

Article

Open Pit Optimization Using the Floating Cone Method: A New Algorithm

Gonzalo Ares ^{1,*}, César Castañón Fernández ², Isidro Diego Álvarez ², Daniel Arias ³ and Arturo Buelga Díaz ¹

¹ Ph.D. Programme on Production, Mining-Environmental and Project Engineering, University of Oviedo, 33004 Oviedo, Spain; arturobuelga@yahoo.es

² School of Mining, Energy and Materials Engineering, University of Oviedo, 33004 Oviedo, Spain; castanoncesar@uniovi.es (C.C.F.); diegoisidro@uniovi.es (I.D.Á.)

³ Department of Geology, University of Oviedo, 33005 Oviedo, Spain; darias@geol.uniovi.es

* Correspondence: gonzaloaresasensio@gmail.com

Abstract: Three-dimensional block models are the most frequently used tool for estimating mineral resources and reserves within a mineral deposit. In open pit mining, the basis of mine design and the long term mining schedule is calculation of the ultimate pit limit. The ultimate pit limit is the pit with the highest profit value. Over the years, different algorithms have been developed that enable us to calculate the final pit: floating or mobile cone, floating cone II and its corrected forms, floating cone III, the Korobov algorithm and its corrected form, the Lerchs–Grossmann 2D algorithm (dynamic programming), and the Lerchs–Grossmann 3D algorithm (graph theory). All these algorithms have advantages and disadvantages. The floating cone method stands out for its simplicity, speed, and easy implementation, even for calculating a pit with a variable slope angle. The main drawback of this method is that it is unable to examine all possible combinations. For this reason, the algorithm does not consistently give optimal results, which is why it has required improvements over time. However, the improved methods still have some problems. To overcome these problems, a new algorithm called the floating cone IV method will be demonstrated in this paper.

Keywords: floating cone; open pit; ultimate pit limits; block model; mine optimization



Citation: Ares, G.; Castañón Fernández, C.; Álvarez, I.D.; Arias, D.; Díaz, A.B. Open Pit Optimization Using the Floating Cone Method: A New Algorithm. *Minerals* **2022**, *12*, 495. <https://doi.org/10.3390/min12040495>

Academic Editor: Yosoon Choi

Received: 14 March 2022

Accepted: 13 April 2022

Published: 18 April 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The algorithms used in the optimization of a mineral deposit work using a mineralization model built by three-dimensional blocks in a quantity large enough to include the entire area of interest. A three-dimensional block model is a database in which each record represents a discrete element of rock where the fields define the block's location and properties (density, lithology, ore grade, etc.).

The discussion of economic models is directly related to the concept of ore cut-off grade. Any block with a grade lower than the cut-off grade is considered waste and without economic value. On the contrary, a block with a grade higher than the cut-off grade will be considered ore and therefore will be of economic interest.

The problem of the ultimate pit limit can be defined as the determination of the subset of reservoir blocks that maximizes the economic value, observing the slope restrictions to ensure the stability of the pit [1]. It is possible to generate pits with variable slope angles assigned to each block [2,3]. It is important to remember that there are no time or extraction capacity considerations at this stage.

In order to calculate the ultimate pit limits, different algorithms have been developed such as the floating cone algorithm [4,5], the Korobov algorithm [6], the corrected form of the Korobov algorithm [7], the Lerchs–Grossmann 2D algorithm (dynamic programming), and the Lerchs–Grossmann 3D algorithm (graph theory) [8]. All of the methods have strengths and weaknesses. For example, the floating cone method is the simplest and easiest technique to implement in order to determine the optimal pit limits. Nonetheless,

this method does not allow the optimal pit to be calculated in all models, as will be discussed below. Therefore, algorithms have been developed that improve the original floating cone algorithm: the floating cone II [9], modified floating cone II, and floating cone III methods [10,11]. Although these algorithms have improved the obtained results, they still do not always allow for the optimal pit calculation.

Throughout this article, a new version of the floating cone algorithm is proposed, to be named “floating cone IV”. This new algorithm tries to solve some of the problems for which the floating cone method does not always obtain the optimal pit.

2. Block Models and Cut-Off Grade

In order to achieve the optimization of the ore deposit, i.e., the optimal pit calculation, it is necessary to have a block model that faithfully represents the area where mining operations will be developed, where each block stores the main parameters that define it such as weight, lithology, and grades. It is also necessary to take into account the following parameters [12]:

C_w = the mining cost of waste rock. This includes the costs of drilling, blasting, loading, hauling, and dumping material in the waste rock dump. This cost is usually given per ton or per cubic metre.

C_o = the mining cost of ore. This includes the costs of drilling, blasting, loading, hauling to the processing plant, and grade control. This cost is usually given per ton or per cubic metre. It tends to be somewhat higher than that of waste rock due to grade control costs and because the mined benches are generally smaller.

C_p = the ore processing cost. This includes all operating costs in the processing plant, including crushing, milling, processing, handling of concentrates, and administrative costs. This cost is usually given per ton of ore processed. It also has the added administrative and other costs here, which are generally calculated per ton of ore processed.

Rec = the average processing recovery. The percentage by weight of metal or ore that is recovered in the processing plant.

P_s = the selling price is the final selling price of the metal or ore. It is equal to the market price minus the costs of transport, freight, fines, smelting, and royalties. Although there are many types of purchase contracts for concentrates produced at the mine, some are complex. The actual price paid is related not only to the metal content of the concentrate but also to the content of other metals. The standard practice is to calculate the real price that is charged minus all of these costs.

Geotechnical angles represent the maximum allowable slope angles that, depending on the characteristics of the rock, ensure a stable mining operation. These angles vary according to lithology, degree of alteration, rock fracture, etc., and are critical for determining the ultimate pit limits since they considerably impact the amount of waste that needs to be extracted.

To discern the ore blocks, i.e., those that provide economic profit, it is mandatory to define the cut-off grade. Two types of cut-off grades can be distinguished:

Break-even cut-off grade is obtained from the null benefit, i.e., the cut-off grade that equates the benefits to the costs. Therefore, it can be calculated by the following equation [13,14]:

$$\text{Cut-off grade}_{\text{Break-even}} = \frac{C_o + C_p}{P_s \cdot Rec} \quad (1)$$

However, taking into account that the waste inside the pit has to be exploited and taken to the dump at a cost C_w (cost of exploitation of the waste per ton). If we eliminate this cost in the previous formula, the internal cut-off grade can be defined, which would equal to the costs of the extraction of the ore and its treatment minus the costs of the waste with the income obtained as:

$$\text{Cut-off grade}_{\text{internal}} = \frac{C_o + C_p - C_w}{P_s \cdot Rec} \quad (2)$$

It is important to note that blocks of ore that are between the break-even cut-off grade and the internal cut-off grade are often referred to as marginal ore. Their value is slightly negative, but less negative than if they are considered waste, so the overall benefit will be higher.

Once the cut-off grade has been determined, the evaluation of the blocks V_b that make up the block model can be carried out. It is important to bear in mind that the value of each block must be calculated assuming that the block is discovered, i.e., the cost required to access the block must not be considered in the total costs [15]. The formula used to generate the value V for each block b is the following:

$$V_b = \begin{cases} T \cdot (G \cdot Rec \cdot P_s - C_o - C_p), & \text{if } G \geq \frac{C_o + C_p - C_w}{P_s \cdot Rec} \\ T \cdot (-C_w) & , \text{ for the rest} \end{cases} \quad (3)$$

where:

T = Tons of the block

G = Mean block cut-off grade

3. Economic Pit Calculation Methods

In the mid-1960s, the mobile or floating cone algorithm was developed [4,5]. As discussed above, the main handicap of this algorithm is that it cannot study all adjacent block combinations. Therefore, the algorithm does not give optimal results in a regular way, which is why several improvements have been developed: floating cone II [9], modified floating cone II, and floating cone III methods [10,11].

At the same time, Lerchs and Grossmann presented two novel methods to try to solve the open-pit-optimization problem: Lerchs–Grossmann 2D, which utilizes dynamic programming, and Lerchs–Grossmann 3D, which utilizes graph theory [8]. Both algorithms allow us to calculate an ultimate pit limit in a block model. In the mid-1980s, Jeff Whittle of Whittle Programming Pty Ltd. developed a practical optimization program called Whittle Three-D where the Lerchs–Grossmann algorithm was implemented [16].

Later, new methods were developed. The Korobov algorithm and the corrected form of the Korobov algorithm [6,7], dynamic programming, and genetic algorithms are some of the several algorithms developed to determine the optimal pit limits [17–20].

4. Evolution of the Floating Cone Algorithm

4.1. Original Floating Cone

This method consists of the economic study of the mineralized and waste blocks that fall within an inverted cone [4,5] that moves systematically through a matrix of blocks, with the vertex of the cone occupying, successively, the center of the blocks (Figure 1). The overall slope angle will depend on the slope angle of the final pit, understood to be from the base of the lower bank to the crest of the upper bank. This angle will be calculated according to the geotechnics of the materials traversed.

The basic premise of this method is that the net benefits obtained from mining the ore found within all the blocks and included in the cone must exceed the mining and treatment expenses in said cone. If the profit is positive, all blocks included within the cone are marked and removed from the block matrix so that they are not part of any future cone. On the contrary, if the profits are negative, it would remain as it is and the vertex of the cone is transferred to the next block whose value is above the minimum mining cut-off grade, repeating the process afterwards.

This method is not capable of producing a true optimum pit limit. There are two situations where optimal solutions cannot be achieved which are described below:

Ore blocks that are analyzed individually. A single ore block may not justify the removal of waste blocks while the combination of these blocks with others that overlap can generate positive values. This situation is shown in Figure 2.

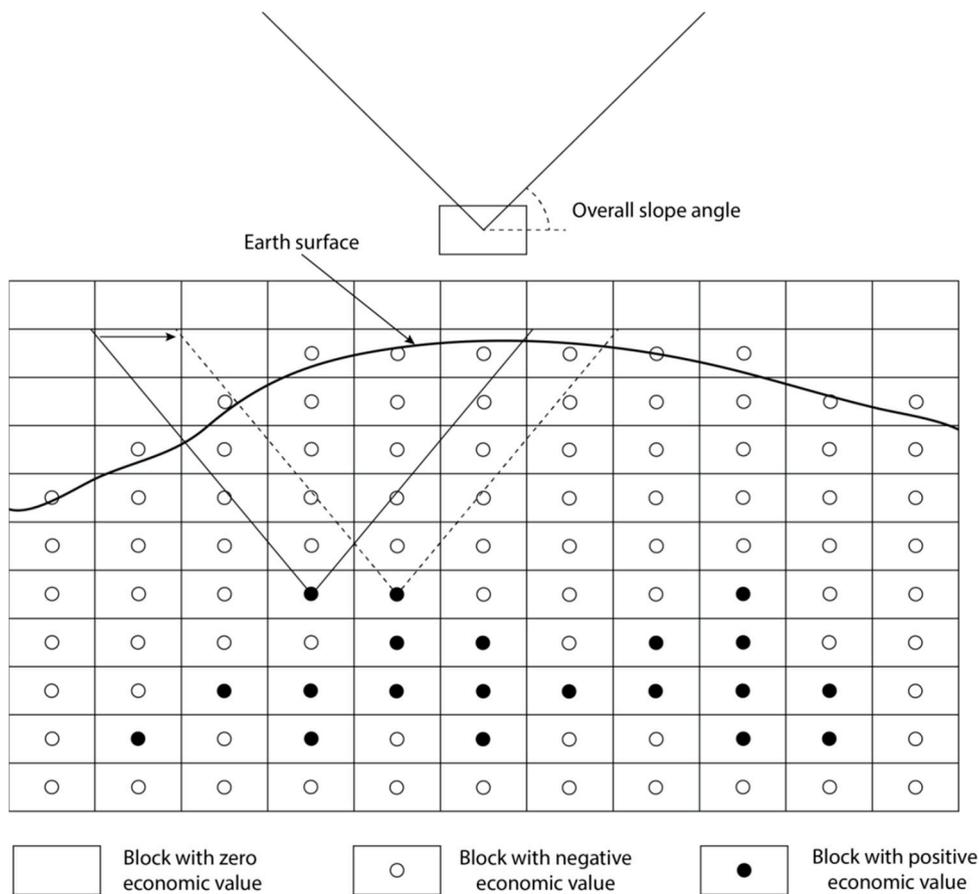


Figure 1. 2D operation of the floating cone algorithm.

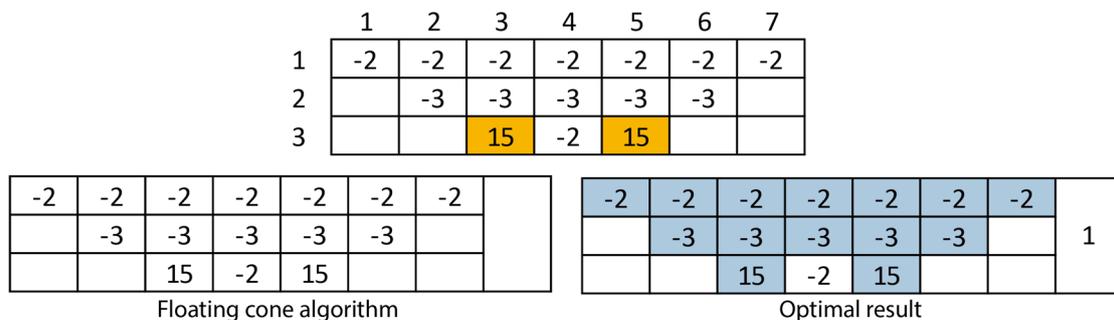


Figure 2. Comparison of the result of the floating cone algorithm against an optimal result. While the floating cone algorithm would not generate a result, an optimal algorithm would generate a pit of +1.

The method includes non-profitable blocks in the final design. Such inclusion reduces the net value of the mine design. A simple example that demonstrates this is the block model shown in Figure 3. The cone of the block of value 20 generates a profit of 50, but if the cones of the blocks of value 70 and 90 are generated before, it would have a profit greater than 60 and the remaining cone of value block 20 would no longer be profitable.

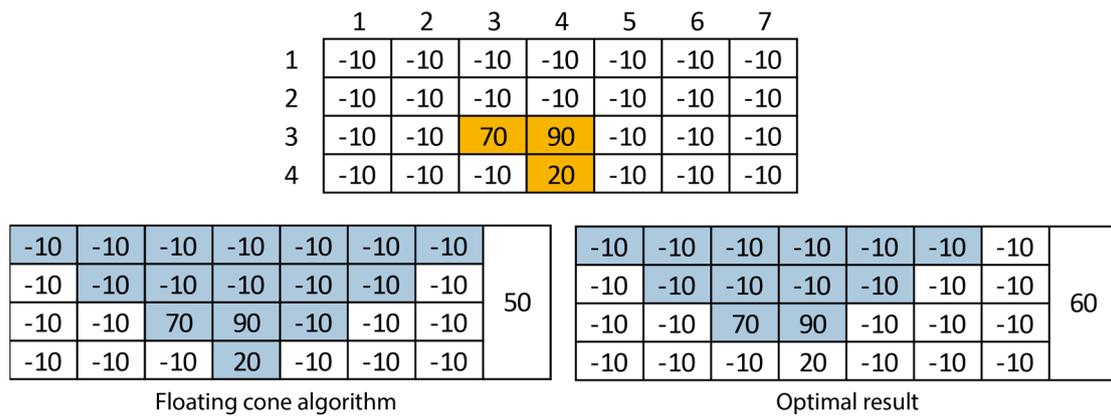


Figure 3. Comparison of the result of the floating cone algorithm against an optimal result. While the floating cone algorithm would yield a +50 pit, an optimal algorithm would yield a +60 pit.

4.2. Floating Cone II

The floating cone II (flowchart in Figure 4) was presented by Wright [9]. It follows a similar methodology to the floating cone approach, except that it goes from level to level from the top down, analyzing them.

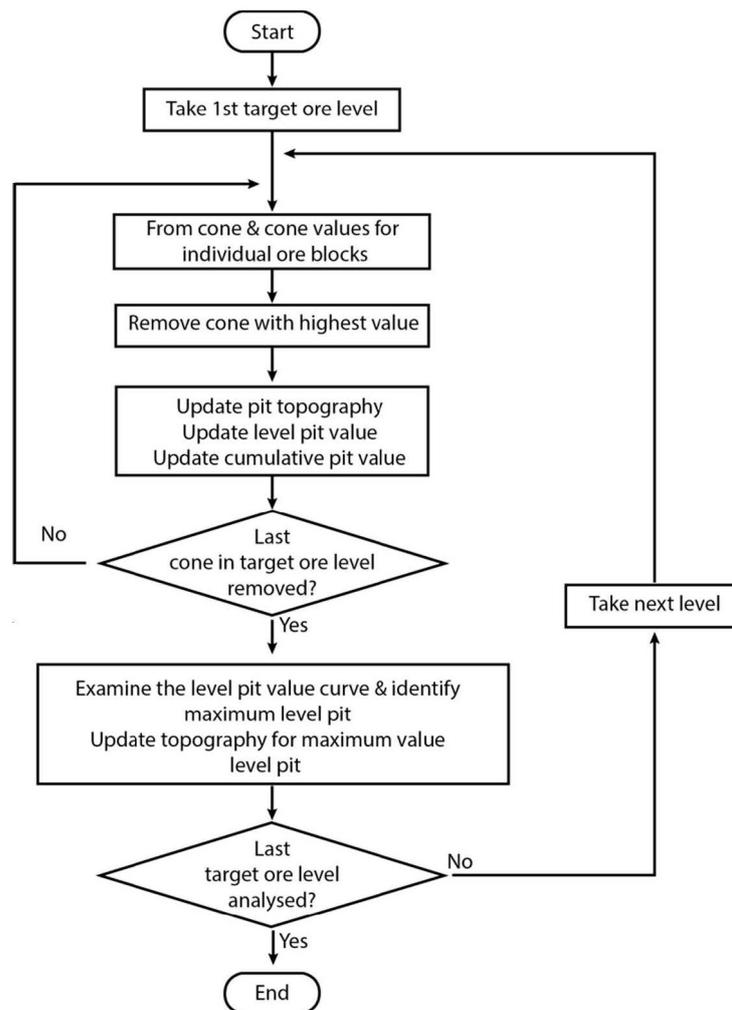


Figure 4. Flowchart of the floating cone II algorithm [9].

The algorithm starts by calculating the values of all potential cones at each level (each ore block has its cone constructed and valued). Following this step, the cones are removed one by one, from highest to the lowest value, until reaching the lowest value of that level. If two cones have the same maximum value, the cone with fewer blocks must be removed.

Finally (flowchart in Figure 4), it analyzes the value curve of all the cones that are removed at that level and the one with the highest value will be chosen as the optimum of that level and will proceed to the next level.

If this method is applied to the block model in Figure 2, the algorithm can start both with the block in the third row and in the third column (3, 3) and with the block (3, 5) that both have the same cone value (−4) and the same size. In this case, block (3, 3) was removed first, giving an accumulated value of −4 (Table 1). Next, the block (3, 5) was removed. By adding this cone to the solution, the accumulated value was +1. Therefore, as can be seen with this example, this method solves the problem of ore blocks that are analyzed individually as long as they are at the same level.

Table 1. Values of the block model of Figure 2 obtained by the floating cone II algorithm.

Stage	Block	Block Value	Cone Value	Cumulative Value	Mineable?
3	(3, 3)	+15	−4	−4	No
	(3, 5)	+15	+5	+1	Yes

In the case of the block model shown in Figure 3, the same process is followed. As shown in Table 2, it begins by calculating the value of the cones of the mineralized blocks on the third level. Once completed, it continues with the next level. At the end of the process, each block in the list represents the highest accumulated value (inclusive) and are the ore blocks that create the optimal pit. In this case, only the blocks belonging to the third level will be included since the blocks of the fourth level decrease the accumulated value.

Table 2. Values of the block model of Figure 3 obtained by the floating cone II algorithm.

Stage	Block	Block Value	Cone Value	Cumulative Value	Mineable?
3	(3, 3)	+70	−10	−10	No
	(3, 4)	+90	+70	+60	Yes
4	(4, 4)	+20	−10	+50	Yes, but not optimal

As has been shown by the previous examples, the floating cone II, in principle, would both solve the problem of mineralized blocks that are analyzed individually, as long as they are at the same level, and include blocks without benefit in the final design. However, the previously studied block models are quite simple; when this method is applied to more complex block models it does not generate optimal solutions. As can be observed in the block model shown in Figure 5, the floating cone II algorithm does not allow us to obtain a solution (Table 3) and therefore no extraction should be carried out, while in an optimal result, a pit value of +2 would be obtained. Due to the inaccuracies of the floating cone II, two modifications have been suggested to improve it, which are explained below.

Table 3. Values of the block model of Figure 4 obtained by the floating cone II algorithm.

Stage	Block	Block Value	Cone Value	Cumulative Value	Mineable?
2	(2, 8)	+8	−1	−1	No
	(2, 2)	+7	−2	−3	No
	(2, 4)	+6	0	−3	No
3	(3, 4)	+15	−2	−2	No

4.2.2. Modified Floating Cone II, Method 2

The second modification to the floating cone method is an enlargement of the previous modification [17]. In this algorithm, all levels are studied together; therefore, the value of each cone is economically evaluated and the cone with the maximum value is included as part of the optimum pit (Figure 7). From this, the accumulated value of the pit is calculated. This process is repeated until no ore blocks remain. The block with the most positive accumulated value and all previously investigated blocks are included as part of the optimal solution.

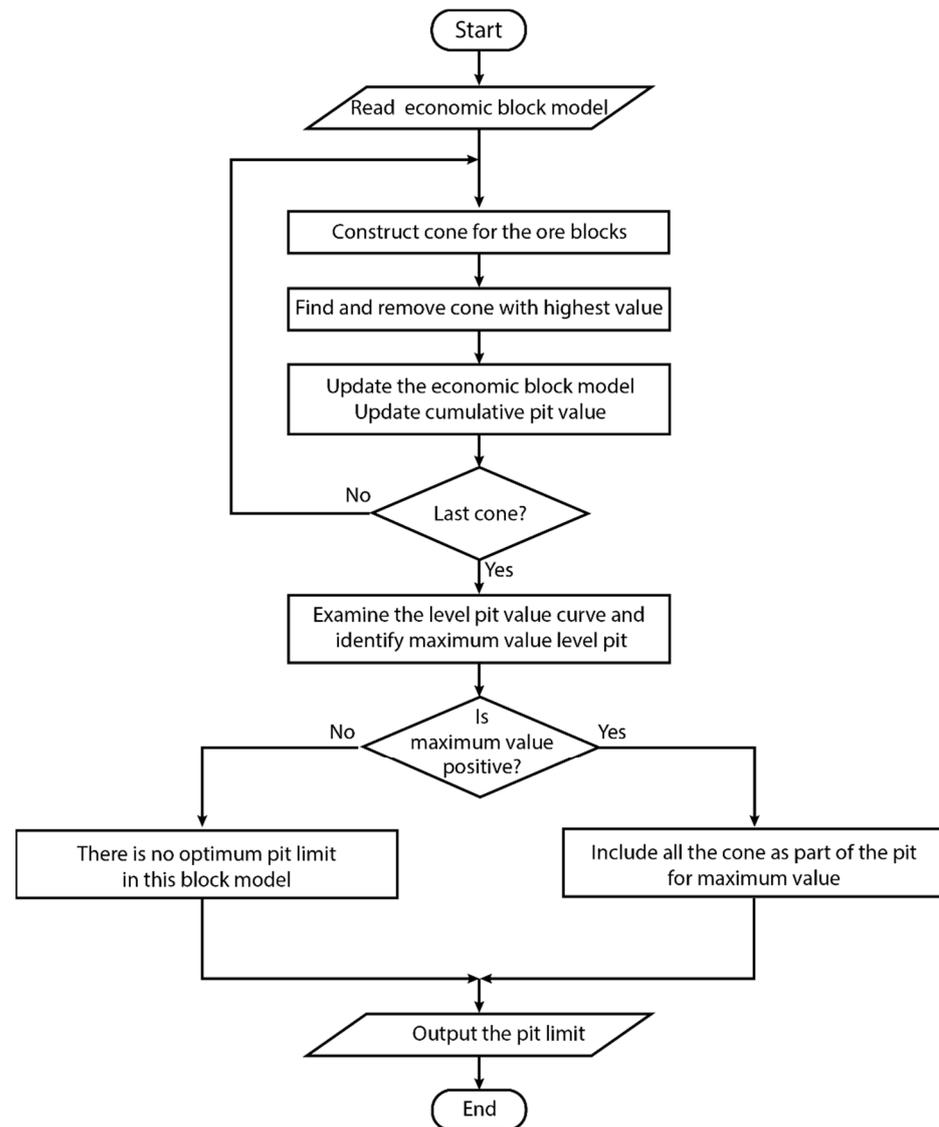


Figure 7. Flowchart of the floating cone II algorithm, method 2 [10].

If this algorithm is used in the block model of Figure 5 the ultimate pit limit has a value of +1 as shown in Figure 8 and Table 4.

	1	2	3	4	5	6	7	8	9	
1	-3	-3	-3	-3	-3	-3	-3	-3	-3	1
2	-4	7	-4	6	-4	-4	-4	8	-4	
3	-5	-5	-5	15	-5	-5	-5	-5	-5	

Figure 8. Limit of the optimal pit calculated by modified floating cone II, method 2.

Table 4. Values of the block model of Figure 4 obtained by the modified floating cone II, method 2.

Stage	Block	Block Value	Cone Value	Cumulative Value	Mineable?
2	(2, 8)	+8	−1	−1	No
	(2, 2)	+7	−2	−3	No
3	(3, 4)	+15	+4	+1	Yes

The problem with this algorithm is that it incorporates blocks to the pit that reduce its final value (problem shown in Figure 3). If the result obtained is compared using this method (Figure 8) with the optimal result of the block model (Figure 5), we can see how the value of the pit has decreased.

4.3. Floating Cone III

A new algorithm was presented in 2012, floating cone III (Figure 9), to solve the problem of the final optimal pit [11]. This floating cone algorithm focuses on the problem of positive blocks that are individually analyzed shown in Figure 2. To do this, it classifies each ore block into two groups: dependent and independent. Ore blocks that share a block with other ore blocks in their cones are called dependent; otherwise, they are classified as independent. These two groups are in turn classified into two other groups according to the value of their cones: effective blocks have positive value cones and ineffective blocks have negative value cones. The cones of the effective blocks are what form the ultimate pit limit. It is important to note that this algorithm does not take into account the cones whose value is null [11]. Once the classification is carried out, the ultimate pit limit is established by the set of cones associated with the effective blocks. The formally established floating cone III algorithm follows the following steps:

1. The algorithm is very similar to the original floating cone algorithm, with the exception that when a cone is removed, the algorithm starts over from the first level. This step ends when all of the independent effective blocks in the model blocks have identified.
2. Levels are analyzed for their effect on each other, e.g., an ore block on overlapping levels can make blocks of ore on underlying levels more or less effective.
3. Following the technical restrictions, cones are built for all ore blocks and ore blocks are classified as dependent or independent.
4. Ineffective independent blocks are identified. These blocks have negative cone value, though they can still form part of a positive block cone.
5. Ineffective and dependent blocks, i.e., the blocks which have negative cone value, are identified. These blocks can be part of the final pit, but only as part of a larger cone, or together with other positive value blocks.
6. Finally, the algorithm studies the remaining positive blocks. To do this, the algorithm continues as follows:
 - a. Identify common blocks for each cone and calculate their weights. The weight of a block equals the number of cones in which said block can be included.
 - b. Calculate the weight of cones. The weight of cone is the sum weight of common blocks of the cone.
 - c. Calculate the value of the cone for each cone.
 - d. Find the importance of each cone. The importance is the ratio between the weight of cones (which is the sum of the weight of the blocks that make up the cone) and cone value.
 - e. Rank the cones at each level in order of importance, then by value in descending order.
 - f. Extract the cones from each level in ascending order, adding the accumulated value.
 - g. If the maximum accumulated value is positive, include the cones as part of the optimal pit. Repeat steps e–g for all levels of the model.

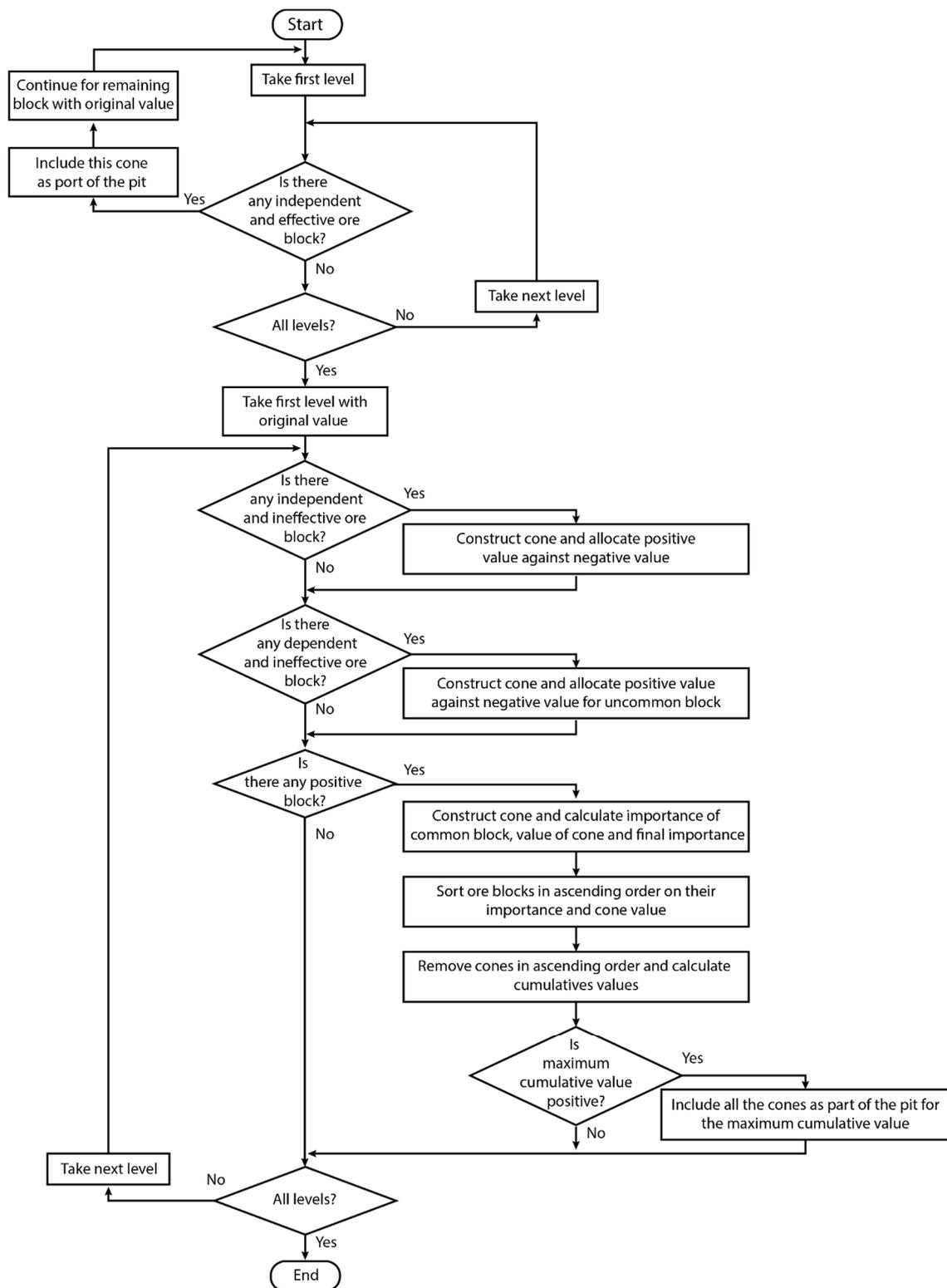


Figure 9. Flowchart of the floating cone III algorithm [11].

The result obtained for the block model shown in Figure 5 with the floating cone III is +2, as can be seen in Figure 10.

-3	-3	-3	-3	-3	-3	-3	-3	-3	2
-4	7	-4	6	-4	-4	-4	8	-4	
-5	-5	-5	15	-5	-5	-5	-5	-5	

Figure 10. Solution generated from the block model presented in Figure 5 by the floating cone III algorithm.

5. Floating Cone IV

A new version of the floating cone algorithm is proposed below, which is called floating cone IV. The floating cone algorithm IV (Figure 11) has two main well differentiated parts. In the first part, cones that are positive are removed from the block model. In the second part, it looks for cones that, although individually they are not positive, the combination of two or more overlapping cones can generate positive values.

The details of the floating cone IV algorithm are explained below:

1. First, the algorithm reads the block model and assigns a value to each block according to Equation (3).
2. In the first part, cones that are positive are removed from the block model following a strict order from top to bottom, taking into account that each time a cone is removed from the set, the process is restarted from the beginning, thus avoiding cones that are profitable due to the positive value of another cone that is above it. Once the first part is finished, the value of all the remaining cones in the block model will be negative.
3. In the second part, the blocks with positive values are crossed again one by one from top to bottom. For each block with a positive value, it will calculate the value of the corresponding cone, which it will call the “cone under study”. If its value is positive, it is removed from the set and this second part is restarted again. The value of these cones should be negative, since the positives were removed in the first part, but when a cone is removed from the set in this second part, it is possible that some cone becomes positive if they have removed blocks that subtracted value, which is why it is returned to ask if the cone is positive before continuing with the process.
4. If the value of the “cone under study” is negative, all positive value blocks that are at the same level or higher whose cones share blocks with the “cone under study” will be selected, ordered from top to bottom, and studied one by one if the value of the corresponding cone, removing the blocks that they share, is ≥ 0 . When the first one that fulfills this condition is identified, it join its blocks, excluding those that they share so as not to repeat them, to the “cone under study”. If the value of the “cone under study” is already positive, it will be removed from the set and the second part will be restarted again. If it is still negative, it will continue with the rest of the selected blocks, repeating the process each time until one with a positive value is found, removing the common ones. If all of the selected blocks have been analyzed and the value of the “cone under study” is still negative, it will go on to the next block with a positive value.
5. When all the blocks with a positive value have been studied in this second part without removing more cones from the set, the process will end. All the blocks that have been removed from the block model will be those that form the ultimate pit limit.

To explain the floating cone algorithm IV (Figure 11), the block model shown in Figure 12 will be used.

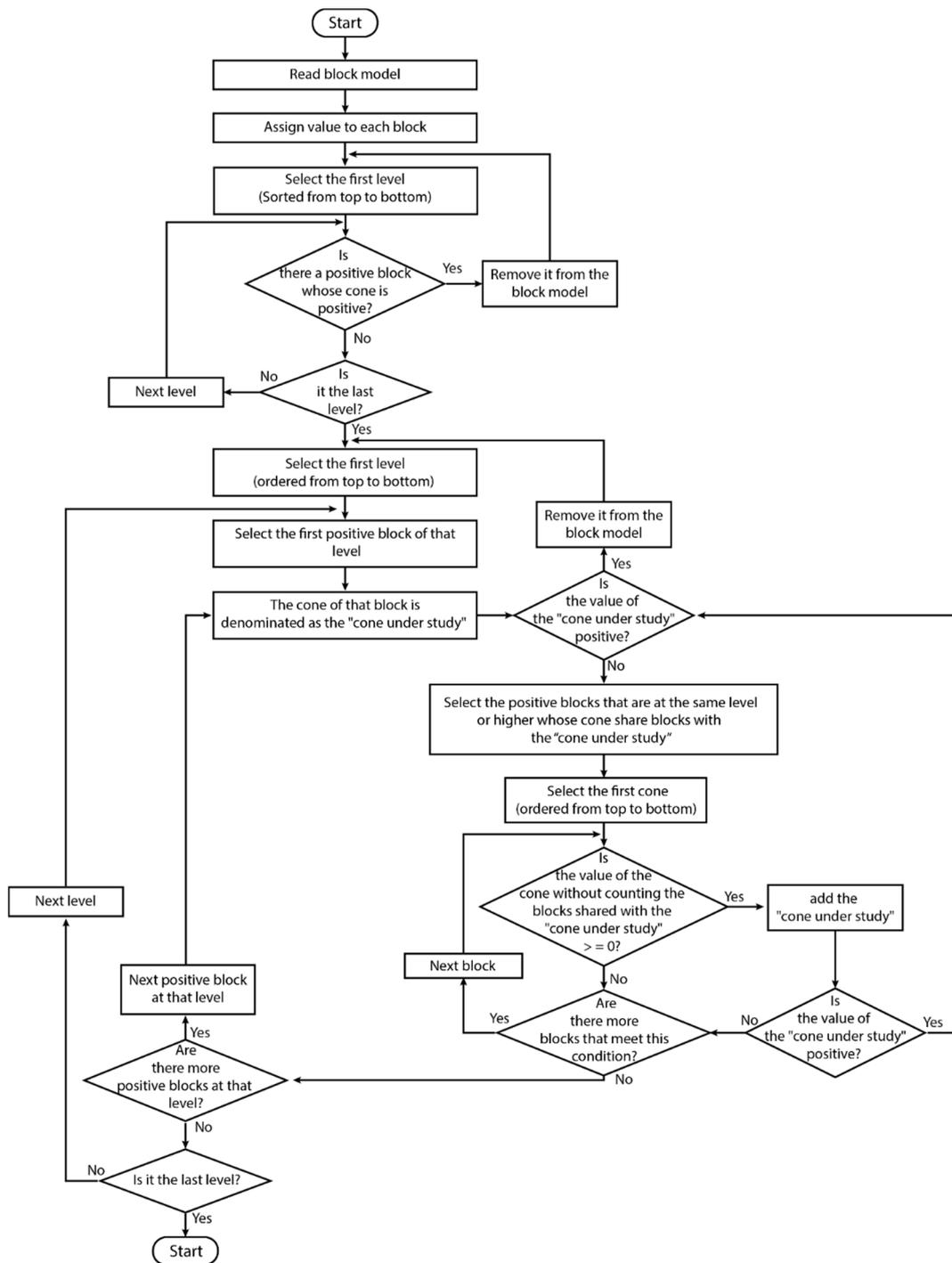


Figure 11. Flowchart of the floating cone IV algorithm.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
2	-10	12	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10
3	-10	-10	-10	-10	-10	-10	-10	-10	41	-10	-10	75	-10	-10
4	-10	-10	-10	-10	-10	145	-10	-10	-10	-10	-10	-10	-10	-10

Figure 12. 2D block model.

The block model in Figure 12 is characterized by the absence of a positive value in any individual cone and only the combination of the block cones (3, 9), (3, 12) and (4, 6) gives the optimal result (+11). If the cone of the block (2, 2) is included in the previous combination, the value of the pit is reduced (+3). The floating cone algorithm IV would work as follows in the block model presented in Figure 12:

1. First, the algorithm analyzes the block model and assigns a value to each block according to Equation (3).
2. Once the value of each block that makes up the model has been calculated, the algorithm follows the same logic as the original floating cone algorithm, as explained above. In this step, cones that are positive are removed from the block model following a strict order from top to bottom. Each time a cone is removed, the process is restarted from the beginning, thus avoiding cones that are profitable due to the positive value of another cone that is above it. In the block model of Figure 12, there is no single cone with a positive result.
3. The remaining cones in the block model will all be negative, otherwise they would have been removed in the previous step. Each of these cones are known as the “cone under study”.
4. The first “cone under study” will be selected and the cones prior to this one (which are above or at the same level) that share blocks with the “cone under study” will be studied. This study will always follow a descending order. In the block model shown in Figure 12, the first “cone under study” would be the cone of block (2, 2) but there are no previous cones or at the same level that share blocks.
5. The next “cone under study” would be the cone formed by the block (3, 9). In this case, it would share blocks with the cone formed by block (3, 12). The value of this second cone (Figure 13), without taking into account the blocks shared with the “cone under study”, would be positive (+15); however, the sum of both cones would still remain negative (-25). Since there are no other cones above or at the same level that share blocks with the cone under study, the next “cone under study” would be passed.
6. The last “cone under study” that remains to be analyzed would be that of the block (4, 6). The first cone that it will share blocks with would be the block cone (2, 2). As can be seen in Figure 14, the value of this second cone, without taking into account the blocks shared with the “cone under study”, is negative (-8), so it is not incorporated into the “cone under study”.
7. The next cone with which the “cone under study” would share blocks would be the cone of the block (3, 9), as shown in Figure 15. In this case, the value of this second cone, not counting the shared blocks with the “cone under study”, is positive (+1), so it is added to the “cone under study” (Figure 15).
8. The new “cone under study” shares blocks with the cone of the block (3, 12). The value of this second cone, not counting the blocks shared with the “cone under study”, is positive (+15) so it would be added to the cone under study (Figure 16). As the “cone under study” is now positive (+11), it is removed from the block model. In the event that the sum of the cones are negative and there are no more cones with which the “cone under study” shares blocks, it would be discarded and the next “cone under study” would be passed. If there is no other “cone under study”, as in this block model, the process will end.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
1	-10	-10	-10	-10	-10	-10									-24
2	-10	12	-10	-10	-10	-10	-10	-39		-10	-10	15		-10	
3	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	
4	-10	-10	-10	-10	-10	145	-10	-10	-10	-10	-10	-10	-10	-10	

Figure 13. The “cone under study”, painted orange, shares blocks with the cone of the block (3, 12), which is painted blue. The sum of both cones gives a negative value (-24).

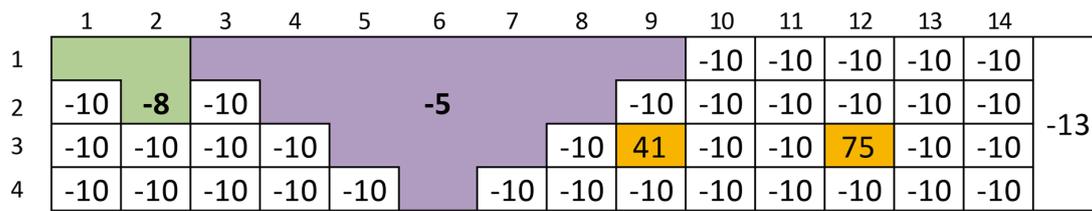


Figure 14. The “cone under study”, painted purple, shares blocks with the cone of the block (2, 2), which is painted green. The sum of both cones gives a negative value (−13).

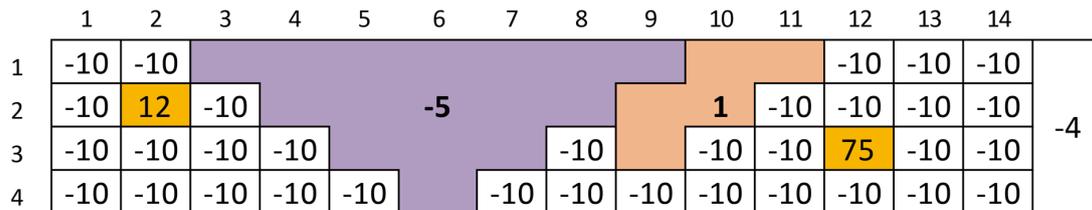


Figure 15. The “cone under study”, painted purple, shares blocks with the cone of the block (3, 9), which is painted orange. The sum of both cones gives a negative value (−5). However, the value of the orange cone is not negative, so it joins the “cone under study”.

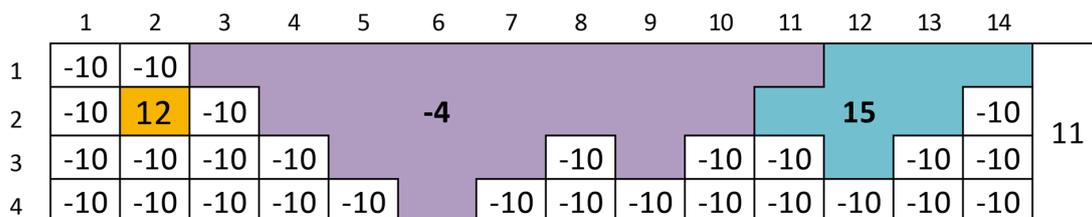


Figure 16. The “cone under study”, painted purple, shares blocks with the cone of the block (3, 12), which is painted blue. The sum of both cones gives a positive value (+11), so it is removed from the block model.

6. Comparison of the Different Algorithms of the Floating Cone

Throughout this section, the results given by the different floating cone algorithms for the block model in Figure 12 are presented.

The original floating cone algorithm is not capable of calculating any pit limit, since, as previously mentioned, no individual cone has a positive value. The floating cone II algorithm (as well as the first modification of it) is not capable of generating a solution either, as can be seen in Table 5.

Table 5. Values of the block model of Figure 12 obtained by the floating cone II.

Stage	Block	Block Value	Cone Value	Cumulative Value	Mineable?
2	(2, 2)	+12	−18	−18	No
3	(3, 12)	+75	−5	−5	No
4	(3, 9)	+41	−39	−24	No
4	(4, 6)	+145	−5	−5	No

The second modification of the floating cone II algorithm, as can be seen in Table 6 and Figure 17, would obtain a pit with a value of +3 (the optimal pit for the block model studied is +11).

Table 6. Values of the block model of Figure 12 obtained by the floating cone II algorithm, method 2.

Stage	Block	Block Value	Cone Value	Cumulative Value	Mineable?
2	(2, 2)	+12	−18	−18	No
3	(3, 12)	+75	−5	−23	No
	(3, 9)	+41	−39	−24	No
4	(4, 6)	+145	−5	+3	Yes

-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	3
-10	12	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	-10	
-10	-10	-10	-10	-10	-10	-10	-10	41	-10	-10	75	-10	-10	
-10	-10	-10	-10	-10	145	-10	-10	-10	-10	-10	-10	-10	-10	

Figure 17. Pit calculated by floating cone II algorithm, method 2 for the block model of Figure 8.

Finally, by means of the floating cone algorithm III, the optimal final pit (+11) was obtained. However, the floating cone III algorithm is more complicated and time consuming than the floating cone IV algorithm. While the floating cone IV algorithm only calculates the value of the cones of the positive blocks and the accumulated value of the “cone under study”, the floating cone III algorithm classifies the positive blocks at each level as either independent or dependent and either effective or ineffective, to be grouped into the following categories: effective independent, independent ineffective and ineffective dependent. After studying the first two groups, we need to calculate the following values for the last group:

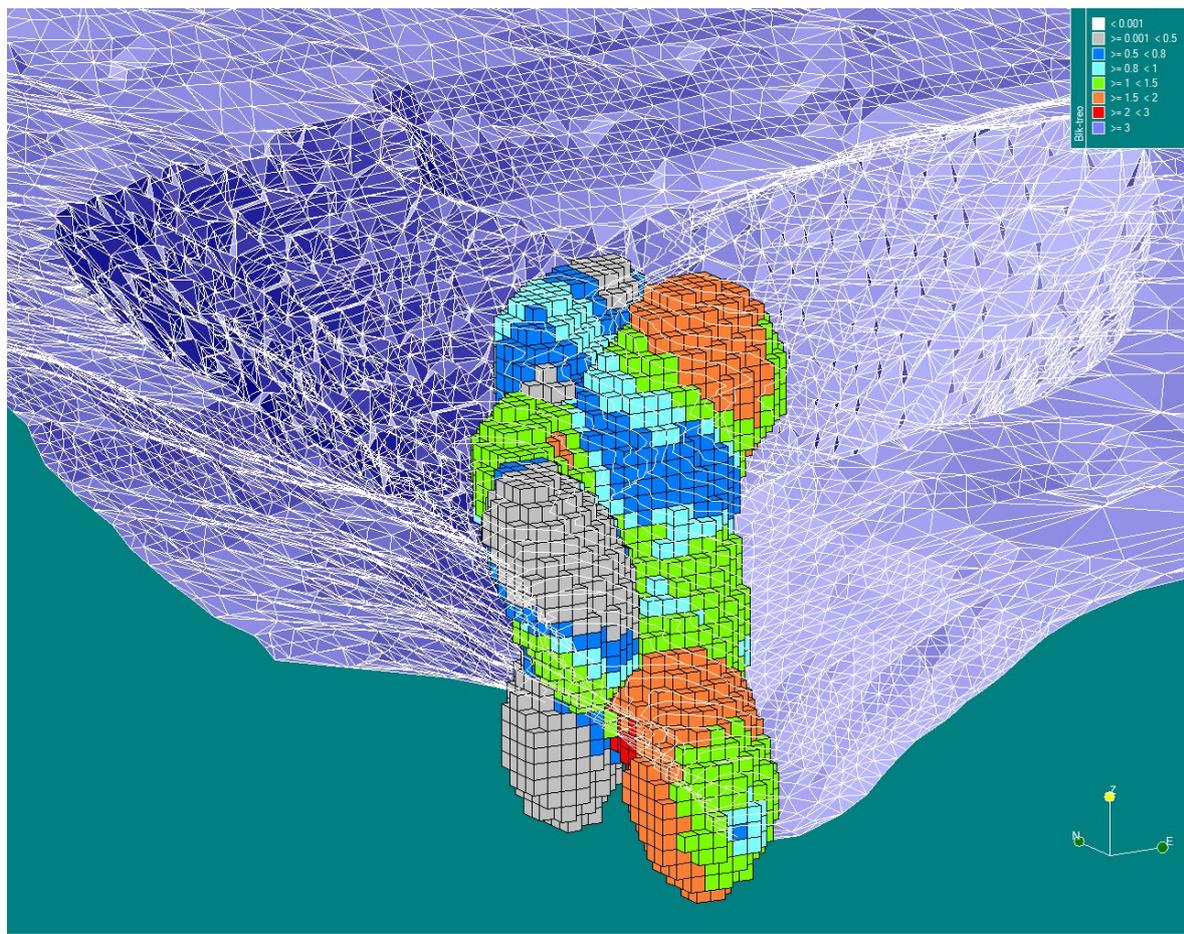
1. Weight of each block (the number of cones in which said block can be included);
2. Weight of the cones (the sum of the weights of all the blocks within each cone);
3. Value of the different cones;
4. Importance of each cone (the relationship between the weight of the cones and the value of the cone).

Once all the aforementioned faces have been calculated, classify the cones in order of importance, then remove the cones in ascending order of importance, adding the accumulated value of the cone. If the maximum accumulated value is positive, include the cones as part of the optimal pit. Once a level is finished, repeat the previous calculations in the next level.

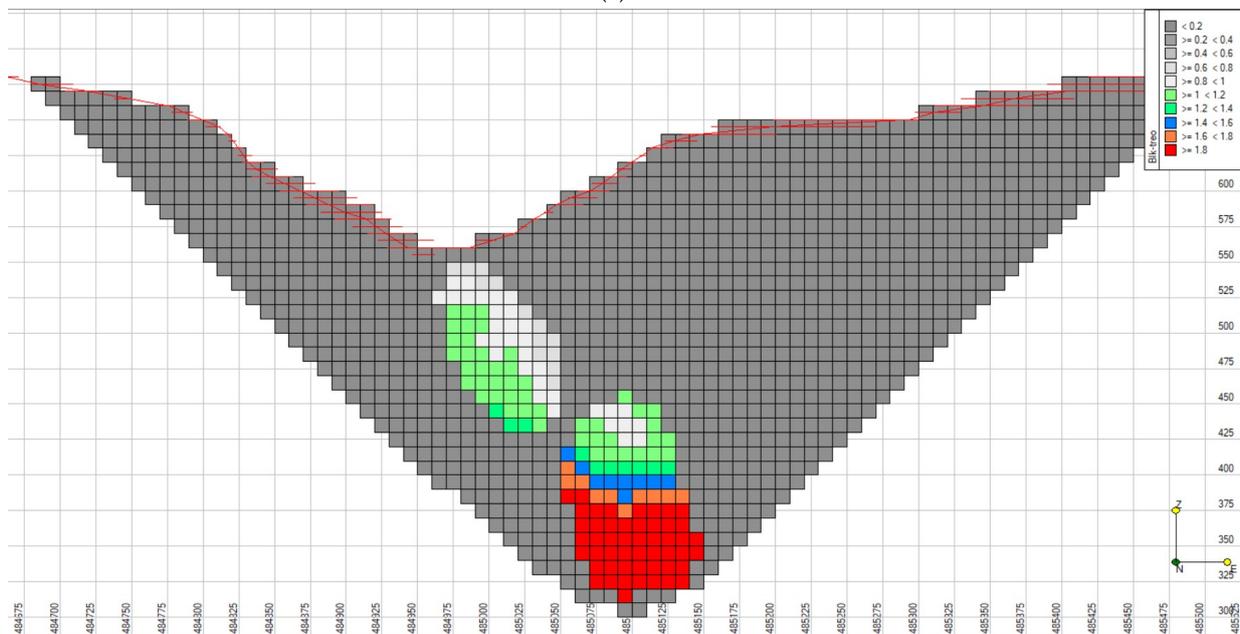
All of the above-mentioned steps and calculations make the floating cone III method much more difficult to understand, complicated to implement, and time consuming than the floating cone IV algorithm. In addition, problems can arise with those cones that have a value equal to zero since the floating cone III algorithm does not specify how these should be classified.

7. Floating Cone IV versus Lerchs–Grossmann

To compare both algorithms, the calculations of the Sarfartoq deposit [21], a rare earth deposit on the southwestern coast of Greenland, were used. (Figure 18).



(a)



(b)

Figure 18. Mineralized body of the Sarfartoq deposit. (a) Mineralized body of the Sarfartoq deposit colored according to the ore grade. The topography has been triangulated using the Recmin software, colored in violet. (b) Ultimate pit limit section obtained using the floating cone IV algorithm in Recmin Pro software. The blocks are colored according to ore grade.

The project is located 60 km southwest of Kangerlussuaq International Airport, in West Greenland. It is a deposit of rare earth carbonatites (rare earth elements, REE), with a very high neodymium content, a key component in the manufacture of permanent magnets directly applied in the construction of electric motors and generators. An estimated 18.1 million tons of ore with an average total rare earth oxide (TREO) content of 1.10%. It is a complex project that involves the creation of energy infrastructure (diesel generators) and transportation (roads, a port in a fjord) in harsh working conditions and only from May to October. The project is in the exploration and development phase [22].

The block model used in the calculations presents a total of 154,849 $10 \times 10 \times 10$ blocks. Figure 18a shows the blocks that define the ore body divided by color based on the ore grade, as well as the digital terrain model of the topography once the pit has been excavated, triangulated using Recmin software.

Starting from the same geological model infused in the block model referred above, three optimization methods were executed and compared. As a result, the inherent uncertainties which are always present when simulating geological or geotechnical materials were cleared. The comparison were made between optimization algorithms, not between different block models.

The calculations of the mining viability were carried out using three procedures (Table 5). Firstly, using Whittle software, which uses the Lerchs and Grossmann algorithm (the de facto standard in the industry, e.g., [23]), although in its latest version it can also apply newer algorithms such as Pseudoflow [16]. The second calculation was performed using the Recmin free software, which uses the floating cone algorithm and the last calculation was performed using the Recmin Pro software, which performed the calculations using the floating cone IV algorithm (Figures 18b and 19).

As can be seen in Table 7, the best result is given by the Lerchs–Grossmann algorithm, with the smallest pit of the three calculated. Although the floating cone method IV improves the results presented by the floating cone algorithm, this method continues to add blocks that reduce the final value of the pit. The differences obtained from the income minus the costs between the different methods barely reached 2%. It is important to note that the acceptable ranges of precision in estimating economic calculations from the preliminary stage to the feasibility stage are the following [24]:

1. Conceptual study: $\pm 30\%$
2. Preliminary study: $\pm 20\%$
3. Feasibility study: $\pm 10\%$

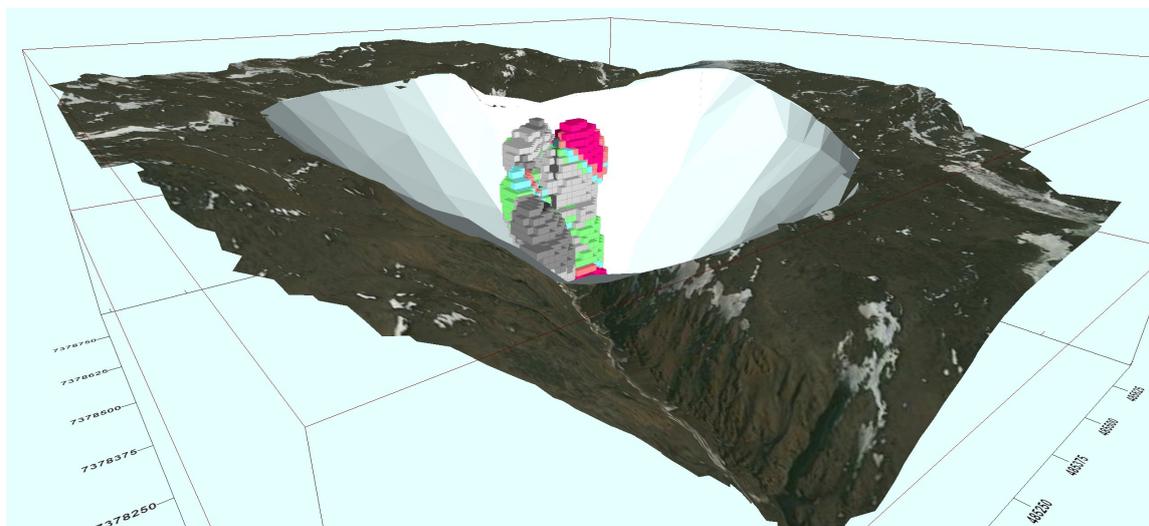


Figure 19. Orthophoto of the area of the Sarfartoq deposit. The ultimate pit limit calculated by the floating cone IV algorithm is represented together with the ore blocks included in the pit. The blocks are colored according to ore grade.

Table 7. Results obtained by the different methods. The values used in the calculation of the economic pits are the following: 43° slope, mining cost \$2.56/t, plant cost \$120.18/t, recovery 76.63%, sale price \$33,470/t, and internal cut-off grade 0.4686.

Date	Lerchs–Grossmann	Floating Cone	Floating Cone IV
No. of pit blocks	53,021	58,211	55,901
No. of ore blocks	5322	5324	5313
No. of waste blocks	47,699	52,887	50,588
Calculation time	3 min 2 s	9 min 47 s	3 min 59 s
Pit volume (m ³)	53,021,000.00	58,211,000.000	55,901,000.000
Pit weight (t)	159,063,000.00	174,633,000.000	167,703,000.00
Ore weight (t)	15,966,000.00	15,972,000.000	15,939,000.00
Waste weight (t)	143,097,000.00	158,661,000.000	151,764,000.000
Ratio	8.963	9.934	9.522
cut-off grade (%)	1.201	1.201	1.201
Metal (t)	191,751.66	191,823.72	191,427.39
Recovered Metal (t)	146,939.30	146,994.52	146,690.81
Meta value (\$)	4,918,058,272.531	4,919,906,471.807	4,909,741,375.79
Ore mining cost (\$)	40,872,960.000	40,888,320.000	40,803,840.00
Waste mining cost (\$)	366,328,320.000	406,172,160.000	388,515,840.00
Processing cost (\$)	1,918,793,880.000	1,919,514,960.000	1,915,549,020.00
Total cost (\$)	2,325,995,160.00	2,366,575,440.00	2,344,868,700.00
Income less cost (\$)	2,592,063,112.531	2,553,331,031.807	2,564,872,675.79

In addition, the acceptable precision for estimating mineral resources and ore reserves is much greater than the differences previously presented [25], i.e., the geological uncertainty in these stages is greater than the differences between the different calculation methods of the optimal pit. Together, the profitable pit must take into account a study of robustness or sensitivity to changes in market conditions, mainly the sale price [21].

The optimal pit, as defined by the different methods, is the basis for designing the final pit in which the mining road will be included. As a result, the final pit will, in reality, have a greater number of blocks. In addition, during the mining operation changes will occur (problems during execution, changes in the market, application of reserves, etc.) that will modify the design of the mine with respect to the optimal pit calculated in these stages.

Finally, it should be noted that the floating cone IV method significantly improves the calculation times of the floating cone, approaching the calculation times presented by the Lerchs–Grossmann algorithm (Table 7). To this last point, it is important to note that in these stages the proper choice of block size is essential, since large blocks with dimensions close to those of the spacing of the sample will improve the reliability of the estimates, while the smaller blocks will have variations of increasingly higher estimates (in addition to much longer optimal pit calculation times). The small block problem has been widely documented [26–28].

It could be said that the use of floating cone methods is out of date. As compared to the Lersch–Grossmann or Pseudoflow methods, it could be slower and give slightly less optimized results. On the contrary, the floating cone method is robust, always gives a solution, and can be easily programmed and its solution checked, as its code can be fully written from scratch with no use of external optimization libraries as is usually done for other methods.

8. Conclusions

The floating cone IV algorithm presented in this article is a notable improvement of the algorithms of the floating cone family, both in terms of results and calculation time, since it is much more precise and faster than other floating cone algorithms in the calculation of cones that are at the limit of being profitable and could become part of the final pit only if they join other cones with which they share blocks. However, the Lerchs and Grossmann method delivers the best results for now.

It is important to highlight that the differences between the different methods that are applied today are very small and are in the blocks whose cones are not cheap but that are joined to other cones. The differences between the different calculations hardly add economic value to the pit, and are not relevant, especially considering that:

1. The calculation of the ultimate pit limit must consider a study of robustness or sensitivity to changes in market conditions, mainly the sale price.
2. The optimal pit defined by the different methods is the basis for designing the final pit in which the transport lanes are included. Therefore, the actual log will implement more blocks in the contour. The floating cone method when making slightly larger pits would already include blocks for the design of the ramps.
3. The geological uncertainty in these stages is greater than the differences between the different methods of calculating the optimal pit.
4. During the mining operation, changes will occur (problems during execution, changes in the market, application of reserves, etc.) that will modify the design of the mine with respect to the ultimate pit limit calculated in these stages.

It is also important to note the notable improvement in the calculation times obtained by the floating cone IV algorithm, bringing it closer to the calculation times obtained by Lerchs and Grossmann.

Due to all of the above, the floating cone method continues to be a fully valid method for calculating the optimal pit of a mineral deposit.

Author Contributions: Conceptualization, C.C.F., I.D.Á. and G.A.; software, C.C.F. and I.D.Á.; investigation, G.A.; writing—original draft preparation, G.A.; writing—review and editing, G.A., C.C.F., I.D.Á., D.A. and A.B.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: No applicable.

Informed Consent Statement: No applicable.

Data Availability Statement: No applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Hartman, H.L. *SME Mining Engineering Handbook*; Society for Mining, Metallurgy, and Exploration: Denver, CO, USA, 1992.
2. Khalokakaie, R.; Dowd, P.A.; Fowell, R.J. Lerchs–Grossmann algorithm with variable slope angles. *Min. Technol.* **2000**, *109*, 77–85. [[CrossRef](#)]
3. Khalokakaie, R.; Dowd, P.A.; Fowell, R.J. A Windows program for optimal open pit design with variable slope angles. *Int. J. Surf. Min. Reclam. Environ.* **2000**, *14*, 261–275. [[CrossRef](#)]
4. Pana, M.T. The simulation approach to open pit design. In Proceedings of the 5th Symposium on the Application of Computers and Operations Research in the Mineral Industries (APCOM), Tucson, Arizona, USA, 10–11 March 1965; pp. ZZ1–ZZ24.
5. Carlson, T.R.; Erickson, J.D.; O’Brain, D.T.; Pana, M.T. Computer techniques in mine planning. *Min. Eng.* **1966**, *18*, 53–56.
6. David, M.; Dowd, P.A.; Korobov, S. Forecasting departure from planning in open pit design and grade control. In Proceedings of the 12th Symposium on the Application of Computers and Operations Research in the Mineral Industries (APCOM), Boulder, CO, USA, April 1974; pp. F131–F142.
7. Dowd, P.A.; Onur, A.H. Open-pit optimization. 1. Optimal open-pit design. *Trans. Inst. Min. Metall.* **1993**, *102*, A95–A104.
8. Lerchs, H. Optimum design of open-pit mines. *Trans CIM* **1965**, *68*, 17–24.
9. Wright, A. MOVING CONE II-A simple algorithm for optimum pit limits design. In Proceedings of the 28th Symposium on the Application of Computers and Operations Research in the Mineral Industries (APCOM), Golden, CO, USA, 20–22 October 1999; pp. 367–374.
10. Khalou, K.R. Optimum Open Pit Design with Modified Moving Cone II Methods. *J. Fac. Eng.* **2007**, *41*, 297–307.
11. Elahi, E.; Kakaie, R.; Yusefi, A. A new algorithm for optimum open pit design: Floating cone method III. *J. Min. Environ.* **2012**, *2*, 118–125. [[CrossRef](#)]
12. Diaz, A.B.; Álvarez, I.D.; Fernández, C.C.; Krzemień, A.; Rodríguez, F.J.I. Calculating ultimate pit limits and determining pushbacks in open-pit mining projects. *Resour. Policy* **2021**, *72*, 102058. [[CrossRef](#)]

13. Githiria, J.; Musingwini, C. Comparison of cut-off grade models in mine planning for improved value creation based on NPV. In Proceedings of the 6th Regional Conference of the Society of Mining Professors (SOMP), Johannesburg, South Africa, 12–13 March 2018; pp. 347–362.
14. Khan, A.; Asad, M.W.A. A method for optimal cut-off grade policy in open pit mining operations under uncertain supply. *Resour. Policy* **2019**, *60*, 178–184. [[CrossRef](#)]
15. Lane, K.F. *The Economic Definition of Ore: Cut-off Grades in Theory and Practice*; Mining Journal Books Limited: London, UK, 1988.
16. Bai, X.; Turczynski, G.; Baxter, N.; Place, D.; Sinclair-Ross, H. Pseudoflow method for pit optimization. In *Whitepaper Geovia Whittle Dassault Systems*; Dassault Systemes: Waltham, MA, USA, 2017.
17. Koenigsberg, E. The optimum contours of an open pit mine: An application of dynamic programming. In Proceedings of the 17th Application of Computers and Operations Research in the Mineral Industry (APCOM), New York, NY, USA, 19–22 April 1982; pp. 274–287.
18. Wilke, F.L. Determining the Optimal Untimate Pit Design for Hard Rock Open Pit Mines Using Dynamic Programming. *Erzmetal* **1984**, *37*, 139–144.
19. Yamatomi, J.; Mogi, G.; Akaike, A.; Yamaguchi, U. Selective extraction dynamic cone algorithm for three-dimensional open pit designs. In Proceedings of the 25th Symposium on the Application of Computers and Operations Research in the Mineral Industries (APCOM), Brisbane, Australia, 9–14 July 1995; Australasian Institute of Mining and Metallurgy: Carlton, VIC, Australia, 1995; pp. 267–274.
20. Denby, B.; Schofield, D. Open-pit design and scheduling by use of genetic algorithms. *Trans. Inst. Min. Metall.* **1994**, *103*, A21–A26.
21. García, M.V.R.; Krzemień, A.; Bárcena, L.C.S.; Álvarez, I.D.; Fernández, C.C. Scoping studies of rare earth mining investments: Deciding on further project developments. *Resour. Policy* **2019**, *64*, 101525. [[CrossRef](#)]
22. Cambon, J. Hudson Receives New Sarfartoq Rare Earth and Niobium Exploration License. Hudson Resources Inc. 9 March 2020. Available online: <https://hudsonresourcesinc.com/hudson-receives-new-sarfartoq-rare-earth-and-niobium-exploration-license/> (accessed on 30 January 2022).
23. Akisa, D.M.; Mireku-Gyimah, D. Application of surpac and whittle software in open pit optimisation and design. *Ghana Min. J.* **2015**, *15*, 35–43.
24. Lee, T.D. Planning and mine feasibility study—An owner perspective. In Proceedings of the 1984 NWMA Short Course “Mine Feasibility—Concept to Completion”, Spokane, WA, USA, 15–17 April 1984.
25. Dominy, S.C.; Noppé, M.A.; Annels, A.E. Errors and uncertainty in mineral resource and ore reserve estimation: The importance of getting it right. *Explor. Min. Geol.* **2002**, *11*, 77–98. [[CrossRef](#)]
26. David, M. *Geostatistical Ore Reserve Estimation*; Elsevier: Amsterdam, The Netherlands, 1977.
27. David, M. *Handbook of Applied Advanced Geostatistical Ore Reserve Estimation*; Elsevier: Amsterdam, The Netherlands, 1988.
28. Annels, A.E. *Mineral Deposit Evaluation: A Practical Approach*; Springer: Berlin/Heidelberg, Germany, 2012.