Fast and Accurate Database Using N-linear Interpolation for Reflectarray Analysis, Layout Design and Crosspolar Optimization

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***Abstract*—This work provides an assessment in terms of computational speed and accuracy at the radiation pattern level of a database to characterize the reflectarray unit cell for use in analysis, layout design and crosspolar pattern optimization. Details of an efficient implementation of the database are provided, and its performance is compared to that provided by a machine learning technique based on support vector regression (SVR). A method of moments based on local periodicity serves as the baseline for the comparison, both in terms of accuracy and acceleration. The results show that, even when the database uses a simple N-linear interpolation, accuracy is similar to that provided by the SVR technique while accelerating the analysis and layout design of reflectarray antennas.**

***Index Terms*—Database, look-up table (LUT), N-linear interpolation, machine learning, surrogate model, support vector regression (SVR), crosspolar optimization, reflectarray antenna**

I. Introduction

An accurate analysis of a reflectarray antenna requires the use of a full-wave analysis tool based on local periodicity, usually a method of moments (MoM-LP), to obtain the reflection coefficients that characterize the electromagnetic response of the unit cell [1]. This tool provides a reasonable computational speed for reflectarray analysis and design compared with a full-wave analysis of the whole antenna, but it is relatively slow for a direct optimization of the layout [2]. Some common techniques for the acceleration of the reflectarray analysis include the use of databases (also known as look-up tables or LUT) [3] or machine learning techniques such as artificial neural networks [4], [5], ordinary kriging [6] or support vector machines applied to regression (SVR) [7]. While both approaches have been shown to greatly accelerate reflectarray layout design and direct layout optimization with regard to MoM-LP, they have yet to be compared against each other.

In this work, we propose a simple database of reflection coefficients with efficient memory access and a fast but effective N-linear interpolation (i.e., a linear interpolation in N dimensions) for the analysis, layout design and crosspolar optimization of reflectarray antennas. Its performance is benchmarked against the baseline provided by the MoM-LP tool from which the database is generated in terms of speed-up and accuracy at the radiation pattern level. In addition, the database is also compared with another tool based on SVR, which has previously been shown to provide substantial acceleration while preserving the accuracy in the analysis of the unit cell [8]. Results show that, as along as the database is conveniently populated, accuracy is similar to that obtained with the SVR while achieving faster design layouts and avoiding the machine learning training phase, which can be computationally expensive.

II. Description of the Database

A. Structure of the Database

The main goal of employing a database for reflectarray analysis, design and optimization is to considerably accelerate those tasks with regard to the use of a MoM-LP. To that end, it is important to consider how the database is going to be stored and accessed in memory. In this regard, we generate the database for N_f frequencies, N_a angles of incidence (θ, φ) (as seen from the reflectarray surface) and N_g combinations of geometrical features (such as patch dimensions, dipole lengths, etc.). The database is stored in a rank-3 array whose first index refers to the frequency, the second to the pair (θ, φ) and the third to the geometrical features of the unit cell. The frequencies are treated as discrete entities in the sense that no interpolation is performed to obtain reflection coefficients at frequencies that are not contained in the database. A similar approach is followed with the angles of incidence (θ, φ), that are discretized into a small set and then the real angle of incidence is approximated with one of the considered (θ, φ) in the database set. This approach was demonstrated in other works when working with SVR, see [9], [10] for more details. Thus, the interpolation of the reflection coefficients will be carried out only with regard to the geometrical features of the unit cell.

Other parameters of the unit cell, such as substrate height, permittivity and the periodicity will remain fixed.

B. Fast Memory Access for the Reflection Coefficients

The determination of the index for the frequency and angle of incidence is not an issue since N_f and N_a are usually small numbers, so a linear search to determine them from a list of available frequencies and (θ, φ) is enough. However, this is not the case with the geometrical features, since the number of total combinations N_g grows exponentially with the number of degrees of freedom (DoF) that the unit cell provides. Thus, a linear search over the N_g entries to determine the reflection coefficients selected to carry out the interpolation is too expensive, especially in the cases of layout design and crosspolar optimization where the database is extensively used (of the order of hundreds of thousands of times for large reflectarrays).

Thus, the fast memory access approach to find the reflection coefficient indices in the database is as follows:

1) In the database initialization, we store D arrays with the values of the geometrical features of the unit cell, where D is the number of DoF. These values are denoted as:
$$x_{i,k_i},\ i = 1,\dots,D;\ k_i = 1,\dots,N_{g_i}. \qquad (1)$$
Notice that $N_g = \prod_{i=1}^{D} N_{g_i}$, where N_{g_i} is the number of points in which the length of the i-th DoF is discretized.
2) The values $x_i^*, i = 1,\dots,D$ are those points at which we want to obtain the reflection coefficients. Note that the x_i^* values are not in general stored in the database and thus interpolation must be carried out.
3) We employ a modified binary search in each dimension i to find an index k_i such that:
$$x_{i,k_i} \le x_i^* \le x_{i,k_i+1},\ i = 1,\dots,D. \qquad (2)$$
4) Then, the general index to directly obtain the desired reflection coefficient from the database is:
$$k = k_1 + \sum_{i=2}^{D}(k_i - 1)\prod_{h=1}^{i-1} N_{g_h}. \qquad (3)$$

Notice that the indices k_i may be employed not only to obtain index k to access the desired reflection coefficient, but also to obtain the coordinates x_{i,k_i} and x_{i,k_i+1} which are employed in the N-linear interpolation as described below.

C. N-Linear Interpolation

The database performs an N-linear interpolation. According to the notation employed in the previous subsection, $N = D$. The general equation for the interpolation is [11]:
$$f(\vec{x}) = \sum_{u=0}^{2^D-1} R_{(k_1+b_1(u),\dots,k_D+b_D(u))}\prod_{i=1}^{D} W_i^{b_i(u)}, \qquad (4)$$
where $\vec{x} = (x_1^*,\dots,x_D^*)$ are the coordinates of the desired interpolated reflection coefficient and they correspond to the physical lengths of the geometrical features of the unit cell; $b_i(u)$ is a function that gives the i-th bit of the integer number u; R is the selected reflection coefficient for a given u and indices k_i; and W_i is the weighting vector
$$W_i = \left(1 - w_i(x_i^*), w_i(x_i^*)\right), \qquad (5)$$
where
$$w_i(x_i^*) = \frac{x_i^* - x_{i,k_i}}{x_{i,k_i+1} - x_{i,k_i}}. \qquad (6)$$

A clarification about the notation of R and W_i in (4) regarding the sub- and super-scripts is in order. First, in the case of $W_i^{b_i(u)}$, the sub-script indicates the current dimension, while the super-script refers to the indization of the array in (5), which has been indexed such as the first component has index zero. Since the function $b_i(u)$ returns a bit, which can only be 0 or 1, either $1 - w_i(x_i^*)$ or $w_i(x_i^*)$ are selected depending on the value of the indices i and u to find the product $\prod_{i=1}^{D} W_i^{b_i(u)}$.

The reflection coefficient R in (4) is indexed with the following notation:
$$(k_1 + b_1(u),\ k_2 + b_2(u),\ \dots,\ k_D + b_D(u)). \qquad (7)$$
As we recall from the previous subsection, k_i are the indices, found by binary search for each dimension i, that correspond to the coordinates that bound [i.e. right above and below, see (2)] the desired coordinate x_i^* where we want to interpolate the reflection coefficient. In a space of D dimensions, we consider a hyper-rectangle of 2^D vertices, which are employed to carry out the interpolation with (4). The indices of the vertex closest to the origin would be $(k_1,\ k_2,\ \dots,\ k_D)$, and it corresponds to $u = 0$, since its binary representation is all zeros. Then we follow the rest of the vertices in a lexicographical order by considering the binary pattern of a number $u = 0, 1, \dots, 2^D - 1$ comprised of D bits. With this idea and using (7) to index the reflection coefficients, we can easily find the general index for the reflection coefficient for an efficient memory access by combining (7) and (3), and then to perform the interpolation with (4).

Finally, it is worth mentioning that since the four reflection coefficients are complex numbers, the multidimensional linear interpolation is applied to their real and imaginary parts, in such a way that (4) is applied eight times every time the database is invoked.

III. Results

A. Testing Conditions

To test the proposed database, the same large rectangular reflectarray of [9] is employed for a fair comparison with other techniques. It is comprised of 7 052 elements and a contoured beam pattern in dual-linear polarization was selected to assess the accuracy at the radiation pattern level. In addition, the same unit cell is employed, which consists in eight coplanar and parallel dipoles, four for each polarization. For the sake of comparison with other works, we reduce the number of DoF to two by imposing a scaling between parallel dipoles as in [9], thus having $N = D = 2$.

In order to compare the accuracy and computing performance of the database, the SVR-based analysis technique described in [8] is employed, with the same 2D SVR used in [9]. The MoM-LP employed as baseline and used to populate the database and generate the training samples of the SVR is fully described in [12]. The total number of samples for the database and the SVR are the same, 380 000 ($N_f = 1$, $N_a = 152$, $N_{g_1} = N_{g_2} = 50, N_g = N_{g_1}N_{g_2}$).

The three tools (database, SVR and MoM-LP) will be compared for three different tasks: obtain a reflectarray layout such that it generates the desired phase-shift distribution, perform a single analysis of a reflectarray layout and carry out a direct layout optimization for the crosspolar optimization.

To assess the computing efficiency, an Intel i9-9900 CPU working at 3.1 GHz and with 32 GB of memory has been

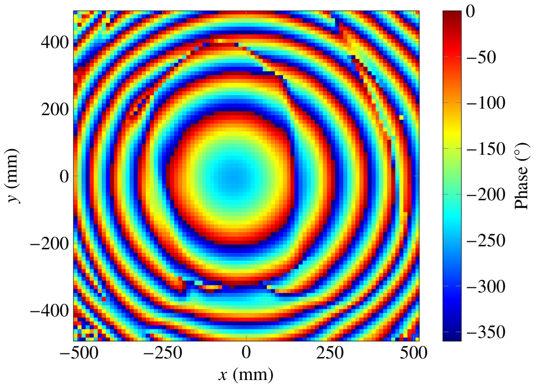

Fig. 1. Phase distribution obtained with a phase-only synthesis algorithm to achieve a European contoured-beam copolar pattern.

Table I
PERFORMANCE OF THE PROPOSED DATABASE WITH REGARD TO SVR AND MOM-LP TOOLS FOR LAYOUT DESIGN.

Tool	Time (s)	Speed-up
MoM-LP	1 572.55	1
SVR	1.11	1 417
Database	0.03	52 418

employed. All computations have been parallelized employing the maximum number of threads allowed by this CPU.

B. Layout Design

For the layout design we consider the phase distribution shown in Fig. 1. It was obtained using a phase-only synthesis algorithm, namely the generalized Intersection Approach [13] to provide a European coverage from a geostationary satellite [9]. From this phase distribution, the layout is obtained by following the procedure detailed in [13], which can be summarized in the following three steps: 1) generate a phase-shift table for each linear polarizaiton, 2) use a linear equation approximation to find the desired geometrical unit cell length for each polarization independently, 3) fine-tune the geometrical length of both polarizations at the same time by employing the Newton-Raphson gradient method.

This methodology was applied with the three tools and the computing performance can be seen in Table I. As it can be seen, both the database and the SVR tool are substantially faster than the MoM-LP, but the database is an order of magnitude faster than the SVR for this task.

For the layout design, the accuracy will be analysed from two different perspectives. First, Fig. 2 shows the relative error in the obtained design when using the database and the SVR tool compared to the design obtained with MoM-LP. As it can be seen, the relative error for both tools is very low and of the same order, confirming the accuracy of the proposed database.

Secondly, we need to assess if those differences in the designed layout have any repercussion in the radiation pattern. To that end, each layout is simulated with the MoM-LP tool

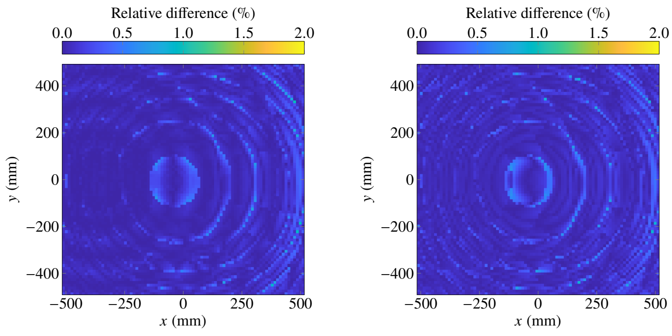

Fig. 2. For the reflectarray layout design, relative error of the layout for polarization X with regard to the design carried out with MoM-LP of the (a) database and (b) SVR.

Table II
FIGURES OF MERIT OF A EUROPEAN-COVERAGE PATTERN WHEN THE DESIGN IS CARRIED OUT USING DIFFERENT TOOLS AND SIMULATED WITH MOM-LP. CP_{MIN} IS IN DBI AND XPD_{MIN} AND XPI ARE IN DB.

	Pol. X			Pol. Y		
Design tool	**CP_{min}**	**XPD_{min}**	**XPI**	**CP_{min}**	**XPD_{min}**	**XPI**
MoM-LP	30.03	32.95	32.91	30.02	32.94	32.90
Database	30.03	32.97	32.92	30.01	32.95	32.90
SVR	30.03	32.95	32.90	30.00	32.94	32.88

and the main figures of merit are compared for both linear polarizations. These are, for the coverage zone: minimum copolar gain (CP_{min}), minimum crosspolar discrimination (XPD_{min}) and crosspolar isolation (XPI). The results are provided in Table II. As it can be seen, the differences in the figures of merit are negligible, proving that the small differences in the layout design shown in Fig. 2 barely affect the radiation pattern in the coverage zone. Indeed, the relative error of the radiation pattern obtained with the design carried out with the database compared with that of the design obtained with MoM-LP is smaller than 0.2% for the copolar pattern and smaller than 0.7% for the crosspolar pattern.

C. Reflectarray Analysis

The previous assessment compared the radiation patterns when three different layouts obtained with MoM-LP, SVR and the database were simulated with the same tool, namely MoM-LP. In this way, we could determine the accuracy of the database to perform a layout design. Now, we will assess the accuracy in the analysis of a given layout. To that end, the layout obtained with the database in the previous subsection is selected and simulated with the three tools to compare again the three figures of merit. These results are gathered in Table III. In this case, there are more differences among the numbers, especially for the cross-polarization figures of merit, although they are still very small. The largest difference is for XPD_{min} for polarization X, with a difference of 0.17 dB between the simulations using the database and MoM-LP.

Table III
FIGURES OF MERIT OF A EUROPEAN-COVERAGE PATTERN WHEN THE DESIGN IS CARRIED OUT USING THE DATABASE AND SIMULATED WITH DIFFERENT TOOLS. CP_{MIN} IS IN DBI, XPD_{MIN} AND XPI ARE IN DB.

	Pol. X			Pol. Y		
Analysis tool	**CP_{min}**	**XPD_{min}**	**XPI**	**CP_{min}**	**XPD_{min}**	**XPI**
MoM-LP	30.03	32.97	32.92	30.01	32.95	32.90
Database	29.97	32.81	32.59	29.94	32.85	32.81
SVR	30.03	32.80	32.58	30.02	32.79	32.76

However, comparing the database and the SVR simulations, the figures of merit are very similar. This is due to the error in the radiation pattern produced by the discretization of the angles of incidence, which has been thoroughly studied in [10].

Fig. 3 shows a visual comparison of the radiation pattern obtained by the analyses carried out with MoM-LP, the database and SVR. This radiation pattern is the same pattern used to collect the data in Table III. As it can be seen, the simulations with the database and SVR offer virtually the same results, and both simulations match with a high degree of accuracy that of the MoM-LP for the copolar pattern. In the case of the crosspolar pattern there are some important discrepancies at levels around -20 dBi, but these levels are 50 dB below the peak gain. More importantly, for higher crosspolar values around and within the coverage area, the database offers good accuracy when compared with the MoM-LP simulation.

D. Direct Layout Optimization for Cross-polarization Improvement

Once we have established the accuracy of the database for the analysis and layout design of reflectarray antennas, we will test its suitability for a direct layout optimization employing the algorithm described in [13]. As starting point for the optimization, the layout designed using the database will be used. Thus, the starting values for CP_{min}, XPD_{min} and XPI are the ones shown in the first row of Table III. In addition, for the sake of comparison, three optimizations are performed, one with each tool. Moreover, the three optimizations are stopped after 16 iterations and the resulting layout is simulated, in all cases, with the MoM-LP tool to assess the difference in performing the optimization with each tool, removing the differences in the final analysis, which has already been assessed in the previous subsection.

From a computing time perspective, the use of the database shows a similar performance to that given by the SVR, as shown in Table IV. The mean time per iteration and the time to compute the Jacobian matrix are very similar using both tools. The explanation for this phenomenon, given that the database is significantly faster than the SVR for analysis and layout design, comes from the fact that both tools are so fast that the dominant time to perform the computation of the Jacobian matrix comes from the rest of operations, which also involve the computation of the radiation pattern and the figures of merit for both linear polarizations.

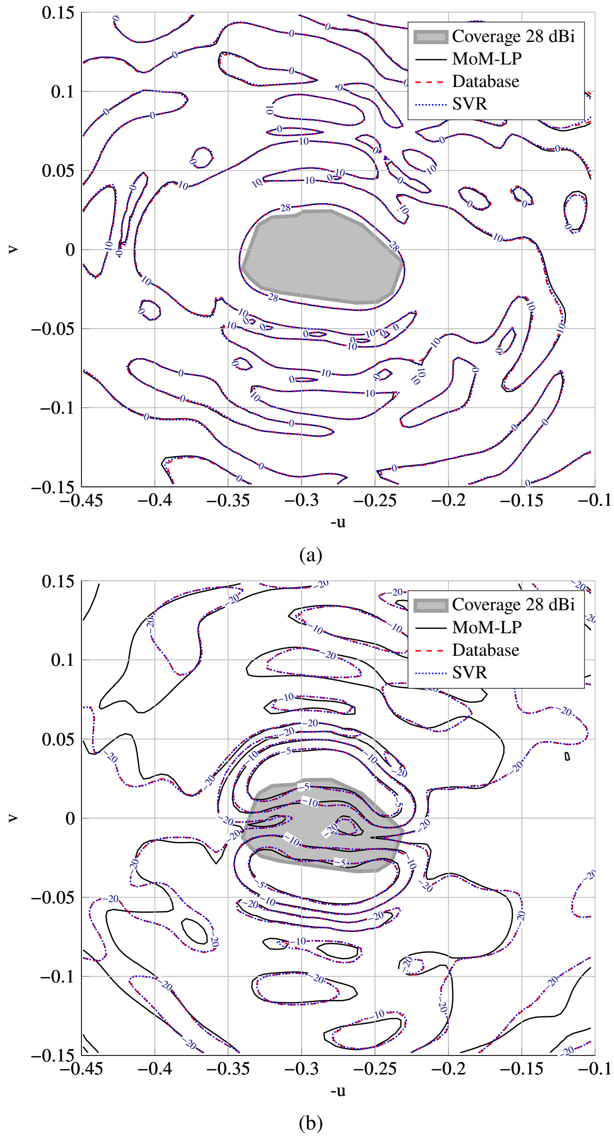

Fig. 3. Comparison of the simulations with MoM-LP, database and SVR for the (a) copolar and (b) crosspolar patterns for polarization X of a reflectarray with European coverage using a layout obtained with the database.

Table IV
COMPUTING TIME PERFORMANCE OF THE DIRECT LAYOUT OPTIMIZATION CONSIDERING THREE DIFFERENT TOOLS FOR THE ANALYSIS OF THE REFLECTARRAY UNIT CELL. TIME IS IN SECONDS.

Tool	**Time per iteration**	**Jacobian time**
MoM-LP	69.39	34.06
SVR	5.06	2.24
Database	5.18	2.13

Finally, Table V shows the results of the three optimized layouts simulated with MoM-LP. As it can be seen, in all cases the cross-polarization performance of the reflectarray is considerably improved while preserving the copolar pattern. The cross-polarization figures of merit after the optimization with the database and the SVR are similar, and in both cases

Table V

COMPARISON OF THE FIGURES OF MERIT OF A EUROPEAN-COVERAGE PATTERN AFTER A DIRECT LAYOUT OPTIMIZATION TO IMPROVE CROSS-POLARIZATION PERFORMANCE. THREE OPTIMIZATIONS HAVE BEEN CARRIED OUT WITH DIFFERENT TOOLS AND, IN ALL CASES, THE LAYOUT AFTER 16 ITERATION OF THE OPTIMIZATION ALGORITHM HAS BEEN SIMULATED WITH MOM-LP. CP_{MIN} IS IN DBI, XPD_{MIN} AND XPI ARE IN DB.

	Pol. X			Pol. Y		
Opt. tool	$\mathbf{CP_{min}}$	$\mathbf{XPD_{min}}$	**XPI**	$\mathbf{CP_{min}}$	$\mathbf{XPD_{min}}$	**XPI**
MoM-LP	29.90	39.80	39.62	30.01	40.07	39.93
Database	29.92	38.62	38.52	30.01	39.27	38.89
SVR	29.94	38.74	38.65	30.01	39.14	38.87

lower than those obtained with MoM-LP. Given the previous results, these discrepancies can again be attributed in part to the discretization of the angles of incidence [10].

Despite the small differences, the use of the database for the analysis, layout design and crosspolar optimization remains highly accurate while considerably reducing computing times with regard to the use of a MoM-LP tool, and does not require the training process that any machine learning technique must go through. The studies carried out in this work have been performed for databases and SVR considering two DoF ($D = 2$), one per polarization. It is possible that for higher dimensionality machine learning techniques require far fewer training samples than a database to maintain the desired accuracy [9]. However, this would require to tackle the problem of the resonances that may appear in the reflection coefficients for a successful training, which is averted in the database at the potential expense of an exponentially increase in the number of samples (i.e., database entries).

IV. CONCLUSION

In this work, we have presented a simple and efficient database for reflectarray analysis, layout design and direct layout optimization for cross-polarization improvement. The database employs a multidimensional N-linear optimization and an efficient access to the reflection coefficients stored in memory for a very fast analysis of the unit cell.

In order to assess the performance of this database both in terms of computational efficiency and accuracy at the radiation pattern level, it has been compared with the MoM-LP tool employed to populate the database and with a machine learning technique based on SVR. In all cases shown in this work, the database presents a high degree of accuracy compared with the simulations carried out with MoM-LP while significantly improving computing times. It is four orders of magnitude faster than MoM-LP to carry out analysis and layout design, while it is an order of magnitude faster than the SVR for the same taks. For the direct layout optimization, both the database and the SVR offer similar computing times with similar accuracy, both being one order of magnitude faster than the MoM-LP.

The use of the database, despite using a simple N-linear interpolation, offers good accuracy and fast computing times compared to the use of MoM-LP. Compared to machine learning techniques such as SVR, it avoids the training phase that can be time consuming [9].

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