times$CY$_2$ solution of Type IIB supergravity. We start reviewing the necessary technology.

Consider a bosonic string $\sigma$-model that supports an SL$(2,\mathbf{R})$ isometry, such that the NS-NS fields can be written as,

$$\begin{aligned} ds^2 &= \frac{1}{4}g_{\mu\nu}(x)L^\mu L^\nu + G_{i\mu}(x)dx^iL^\mu + G_{ij}(x)dx^idx^j, \\ B_2 &= \frac{1}{8}b_{\mu\nu}(x)L^\mu\wedge L^\nu + \frac{1}{2}B_{i\mu}(x)dx^i\wedge L^\mu + B_{ij}(x)dx^i\wedge dx^j, \qquad \Phi=\Phi(x), \end{aligned} \tag{2.1}$$

where $x^i$ are the coordinates in the internal manifold, for $i,j=1,2,\ldots,7$, and $L^\mu$ are the SL$(2,\mathbf{R})$ left-invariant Maurer-Cartan forms,

$$L^\mu = -i\mathrm{Tr}(t^\mu g^{-1}\mathrm{d}g), \qquad \text{which obey,} \quad \mathrm{d}L^\mu = \frac{1}{2}f^\mu{}_{\alpha\nu}L^\alpha\wedge L^\nu, \tag{2.2}$$

where $f^\mu{}_{\alpha\nu}$ are the structure constants of SL(2,$\mathbf{R}$). The generators of the $sl(2,\mathbf{R})$ algebra can be obtained by analytically continuing the $su(2)$ generators as,

$$t^a = \frac{\tau_a}{\sqrt{2}}, \qquad \text{with} \quad \tau_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{2.3}$$

These generators satisfy,[2]

$$\mathrm{Tr}(t^at^b) = (-1)^a\delta^{ab}, \qquad [t^1,t^2] = i\sqrt{2}t^3, \quad [t^2,t^3] = i\sqrt{2}t^1, \quad [t^3,t^1] = -i\sqrt{2}t^2. \tag{2.4}$$

The group element $g\in$ SL(2,$\mathbf{R}$) depends on the target space isometry directions, realising an SL(2,$\mathbf{R}$) group manifold. Here the group manifold is an AdS$_3$ space. The geometry described by (2.1) is then manifestly invariant under $g\to\lambda^{-1}g$ for $\lambda\in$ SL(2,$\mathbf{R}$). We parametrise an SL(2,$\mathbf{R}$) group element in the following fashion,

$$g = e^{\frac{i}{2}t\tau_3}e^{\frac{i}{2}\theta\tau_2}e^{\frac{i}{2}\eta\tau_3}, \qquad \text{with} \qquad 0\le\theta\le\pi, \;\; 0\le t<\infty, \;\; 0\le\eta<\infty, \tag{2.5}$$

[1] In [41], SL$(2,\mathbf{R})$-NATD was used to find an explicit example — with brane sources — in the class of AdS$_2\times$S$^2\times$CY$_2$ solutions fibered over a 2d Riemann surface constructed in [14].

[2] We take $g^{\mu\nu} = -\mathrm{Tr}(t^\mu t^\nu)$ in order to have $(+,-,+)$ signature.



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which is closely related to the Euler angles parametrising SU(2). Thus, the left-invariant forms (2.2) are given by,

$$\begin{aligned} &L^1 = \sinh\eta \mathrm{d}\theta - \cosh\eta\sin\theta \mathrm{d}t, \qquad L^2 = \cosh\eta \mathrm{d}\theta - \sinh\eta\sin\theta \mathrm{d}t, \\ &L^3 = -\cos\theta \mathrm{d}t - \mathrm{d}\eta. \end{aligned} \tag{2.6}$$

A string propagating in the geometry given by (2.1) is described by the non-linear $\sigma$-model,

$$S = \int \mathrm{d}\sigma^2 \left( E_{\mu\nu} L_+^\mu L_-^\nu + Q_{i\mu}\partial_+ x^i L_-^\mu + Q_{\mu i} L_+^\mu \partial_- x^i + Q_{ij}\partial_+ x^i \partial_- x^j \right), \tag{2.7}$$

$$\text{with} \quad E_{\mu\nu} = g_{\mu\nu} + b_{\mu\nu}, \quad Q_{i\mu} = G_{i\mu} + B_{i\mu}, \quad Q_{\mu i} = G_{\mu i} + B_{\mu i}, \quad Q_{ij} = G_{ij} + B_{ij}.$$

and $L_\pm^\mu$ are the left-invariant forms pulled back to the worldsheet. This $\sigma$-model is also invariant under $g \to \lambda^{-1} g$ for $\lambda \in$ SL(2,$\mathbf{R}$).

The SL(2,$\mathbf{R}$) non-Abelian T-dual solution for the $\sigma$-model (2.7) is constructed as in [45], introducing covariant derivatives, $\partial_\pm g \to D_\pm g = \partial_\pm g - A_\pm g$, in the Maurer-Cartan forms but enforcing the condition that the gauge field is non-dynamical with the addition to the action of a Lagrange multiplier term,

$$-i\mathrm{Tr}(vF_\pm), \tag{2.8}$$

where $F_\pm = \partial_+ A_- - \partial_- A_+ - [A_+, A_-]$ is the field strength for the gauged fields $A_\pm$. $v$ is a vector that takes values in the Lie algebra of the SL(2,$\mathbf{R}$) group and it is coupled to the field strength, $F_\pm$. In this way, the total action is invariant under,

$$g \to \lambda^{-1} g, \quad A_\pm \to \lambda^{-1}(A_\pm \lambda - \partial_\pm \lambda), \quad v \to \lambda^{-1} v \lambda, \quad \text{with} \quad \lambda(\sigma^+, \sigma^-) \in \mathrm{SL}(2, \mathbf{R}). \tag{2.9}$$

After integrating out the Lagrange multiplier and fixing the gauge, we recover the original non-linear $\sigma$-model. On the other hand, by integrating by parts the Lagrange multiplier term one can solve for the gauge fields and obtain the dual $\sigma$-model, that still relies on the parameters $t, \theta, \eta$ and the Lagrange multipliers. In order to preserve the number of degrees of freedom, the redundancy is fixed by choosing a gauge fixing condition, for instance $g = \mathbb{I}$, which implies $t = \theta = \eta = 0$. The resulting action reads,

$$\hat{S} = \int \mathrm{d}\sigma^2 \left[ Q_{ij}\partial_+ x^i \partial_- x^j + (\partial_+ v_\mu + \partial_+ x^i Q_{i\mu}) M_{\mu\nu}^{-1} (\partial_- v_\nu - Q_{\nu i}\partial_+ x^i) \right], \tag{2.10}$$

$$\text{with} \quad M_{\mu\nu} = E_{\mu\nu} + f^\alpha{}_{\mu\nu} v_\alpha.$$

In this action the parameters $t, \theta, \eta$ have been replaced by the Lagrange multipliers $v_i$, $i = 1, 2, 3$, which live in the Lie algebra of SL(2,$\mathbf{R}$), this is non-compact, by its construction as a vector space.

In particular, the solutions generated by SU(2)-NATD are non-compact manifolds even if the group used in the dualisation procedure is compact, this is because the new variables live in the Lie algebra of the dualisation group. As we see, the SL(2,$\mathbf{R}$)-NATD solution generating technique inherits this non-compactness. At the level of the metric and using the following parametrisation for the Lagrange multipliers,

$$v = (\rho \cos\tau \cosh\xi, \rho \sinh\xi, \rho \sin\tau \cosh\xi), \tag{2.11}$$

the original $AdS_3$ space is replaced by an $AdS_2 \times \mathbf{R}^+$ space, where besides the $AdS_2$ factor (in which the remaining SL(2,**R**) symmetry is reflected) a non-compact radial direction is generated in the internal space.

Furthermore, from the path integral derivation the dilaton receives a 1-loop shift, leading to a non-trivial dilaton in the dual theory, given by,

$$\hat{\Phi}(x, v) = \Phi(x) - \frac{1}{2}\log(\det M). \tag{2.12}$$

A similar shift in the dilaton was obtained in Abelian T-duality [45], in such a case $M$ is the metric component in the direction where the dualisation is carried out.

The transformation rules for the RR fields was the new input in [56] which allowed to use NATD as a solution generating technique in supergravity. This was done using a spinor representation approach. The derivation relied on the fact that left and right movers transform differently under NATD, and therefore lead to two different sets of frame fields for the dual geometry. In the SL(2,**R**)-NATD case, we also have two different sets of frame fields, which define the same dual metric obtained from (2.10), and must therefore be related by a Lorentz transformation, $\Lambda^\alpha{}_\beta$. In turn, this Lorentz transformation acts on spinors through a matrix $\Omega$, defined by the invariance property of gamma matrices,

$$\Omega^{-1}\Gamma^\alpha\Omega = \Lambda^\alpha{}_\beta\Gamma^\beta, \tag{2.13}$$

and given that the RR fluxes can be combined to form bispinors,

$$P = \frac{e^\Phi}{2}\sum_{n=0}^{4} \not{F}_{2n+1}, \qquad \hat{P} = \frac{e^{\hat{\Phi}}}{2}\sum_{n=0}^{5} \hat{\not{F}}_{2n}, \qquad \text{with} \quad \not{F}_p = \frac{1}{p!}\Gamma_{\nu_1\ldots m_p} F_p^{\nu_1\nu_2\ldots m_p}, \tag{2.14}$$

one can finally extract their transformation rules by right multiplication with the $\Omega^{-1}$ matrix on the RR bispinors,

$$\hat{P} = P \cdot \Omega^{-1}, \tag{2.15}$$

where $\hat{P}$ are the dual RR bispinors. Notice that the action (2.15) on the RR sector is from a Type IIB to a IIA solution. If starting from a Type IIA to a IIB solution instead, the rôle of $P$ and $\hat{P}$ is swapped. The knowledge of the transformation rules for the RR sector guarantees that starting with a solution to Type II supergravity the dual background is also a solution.

The technology reviewed in this section allows us to consider a non-compact space like $AdS_3$, which posses an $SO(2,2) \cong SL(2,\mathbf{R})_L \times SL(2,\mathbf{R})_R$ isometry group. After performing the dualisation with respect to a freely acting $SL(2,\mathbf{R})$ group the isometry gets reduced to just $SL(2,\mathbf{R})$, which is geometrically realised by an $AdS_2$ factor in the dual geometry.



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| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D1 | x | | | | | x | | | | |
| D5 | x | x | x | x | x | x | | | | |

**Table 1**. The set-up for $Q_{\mathrm{D1}}$ D1-branes wrapped on $x^5$ and $Q_{\mathrm{D5}}$ D5-branes wrapped on $x^5$ and $\mathrm{CY}_2$. This system preserves (4,4) supersymmetry. The field theory lives in the $x^0$ and $x^5$ directions and $x^1, x^2, x^3, x^4$ parameterise the $\mathrm{CY}_2$. The SO(4) R-symmetry is geometrically realised in the $x^6, x^7, x^8, x^9$ directions.

Further, as we explained before, the dual geometry acquires a non-compact direction, that now belongs to the internal space.

In the next section we will apply this technology to the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ geometry to produce an $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2\times\mathrm{I}$ solution in massive Type IIA supergravity, which fits in the classification in [1].

### 2.2 Dualisation of the $\mathbf{AdS}_3\times\mathbf{S}^3\times\mathbf{CY}_2$ background

We consider IIB string theory on $\mathbf{R}^{1,1}\times\mathbf{R}^4\times\mathrm{CY}_2$ where we include $Q_{\mathrm{D1}}$ D1-branes and $Q_{\mathrm{D5}}$ D5-branes as is shown in the brane set-up depicted in table 1.

The $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ background arising in the near horizon limit of the D1-D5 system depicted in table 1 is,

$$\begin{aligned}
&ds^2_{10} = 4L^2 ds^2_{\mathrm{AdS}_3} + M^2 ds^2_{\mathrm{CY}_2} + 4L^2 ds^2_{\mathrm{S}^3}, \qquad e^{2\Phi}=1,\\
&F_3 = 8L^2(\mathrm{vol}_{\mathrm{S}^3}+\mathrm{vol}_{\mathrm{AdS}_3}), \qquad\qquad F_7 = -8L^2M^4(\mathrm{vol}_{\mathrm{S}^3}+\mathrm{vol}_{\mathrm{AdS}_3})\wedge \mathrm{vol}_{\mathrm{CY}_2}\,.
\end{aligned} \tag{2.16}$$

Here we will use $\mathrm{Vol}_{\mathrm{CY}_2}=(2\pi)^4$.

Following the rules explained in the previous section, the $\mathrm{SL}(2,\mathbf{R})$-NATD transformation of the background (2.16) gives rise to the following geometry,

$$\begin{aligned}
&ds^2_{10} = \frac{L^2\rho^2}{\rho^2-4L^4} ds^2_{\mathrm{AdS}_2} + 4L^2 ds^2_{\mathrm{S}^3} + M^2 ds^2_{\mathrm{CY}_2} + \frac{d\rho^2}{4L^2},\\
&e^{2\Phi} = \frac{4}{L^2(\rho^2-4L^4)}, \qquad\qquad B_2 = -\frac{\rho^3}{2(\rho^2-4L^4)}\mathrm{vol}_{\mathrm{AdS}_2},\\
&F_0 = L^2, \quad F_2 = -\frac{L^2\rho^3}{2(\rho^2-4L^4)}\mathrm{vol}_{\mathrm{AdS}_2}, \quad F_4 = -L^2(M^4\mathrm{vol}_{\mathrm{CY}_2} - 2\rho\, \mathrm{d}\rho\wedge\mathrm{vol}_{\mathrm{S}^3}),\\
&F_6 = \frac{L^2\rho^2}{2(\rho^2-4L^4)}(\rho M^4 \mathrm{vol}_{\mathrm{CY}_2} - 8L^4\mathrm{d}\rho\wedge\mathrm{vol}_{\mathrm{S}^3})\wedge\mathrm{vol}_{\mathrm{AdS}_2},\\
&F_8 = 2L^2M^4\rho\,\mathrm{vol}_{\mathrm{S}^3}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho, \qquad F_{10} = -\frac{4L^6\rho^2M^4}{\rho^2-4L^4}\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{S}^3}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho.
\end{aligned} \tag{2.17}$$

Here we have parametrised the Lagrange multipliers as in (2.11) in order to manifestly realise the $\mathrm{SL}(2,\mathbf{R})$ residual global symmetries. Indeed, from the original $\mathrm{SO}(2,2)$ isometry group, after the dualisation, an $\mathrm{SL}(2,\mathbf{R})$ subgroup survives, which is geometrically realised by a warped $\mathrm{AdS}_2\times\mathbf{R}^+$ subspace.



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The background (2.17) is a solution to the massive Type IIA supergravity EOMs. As we will see in section 4, it is an explicit solution in the classification provided in [1]. In order to have the right signature and avoid singularities we are forced to set $\rho^2 - 4L^4 > 0$. Namely, we get a well-defined geometry for,

$$\rho > \rho_0 = 2L^2, \tag{2.18}$$

where the $\rho$ coordinate begins.

The asymptotic behaviour of the metric and dilaton in (2.17) at the beginning of the space, around $\rho = \rho_0$ is,

$$ds^2 \sim \frac{a_1}{x} ds^2_{\mathrm{AdS}_2} + a_2 ds^2_{\mathrm{S}^3} + M^2 ds^2_{\mathrm{CY}_2} + a_3 dx^2, \qquad e^{\Phi} \sim a_4 x^{-1/2}, \tag{2.19}$$

with $x = \rho - \rho_0 > 0$ and $a_i$ are constants. Here the warp factor reproduces the behaviour of an OF1 plane extended in $\mathrm{AdS}_2$ and smeared over $\mathrm{S}^3$, this is also consistent with additional coincident fundamental strings if they are smeared on the $\mathrm{S}^3$ and the $\mathrm{CY}_2$. Further, in section 4.1, we will provide a concrete completion for the background (2.17), where at both ends of the space the behaviour given in (2.19) is identified.

We conclude this section with some comments about the supersymmetry of the solution (2.17). On one hand, as we mentioned before (and we will show in section 4) the background (2.17) fits in the class of $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2\times\mathrm{I}$ solutions to massive Type IIA constructed in [1], which contain eight supersymmetries, four Poincaré and four conformal. Second, it is well established by now [56, 60] that performing non-Abelian T-duality on a round 3-sphere projects out the spinors charged under either the $\mathrm{SU}(2)_L$ or $\mathrm{SU}(2)_R$ subgroup of the global SO(4) factor of $\mathrm{S}^3$, leaving the rest intact. This amounts to a halving of supersymmetry in the non-Abelian T-dual of $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ [56]. The SL(2,$\mathbf{R}$)-NATD works analogously, this time one projects out the spinors charged under one of the SL(2,$\mathbf{R}$) factors of the global $\mathrm{SO}(2,2) \cong \mathrm{SL}(2,\mathbf{R})_L \times \mathrm{SL}(2,\mathbf{R})_R$ isometry, keeping the rest intact. As such SL(2,$\mathbf{R}$)-NATD on the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ solution also reduces the supersymmetry by half. That this mirrors the halving of the supersymmetries as in the SU(2)-NATD case is hardly surprising, the solutions are after all related by a double analytic continuation (as we will explain around the equation (4.2)).

### 2.3 Brane set-up and charges

Non-Abelian T-dualisation under a freely acting SU(2) subgroup of an SO(4) symmetry reduces the isometry group to SU(2). Geometrically, the $\mathrm{S}^3$ is replaced by its Lie algebra, $\mathbf{R}^3$, which is locally $\mathbf{R}\times\mathrm{S}^2$. This isometry is reflected in the dual fields, for instance a $B_2$ over the $\mathrm{S}^2$ is generated after the dualisation, which is $\rho$ dependent (like the $B_2$ in (2.17)). This $\rho$ dependence in $B_2$ implies that large gauge transformations must be included such that $\frac{1}{4\pi^2}|\int B_2|$ remains in the fundamental region as we move in the $\rho$ direction. This argument was developed in [64, 66, 74] where the non-compactness in the $\rho$ coordinate — in backgrounds like (2.17) — was addressed with the introduction of large gauge transformations in the dual geometry.

| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D0 | x | | | | | | | | | |
| D4 | x | x | x | x | x | | | | | |
| D4′ | x | | | | | | x | x | x | x |
| D8 | x | x | x | x | x | | x | x | x | x |
| F1 | x | | | | | x | | | | |

**Table 2**. Brane set-up associated to our solution. Here x denotes the spacetime directions spanned by the various branes. $x^0$ corresponds to the time direction of the ten dimensional spacetime, $x^1, \ldots, x^4$ are the coordinates spanned by the $\mathrm{CY}_2$, $x^5$ is the direction where the F1-strings are stretched, and $x^6, x^7, x^8, x^9$ are the coordinates where the SO(4) symmetry is realised.

The $\mathrm{SL}(2, \mathbf{R})$-NATD, as shown in the previous section, produces an antisymmetric Kalb-Ramond tensor over the $\mathrm{AdS}_2$ directions, signaling the presence of fundamental strings in the solution. We use the same argument as in the SU(2)-NATD case to determine the range of the $\rho$ coordinate, (see [1] for more details). Namely, we impose that the quantity,

$$\frac{1}{4\pi^2}\Big|\int_{\mathrm{AdS}_2} B_2\Big| \in [0,1), \tag{2.20}$$

is bounded and use a regularised volume for $\mathrm{AdS}_2$,[3]

$$\mathrm{Vol}_{\mathrm{AdS}_2} = 4\pi^2. \tag{2.21}$$

For $B_2$ in (2.17) to satisfy (2.20) a large gauge transformation is needed as we move along $\rho$. Namely, for $\rho \in [\rho_k, \rho_{k+1}]$ we need to perform $B_2 \to B_2 + \pi k \mathrm{vol}_{\mathrm{AdS}_2}$, with

$$\frac{\rho_k^3}{\rho_k^2 - \rho_0^2} = 2\pi k. \tag{2.22}$$

We continue the study of the background (2.17) by computing the associated charges, obtained from the Page fluxes, defined by $\hat{F} = e^{-B_2} \wedge F$, given by,

$$\begin{aligned}
\hat{F}_0 =& L^2, \qquad \hat{F}_2 = -L^2 k\pi \mathrm{vol}_{\mathrm{AdS}_2}, \qquad \hat{F}_4 = -L^2(M^4 \mathrm{vol}_{\mathrm{CY}_2} - 2\rho\, \mathrm{d}\rho \wedge \mathrm{vol}_{\mathrm{S}^3}),\\
\hat{F}_6 =& L^2(\pi k M^4 \mathrm{vol}_{\mathrm{CY}_2} + \rho(\rho - 2\pi k)\, \mathrm{d}\rho \wedge \mathrm{vol}_{\mathrm{S}^3}) \wedge \mathrm{vol}_{\mathrm{AdS}_2},\\
\hat{F}_8 =& 2L^2 M^4 \rho\, \mathrm{vol}_{\mathrm{S}^3} \wedge \mathrm{vol}_{\mathrm{CY}_2} \wedge \mathrm{d}\rho,\\
\hat{F}_{10} =& L^2 M^4 \rho(\rho - 2\pi k) \mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{vol}_{\mathrm{S}^3} \wedge \mathrm{vol}_{\mathrm{CY}_2} \wedge \mathrm{d}\rho,
\end{aligned} \tag{2.23}$$

where we have taken into account the large gauge transformations $B_2 \to B_2 + \pi k \mathrm{vol}_{\mathrm{AdS}_2}$. Inspecting the Page fluxes (2.23), we determine the type of branes that we have in the system. This is the D0-D4-D4′-D8-F1 brane intersection depicted in table 2.

Using the expressions for the Page fluxes (2.23) we compute the magnetic charges of Dp-branes using,

$$Q^m_{\mathrm{Dp}} = \frac{1}{(2\pi)^{7-p}} \int_{\Sigma_{8-p}} \hat{F}_{8-p}, \tag{2.24}$$

[3]This reguralisation prescription is taken from [1].

where $\Sigma_{8-p}$ is a $(8-p)$-dimensional manifold transverse to the directions of the Dp-brane. Furthermore, we define the electric charge of a Dp-brane as follows,

$$Q^e_{\rm Dp} = \frac{1}{(2\pi)^{p+1}} \int_{{\rm AdS}_2\times\tilde{\Sigma}_p} \hat{F}_{p+2}, \tag{2.25}$$

here $\tilde{\Sigma}_p$ is defined as a $p-$dimensional manifold on which the brane extends. Both expressions, (2.24) and (2.25) are written in units of $\alpha' = g_s = 1$.

As we anticipated, the background (2.17) fits in the class of solutions presented in [1], that we briefly summarise in the next section. In such geometries, the D0 and D4-branes are interpreted in the dual field theory as instantons carrying electric charge. In turn, the D4$'$ and D8-branes have an interpretation as magnetically charged branes where the instantons lie. In the interval $[\rho_k, \rho_{k+1}]$, these charges look in the following fashion,

$$\begin{aligned}
Q^m_{\rm D8} &= 2\pi \hat{F}_0 = 2\pi L^2,\\
Q^m_{\rm D4'} &= \frac{1}{(2\pi)^3}\int_{{\rm CY}_2} \hat{F}_4 = 2\pi L^2 M^4,\\
Q^e_{\rm D0} &= \frac{1}{2\pi}\int_{{\rm AdS}_2} \hat{F}_2 = 2\pi k L^2 = k\, Q^m_{\rm D8},\\
Q^e_{\rm D4} &= \frac{1}{(2\pi)^5}\int_{{\rm AdS}_2\times{\rm CY}_2} \hat{F}_6 = 2\pi k L^2 M^4 = k\, Q^m_{\rm D4'}\ ,
\end{aligned} \tag{2.26}$$

where we have used $\mathrm{Vol}_{{\rm CY}_2} = 16\pi^4$. Furthermore, the fundamental strings are electrically charged with respect to the 3-form $H_3$,

$$Q^e_{\rm F1} = \frac{1}{(2\pi)^2}\int_{{\rm AdS}_2\times {\rm I}_\rho} H_3 = \frac{1}{\pi} B_2 \Big|^{\rho_{k+1}}_{\rho_k} = 1. \tag{2.27}$$

One fundamental string is produced every time we cross the value $\rho = \rho_k$. Therefore in the interval $[0, \rho_k]$ there are $k$ F1-strings.

### 2.4 Holographic central charge

In the spirit of the AdS/CFT correspondence, the study of AdS$_2$ geometries leads to consider one-dimensional dual field theories, where the definition of the central charge is subtle. In a conformal quantum mechanics the energy momentum tensor has only one component, and as the theory is conformal, it must vanish. We will interpret the central charge as counting the number of vacuum states in the dual superconformal quantum mechanics, along the lines of [1, 38, 41].

We compute the holographic central charge following the prescription in [69, 79], where this quantity is obtained from the volume of the internal manifold, accounting for a non-trivial dilaton,

$$\begin{aligned}
V_{\rm int} &= \int d^8x\, e^{-2\Phi}\sqrt{\det g_{8,{\rm ind}}} = 2^5\pi^6 L^4 M^4 \int_{{\rm I}_\rho} (\rho^2 - 4L^4)\, {\rm d}\rho,\\
c_{\rm hol} &= \frac{3V_{\rm int}}{4\pi G_N} = \frac{3L^4 M^4}{\pi}\int_{{\rm I}_\rho} (\rho^2 - 4L^4)\, {\rm d}\rho,
\end{aligned} \tag{2.28}$$

where $G_N = 8\pi^6$ in units $g_s = \alpha' = 1$. Since the dual manifold is non-compact the new background has an internal space of infinite volume that leads to an infinite holographic central charge, which points the solution needs a completion, as is shown in expression (2.28).

In the next section, we review the solutions constructed in [1] in order to see that the background (2.17) fits in that class of solutions. In turn, using the developments of [1], a concrete completion to the background (2.17) generated by SL(2,**R**)-NATD is proposed. Such completion in the geometry implies also a completion in the quiver, letting us to describe a well-defined CFT.

## 3 The $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2$ solutions to massive IIA and their dual SCQM

In [26] a classification of $\mathrm{AdS}_3\times\mathrm{S}^2$ solutions to massive IIA supergravity with small (0,4) supersymmetry and SU(2)-structure was obtained. These solutions are warped products of the form $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{M}_4\times\mathrm{I}$ preserving an SU(2) structure on the internal five-dimensional space. The $\mathrm{M}_4$ is either a $\mathrm{CY}_2$ or a 4d Kähler manifold. The respective classes of solutions are referred as class I and class II. In this section we briefly discuss the $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2$ solutions obtained via a double analytical continuation of the class I solutions above. These solutions were first constructed in [34] and then studied in detail in [1]. These backgrounds are dual to SCQMs which were also studied in [1], and that we also review. The study of the solutions constructed in [1, 34] allows us to propose a concrete completion for the solution (2.17) and therefore a well-defined central charge. We present the details of this completion in section 4.

A subset of the backgrounds studied in [1, 34] — where we assume that the symmetries of the $\mathrm{CY}_2$ are respected by the full solution — read,

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{h_4 h_8}}\left(\frac{h_4 h_8}{\Delta}\mathrm{d}s^2_{\mathrm{AdS}_2} + \mathrm{d}s^2_{\mathrm{S}^3}\right) + \sqrt{\frac{h_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{h_4 h_8}}{u}\mathrm{d}\rho^2\,, \quad \Delta = 4h_4 h_8 - (u')^2,\\
e^{-2\Phi} &= \frac{h_8^{3/2}\Delta}{4h_4^{1/2}u}\,, \qquad\qquad B_2 = -\frac{1}{2}\left(\rho - 2\pi k + \frac{uu'}{\Delta}\right)\mathrm{vol}_{\mathrm{AdS}_2},\\
\hat{F}_0 &= h_8'\,, \qquad\qquad \hat{F}_2 = -\frac{1}{2}\Big(h_8 - h_8'(\rho - 2\pi k)\Big)\mathrm{vol}_{\mathrm{AdS}_2}\,,\\
\hat{F}_4 &= \left(2h_8\mathrm{d}\rho - \mathrm{d}\left(\frac{u'u}{2h_4}\right)\right)\wedge \mathrm{vol}_{\mathrm{S}^3} - \partial_\rho h_4 \mathrm{vol}_{\mathrm{CY}_2}\,.
\end{aligned} \tag{3.1}$$

Here $\Phi$ is the dilaton and $B_2$ is the Kalb-Ramond field. The warping functions $h_8$, $h_4$ and $u$ have support on $\rho$, with $u' = \partial_\rho u$. We have quoted the Page fluxes, $\hat{F} = e^{-B_2}\wedge F$, and included large gauge transformations[4] of $B_2$ of parameter $k$, $B_2 \to B_2 + \pi k \mathrm{vol}_{\mathrm{AdS}_2}$. The higher dimensional fluxes can be obtained as $F_p = (-1)^{[p/2]}\star_{10} F_{10-p}$. Note that $\Delta > 0$, in order to guarantee a real dilaton and a metric with the correct signature.

[4] Like those that were studied in section 2.3.

Supersymmetry holds whenever $u'' = 0$. In turn, the Bianchi identities of the fluxes impose,

$$h_8'' = 0\,, \qquad h_4'' = 0, \tag{3.2}$$

away from localised sources, which makes $h_8$ and $h_4$ are piecewise linear functions of $\rho$.

Particular solutions were studied in [1] where the functions $h_4$ and $h_8$ are piecewise continuous as follows,

$$h_4(\rho) = \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \le \rho \le 2\pi, \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1), \quad k = 1, \dots, P-1 \\ \alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1), \end{cases} \tag{3.3}$$

$$h_8(\rho) = \begin{cases} \frac{\nu_0}{2\pi}\rho & 0 \le \rho \le 2\pi, \\ \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k) & 2\pi k \le \rho \le 2\pi(k+1), \quad k = 1, \dots, P-1 \\ \mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P) & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{3.4}$$

For $u' = 0$ the previous functions vanish at $\rho = 0$ and $\rho = 2\pi(P+1)$, where the space begins and ends. The $\alpha_k$, $\beta_k$, $\mu_k$ and $\nu_k$ are integration constants, which are determined by imposing continuity of the NS sector as,

$$\mu_k = \sum_{j=0}^{k-1} \nu_j, \qquad \alpha_k = \sum_{j=0}^{k-1} \beta_j. \tag{3.5}$$

Using the piecewise functions (3.3) and (3.4) in the $[\rho_k, \rho_{k+1}]$ interval and the definitions (2.24)–(2.25), the expressions for the charges are,

$$\begin{aligned} Q^e_{\mathrm{D0}} = h_8 - (\rho - 2\pi k)h_8' = \mu_k, \qquad & Q^e_{\mathrm{D4}} = h_4 - (\rho - 2\pi k)h_4' = \alpha_k, \\ Q^m_{\mathrm{D4'}} = 2\pi h_4' = \beta_k, \qquad & Q^m_{\mathrm{D8}} = 2\pi h_8' = \nu_k, \end{aligned} \tag{3.6}$$

and given that,

$$\mathrm{d}\hat{F}_0 = h_8''\mathrm{d}\rho, \qquad \mathrm{d}\hat{F}_4 = h_4''\mathrm{d}\rho \wedge \mathrm{vol}_{\mathrm{AdS}_2}, \tag{3.7}$$

with,

$$h_8'' = \frac{1}{2\pi}\sum_{j=1}^{P}(\nu_{j-1} - \nu_j)\delta(\rho - 2\pi j), \qquad h_4'' = \frac{1}{2\pi}\sum_{j=1}^{P}(\beta_{j-1} - \beta_j)\delta(\rho - 2\pi j), \tag{3.8}$$

there are D8 and D4$'$ brane sources localised in the $\rho$ direction. In turn, both $\mathrm{d}\hat{F}_8$ and the $\mathrm{vol}_{\mathrm{S}^3}$ component of $\mathrm{d}\hat{F}_4$ vanish identically, which implies that D0 and D4 branes play the rôle of colour branes. The brane set-up associated to the solution (3.1) consists of a D0-F1-D4-D4$'$-D8 brane intersection, as depicted in table 2.

In addition, in [1] the number of vacua was computed. For the solutions defined by the above functions, it was shown that the holographic central charge is given by,

$$c_{\mathrm{hol,1d}} = \frac{3V_{\mathrm{int}}}{4\pi G_N} = \frac{3}{4\pi}\frac{\mathrm{Vol}_{\mathrm{CY}_2}}{(2\pi)^4}\int_0^{2\pi(P+1)}(4h_4h_8 - (u')^2)\,\mathrm{d}\rho. \tag{3.9}$$

In the next section we briefly describe the SCQM proposed in [1] in order to extract information about the field theory associated to the background (2.17).

### 3.1 The dual superconformal quantum mechanics

In [1], a proposal for the quantum mechanics living on the D0-D4-D4′-D8-F1 brane intersection was given in terms of an ADHM quantum mechanics that generalises the one discussed in [80]. This quantum mechanics was interpreted as describing the interactions between instantons and Wilson lines in 5d gauge theories with 8 Poincaré supersymmetries living in D4-D8 intersections. The complete D0-D4-D4′-D8-F1 brane intersection was split into two subsystems, D4-D4′-F1 and D0-D8-F1, that were first studied independently.

Let us start considering the D4-D4′-F1 brane subsystem. This subsystem was interpreted as a BPS Wilson line in the 5d theory living on the D4-branes. When probing the D4-branes with fundamental strings, D4′-branes transverse to the D4-branes are originated. These orthogonal D4′-branes carry a magnetic charge $Q^m_{\mathrm{D4}'} = 2\pi h'_4$ proportional to the number of fundamental strings dissolved in the world-volume of the D4′-branes. Additionally, the D4-branes can be seen as instantons in the world-volume of the D8-branes [81], where the D4-brane wrapped on the $\mathrm{CY}_2$ can be absorbed by a D8-brane and converted into an instanton.

The D0-D8-F1 brane subsystem is distributed as the D4-D4′-F1 previous case. Here a Wilson line is introduced into the QM living on the D0-branes, in this case D8-branes are originated by probing D0-branes with fundamental strings. The number of fundamental strings dissolved in the worlvolume of D8-branes is in correspondence with the magnetic charge of the D8-branes, $Q^m_{\mathrm{D8}} = 2\pi h'_8$. In terms of instantons, the D0-brane is absorbed by a D4′-brane and converted into an instanton.

The proposal in [1] is that the one dimensional $\mathcal{N} = 4$ quantum mechanics living on the complete D0-D4-D4′-D8-F1 brane intersection describes the interactions between the two types of instantons and two types of Wilson loops in the $Q^m_{\mathrm{D4}'} \times Q^m_{\mathrm{D8}}$ antisymmetric representation of $\mathrm{U}(Q^e_{\mathrm{D4}}) \times \mathrm{U}(Q^e_{\mathrm{D0}})$.

The SCQMs that live on these brane set-ups were analysed in [1]. They are described in terms of a set of disconnected quivers as shown in figure 1, with gauge groups associated to the colour D0 and D4 branes (the latter wrapped on the $\mathrm{CY}_2$) coupled to the D4′ and D8 flavour branes. The dynamics is described in terms of (4,4) vector multiplets, associated to gauge nodes (circles); (4,4) hypermultiplets in the adjoint representation connecting one gauge node to itself (semicircles in black lines); and (4,4) hypermultiplets in the bifundamental representation of the two gauge groups (vertical black lines). The connection with the flavour groups is through twisted (4,4) bifundamental hypermultiplets, connecting the D0-branes with the D4′-branes and the D4-branes with the D8-branes (bent black lines), and (0,2) bifundamental Fermi multiplets, connecting the D4-branes with the D4′-branes and the D0-branes with the D8-branes (dashed lines) — see [1] for more details.

Such quivers, depicted in figure 1, can be read from the Hanany-Witten like brane set-up depicted at the top of figure 2. Here in each $[\rho_k, \rho_{k+1}]$ interval there are $\mu_k$ D0-branes and $\alpha_k$ D4-branes, playing the rôle of colour branes. Orthogonal to them there are $\nu_k$ D8-branes and $\beta_k$ D4′-branes, interpreted as flavour branes. In order to see the interpretation as Wilson lines one can proceed as follows (see [1]). The D0-D4-D4′-D8-F1 brane set-up is taken to an F1-D3-NS5-NS7-D1 system in Type IIB through a T+S duality transfor-

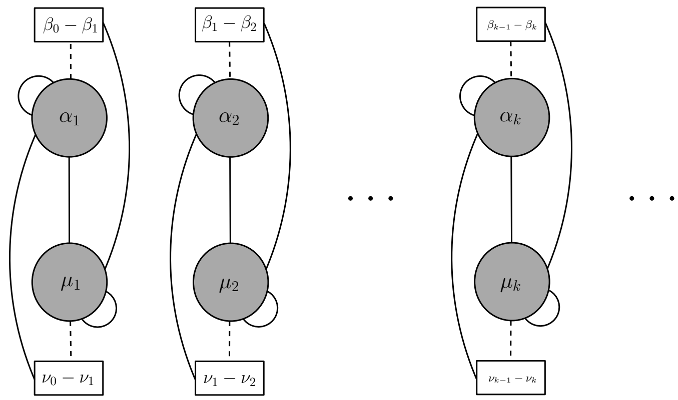


**Figure 1**. A generic one dimensional quiver field theory whose IR limit is dual to the $\mathrm{AdS}_2$ backgrounds given in [1].

mation. In this set-up, Hanany-Witten moves can be performed, which upon T-duality give the Type IIA configuration depicted at the bottom of figure 2. This configuration consists of $\nu_k$ coincident stacks of D8-branes and $\beta_k$ coincident stacks of D4′-branes, with $\mu_k$ and $\alpha_k$ F1-strings originating in the different $(\nu_0, \nu_1, \ldots \nu_{k-1}; \beta_0, \beta_1, \ldots, \beta_{k-1})$ coincident stacks of D8- and D4′-branes. The other endpoint of the F1-strings is on each stack of $\mu_k$ D0-branes and $\alpha_k$ D4-branes. From this picture the description of Wilson loops in the $(\nu_0, \nu_1, \ldots \nu_{k-1}; \beta_0, \beta_1, \ldots, \beta_{k-1})$ completely antisymmetric representation of $\mathrm{U}(\mu_k)$ and $\mathrm{U}(\alpha_k)$, respectively, is recovered. In [1], this was interpreted as describing Wilson lines for each of the D0 and D4 gauge groups, given that they are in the completely antisymmetric representation they actually described backreacted D4-D0 baryon vertices [82] within the 5d CFT living in D4′-D8 brane intersections. The reader is referred to [1] for more details on this construction.

In [1], it was shown that the holographic central charge (given by (3.9)), matches the field theory central charge, computed using the expression,

$$c_{\mathrm{ft}} = 6(n_{\mathrm{hyp}} - n_{\mathrm{vec}}), \tag{3.10}$$

where $n_{\mathrm{hyp}}$ counts the number of bifundamental, fundamental and adjoint (0,4) hypermultiplets and $n_{\mathrm{vec}}$ counts the number of (0,4) vector multiplets, both in the UV description. The equation (3.10) was obtained in [83] for two-dimensional conformal field theories, and was determined by identifying the right-handed central charge with the $\mathrm{U}(1)_R$ current two-point function. With the expression (3.10), both results, holographic and field theory central charge have been shown to agree for the 2d $\mathcal{N} = (0, 4)$ quiver CFTs constructed in [27–29], as well as for the $\mathrm{AdS}_2$/SCQM pairs proposed in [1, 38, 41]. In [38], the agreement is kept since the one-dimensional quiver QMs are originated from the two-dimensional $\mathcal{N} = (0, 4)$ CFTs upon dimensional reduction. However, in [1, 41], the equation (3.10) matches with the holographic result even though the 1d CFTs have not originated from the 2d "mother" CFTs.

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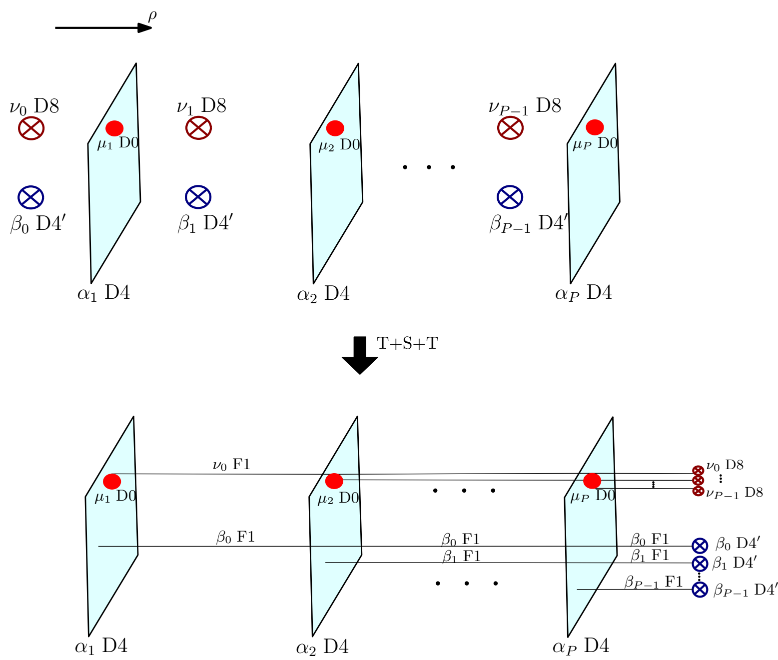


**Figure 2**. (Top) Hanany-Witten like brane set-up associated with the quivers depicted in figure 1. Brane set-up equivalent to the previous one after a T+S+T duality transformation and Hanany-Witten moves (bottom).

As we anticipated, the background (2.17) belongs to the classification provided in [1]. Therefore, we will use the expression (3.10) to obtain the number of vacua of the superconformal quantum mechanics dual to our solution. The previous analysis guarantees its agreement with the holographic result.

After this summary, we turn to the solution (2.17), the main focus of this paper and show that it fits locally in the previous class of $\mathrm{AdS}_2\times\mathrm{S}^3\times\mathrm{CY}_2\times\mathrm{I}$ solutions to Type IIA supergravity constructed in [1].

## 4 SCQM dual to the non-Abelian T-dual solution

In this section we show that the solution (2.17), obtained as the $\mathrm{SL}(2,\mathbf{R})$-NATD of the $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ solution to Type IIB supergravity, fits in the class of geometries constructed in [1], that we have just reviewed. We will also provide a global completion to this solution by glueing it to itself.

Consider the backgrounds (3.1). It is easy to see that the background (2.17) fits locally in this class of solutions, with the simple choices,

$$u = 4L^4M^2\rho, \qquad h_4 = L^2M^4\rho, \qquad h_8 = F_0\rho\,. \tag{4.1}$$

In [29], it was studied that the $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2\times\mathrm{I}$ solution constructed in [56], by acting SU(2)-NATD on the near horizon limit of the D1-D5 system, belongs to a subset of the geometries classified in [26]. Therefore, since both classifications, [26] and [1, 34], are related

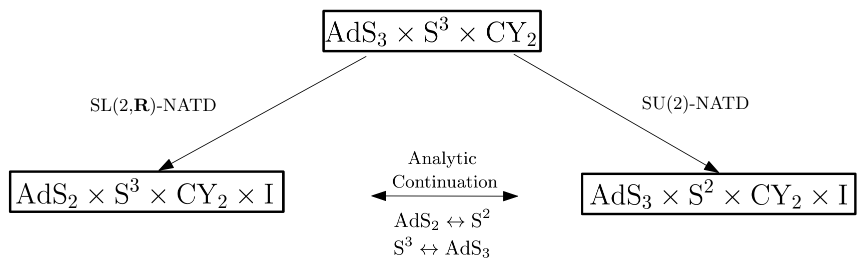


**Figure 3**. Relation between the solution (2.17) and the solution obtained in [56] through SU(2)-NATD.



by a double analytical continuation, this fact strongly suggests that the background (2.17) should be related to the solution obtained in [56], upon an analytical continuation prescription.

This double analytical continuation works as follows, we focus on the Type IIA background given by (2.17) and the $AdS_2$ and $S_3$ factors are interchanged as,

$$ds^2_{AdS_2} \to -ds^2_{S^2}, \qquad ds^2_{S^3} \to -ds^2_{AdS_3}. \tag{4.2}$$

In order to get well-defined supergravity fields, we also need to analytically continue the following terms,

$$\rho \to i\rho, \qquad L \to iL, \qquad F_i \to -F_i, \tag{4.3}$$

where $F_i$ are the RR fluxes. Thus, applying this set of transformations one finds the $AdS_3 \times S^2 \times CY_2 \times I$ solution to massive Type IIA supergravity with four Poincaré supersymmetries constructed for the first time in [56]. We summarise these connections in figure 3.

### 4.1 Completed NATD solution

According to (3.3)–(3.4) one can choose a profile for the piecewise linear functions $h_4$, $h_8$ and propose a concrete way to complete the solution (2.17). In turn, completing the geometry implies a completion in the quiver, allowing us to match between holographic and field theory computations.

We can complete the solution (2.17) by terminating the $\rho$ interval at a certain value $\rho_{2P}$ with $P \in \mathbb{Z}$.[5] Then, the piecewise functions (3.3)–(3.4) read,

$$u = 4L^4 M^2 \rho, \tag{4.4}$$

$$h_4(\rho) = L^2 M^4 \begin{cases} \rho & \rho_0 \leq \rho \leq \rho_P\ , \\ \rho_0 - (\rho - \rho_{2P}) & \rho_P \leq \rho \leq \rho_{2P}, \end{cases} \tag{4.5}$$

$$h_8(\rho) = L^2 \begin{cases} \rho & \rho_0 \leq \rho \leq \rho_P, \\ \rho_0 - (\rho - \rho_{2P}) & \rho_P \leq \rho \leq \rho_{2P}. \end{cases} \tag{4.6}$$

[5] We choose the value $\rho_{2P}$ due to the completion is composed by two copies of the SL(2,**R**)-NATD solution, glued between them.

The previous functions reproduce the behaviour (2.19) for the metric and dilaton at both ends of the space and one can check that the NS sector is continuous at $\rho_P$ when $\rho_P = \frac{\rho_0+\rho_{2P}}{2}$. Hereinafter, we take the value $\rho_{2P} = \rho_0\,(2P-1)$ and use $\frac{\rho_0}{2\pi}$ dimensionless, namely $\frac{\rho_0}{2\pi} \to 1$, in order to obtain well-quantised charges. Thus, we get $\rho_P = 2\pi P$.

Notice that the functions (4.5)–(4.6) are a simple example, with $\beta_k = \beta$ and $\nu_k = \nu$, for all intervals. This implies there are no flavour branes at the different intervals — with the exception of the $[\rho_{P-1}, \rho_P]$ interval, that we will analyse later.

The Page fluxes (2.23) in each $[\rho_k, \rho_{k+1}]$ interval then read as follows,

$$\hat{F}_0 = L^2 \begin{cases} 1 & k = 0, \ldots, P-1, \\ -1 & k = P, \ldots, (2P-1), \end{cases} \tag{4.7}$$

$$\hat{F}_2 = \pi L^2 \mathrm{vol}_{\mathrm{AdS}_2} \begin{cases} -k & k = 0, \ldots, P-1, \\ -(2P-k) & k = P, \ldots, (2P-1), \end{cases} \tag{4.8}$$

$$\hat{F}_4^{\mathrm{CY}_2} = L^2 M^4 \mathrm{vol}_{\mathrm{CY}_2} \begin{cases} -1 & k = 0, \ldots, P-1, \\ 1 & k = P, \ldots, (2P-1), \end{cases} \tag{4.9}$$

$$\hat{F}_6^{\mathrm{CY}_2} = \pi L^2 M^4 \mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{vol}_{\mathrm{CY}_2} \begin{cases} k & k = 0, \ldots, P-1, \\ (2P-k) & k = P, \ldots, (2P-1), \end{cases} \tag{4.10}$$

Here we show the component over $\mathrm{CY}_2$ for $\hat{F}_4$ and $\hat{F}_6$. The 2-form and 6-form Page fluxes are continuous at $\rho_P$ and the change of sign in the 0-form and 4-form Page fluxes is due to the presence of D8 and D4′ flavour branes at $[\rho_{P-1}, \rho_P]$ interval.

The corresponding quantised charges read,

$$Q^m_{\mathrm{D8}} = 2\pi L^2 \begin{cases} 1 & k = 0, \ldots, P-1, \\ -1 & k = P, \ldots, (2P-1), \end{cases} \tag{4.11}$$

$$Q^e_{\mathrm{D0}} = Q^m_{\mathrm{D8}} \begin{cases} -k & k = 0, \ldots, P-1, \\ -(2P-k) & k = P, \ldots, (2P-1), \end{cases} \tag{4.12}$$

$$Q^m_{\mathrm{D4}'} = 2\pi L^2 M^4 \begin{cases} -1 & k = 0, \ldots, P-1, \\ 1 & k = P, \ldots, (2P-1), \end{cases} \tag{4.13}$$

$$Q^e_{\mathrm{D4}} = Q^m_{\mathrm{D4}'} \begin{cases} k & k = 0, \ldots, P-1, \\ (2P-k) & k = P, \ldots, (2P-1). \end{cases} \tag{4.14}$$

Thus, the D0 and D4 brane charges increase linearly in the $0 \le k \le P$ region, and decrease linearly in the $P+1 \le k \le 2P-1$ region, until the value $k = 2P-1$ is reached. Here the minus sign in the charges denotes anti-Dp brane charge. The quiver for the configuration (4.5)–(4.6) is depicted in figure 4.

The discontinuities at $\rho_P$ are translated into $2Q^m_{\mathrm{D4}'}$ and $2Q^m_{\mathrm{D8}}$ flavour groups according to,

$$N^{[P-1,P]}_{\mathrm{D4}'} = \frac{1}{(2\pi)^3} \int_{\mathrm{CY}_2} \hat{F}_4 = \frac{1}{(2\pi)^3} \int_{\mathrm{CY}_2 \times \mathrm{I}_\rho} \mathrm{d}\hat{F}_4 = \beta_{P-1} - \beta_P = 2Q^m_{\mathrm{D4}'} \tag{4.15}$$

$$N^{[P-1,P]}_{\mathrm{D8}} = 2\pi \hat{F}_0 = 2\pi \int_{\mathrm{I}_\rho} \mathrm{d}\hat{F}_0 = \nu_{P-1} - \nu_P = 2Q^m_{\mathrm{D8}} \tag{4.16}$$

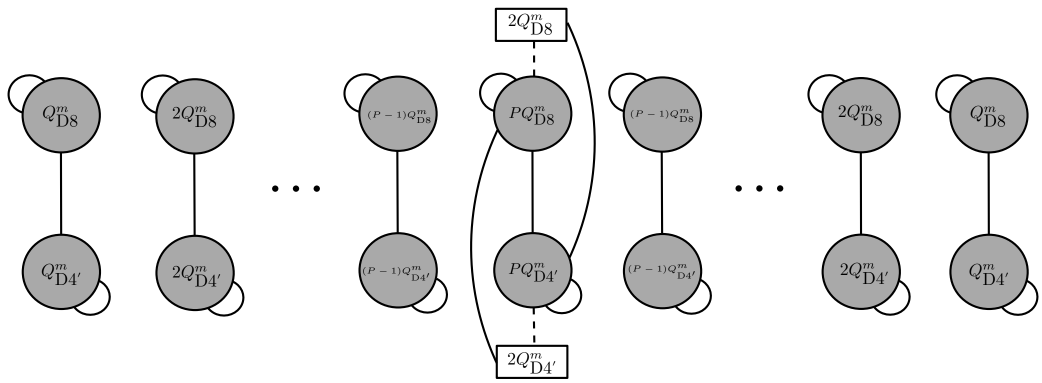


**Figure 4**. Symmetric completed quiver associated to the NATD solution.

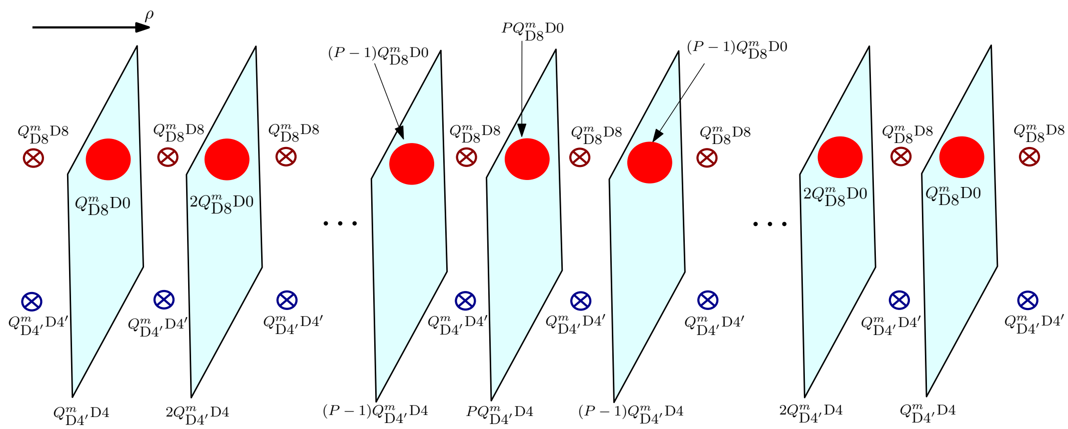


**Figure 5**. The Hanany-Witten like brane set-up for the completed non-Abelian T-dual solution, underlying the quiver depicted in figure 4.

where we used the expressions (3.7)–(3.8) with $\beta_{P-1} = 2\pi L^2 M^4$, $\beta_P = -2\pi L^2 M^4$, $\nu_{P-1} = 2\pi L^2$ and $\nu_P = -2\pi L^2$.

The quiver shown in figure 4 can be translated to the description reviewed in section 3.1. The Hanany-Witten like brane set-up is shown in figure 5. In each $[\rho_k, \rho_{k+1}]$ interval, for $k = 0, \dots, P-1$, we have $kQ^m_{\mathrm{D8}}$ D0-branes and $kQ^m_{\mathrm{D4'}}$ D4-branes. For $k = P, \dots, 2P-1$ we have $(2P-k)Q^m_{\mathrm{D8}}$ D0-branes and $(2P-k)Q^m_{\mathrm{D4'}}$ D4-branes. Orthogonal to them, in each interval, there are $Q^m_{\mathrm{D8}}$ D8-branes and $Q^m_{\mathrm{D4'}}$ D4′-branes, playing the rôle of flavour branes.

As proposed in [1] and we reviewed in section 3.1, one can perform a T-S-T duality transformation[6] to the D0-D4-D4′-D8-F1 system. Consider the left-hand side of the Hanany-Witten like brane set-up shown in figure 5, from the first $Q^m_{\mathrm{D8}}$D8- and $Q^m_{\mathrm{D4'}}$D4′-branes until the $PQ^m_{\mathrm{D8}}$D0- and $PQ^m_{\mathrm{D4'}}$D4-branes. It is easy to see that this subsystem is equivalent to the brane set-up depicted on the top of figure 2 (with $\nu_i = Q^m_{\mathrm{D8}}$, $\beta_i = Q^m_{\mathrm{D4'}}$, $\mu_j = jQ^m_{\mathrm{D8}}$ and $\alpha_j = jQ^m_{\mathrm{D4'}}$, for $i = 0, 1, \dots, P-1$ and $j = 1, \dots, P$). When we perform the T+S+T transformation on the left-hand configuration an equivalent system to the bottom

[6]That is a T-(S-duality)-T transformation.

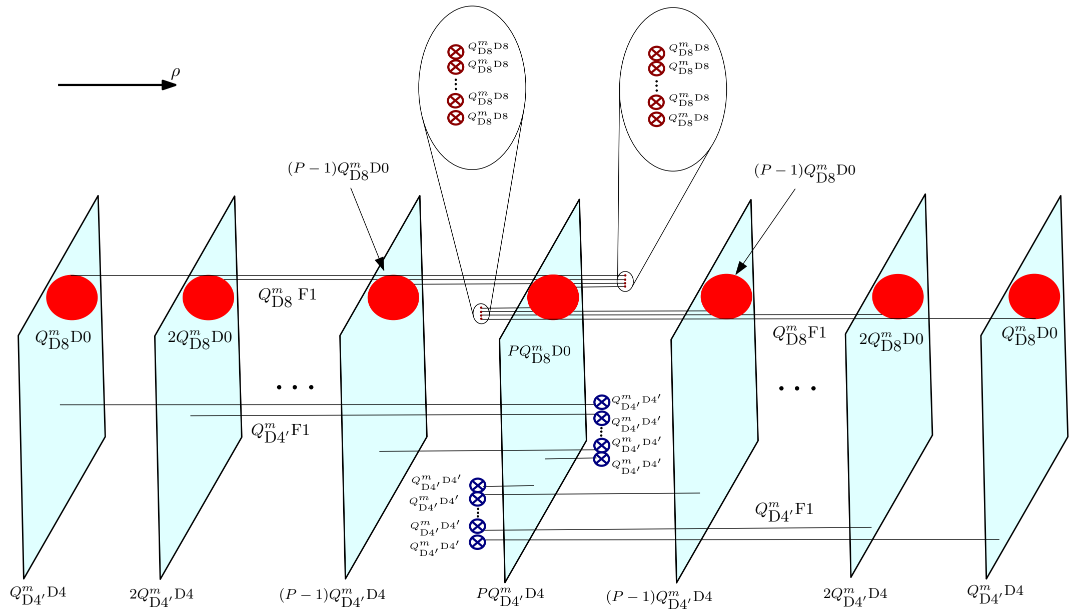


**Figure 6**. Symmetric brane set-up after a T+S+T duality transformation and Hanany-Witten moves from the brane set-up depicted in figure 5.

in figure 2 is obtained. This is depicted on the left-hand side in figure 6, here the now coincident D8-branes and D4′-branes are to the right of the $PQ^m_{\mathrm{D8}}$D0 and $PQ^m_{\mathrm{D4}'}$D4 stacks. On the right-hand side of the Hanany-Witten like brane set-ups, shown in figure 5 and figure 6, we have the same configuration that on the left-hand side, since the right-hand side is the symmetric part of the left-hand side. That is, the complete configuration is the left-hand side glued to itself.

Let us focus on the D0-D8-F1 system on the left-hand side of the Hanany-Witten like brane set-up shown in figure 6 (from $Q^m_{\mathrm{D8}}$D0 until $PQ^m_{\mathrm{D8}}$D0).[7] After the T+S+T transformation, we obtain $P$ stacks of $Q^m_{\mathrm{D8}}$D8-branes — depicted in figure 6 to the right of the $PQ^m_{\mathrm{D8}}$D0-branes — with $PQ^m_{\mathrm{D8}}$F1-strings originating in the different coincident stacks of D8-branes. The other endpoint of the F1-strings is on each stack of $kQ^m_{\mathrm{D8}}$D0-branes. For the D4-D4′-F1 system we have a similar configuration, namely $P$ stacks of $Q^m_{\mathrm{D4}'}$D4′-branes, with $PQ^m_{\mathrm{D4}'}$F1-strings attached to them. These F1-strings have the other end point on the different $kQ^m_{\mathrm{D4}'}$ stacks of D4-branes. Thus, as we reviewed in section 3.1, the system can be interpreted as Wilson loops in the $Q^m_{\mathrm{D8}} \times Q^m_{\mathrm{D4}'}$ completely antisymmetric representation of the gauge groups $\mathrm{U}(kQ^m_{\mathrm{D8}})\times\mathrm{U}(kQ^m_{\mathrm{D4}'})$, that we interpret as describing the backreaction of the D4-D0 baryon vertices of a D4′-D8 brane intersection.

To be concrete, consider the SCQM that arises in the very low energy limit of a D4′-D8 brane intersection, dual to a 5d QFT, where D4- and D0-brane baryon vertices are introduced. Namely, D4-brane (D0-brane) baryon vertices are linked to D4′-branes (D8-branes) with fundamental strings. In the IR these branes change their rôle, that is the

[7]Since on the right-hand side we have the same configuration.

gauge symmetry on both D4′- and D8-branes becomes global, shifting D4′ and D8 from colour to flavour branes and the D0- and D4-branes play now the rôle of colour branes of the backreacted geometry.

Furthermore, with the piecewise linear functions (4.4), (4.5) and (4.6) we can compute the holographic central charge,[8]

$$\begin{aligned} c_{\text{hol}} &= \frac{3}{\pi} L^4 M^4 \left( \int_{\rho_0}^{\rho_P} (\rho^2 - \rho_0^2)\, \mathrm{d}\rho + \int_{\rho_P}^{\rho_{2P}} \left( (\rho_0 - (\rho - \rho_{2P}))^2 - \rho_0^2 \right) \mathrm{d}\rho \right) \\ &= Q^m_{\mathrm{D4'}} Q^m_{\mathrm{D8}} (4P^3 - 12P + 8). \end{aligned} \tag{4.17}$$

In order to compare this result with the field theory computation in (3.10), we need to compute the number of hypermultiplets and vector multiplets. For the quiver in figure 4 we obtain,

$$\begin{aligned} n_{\text{hyp}} &= \left( (Q^m_{\mathrm{D4'}})^2 + (Q^m_{\mathrm{D8}})^2 + Q^m_{\mathrm{D4'}} Q^m_{\mathrm{D8}} \right) \left( P^2 + 2 \sum_{i=1}^{P-1} i^2 \right), \\ n_{\text{vec}} &= \left( (Q^m_{\mathrm{D4'}})^2 + (Q^m_{\mathrm{D8}})^2 \right) \left( P^2 + 2 \sum_{i=1}^{P-1} i^2 \right). \end{aligned} \tag{4.18}$$

Thus, when the sums are performed we get the following expression for the field theory central charge,

$$\begin{aligned} c_{\text{ft}} = 6(n_{\text{hyp}} - n_{\text{vec}}) &= 6 Q^m_{\mathrm{D4'}} Q^m_{\mathrm{D8}} \left( P^2 + 2 \sum_{i=1}^{P-1} i^2 \right) \\ &= Q^m_{\mathrm{D4'}} Q^m_{\mathrm{D8}} (4P^3 + 2P). \end{aligned} \tag{4.19}$$

We see that at large $Q^m_{\mathrm{D8}}$, $Q^m_{\mathrm{D4'}}$ and $P$ (in the holographic limit, which is long quivers with large ranks) the results (4.17) and (4.19) coincide.

## 5 Conclusions

In this paper we developed the implementation of NATD in supergravity backgrounds supporting an SL(2,$\mathbf{R}$) subgroup as part of their full isometry group. Namely, we implemented the solution generating technique in non-compact spaces exhibiting an SO(2,2) $\cong$ SL(2,$\mathbf{R}$)$_L$×SL(2,$\mathbf{R}$)$_R$ isometry group geometrically realised by an AdS$_3$ space. After the dualisation, the resultant dual geometry exhibits an SL(2,$\mathbf{R}$) isometry reflected geometrically as an AdS$_2$ space plus a non-compact new direction in the internal space. This non-compact direction arises since the Lagrange multipliers live in the Lie algebra of the SL(2,$\mathbf{R}$) group, which is by construction a vector space, $\mathbf{R}^3$. That is, the space dual to AdS$_3$ is locally AdS$_2 \times \mathbf{R}^+$.

We worked out in detail the SL(2,$\mathbf{R}$)-NATD solution of the AdS$_3$×S$^3$×CY$_2$ solution that arises in the near horizon limit of the D1-D5 brane intersection. We found that the SL(2,$\mathbf{R}$)-NATD solution is a simple example in the classification constructed in [1].

[8] We used $\frac{\rho_0}{2\pi} \to 1$ as explained above.

Further, our background (2.17) is related through an analytic continuation prescription to the $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2\times\mathrm{I}$ solution obtained in [56], as one of the first examples of AdS backgrounds generated through SU(2) non-Abelian T-duality.

An important drawback of non-Abelian T-duality is the lack of global information about the dual geometry, which cannot be inferred from the transformation itself. For this we used the fact that our solution (2.17) fits in the classification constructed in [1] which allowed us to propose an explicit completion for the geometry. Unlike the two completions worked out in [29] for the SU(2)-NATD solution constructed therein, continuity of the NS sector allows only one possible completion for the geometry given in (2.17). Our completion, shown in section 4.1, is obtained by glueing the SL(2,**R**)-NATD solution to itself. We proposed a well-defined quiver quantum mechanics, dual to our $\mathrm{AdS}_2$ solution, that flows in the IR to a superconformal quantum mechanics (based on the Hanany-Witten brane set-ups and Page charges), which admits an interpretation in terms of backreacted D4-D0 baryon vertices within the 5d QFT living in a D4′-D8 brane intersection. In support of our proposal we checked the agreement between the holographic and field theory central charges.

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# 8. AdS$_2$/SCQM in Type IIB

## 8.1 AdS$_2\times$S$^2\times$CY$_2 \times \Sigma_2$ solutions and their QM as a DLCQ

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# New $\mathrm{AdS}_2$ backgrounds and $\mathcal{N}=4$ conformal quantum mechanics

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Abstract: We present a new infinite family of Type IIB backgrounds with an $\mathrm{AdS}_2$ factor, preserving $\mathcal{N}=4$ SUSY. For each member of the family we propose a precise dual Super Conformal Quantum Mechanics (SCQM). We provide holographic expressions for the number of vacua (the "central charge"), Chern-Simons terms and other non-perturbative aspects of the SCQM. We relate the "central charge" of the one-dimensional system with a combination of electric and magnetic fluxes in Type IIB. The Ramond-Ramond fluxes are used to propose an extremisation principle for the central charge. Other physical and geometrical aspects of these conformal quantum mechanics are analysed.

Keywords: AdS-CFT Correspondence, D-branes, Gauge-gravity correspondence

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## Contents



## 1 Introduction and general idea

A major line of research motivated by the Maldacena conjecture [1] is the study of supersymmetric and conformal field theories in diverse dimensions. Since the early 2000's efforts have been dedicated to the classification of Type II or M-theory backgrounds with $\mathrm{AdS}_{d+1}$ factors. These backgrounds are conjecturally dual to SCFTs in $d$ dimensions with different amounts of SUSY. For the case in which the solutions are half-maximal supersymmetric, important progress in classifying string backgrounds and the mapping to quantum field theories has been achieved.

Indeed, for $\mathcal{N} = 2$ SCFTs in four dimensions, the field theories studied in [2] have holographic duals first discussed in [3], and further elaborated (among other works) in [4–10].

The case of five dimensional SCFTs was analysed from the field theoretical and holographic viewpoints in [11–25], among many other interesting works. An infinite family of six-dimensional $\mathcal{N} = (1, 0)$ SCFTs was discussed both from the field theoretical and holographic points of view in [26–34]. For three-dimensional $\mathcal{N} = 4$ SCFTs, the field theoretical aspects presented in [35] were discussed holographically in [36–41], among other works. The case of two-dimensional SCFTs and their $\mathrm{AdS}_3$ duals is very attractive, not only for the rich landscape of two dimensional CFTs, but also for the connection with the Physics of black holes. In this case, recent progress was reported for half-maximal supersymmetric (for $\mathrm{AdS}_3$) backgrounds, see for example [42–56]. All these solutions geometrise various perturbative and non-perturbative aspects of conformal field theories in diverse dimensions.

A natural extension is the study of backgrounds with an $\mathrm{AdS}_2$ factor [57–72]. These should be dual to superconformal quantum mechanics (SCQM). The similarities between the superconformal algebras in one and two dimensions or, by duality, the geometric relations between $\mathrm{AdS}_2$ and $\mathrm{AdS}_3$-spaces, suggest in particular that the studies of [49–56] could be extended to the $\mathrm{AdS}_2$ case. Some studies involving $\mathrm{AdS}_2$ geometries were motivated by developments in black holes Physics, whilst others drew inspiration from a purely geometric or field theoretical viewpoint, or both [73–85].

Surprisingly, the case of $\mathrm{AdS}_2/\mathrm{CFT}_1$ is less understood than its higher dimensional cousins. Indeed, various subtleties take place in the study of $\mathrm{AdS}_2$ backgrounds [86–90]. Let us summarise some of them.

A conformal quantum mechanical theory needs to have only $\mathrm{SL}(2, \mathbb{R})$ global symmetry (aside from possible supersymmetry and associated R-symmetry). Nevertheless, the analysis of [74–76], implies that whilst the isometry of $\mathrm{AdS}_2$ is $\mathrm{SL}(2, \mathbb{R})$, asymptotically the group of symmetry is one-copy of the Virasoro algebra. The central charge of the algebra is proportional to the inverse Newton's constant in two dimensions.

The connection between $\mathrm{AdS}_3$ and $\mathrm{AdS}_2$ geometries was discussed from the field theory perspective in [77, 78]. These authors prove that quantising a two dimensional CFT using Discrete Light Cone Quantisation (DLCQ) is equivalent to decoupling one of the chiral sectors of a CFT. In this paper we use these ideas to connect $\mathrm{AdS}_3$ and $\mathrm{AdS}_2$ string solutions in geometrical fashion.

In the context of JT-gravity, the authors of [80] found flows interpolating between $\mathrm{AdS}_3$ and $\mathrm{AdS}_2$ spaces. These correspond to the reduction of $\mathrm{AdS}_3$ along a space-like direction. There may be a relation between those solutions at the IR fixed point, and the backgrounds we find in this work. The authors of [78] found that black holes with generic $\mathrm{AdS}_2$ near horizon geometry have an entanglement entropy related to the two-dimensional Newton's constant, according to,

$$S_{\mathrm{EE}} = \frac{1}{G_N^{(2)}}.$$

They show that this entanglement entropy coincides with the entropy of a black hole whose near horizon contains the $\mathrm{AdS}_2$. In the present paper we perform explicit holographic calculations that hint at a relation between three quantities: the number of vacuum states of the SCQM, the partition function for the one dimensional SCFT when formulated on a circle and the entropy of a black hole that has $\mathrm{AdS}_2$ near horizon geometry.



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We enlarge the classification of SCFTs and $AdS_2$-string backgrounds, dealing with the case of $\mathcal{N} = 4$ SCQMs and $AdS_2$ string geometries with an SU(2)-isometry. This leads us to the study of SCQM that are more elaborated than those usually analysed in the bibliography. We define our SCQM to be the strongly coupled IR fixed point of $\mathcal{N} = 4$ UV-finite quantum mechanical quiver theories, that we precisely specify. Our new $\mathcal{N} = 4$ $AdS_2$ background solutions in Type IIB, are a trustable dual description of the $CFT_1$ dynamics, whenever the number of nodes of the quiver and the ranks of each gauge group are large. We also need that the flavour groups (geometrically realised on source-branes) are widely separated in the geometry (we refer to this as the flavour groups being "sparse").

We present precise proposals for the $\mathcal{N} = 4$ conformal quantum mechanics. We study various aspects of the SCQMs using the dual backgrounds. These include: number of vacua, Chern Simons coefficients, symmetry breaking, expected values of Wilson lines, couplings, etc. We uncover a novel and intriguing relation between a suitably defined "central charge" (associated with the number of vacua above mentioned) and the product of electric and magnetic charges for each Type IIB background.

The contents of this work are distributed as follows. In section 2 we review the $AdS_3$ backgrounds in massive IIA that act as "seed" for our new infinite family of $AdS_2$ solutions in Type IIB. We revisit the two-dimensional $\mathcal{N} = (0, 4)$ SCFTs dual to these backgrounds and improve on the existing bibliography by discussing the superpotential terms. In section 3 we present our new family of $AdS_2$ backgrounds and study in detail various geometrical aspects. In section 4 we present a concrete proposal for our $\mathcal{N} = 4$ SCQM and perform holographic calculations that encode field theoretical aspects of our strongly coupled $CFT_1$s, with some emphasis on the holographic central charge above mentioned.

In section 5 we discuss a connection between the number of vacua of the SCQM and the RR sector of our supergravity solutions. We show that the holographic central charge is related to a product of electric and magnetic charges of the D-branes present in the background. We also present a new extremal principle in supergravity from which the central charge of the SCQM can be obtained. Our results extend and generalise those in the existing literature by the inclusion of sources and boundaries. Moreover they suggest new ways for the construction of the extremising functionals. In section 6 we present our conclusions, with an invitation to colleagues working on field theoretical aspects of $\mathcal{N} = 4$ SCQM to check some of our predictions using their favourite exact methods. Various appendices complement geometrical aspects of the backgrounds. Field theoretical observables of the strongly coupled quantum mechanical system are also holographically computed.

## 2 Seed backgrounds and associated CFTs

In this section we review discuss the solutions to massive IIA supergravity (with localised sources) obtained in the recent work [49]. These backgrounds provide the "seed" from which the new $AdS_2$ supergravity solutions presented in this work are derived. New results will also be presented.

For brevity, we restrict ourselves to a particular case of the generic backgrounds in [49]. The generic case is analysed in appendix A. The Neveu-Schwarz (NS) sector of these

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solutions reads,

$$ds^2 = \frac{u}{\sqrt{\widehat{h}_4 h_8}}\Big(ds^2_{\mathrm{AdS}_3} + \frac{h_8\widehat{h}_4}{4h_8\widehat{h}_4+(u')^2}ds^2_{\mathrm{S}^2}\Big) + \sqrt{\frac{\widehat{h}_4}{h_8}}ds^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}d\rho^2, \tag{2.1}$$

$$e^{-\Phi} = \frac{h_8^{\frac{3}{4}}}{2\widehat{h}_4^{\frac{1}{4}}\sqrt{u}}\sqrt{4h_8\widehat{h}_4+(u')^2}, \qquad H_3 = \frac{1}{2}d\left(-\rho + \frac{uu'}{4\widehat{h}_4 h_8+(u')^2}\right)\wedge \mathrm{vol}_{\mathrm{S}^2},$$

where $\Phi$ is the dilaton, $H = dB_2$ is the NS 3-form and the metric is written in string frame. The warping functions $\widehat{h}_4$, $h_8$ and $u$ have support on $\rho$. We denote $u' = \partial_\rho u$ and similarly for $\widehat{h}'_4, h'_8$. The RR fluxes are

$$F_0 = h'_8, \quad F_2 = -\frac{1}{2}\Big(h_8 - \frac{h'_8 u' u}{4h_8\widehat{h}_4+(u')^2}\Big)\mathrm{vol}_{\mathrm{S}^2}, \tag{2.2a}$$

$$F_4 = -\Big(d\left(\frac{uu'}{2\widehat{h}_4}\right) + 2h_8 d\rho\Big)\wedge \mathrm{vol}_{\mathrm{AdS}_3} - \partial_\rho \widehat{h}_4 \mathrm{vol}_{\mathrm{CY}_2}, \tag{2.2b}$$

with the higher fluxes related to them as $F_6 = -\star_{10} F_4$, $F_8 = \star_{10} F_2$, $F_{10} = -\star_{10} F_0$. The background in (2.1)–(2.2b) is a SUSY solution of the massive IIA equations of motion if the functions $\widehat{h}_4, h_8, u$ satisfy (away from localised sources),

$$\widehat{h}''_4(\rho) = 0, \quad h''_8(\rho) = 0, \quad u''(\rho) = 0. \tag{2.3}$$

The first two are Bianchi identities. Hence the presence of localised sources will be indicated by delta-function inhomogeneities. In contrast, $u'' = 0$ is a BPS equation.

The Page fluxes, defined as $\widehat{F} = e^{-B_2}\wedge F$, are

$$\begin{aligned}\widehat{F}_0 &= h'_8, \quad \widehat{F}_2 = -\frac{1}{2}\Big(h_8 - h'_8(\rho - 2\pi k)\Big)\mathrm{vol}_{\mathrm{S}^2},\\ \widehat{F}_4 &= -\Big(\partial_\rho\left(\frac{uu'}{2\widehat{h}_4}\right) + 2h_8\Big)d\rho\wedge \mathrm{vol}_{\mathrm{AdS}_3} - \partial_\rho \widehat{h}_4 \mathrm{vol}_{\mathrm{CY}_2}.\end{aligned} \tag{2.4}$$

We have allowed for large gauge transformations $B_2 \to B_2 + \pi k \mathrm{vol}_{\mathrm{S}^2}$, for $k = 0, 1, \ldots, P$. The transformations are performed every time we cross an interval $[2\pi k, 2\pi(k+1)]$. The $\rho$-direction begins at $\rho = 0$ and ends at $\rho = 2\pi(P+1)$. This will become apparent once the functions $\widehat{h}_4, h_8, u$ are specified below.

Various particular solutions were analysed in [49]. Here we consider an infinite family of backgrounds for which the $\widehat{h}_4$, $h_8$ functions are piecewise continuous. These were carefully studied in [50–52], where a precise dual field theory was proposed. The above mentioned range of the $\rho$-coordinate is determined by the vanishing of the functions $\widehat{h}_4$ and $h_8$. Generically these functions read,

$$\widehat{h}_4(\rho) = \Upsilon\, h_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi}\rho & 0\le\rho\le 2\pi \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho-2\pi k) & 2\pi k\le\rho\le 2\pi(k+1),\ k=1,..,P-1 \\ \alpha_P - \frac{\alpha_P}{2\pi}(\rho-2\pi P) & 2\pi P\le\rho\le 2\pi(P+1), \end{cases} \tag{2.5}$$

$$h_8(\rho) = \begin{cases} \frac{\nu_0}{2\pi}\rho & 0\le\rho\le 2\pi \\ \mu_k + \frac{\nu_k}{2\pi}(\rho-2\pi k) & 2\pi k\le\rho\le 2\pi(k+1), \quad k:=1,\ldots,P-1 \\ \mu_P - \frac{\mu_P}{2\pi}(\rho-2\pi P) & 2\pi P\le\rho\le 2\pi(P+1). \end{cases} \tag{2.6}$$



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| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D2 | x | x | | | | | x | | | |
| D4 | x | x | | | | | | x | x | x |
| D6 | x | x | x | x | x | x | x | | | |
| D8 | x | x | x | x | x | x | | x | x | x |
| NS5 | x | x | x | x | x | x | | | | |

**Table 1**. BPS brane intersection underlying the geometry in (2.1)–(2.3). The directions $(x^0, x^1)$ are the directions where the 2d dual CFT lives. The directions $(x^2, \dots, x^5)$ span the $\mathrm{CY}_2$, on which the D6 and the D8-branes are wrapped. The coordinate $x^6$ is the direction associated with $\rho$. Finally $(x^7, x^8, x^9)$ are the transverse directions realising an SO(3)-symmetry associated with the isometries of $\mathrm{S}^2$.

The quantities $(\alpha_k, \beta_k, \mu_k, \nu_k)$ are integration constants. By imposing continuity we determine,

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad \mu_k = \sum_{j=0}^{k-1} \nu_j. \tag{2.7}$$

Below, we summarise aspects of the two dimensional field theories dual to the backgrounds (2.1)–(2.3) for the solutions determined by eqs. (2.5)–(2.6). We also present new aspects of these field theories.

### 2.1 The associated dual SCFTs

As was explained in the papers [50–52], for the functions $\widehat{h}_4, h_8, u$ in eqs. (2.5)–(2.6), the backgrounds in eqs. (2.1)–(2.3) are associated with a Hanany-Witten [91] set-up indicated in table 1. Using this brane set-up, dual two-dimensional CFTs with $\mathcal{N} = (0, 4)$ SUSY were proposed. These CFTs describe the low energy, strongly coupled dynamics of two dimensional quantum field theories. The field theories are encoded by the quiver in figure 1. The difference between this quiver and those proposed in [50–52] is the presence of $(4, 4)$ matter connecting flavour and colour groups. These correspond in figure 1 to the vertical (bent) lines. In the limit that makes the holographic backgrounds trustable (that is, long quivers with large ranks and sparse flavour groups), these (4,4) hypermultiplets do not affect the matching of observables discussed in [50–52]. In fact, their contribution is subleading and not captured by supergravity.

The absence of gauge anomalies constrains the ranks of the flavour groups to be

$$F_k = \nu_{k-1} - \nu_k, \quad \tilde{F}_k = \beta_{k-1} - \beta_k. \tag{2.8}$$

These are precisely the quantised numbers of D8 and D4 flavour (source) branes derived from eq. (2.4) These conditions are unchanged by the presence of the $\mathcal{N} = (4, 4)$ bifundamentals connecting flavour and colour groups, which (being vectorial) do not count towards the anomaly.

Numerous checks for the validity of this proposal were presented in [50–52]. The right-handed central charge of the SCFTs is computed by identifying it with the $\mathrm{U}(1)_R$



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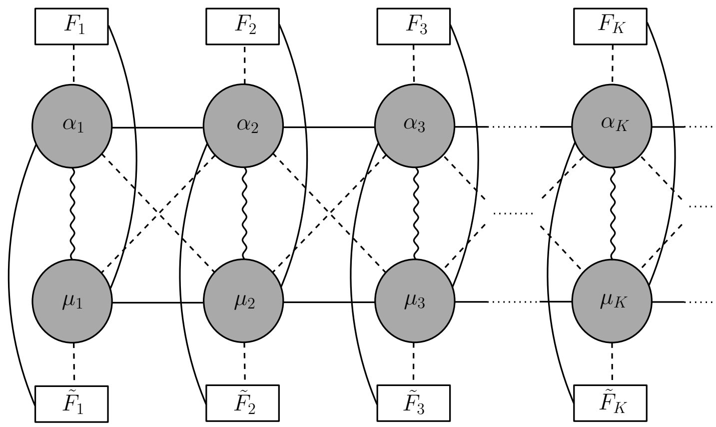


**Figure 1**. A generic quiver field theory whose IR is dual to the holographic background defined by the functions in (2.5)–(2.6). The solid black lines represent $(4,4)$ hypermultiplets, the wavy lines represent $(0,4)$ hypermultiplets and the dashed lines represent $(0,2)$ Fermi multiplets. $\mathcal{N}=(4,4)$ vector multiplets are the degrees of freedom in each gauged node.

current two-point function. The works [92, 93] found that for a generic quiver with $n_{\text{hyp}}$ hypermultiplets and $n_{\text{vec}}$ vector multiplets the central charge is,

$$c_{\text{CFT}} = 6(n_{\text{hyp}} - n_{\text{vec}}). \tag{2.9}$$

The papers [50–52] present a variety of examples of long linear quivers with sparse flavour groups and large ranks for each of the nodes. In each of these qualitatively different examples, it was found that the field theoretical central charge of eq. (2.9) coincides with the holographic central charge (at leading order, when the background is a trustable dual description to the CFT). Note that this matching is not changed by the presence of the extra $(4,4)$ hypermutiplets mentioned above. The "sparse" character of the flavour groups makes their contribution subleading.

The expression for the holographic central charge derived in [50–52] is,

$$c_{\text{hol}} = \frac{3\pi}{2G_N}\text{Vol}_{\text{CY}_2}\int_0^{2\pi(P+1)}\widehat{h}_4 h_8 \mathrm{d}\rho = \frac{3}{\pi}\int_0^{2\pi(P+1)} h_4 h_8 \mathrm{d}\rho. \tag{2.10}$$

We used that $G_N = 8\pi^6$ (with $g_s = \alpha' = 1$) and that $\Upsilon \text{Vol}_{\text{CY}_2} = 16\pi^4$.

### 2.1.1 Superpotential

Now, we present a new development, adding value to this review-section. Let us discuss the superpotential terms that can be written due to the presence of the $(4,4)$ hypermultiplets connecting D2-D4 and D6-D8 branes. In two dimensions with $\mathcal{N}=(0,4)$ SUSY, we can write interactions in terms of a superpotential $W$ [92–95],

$$S = \int \mathrm{d}^2x \mathrm{d}\theta^+ W, \qquad W = \Psi_a J^a(\Phi_i). \tag{2.11}$$

Studying the strings stretched between the different branes in the Hanany-Witten set-up, we find the massless fields described in figure 2 (left side). We depict only one "interval"

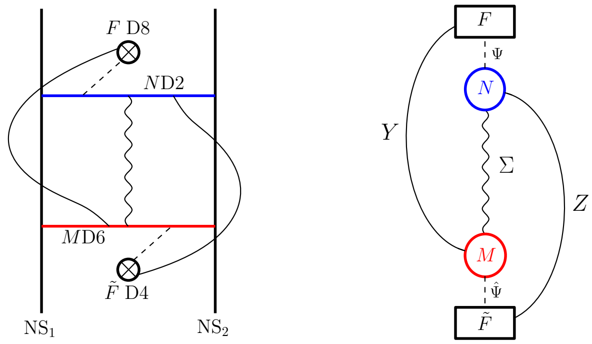


**Figure 2**. On the left, we plot one cell in the Hanany-Witten set-up, in between the NS-five branes $\mathrm{NS}_1$ and $\mathrm{NS}_2$. The solid black lines represent $(4,4)$ hypermultiplets, the curvy lines $(0,4)$ hypermultiplets and the dashed lines $(0,2)$ Fermi multiplets. On the right, we plot the field content of one cell in the quiver (same convention). $Y, Z$ are $(4,4)$ hypers, $\Sigma$ denotes a $(0,4)$ hyper. The $(0,2)$ Fermi fields are denoted by $(\Psi, \widehat{\Psi})$. The gauge nodes contain $(4,4)$ vector multiplets.

of the whole Hanany-Witten set-up. The blue and red lines denote $N$-D2 branes and $M$-D6 branes, there are also $F$ D8 and $\tilde{F}$ D4 flavour branes. The D2 and D6 colour branes are joined by a wavy line, representing a $(0,4)$ hyper (denoted by $\Sigma$ in the right figure). Dashed lines represent $(0,2)$ Fermi multiplets, joining D2-D8 and D4-D6 pairs. These are denoted by $\Psi, \widehat{\Psi}$ in the right figure. We also have solid black lines, representing $(4,4)$ hypermultiplets, joining D2-D4 and D6-D8 branes and denoted by $Y, Z$ on the right panel of figure 2. This "interval" is connected via $(0,2)$ Fermi multiplets and $(4,4)$ hypermultiplets with a similar next-interval as indicated in figure 1.

As discussed above, these new $(4,4)$ matter fields $Y, Z$ have no-effect on anomalies and their effect on the central charge is subleading. Their presence was emphasised in [56]. They allow to write a superpotential term.

The superpotential interaction is obtained by closing the "triangle", contracting indexes appropriately in the circuit D8-D2-D6-D8 and D4-D6-D2-D4. This suggests that we should include cubic superpotential terms of the form,

$$W \sim Y\Sigma\Psi + Z\Sigma\widehat{\Psi}. \tag{2.12}$$

In appendix B we give more details about the Lagrangian associated with the quiver QFT in figure 1. Putting together all this information, the full Lagrangian describing the UV dynamics is written there. This dynamics conjecturally flows in the IR to a CFT with small $\mathcal{N} = (0,4)$ SUSY [50–52].

After this summary of the "seed" backgrounds and dual SCFTs, let us now focus on the new infinite family of backgrounds and the associated SCQMs.

## 3 New Type IIB backgrounds

In this section we present a new infinite family of $\mathrm{AdS}_2$ backgrounds of Type IIB supergravity. They are obtained by applying T-duality on the seed backgrounds defined by eqs. (2.1)–(2.3), along a direction inside $\mathrm{AdS}_3$.

Consider the backgrounds of eqs. (2.1)–(2.3). Write $\mathrm{AdS}_3$ as a fibration over $\mathrm{AdS}_2$,

$$\begin{aligned}
&\mathrm{d}s^2_{\mathrm{AdS}_3} = \frac{1}{4}\left[\left(\mathrm{d}\tilde{\psi}+\eta\right)^2 + \mathrm{d}s^2_{\mathrm{AdS}_2}\right] \qquad \text{with} \qquad \mathrm{d}\eta = \mathrm{vol}_{\mathrm{AdS}_2}. \\
&\mathrm{d}s^2_{\mathrm{AdS}_2} = -\mathrm{d}t^2\cosh^2 r + \mathrm{d}r^2, \quad \eta = -\sinh r \mathrm{d}t.
\end{aligned} \tag{3.1}$$

We T-dualise on the fibre direction to obtain the new solutions (more general configurations are discussed in appendix A). These backgrounds have the structure $\mathrm{AdS}_2\times \mathrm{S}^2\times \mathrm{CY}_2\times \mathrm{I}_\rho\times \mathrm{S}^1_\psi$. The NS sector reads,

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\frac{1}{4}\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 + (u')^2}\mathrm{d}s^2_{\mathrm{S}^2}\right) + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}(\mathrm{d}\rho^2+\mathrm{d}\psi^2)\,,\\
e^{-2\Phi} &= \frac{h_8}{4\widehat{h}_4}(4\widehat{h}_4 h_8 + (u')^2)\,, \quad H_3 = \frac{1}{2}\mathrm{d}\Big(-\rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2}\Big)\wedge \mathrm{vol}_{\mathrm{S}^2} + \frac{1}{2}\mathrm{vol}_{\mathrm{AdS}_2}\wedge \mathrm{d}\psi\,,
\end{aligned} \tag{3.2}$$

where $\psi$ is the T-dual-coordinate, with range $[0,2\pi]$.

The RR sector is given by

$$\begin{aligned}
F_1 &= h_8'\mathrm{d}\psi\,, \quad F_3 = -\frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4h_8\widehat{h}_4 + (u')^2}\right)\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\psi + \frac{1}{4}\left(\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right) + 2h_8\mathrm{d}\rho\right)\wedge \mathrm{vol}_{\mathrm{AdS}_2}\,,\\
F_5 &= -(1+\star)\,\widehat{h}_4'\,\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi = -\widehat{h}_4'\,\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi + \frac{\widehat{h}_4' h_8 u^2}{4\widehat{h}_4(4\widehat{h}_4 h_8 + (u')^2)}\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho\,,\\
F_7 &= \frac{4\widehat{h}_4^2 h_8 - uu'\widehat{h}_4' + \widehat{h}_4(u')^2}{8\widehat{h}_4 h_8 + 2(u')^2}\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\psi\\
&\quad - \frac{4\widehat{h}_4 h_8^2 - uu'h_8' + h_8(u')^2}{8h_8^2}\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho\,,\\
F_9 &= -\frac{\widehat{h}_4 h_8' u^2}{4\widehat{h}_8(4\widehat{h}_4 h_8 + (u')^2)}\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho\,,
\end{aligned} \tag{3.3}$$

where $F_7 = -\star F_3 =$ and $F_9 = \star F_1$. We also quote the explicit expression of $\star H_3$,

$$\begin{aligned}
\star H_3 = &\frac{2\widehat{h}_4^2}{4\widehat{h}_4 h_8 + (u')^2}\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho\\
&- \frac{\widehat{h}_4' h_8 uu' + \widehat{h}_4 u'\,(uh_8' + h_8 u') + 4\widehat{h}_4^2 h_8^2}{2h_8^2(4\widehat{h}_4 h_8 + u'^2)}\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi\,.
\end{aligned}$$

One can check that the Type IIB equations of motion are satisfied imposing the BPS equations and Bianchi identities: $u''=0$ and $\widehat{h}_4''=0$, $h_8''=0$. A violation of the Bianchi identities is admissible at points where brane sources are located. We consider solutions like those in eqs. (2.5)–(2.6).

We perform a large gauge transformation $B_2 \to B_2 + k\pi \mathrm{vol}_{S^2}$. This naturally divides the interval $I_\rho$ in $(P+1)$-cells of size $2\pi$. The Page forms $\widehat{F} = e^{-B_2} \wedge F$ are,

$$\begin{aligned}
\widehat{F}_1 &= h_8' \, \mathrm{d}\psi\,,\\
\widehat{F}_3 &= \frac{1}{2}\left(h_8'(\rho-2\pi k)-h_8\right)\mathrm{vol}_{S^2}\wedge \mathrm{d}\psi + \frac{1}{4}\left(\frac{u'(\widehat{h}_4 u' - u\widehat{h}_4')}{2\widehat{h}_4^2}+2h_8\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{d}\rho\,,\\
\widehat{F}_5 &= \frac{1}{16}\left(\frac{(u-(\rho-2\pi k)u')(u\widehat{h}_4'-\widehat{h}_4 u')}{\widehat{h}_4^2}+4(\rho-2\pi k)h_8\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{S^2}\wedge\mathrm{d}\rho\\
&\quad -\widehat{h}_4'\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi\,,\\
\widehat{F}_7 &= \frac{1}{2}\left(\widehat{h}_4-(\rho-2\pi k)\widehat{h}_4'\right)\mathrm{vol}_{S^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi\\
&\quad -\left(\frac{4\widehat{h}_4 h_8^2 - uu'h_8' + h_8(u')^2}{8h_8^2}\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho\,,\\
\widehat{F}_9 &= -\left(\frac{u^2h_8' - h_8uu' + (\rho-2\pi k)(h_8u'^2 - uu'h_8' + 4\widehat{h}_4 h_8^2)}{16h_8^2}\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{S^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho.
\end{aligned}\tag{3.4}$$

To describe the brane set-up, we use that $\widehat{h}_4$ and $h_8$ are continuous polygonal functions with discontinuous derivatives, as in eqs. (2.5)–(2.6). We compute,

$$\mathrm{d}\widehat{F}_1 = h_8''\mathrm{d}\rho\wedge\mathrm{d}\psi, \qquad \mathrm{d}\widehat{F}_3 = -\frac{1}{2}h_8''\times(\rho-2\pi k)\mathrm{d}\rho\wedge\mathrm{vol}_{S^2}\wedge\mathrm{d}\psi, \tag{3.5}$$

$$\begin{aligned}
&\mathrm{d}\widehat{F}_5 = -\widehat{h}_4''\mathrm{d}\rho\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi, \qquad \mathrm{d}\widehat{F}_7 = -\frac{1}{2}\widehat{h}_4''\times(\rho-2\pi k)\mathrm{d}\rho\wedge\mathrm{vol}_{S^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi,\\
&\mathrm{d}\widehat{F}_9 = 0,
\end{aligned}\tag{3.6}$$

with

$$\begin{aligned}
&\widehat{h}_4'' = \frac{1}{2\pi}\sum_{j=1}^{P}(\beta_{j-1}-\beta_j)\delta(\rho-2\pi j), \qquad h_8'' = \frac{1}{2\pi}\sum_{j=1}^{P}(\nu_{j-1}-\nu_j)\delta(\rho-2\pi j),\\
&\widehat{h}_4''\times(\rho-2\pi k) = h_8''\times(\rho-2\pi k) = x\delta(x) = 0.
\end{aligned}\tag{3.7}$$

Inspecting the Page fluxes, the electric parts tell us what type of branes we have in the system. The exterior derivative of the dual magnetic form $\mathrm{d}\widehat{F}_{8-p}$ being nonzero, indicates that the Dp brane is a source in the background (flavour branes). In contrast, $\mathrm{d}\widehat{F}_{8-p} = 0$ indicates that these branes are dissolved into fluxes (colour branes). We then find a brane set-up consisting of (colour) D1 and D5 branes, extending in between NS-five branes. This is complemented by (sources) D3 and D7 branes. There are also fundamental strings dissolved into flux. We list the brane content in table 2 and the associated Hanany-Witten set-up in figure 3.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D1 | x | | | | | x | | | | |
| D3 | x | | | | | | x | x | x | |
| D5 | x | x | x | x | x | x | | | | |
| D7 | x | x | x | x | x | | x | x | x | |
| NS5 | x | x | x | x | x | | | | | x |
| F1 | x | | | | | | | | | x |

**Table 2**. Brane set-up underlying the geometry in (3.2)–(3.3). $x^0$ is the time direction of the ten dimensional spacetime. The directions $(x^1,\dots,x^4)$ span the $\mathrm{CY}_2$, $x^5$ is the direction associated with $\rho$, $(x^6,x^7,x^8)$ are the transverse directions realising the SO(3)-symmetry of the $S^2$, and $x^9$ is the $\psi$ direction.

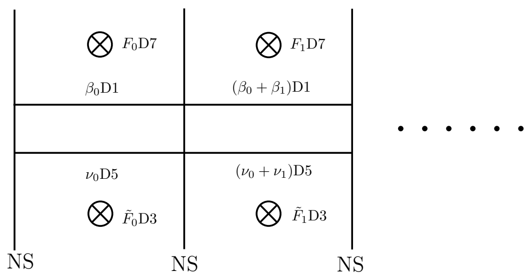


**Figure 3**. The Hanany-Witten set-up corresponding to the background in eqs. (3.2)–(3.3).

Let us study the quantised Page charges, defined by integrating the Page magnetic flux,[1]

$$Q_{\mathrm{Dp}} = \frac{1}{2\kappa_{10}^2 T_{\mathrm{Dp}}}\int \widehat{F}_{8-p} = \frac{1}{(2\pi)^{7-p}}\int \widehat{F}_{8-p}\,. \tag{3.8}$$

The functions $\widehat{h}_4, h_8$ are as those in eqs. (2.5)–(2.6). The integrals over volumes are,

$$\Upsilon \mathrm{Vol}_{\mathrm{CY}_2} = 16\pi^4, \quad \int \mathrm{d}\psi = \mathrm{Vol}_\psi = 2\pi, \quad \mathrm{Vol}_{\mathrm{S}^2} = 4\pi.$$

The different brane charges in each interval $[2\pi k, 2\pi(k+1)]$ are

$$\begin{aligned}
Q_{\mathrm{D1}} &= \frac{1}{(2\pi)^6}\int_{\Sigma_7}\widehat{F}_7 = \left(\frac{\Upsilon \mathrm{Vol}_{\mathrm{CY}_2}}{16\pi^4}\right)\times\left(\frac{\mathrm{Vol}_{\mathrm{S}^2}}{4\pi}\right)\times\left(\frac{\mathrm{Vol}_\psi}{2\pi}\right)(h_4 - h_4'(\rho-2\pi k)) = \alpha_k,\\
Q_{\mathrm{D3}} &= \frac{1}{16\pi^4}\int_{\Sigma_5}\widehat{F}_5 = \frac{1}{16\pi^4}\int_{[\rho,\Sigma_5]}\mathrm{d}\widehat{F}_5 = \left(\frac{\Upsilon \mathrm{Vol}_{\mathrm{CY}_2}}{16\pi^4}\right)\times \mathrm{Vol}_\psi \int \mathrm{d}\rho h_4'' = \beta_{k-1}-\beta_k,\\
Q_{\mathrm{D5}} &= \frac{1}{4\pi^2}\int_{\Sigma_3}\widehat{F}_3 = \left(\frac{\mathrm{Vol}_{\mathrm{S}^2}}{4\pi}\right)\times\left(\frac{\mathrm{Vol}_\psi}{2\pi}\right)(h_8 - h_8'(\rho-2\pi k)) = \mu_k,\\
Q_{\mathrm{D7}} &= \int_{\Sigma_1} F_1 = \int_{[\rho,\Sigma_1]}\mathrm{d}F_1 = \mathrm{Vol}_\psi \int h_8''\mathrm{d}\rho = \nu_{k-1}-\nu_k.
\end{aligned} \tag{3.9}$$

Notice that we have used the expression for the second derivatives in eq. (3.7).

[1] The relevant constants are,

$$T_{\mathrm{Dp}} = \frac{1}{(2\pi)^p g_s \alpha'^{\frac{p+1}{2}}}, \quad 2\kappa_{10}^2 = (2\pi)^7 g_s^2\alpha'^4, \quad T_{\mathrm{NS5}} = \frac{1}{(2\pi)^5 g_s^2 \alpha'^3}, \quad \alpha' = g_s = 1.$$

The structure of singularities in the two ends of the $\rho$-interval is studied in appendix C. Referring to the set-up in table 2 and figure 3, in the $[2\pi k, 2\pi(k+1)]$ interval we have $\alpha_k = \sum_{j=0}^{k-1} \beta_j$ D1 colour branes and $\mu_k = \sum_{j=0}^{k-1} \nu_j$ D5 branes. We also have $(\beta_{k-1} - \beta_k)$ D3 and $(\nu_{k-1} - \nu_k)$ D7 sources (flavour branes).

We close here our analysis of the new $\mathrm{AdS}_2$ geometries. Below, we present a proposal for the dual super conformal quantum mechanics. Matchings between holographic and field theoretical calculations, together with some holographic predictions for these quantum mechanical systems at strong coupling, are discussed in the next section.

## 4 Field theory and holography

In this section we discuss the $\mathcal{N} = 4$ super-conformal quantum mechanical theories proposed as duals to our backgrounds in eqs. (3.2)–(3.3). As anticipated in section 2.1, we provide a UV $\mathcal{N} = 4$ quantum mechanics, that conjecturally flows to a super conformal quantum mechanics dual to our $\mathrm{AdS}_2$ backgrounds.

The bottomline is that the quantum mechanical quiver is the dimensional reduction of the two dimensional QFTs presented in section 2.1. Let us discuss two approaches into the quantum mechanical theory.

One approach is based on the works [73, 77, 78]. In these papers it is suggested that the transition from $\mathrm{AdS}_3$ to $\mathrm{AdS}_2$ should be thought of in $\mathrm{CFT}_2 \to \mathrm{CFT}_1$ language as a discrete light-cone quantisation of the two dimensional CFT. This is to be taken in a limit such that, of the original $\mathrm{SL}(2,\mathbb{R})\times$ $\mathrm{SL}(2,\mathbb{R})$ symmetry of the seed $\mathrm{CFT}_2$, only one of the sectors is kept. The other sector needs infinite energy to be excited. Writing the boundary metric of $\mathrm{AdS}_3$ as a cylinder, $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\phi^2$, and changing coordinates to $u = t + \phi$ and $v = t - \phi$, we have $\mathrm{d}s^2 = -\mathrm{d}u\mathrm{d}v$. In these variables the identification of coordinates $[t, \phi] \to [t, \phi + 2\pi R]$ demands $[u, v] \to [u - 2\pi R, v + 2\pi R]$. The scaling $u \to e^{\gamma} u$, $v \to e^{-\gamma} v$ keeps the metric invariant. In the limit $\gamma \to \infty$, keeping $Re^{\gamma} = \tilde{R}$ fixed, the $\mathrm{CFT}_2$ then lives on a space consisting on time and a null-circle. The energies scale in such a way that the left sector decouples and the right sector has $E_n = \frac{n}{\tilde{R}}$ (see [77, 78] for the details). The T-dualisation along the $\tilde{\psi}$-direction performed in section 3 is equivalent to starting with a given $\mathcal{N} = (0, 4)$ $\mathrm{SCFT}_2$ as those described in section 2.1 and DLCQ it, keeping the $\mathcal{N} = 4$ SUSY right sector. Similar ideas have been discussed recently in [85]. In purely field-theoretical terms, we start with the Lagrangian alluded to in section 2.1 (written in appendix B) and dimensionally reduce it to a matrix model where we keep only the time dependence and the zero modes in the $\tilde{\psi}$-direction.

A second interesting way to think about our quantum mechanical theory is inspired by the works [96, 97]. In these references the same brane set-up depicted in table 2 was proposed in order to describe half-BPS Wilson and 't Hooft loops in 5d gauge theories with 8 supercharges. These defects were described by quiver quantum mechanics with the same field content that we described in section 2.1, after dimensional reduction. Our quiver quantum mechanics exhibit however additional constraints, that are inherited from the anomaly cancelation conditions of the seed 2d CFT. We will see in [98, 99] that more general quivers such as the ones constructed in [96, 97] are dual in the IR to $\mathrm{AdS}_2$ solutions not related to $\mathrm{AdS}_3$ upon T-duality.

In summary, our proposal is that the dynamics of the UV quantum mechanical systems of interest, is described by the dimensional reduction along the space-direction of the $\mathcal{N} = (0,4)$ $\mathrm{SCFT}_2$ discussed in section 2.1. To be concrete: consider the Type IIB backgrounds described in eqs. (3.2)–(3.3) with the functions $\widehat{h}_4, h_8$ given in eqs. (2.5)–(2.6). These solutions are dual to an $\mathcal{N} = 4$ superconformal quantum mechanics that arises in the IR of a generic quiver quantum mechanics, with the matter content depicted in figure 1. The dynamics is inherited from the two-dimensional $\mathcal{N} = (0,4)$ Lagrangian by dimensional reduction. We can read the ranks of colour and flavour groups from the Page charges computed in eqs. (3.9). In the $k^{th}$ entry, corresponding with the $[2\pi k, 2\pi(k+1)]$ interval of the geometry, we have $\mathrm{U}(\alpha_k)$ and $\mathrm{U}(\mu_k)$ colour groups — with $\alpha_k = \sum_{j=0}^{k-1}\beta_j$, $\mu_k = \sum_{j=0}^{k-1}\nu_j$. These are coupled via bifundamental hypermultiplets and Fermi multiplets with the adjacent nodes. The connections with the $k^{th}$ flavour groups of ranks $\mathrm{SU}(\nu_{k-1}-\nu_k)$ and $\mathrm{SU}(\beta_{k-1}-\beta_k)$ is mediated by Fermi fields and by bifundamental hypermultiplets.

The authors of [97] impose that the numbers of D3 and D7 (sources/flavour) branes must equal the difference of two integers. In our formalism this is automatic. The integers are identified with the ranks of the colour groups (be it D1 or D5) on each side of the interval. We have that the number of D3 sources is $(\beta_{k-1}-\beta_k)$ and analogously $(\nu_{k-1}-\nu_k)$ for the number of D7 flavours. These numbers are positive as guaranteed by the convex character of our polygonal functions $\widehat{h}_4, h_8$. The quiver is identical to and inherited from that of the two-dimensional "mother" theory — see figure 1. The superfields involved in writing the Lagrangian are also inherited, as explained in appendix B.

In what follows, we perform some holographic calculations that inform us about the strong dynamics of these conformal quantum mechanical quivers.

### 4.1 The holographic central charge

The definition of central charge in conformal quantum mechanics is subtle. In a one-dimensional theory, we have only one component of $T_{\mu\nu}$. If the theory is conformal, the trace of this quantity must vanish, and this implies that $T_{tt} = 0$. One way to think about central charge is to consider a conformal quantum mechanics which has many ground states (but no excitations). One may associate this quantity with the "central extension" of the Virasoro algebra that appears in the global charges of the two-dimensional dual gravity, as discussed in [74–76]. We can also associate this "central charge" with the partition function of the quantum mechanics when formulated on a circle, as discussed for example in [83].

Though we refer to it as "holographic central charge" the quantity that we present below should be interpreted as a "number of vacuum states" in the associated SCQM. To define it, we shall use the logic discussed in [100–102].

We follow the prescription in [101, 102]. Being the field theory zero-dimensional, some of the steps in the calculation need some care. The relevant quantity in this case is the volume of the internal space (the part not belonging to $\mathrm{AdS}_2$). Analogously, we are



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computing Newton's constant in an effective two-dimensional gravity theory,

$$\frac{1}{G_{N,2}} = \frac{V_{\text{int}}}{G_{N,10}}. \tag{4.1}$$

Following the formalism of [101, 102] for the backgrounds described in eqs. (3.2)–(3.4), we find

$$V_{\text{int}} = \int \mathrm{d}^8 x \sqrt{e^{-4\Phi} \det g_{8,\text{ind}}} = \left( \frac{\text{Vol}_{\text{CY}_2} \text{Vol}_{\text{S}^2} \text{Vol}_{\psi}}{4} \right) \int_0^{2\pi(P+1)} \widehat{h}_4 h_8 \mathrm{d}\rho. \tag{4.2}$$

The comparison with eq. (2.10) indicates that, under suitable rescaling, this quantity is related to the central charge of the seed 2d SCFT.

We define the "holographic central charge" of the conformal quantum mechanics to be

$$c_{\text{hol},1d} = \frac{3}{4\pi G_2} = \frac{3 V_{\text{int}}}{4\pi G_N}. \tag{4.3}$$

Computing explicitly with eq. (4.2) and using that (in the units $g_s = \alpha' = 1$) $G_N = 8\pi^6$, we find

$$c_{\text{hol},1d} = \frac{3}{\pi} \int_0^{2\pi(P+1)} h_4 h_8 \mathrm{d}\rho, \tag{4.4}$$

in agreement with the two-dimensional result in eq. (2.10). This is compatible with the findings of the paper [77], that suggest that the chiral sector remaining when DLCQ is applied to a 2d CFT has the same central extension in the Virasoro algebra.

On purely field theoretical terms, this result tells us that the number of vacua of the $\mathcal{N} = 4$ SCQM obtained by dimensional reduction of the two-dimensional "mother theory", responds to the expression obtained in [93], namely

$$c_{\text{qm}} = 6(n_{\text{hyp}} - n_{\text{vec}}). \tag{4.5}$$

The numbers of $\mathcal{N} = 4$ hyper and vector multiplets in the one dimensional theory are inherited from those in the two dimensional "mother" theory. The agreement between $c_{\text{qm}}$ in eq. (4.5) and $c_{\text{CFT}}$ in eq. (2.9) is the field theoretical translation of the equality of the holographic central charges in two dimensions, eq. (2.10), and in one dimension, eq. (4.4).

It is interesting to draw a comparison with the works [103–106]. These papers make crucial use of the dimension of the Higgs branch for a quiver quantum mechanics with gauge group $\Pi_v \text{U}(N_v)$ and bifundamentals joining each colour group with the adjacent ones. This quantity is given by,

$$\mathcal{M} = \sum_{v,w} N_v N_w - \sum_v N_v^2 + 1. \tag{4.6}$$

We propose that the calculation in eq. (4.4) captures the same information as eq. (4.6). Note that our quantum mechanical theories have a field content that is more involved than the ones considered in [103–106]. Let us illustrate this with an example (similar calculations

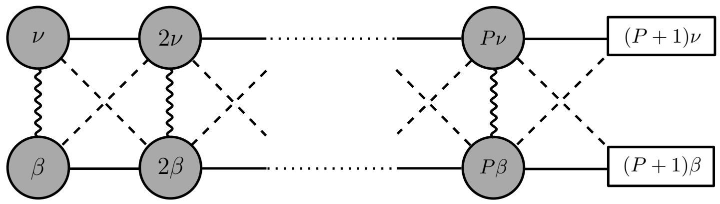


**Figure 4**. The quantum mechanical system that conjecturally flows in the IR to the SCQM described by the backgrounds obtained from eqs. (4.7)–(4.8).

can be done for other quivers). The example we choose is represented by the functions,

$$h_8(\rho) = \begin{cases} \frac{\nu}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\nu P}{2\pi}(2\pi(P+1)-\rho), & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.7}$$

$$\widehat{h}_4(\rho) = \Upsilon h_4(\rho) = \Upsilon \begin{cases} \frac{\beta}{2\pi}\rho & 0 \le \rho \le 2\pi P \\ \frac{\beta P}{2\pi}(2\pi(P+1)-\rho), & 2\pi P \le \rho \le 2\pi(P+1). \end{cases} \tag{4.8}$$

According to the rules presented in section 4, the $\mathcal{N}=4$ quantum mechanical quiver is the one depicted in figure 4. We calculate the expressions for the one dimensional central charge $c_{\text{qm}}$ in eq. (4.5) and its holographic counterpart $c_{\text{hol},1d}$ in eq. (4.4). These expressions should coincide in the holographic limit with the dimension of the Higgs branch in eq. (4.6). Using the definitions in eqs. (4.4) and (4.5) we calculate

$$\begin{aligned} n_{\text{hyp}} &= \sum_{j=1}^{P}\Big(j(j+1)(\nu^2+\beta^2)+j^2\nu\beta\Big), \quad n_{\text{vec}} = \sum_{j=1}^{P} j^2(\nu^2+\beta^2), \\ c_{\text{qm}} &= 3P(P+1)(\nu^2+\beta^2)+(2P+1)(P+1)P\nu\beta \sim 2\beta\nu P^3 \\ c_{\text{hol},1d} &= 2\beta\nu(P^3+P^2) \sim 2\beta\nu P^3. \end{aligned} \tag{4.9}$$

We see that in the holographic limit (large $P, \nu, \beta$) the results of eqs. (4.4) and (4.5) coincide. At the same time we see that the dimension of the Higgs branch moduli space in eq. (4.6) is precisely counting the number of hypers minus the number of vectors. Note that our quiver has hypers joining the links not only "horizontally" but also "vertically", in comparison with the quivers considered in [103–106].

Following [50–52], the reader can produce a variety of test-examples showing the coincidence of the calculations of eqs. (4.4), (4.5) and (4.6) in the holographic limit (it should be interesting to explore sub-leading corrections!). We shall come back to the holographic central charge and relate it to an extremisation principle in section 5.

Let us now discuss predictions for the strong coupling dynamics of our SCQMs.

### 4.2 Chern-Simons terms

Let us discuss the possible "dynamical" term for the gauge multiplet. In $(0+1)$ dimensions this is a Chern-Simons (CS) term. Let us motivate their presence with a small detour on anomalies.

The authors of [107] present a detailed study on the conflict between gauge symmetry and global symmetry (charge conjugation in this case). They study the action of $l$-fermions

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on a time circle of size $T$, in the presence of a U(1) gauge field $A_t(t)$. The system has Lagrangian, gauge and charge conjugation transformations given by,

$$\begin{aligned} & L = \bar{\psi}(i\partial_t + A_t)\psi. \\ & \psi \to e^{i\Lambda}\psi, \quad \bar{\psi} \to \bar{\psi}e^{-i\Lambda}, \quad A_t \to A_t + \partial_t\Lambda(t), \quad A_t \to -A_t. \end{aligned} \tag{4.10}$$

For configurations that are periodic in the circle, the partition function (for $l$ fermions with the above Lagrangian) is,

$$\begin{aligned} Z &= \int D\psi D\bar{\psi} e^{-i\int_0^T \mathrm{d}t L} = \det(i\partial_t + A_t)^l = (1 + e^{ia_0 T})^l, \\ a_0 T &= \int_0^T A_t(t)\mathrm{d}t. \end{aligned} \tag{4.11}$$

This is invariant under both large and small gauge transformations, but not under charge conjugation. A way to recover the charge conjugation invariance is through the introduction of a counterterm

$$L_{\mathrm{ct}} = e^{-ik\int_0^T A_t \mathrm{d}t} = e^{-ika_0 T}. \tag{4.12}$$

This is a CS-term. In itself, it is gauge invariant but not charge conjugation invariant. If $2k = l$, its presence cancels the lack of invariance under charge conjugation in eq. (4.11). We can regularise the partition function of an even number of fermions, such that gauge invariance and charge conjugation are both preserved. If the number of fermions is odd, we just loose the charge conjugation invariance.

In summary, for the case of $(0+1)$-dimensions the Chern-Simons term is of the form

$$S_{\mathrm{CS}} = \kappa_{\mathrm{CS}} \int \mathrm{d}t A_t.$$

The coefficient $\kappa_{\mathrm{CS}}$ must be quantised. As above, consider the theory on a circle of length $T$. Performing a large gauge transformation, $A_t \to A_t + \partial_t\Lambda$ with parameter $\Lambda = \frac{2\pi n}{T}t$, we find that the Chern-Simons action changes,

$$S_{\mathrm{CS}} \to S_{\mathrm{CS}} + \kappa_{\mathrm{CS}} 2\pi n.$$

Imposing that $e^{iS_{\mathrm{CS}}}$ is single-valued under large gauge transformations, we find that $e^{i2\pi n\kappa_{\mathrm{CS}}} = 1$, which quantises the Chern-Simons coefficient.

### 4.2.1 Holographic calculation of the Chern-Simons coefficients

Let us holographically compute the Chern Simons coefficients for each gauge group in the quantum mechanical quiver derived by dimensional reduction of that in figure 1. The presence of the CS term is of non-perturbative origin. We calculate it using the Type IIB $\mathrm{AdS}_2$ description of the system. To do so we use a D1 brane probe extended in $[t, \rho]$, with a gauge field (of curvature $F_{t\rho}$) excited on it. The Wess-Zumino term for the D1 brane probe reads,

$$S_{\mathrm{WZ}} = T_{\mathrm{D1}}\int C_p \wedge e^{2\pi F_2} = T_{\mathrm{D1}}\left(\int C_2 + 2\pi\int C_0 F_{t\rho}\mathrm{d}t\mathrm{d}\rho\right) = -2\pi T_{\mathrm{D1}}\int \mathrm{d}t\int \mathrm{d}\rho A_t\partial_\rho C_0. \tag{4.13}$$



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In the last equality we have used that the RR field $C_2$ has no pull-back on this probe. Moreover, we have set the gauge $A_\rho = 0$ and imposed that the gauge field $A_t$ takes the same values at the extrema of the interval. Keeping in mind that the axion field $C_0$ is only well-defined in regions where $h_8'$ is a constant,[2] where it reads $C_0 = h_8'\psi$, we find,

$$S_{\rm WZ} = -2\pi T_{\rm D1}\int {\rm d}t{\rm d}\rho A_t(t,\rho)\psi h_8'' = -2\pi T_{\rm D1}\psi(\nu_{k-1}-\nu_k)\int {\rm d}t A_t(t,2\pi k) = \kappa_{{\rm CS},I}\int A_t{\rm d}t. \tag{4.14}$$

Using eq. (4.13) we have that the Chern-Simons coefficient in the interval $[2\pi k, 2\pi(k+1)]$ is then given by,

$$\kappa_{{\rm CS},I}[k,k+1] = \psi\frac{(\nu_{k-1}-\nu_k)}{2\pi}. \tag{4.15}$$

Therefore, in order to keep the CS coefficient well quantised, we can allow discrete changes of the coordinate $\psi$,

$$\psi \to \psi + \left(\frac{2\pi l}{\nu_{k-1}-\nu_k}\right), \quad \text{with} \;\; l = 1,\ldots,(\nu_{k-1}-\nu_k). \tag{4.16}$$

These changes indicate that not all positions in $\psi$ are allowed for the D1 probes. In other words, the $\mathrm{U}(1)_\psi$ isometry of the background is broken to $\mathbb{Z}_{\nu_{k-1}-\nu_k}$. On the other hand, the presence of the source D7 branes implies a change in the Chern-Simons coefficient, as the slopes of the function $h_8$ change.

A very similar calculation for a D5 brane that extends on $[t,\rho,\mathrm{CY}_2]$ gives a Chern Simons coefficient for the gauge groups in the lower row that is

$$\kappa_{\rm CS,II}[k,k+1] = \psi\frac{(\beta_{k-1}-\beta_k)}{2\pi}. \tag{4.17}$$

We find that the $\mathrm{U}(1)_\psi$ is broken to $\mathbb{Z}_{\beta_{k-1}-\beta_k}$ and $\mathbb{Z}_{\nu_{k-1}-\nu_k}$, by the Chern-Simons terms in the lower and upper rows respectively. If they have no common subgroups the $\mathrm{U}(1)_\psi$ is completely broken. Notice also that the sum of all the Chern-Simons coefficients gives $\sum_k \kappa_{\rm CS}[k,k+1] = N_F\psi$, where $N_F$ is the sum of the total number of D7 brane sources in the upper row and the total number of D3 branes sources in the lower row.

These are non-trivial predictions for the strongly coupled dynamics of our $\mathcal{N}=4$ SCQM. In appendix D we discuss additional ones. We now go back to discussing the holographic central charge from two different perspectives.

## 5 Holographic central charge, electric-magnetic charges and a minimisation principle

In this section we present different ways of understanding the holographic central charge given by eq. (4.4). We give a two-fold presentation. In section 5.1, that is more physically inspired, we show that the expression in eq. (4.4) is related to a product of electric and magnetic charges associated with our backgrounds. In section 5.2 we present a more geometrical approach, finding that the expression (4.4) can be obtained via an extremisation principle.

[2] Due to the presence of sources, see eqs. (3.4)–(3.5), the axion field is not globally defined.



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### 5.1 The relation between central charge and Page fluxes

We study the link between the holographic central charge of the SCQM in eqs. (4.2)–(4.4) with the integral of electric and magnetic fluxes in the ten dimensional space. We see this explicitly by working with the Page fluxes in eqs. (3.4).

This calculation is the string theoretic realisation of an argument presented in [74] for two dimensional $\mathrm{AdS}_2$ gravity. In this reference it was proposed that the central charge of the SCQM should be related to the (square of the) electric field in an effective $\mathrm{AdS}_2$ gravity theory coupled to a gauge field.

Consider a Dp brane, to which we can associate electric $\widehat{F}_{p+2}$ and magnetic $\widehat{F}_{8-p}$ Page field strengths. We define the "density of electric and magnetic charges", $\rho^e_{\mathrm{Dp}}$ and $\rho^m_{\mathrm{Dp}}$, as the forms

$$\rho^e_{\mathrm{Dp}} = \frac{1}{(2\pi)^p}\widehat{F}_{p+2}, \qquad \rho^m_{\mathrm{Dp}} = \frac{1}{(2\pi)^{7-p}}\widehat{F}_{8-p}. \tag{5.1}$$

The electric charge, obtained by integration of the charge density form, will turn out to be infinite, as it involves the integration of the volume form of the non-compact $\mathrm{AdS}_2$ spacetime. We will work with these definitions, having in mind that a regularisation will be necessary after the integrations are performed, see for example [108].

Consider the product of electric and magnetic charge densities in eq. (5.1), and its integration over all space for the D-branes present in our backgrounds. We show that (after regularisation) this product is proportional to the holographic central charge given by eq. (4.4).[3] We calculate the integral of electric and magnetic densities in eq. (5.1) using the Page fluxes derived in eq. (3.4), and the ordered basis $[t, r, \mathrm{S}^2, \mathrm{CY}_2, \rho, \psi]$. The calculation leads to

$$\begin{aligned}
&\int \sum_{k=0}^{3} (-1)^k \rho^e_{\mathrm{D}(2k+1)} \rho^m_{\mathrm{D}(2k+1)} \qquad (5.2)\\
&\quad = \int \mathrm{d}\rho \left( \frac{\widehat{h}_4 h_8}{2} + \frac{1}{16}\partial_\rho \left[ 2uu' - u^2 \left( \frac{(\widehat{h}_4 h_8)'}{\widehat{h}_4 h_8} \right) \right] \right) \mathrm{Vol}_{\mathrm{AdS}_2} \left( \frac{\mathrm{Vol}_{\mathrm{S}^2}}{4\pi^2} \right) \left( \frac{\mathrm{Vol}_{\mathrm{CY}_2}}{16\pi^4} \right) \left( \frac{\mathrm{Vol}_\psi}{2\pi} \right).
\end{aligned}$$

Up to a boundary term, this is proportional to eq. (4.4), the expression for the holographic central charge of our $\mathrm{AdS}_2$ backgrounds.

Hence, we learn that the holographic central charge in eq. (4.4), measuring the number of vacua of the associated SCQM, is proportional to the (regularised) product of electric and magnetic charge densities. We see this relation as a generalisation of the proposal in [74], showing that the central charge in the algebra of symmetry generators of $\mathrm{AdS}_2$ with an electric field is proportional to the square of the electric field. In our case, for a fully string theoretic set-up, we have objects with electric and magnetic charges and both enter the calculation.

This links the holographic central charge, usually calculated from the dilaton and the metric of the internal space, as shown by equations (4.1) and (4.2), with an expression

[3] In order to show this we use that only one of the components of $\widehat{F}_5 \wedge \widehat{F}_5$ needs to be taken into account, due to its self-duality, and that some sign flips are necessary in order to work with the absolute values of the charges and avoid unwanted cancellations.

purely in terms of the RR-sector. It would be nice to see a similar logic at work in higher dimensional AdS-solutions.

### 5.2 An extremisation principle

In this section we present a minimisation principle in supergravity that will lead to the expression for the holographic central charge in eq. (4.4). Our presentation falls in line with the ideas that extremisation problems in quantum field theory are realised in supergravity through the extremisation of certain geometrical quantities. Various examples exist of this mirroring of extremal principles. The most relevant to us are the ones studied in [83, 109–113]. In these papers a geometrical quantity is defined in supergravity that coincides upon extremisation with the holographic central charge of the systems under study. In some cases this defines the central charge of the dual field theory. We point out some extensions and differences with the approach of [83, 109–113].

Let us follow the idea of [109, 110]. These authors consider a particular family of backgrounds (in eleven dimensional supergravity) of the form $\mathrm{AdS}_2 \times \mathrm{Y}_9$, containing an electric flux $F_4$ and preserving $\mathcal{N} = (0, 2)$ SUSY. Aside from the $\mathrm{AdS}_2$ factor, these backgrounds are quite different from the ones we discuss here (or their lift to M-theory, in the case in which $h_8$ is constant [55]). Nevertheless, the lesson from [109, 110] is that the central charge can be written in terms of an extremised functional. This functional is defined as an integral of various forms in the geometry $\mathrm{Y}_9$, and it is such that, once extremised, equals a weighted volume of the internal space. Importantly, the manifold $\mathrm{Y}_9$ in [109, 110] has no boundary. In our case, we have a boundary and we allow for the presence of sources.

In order to implement these ideas we define certain differential forms on the $X_8$ internal space tranverse to our $\mathrm{AdS}_2$ solutions, $X_8 = [S^2, \mathrm{CY}_2, \rho, \psi]$. We construct these forms *restricting* the Page forms in eq. (3.4) to the manifold $X_8$. For example, from $\widehat{F}_1$ we generate the one-form

$$\widehat{F}_1 \longrightarrow J_1 = h_8' \mathrm{d}\psi. \tag{5.3}$$

From the Page form $\widehat{F}_3$ in eq. (3.4) we generate a second one-form, plus a three-form,

$$\widehat{F}_3 \longrightarrow \mathcal{F}_1 = \left(\frac{h_8}{2} + \frac{u'^2 \widehat{h}_4 - uu'\widehat{h}_4}{8\widehat{h}_4^2}\right) \mathrm{d}\rho, \;\; J_3 = -\frac{1}{2}(h_8 - h_8'(\rho - 2\pi k))\mathrm{vol}_{\mathrm{S}^2} \wedge \mathrm{d}\psi. \tag{5.4}$$

The other forms generated from the Page fluxes are,

$$\begin{aligned}
\mathcal{F}_3 &= \frac{1}{16}\left(\frac{(u-(\rho-2\pi k)u')\,(u\widehat{h}_4' - \widehat{h}_4 u')}{\widehat{h}_4^2} + 4(\rho-2\pi k)h_8\right)\mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\rho,\\
J_5 &= -\widehat{h}_4' \mathrm{vol}_{\mathrm{CY}_2}\wedge \mathrm{d}\psi, \quad \mathcal{F}_5 = -\left(\frac{4\widehat{h}_4 h_8^2 - uu'h_8' + h_8(u')^2}{8h_8^2}\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge \mathrm{d}\rho,\\
J_7 &= \frac{1}{2}(\widehat{h}_4 - \widehat{h}_4'(\rho-2\pi k))\mathrm{vol}_{\mathrm{CY}_2}\wedge \mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\psi,\\
\mathcal{F}_7 &= -\left(\frac{4(\rho-2\pi k)\widehat{h}_4 h_8^2 + u^2 h_8' - h_8 uu' - (\rho-2\pi k)uu'h_8' + (\rho-2\pi k)h_8 u'^2}{16h_8^2}\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge \mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\rho.
\end{aligned} \tag{5.5}$$



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With the forms in eqs. (5.3)–(5.5), we define the functional,

$$
\begin{aligned}
\mathcal{C} &= i\int_{X_8} (J_3+i\mathcal{F}_3)\wedge(J_5+i\mathcal{F}_5)-(J_1+i\mathcal{F}_1)\wedge(J_7+i\mathcal{F}_7) \\
&= \frac{1}{16}\int_{X_8}\left(8\widehat{h}_4 h_8+u^2\left(\frac{\widehat{h}_4'^2}{\widehat{h}_4^2}+\frac{h_8'^2}{h_8^2}\right)-2uu'\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right)+2u'^2\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho\wedge\mathrm{d}\psi.
\end{aligned}
\tag{5.6}
$$

Let us remind that the functional $\mathcal{C}$ is defined in terms of the *restriction* on $X_8$ of the Page fluxes. This can be minimised by imposing the Euler-Lagrange equation for $u(\rho)$ from the "Lagrangian" in eq. (5.6). This equation reads

$$
2u''=u\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right). \tag{5.7}
$$

Imposing the Bianchi identities

$$
h_8''=0, \qquad \widehat{h}_4''=0, \tag{5.8}
$$

this leads us to the BPS equation of our class of solutions (3.2)–(3.3),[4]

$$
u''=0. \tag{5.9}
$$

Note that the fluxes are quantised, due to the type of solutions we consider for $\widehat{h}_4, h_8$ — see eqs. (3.9). It is interesting that here we impose the Bianchi identities in eq. (5.8) leading to the BPS eq. (5.9). This is different from the procedure followed in previous bibliography.

On the solutions to eqs. (5.8), (5.9), which we refer to as "on-shell', the functional is extremised to be,

$$
\begin{aligned}
&\mathcal{C}|_{\text{on-shell}}=\left(\frac{\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{\mathrm{S}^2}\mathrm{Vol}_{\psi}}{2}\right)\int_0^{2\pi(P+1)}\left(\widehat{h}_4 h_8+\partial_\rho\mathcal{M}\right)\mathrm{d}\rho. \\
&\text{with}\quad \mathcal{M}=\frac{1}{8}\left(2uu'-u^2\Big(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\Big)\right).
\end{aligned}
\tag{5.10}
$$

We now compare equations (4.4) and (5.10). Up to a boundary term (present in $X_8$ but not in the boundary-less $Y_9$ of [109, 110]), the inverse Newton's constant in two dimensions in eq. (4.1), the internal volume $V_{\text{int}}$ in eq. (4.2) and the $\mathcal{C}|_{\text{on-shell}}$ in eq. (5.10), all converge into the same calculation. Our proposed duality implies that these geometrical quantities count the number of vacua of the dual SCQM, according to eqs. (4.4)–(4.5).

Let us come back to eq. (5.6), and analyse it for the case in which we have sources. After integration by parts we write it as,

$$
\begin{aligned}
&\mathcal{C}=\frac{1}{16}\int_{X_8}\left(8\widehat{h}_4 h_8+\mathcal{N}+\partial_\rho\mathcal{M}\right)\mathrm{d}\rho, \qquad \text{with} \\
&\mathcal{N}=u^2\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right), \quad \mathcal{M}=2uu'-u^2\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right).
\end{aligned}
$$

[4] Below, we analyse the situation in which sources are present. This implies the presence of delta-function sources as in eq. (3.7).



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Using eq. (3.7), the term $\mathcal{N}$ can be seen to give a finite result proportional to the quotient of the number of flavours by the number of colours in each node. Let us understand the boundary term in more detail. For the solutions in eqs. (2.5)–(2.6) the boundary term is divergent. We can associate this divergence with the presence of sources. Consider for example a solution for $\widehat{h}_4, h_8$ of the type in eqs. (2.5)–(2.6) with $u = u_0$ (a constant). In that case evaluating the boundary term we find

$$\int_0^{2\pi(P+1)} \partial_\rho \mathcal{M} = \lim_{\epsilon\to 0} \frac{u_0^2}{\epsilon}\left(\nu_0 + \beta_0 + \alpha_P + \mu_P\right) = \lim_{\epsilon\to 0} \frac{u_0^2}{\epsilon}\left(N_{\rm D3}^{total} + N_{\rm D7}^{total}\right). \tag{5.11}$$

We have used that $\widehat{h}_4(\rho = 0) = h_8(\rho = 0) = \epsilon$ and the same for the corresponding values at $\rho = 2\pi(P+1)$. Then we take $\epsilon \to 0$, finding a divergent result in terms of the total number of sources present in the background. Notice that in the limit of long quivers ($P$ large) with large ranks for colour group nodes $\mathrm{U}(\alpha_k)$ and $\mathrm{U}(\mu_k)$ and sparse flavour groups, both the boundary term $\mathcal{M}$ and the bulk term $\mathcal{N}$, conveniently renormalised, are subleading in these numbers $(P, \alpha_k, \mu_k)$ with respect to the first term, proportional to the holographic central charge in eq. (4.4). For this we need to define the functional $\mathcal{C}$ in eq. (5.6) with a suitable counter-term that removes the divergence when $\epsilon \to 0$.

It should be interesting to attempt the calculation presented here in different systems, like those in [51] or in higher dimensional AdS-spaces, to check if similar extremisation principles are at work. In particular, it would be interesting to understand the geometrical meaning of the forms in eqs. (5.3)–(5.5).

In summary, the presentation above shows that the holographic central charge, originally defined purely in terms of the NS-NS sector — see eqs. (4.2)–(4.4), is also encoded in the forms of eq. (5.5) and the functional (5.6). The contents of this section link the holographic central charge with the product of electric and magnetic brane charges and with an extremisation principle. These geometrical quantities are capturing the number of vacua of the $\mathcal{N} = 4$ SCQM.

## 6 Summary and conclusions

Given that this is a long and dense paper, the reader may find useful to start with a summary. We describe the main new ideas and calculations presented, pointing to the sections and equations that best describe them.

We start in section 2 with a summary of the seed-backgrounds in massive IIA, dual to two-dimensional $\mathcal{N} = (0, 4)$ SCFTs. The new material is written in section 2.1. There we discuss in detail the field content of the two dimensional field theories. Also, in section 2.1.1 we presented the superpotential for these two-dimensional field theories. Details and generalisations are given in appendices A), (B.

In section 3, we have constructed new $\mathrm{AdS}_2$ solutions to Type IIB supergravity with $\mathcal{N} = 4$ supersymmetry. This infinite family of solutions is precisely written in eqs. (3.2)–(3.7). The Page charges are calculated and the Hanany Witten set-up summarised in table 2.



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In section 4, we propose explicit quiver quantum mechanics that should flow in the IR to the SCQM dual to the backgrounds in section 3. Some aspects of the dynamics of the SCQM have been calculated using the dual description. For example the number of vacua, that we equated with the holographic central charge of the SCQM. This quantity is derived in eqs. (4.1)–(4.4). Our expressions are tested with an example in eqs. (4.7)–(4.9), showing the precise match between a field theory and holography calculations. The holographic central charge is in turn identified with the partition function of the quantum mechanics when formulated on a circle [83]. We have seen that this quantity is related to the "seed" two-dimensional $\mathcal{N} = (0, 4)$ SCFT right-moving central charge. This is an explicit manifestation of the DLCQ upon which both CFTs are related. In this section we also presented predictions for the conformal quantum mechanics. For example, we calculated the Chern-Simons coefficients in eqs. (4.13)–(4.17). This lead to a prediction for the number of vacua and the anomalous breaking of the symmetry $\mathrm{U}(1)_\psi$. Wilson loops, baryon vertices and gauge couplings have been studied in appendix D.

In section 5, we link the holographic central charge (a quantity originally defined in terms of the NS-NS sector of the solutions) to the RR sector of our $\mathrm{AdS}_2$ backgrounds. In particular, we have shown that it is related to the integral of the product of the electric and magnetic charge densities of the D-branes present in the system — see eq. (5.2). This generalises the proposal in [74], where the central charge in the algebra of symmetry generators of $\mathrm{AdS}_2$ is related to the square of the electric field. In our controlled string theory set-up, all electric and magnetic charges of the D-branes present in the solution enter the calculation. In this same section, we have presented an extremisation principle following the general ideas about geometric extremisation in [83, 109–113], from which we have derived the holographic central charge. Our extremising functional is constructed in terms of the electric and magnetic RR fluxes associated to the solutions, see eqs. (5.3)–(5.6). Our results extend those in [83, 109–113], by the presence of sources and boundaries.

Let us end with some proposed research for the future. It would be interesting to see if a similar relation between the holographic central charge and products of Ramond fluxes, holds for other classes of solutions, especially higher dimensional ones. That would allow for a physical principle underlying the construction of the purely geometric extremising functional. Moreover, it would be interesting to find a field theory interpretation for the extremisation construction found for our $\mathrm{AdS}_2$ solutions. Being the R-symmetry non-Abelian it is not clear why an extremisation should be necessary in order to identify the right R-symmetry from which the central charge should be constructed. This issue clearly deserves a more careful investigation.

It would be interesting to relate, in the holographic regime, the calculations of an index (at leading order) with our holographic central charge. More generally, it would be interesting to apply exact calculation techniques to our quiver quantum mechanics in order to understand the properties of the SCQM in the infrared. Related to this is the possibility of learning about supergravity using exact results, along the lines of [114].

It would be interesting to find a defect interpretation for our $\mathrm{AdS}_2$ solutions, possibly along the lines in [115]. In this reference the $\mathrm{AdS}_3$ "seed" solutions from which our $\mathrm{AdS}_2$ solutions have been constructed were interpreted as surface defects within the 5d Sp(N)



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gauge theory dual to the $\mathrm{AdS}_6$ Brandhuber-Oz background [11]. It is likely that, upon T-duality, our solutions would find a similar interpretation, this time in terms of line defects, within the T-dual of the Brandhuber-Oz background [13, 23]. It would be interesting to find a flow interpolating these solutions with this $\mathrm{AdS}_6$ background. These flows may be found in six dimensional supergravity, like those in [47, 48, 66, 115–117].

It should be possible to try to find compactifications of Type IIB supergravity to $\mathrm{AdS}_2$ times an eight manifold, of the form $M_8 = \mathrm{S}^2 \times \mathrm{CY}_2 \times \mathrm{S}^1_\psi \times \mathrm{I}_\rho$. Having these gauged supergravities may allow to study flows away from $\mathrm{AdS}_2$, along the lines of those in [118].

Finding an interpretation of our solutions in the context of 4d black holes is clearly a direction that should be investigated, possibly along the different lines of [85, 119–122]. It should be important to clarify the relation between the number of vacua/holographic central charge and the entropy of these black holes, as advanced around eq. (4.1). It would be interesting to understand the role of the freedom in choosing $\widehat{h}_4, h_8$ and their implications for black holes. Similarly, it would be interesting to explore the uses of the formalism developed in [123, 124], applied to our particular systems. Along this line, the possibility of understanding our $\mathrm{AdS}_2$ backgrounds as the emergent dynamics in [125, 126] is interesting to explore.

We hope to tackle some of these problems in the near future.


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## A AdS$_3$ and AdS$_2$ backgrounds in full generality

In section 2 we discussed a particular set of solutions in class I of the paper [49] and in section 3 we discussed the T-dual of these "seed" backgrounds. In this appendix we summarise the general backgrounds in class I of [49] and perform the T-duality on the $\mathrm{AdS}_3$ fibre, generating $\mathrm{AdS}_2$ backgrounds that generalise those of section 3.

The Neveu-Schwarz sector of the generic $\mathrm{AdS}_3$ backgrounds in [49] reads,

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 + (u')^2}\mathrm{d}s^2_{\mathrm{S}^2}\right) + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2\,,\\
e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4}\sqrt{u}}\sqrt{4\widehat{h}_4 h_8 + (u')^2}\,, \quad H_3 = \frac{1}{2}\mathrm{d}\left(-\rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2}\right)\wedge \mathrm{vol}_{\mathrm{S}^2} + \frac{1}{h_8^2}\mathrm{d}\rho \wedge H_2\,.
\end{aligned} \tag{A.1}$$

Here the metric is given in the string frame, $\Phi$ is the dilaton and $H_3 = \mathrm{d}B_2$ is the NS three-form. In the general case the warping function $\widehat{h}_4$ has support on $(\rho, \mathrm{CY}_2)$. The RR fluxes are,

$$
\begin{aligned}
&F_0 = h_8' \,, \quad F_2 = -\frac{1}{h_8} H_2 - \frac{1}{2}\left(h_8 - \frac{h_8' u u'}{4 h_8 \widehat{h}_4 + (u')^2}\right) \mathrm{vol}_{\mathrm{S}^2} \,, \\
&F_4 = -\left(\mathrm{d}\left(\frac{u u'}{2\widehat{h}_4}\right) + 2 h_8 \mathrm{d}\rho\right) \wedge \mathrm{vol}_{\mathrm{AdS}_3} - \partial_\rho \widehat{h}_4 \mathrm{vol}_{\mathrm{CY}_2} - \frac{h_8}{u} (\widehat{\star}_4 \mathrm{d}_4 \widehat{h}_4) \wedge \mathrm{d}\rho \\
&\qquad - \frac{u u'}{2 h_8 (4 \widehat{h}_4 h_8 + (u')^2)} H_2 \wedge \mathrm{vol}_{\mathrm{S}^2} \,,
\end{aligned}
\tag{A.2}
$$

with the higher fluxes related to these as $F_6 = -\star_{10} F_4$, $F_8 = \star_{10} F_2$, $F_{10} = -\star_{10} F_0$, and where $\widehat{\star}_4$ is the Hodge dual on the $\mathrm{CY}_2$. It was shown in [49] that supersymmetry holds whenever,

$$
u'' = 0 \,, \qquad H_2 + \widehat{\star}_4 H_2 = 0 \,, \tag{A.3}
$$

which makes $u$ a linear function of $\rho$. $H_2$ can be defined in terms of three functions $g_{1,2,3}$ on $\mathrm{CY}_2$,

$$
H_2 = g_1(\hat{e}^1 \wedge \hat{e}^2 - \hat{e}^3 \wedge \hat{e}^4) + g_2(\hat{e}^1 \wedge \hat{e}^3 + \hat{e}^2 \wedge \hat{e}^4) + g_3(\hat{e}^1 \wedge \hat{e}^4 - \hat{e}^2 \wedge \hat{e}^3), \tag{A.4}
$$

where $\hat{e}^i$ are a canonical vielbein on $\mathrm{CY}_2$ (see section 3.1. of [49]). Hence, the Bianchi identities of the fluxes impose (away from localised sources),

$$
\begin{aligned}
&h_8'' = 0 \,, \qquad \mathrm{d}H_2 = 0, \\
&\frac{h_8}{u} \nabla^2_{\mathrm{CY}_2} \widehat{h}_4 + \partial_\rho^2 \widehat{h}_4 + \frac{2}{h_8^3}(g_1^2 + g_2^2 + g_3^2) = 0.
\end{aligned}
\tag{A.5}
$$

In the case when $H_2$ vanishes and $\widehat{h}_4$ has support on the $\rho$ coordinate only, we are in the case of the solutions reviewed in section 2.

We T-dualise the previous backgrounds on the Hopf direction of $\mathrm{AdS}_3$ by parametrising it as in (3.1). Performing T-duality on $\tilde{\psi}$ results in the dual NS sector,

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}}\left(\frac{1}{4}\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 + (u')^2}\mathrm{d}s^2_{\mathrm{S}^2}\right) + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}(\mathrm{d}\rho^2 + \mathrm{d}\psi^2) \,, \\
e^{-2\Phi} &= \frac{h_8}{4\widehat{h}_4}(4\widehat{h}_4 h_8 + (u')^2) \,, \\
H_3 &= \frac{1}{2}\mathrm{d}\left(-\rho + \frac{u u'}{4\widehat{h}_4 h_8 + (u')^2}\right) \wedge \mathrm{vol}_{\mathrm{S}^2} + \frac{1}{h_8^2}\mathrm{d}\rho \wedge H_2 + \frac{1}{2}\mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{d}\psi \,,
\end{aligned}
\tag{A.6}
$$

and the RR sector is,

$$
\begin{aligned}
F_1 &= h_8' \mathrm{d}\psi \,, \\
F_3 &= -\frac{1}{2}\left(h_8 - \frac{h_8' u' u}{4 h_8 \widehat{h}_4 + (u')^2}\right) \mathrm{vol}_{\mathrm{S}^2} \wedge \mathrm{d}\psi - \frac{1}{h_8} H_2 \wedge \mathrm{d}\psi + \frac{1}{4}\left(\mathrm{d}\left(\frac{u' u}{2\widehat{h}_4}\right) + 2 h_8 \mathrm{d}\rho\right) \wedge \mathrm{vol}_{\mathrm{AdS}_2} \,, \\
F_5 &= -(1 + \star_{10})\left(\partial_\rho \widehat{h}_4 \mathrm{vol}_{\mathrm{CY}_2} + \frac{h_8}{u}(\widehat{\star}_4 \mathrm{d}_4 \widehat{h}_4) \wedge \mathrm{d}\rho + \frac{u u'}{2 h_8 (4\widehat{h}_4 h_8 + (u')^2)} H_2 \wedge \mathrm{vol}_{\mathrm{S}^2}\right) \wedge \mathrm{d}\psi \,,
\end{aligned}
\tag{A.7}
$$

where $F_7 = -\star_{10} F_3 =$ and $F_9 = \star_{10} F_1$.

In the case when $H_2$ vanishes and $\widehat{h}_4$ has support on the $\rho$ coordinate only, we are in the case of the solutions constructed in section 3.

## B $\mathcal{N}=(0,2)$ and $\mathcal{N}=(0,4)$ theories and quantum mechanics

In this appendix we briefly discuss the QFTs that conjecturally flow in the IR to a strongly coupled CFT. The discussion requires some standard aspects of two dimensional $\mathcal{N}=(0,2)$ and $\mathcal{N}=(0,4)$ supersymmetric theories. As these are well summarised in other works — see for example [92–94] — we shall not give too many details.

**$\mathcal{N}=(0,2)$ multiplets.** Let us list the field components of the three types of $\mathcal{N}=(0,2)$ multiplets, namely the vector $U$, chiral $\Phi$ and the Fermi $\Psi$ multiplets

$$U:(u_\mu,\zeta_-,D)\,,\qquad \Phi:(\phi,\psi_+)\,,\qquad \Psi:(\psi_-,G)\,. \tag{B.1}$$

The subscript on the fermions refers to their chiralities under the $\mathrm{SO}(1,1)$ Lorentz group. $D$ is a real and $G$ a complex auxiliary field.

A *vector* $U$ has the following expansion in superspace[5]

$$U=u_0-u_1-2i\theta^+\bar{\zeta}_--2i\bar{\theta}^+\zeta_-+2\theta^+\bar{\theta}^+D\,. \tag{B.2}$$

The corresponding field strength is obtained by means of

$$\Upsilon=[\overline{\mathcal{D}}_+,\mathcal{D}_-]=-\zeta_--i\theta^+(D-iu_{01})-i\theta^+\bar{\theta}^+(\mathcal{D}_0+\mathcal{D}_1)\zeta_-\,, \tag{B.3}$$

where $\overline{\mathcal{D}}_+$ and $\mathcal{D}_-$ are the supercovariant gauge derivatives [95]. It turns out that $\Upsilon$ is a Fermi multiplet — it satisfies $\overline{\mathcal{D}}_+\Upsilon=0$. We shall give a more precise definition of a Fermi multiplet momentarily.

A *chiral field* $\Phi$ is a superfield that satisfies the following equation

$$\overline{\mathcal{D}}_+\Phi=0\,, \tag{B.4}$$

and therefore expands out in components as

$$\Phi=\phi+\sqrt{2}\theta^+\psi_+-i\theta^+\bar{\theta}^+(D_0+D_1)\phi\,, \tag{B.5}$$

where $D_0$ and $D_1$ stand for the time- and space-components of the usual covariant derivative.

A *Fermi superfield* instead satisfies the following equation

$$\overline{\mathcal{D}}_+\Psi=E(\Phi_i)\,, \tag{B.6}$$

where $E(\Phi_i)$ is a holomorphic function of the chiral superfields $\Phi_i$. $E$ should be chosen in such a way that it transforms as $\Psi$ under all symmetries. Solving (B.6) leads to the following expansion for $\Psi$

$$\Psi=\psi_--\theta^+G-i\theta^+\bar{\theta}^+(D_0+D_1)\psi_--\bar{\theta}^+E(\phi_i)-\theta^+\bar{\theta}^+\frac{\partial E}{\partial\phi^i}\psi_{+i}\,, \tag{B.7}$$

[5] $\mathcal{N}=(0,2)$ superspace is parametrised by two real spacetime coordinates, $x_\pm=x^0\pm x^1$, and two complex Grassmann variables $\theta^+$ and $\overline{\theta}^+$ subject to a reality constraint.



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where $G$ is an auxiliary complex field. The holomorphic function $E$ can be shown to appear in the Lagrangian as a potential term. There is also another type of superpontential we can consider for $\mathcal{N} = (0,2)$ theories. For each Fermi multiplet $\Psi_a$ we can introduce a holomorphic function $J^a(\Phi_i)$ such that

$$S_J = \int \mathrm{d}^2x \mathrm{d}\theta^+ \sum_a J^a(\Phi_i)\Psi_a + \text{h.c.}\,. \tag{B.8}$$

It must be stressed that the superpotentials $E$ and $J$ cannot be introduced independently. It turns out that, in order for supersymmetry to be preserved, they have to satisfy the following constraint

$$E \cdot J = \sum_a E_a J^a = 0\,. \tag{B.9}$$

Let us now move on to listing $\mathcal{N} = (0,4)$ supermultiplets.

**$\mathcal{N} = (0,4)$ multiplets.** $\mathcal{N} = (0,4)$ supermultiplets are usually given in terms of $\mathcal{N} = (0,2)$ supermultiplets, pretty much as in 4 dimensions $\mathcal{N} = 2$ superfields are built from $\mathcal{N} = 1$ superfields. Again, let us list them first.

| Multiplets | $\mathcal{N} = (0,2)$ building blocks | component fields | $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ |
|---|---|---|---|
| Vector | Vector + Fermi $(U, \Theta)$ | $(u_\mu, \zeta_-^a, G^A)$ | $(1,1), (2,2), (3,1)$ |
| Hyper | Chiral + Chiral $(\Phi, \tilde{\Phi})$ | $(\phi^a, \psi_+^b)$ | $(2,1), (1,2)$ |
| Twisted hyper | Chiral + Chiral $(\Phi', \tilde{\Phi}')$ | $(\phi'^a, \psi'^b_+)$ | $(1,2), (2,1)$ |
| Fermi | Fermi + Fermi $(\Gamma, \tilde{\Gamma})$ | $(\psi'^a_-, G^b)$ | $(1,1), (2,2)$ |

The $\mathcal{N} = (0,4)$ vector multiplet is made of an $\mathcal{N} = (0,2)$ vector multiplet and an adjoint $\mathcal{N} = (0,2)$ Fermi multiplet $\Theta$. The field content is that of a gauge field $u_\mu$ and two left-handed fermions $\zeta_-^a$, $a = 1,2$, in addition to a triplet of auxiliary fields $G^A$, $A = 1,2,3$. The gauge field is a singlet under the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ R-symmetry while the two fermions transform as $(\mathbf{2},\mathbf{2})$. The triplet of auxiliary fields transforms as $(\mathbf{3},\mathbf{1})$ under the R-symmetry.

There are two different types of hypermultiplets, the hypermultiplet and the twisted hypermultiplet. Both of them are formed by two $\mathcal{N} = (0,2)$ chiral multiplets, therefore they both contain two complex scalars $(\phi^a)$ and two right-handed fermions $(\psi_+^b)$. They differ from each other because of the different representations under the R-symmetry group, as we can see from the table above.

If we want to couple the hypermultiplet to the vector multiplet, we should consider the following coupling between the hyper $(\Phi, \tilde{\Phi})$ and the adjoint Fermi field $\Theta$

$$J^\Theta = \Phi\tilde{\Phi} \Rightarrow \mathcal{W} = \tilde{\Phi}\Theta\Phi\,. \tag{B.10}$$

On the other hand, coupling a twisted hypermultiplet to the gauge sector requires an E-type of superpotential

$$E_\Theta = \Phi'\tilde{\Phi}'\,, \tag{B.11}$$

with indices in $\Phi'\tilde{\Phi}'$ set to have $E_\Theta$ transforming in the adjoint of the gauge group.



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Finally, we can have an $\mathcal{N}=(0,4)$ Fermi multiplet, which is made of two $\mathcal{N}=(0,2)$ Fermi multiplets. It contains two left-handed fermions which are singlets of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ R-symmetry. No further coupling between $\Gamma$, $\tilde{\Gamma}$ and $\Theta$ is possible.

As in the quiver of figure 1, there appear also $\mathcal{N}=(4,4)$ vector and chiral multiplets, it is worth mentioning how $\mathcal{N}=(4,4)$ superfields decompose in $\mathcal{N}=(0,4)$ language.

**$\mathcal{N}=(4,4)$ multiplets.** There are two types of $\mathcal{N}=(4,4)$ superfields, the vector and the hypermultiplet.

| Multiplets | $\mathcal{N}=(0,4)$ building blocks | $\mathcal{N}=(0,2)$ building blocks |
|---|---|---|
| Vector | Vector + Twisted Hyper | $(U,\Theta),(\Sigma,\tilde{\Sigma})$ |
| Hyper | Hyper + Fermi | $(\Phi,\tilde{\Phi}),(\Gamma,\tilde{\Gamma})$ |

The $\mathcal{N}=(4,4)$ vector multipled is comprised of an $\mathcal{N}=(0,4)$ vector multiplet and a $\mathcal{N}=(0,4)$ twisted hypermultiplet. The twisted hypermultiplet is usually denoted as $(\Sigma,\tilde{\Sigma})$. They are coupled to the gauge sector via the E-type potential

$$E_\Theta = [\Sigma,\tilde{\Sigma}]\,. \tag{B.12}$$

$\mathcal{N}=(4,4)$ hypermultiplets are made of an $\mathcal{N}=(0,4)$ hypermultiplet and an $\mathcal{N}=(4,4)$ Fermi multiplet, all in all $(\Phi,\tilde{\Phi}),(\Gamma,\tilde{\Gamma})$. As before, $\Phi$ and $\tilde{\Phi}$ are coupled to the gauge sector via

$$\mathcal{W} = \tilde{\Phi}\Theta\Phi\,. \tag{B.13}$$

We conclude this part by stressing out that there are couplings between $\mathcal{N}=(0,4)$ Fermi multiplets $\Gamma$, $\tilde{\Gamma}$, hypermultiplets $\Phi$, $\tilde{\Phi}$ and twisted hypers $\Sigma$, $\tilde{\Sigma}$. They involve both superpotential and E-terms

$$\mathcal{W} = \tilde{\Gamma}\tilde{\Sigma}\Phi + \tilde{\Phi}\tilde{\Sigma}\Gamma\,, \tag{B.14}$$

and

$$E_\Gamma = \Sigma\Phi\,, \quad E_{\tilde{\Gamma}} = -\tilde{\Phi}\Sigma\,. \tag{B.15}$$

It is easy to see that

$$E\cdot J = \tilde{\Phi}[\Sigma,\tilde{\Sigma}]\Phi + \tilde{\Phi}\tilde{\Sigma}\Sigma\Phi - \tilde{\Phi}\Sigma\tilde{\Sigma}\Phi = 0\,. \tag{B.16}$$

### B.1 $\mathcal{N}=4$ quantum mechanics

As we argued in the main text, the $\mathcal{N}=4$ superconformal quantum mechanics dual to the IIB backgrounds discussed around (3.2) and (3.3) is given by the dimensional reduction of the CFT in figure 1. Thus, we start with a general discussion on compactification of 2d $\mathcal{N}=(0,4)$ theories. These are usually formulated in terms of $\mathcal{N}=(0,2)$ multiplets, so we start by reducing them first.

**$\mathcal{N} = 2$ supersymmetry multiplets.** In $\mathcal{N} = 2$ quantum mechanics we have two real supercharges with an SO(2) R-symmetry. Equivalently, they can be rearranged as two complex supercharges $Q$ and $\overline{Q}$ with a reality constraint, and U(1) R-symmetry. They satisfy the algebra

$$Q^2 = \overline{Q}^2 = 0\,, \quad \{Q, \overline{Q}\} = H\,, \tag{B.17}$$

with $H$ the hamiltonian. Moreover, if we denote by $J$ the R-symmetry generator we have

$$[J, Q] = -Q\,, \quad [J, \overline{Q}] = \overline{Q}\,, \quad [J, H] = 0\,. \tag{B.18}$$

Let us now see what $\mathcal{N} = 2$ supermultiplets in quantum mechanics are relevant for us. Much of the construction is obtained from the dimensional reduction of 2d $\mathcal{N} = (0, 2)$ reviewed above.

As we have seen in the previous section, the 2d $\mathcal{N} = (0, 2)$ vector multiplet consists of a two-dimensional gauge field $u_\mu$, a left-handed (complex) fermionic field $\zeta_-$ and a real auxiliary field $D$. They are all valued in the adjoint representation of the corresponding gauge group. In the following we will just set $\zeta_- \equiv \zeta$, as there is no chirality in 1d. After reduction, we have $u_\mu = (u_t, \sigma)$, where $u_t$ is the one dimensional gauge field and $\sigma$ a real scalar. The supersymmetric kinetic term for an $\mathcal{N} = 2$ vector multiplet in quantum mechanics is

$$L_{\text{vector}} = \frac{1}{2g^2}\text{tr}\left[(D_t\sigma)^2 + i\bar{\zeta}D_t^{(+)}\zeta + D^2\right]\,, \tag{B.19}$$

where $D_t^{(\pm)} = D_t \pm i\sigma$ and $D_t$ is the usual covariant derivative $D_t = \partial_t + iu_t$ for fields in a generic representation of the gauge group.

A 2d $\mathcal{N} = (0, 2)$ chiral multiplet consists of a complex scalar boson $\phi$ and a right-handed (complex) fermionic field $\psi_+$ in some unitary representation of the gauge group. As before, we will only be concerned with the fundamental and adjoint representations. Again, in going down to 1d we will drop the sub-index. The supersymmetric kinetic term for an $\mathcal{N} = 2$ chiral multiplet in quantum mechanics reads

$$L_{\text{chiral}} = D_t\bar{\phi}D_t\phi + i\bar{\psi}D_t^{(-)}\psi + \bar{\phi}(D - \sigma^2)\phi - i\sqrt{2}\bar{\phi}\zeta\psi + i\sqrt{2}\bar{\psi}\bar{\zeta}\phi\,. \tag{B.20}$$

A 2d $\mathcal{N} = (0, 2)$ Fermi multiplet consists on a left-handed (complex) fermion $\psi_-$ and an auxiliary field $G$. In the following, we will make the identification $\psi_- \equiv \eta$ and $\psi_{+,i} \equiv \psi_i$ if $(\phi_i, \psi_{+,i})$ is a chiral multiplet. The Lagrangian for a generic Fermi multiplet reads

$$L_{\text{fermi}} = i\bar{\eta}D_t^{(+)}\eta + |G^2| - |E(\phi_i)|^2 - \bar{\eta}\frac{\partial E}{\partial\phi_i}\psi_i - \bar{\psi}_i\frac{\partial E}{\partial\bar{\phi}_i}\eta\,. \tag{B.21}$$

In addition to the $E$-term potentials it is possible, for each Fermi multiplet $\Psi_a$, to introduce a holomorphic function $J^a(\Phi_i)$ which gives rise to interactions of the form

$$L_J = G^a J_a(\phi_i) + \sum_i \eta_a \frac{\partial J^a}{\partial\phi^i}\psi_i + \text{h.c.}\,. \tag{B.22}$$

As remarked already, the superpotentials $E$ and $J$ cannot be introduced independently. In order for supersymmetry to be preserved, they must satisfy $\sum_a E_a J^a = 0$.

**$\mathcal{N} = 4$ supersymmetric systems.** The $\mathcal{N} = 4$ supermultiplets that are relevant to our construction are given just by dimensional reduction of $\mathcal{N} = (0, 4)$ and $\mathcal{N} = (4, 4)$ supermultiplets. Two-dimensional $\mathcal{N} = (0, 4)$ and $\mathcal{N} = (4, 4)$ supermultiplets are given in terms of $\mathcal{N} = (0, 2)$ multiplets, according to the discussion in the previous section, summarised in the two tables above. The dimensional reduction of the 2d theory depicted in figure 1 is then readily done according to the rules above. In particular, a two-dimensional gauge field always reduces to one-component gauge field plus a scalar in one dimension. Scalars and fermions remain untouched. In the case of the fermions, this is due to the fact that in both one and two dimensions the minimal spinor representation is one-dimensional. Supersymmetric interactions for the UV Lagrangian can be added as long as the condition (B.9) is satisfied. See for instance [92].

Before ending this section, let us give one remark about the R-symmetry of the IR theory.

**R-symmetry.** The R-symmetry group of supersymmetric $\mathcal{N} = 4$ quantum mechanics is $\mathrm{SO}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)$. As we flow to the IR and hit a fixed point, given that it exists, we should find that our quantum mechanics realises some classified superconformal algebra. When $\mathcal{N} = 4$, we have essentially two possibilities: $\mathfrak{d}(2, 1; \alpha)$, with two $\mathfrak{su}(2)$'s R-symmetries, or $\mathfrak{su}(1, 1|2)$, with one $\mathfrak{su}(2)$ only.

The $\mathfrak{d}(2, 1; \alpha)$ global algebra is often referred to as *large* superconformal algebra and $\alpha$ is a parameter which parametrises the relative strength of the two Kac-Moody levels, $k_-$ and $k_+$ of the SU(2) R-symmetries. Given that we have only one SU(2) (realised geometrically on the $\mathrm{S}^2$) and given also that in the parent $\mathrm{AdS}_3 \times \mathrm{S}^2$ backgrounds supersymmetries were in the $(\mathbf{1}, \mathbf{2}; \mathbf{2})$ of $\mathrm{SL}(2, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R}) \times \mathrm{SU}(2)_R$, we are naturally led to the conclusion that the (global) superalgebra realised by our backgrounds and the dual field theories is the $\mathfrak{su}(1, 1|2)$ superalgebra.[6] Also, superalgebras in one and two dimensions are closely related — each chiral sector of a 2d SCFT provides a superalgebra and its realisation for a 1d superconformal QM — and this makes it possible to identify central charges in 1d and 2d [77].

## C Singularity structure at the ends of the $\rho$-interval

In this appendix we study the asymptotic behaviour of the backgrounds in eq. (3.2), for defining functions $\widehat{h}_4, h_8$ given by eqs. (2.5)–(2.6). Other possible ways of bounding the space can be considered following [49, 50]. We distinguish two cases:

- $u = c_u \rho$: At $\rho = 0$, we find a regular background. In turn, at the end of the $\rho$-interval (which we denote by $\rho = 2\pi(P + 1) - x$ for small $x$) we find a metric and dilaton that behave as,

$$\mathrm{d}s^2 \sim \frac{1}{x}\mathrm{d}s^2_{\mathrm{AdS}_2} + x(\mathrm{d}x^2 + \mathrm{d}\psi^2 + \mathrm{d}s^2_{\mathrm{S}^2}) + \mathrm{d}s^2_{\mathrm{CY}_2}, \quad e^{-2\Phi} \sim 1. \tag{C.1}$$

[6] The $\mathfrak{su}(1, 1|2)$ is also realised by taking the limit $\alpha \to \infty$ in the $\mathfrak{d}(2, 1; \alpha)$ algebra.

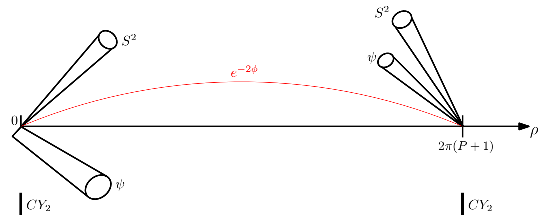


**Figure 5**. Behaviour of the solutions at both ends of the $\rho$-interval for $u = u_0\rho$. The $\mathrm{S}^2$ vanishes, while the $\mathrm{S}^1_\psi$ is finite at $\rho = 0$ but shrinks to zero size at $\rho = 2\pi(P+1)$. The $\mathrm{CY}_2$ has finite size at both ends.

This is a superposition of O1 and O5 planes, extended on $\mathrm{AdS}_2$ (and smeared on the $\mathrm{CY}_2$) and $\mathrm{AdS}_2 \times \mathrm{CY}_2$ respectively (see for example around equation (3.38) of the paper [49]).

- $u = u_0$: At both ends of the interval, the metric and dilaton asymptote similarly. The expansion of the background at both ends is,

$$\mathrm{d}s^2 \sim \frac{1}{\rho}(\mathrm{d}s^2_{\mathrm{AdS}_2} + \mathrm{d}s^2_{\mathrm{S}^2}) + \rho(\mathrm{d}\rho^2 + \mathrm{d}\psi^2) + \mathrm{d}s^2_{\mathrm{CY}_2}, \quad e^{-2\Phi} \sim \rho^2. \tag{C.2}$$

This indicates the superposition of O3 and O7 planes, extended on $\mathrm{AdS}_2 \times \mathrm{S}^2$ (and smeared on the $\mathrm{CY}_2$) and $\mathrm{AdS}_2 \times \mathrm{S}^2 \times\, \mathrm{CY}_2$ respectively.

In both cases we find that in approaching the end of the interval, the $\psi$-cycle becomes of vanishing size. T-dualising in this direction we recover the seed backgrounds discussed in section 2.

Analysing the volume of the compact submanifolds of the solutions in eqs. (2.5)–(2.6) with $u' = 0$, we run into the possibility that some of these submanifolds have infinite size. However, in spite of a divergent warp factor, the "stringy size" of the submanifold is actually finite or vanishing at the ends of the space. The finite stringy-volume case does not pose any problem in interpreting a D-brane wrapping such cycle. The case in which the cycle shrinks may suggest an interpretation of the singularity in terms of new massless degrees of freedom (branes wrapping the shrinking cycles) that the supergravity solution is not encoding.

For the case in eqs. (2.5)–(2.6), with $u = u_0\rho$ and hence non-vanishing $u'$, the background is smooth at $\rho = 0$, but it presents a singularity at $\rho = 2\pi(P+1)$. We analysed this singularity around eq. (C.1). A pictorial view of this background is given in figure 5. In the case in which $u' = 0$, the asymptotic behaviour given in eq. (C.2) is sketched in figure 6.

In spite of the two sphere having divergent volume, we find that the stringy volume of the $\mathrm{S}^2$ calculated as,

$$V_s[\mathrm{S}^2] = \int \mathrm{vol}_{\mathrm{S}^2}\, e^{-\Phi}\sqrt{\det[g+B]} = 2\pi\sqrt{h_8^2\left(\frac{u^2}{16\widehat{h}_4 h_8} + \frac{(\rho-2\pi k)^2}{4}\right)}, \tag{C.3}$$

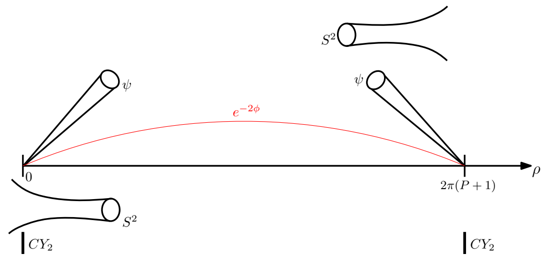


**Figure 6**. Behaviour of the solutions at both ends of the $\rho$-interval for $u = u_0$. The $\mathrm{S}^2$ diverges while the $\mathrm{S}^1_\psi$ shrinks at both ends. The $\mathrm{CY}_2$ remains finite.

is finite for $\rho = 0$ and $\rho = 2\pi(P + 1)$. A brane wrapped on $\mathrm{S}^2$ will then have finite energy and will not pose problems when considering its backreaction.

## D Holographic calculation of QFT observables

In this appendix we discuss the holographic calculation of various field theoretical observables of the strongly coupled quantum mechanics. We focus on Wilson loops, baryon vertices and gauge couplings.

### D.1 Wilson loops

As we mentioned in the main text we expect that our conformal quantum mechanics are related to the more general theories describing line defects inside five dimensional $\mathcal{N} = 2$ SCFTs, studied in [96, 97]. This opens the possibility that the VEV of a Wilson (or 't Hooft) line can be exactly computed using localisation, along the lines described in [97, 108]. Here we discuss the holographic calculation of a particular Wilson line that can potentially be checked with some exact methods.

Consider a fundamental string extended on $\mathrm{AdS}_2$, parametrised with coordinates $(t, r)$ as in eq. (3.1). The string has a profile $\rho = \rho(r)$. The induced metric and NS-NS 2-form field, as well as the action for the string, are obtained from eqs. (3.1)–(3.2). They read,

$$\begin{aligned}
\mathrm{d}s^2_{\text{ind}} &= \frac{u}{4\sqrt{\widehat{h}_4 h_8}}(-\mathrm{d}t^2\cosh^2 r + \mathrm{d}r^2) + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\rho'^2\mathrm{d}r^2,\\
B_2 &= \frac{\psi_0}{2}\cosh r\,\mathrm{d}t\wedge\mathrm{d}r,\\
S_{F1} &= \frac{1}{2\pi}\int \mathrm{d}t\mathrm{d}r\cosh r\left[\frac{u}{4\sqrt{\widehat{h}_4 h_8}}\sqrt{1+\frac{4\widehat{h}_4 h_8}{u^2}\rho'^2} - \frac{\psi_0}{2}\right].
\end{aligned} \tag{D.1}$$

We solve the equations of motion for this probe string if

$$\partial_\rho\left(\frac{u}{4\sqrt{\widehat{h}_4 h_8}}\right) = 0, \qquad u = u_0\rho, \qquad \widehat{h}_4 = \frac{\beta}{2\pi}\rho, \qquad h_8 = \frac{\nu}{2\pi}\rho. \tag{D.2}$$

The solution in eq. (D.2) implies that the string is sitting close to the beginning of a generic quiver, for the functions $\widehat{h}_4, h_8$ in eqs. (2.5)–(2.6). The on-shell action for this string is,

$$S_{\text{on-shell}} = \frac{1}{2\pi}\int \mathrm{d}t\mathrm{d}r \cosh r\left(\frac{\pi u_0}{2\sqrt{\nu\beta}} - \frac{\psi_0}{2}\right) = \frac{1}{2\pi}\left(\frac{\pi u_0}{2\sqrt{\nu\beta}} - \frac{\psi_0}{2}\right)\mathrm{Vol}_{\mathrm{AdS}_2}. \tag{D.3}$$

This is the quantity that we associate with the expected value of this particular Wilson loop.

There is another solution with constant profiles $u \sim h_8 \sim \widehat{h}_4 \sim 1$. This solution does not fall within the analysis of this paper. Instead, it can be thought of as the reduction and T-dual, along the fibration in $\mathrm{S}^3$, of the background $\mathrm{AdS}_3 \times \mathrm{S}^3 \times K3$. Other interesting defect-like operator, the baryonic vertex, is discussed below.

### D.2 Baryonic vertex

We study here a couple of probe branes that we can identify with baryonic vertices. As in the paper [127], there will be an integer number of fundamental strings ending on them and their tension will be of order $T_{bar} \sim 1/g_s$.

We consider first the gauge groups that come from D5 branes in the interval $[2\pi k, 2\pi(k+1)]$, for which the number of branes is $\mu_k$. For this gauge group, we consider a probe consisting on a D3 brane extended along $[t, \mathrm{S}^2, \psi]$ at some fixed value $\rho = 2\pi k$. The induced metric, NS-NS B-field and BIWZ action read,

$$\begin{aligned}
\mathrm{d}s^2_{\text{ind,D3}} &= -\frac{u}{4\sqrt{\widehat{h}_4 h_8}}\cosh^2 r\mathrm{d}t^2 + \frac{u\sqrt{\widehat{h}_4 h_8}}{\Delta}\mathrm{d}s^2_{\mathrm{S}^2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\psi^2, \quad \Delta = 4\widehat{h}_4 h_8 + u'^2,\\
B_2 &= \frac{1}{2\Delta}\left[(2\pi k - \rho)\Delta + uu'\right]\mathrm{vol}_{S^2} - \frac{\sinh r}{2}\mathrm{d}t\wedge\mathrm{d}\psi, \qquad\qquad (\mathrm{D}.4)\\
S_{\mathrm{BI}} &= T_{\mathrm{D3}}\int e^{-\Phi}\sqrt{\det[g+B]}\,\mathrm{d}t\wedge\mathrm{d}\psi\wedge\mathrm{vol}_{\mathrm{S}^2} = T_{\mathrm{D3}}\left(\frac{u}{8}\sqrt{\frac{h_8}{\widehat{h}_4}}\right)|_{\rho=2\pi k}(8\pi^2)\int \mathrm{d}t,\\
S_{\mathrm{WZ}} &= T_{\mathrm{D3}}\int C_4 + f_2\wedge C_2 = -T_{\mathrm{D3}}\int_t a_t\int_{\mathrm{S}^2\times\mathrm{S}^1_\psi}\widehat{F}_3 = \frac{\mu_k}{2\pi}\int_t a_t.
\end{aligned}$$

We used that $f_2 = da_1$. We see that the mass of this particle is given by $M_{bar} = \pi^2 T_{\mathrm{D3}}\left(u\sqrt{\frac{h_8}{\widehat{h}_4}}\right)|_{\rho=2\pi k}$. We also observe that $\mu_k$ strings must end on it, to cancel the charge of the object on $\mathrm{S}^2\times\mathrm{S}^1_\psi$. This is the baryonic vertex for the gauge group $\mathrm{U}(\mu_k)$.

With a probe D7 brane extended in $[t, \mathrm{CY}_2, \mathrm{S}^2, \psi]$ at some fixed value of $\rho = 2\pi k$, we find analogously,

$$\begin{aligned}
\mathrm{d}s^2_{\text{ind, D7}} &= -\frac{u}{4\sqrt{\widehat{h}_4 h_8}}\cosh^2 r\mathrm{d}t^2 + \frac{u\sqrt{\widehat{h}_4 h_8}}{\Delta}\mathrm{d}s^2_{\mathrm{S}^2} + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\psi^2,\\
\Delta &= 4\widehat{h}_4 h_8 + u'^2, \quad B_2 = \frac{1}{2\Delta}\left[(2\pi k - \rho)\Delta + uu'\right]\mathrm{vol}_{S^2} - \frac{\sinh r}{2}\mathrm{d}t\wedge\mathrm{d}\psi, \qquad (\mathrm{D}.5)\\
S_{\mathrm{BI}} &= T_{\mathrm{D7}}\int e^{-\Phi}\sqrt{\det[g+B]}\,\mathrm{d}t\wedge\mathrm{d}\psi\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\\
&= T_{\mathrm{D7}}\,\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{\mathrm{S}^2}\mathrm{Vol}_\psi\left(\frac{u}{8}\sqrt{\frac{\widehat{h}_4}{h_8}}\right)|_{\rho=2\pi k}\int \mathrm{d}t,\\
S_{\mathrm{WZ}} &= T_{\mathrm{D7}}\int C_8 + f_2\wedge C_6 = -T_{\mathrm{D7}}\int_t a_t\int_{\mathrm{S}^2\times\mathrm{CY}_2\times\mathrm{S}^1_\psi}\widehat{F}_7 = \frac{\alpha_k}{2\pi}\int_t a_t.
\end{aligned}$$

In this case we find that the mass of the particle is $M = T_{\mathrm{D7}}\,\mathrm{Vol}_{\mathrm{CY}_2}\mathrm{Vol}_{\mathrm{S}^2}\mathrm{Vol}_\psi\left(\frac{u}{8}\sqrt{\frac{\widehat{h}_4}{h_8}}\right)|_{\rho=2\pi k}$, and that there should be $\alpha_k$ fundamental strings ending on it.

### D.3 Gauge couplings

The gauge coupling of each node can also be computed holographically. We follow a prescription that works in higher dimensional systems. For $\alpha_k$ gauge groups we study the action of D1 branes extending in $[t,\rho]$ with $\rho\in[2\pi k, 2\pi(k+1)]$ (at fixed values of the other coordinates). For the $\mu_k$ gauge groups, we study D5 branes that extend on $[t,\rho,\mathrm{CY}_2]$ with $\rho\in[2\pi k, 2\pi(k+1)]$, also at fixed values of all other coordinates.

We compute the Born-Infeld and Wess-Zumino actions of these branes. We associate the coefficient of the BI parts with the gauge couplings. For the D1 brane probe we consider here, for which the gauge field is taken to be zero and for which the NS two form has zero pull-back on the brane worldvolume, the induced metric, Born-Infeld-Wess-Zumino action and gauge coupling $g^2_{\mathrm{YM},1}$ read,

$$
\begin{aligned}
S_{\mathrm{BIWZ}} &= T_{\mathrm{D1}}\int \mathrm{d}t\mathrm{d}\rho\, e^{-\Phi}\sqrt{-\det[g_{\mathrm{ind}}]} - T_{\mathrm{D1}}\int C_2 \qquad \text{(D.6)}\\
\mathrm{d}s^2_{\mathrm{ind,D1}} &= -\frac{u}{4\sqrt{\widehat{h}_4 h_8}}\cosh^2 r_0\mathrm{d}t^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2,\quad C_2 = \frac{\sinh(r_0)}{4}\left(\partial_\rho\left(\frac{uu'}{2\widehat{h}_4}\right) + 2h_8\right)\mathrm{d}\rho\wedge\mathrm{d}t,\\
S_{\mathrm{BI}} &= -T_{\mathrm{D1}}\int_{2\pi k}^{2\pi(k+1)}\mathrm{d}\rho\sqrt{\frac{h_8}{\widehat{h}_4}(4\widehat{h}_4 h_8 + u'^2)}\int \mathrm{d}t\,\frac{\cosh r_0}{4},\\
S_{\mathrm{WZ}} &= T_{\mathrm{D1}}\int_{2\pi k}^{2\pi(k+1)}\mathrm{d}\rho\left(\partial_\rho\left(\frac{uu'}{\widehat{h}_4}\right) + 2h_8\right)\int \mathrm{d}t\,\frac{\sinh r_0}{4},
\end{aligned}
$$

Notice that this probe D1 brane becomes extremal (its tension equals its charge) when $u' = 0$ and when the brane is placed near the boundary of $\mathrm{AdS}_2$ (that is, $r_0\to\infty$). Probably under these circumstances the branes are calibrated. We can define the gauge coupling from the coefficient of the Born-Infeld term. We find for $\alpha_k$ gauge groups,

$$
\frac{1}{g^2_{\mathrm{YM},1}[k,k+1]} = \frac{1}{2\pi}\int_{2\pi k}^{2\pi(k+1)} h_8\mathrm{d}\rho = \frac{2\mu_k + \nu_k}{2}. \qquad \text{(D.7)}
$$

Notice that this coupling is dimensionless.

Similarly for the $\mu_k$ gauge groups we find, using a D5 brane in $[t,\rho,\mathrm{CY}_2]$ (at fixed values for all other coordinates),

$$
\begin{aligned}
\mathrm{d}s^2_{\mathrm{ind,D5}} &= -\frac{u}{4\sqrt{\widehat{h}_4 h_8}}\cosh^2 r_0\mathrm{d}t^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u}\mathrm{d}\rho^2 + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2}, \qquad \text{(D.8)}\\
C_6 &= \left(\frac{4\widehat{h}_4 h_8^2 - uu'h_8' + h_8(u')^2}{8h_8^2}\right)\sinh r_0\mathrm{d}\rho\wedge\mathrm{d}t\wedge\mathrm{vol}_{\mathrm{CY}_2},\\
S_{\mathrm{BI}} &= -T_{\mathrm{D5}}\mathrm{Vol}_{\mathrm{CY}_2}\int_{2\pi k}^{2\pi(k+1)}\mathrm{d}\rho\sqrt{\frac{\widehat{h}_4}{h_8}(4\widehat{h}_4 h_8 + u'^2)}\int \mathrm{d}t\,\frac{\cosh r_0}{4},
\end{aligned}
$$

$$S_{\mathrm{WZ}} = T_{\mathrm{D5}}\mathrm{Vol}_{\mathrm{CY}_2} \int_{2\pi k}^{2\pi(k+1)} \mathrm{d}\rho \left( \frac{4\widehat{h}_4 h_8^2 - u u' h_8' + h_8 (u')^2}{8h_8^2} \right) \int \mathrm{d}t \sinh r_0,$$
$$\frac{1}{g^2_{\mathrm{YM},2}[k,k+1]} = \frac{1}{2\pi} \int_{2\pi k}^{2\pi(k+1)} h_4 \mathrm{d}\rho = \frac{2\alpha_k + \beta_k}{2}.$$

In the last line we observe that this particular D5 brane probe is extremal for the solutions with $u' = 0$ and at $r_0 \to \infty$.



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## 8.2 A second $\mathbf{AdS_2 \times S^2 \times CY_2 \times \Sigma_2}$ family of solutions via AC and their QM

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# $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2$ solutions in Type IIB with 8 supersymmetries

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Abstract: We present a new infinite family of Type IIB supergravity solutions preserving eight supercharges. The structure of the space is $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \mathrm{S}^1$ fibered over an interval. These solutions can be related through double analytical continuations with those recently constructed in [1]. Both types of solutions are however dual to very different superconformal quantum mechanics. We show that our solutions fit locally in the class of $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2$ solutions fibered over a 2d Riemann surface $\Sigma$ constructed by Chiodaroli, Gutperle and Krym, in the absence of D3 and D7 brane sources. We compare our solutions to the global solutions constructed by Chiodaroli, D'Hoker and Gutperle for $\Sigma$ an annulus. We also construct a cohomogeneity-two family of solutions using non-Abelian T-duality. Finally, we relate the holographic central charge of our one dimensional system to a combination of electric and magnetic fluxes. We propose an extremisation principle for the central charge from a functional constructed out of the RR fluxes.

Keywords: AdS-CFT Correspondence, Superstring Vacua

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## Contents



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# 1 Introduction

The Maldacena conjecture [2] and its extensions motivate the search for AdS backgrounds preserving different amounts of Supersymmetry (SUSY), in different dimensions.

The half-maximal SUSY case is especially fructiferous. The correspondence between linear quiver conformal field theories preserving half-maximal SUSY and half-maximal BPS solutions with an AdS factor, leads to a precise map between infinite families of string backgrounds and their dual super-conformal field theories (SCFTs). Indeed, various works have developed the dictionary between $d$-dimensional SCFTs, the associated Hanany-Witten brane set-ups [3] and the dual $AdS_{d+1}$ string theory backgrounds.

For the case $d = 6$, for which the strongly coupled conformal point is reached at high energies, the papers [4–7] have outlined the holographic dictionary and many other works have developed it. For $d = 5$, the works [8–14] presented backgrounds with an $AdS_6$ factor and their UV-dual SCFTs. The dictionary for the case of four dimensional $\mathcal{N} = 2$ linear quiver SCFTs and their $AdS_5$ dual backgrounds was studied in [15–18] among other works. The case of $d = 3$ SCFTs (arising at low energies after a RG flow) and the dual $AdS_4$ backgrounds is studied in [19–21] among other works. The correspondence for the case of two-dimensional (half-maximal BPS) low-energy SCFTs is particularly rich and has received a lot of attention recently. With the lens described above (linear quivers, Hanany-Witten set-ups and dual backgrounds), we encounter the works [22–30] among various other papers.

The study of $AdS_2$ backgrounds in string/M-theory has a long and illustrious history. With the point of view described above, partial aspects of the correspondence between super-conformal quantum mechanics theories (SCQMs) of the quiver type and half-maximal BPS backgrounds containing an $AdS_2$ factor were initially studied in [31–37]. The recent works [1, 38, 39] made precise and concrete the viewpoint advertised above for different infinite families of string backgrounds containing an $AdS_2$ factor.

This work presents a new infinite family of backgrounds with an $AdS_2$ factor. We focus our presentation mostly on geometrical aspects of the new type IIB solutions. The contents of this paper are distributed as follows. In section 2, we present the new backgrounds preserving eight supersymmetries (four Poincaré and four conformal SUSYs). We study the conserved brane charges and deduce the associated brane set-up, consisting on D1 and D5 'colour' branes (dissolved into fluxes) with D3 and D7 'source' branes (present in the background and violating Bianchi identities). NS-five branes and fundamental strings complete this configuration. We define the holographic central charge following the procedure and physical meaning advanced in [1]. The section is closed with a brief discussion of the dual SCQM. In section 3 we connect our backgrounds with those presented in [34, 35]. We point out that the presence of sources in our solutions extend (for the $AdS_2$ fixed point) the results of [34, 35]. We also link the solutions in [1] with those of [34, 35] (under the above mentioned restrictions). These links require a zoom-in procedure that we discuss in detail. In section 4 we uncover a new and explicit infinite family of solutions of cohomogeneity-two, by applying non-Abelian T-duality on the $AdS_3$ backgrounds of [23–26]. The study of these backgrounds and their 'completion' following the ideas of [14, 40–43] is reserved for a future study. We extend to the families of backgrounds discussed in this work a relation uncovered in [1, 39] between the Ramond-Ramond sector of the backgrounds and the holographic central charge. Such relation is discussed In section 5. A functional whose extremisation yields the central charge is also presented. Finally, section 6 gives a short summary of the work, together with some ideas to work on the future.

## 2 New $AdS_2 \times S^2 \times CY_2$ backgrounds

In this section we present a new family of $AdS_2$ solutions with $\mathcal{N} = 4$ Poincaré supersymmetry in Type IIB supergravity. These geometries are foliations of $AdS_2 \times S^2 \times CY_2 \times S^1$ over an interval. Alternatively, they can be considered as foliations of $AdS_2 \times S^2 \times CY_2$ over a 2d Riemann surface $\Sigma$ with the topology of an annulus. The NS-NS sector of our



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solutions reads,

$$
\begin{aligned}
\mathrm{d}s^2_{st} &= \frac{u\sqrt{\widehat{h}_4 h_8}}{4\widehat{h}_4 h_8-(u')^2}\mathrm{d}s^2_{\mathrm{AdS}_2} + \frac{u}{4\sqrt{\widehat{h}_4 h_8}}\mathrm{d}s^2_{\mathrm{S}^2} + \sqrt{\frac{\widehat{h}_4}{h_8}}\mathrm{d}s^2_{\mathrm{CY}_2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u}(\mathrm{d}\psi^2+\mathrm{d}\rho^2)\,,\\
e^{-2\phi} &= \frac{h_8}{4\widehat{h}_4}(4\widehat{h}_4 h_8-(u')^2)\,,\\
H_3 &= -\frac{1}{2}\mathrm{d}\left(\rho+\frac{uu'}{4\widehat{h}_4 h_8-(u')^2}\right)\wedge \mathrm{vol}_{\mathrm{AdS}_2} + \frac{1}{h_8^2}\mathrm{d}\rho\wedge H_2 + \frac{1}{2}\mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\psi\,.
\end{aligned}
\tag{2.1}
$$

Here $\phi$ is the dilaton, $H_3$ the NS-NS three-form and the metric is given in string frame. A prime denotes a derivative with respect to $\rho$. The two-form $H_2$ is defined on the $\mathrm{CY}_2$. The coordinate $\psi$ ranges in $[0, 2\pi]$, while the $\rho$ coordinate describes an interval that we will take to be bounded between 0 and $2\pi(P + 1)$ (see below). Note that $u \geq 0$ and $4\widehat{h}_4 h_8 - (u')^2 \geq 0$ must be imposed to have a positive definite metric. The background in eq. (2.1) is supported by the RR fluxes,

$$
\begin{aligned}
F_1 &= h_8'\mathrm{d}\psi\,,\\
F_3 &= -\frac{1}{h_8}H_2\wedge\mathrm{d}\psi - \frac{1}{2}\left(h_8+\frac{h_8' u' u}{4h_8\widehat{h}_4-(u')^2}\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{d}\psi + \frac{1}{4}\left(-\mathrm{d}\left(\frac{u'u}{2\widehat{h}_4}\right)+2h_8\mathrm{d}\rho\right)\wedge\mathrm{vol}_{\mathrm{S}^2}\,,\\
F_5 &= -(1+\star_{10})\left(\partial_\rho\widehat{h}_4\mathrm{vol}_{\mathrm{CY}_2} + \frac{h_8}{u}\widehat{\star}_4\mathrm{d}_4 h_4\wedge d\rho - \frac{u'u}{2h_8(4\widehat{h}_4 h_8-(u')^2)}H_2\wedge\mathrm{vol}_{\mathrm{AdS}_2}\right)\wedge\mathrm{d}\psi.\\
F_7 &= \left(\frac{1}{2}\left(\widehat{h}_4+\frac{uu'\widehat{h}_4'}{4h_8\widehat{h}_4-(u')^2}\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{d}\psi - \frac{1}{4}\left(2\widehat{h}_4\mathrm{d}\rho-\mathrm{d}\left(\frac{uu'}{2h_8}\right)\right)\wedge\mathrm{vol}_{\mathrm{S}^2}\right)\wedge\mathrm{vol}_{\mathrm{CY}_2}\\
&\quad -\star_{10}\left(\frac{1}{h_8}H_2\wedge\mathrm{d}\psi\right),\\
F_9 &= \frac{\widehat{h}_4 h_8' u^2}{4\widehat{h}_8(4\widehat{h}_4 h_8-(u')^2)}\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho.
\end{aligned}
\tag{2.2}
$$

Supersymmetry holds whenever,

$$
u''=0, \qquad H_2+\widehat{\star}_4 H_2 = 0, \tag{2.3}
$$

where $\widehat{\star}_4$ is the Hodge dual on $\mathrm{CY}_2$. In turn, the Bianchi identities of the fluxes impose —away from localised sources — that,

$$
h_8''=0\,, \qquad \mathrm{d}H_2=0, \qquad \frac{h_8}{u}\nabla^2_{\mathrm{CY}_2}\widehat{h}_4 + \partial^2_\rho\widehat{h}_4 - \frac{1}{h_8^3}H_2\wedge H_2 = 0. \tag{2.4}
$$

In what follows we will concentrate on backgrounds for which $H_2 = 0$ and $\widehat{h}_4 = \widehat{h}_4(\rho)$. These backgrounds are supersymmetric solutions of the Type IIB equations of motion if the warping functions satisfy (away from localised sources),

$$
\widehat{h}_4''=0, \qquad h_8''=0, \qquad u''=0, \tag{2.5}
$$

which makes them linear functions of $\rho$.



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We focus on the solutions defined by the piecewise linear functions $\widehat{h}_4, h_8$ considered in [24, 25]. These are continuous functions with discontinuous derivatives. These imply discontinuities in the RR-sector that are interpreted as generated by sources in the background. The solutions in [24, 25] have well-defined 2d dual CFTs. This requires a global definition of the $\rho$-interval. We achieve this imposing that $\widehat{h}_4$ and $h_8$ vanish at both ends of the $\rho$-interval, that we take at $\rho = 0, 2\pi(P+1)$. Ending the space in this fashion, introduces extra source branes in the configuration. For the backgrounds to be trustable (in view of holographic applications), we need to impose that the sources are 'sparse', namely that they occur separated enough in the $\rho$-interval. This imposes that $P$ (the length of the $\rho$-interval) is large.

The functions $\widehat{h}_4$ and $h_8$ are then defined as,

$$\widehat{h}_4(\rho) = \Upsilon h_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi}\rho & 0 \leq \rho \leq 2\pi, \\ \alpha_k + \frac{\beta_k}{2\pi}(\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi(k+1), \quad k = 1, \dots, P-1 \\ \alpha_P - \frac{\alpha_P}{2\pi}(\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1), \end{cases} \tag{2.6}$$

$$h_8(\rho) = \begin{cases} \frac{\nu_0}{2\pi}\rho & 0 \leq \rho \leq 2\pi, \\ \mu_k + \frac{\nu_k}{2\pi}(\rho - 2\pi k) & 2\pi k \leq \rho \leq 2\pi(k+1), \quad k = 1, \dots, P-1 \\ \mu_P - \frac{\mu_P}{2\pi}(\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1). \end{cases} \tag{2.7}$$

The choice of constants is imposed by continuity of the metric and dilaton. This implies that

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j, \qquad \mu_k = \sum_{j=0}^{k-1} \nu_j. \tag{2.8}$$

In turn, $\beta_k$ and $\nu_k$ must be integer numbers to give well defined quantised charges (see the next subsection). In (2.6) the number $\Upsilon$ is chosen such that,

$$\Upsilon \mathrm{Vol}_{\mathrm{CY}_2} = 16\pi^4. \tag{2.9}$$

In most of our analysis in this paper we will concentrate on solutions for which $u = u_0 =$ constant. In that case the behaviour of the metric and dilaton at both ends of the $\rho$-interval is

$$ds^2 \sim \frac{1}{x}(ds^2_{\mathrm{AdS}_2} + ds^2_{\mathrm{S}^2}) + ds^2_{\mathrm{CY}_2} + x\,(dx^2 + d\psi^2)\,, \qquad e^{-\phi} \sim x\,, \tag{2.10}$$

where $x = \rho$ close to $\rho = 0$ and $x = 2\pi(P+1) - \rho$ close to $\rho = 2\pi(P+1)$. This corresponds to a superposition of D3-branes, extended on $\mathrm{AdS}_2 \times \mathrm{S}^2$ and smeared on $\psi$ and the $\mathrm{CY}_2$, and D7-branes, extended on $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2$ and smeared on $\psi$.

The backgrounds in eqs. (2.1)–(2.2) can be obtained applying the usual T duality rules over the Hopf fibre of the three sphere of the $\mathrm{AdS}_2 \times \mathrm{S}^3$ backgrounds in [39]. Additionally, these solutions have the same structure as the geometries in [1], namely $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \mathrm{S}^1$ foliated over an interval. The relation with the backgrounds in [1] is through an analytic continuation,

$$\begin{aligned} &ds^2_{\mathrm{AdS}_2} \to -ds^2_{\mathrm{S}^2}, \quad ds^2_{\mathrm{S}^2} \to -ds^2_{\mathrm{AdS}_2}, \quad e^{\phi} \to ie^{\phi}, \quad F_i \to -iF_i, \\ &u \to -iu, \quad \widehat{h}_4 \to i\widehat{h}_4, \quad h_8 \to ih_8, \quad \rho \to i\rho, \quad \psi \to -i\psi, \quad g_i \to ig_i. \end{aligned} \tag{2.11}$$

$$
\begin{tikzcd}
\mathrm{AdS}_3\times \mathrm{S}^2\times \mathrm{CY}_2\times \mathrm{I}_\rho \arrow[rr, leftrightarrow, "\substack{\mathrm{AdS}_3\leftrightarrow \mathrm{S}^3\\ \mathrm{S}^2\leftrightarrow \mathrm{AdS}_2}", "\substack{\text{Analytic}\\ \text{Continuation}}"'] \arrow[d, "\text{T-duality}"'] & & \mathrm{AdS}_2\times \mathrm{S}^3\times \mathrm{CY}_2\times \mathrm{I}_\rho \arrow[d, "\text{T-duality}"] \\
\mathrm{AdS}_2\times \mathrm{S}^2\times \mathrm{CY}_2\times \mathrm{I}_\rho\times \mathrm{S}_\psi \arrow[rr, leftrightarrow, "\substack{\text{Analytic}\\ \text{Continuation}}", "\substack{\mathrm{AdS}_2\leftrightarrow \mathrm{S}^2\\ \mathrm{S}^2\leftrightarrow \mathrm{AdS}_2}"'] & & \mathrm{AdS}_2\times \mathrm{S}^2\times \mathrm{CY}_2\times \mathrm{I}_\rho\times \mathrm{S}_\psi
\end{tikzcd}
$$

**Figure 1**. Relations between the infinite family of $\mathrm{AdS}_3$ backgrounds to massive IIA constructed in [23] (top left), the IIB $\mathrm{AdS}_2$ backgrounds studied in [1] (bottom left), the IIA $\mathrm{AdS}_2$ backgrounds constructed in [39] (top right), and the new $\mathrm{AdS}_2$ solutions in Type IIB given by eqs. (2.1)–(2.2) (bottom right).

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These relations are summarised in figure 1.

Next we study the charges associated with the backgrounds in eqs. (2.1)–(2.2) and the associated brane set-up.

### 2.1 Brane charges and brane set-up

We compute the charges associated to our backgrounds using that the magnetic charge for a Dp brane is given by,

$$Q^m_{\mathrm{Dp}} = \frac{1}{(2\pi)^{7-p}}\int_{\mathcal{M}_{8-p}} \widehat{F}_{8-p}, \tag{2.12}$$

where $\mathcal{M}_{8-p}$ is any $(8-p)$-dimensional compact manifold transverse to the branes. In turn, the electric charge of Dp branes is defined by,

$$Q^e_{\mathrm{Dp}} = \frac{1}{(2\pi)^{p+1}}\int_{\mathrm{AdS}_2\times\Sigma_p} \widehat{F}_{p+2}, \tag{2.13}$$

where $\Sigma_p$ is the $p$-dimensional manifold on which the brane extends. In both expressions we have set $\alpha' = g_s = 1$. For the electric charges we need to regularise the volume of the $\mathrm{AdS}_2$ space. We take it to be the analytical continuation of the volume of the two-sphere,

$$\mathrm{Vol}_{\mathrm{AdS}_2} = 4\pi\,. \tag{2.14}$$

In the previous expressions $\widehat{F}$ are the Page fluxes, defined as $\widehat{F} = F\wedge e^{-B_2}$. They read, for our backgrounds

$$
\begin{aligned}
\widehat{F}_1 &= h_8'\,\mathrm{d}\psi\,,\\
\widehat{F}_3 &= \frac{1}{2}\left(h_8'(\rho-2\pi k)-h_8\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{d}\psi+\frac{1}{4}\left(2h_8+\frac{u'(u\widehat{h}_4'-\widehat{h}_4u')}{2\widehat{h}_4^2}\right)\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho\,,\\
\widehat{F}_5 &= \frac{1}{4}\left(h_8(\rho-2\pi k)-\frac{(u-(\rho-2\pi k)u')\,(u\widehat{h}_4'-\widehat{h}_4u')}{4\widehat{h}_4^2}\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho-\widehat{h}_4'\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi\,,\\
\widehat{F}_7 &= \frac{1}{2}\left(\widehat{h}_4-(\rho-2\pi k)\,\widehat{h}_4'\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi-\frac{1}{4}\left(2\widehat{h}_4+\frac{u'(uh_8'-h_8u')}{2h_8^2}\right)\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\rho\,,\\
\widehat{F}_9 &= -\frac{1}{4}\left((\rho-2\pi k)\widehat{h}_4-\frac{(u-(\rho-2\pi k)u')\,(uh_8'-h_8u')}{4h_8^2}\right)\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge d\rho.
\end{aligned}
\tag{2.15}
$$

| | $x^0=t$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5=\rho$ | $x^6=r$ | $x^7=\theta_1$ | $x^8=\theta_2$ | $x^9=\psi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D1 | x | | | | | | | | | x |
| D3 | x | | | | | | x | x | x | |
| D5 | x | x | x | x | x | | | | | x |
| D7 | x | x | x | x | x | | x | x | x | |
| NS5 | x | x | x | x | x | | x | | | |
| F1 | x | | | | | | x | | | |

**Table 1**. Brane set-up associated to our solutions. $x^0$ corresponds to the time direction of the ten dimensional spacetime, $x^1, \ldots, x^4$ are the coordinates spanned by the $\mathrm{CY}_2$ and $x^7, x^8$ are the coordinates parametrising the $S^2$.

In these expressions we have allowed for large gauge transformations of the $B_2$-field, $B_2 \to B_2 + k\pi \mathrm{vol}_{\mathrm{AdS}_2}$, as in [39] (see this reference for more details).

Before calculating the quantised charges associated to these fluxes it is useful to compute the following quantities,

$$\begin{aligned}
&\mathrm{d}\widehat{F}_1 = h_8'' \mathrm{d}\rho \wedge \mathrm{d}\psi\,, \quad \mathrm{d}\widehat{F}_3 = \frac{1}{2} h_8''(\rho - 2\pi k)\mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{d}\rho \wedge \mathrm{d}\psi\,, \quad \mathrm{d}\widehat{F}_5 = -h_4''\, \mathrm{vol}_{\mathrm{CY}_2} \wedge \mathrm{d}\rho \wedge \mathrm{d}\psi\,, \\
&\mathrm{d}\widehat{F}_7 = -\frac{1}{2} h_4''(\rho - 2\pi k)\mathrm{vol}_{\mathrm{AdS}_2} \wedge \mathrm{vol}_{\mathrm{CY}_2} \wedge \mathrm{d}\rho \wedge \mathrm{d}\psi\,, \quad \mathrm{d}\widehat{F}_9 = 0\,.
\end{aligned} \tag{2.16}$$

In these expressions $\widehat{h}_4''$ and $h_8''$ are the ones that follow from eqs. (2.6)–(2.7),

$$\begin{aligned}
&\widehat{h}_4'' = \frac{1}{2\pi} \sum_{k=1}^{P} (\beta_{k-1} - \beta_k)\delta(\rho - 2\pi k), \qquad h_8'' = \frac{1}{2\pi} \sum_{k=1}^{P} (\nu_{k-1} - \nu_k)\delta(\rho - 2\pi k), \\
&\widehat{h}_4'' \times (\rho - 2\pi k) = h_8'' \times (\rho - 2\pi k) = x\delta(x) = 0.
\end{aligned} \tag{2.17}$$

We then obtain

$$\mathrm{d}\widehat{F}_3 = \mathrm{d}\widehat{F}_7 = 0, \tag{2.18}$$

and

$$\mathrm{d}\widehat{F}_1 = \frac{1}{2\pi} \sum_{k=1}^{P} (\nu_{k-1} - \nu_k)\delta(\rho - 2\pi k)\, \mathrm{d}\rho \wedge \mathrm{d}\psi \tag{2.19}$$

$$\mathrm{d}\widehat{F}_5 = -\frac{1}{2\pi} \sum_{k=1}^{P} (\beta_{k-1} - \beta_k)\delta(\rho - 2\pi k)\, \mathrm{vol}_{\mathrm{CY}_2} \wedge \mathrm{d}\rho \wedge \mathrm{d}\psi. \tag{2.20}$$

These results can be put in correspondence with the brane set-up summarised in table 1. The fact that $\mathrm{d}\widehat{F}_3 = 0$ and $\mathrm{d}\widehat{F}_7 = 0$ indicates that the D5 and D1 branes play the role of colour branes (dissolved in fluxes) in the brane set-up. On the other hand, $\mathrm{d}\widehat{F}_1$ and $\mathrm{d}\widehat{F}_5$ being nonzero indicate that the D7 and D3 branes are flavour branes, that is, explicit sources with dynamics described by the Born-Infeld-Wess-Zumino action.



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Substituting $\widehat{h}_4$ and $h_8$ as defined by eqs. (2.6) and (2.7), together with eqs. (2.9) and (2.14), we find, in each $\rho$-interval $[2\pi k, 2\pi(k+1)]$

$$
\begin{aligned}
Q^e_{\rm D1} &= \frac{1}{(2\pi)^2}\int_{\rm AdS_2\times S_\psi}\widehat{F}_3 = \left(\frac{\rm Vol_{AdS_2}}{4\pi}\right)\left(\frac{\rm Vol_\psi}{\pi}\right)\frac{h_8-h_8'(\rho-2\pi k)}{2} = \mu_k,\\
Q^m_{\rm D3} &= \frac{1}{16\pi^4}\int_{\rm CY_2\times S_\psi}\widehat{F}_5 = \frac{1}{16\pi^4}\int_{\rm CY_2\times S_\psi\times I_\rho}{\rm d}\widehat{F}_5 = \left(\frac{\Upsilon{\rm Vol_{CY_2}}}{16\pi^4}\right)\times{\rm Vol}_\psi\int h_4''{\rm d}\rho = (\beta_{k-1}-\beta_k),\\
Q^e_{\rm D5} &= \frac{1}{(2\pi)^6}\int_{\rm AdS_2\times CY_2\times S_\psi}\widehat{F}_7 = \left(\frac{\rm Vol_{AdS_2}}{4\pi}\right)\left(\frac{\Upsilon{\rm Vol_{CY_2}}}{16\pi^4}\right)\left(\frac{\rm Vol_\psi}{\pi}\right)\frac{h_4-h_4'(\rho-2\pi k)}{2} = \alpha_k,\\
Q^m_{\rm D7} &= \int_{\rm S_\psi}\widehat{F}_1 = \int_{\rm S_\psi\times I_\rho}{\rm d}\widehat{F}_1 = {\rm Vol}_\psi\int h_8''{\rm d}\rho = (\nu_{k-1}-\nu_k).
\end{aligned}
\tag{2.21}
$$

Further, in the brane set-up the F1-strings are electrically charged with respect to the NS-NS 3-form $H_3$ while the NS5 branes are magnetically charged,

$$
\begin{aligned}
Q^e_{\rm F1} &= \frac{1}{(2\pi)^2}\int_{\rm AdS_2\times I_\rho}H_3 = \left(\frac{\rm Vol_{AdS_2}}{4\pi}\right)\left(\frac{1}{2\pi}\right)\int_{2\pi k}^{2\pi(k+1)}{\rm d}\rho = 1,\\
Q^m_{\rm NS5} &= \frac{1}{(2\pi)^2}\int_{\rm S^2\times S_\psi}H_3 = \left(\frac{\rm Vol_{S^2}}{4\pi}\right)\left(\frac{\rm Vol_\psi}{2\pi}\right) = 1.
\end{aligned}
\tag{2.22}
$$

## 2.2 Holographic central charge

To close this part of our study we compute the holographic central charge associated to our solutions. Being the field theory zero-dimensional, the previous quantity should be interpreted as the number of vacuum states in the dual superconformal quantum mechanics (see [1, 39] for a further discussion of the physical meaning of this quantity). We follow the prescription in [44, 45]. We get for the internal volume,

$$
V_{\rm int} = \int {\rm d}^8x\sqrt{e^{-4\phi}\det g_{8,ind}} = \frac{\rm Vol_{CY_2}Vol_{S^2}Vol_\psi}{4^2}\int_0^{2\pi(P+1)}(4\widehat{h}_4h_8-(u')^2)\,{\rm d}\rho\,, \tag{2.23}
$$

and, finally, for the central charge

$$
c_{\rm hol,1d} = \frac{3V_{\rm int}}{4\pi G_N} = \frac{3}{\pi}\int_0^{2\pi(P+1)}\left(h_4h_8-\frac{(u')^2}{4\Upsilon}\right){\rm d}\rho\,. \tag{2.24}
$$

We have used that $G_N = 8\pi^6$ and set units so that $\alpha' = g_s = 1$.

We would like to stress that in the usual calculations, such as the previous one, giving rise to the holographic central charge, only the NS-NS sector of the backgrounds needs to be taken into account. We will point out an interesting relation between the holographic central charge and the RR sector of our AdS$_2$ solutions in section 5. Such relation has been previously encountered in the AdS$_2$ solutions constructed in [1, 39].

## 2.3 Aspects of the dual Conformal Quantum Mechanics

Whilst the main focus of this work is not the Quantum Mechanical analysis of the duals to the backgrounds in eqs. (2.1)–(2.2), we add below some thoughts along this direction.

In the papers [1, 24, 25, 39] concrete quivers were proposed as UV-descriptions of weakly coupled 2d QFTs or 1d Quantum Mechanics. It was conjectured that these quivers



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become strongly coupled at low energies and a conformal fixed point arises. Checks for these proposals were presented in each of the works [1, 24, 25, 39], for the different systems under study. These checks deal with RG-invariant quantities that can be well-identified in the UV and IR descriptions.

As we indicated around eq. (2.11) and summarised in figure 1, the backgrounds of section 2 arise after an Abelian T-duality on the backgrounds of [39]. This suggests that the quantum mechanical system proposed in [39] should also apply here. We are in fact T-dualising across a non-R-symmetry-direction, hence we expect the amount of SUSY to be the same. The R-symmetry of the quivers in [39] is $\mathrm{SU}(2)_R$, and there is also a global $\mathrm{SU}(2)_g$ symmetry. We are choosing a $\mathrm{U}(1)_g$ inside $\mathrm{SU}(2)_g$ for our dualisation. Therefore, our dual quantum mechanical system should have $\mathrm{SU}(2)_R \times \mathrm{U}(1)_g$ symmetry. This is in fact geometrically realised by the presence of the round $\mathrm{S}^2$ and the circle $\mathrm{S}^1_\psi$ in the backgrounds of section 2.

Since the string sigma model in a background and in its T-dual is the same, we expect the same dual quantum mechanical systems for our backgrounds as those for the backgrounds [39] (only that perhaps it will be written in a different language).

Using this reasoning, we may think about the SCQM as that arising in the very low energy limit of a system of D3-D7 branes — dual to a four dimensional $\mathcal{N} = 2$ QFT. This system is 'polluted' by one-dimensional defects. These are Wilson loops (arising from F1-D5) and 't Hooft loops (arising from NS5-D1) added to the background, see for example [46]. Note that both the D1's and the D5's extend on the $\psi$-isometric direction. From the discussion above, it follows that the dual SCQM to our backgrounds is the description of these one-dimensional defects inside a four dimensional $\mathcal{N} = 2$ QFT. In fact, in the IR the gauge symmetry on both D7 and D3 branes should become global. This implies that these branes must be sources/flavours, as it occurs in the backgrounds of section 2. By the same token we have two lines of one dimensional gauge groups: $\Pi_{i=1}^P \mathrm{U}(\alpha_i)$ and $\Pi_{i=1}^P \mathrm{U}(\mu_i)$ realised on D5 and D1 branes in each $\rho$-interval. This is reflected by the counting of branes of eq. (2.21). The nodes in the $[2\pi k, 2\pi(k+1)]$ interval will have $\mathrm{SU}(\beta_k - \beta_{k+1})$ and $\mathrm{SU}(\nu_k - \nu_{k+1})$ flavour groups, realised on the D3 and D7 branes, as also reflected by eq. (2.21). The brane set-up is the one described in table 1.

As was found in [39], our 1d system should also have Wilson lines (in an antisymmetric representation) inserted in the different gauge nodes of the quiver. These Wilson lines arise from the massive fermionic strings that stretch between D1s in the $k$-th interval and D7s in all other intervals. The Wilson lines would be in the $(\nu_0, \ldots, \nu_{k-1})$ antisymmetric representation of the $\mathrm{U}(\mu_k)$ gauge group. The same applies to the massive D3-D5 fermionic strings and the antisymmetric Wilson lines on the $\mathrm{U}(\alpha_k)$ groups. As in [39], this information can be encoded in Young diagrams.

We would also have a dynamical CS-term of each gauge group. This comes from the massless fermionic strings stretched between D1-D7 and D5-D3 branes. The coefficient can be extracted studying the WZ action for a D1 along $[t, \psi]$ and a D5 along $[t, \mathrm{CY}_2, \psi]$. As expected, these coefficients are quantised.

The field content of the UV-quantum mechanical quiver follows directly from the analysis of Appendix B in [39]. In fact, each node contains a $(4, 4)$ vector multiplet and a $(4, 4)$



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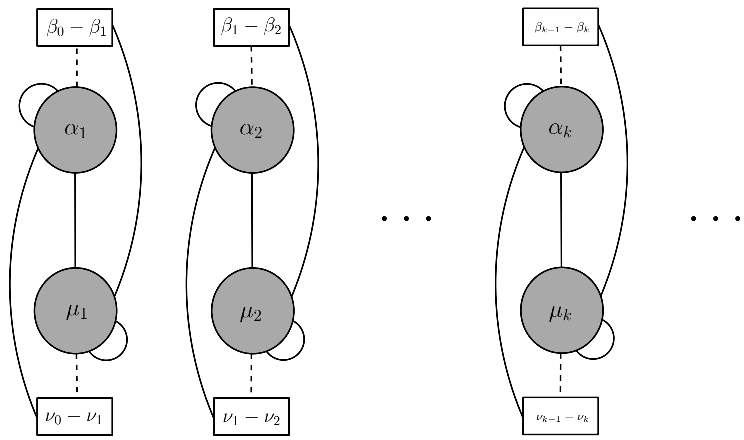


**Figure 2**. The proposed quantum mechanical quiver. This follows from the analysis of open strings in the Hanany-Witten set-up.

adjoint hyper, $(0,4)$ bifundamental hypers join the two types of colour, D5 and D1, branes, (4,4) twisted bifundamental hypers join the D7 sources with D5-branes, and the D3 sources with D1 branes, respectively. Finally, $(0,2)$ Fermi multiplets join source D7 with colour D1s and source D3 with colour D5 branes. The quiver diagram is depicted in figure 2.

## 3 Connection with the $\mathbf{AdS_2 \times S^2 \times CY_2 \times \Sigma}$ backgrounds of Chiodaroli-Gutperle-Krym

In this section we relate our backgrounds to the general class of $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \Sigma$ solutions to Type IIB supergravity with 8 supercharges found by Chiodaroli, Gutperle and Krym (CGK) in [34]. We show that our solutions fit locally in their classification in the absence of D3 and D7 brane sources (in this sense our backgrounds extend those in [34] at the $\mathrm{AdS}_2$ point). A similar analysis shows that the family of $\mathrm{AdS}_2$ solutions to Type IIB supergravity recently found in [1] also fits in their general class.

### 3.1 Review of the CGK geometries

The CGK backgrounds are dual to one dimensional conformal interfaces inside the two dimensional CFT associated to the D1-D5 system. These solutions (unlike ours) interpolate between $\mathrm{AdS}_2$ in the IR (at the interface) and the $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{CY}_2$ solution of Type IIB supergravity in the UV. We shall focus on the $\mathrm{AdS}_2$ fixed points and compare them with both the backgrounds discussed in section 2 and the solutions found in [1].

In [34], the authors used techniques developed in [19, 47] to find half BPS solutions that preserve eight of the sixteen supersymmetries of the $\mathrm{AdS}_3{\times}\mathrm{S}^3{\times}\mathrm{CY}_2$ vacuum, and are locally asymptotic to this vacuum solution. They provided an ansatz for the bosonic fields in Type IIB supergravity for a foliation of $\mathrm{AdS}_2{\times}\mathrm{S}^2{\times}\mathrm{CY}_2$ over a two-dimensional Riemann surface $\Sigma$ with a boundary, and found that the local solutions of the BPS equations can be written in terms of two harmonic and two holomorphic functions defined on $\Sigma$. The solutions corresponds to a D1-D5 configuration with extra NS5 branes and fundamental strings, but vanishing D3 and D7 brane charges. We will see that our solutions fit locally within this class of solutions in the absence of D3 and D7 brane sources. Our D3 and D7 sources are localised in $\rho$ and smeared in the $\psi$-coordinate. The mapping explained below is valid at points in $\rho$ where the sources are not present.

We start summarising the local solutions constructed in [34]. The metric for the ten-dimensional spacetime is given, in Einstein frame, by

$$\mathrm{d}s^2 = f_1^2 \mathrm{d}s^2_{\mathrm{AdS}_2} + f_2^2 \mathrm{d}s^2_{\mathrm{S}^2} + f_3^2 \mathrm{d}s^2_{\mathrm{CY}_2} + \widetilde{\rho}^2 \mathrm{d}z \mathrm{d}\overline{z}\,, \tag{3.1}$$

where the warping factors $f_i$ $(i = 1, \ldots, 3)$ and $\widetilde{\rho}$ are functions of $z$ and $\overline{z}$, the local holomorphic coordinates of $\Sigma$. The orthonormal frames can be written as,

$$\begin{aligned}
f_1^2 \mathrm{d}s^2_{\mathrm{AdS}_2} &= \eta_{i_1 i_2} e^{i_1} \otimes e^{i_2}\,, && i_{1,2} = 0,1\,,\\
f_2^2 \mathrm{d}s^2_{\mathrm{S}^2} &= \delta_{j_1 j_2} e^{j_1} \otimes e^{j_2}\,, && j_{1,2} = 2,3\,,\\
f_3^2 \mathrm{d}s^2_{\mathrm{CY}_2} &= \delta_{k_1 k_2} e^{k_1} \otimes e^{k_2}\,, && k_{1,2} = 4,5,6,7\,,\\
\widetilde{\rho}^2 \mathrm{d}z \mathrm{d}\overline{z} &= \delta_{ab} e^{a} \otimes e^{b}\,, && a,b = 8,9\,.
\end{aligned} \tag{3.2}$$

The NS-NS and RR three-forms are written as a complex three-form, defined as $G = e^{\Phi} H_3 + i e^{-\Phi}(F_3 - \chi\, H_3)$. This form is given by,

$$G = g_a^{(1)} e^{a01} + g_a^{(2)} e^{a23}\,. \tag{3.3}$$

In turn, the self-dual five-form flux is,

$$F_5 = h_a e^{a0123} + \widetilde{h}_a e^{a4567}\,, \quad a = z, \overline{z}\,, \tag{3.4}$$

where the self-duality condition implies $h_a = -\epsilon_a{}^{b} \widetilde{h}_b$.

The local solutions of the BPS equations and Bianchi identities admit a description in terms of four functions, $A$, $B$, $H$ and $K$. The analysis in [34] shows that the functions $A$ and $B$ must be holomorphic on the Riemann surface $\Sigma$, whilst $H$ and $K$ must be harmonic. The supergravity fields can be written in terms of these functions as,

$$\begin{aligned}
f_1^2 &= \frac{e^{-2\Phi}|H|}{2 f_3^2 K}\left((A+\overline{A})K - (B-\overline{B})^2\right), \qquad f_2^2 = \frac{e^{-2\Phi}|H|}{2 f_3^2 K}\left((A+\overline{A})K - (B+\overline{B})^2\right),\\
f_3^4 &= 4\frac{e^{2\Phi}K}{A+\overline{A}}\,, \qquad e^{4\Phi} = \frac{1}{4K^2}\left((A+\overline{A})K - (B+\overline{B})^2\right)\left((A+\overline{A})K - (B-\overline{B})^2\right),\\
\chi &= \frac{1}{2iK}\left(B^2 - \overline{B}^2 - (A-\overline{A})K\right), \qquad \widetilde{\rho}^4 = e^{2\Phi} K \frac{(A+\overline{A})}{H^2}\frac{|\partial_z H|^4}{|B|^4}\,.
\end{aligned} \tag{3.5}$$

Here $\Phi = -\phi/2$, where $\phi$ is the dilaton. For the five-form field strength, we define a four-form potential, along $\mathrm{CY}_2$,

$$C_{\mathrm{CY}_2} = -\frac{i}{2}\frac{B^2 - \overline{B}^2}{A+\overline{A}} - 2\widetilde{K}, \qquad \partial_z C_{\mathrm{CY}_2} = f_3^4 \widetilde{\rho}\, \widetilde{h}_z\,, \tag{3.6}$$

where $\widetilde{K}$ is the harmonic function conjugate[1] to $K$.

[1]The harmonic conjugate of $g$ is denoted as $\widetilde{g}$ and satisfies $i\partial_z \widetilde{g} = \partial_z g$.

The potentials for the field strengths in equation (3.3) are written in terms of the holomorphic and harmonic functions as

$$
\begin{aligned}
b^{(1)} &= -\frac{H(B+\overline{B})}{(A+\overline{A})K-(B+\overline{B})^2} - h_1, & h_1 &= \frac{1}{2}\int \frac{\partial_z H}{B} + \text{c.c.}\,,\\
b^{(2)} &= -i\frac{H(B-\overline{B})}{(A+\overline{A})K-(B-\overline{B})^2} + \widetilde{h}_1, & \widetilde{h}_1 &= -\frac{i}{2}\int \frac{\partial_z H}{B} + \text{c.c.}\,,\\
c^{(1)} &= -i\frac{H(A\overline{B}-\overline{A}B)}{(A+\overline{A})K-(B+\overline{B})^2} + \widetilde{h}_2, & \widetilde{h}_2 &= -\frac{i}{2}\int \frac{A\,\partial_z H}{B} + \text{c.c.}\,,\\
c^{(2)} &= -\frac{H(A\overline{B}+\overline{A}B)}{(A+\overline{A})K-(B-\overline{B})^2} + h_2, & h_2 &= \frac{1}{2}\int \frac{A\,\partial_z H}{B} + \text{c.c.},
\end{aligned}
\tag{3.7}
$$

where $\widetilde{h}_i$ and $h_i$ are harmonic functions conjugate to each other. In the previous expression $b^{(1)}$ and $b^{(2)}$ are the potentials of the NS-NS three-form $H_3$ and $c^{(1)}$ and $c^{(2)}$ are the potentials related to the RR three-form $F_3$. These read,

$$
\begin{aligned}
H_3 &= \mathrm{d}b^{(1)} \wedge \mathrm{vol}_{\mathrm{AdS}_2} + \mathrm{d}b^{(2)} \wedge \mathrm{vol}_{\mathrm{S}^2}\\
F_3 &= \mathrm{d}C_2 - \chi H_3 = (\mathrm{d}c^{(1)} - \chi \mathrm{d}b^{(1)}) \wedge \mathrm{vol}_{\mathrm{AdS}_2} + (\mathrm{d}c^{(2)} - \chi \mathrm{d}b^{(2)}) \wedge \mathrm{vol}_{\mathrm{S}^2}.
\end{aligned}
\tag{3.8}
$$

The existence of sensible regular solutions imposes the following conditions on the functions $A$, $B$, $H$ and $K$,

- The harmonic functions $A+\overline{A}, B+\overline{B}$ and $K$ must have common singularities.
- No singular points should appear in the bulk of the Riemann surface $\Sigma$.
- The functions $A+\overline{A}, K$ and $H$ cannot have any zero in the bulk of the Riemann surface.
- The holomorphic functions $B$ and $\partial_z H$ must have common zeros.

The previous conditions guarantee a non-vanishing and finite everywhere $f_1$ (except at isolated singular points at the boundary), a finite $f_2$ in the interior of the Riemann surface and vanishing at the boundary, and, finally, finite and non-vanishing $f_3$ and $e^{2\Phi}$ functions everywhere on the Riemann surface, including the boundary.

The equations in (3.5) can be inverted to find $A$, $B$, $H$ and $K$ in terms of $f_i$ ($i = 1, \ldots, 3$), $\chi$ and $\Phi$. One finds two possibilities, that we will refer as the "plus and minus solutions",[2]

$$
\mathrm{Sol}_+ :\; H = f_1 f_2 f_3^2, \quad K_+ = \frac{f_1 f_3^4}{2f_2}, \quad A_+ = \frac{f_1}{f_2}e^{2\Phi} - i\chi, \quad B_+ = \frac{e^{\Phi} f_3^2}{2f_2}\sqrt{f_1^2 - f_2^2}\,, \tag{3.9}
$$

$$
\mathrm{Sol}_- :\; H = f_1 f_2 f_3^2, \quad K_- = \frac{f_2 f_3^4}{2f_1}, \quad A_- = \frac{f_2}{f_1}e^{2\Phi} - i\chi, \quad B_- = i\,\frac{e^{\Phi} f_3^2}{2f_1}\sqrt{f_1^2 - f_2^2}. \tag{3.10}
$$

[2]The "plus solution" corresponds to our $\mathrm{AdS}_2$ backgrounds and the " minus solutions" to the $\mathrm{AdS}_2$ geometries of [1]. Both these solutions are related through an analytical continuation, as explained around eq. (2.11).

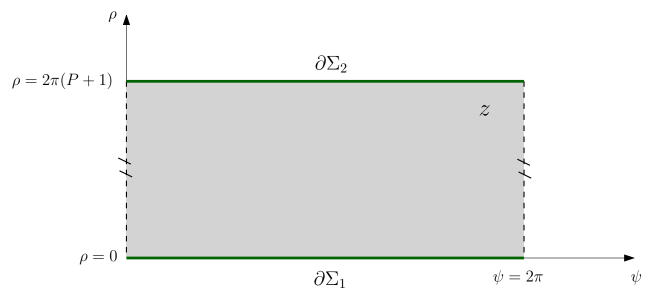


**Figure 3**. Riemann surface associated to our $\mathrm{AdS}_2$ geometries. Given the periodicity of $\psi$ it defines an annulus.

Inserting the "plus-solution", equation (3.9), or the "minus-solution", equation (3.10), in the first expression of (3.6) one obtains, in both cases, the function associated to the 4-form potential,

$$C_{\mathrm{CY}_2} = -2\widetilde{K}, \tag{3.11}$$

where $\widetilde{K}$ is the harmonic function conjugate to $K$, according the footnote 1.

In the next subsection we obtain the harmonic and holomorphic functions that give rise to our backgrounds in eqs. (2.1)–(2.2), as well as to the geometries in [1].

### 3.2 Our $\mathrm{AdS}_2$ geometries and the "plus-solution"

In order to compare the generic backgrounds given by eqs. (2.1)–(2.2) with the solutions in [34, 35] we express our solutions in Einstein frame, to agree with their conventions. We obtain,

$$\begin{aligned}
&f_1^2 = \frac{u}{\sqrt{2}}\left(\frac{\widehat{h}_4 h_8^3}{(4\widehat{h}_4 h_8 - (u')^2)^3}\right)^{1/4}, \qquad f_2^2 = \frac{u}{\sqrt{2^5}}\left(\frac{4\widehat{h}_4 h_8 - (u')^2}{\widehat{h}_4^3 h_8}\right)^{1/4},\\
&f_3^2 = \left(\frac{\widehat{h}_4(4\widehat{h}_4 h_8 - (u')^2)}{2^2 h_8}\right)^{1/4}, \qquad e^{2\Phi} = e^{-\phi} = \frac{1}{2}\sqrt{\frac{h_8}{\widehat{h}_4}}\sqrt{4\widehat{h}_4 h_8 - (u')^2},\\
&\chi = h_8'\psi, \qquad \widetilde{\rho}^2 = \frac{1}{\sqrt{2}u}\left(\widehat{h}_4 h_8^3(4\widehat{h}_4 h_8 - (u')^2)\right)^{1/4}, \qquad C_{\mathrm{CY}_2} = -h_4'\psi.
\end{aligned} \tag{3.12}$$

We emphasise that these expressions are valid at the points where $h_8'' = \widehat{h}_4'' = 0$.

We take the $\rho$ and $\psi$ coordinates to define the real and imaginary parts of the $z$ variable. With this parametrisation, $\Sigma$ is an annulus, defined in the complex plane (see figure 3),

$$z = \psi + i\rho \qquad \text{with} \qquad \psi \in [0, 2\pi] \qquad \text{and} \qquad \rho \in [0, 2\pi(P+1)]\,. \tag{3.13}$$

Locally, our solutions are defined by the three functions $u, \widehat{h}_4, h_8$, which must be linear in $\rho$. We take

$$u = u_0 + u_1\rho, \qquad h_8 = \mu + \nu\rho, \qquad \widehat{h}_4 = \alpha + \beta\rho. \tag{3.14}$$

Substituting (3.12) in (3.9) and taking into account (3.14), we find for the functions $A, B, H, K$,

$$A = h_8 - i\psi h_8' = \mu - i\nu z, \qquad B = \frac{u'}{4} = \frac{u_1}{4}\,,$$
$$H = \frac{u}{4} = \frac{u_0}{4} - i\frac{u_1}{8}(z-\overline{z})\,, \qquad K = \frac{\widehat{h}_4}{2} = \frac{\alpha}{2} - i\frac{\beta}{4}(z-\overline{z})\,. \tag{3.15}$$

It is easy to check that $H$, $K$, $A+\overline{A}$ and $B+\overline{B}$ are harmonic functions and $A$ and $B$ are holomorphic. The harmonic function $\widetilde{K}$ reads, in turn,

$$\widetilde{K} = -\frac{\widehat{h}_4'}{4}(z+\overline{z}) = -\frac{\beta}{4}(z+\overline{z})\,, \tag{3.16}$$

which is the harmonic function conjugate to the expression for $K$ in (3.15).

From the equations (3.7), and using (3.15), we can then obtain the harmonic functions and potentials associated with the NS-NS three-form,

$$h_1 = -\frac{i}{4}(z-\overline{z})\,, \qquad \widetilde{h}_1 = -\frac{1}{4}(z+\overline{z})\,,$$
$$b^{(1)} = \frac{u_1(2u_0 - iu_1(z-\overline{z}))}{u_1^2 + (2i\alpha + (z-\overline{z})\beta)(2i\mu + (z-\overline{z})\nu)} - h_1\,, \qquad b^{(2)} = -\frac{1}{4}(z+\overline{z})\,, \tag{3.17}$$

as well as those associated with the RR three-form,

$$h_2 = -\frac{\nu}{8}(z^2+\overline{z}^2) - \frac{i}{4}\mu(z-\overline{z})\,, \qquad \widetilde{h}_2 = i\frac{\nu}{8}(z^2-\overline{z}^2) - \frac{\mu}{4}(z+\overline{z})\,,$$
$$c^{(1)} = \frac{u_1(2u_0 - iu_1(z-\overline{z}))(z+\overline{z})\nu}{8(u_1^2 + (2i\alpha + (z-\overline{z})\beta)(2i\mu + (z-\overline{z})\nu))} + \widetilde{h}_2\,, \qquad c^{(2)} = -\frac{u_1(2iu_0 + u_1(z-\overline{z}))}{8(\beta(z-\overline{z}) + 2i\alpha)} + h_2\,. \tag{3.18}$$

From these expressions we can recover $H_3$ and $F_3$ as given in eqs. (2.1)–(2.2). Note that $h_i$ and $\widetilde{h}_i$ are harmonic functions conjugate to each other.

We have thus shown that our solutions can be obtained, locally, from the class of solutions constructed in [34]. Note that in our analysis we have implicitly assumed that $h_8'' = 0$ and $\widehat{h}_4'' = 0$ also hold globally. This is necessary in order to match the axion and the 4-form RR potential given in (3.12). This assumption — translated to our geometries — indicates that we are not allowing for D7 and D3 brane sources, according to equations (2.16)–(2.17). This agrees with the analysis in [34], which does not include either these types of branes.

We will show in subsection 3.4 that D3-brane sources can be included in the two boundaries of the annulus following the formalism for the annulus derived in [35]. This allows to recover the solutions in our class where D3-branes terminate the space at $\rho = 2\pi(P+1)$. Quite surprisingly, we will also see that, even if not included in the analysis in [35], D7-brane sources can also be allowed at the end of the space. We will show that they also manifest as (smeared) singularities of the basic harmonic function defined in the annulus in [35].

Before that, we show in the next subsection that the $\mathrm{AdS}_2$ geometries found in [1], that we will refer as LNRS geometries, fit as well in the CGK class.

### 3.3 The LNRS geometries and the "minus-solution"

As we already mentioned in section 2, our class of geometries can be obtained through a double analytic continuation from the $\mathrm{AdS}_2$ solutions studied in [1]. In this section we show that the latter fit within the class of solutions referred as "minus solutions" in [34].

The warping factors, dilaton, axion and RR 4-form potential associated to the $\mathrm{AdS}_2$ geometries constructed in [1] (in Einstein frame) are given by,

$$\begin{aligned} f_1^2 &= \frac{u}{\sqrt{2^5}}\left(\frac{4\widehat{h}_4 h_8 + (u')^2}{\widehat{h}_4^3 h_8}\right)^{1/4}, \qquad & f_2^2 &= \frac{u}{\sqrt{2}}\left(\frac{\widehat{h}_4 h_8^3}{(4\widehat{h}_4 h_8 + (u')^2)^3}\right)^{1/4}, \\ f_3^2 &= \frac{1}{\sqrt{2}}\left(\frac{\widehat{h}_4(4\widehat{h}_4 h_8 + (u')^2)}{h_8}\right)^{1/4}, \qquad & e^{2\Phi} &= e^{-\phi} = \frac{1}{2}\sqrt{\frac{h_8}{\widehat{h}_4}}\sqrt{4\widehat{h}_4 h_8 + (u')^2}, \\ \chi &= h_8'\psi, \qquad \widetilde{\rho}^2 = \frac{1}{\sqrt{2}u}\left(\widehat{h}_4 h_8^3(4\widehat{h}_4 h_8 + (u')^2)\right)^{1/4}, & C_{\mathrm{CY}_2} &= -h_4'\psi. \end{aligned} \tag{3.19}$$

The Riemann surface is the same one defined in equation (3.13) and figure 3, and, as in the previous subsection, we are also taking $h_8''=0$ and $\widehat{h}_4''=0$ globally, i.e. solutions without D7 and D3 brane sources. This is needed to obtain the axion and RR 4-form potential of the previous equations.

In this case the matching with the solutions in [34] is with the "minus-solutions" defined by equation (3.10). Taking into account (3.14), the harmonic and holomorphic functions read,

$$\begin{aligned} A &= h_8 - i\psi h_8' = \mu - i\,\nu z, \qquad & B &= i\,\frac{u'}{4} = i\,\frac{u_1}{4}, \\ H &= \frac{u}{4} = \frac{u_0}{4} - i\frac{u_1}{8}(z-\bar{z}), \qquad & K &= \frac{\widehat{h}_4}{2} = \frac{\alpha}{2} - i\,\frac{\beta}{4}(z-\bar{z}). \end{aligned} \tag{3.20}$$

As in the previous subsection, the functions $H$, $K$, $A+\overline{A}$ $B+\overline{B}$ are harmonic and $A$ and $B$ holomorphic. The harmonic function $\widetilde{K}$ reads exactly as in (3.16).

In turn, the harmonic functions that give rise to the NS-NS and RR three-forms read,

$$\begin{aligned} h_1 &= -\frac{1}{4}(z+\bar{z}), & \widetilde{h}_1 &= \frac{i}{4}(z-\bar{z}), \\ h_2 &= -\frac{\mu}{4}(z+\bar{z}) + i\frac{\nu}{8}(z^2-\bar{z}^2), & \widetilde{h}_2 &= i\frac{\mu}{4}(z-\bar{z}) + \frac{\nu}{8}(z^2+\bar{z}^2), \\ b^{(1)} &= \frac{1}{4}(z+\bar{z}), & b^{(2)} &= \frac{u_1(2u_0 - iu_1(z-\bar{z}))}{4(u_1^2 - (2i\mu+\nu(z-\bar{z}))(2i\alpha+\beta(z-\bar{z})))} + \widetilde{h}_1, \\ c^{(1)} &= -\frac{u_1(u_1(z-\bar{z})+2iu_0)}{8(2i\alpha+\beta(z-\bar{z}))} + \widetilde{h}_2, & c^{(2)} &= \frac{\nu u_1(2u_0 - iu_1(z-\bar{z}))(z+\bar{z})}{8(u_1^2 - (2i\mu+\nu(z-\bar{z}))(2i\alpha+\beta(z-\bar{z})))} + h_2. \end{aligned} \tag{3.21}$$

From these expressions we recover the NS-NS and RR field strengths, $H_3$ and $F_3$, of the solutions in [1].

### 3.4 The annulus

As we have already mentioned, the class of solutions constructed in [34] have vanishing D3 and D7-brane charges. Those solutions have a Riemann surface with a single boundary component. In the follow-up paper [35], the authors constructed solutions in which the Riemann surface $\Sigma$ has an arbitrary number of boundaries and non-vanishing D3 brane charges. The D3-branes occur as poles of a basic harmonic function at the boundaries. In this section we consider the simplest case of a Riemann surface with two disconnected boundary components, namely the annulus. We will then see in subsection 3.5 that we can recover the solutions with D3-brane sources at $\rho = 2\pi(P+1)$, the end of the space. Quite surprisingly, we will see that D7-branes seem also allowed at the end of the space.

The annulus is defined as,

$$\Sigma \equiv \left\{ w \in C, 0 \leq \mathrm{Re}(w) \leq 1, 0 \leq \mathrm{Im}(w) \leq \frac{t}{2} \right\}, \tag{3.22}$$

with $t \in \mathbb{R}^+$. The points $w+1$ and $w$ are identified, thus giving the topology of an annulus. Its two boundaries, $\partial\Sigma_{1,2}$, are located at $\mathrm{Im}(w) = 0$ and $\mathrm{Im}(w) = \frac{t}{2}$. The annulus can be constructed from a double surface $\widehat{\Sigma}$, which is defined as a rectangular torus with periods 1 and $\tau$, where $\tau$ is a purely imaginary parameter, $\tau = it$. The original surface $\Sigma$ is obtained as the quotient $\Sigma = \widehat{\Sigma}/\mathcal{J}$ where $\mathcal{J}(z) = \overline{z}$.

The construction of the solutions for the annulus in [35] proceeds in three steps. First, a basic harmonic function with singularities and suitable boundary conditions is constructed. Second, the harmonic functions, $A + \overline{A}$, $H$ and $K$, are expressed as linear superpositions of the basic harmonic function, evaluated at the various poles in the two boundaries. Finally, the meromorphic function $B$ is constructed such that it satisfies certain regularity conditions. Some of these conditions come from imposing that the solutions asymptote locally to the $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{CY}_2$ background. These regularity conditions will not be satisfied by our solutions, first because they do not asymptote to this geometry and, second, because the D3-branes (also the D7-branes) are smeared in the $\psi$ direction. This introduces significant changes in the regularity analysis. For this reason we will not give a detailed account of the regularity conditions imposed in [35]. We will see however in the next subsection that our solutions can still be recovered from the general formalism in [35] in an appropriate limit.

The construction in [35] of the basic harmonic function is carried out in terms of elliptic functions and their related Jacobi theta function of the first kind,

$$\theta_1(w|\tau) = 2\sum_{n=0}^{\infty} (-1)^n e^{i\pi\tau(n+\frac{1}{2})^2} \sin[(2n+1)w], \tag{3.23}$$

as follows,

$$h_0(w, \overline{w}) = i\left(\frac{\partial_w \theta_1(\pi w|\tau)}{\theta_1(\pi w|\tau)} + \frac{2\pi i w}{\tau}\right) + \mathrm{c.c.} \tag{3.24}$$

This function has the following simple properties: it has a single simple pole on $\partial\Sigma$, it satisfies Dirichlet conditions away from the pole, and it is positive in the interior of $\Sigma$.

Notice that $h_0(w, \overline{w})$ has a singularity at $w = 0$, on the first boundary. This pole can be shifted to any point on $\partial\Sigma_1$ by a real translation, so that $h_0(w - x, \overline{w} - x)$ has a singularity at $w = x$. Instead, to obtain the harmonic functions with singularities at $\partial\Sigma_2$ one needs to define,

$$w' \equiv \frac{\tau}{2} - w. \tag{3.25}$$

Then the function $h_0(w'+y,\overline{w}'+y)$ has a pole at $w'=-y$, on the second boundary, for a real $y$. In other words the pole is localised at $w=y+it/2$.

In the annulus the harmonic functions $A+\overline{A}$, $B+\overline{B}$, $H$ and $K$ are expressed as linear combinations of $h_0$ harmonic functions with poles on both boundaries,

$$\begin{aligned}
A+\overline{A} &= \sum_{\ell_A=1}^{M_A} r_{\ell_A} h_0(w-x_{\ell_A},\overline{w}-x_{\ell_A}) + \sum_{j_A=1}^{M'_A} r'_{j_A} h_0(w'+y_{j_A},\overline{w}'+y_{j_A}),\\
B+\overline{B} &= \sum_{\ell_B=1}^{M_B} r_{\ell_B} h_0(w-x_{\ell_B},\overline{w}-x_{\ell_B}) + \sum_{j_B=1}^{M'_B} r'_{j_B} h_0(w'+y_{j_B},\overline{w}'+y_{j_B}),\\
H &= \sum_{\ell_H=1}^{M_H} r_{\ell_H} h_0(w-x_{\ell_H},\overline{w}-x_{\ell_H}) + \sum_{j_H=1}^{M'_H} r'_{j_H} h_0(w'+y_{j_H},\overline{w}'+y_{j_H}),\\
K &= \sum_{\ell_K=1}^{M_K} r_{\ell_K} h_0(w-x_{\ell_K},\overline{w}-x_{\ell_K}) + \sum_{j_K=1}^{M'_K} r'_{j_K} h_0(w'+y_{j_K},\overline{w}'+y_{j_K}).
\end{aligned} \tag{3.26}$$

Each harmonic function is taken to have $M_i$ poles $x_{\ell_i}$ with $\ell_i=1,\dots,M_i$ on $\partial\Sigma_1$, and $M'_i$ poles $y_{j_i}$ with $j_i=1,\dots,M'_i$ on $\partial\Sigma_2$. The corresponding residues are $r_{\ell_i}$ and $r_{j_i}$.

In addition to the regularity conditions given in subsection 3.1, the harmonic functions (3.26) satisfy an extra condition coming from the requirement that $e^{4\Phi}>0$. Namely, $(A+\overline{A})K-(B+\overline{B})^2>0$ must be obeyed throughout $\Sigma$. Furthermore, in this language the first regularity condition can be written in terms of the residues as $r_A r_K = r_B^2$.

### 3.5 Zoom-in to our solutions

In this subsection we show that it is possible to recover well-defined global solutions with source branes at the ends of the space from the general analysis above for the annulus. These solutions do not satisfy most of the regularity conditions imposed in [34, 35], and, moreover, contain not only D3 but also D7-brane sources at the ends of the space. Still, we will be able to recover them in a particular limit from the formalism in [35].

As we have already stressed, the choice of constants in the general $\widehat{h}_4$ and $h_8$ functions defined by equations (2.6) and (2.7) allows for discontinuities in the RR sector of our backgrounds at each $\rho=2\pi k$ value, with $k=1,\dots,P$. The discontinuities in $\widehat{h}'_4$ are interpreted as generated by D3-brane sources, while the discontinuities in $h'_8$ are interpreted as generated by D7-branes. Both types of branes are smeared in the $\psi$ direction. The space is terminated in the $\rho$ direction by imposing that $\widehat{h}_4=h_8=0$ at $\rho=0,2\pi(P+1)$. When $u=$ constant the closure of the space by setting $\widehat{h}_4=h_8=0$ generates D3 and D7 sources, in the boundary of the space, as explained around eq. (2.10).

Instead, in the general discussion for the annulus in [35] the D3-branes occur as poles of a basic harmonic function at its two boundaries. The basic harmonic function must however be regular in the interior. Therefore, in order to fit in the discussion for the annulus we need continuous $\widehat{h}'_4$ and $h'_8$ functions. This is imposed taking

$$\beta_k \equiv \beta, \qquad \nu_k \equiv \nu, \qquad \text{for} \quad k=0,1,\dots,P, \tag{3.27}$$



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in (2.6), (2.7), which implies

$$\alpha_k = k\,\beta, \qquad \mu_k = k\,\nu, \qquad \text{for} \quad k = 0, \dots, P. \tag{3.28}$$

The solutions are then defined by the functions

$$\widehat{h}_4 = \frac{\beta}{2\pi}\,\rho, \qquad h_8 = \frac{\nu}{2\pi}\,\rho \tag{3.29}$$

at all $\rho$-intervals. Yet, the closure of the space at $\rho = 2\pi(P+1)$ requires that $(P+1)\beta$ D3-branes and $(P+1)\nu$ D7-branes are present at the end of the space. Instead of closing the space by introducing sources as we did with the choice of $\widehat{h}_4$ and $h_8$ functions given by (2.6) and (2.7), these branes will be automatically present at the end of the space in the annulus construction.

Let us now see how these solutions arise from the general formalism in [35]. We take the annulus in (3.22) as defined from,

$$w = \frac{z}{2\pi} = \widetilde{\psi} + i\widetilde{\rho}, \qquad \text{with} \quad \widetilde{\psi} = \frac{\psi}{2\pi}, \quad \widetilde{\rho} = \frac{\rho}{2\pi}. \tag{3.30}$$

Then $\widetilde{\psi} \in [0,1]$ and the parameter $t$ in the definition of the annulus is $t = 2(P+1)$. As recalled in section 2, our class of solutions is valid when $P$ is large. This allows us to approximate the Jacobi theta function introduced in (3.23) by its asymptotic expansion when $t \to \infty$,

$$\theta_1(\pi w|\tau)|_{t\to\infty} \approx 2e^{-\frac{\pi}{4}t}\sin\pi w \approx ie^{-\frac{\pi}{4}t}e^{-i\pi w}. \tag{3.31}$$

This approximation will be key in showing the matching with our solutions. Indeed, in this approximation it is easy to see that the basic harmonic function defined by (3.24) reads,

$$h_0(w,\overline{w}) \approx 2\pi + \frac{i\pi}{P+1}(w-\overline{w}). \tag{3.32}$$

This gives, at the two boundaries $\partial\Sigma_1$ and $\partial\Sigma_2$,

$$\begin{aligned} h_0(w - x_{\ell_i}, \overline{w} - x_{\ell_i}) &\approx 2\pi + \frac{i\pi}{P+1}(w-\overline{w}), \\ h_0(w' + y_{j_i}, \overline{w}' + y_{j_i}) &\approx -\frac{i\pi}{P+1}(w-\overline{w}), \end{aligned} \tag{3.33}$$

respectively, where for the second boundary we have used the relation (3.25). These expressions are thus independent of the positions of the poles at both boundaries. This is in agreement with the fact that our D3/D7 branes are smeared in the $\psi$-direction. We then get for the harmonic functions in eq. (3.26),

$$\begin{aligned}
A+\overline{A} &= 2\pi \sum_{\ell_A=1}^{M_A} r_{\ell_A} - \frac{i\pi}{P+1}(w-\overline{w})\left(\sum_{j_A=1}^{M'_A} r'_{j_A} - \sum_{\ell_A=1}^{M_A} r_{\ell_A}\right), \\
B+\overline{B} &= 2\pi \sum_{\ell_B=1}^{M_B} r_{\ell_B} - \frac{i\pi}{P+1}(w-\overline{w})\left(\sum_{j_B=1}^{M'_B} r'_{j_B} - \sum_{\ell_B=1}^{M_B} r_{\ell_B}\right), \\
H &= 2\pi \sum_{\ell_H=1}^{M_H} r_{\ell_H} - \frac{i\pi}{P+1}(w-\overline{w})\left(\sum_{j_H=1}^{M'_H} r'_{j_H} - \sum_{\ell_H=1}^{M_H} r_{\ell_H}\right), \\
K &= 2\pi \sum_{\ell_K=1}^{M_K} r_{\ell_K} - \frac{i\pi}{P+1}(w-\overline{w})\left(\sum_{j_K=1}^{M'_K} r'_{j_K} - \sum_{\ell_K=1}^{M_K} r_{\ell_K}\right).
\end{aligned} \tag{3.34}$$

In order to match these expressions with the expressions for $A+\overline{A}$ and $K$ given in eq. (3.15) we take into account that $w = z/(2\pi)$, and we obtain,

$$\sum_{\ell_A=1}^{M_A} r_{\ell_A} = 0 \qquad \text{and} \qquad \sum_{j_A=1}^{M'_A} r'_{j_A} = \frac{(P+1)\nu}{4\pi}, \tag{3.35}$$

for the matching of $A+\overline{A}$, and

$$\sum_{\ell_K=1}^{M_K} r_{\ell_K} = 0 \qquad \text{and} \qquad \sum_{j_K=1}^{M'_K} r'_{j_K} = \frac{(P+1)\beta}{\pi}, \tag{3.36}$$

for the matching of $K$. Rescaling the residues as[3]

$$r'_{j_A} \to 2r'_{j_A}, \qquad r'_{j_K} \to \frac{r'_{j_K}}{4}. \tag{3.37}$$

and replacing the sums by

$$\sum_{j_A=1}^{M'_A} r'_{j_A} \to \frac{1}{2\pi}\int_0^{2\pi} dr'_{j_A}, \tag{3.38}$$

as implied by the smearing of the branes in the $\psi$-direction, we can finally interpret the residues as the charge-densities of D7 and D3 brane sources at both boundaries of the annulus. We would like to stress that even if the general formalism in [35] does not account for D7-branes at the boundaries of the annulus, we have associated these to the (smeared) poles of the basic harmonic function for $A+\overline{A}$. The analysis goes in complete parallelism to the analysis of the residues and poles of the $K$ function, associated to the D3-brane sources at both boundaries of the annulus. It is unclear to us the precise reason why this seems to work in the presence of D7-branes.

[3] Note that a rescaling is also necessary in order to interpret the residues of the solutions in [48] as charges of $(p,q)$ 5-branes.

Finally, from the matching of the $B+\overline{B}$ and $H$ functions we find

$$\sum_{\ell_B=1}^{M_B} r_{\ell_B} = \sum_{j_B=1}^{M'_B} r'_{j_B} = 0 \qquad \text{and} \qquad \sum_{\ell_H=1}^{M_H} r_{\ell_H} = \sum_{j_H=1}^{M'_H} r'_{j_H} = \frac{u_0}{8\pi}. \tag{3.39}$$

These expressions do not seem to have however a direct interpretation in terms of charges of our solutions.

The previous analysis holds true as well for the LNRS backgrounds discussed in [1]. The matching of the $A+\overline{A}$, $H$ and $K$ functions is valid for both solutions, while the harmonic function $B+\overline{B}$ vanishes. Again, there are smeared D3 and D7-branes at the end of the space with the same relations between residues and charges.

## 4 A new class of $\mathbf{AdS_2\times S^2\times CY_2\times \Sigma}$ solutions with $\mathbf{\Sigma}$ an infinite strip

In this section we construct a new class of $\mathrm{AdS}_2$ solutions to Type IIB supergravity with 8 supercharges by acting with non-Abelian T-duality (NATD) on the $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{CY}_2\times\mathrm{I}_\rho$ solutions obtained in [23]. The non-Abelian T-duality transformation is performed with respect to a freely acting $\mathrm{SL}(2,\mathbb{R})$ isometry group of the $\mathrm{AdS}_3$ subspace. This transformation gives rise to a new class of solutions in which the $\mathrm{AdS}_3$ subspace is replaced by $\mathrm{AdS}_2$ times an interval. These solutions fit in the classification of [34] for a Riemann surface with a single boundary, equivalent to an infinite strip.

### 4.1 NATD of the $\mathbf{AdS_3\times S^2\times CY_2}$ solutions

The study of NATD as a solution generating technique of supergravity was initiated in [49]. Further works include [50–53]. In all these examples the dualisation took place with respect to a freely acting SU(2) subgroup of the entire symmetry group of the solutions. Instead, in this section we perform the non-Abelian T-duality transformation with respect to one of the freely acting $\mathrm{SL}(2,\mathbb{R})$ isometry groups of the $\mathrm{AdS}_3$ subspace.

In order to perform the dualisation with respect to the $\mathrm{SL}(2,\mathbb{R})$ isometry group we follow the derivation in [54]. We take the $sl(2,\mathbb{R})$ generators analytically continuing the $su(2)$ generators, as $t^a=\tau_a/\sqrt{2}$, with

$$\tau_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \qquad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \tag{4.1}$$

These generators satisfy,

$$\mathrm{Tr}(t^a t^b) = (-1)^a \delta^{ab}, \qquad [t^1, t^2] = i\sqrt{2}t^3, \qquad [t^2, t^3] = i\sqrt{2}t^1, \qquad [t^3, t^1] = -i\sqrt{2}t^2. \tag{4.2}$$

A group element in the Euler parametrisation is given by,

$$g = e^{\frac{i}{2}\phi\tau_3} e^{\frac{i}{2}\theta\tau_2} e^{\frac{i}{2}\psi\tau_3}, \qquad \text{where} \qquad 0 \le \theta \le \pi, \;\; 0 \le \psi < \infty, \;\; 0 \le \phi < \infty, \tag{4.3}$$

from which we write the left invariant one forms, $L^a = -i\mathrm{Tr}(t^a g^{-1} \mathrm{d}g)$, in the following fashion,

$$\begin{aligned} L^1 &= \sinh\psi \mathrm{d}\theta - \cosh\psi \sin\theta \mathrm{d}\phi \\ L^2 &= \cosh\psi \mathrm{d}\theta - \sinh\psi \sin\theta \mathrm{d}\phi \\ L^3 &= -\cos\theta \mathrm{d}\phi - \mathrm{d}\psi. \end{aligned} \tag{4.4}$$

The backgrounds in [23] support an $\mathrm{SL}(2,\mathbb{R})$ isometry such that the metric, the Kalb-Ramond field and the dilaton can be written as,[4]

$$ds^2 = \frac{1}{4} g_{\mu\nu}(x) L^\mu L^\nu + G_{ij}(x) dx^i dx^j, \qquad B_2 = B_{ij}(x) dx^i \wedge dx^j, \qquad \phi = \phi(x), \tag{4.5}$$

where $x^i$ are the coordinates in the internal manifold, for $i, j = 1, 2, \ldots, 7$, and $L^\mu$ are the forms given by (4.4). All the coordinate dependence on the $\mathrm{SL}(2,\mathbb{R})$ group is contained in these forms. The subsequent details on how to technically compute the NATD transformation have been developed extensively in the literature [49, 53] (see these reference for more details).

The geometries obtained through NATD with respect to a freely acting $\mathrm{SL}(2,\mathbb{R})$ group on the $\mathrm{AdS}_3$ of the solutions in [23] are given by,

$$\begin{aligned} \mathrm{d}s^2_{st} &= \frac{u\sqrt{\widehat{h}_4 h_8}}{4r^2\widehat{h}_4 h_8 - u^2} r^2 \mathrm{d}s^2_{\mathrm{AdS}_2} + \sqrt{\frac{\widehat{h}_4}{h_8}} \mathrm{d}s^2_{\mathrm{CY}_2} + \frac{u\sqrt{\widehat{h}_4 h_8}}{4\widehat{h}_4 h_8 + u'^2} \mathrm{d}s^2_{\mathrm{S}^2} + \frac{\sqrt{\widehat{h}_4 h_8}}{u} (\mathrm{d}\rho^2 + \mathrm{d}r^2), \\ e^{-2\phi} &= \frac{\left(4r^2\widehat{h}_4 h_8 - u^2\right)\left(4\widehat{h}_4 h_8 + u'^2\right)}{2^8 \widehat{h}_4^2}, \\ B_2 &= -\frac{2r^3\widehat{h}_4 h_8}{4r^2\widehat{h}_4 h_8 - u^2} \mathrm{vol}_{\mathrm{AdS}_2} - \frac{4\rho\widehat{h}_4 h_8 - u'(u - \rho u')}{2\left(4\widehat{h}_4 h_8 + u'^2\right)} \mathrm{vol}_{\mathrm{S}^2}. \end{aligned} \tag{4.6}$$

[4] We have taken $g^{\mu\nu} = -\mathrm{Tr}(t^\mu t^\nu)$ to have signature $(+,-,+)$.

Additionally, the background is supported by the RR fluxes,

$$\begin{aligned}
F_1 &= -\frac{rh_8'}{4}\mathrm{d}r + \frac{1}{2^4}\left(4h_8 + \partial_\rho\left[\frac{uu'}{\widehat{h}_4}\right]\right)\mathrm{d}\rho,\\
F_3 &= \frac{h_8}{8\left(4\widehat{h}_4h_8+u'^2\right)}\left(\frac{\widehat{h}_4'u^2}{\widehat{h}_4}\mathrm{d}\rho + rh_8\left(4\widehat{h}_4+\partial_\rho\left[\frac{uu'}{h_8}\right]\right)\mathrm{d}r\right)\wedge\mathrm{vol}_{\mathrm{S}^2}\\
&\quad+\frac{r^2h_8}{8\left(4r^2\widehat{h}_4h_8-u^2\right)}\left(\frac{h_8'u^2}{h_8}\mathrm{d}r - r\widehat{h}_4\left(4h_8+\partial_\rho\left[\frac{uu'}{\widehat{h}_4}\right]\right)\mathrm{d}\rho\right)\wedge\mathrm{vol}_{\mathrm{AdS}_2},\\
F_5 &= \frac{1}{2^4}\left(4r\widehat{h}_4'\mathrm{d}r - \left(4\widehat{h}_4+\partial_\rho\left[\frac{uu'}{h_8}\right]\right)\mathrm{d}\rho\right)\wedge\mathrm{vol}_{\mathrm{CY}_2}\\
&\quad-\frac{r^2u^2h_8^2}{2^4\left(4r^2\widehat{h}_4h_8-u^2\right)\left(4\widehat{h}_4h_8+u'^2\right)}\left(4r\widehat{h}_4'\mathrm{d}\rho + \left(4\widehat{h}_4+\partial_\rho\left[\frac{uu'}{h_8}\right]\right)\mathrm{d}r\right)\wedge\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{S}^2},\\
F_7 &= -\frac{\widehat{h}_4}{8\left(4\widehat{h}_4h_8+u'^2\right)}\left(\frac{h_8'u^2}{h_8}\mathrm{d}\rho + r\widehat{h}_4\left(4h_8+\partial_\rho\left[\frac{uu'}{\widehat{h}_4}\right]\right)\mathrm{d}r\right)\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\\
&\quad-\frac{r^2\widehat{h}_4}{8\left(4r^2\widehat{h}_4h_8-u^2\right)}\left(\frac{\widehat{h}_4'u^2}{\widehat{h}_4}\mathrm{d}r - rh_8\left(4\widehat{h}_4+\partial_\rho\left[\frac{uu'}{h_8}\right]\right)\mathrm{d}\rho\right)\wedge\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2},\\
F_9 &= \frac{r^2u^2\widehat{h}_4^2}{4\left(4r^2\widehat{h}_4h_8-u^2\right)\left(4\widehat{h}_4h_8+u'^2\right)}\left(rh_8'\mathrm{d}\rho + \frac{1}{4}\left(4h_8+\partial_\rho\left[\frac{uu'}{\widehat{h}_4}\right]\right)\mathrm{d}r\right)\wedge\mathrm{vol}_{\mathrm{AdS}_2}\wedge\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}.
\end{aligned} \tag{4.7}$$

The previous background is a solution to the Type IIB supergravity EOM whenever $4r^2\widehat{h}_4h_8 - u^2 > 0$. Namely we get a well-defined geometry for

$$r > r_0 = \frac{u}{2\sqrt{\widehat{h}_4h_8}}. \tag{4.8}$$

In the next section we show that a subset of the solutions defined by (4.6) and (4.7) fit in the general classification of $\mathrm{AdS}_2\times\mathrm{S}^2\times\mathrm{CY}_2\times\Sigma$ geometries given in [34] with $\Sigma$ an infinite strip.

### 4.2 The NATD solution as a CGK geometry

In this section we discuss how the solutions given by (4.6)–(4.7) fit in the class of CGK. Going to Einstein frame we get the warp factors of the metric, dilaton and axion,

$$\begin{aligned}
f_1^2 &= \frac{ur^2\sqrt{h_8}(4\widehat{h}_4h_8+u'^2)^{1/4}}{4(4\widehat{h}_4h_8r^2-u^2)^{3/4}}, & f_2^2 &= \frac{u\sqrt{h_8}(4\widehat{h}_4h_8r^2-u^2)^{1/4}}{4(4\widehat{h}_4h_8+u'^2)^{3/4}},\\
f_3^2 &= \frac{(4\widehat{h}_4h_8r^2-u^2)^{1/4}(4\widehat{h}_4h_8+u'^2)^{1/4}}{4\sqrt{h_8}}, & e^{2\Phi} &= e^{-\phi} = \frac{\sqrt{(4\widehat{h}_4h_8r^2-u^2)(4\widehat{h}_4h_8+u'^2)}}{2^4\widehat{h}_4},\\
\chi &= \frac{1}{2^4}\left(2\nu(\rho^2-r^2)+4\mu\rho+\frac{uu'}{\widehat{h}_4}\right), & \widetilde{\rho}^2 &= \frac{\sqrt{h_8}(4\widehat{h}_4h_8r^2-u^2)^{1/4}(4\widehat{h}_4h_8+u'^2)^{1/4}}{2^2u}.
\end{aligned} \tag{4.9}$$


**Figure 4**. Infinite strip associated to the NATD solution.

In $\chi$, the axion field, we have taken $h_8 = \mu + \nu\rho$, with $\mu$, $\nu$ constants. This choice corresponds to backgrounds without D7-branes, as those constructed in [34].

The 2d Riemann surface associated to the solutions is the strip depicted in figure 4, parametrised as,

$$z = \rho + i\, r \qquad \text{where} \qquad \rho \in [0, 2\pi(P+1)] \qquad \text{and} \qquad r \in [r_0, \infty], \tag{4.10}$$

where the value of $r_0$ is determined in eq. (4.8).

Taking the 'plus-solution' defined by equation (3.9) we obtain the $A$, $B$, $H$ and $K$ functions in terms of the defining functions of our backgrounds, $\widehat{h}_4$, $h_8$ and $u$,

$$A = \frac{1}{2^4}\left(4\mu(r-i\rho)+2i\nu(r-i\rho)^2+\frac{u'}{\widehat{h}_4}(ru'-iu)\right), \qquad B = \frac{1}{2^5}\frac{\sqrt{4\widehat{h}_4 h_8+u'^2}\sqrt{u^2+r^2u'^2}}{\sqrt{\widehat{h}_4 h_8}},$$
$$H = \frac{ru}{2^4}, \qquad K = \frac{r(4\widehat{h}_4 h_8+u'^2)}{2^5 h_8}. \tag{4.11}$$

We anticipate these functions are neither harmonic nor holomorphic. In order to ensure harmonicity -in $H$ and $K$- and holomorphicity -in $A$ and $B$- we need to choose $u' = 0$. In that case we obtain,

$$A = \frac{4\mu(r-i\rho)+2i\nu(r-i\rho)^2}{2^4} = -iz\frac{2\mu+z\nu}{8}, \qquad B = \frac{u}{2^4} = \frac{u_0}{2^4},$$
$$H = \frac{ru}{2^4} = -i\frac{u_0}{2^5}(z-\overline{z}), \qquad K = \frac{r\widehat{h}_4}{2^3} = -i\frac{(z-\overline{z})}{2^5}(\beta(z+\overline{z})+2\alpha), \tag{4.12}$$

where we have used (3.14). The harmonic function conjugated to $K$ is,

$$\widetilde{K} = -\frac{1}{2^5}(\beta(z^2+\overline{z}^2)+2\alpha(z+\overline{z})), \qquad \text{with} \qquad C_{\mathrm{CY}_2} = \frac{1}{2^3}(\beta(r^2-\rho^2)+2\alpha\rho). \tag{4.13}$$

Note that we have taken $\widehat{h}_8 = \alpha + \beta\rho$, with $\alpha$, $\beta$ constants, which corresponds to backgrounds without D3-branes, as those constructed in [34].

The functions associated to the complex three-form are,

$$h_1 = -\frac{i}{4}(z-\overline{z}), \qquad \widetilde{h}_1 = -\frac{1}{4}(z+\overline{z}),$$
$$h_2 = -\frac{1}{2^5}\left(\mu(z^2+\overline{z}^2)+\frac{\nu}{3}(z^3+\overline{z}^3)\right), \qquad \widetilde{h}_2 = \frac{i}{2^5}\left(\mu(z^2-\overline{z}^2)+\frac{\nu}{3}(z^3-\overline{z}^3)\right). \tag{4.14}$$

Notice that $h_i$ and $\widetilde{h}_i$ are harmonic functions conjugate to each other. The potentials given in (3.7) are,

$$
\begin{aligned}
b^{(1)} &= -i\frac{u_0^2(z-\overline{z})}{(z-\overline{z})^2(\beta(z+\overline{z})+2\alpha)(\nu(z+\overline{z})+2\mu)+4u_0^2} - h_1, \qquad b^{(2)} = -\frac{1}{4}(z+\overline{z}),\\
c^{(1)} &= -i\frac{u_0^2(z-\overline{z})(2\mu(z+\overline{z})+\nu(z^2+\overline{z}^2))}{16((z-\overline{z})^2(\beta(z+\overline{z})+2\alpha)(\nu(z+\overline{z})+2\mu)+4u_0^2)} + \widetilde{h}_2,\\
c^{(2)} &= -\frac{u_0^2}{16(\beta(z+\overline{z})+2\alpha)} + h_2,
\end{aligned} \tag{4.15}
$$

which agree with the expressions (4.6) and (4.7) for $u'=0$.

The previous analysis shows that the new class of solutions constructed through non-Abelian T-duality provide an explicit example of CGK geometries where the Riemann surface is an infinite strip. We will provide a more detailed global study of these solutions in a future publication.

## 5 Electric-magnetic charges and a minimisation principle

In this section we extend two results discussed in [1, 39] to our new infinite family of $\mathrm{AdS}_2$ solutions.

The first result is a relation between the holographic central charge in eq. (2.24) and an integral of the product of the electric and magnetic fluxes of the Dp-branes present in the background. This relates the holographic central charge in section 2.2, computed purely in terms of the NS-NS sector of the background, with a calculation purely in terms of the Ramond-Ramond sector.

Furthermore, in section 5.2, we explore this relation from a geometrical point of view. We define a quantity in terms of geometric forms in our geometries and through an extremisation principle relate it to the holographic central charge in eq. (2.24). In summary, in this section we present a connection between the holographic central charge, the product of the electric and magnetic charges and an extremised functional.

### 5.1 A relation between the holographic central charge and the RR fluxes

We provide a relation between the holographic central charge found in eq. (2.24) and the fluxes of the Ramond-Ramond sector in eq. (2.2). Consider a Dp brane and the associated electric $\widehat{F}_{p+2}$ and magnetic $\widehat{F}_{8-p}$ Page field strengths. We define the "density of electric and magnetic charges", $\rho^e_{\mathrm{Dp}}$ and $\rho^m_{\mathrm{Dp}}$, as follows,

$$
\rho^e_{\mathrm{Dp}} = \frac{1}{(2\pi)^p}\widehat{F}_{p+2}, \qquad \rho^m_{\mathrm{Dp}} = \frac{1}{(2\pi)^{7-p}}\widehat{F}_{8-p}. \tag{5.1}
$$

From these we construct the quantity,

$$
\begin{aligned}
&\int \sum_{p=1,3,5,7} \rho^e_{\mathrm{D}p}\rho^m_{\mathrm{D}p} =\\
&= \frac{1}{\pi}\mathrm{Vol}_{\mathrm{AdS}_2}\left(\frac{\mathrm{Vol}_{\mathrm{CY}_2}}{16\pi^4}\right)\int \mathrm{d}\rho \left[\frac{4\widehat{h}_4 h_8-(u')^2}{8} + \frac{1}{16}\partial_\rho\left(u^2\frac{(h_4h_8)'}{h_4h_8}\right) - \frac{u^2}{16}\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right)\right].
\end{aligned} \tag{5.2}
$$

In the absence of sources $\widehat{h}_4'' = h_8'' = 0$ and, up to a boundary term, this is proportional to the expression for the holographic central charge in equation (2.24). We explore below the contribution of the sources to this expression. Notice that eq. (5.2) links the holographic central charge in eq. (2.24)—a calculation purely in terms of the NS-NS sector — with one purely in terms of the Ramond-Ramond sector.

### 5.2 An action functional for the central charge

Following the ideas of [55, 56] and the lead of the works [1, 39], we construct a functional in terms of an integral of forms defined in the internal space. Once such functional is extremised the holographic central charge in eq. (2.24) is recovered, up to a boundary term.

We define forms $J_i$ and $\mathcal{F}_i$ (for $i = 1, 3, 5, 7$) on the internal space $X_8 =$[$\mathrm{S}^2$, $\mathrm{CY}_2$, $\mathrm{S}_\psi$, $\mathrm{I}_\rho$]. These forms are inherited from the Page fluxes (2.15).[5] As explained in [1, 39], they are the *restriction* of the fluxes to the internal space. Writing the Page fluxes in eqs. (2.15) in terms of forms $J_i$ and $\mathcal{F}_i$ as,

$$
\begin{aligned}
&\widehat{F}_1 = J_1, && \widehat{F}_3 = \mathcal{F}_1 \wedge \mathrm{vol}_{\mathrm{AdS}_2} + J_3, && \widehat{F}_5 = \mathcal{F}_3 \wedge \mathrm{vol}_{\mathrm{AdS}_2} + J_5,\\
&\widehat{F}_7 = \mathcal{F}_5 \wedge \mathrm{vol}_{\mathrm{AdS}_2} + J_7, && \widehat{F}_9 = \mathcal{F}_7 \wedge \mathrm{vol}_{\mathrm{AdS}_2}.
\end{aligned} \tag{5.3}
$$

The forms $J_i$ and $\mathcal{F}_i$ are,

$$
\begin{aligned}
&J_1 = h_8'\,\mathrm{d}\psi\,, \qquad J_3 = \frac{1}{4}\left(2h_8 + \frac{u'(u\widehat{h}_4' - \widehat{h}_4 u')}{2\widehat{h}_4^2}\right)\mathrm{vol}_{\mathrm{S}^2}\wedge \mathrm{d}\rho, \qquad J_5 = -\widehat{h}_4'\mathrm{vol}_{\mathrm{CY}_2}\wedge \mathrm{d}\psi\\
&J_7 = -\frac{1}{4}\left(2\widehat{h}_4 + \frac{u'(uh_8' - h_8 u')}{2h_8^2}\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho, \quad \mathcal{F}_1 = \frac{1}{2}\left(h_8'(\rho - 2\pi k) - h_8\right)\mathrm{d}\psi\,,\\
&\mathcal{F}_3 = \frac{1}{4}\left((\rho - 2\pi k)h_8 - \frac{\left(u - (\rho - 2\pi k)u'\right)\left(u\widehat{h}_4' - \widehat{h}_4 u'\right)}{4\widehat{h}_4^2}\right)\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho,\\
&\mathcal{F}_5 = \frac{1}{2}(\widehat{h}_4 - (\rho - 2\pi k)\widehat{h}_4')\,\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{d}\psi\,,\\
&\mathcal{F}_7 = -\frac{1}{4}\left((\rho - 2\pi k)\widehat{h}_4 - \frac{\left(u - (\rho - 2\pi k)u'\right)\left(uh_8' - h_8 u'\right)}{4h_8^2}\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\rho.
\end{aligned} \tag{5.4}
$$

With the forms in eqs. (5.4), we construct the functional,

$$
\begin{aligned}
\mathcal{C} &= \int_{X_8} \mathcal{F}_1 \wedge J_7 + \mathcal{F}_3 \wedge J_5 - (J_1 \wedge \mathcal{F}_7 + J_3 \wedge \mathcal{F}_5)\\
&= \int_{X_8} \left(\frac{4\widehat{h}_4 h_8 - (u')^2}{8} - \frac{u^2}{16}\left(\frac{\widehat{h}_4'^2}{\widehat{h}_4^2} + \frac{h_8'^2}{h_8^2}\right) + \frac{uu'}{8}\left(\frac{\widehat{h}_4'}{\widehat{h}_4} + \frac{h_8'}{h_8}\right)\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\psi\wedge\mathrm{d}\rho,
\end{aligned} \tag{5.5}
$$

We minimise the functional $\mathcal{C}$ by imposing the Euler-Lagrange equation for $u(\rho)$,

$$
2u'' = u\left(\frac{\widehat{h}_4''}{\widehat{h}_4} + \frac{h_8''}{h_8}\right). \tag{5.6}
$$

[5] The same result can be obtained considering the Maxwell fluxes in (2.2).

This equation of motion is solved if,

$$h_8''=0, \qquad \widehat{h}_4''=0, \qquad u''=0, \tag{5.7}$$

the first two are Bianchi identities for the background and the last is a BPS equation. The functional in eq. (5.5) can be rewritten as,

$$\mathcal{C}=\frac{1}{8}\int_{X_8}\left(4\widehat{h}_4h_8-(u')^2+\partial_\rho\left[\frac{u^2}{2}\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right)\right]-\frac{u^2}{2}\left(\frac{\widehat{h}_4''}{\widehat{h}_4}+\frac{h_8''}{h_8}\right)\right)\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\psi\wedge\mathrm{d}\rho. \tag{5.8}$$

The last term (that would vanish in the absence of sources), is proportional to the quotient of the number of flavours by the number of colours in each node. Using the condition that the flavours are sparse, as explained below eq. (2.17), we see that its contribution is subleading in front of the other terms. Furthermore, the boundary term gives a divergent contribution. Indeed, for the case $u=u_0$ and $\widehat{h}_4, h_8$ in eqs. (2.6)–(2.7) the boundary term reads,

$$\begin{aligned}\int_0^{2\pi(P+1)}\partial_\rho\left[\frac{u^2}{2}\left(\frac{\widehat{h}_4'}{\widehat{h}_4}+\frac{h_8'}{h_8}\right)\right]\mathrm{vol}_{\mathrm{CY}_2}\wedge\mathrm{vol}_{\mathrm{S}^2}\wedge\mathrm{d}\psi\wedge\mathrm{d}\rho&=-\lim_{\epsilon\to 0}\frac{2\pi u_0^2}{\epsilon}(\alpha_P+\mu_P+\beta_0+\nu_0)\mathrm{Vol}_{\mathrm{CY}_2}\\ &=-\lim_{\epsilon\to 0}\frac{2\pi u_0^2}{\epsilon}(Q_{\mathrm{D3}}^{total}+Q_{\mathrm{D7}}^{total})\mathrm{Vol}_{\mathrm{CY}_2},\end{aligned} \tag{5.9}$$

where we regularised $\widehat{h}_4(0)=h_8(0)=\widehat{h}_4(2\pi(P+1))=h_8(2\pi(P+1))=\epsilon$. The divergence in eq. (5.9) is associated with the presence of sources in the background as was found in [1, 39].

In summary, the functional in eq. (5.5) is proportional to the holographic central charge of eq. (2.24), plus a subleading contribution and a boundary term. For our infinite family of backgrounds, we have linked a calculation purely in terms of the NS-NS sector — eq. (2.24), with a calculation purely in terms of the Ramond-Ramond sector — eq. (5.2), with the extremisation of a functional constructed as a restriction of the Ramond-Ramond forms to the internal space — eq. (5.5). We believe that this may be a generic feature, worth exploring in backgrounds dual to various SCFTs in different dimensions.

## 6 Conclusions

We close this paper by presenting a short summary of the contents of this work and proposing future lines of investigation.

This work presents two new infinite families of backgrounds with an $\mathrm{AdS}_2$ factor. The presentation focuses mostly on geometrical aspects of the new solutions. The new family of backgrounds in section 2 can be obtained by analytically continuing the backgrounds of [1] or via T-duality, on the Hopf-fibre of the $\mathrm{S}^3$, from the solutions in [39]. These connections are summarised in figure 1. A precise brane set-up was proposed for these backgrounds and the holographic central charge was calculated. We used the brane set-up to argue for a precise quiver. The IR dynamics of such quivers should be the SCQMs dual to our backgrounds.

The family of $AdS_2$ backgrounds in section 2 and that in the paper [1] have been shown to be connected to the solutions of [34, 35]. In fact, under certain circumstances they extend this class of solutions. The connection between these qualitatively different backgrounds requires of a subtle zoom-in procedure that we explained in detail in section 3.

A second family of new backgrounds is presented in section 4. These interesting solutions depend explicitly on two coordinates (labelled as $\rho$ and $r$ in section 4) and were obtained by the application of non-Abelian T-duality on the $AdS_3$ factor of the backgrounds in [23]. We leave for future work to discuss the associated brane set-up, though it seems clear that the ideas described in [14, 40–43] will play an essential role in the global-definition of these solutions. By the same token, it would be interesting to study the integrability (or not) of the backgrounds presented here, as well as those in [1, 39]. Integrable string backgrounds dual to field theories described by linear quivers in dimensions $d = 2, 4, 6$, have been found in [57–60]. Similar techniques should probably apply for the $d = 1$ case.

Finally, in section 5 the holographic central charge defined in section 2—a quantity computed solely in terms of the NS-NS sector of the backgrounds, has been connected with a calculation purely in terms of the Ramond-Ramond sector of our solutions. A functional whose extremisation yields the holographic central charge was also discussed. It should be interesting to find out if a similar structure occurs generically for other $AdS_{d+1}$ backgrounds.

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# 9. Conclusions

Let us now conclude with the results of this thesis. In this work we have presented new entries in the mapping between AdS backgrounds and SCFTs, for the particular case of $\mathrm{AdS}_3/\mathrm{CFT}_2$ and $\mathrm{AdS}_2/\mathrm{SCQM}$.

We have used a well-structured methodology that can be summarised in the following way: first of all, we have constructed new type II and M-theory supergravity solutions with AdS factor, using diverse techniques like G-structures, string dualities and analytical continuations. We have carried out a careful study of the backgrounds and shown that there are physical sources in the geometry, providing flavour symmetry. We have also counted the number of branes, flavour and colour, and analysed the holographic central charge. With these geometric ingredients, we have assembled a detailed description of the underlying brane intersection and Hanany-Witten brane setups. These Hanany-Witten brane setups have been mapped to quiver field theories, which are a product of many gauge groups connected with other symmetry groups through hypermultiplets. Since we have considered theories in lower dimensions, the field theory described by these quivers conjecturally flow in the IR to a strongly coupled CFT, that we have proposed as dual to our AdS geometries. In each case, the proposed duality has been tested, comparing the field theory and holographic central charges and showing a precise match between both quantities. We have complemented our analysis with the construction of concrete examples in the classes obtained, a discussion of the connection with existing classifications, and the identification of a striking relation that allows to reproduce the holographic central charge from the RR sector of the theories.

Chapters 1, 2, and 3 have been dedicated to draft the main concepts used in this work, both geometric and field theoretical. In Chapter 4, we have provided a summary of the contents presented in the different papers that compose this thesis.

In Chapter 5 we have presented our 'seed' geometries. Here, using the Killing spinor techniques, we have constructed four new $\mathrm{AdS}_3\times\mathrm{S}^2$ classes preserving small $\mathcal{N}=(0,4)$ supersymmetry in type II supergravity. Two of them are in massive IIA and have SU(2) structure on the $\mathrm{M}_5=\mathrm{M}_4\times\mathrm{I}$ internal space; the class I has $\mathrm{M}_4=\mathrm{CY}_2$ and the class II a 4d Kähler manifold. The other two solutions are in type IIB, have identity structure on $\mathrm{M}_5$ and are distinguishable by having or not D7-branes. We have focused in the class I in massive type IIA, in the subset with a compact $\mathrm{CY}_2$. We have proposed two dimensional quiver field theories dual to these solutions, based on the Hanany-Witten setups implied by the charges of the solutions. In order to check the proposed duality we have matched the field theory and holographic central charges finding agreement between both quantities in Section 5.3. We have also found that the solution obtained via SU(2)-NATD on $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ fits in this class, and we have provided explicit completions for this background that allow to define a consistent dual CFT.

In Chapter 6, we have taken the massless case of the previous geometries and uplifted it to M-theory, obtaining a new class of warped $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbf{Z}_k\times\mathrm{CY}_2\times\mathrm{I}$ geometries. We have shown that these solutions are dual to two dimensional SCFTs with small $\mathcal{N}=(0,4)$ supersymmetry that describe self-dual strings in 6d (1,0) CFTs. A further family of new solutions in M-theory has been obtained through analytic continuations, giving rise to a new class where the modding acts on the $\mathrm{AdS}_3$ factor.

The reduction on $\mathrm{AdS}_3$ of the $\mathrm{AdS}_3/\mathbf{Z}_k\times\mathrm{S}^3$ solutions has led a new class of $\mathrm{AdS}_2$ solutions in massless type IIA supergravity with four Poincaré supercharges. We have shown that these solutions can be extended to the massive case noticing that they are related, via analytical continuations, with the 'seed' solutions. Their superconformal quantum mechanics has been studied carefully in Section 7.1, where we have identified the underlying $\frac{1}{8}$-BPS brane intersection. This has allowed to interpret the dual CFT as describing one dimensional defects consisting on D4-D0 baryon vertices in 5d D4'-D8 systems. In the presence of the defects the D4'- and D8-branes, of the 5d theory, become flavour branes, and the D0- and

D4-branes play the role of colour branes. Accordingly, we have seen that, the SCQM that lives on these systems is given in terms of a set of disconnected quivers with gauge groups associated to D0- and D4-branes coupled to D4' and D8 flavour branes.

We have also shown that, performing SL$(2,\mathbb{R})$-NATD on the $\text{AdS}_3\times\text{S}^3\times\text{CY}_2$ solution in type IIB, an explicit solution in the class given in Section 7.1 is obtained. It is worth to mention that this has been the first time that NATD with respect to a non-compact isometry group has been applied as a solution generating technique in supergravity. A careful analysis of this example has been presented in Section 7.2 with a precise completion of the solution and a concrete quiver quantum mechanics proposal. It has been shown that this solution is connected via analytical continuations, with the $\text{AdS}_3\times\text{S}^2\times\text{CY}_2\times\text{I}$ solution, obtained in [81] and studied in Section 5.4.

Chapter 8 has been dedicated to the study of three new $\text{AdS}_2$/SCQM pairs in type IIB supergravity. The three families of $\text{AdS}_2$ solutions have been obtained through ATD and NATD. Type A and Type B have the same warped form, $\text{AdS}_2\times\text{S}^2\times\text{CY}_2\times\Sigma_2$, with $\Sigma_2$ a 2d Riemann surface with the topology of an annulus. The Type A solutions arise acting with ATD on the Hopf-fibre of the $\text{AdS}_3$ subspace of the $\text{AdS}_3\times\text{S}^2$ solutions to massive IIA. In turn, Type B solutions arise acting with ATD on the Hopf-fibre of the $\text{S}^3$ in the $\text{AdS}_2\times\text{S}^3$ backgrounds of massive IIA. We have seen that, both solutions are also related through analytical continuations and that they share the same D1-D3-D5-D7-NS5-F1 brane setup. In spite of their similar features, they have however different SQCM interpretations –due to their different origins. The SCQM dual to Type A solutions arises as a DLC compactification of the 2d CFTs dual to the $\text{AdS}_3\times\text{S}^2$ solutions. In turn, the Type B solutions are dual to backreacted D1-D5 baryon vertices within the 4d $\mathcal{N}=2$ QFT living in D3-D7 branes. We have also discussed some features of these backgrounds, like the existence of an interesting connection between the holographic central charge and the RR sector of the solutions. The third solution in type IIB has been obtained applying SL$(2,\mathbb{R})$-NATD on the 'seed' solutions. This new family has the same form as the previous Type A and Type B geometries, but in this case $\Sigma_2$ has the topology of an infinite strip. The detailed study of these backgrounds and their completion will be the subject of future work.

In Section 8.2, we have shown that the solutions in type IIB extend the classification of [47, 48] to include D3- and D7-brane sources.

This work has interesting open avenues that deserve further investigation. For instance, lower-dimensional AdS solutions are particularly relevant for the development of the black hole microscopic counting programme and, on the other hand, these theories can be studied as defect branes ending on a given brane setup and producing the lower dimensional AdS backgrounds.

In the spirit of [41, 102, 103], finding an interpretation of our solutions in the context of 4d and 5d black holes is clearly a direction that should be investigated. It would be interesting to understand the role of the freedom in choosing $h_4$, $h_8$ and their implications for black holes. It should be important to clarify the relation between the number of vacua and the entropy of these black holes.

In particular, it would be interesting to continue exploiting the G-structures technique to explore the $\mathcal{N} = (0, 2)$ $\text{AdS}_2$ and $\text{AdS}_3$ solutions which have been little studied and, furthermore, propose their CFT duals with the tools showed in this work.

It would be worthwhile to use the SL$(2, \mathbf{R})$-NATD to construct new $\text{AdS}_2$ solutions and explore their dual SCQMs. In the previous examples where NATD has been used –with respect o a freely acting SU(2) isometry group– the setup to engineer the field theory was determined by Dp-branes extended between NS5 branes, whereas SL$(2, \mathbf{R})$-NATD seems to generate fundamental strings stretched between the different Dp-branes. With this reasoning, the SCQMs might be arising as Wilson lines defects or baryon vertices within higher dimensional conformal theories. In this vein, the first candidate to study are the Type C solutions obtained using SL$(2, \mathbf{R})$-NATD on the $\text{AdS}_3$ subspace of our 'seed' geometries, which are missing a CFT description.

It would be interesting to see if a similar relation between the holographic central charge and products of Ramond-Ramond fluxes holds for other classes of solutions, especially higher dimensional AdS spaces. This study would clearly be of benefit because it falls in line with the notion that extremisation problems in quantum field theory are realised in supergravity via the extremisation of certain geometrical quantities. Accordingly, the relevance is doubly so, first, the functional is constructed in terms of precise electric/magnetic forms determined by the Ramond-Ramond sector of the solution, whereas the holographic central charge is usually computed with the NS-NS sector of the solution. In this sense, it would be interesting to understand the geometrical meaning of these forms and see whether they are connected to the SU(2)-structures. And second, the extremal principle generalises those in the existing literature by the inclusion of sources and boundaries. Thus, a way forward is to extend these results to other geometries which also contain sources.

In addition, an obvious open problem is to discuss the CFTs duals of the solutions referred as class II, where the $\mathrm{CY}_2$ is replaced by a 4d Kähler manifold, and the CFTs duals to the solutions $\mathrm{AdS}_3\times\mathrm{S}^2\times\mathrm{M}_5$ in type IIB. Besides, it would be interesting to study the integrability (or not) of the backgrounds presented in this thesis. It would be nice to explore other tests and find predictions of our proposed duality. In particular it would be interesting to apply exact calculational techniques, as in [41], to the new classes of solutions, since this would provide for a deeper understanding of the IR regime of the different theories.

# Conclusiones

Concluimos ahora con los resultados de esta tesis. En este trabajo hemos presentado nuevas entradas en el mapeo entre geometrias AdS y SCFTs, para el caso particular de $AdS_3/CFT_2$ y $AdS_2$/SCQM.

Hemos utilizado una metodología bien estructurada que se resume de la siguiente manera: en primer lugar, hemos construido nuevas soluciones de supergravedad tipo II y de teoría M con factores AdS, utilizando diversas técnicas como *G-structures*, dualidades de teoría de cuerdas y continuaciones analíticas. Hemos llevado a cabo un estudio cuidadoso de las soluciones y mostrado que tenemos fuentes físicas en la geometría, proporcionando simetría de sabor. Igualmente, hemos contado el número de branas, de sabor y de color, y analizado la carga central holográfica. Con estos ingredientes geométricos, hemos ensamblado una descripción detallada de la intersección de branas subyacente y construido configuraciones de brana de Hanany-Witten. Estas configuraciones de brana de Hanany-Witten han sido mapeadas a teorías de campo de *quiver*, que son un producto de varios grupos de gauge conectados con otros grupos de simetría a través de hipermultipletes. Dado que hemos considerado teorías en dimensiones bajas, las teorías de campo descritas por estos *quiver* fluyen conjeturalmente en el IR a CFTs fuertemente acopladas, que hemos propuesto como duales a nuestras geometrías AdS. En cada caso, la dualidad propuesta ha sido verificada comparando la carga central de la teoría de campo y la carga central holográfica, mostrando una coincidencia precisa entre ambas cantidades. Hemos complementado nuestro análisis con la construcción de ejemplos concretos en las clases obtenidas, con una discusión de la conexión con las clasificaciones existentes, y con la identificación de una interesante relación, que permite reproducir la carga central holográfica con el sector RR de las teorías.

Los Capítulos 1, 2 y 3 han sido dedicados a bosquejar los principales conceptos utilizados en este trabajo, tanto conceptos geométricos como conceptos teóricos de campo. En el Capítulo 4, hemos proporcionado un resumen de los artículos que componen esta tesis.

En el Capítulo 5 hemos presentado nuestras geometrías 'semilla'. Aquí, usando las técnicas *Killing spinor*, hemos construido cuatro nuevas clases $\mathrm{AdS}_3\times\mathrm{S}^2$ preservando pequeña $\mathcal{N} = (0,4)$ supersimetría en supergravedad tipo II. Dos de ellas están en tipo IIA masiva y tienen estructura SU(2) en el espacio interno $\mathrm{M}_5 = \mathrm{M}_4\times\mathrm{I}$; la clase I tiene $\mathrm{M}_4 = \mathrm{CY}_2$ y la clase II tiene una variedad Kähler de cuatro dimensiones. Las otras dos soluciones están en tipo IIB, tienen estructura identidad en $\mathrm{M}_5$ y se distinguen por tener o no D7-branas. Nos hemos enfocado en la clase I en tipo IIA masiva, en un subconjunto con una $\mathrm{CY}_2$ compacta. Hemos propuesto teorías bidimensionales de campos de *quiver* duales para estas soluciones, basadas en las configuraciones de Hanany-Witten las cuales son implícadas por las cargas de las soluciones. Para comprobar la dualidad propuesta hemos comparado la carga central de la teoría de campo y la carga central holográfica encontrando concordancia entre ambas cantidades, estos resultados son presentados en la Sección 5.3. Además, hemos encontrado que la solución obtenida a través de SU(2)-NATD sobre la solución $\mathrm{AdS}_3\times\mathrm{S}^3\times\mathrm{CY}_2$ encaja en esta clase, y hemos proporcionado terminaciones explícitas para este fondo que permiten definir consistentemente una CFT dual.

En el Capítulo 6, hemos tomado el caso sin masa de las geometrías anteriores y elevado a la teoría M, obteniendo una nueva clase de geometrías deformadas del tipo $\mathrm{AdS}_3\times\mathrm{S}^3/\mathbf{Z}_k\times\mathrm{CY}_2\times\mathrm{I}$. Hemos mostrado que estas soluciones son duales a SCFT dos dimensionales con pequeña $\mathcal{N} = (0,4)$ supersimetría que describen cuerdas autoduales en (1,0) CFTs seis dimensionales. Hemos obtenido también otra familia de nuevas soluciones en la teoría M a través de continuaciones analíticas, dando lugar a una nueva clase donde el *modding* actúa ahora sobre el factor $\mathrm{AdS}_3$.

La reducción en $\mathrm{AdS}_3$ de las soluciones $\mathrm{AdS}_3/\mathbf{Z}_k\times\mathrm{S}^3$ ha conducido a una nueva clase de soluciones $\mathrm{AdS}_2$ en supergravedad tipo IIA sin masa con cuatro supercargas de Poincaré. Hemos mostrado que estas soluciones se pueden extender al caso masivo notando que están relacionadas, vía continuaciones analíticas, con las soluciones 'semilla'. Su mecánica cuántica superconforme se ha estudiado detenidamente en la Sección 7.1, donde hemos identificado la intersección de brana $\frac{1}{8}$-BPS subyacente. Esto ha permitido interpretar la CFT dual como una descripción de defectos unidimensionales que consisten en vértices bariónicos, D4-D0, en sistemas cinco dimensionales, D4'-D8. En presencia de los defectos, las branas D4' y D8, de la teoría cinco dimensional, se convierten en branas de sabor, y las branas D0 y D4 desempeñan ahora el papel de branas de color. En consecuencia, hemos visto que, la SCQM que vive en estos sistemas se da en términos de un conjunto de *quiver* desconectados con grupos de gauge asociados a las D0- y D4-branas acopladas a las branas de sabor D4' y D8.

Además hemos mostrado que, realizando SL$(2, \mathbf{R})$-NATD sobre la solución $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{CY}_2$ en tipo IIB, una solución explícita de la clase dada en la Sección 7.1 es obtenida. Cabe mencionar que esta ha sido la primera vez que se aplicó NATD con respecto a un grupo de isometría no compacto como técnica de generación de soluciones en supergravedad. Un cuidadoso análisis de este ejemplo ha sido presentado en la Sección 7.2 con una terminación precisa de la solución y una propuesta concreta de su mecánica cuántica. Ha sido mostrado que esta solución está conectada a través de continuaciones analíticas, con la solución $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \mathrm{I}$, obtenida en [81] y estudiada en la Sección 5.4.

El Capítulo 8 ha sido dedicado al estudio de tres nuevos pares $\mathrm{AdS}_2$/SCQM en supergravedad tipo IIB. Las tres familias de soluciones $\mathrm{AdS}_2$ se han obtenido a través de ATD y NATD. El Tipo A y el Tipo B tienen la misma forma, $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{CY}_2 \times \Sigma_2$, con $\Sigma_2$ una superficie de Riemann 2d con la topología de un anillo. Las soluciones Tipo A surgen actuando con ATD sobre la fibra de Hopf del subespacio $\mathrm{AdS}_3$ de las soluciones $\mathrm{AdS}_3 \times \mathrm{S}^2$ en tipo IIA masiva. A su vez, las soluciones Tipo B surgen actuando con ATD sobre la fibra de Hopf del $\mathrm{S}^3$ en las soluciones $\mathrm{AdS}_2 \times \mathrm{S}^3$ en tipo IIA masiva. Hemos visto que ambas soluciones también están relacionadas a través de continuaciones analíticas y que comparten la misma configuración de brana D1-D3-D5-D7-NS5-F1. A pesar de sus características similares, tienen diferentes interpretaciones en su SQCM, debido a sus diferentes orígenes. Las SCQMs duales a las soluciones Tipo A surgen como una compactificación (*DLC*) de las soluciones 2d CFT duales a las $\mathrm{AdS}_3 \times \mathrm{S}^2$. A su vez, las soluciones de Tipo B son duales a los vértices bariónicos D1-D5 que *backreacted* dentro de las $\mathcal{N} = 2$ QFT en 4d que viven en las branas D3-D7. También hemos comentado algunas características de estos fondos, como la existencia de una interesante conexión entre la carga central holográfica y el sector RR de las soluciones. La tercera solución en tipo IIB se ha obtenido aplicando SL$(2, \mathbf{R})$-NATD sobre las soluciones 'semilla'. Esta nueva familia tiene la misma forma que las geometrías Tipo A y Tipo B anteriores, pero en este caso $\Sigma_2$ tiene la topología de una franja infinita. El estudio detallado de esta solución y su terminación es objeto para un trabajo futuro.

En la Sección 8.2, hemos mostrado que las soluciones en tipo IIB extienden la clasificación de [47, 48] al incluir fuentes de branas, D3- y D7-branas.

Este trabajo tiene interesantes caminos abiertos que merecen una mayor investigación. Por ejemplo, las soluciones AdS en bajas dimensiones son particularmente relevantes para el desarrollo del programa de conteo microscópico de agujeros negros y, por otro lado, estas teorías pueden estudiarse como branas defecto que terminan en una configuración de brana dada y producen las geometrías AdS de menor dimensión.

En el espíritu de [41, 102, 103], encontrar una interpretación de nuestras soluciones en el contexto de 4d y 5d agujeros negros es claramente una dirección que debe investigarse. Sería interesante entender la libertad existente al elegir $h_4$, $h_8$ y sus implicaciones para los agujeros negros. Debería ser importante aclarar la relación entre el *number of vacua* y la entropía de estos agujeros negros.

En particular, sería interesante seguir explotando la técnica de *G-structures* para explorar las soluciones $\mathcal{N} = (0, 2)$ AdS$_2$ y AdS$_3$ que han sido poco estudiadas y, además, proponer sus duales CFT con las herramientas mostradas en este trabajo.

Valdría la pena seguir usando SL$(2, \mathbf{R})$-NATD para construir nuevas soluciones con factor AdS$_2$ y explorar sus SCQMs duales. En los ejemplos anteriores en los que se ha utilizado NATD –con respecto a un grupo de isometría SU(2) de acción libre– la configuración para diseñar la teoría de campo estaba determinada por Dp-branas extendidas entre NS5-branas, mientras que SL$(2, \mathbf{R})$-NATD parece generar cuerdas fundamentales estiradas entre las diferentes Dp-branas. Con este razonamiento, las SCQMs podrían estar surgiendo como defectos de líneas de Wilson o vértices de barionicos dentro de teorías conformes de dimensiones superiores. En este sentido, el primer candidato a estudiar son las soluciones Tipo C obtenidas usando $\mathrm{SL}(2,\mathbb{R})$-NATD sobre el subespacio $\mathrm{AdS}_3$ de nuestras geometrías 'semilla', a las que les falta una descripción CFT.

Sería interesante ver si una relación similar entre la carga central holográfica y los productos de los flujos de Ramond-Ramond se mantiene para otras clases de soluciones, especialmente para espacios AdS de mayor dimensión. Este estudio sería claramente beneficioso porque coincide con la noción de que los problemas de extremización en teoría cuántica de campos se realizan en supergravedad a través de la extremización de ciertas cantidades geométricas. En consecuencia, la relevancia es doble, primero, el funcional se construye en términos de formas eléctricas/magnéticas precisas determinadas por el sector Ramond-Ramond de la solución, mientras que la carga central holográfica generalmente se calcula con el sector NSNS de la solución. En este sentido, sería interesante comprender el significado geométrico de estas formas y ver si están conectadas a las estructuras SU(2). Y segundo, el principio extremal generaliza aquellos en la literatura existente mediante la inclusión de fuentes y fronteras. Por lo tanto, es interesante extender estos resultados a otras geometrías que también contienen fuentes.

Además, un problema abierto obvio es discutir los CFTs duales de las soluciones referidas como clase II, donde el $\mathrm{CY}_2$ es reemplazado por una variedad 4d de Kähler, y los CFTs duales a las soluciones $\mathrm{AdS}_3 \times \mathrm{S}^2 \times \mathrm{M}_5$ en tipo IIB. Además, sería interesante estudiar la integrabilidad (o no) de los fondos presentados en esta tesis. Sería bueno explorar otras pruebas y encontrar predicciones de nuestra dualidad propuesta. En particular, sería interesante aplicar técnicas de cálculo exacto, como en [41], a las nuevas clases de soluciones, ya que esto proporcionaría una comprensión más profunda del régimen IR de las diferentes teorías

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