Multivariate Look-Up Table Based on N-Linear Interpolation for General Reflectarray Design

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***Abstract*—We present a multivariate look-up table (LUT) based on N-linear interpolation for a fast general reflectarray design. In order to achieve an efficient implementation of the LUT, details are provided on its structure and a technique for a fast memory access of the reflection coefficients is described. A general formulation for the N-linear interpolation is given and then it is applied to a general reflectarray design procedure, including the improvement of the cross-polarization figures of merit. In order to benchmark the computational efficiency of the LUT, it is compared with other two tools, a method of moments based on local periodicity and surrogate models based on support vector regression. Results show that the LUT achieves a high computational efficiency while preserving a high degree of accuracy.**

I. Introduction

The most common approach to analyse reflectarray antennas is to employ a full-wave analysis tool based on local periodicity to obtain the electromagnetic response of the constituent unit cell. Typically, a method of moments based on local periodicity (MoM-LP) is employed [1], [2], which provides a good trade-off between the accuracy of a full-wave analysis of the whole antenna —which results in very high computing times—, and computational efficiency. However, even using a MoM-LP may result relatively slow in some situations, such as the direct optimization of very large reflectarray antennas [3]. Thus, it is useful to find alternatives approaches to further accelerate computations while preserving a high degree of accuracy. To that end, some new methods include the use of machine learning techniques, such as artificial neural networks [4], [5], ordinary kriging [6] or support vector machines applied to regression (SVR) [7] to generate surrogate models of the unit cell. Another interesting approach is to the use of look-up tables (LUT) [8], which do not require a training stage as the surrogate model generation does. Both methodologies have proven to provide accurate results while speeding-up reflectarray analysis with regard to a MoM-LP tool. However, they have yet to be measured against each other.

This paper presents an efficient LUT of reflection coefficients with a very fast memory access and an effective N-linear interpolation for the analysis of reflectarrays. Details about the structure of the database are provided, as well as the general formulation for the N-linear interpolation. In addition, the performance of the LUT is compared with that of surrogate models based on SVR as well as the MoM-LP employed to generate samples for the LUT and SVR model training. Results show that a high degree of accuracy is achieved for reflectarray analysis while considerably accelerating computing times.

II. Description of the Look-Up Table

A. Structure of the Look-Up Table

The objective of using a look-up table for reflectarray analysis is to significantly speed-up computations when compared with a MoM-LP tool. It is thus significant to consider how the LUT is stored and accessed in memory. When tackling reflectarray design, the frequency is fixed, and depending on the application it may work on a given bandwidth. Thus, we will consider a set of N_f discrete frequencies. In addition, the substrate is usually fixed and chosen beforehand, so it will not be taken into account for the LUT. And similarly for the periodicity. The angles of incidence at each reflectarray element are fixed once the antenna optics is selected. For the present work, we consider a set of N_a angles of incidence (θ, φ). Finally, we also consider a set of N_g combinations of geometrical features (such as patch dimensions, dipole lengths, etc.). Thus, the total of entries that will be stored in the LUT is

$$N_t = N_f N_a N_g. \tag{1}$$

The LUT is stored in a rank-4 array whose indices refer to the reflection coefficient, frequency, (θ, φ) pair and geometrical features of the unit cell. Moreover, both the frequency and angle of incidence are treated as discrete entities in the sense that no interpolation is performed to obtain reflection coefficients at frequencies or angles of incidence that are not stored in the LUT. In the case of the angles of incidence, given the real (θ, φ) on a given reflectarray element, it is approximated by the closest (θ, φ) stored in the LUT. This approach has proven accurate when the discretization is fine enough [9]. Thus, the interpolation of the reflection coefficients will be carried out only with regard to the geometrical features of the unit cell.

B. Fast Memory Access for the Reflection Coefficients

Since the LUT is stored as a 4-rank array, with separate indices for the frequency, angle of incidence and geometrical features, selecting the index for the frequency and angle of incidence is very fast since N_f and N_a are usually small numbers. For these variables, a linear search will suffice. However, in the case of the geometrical features of the unit cell, N_g can be a very large number, and so using a linear search is too expensive, particularly in cases of optimization where the LUT is invoked hundreds of thousands of times.

A strategy for a fast memory access of the reflection coefficients in the LUT is the following:

1) During the LUT initialization, D arrays are stored with the values of the unit cell geometrical features, being D

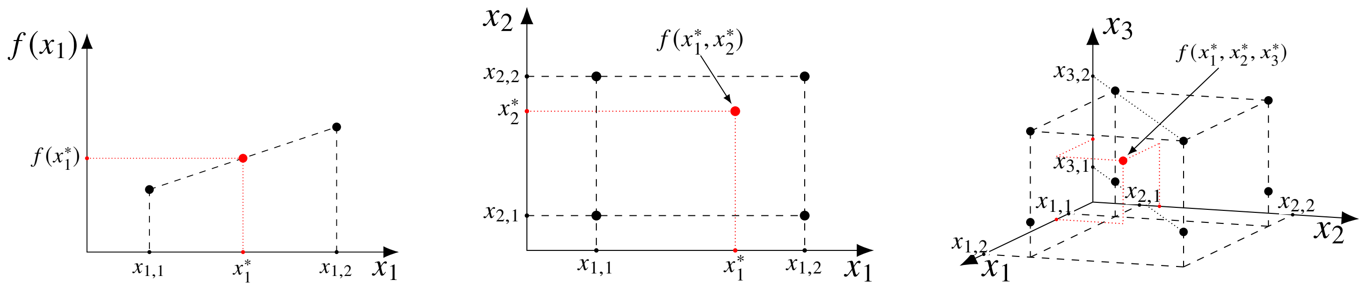


Fig. 1. Examples of regular grids for (a) $D = 1$, (b) $D = 2$ and (c) $D = 3$ showing the number of vertices surrounding the point where the interpolant must be calculated. The number of vertices surrounding a point $x_i^*, i = 1, \ldots, D$ where the interpolant is to be calculated is 2^D.

the number of degrees of freedom (DoF). These values are denoted as:

$$x_{i,k_i},\ i = 1, \ldots, D;\ k_i = 1, \ldots, N_{g_i}. \tag{2}$$

Notice that $N_g = \prod_{i=1}^{D} N_{g_i}$, where N_{g_i} is the number of points in which the length of the i-th DoF is discretized.

2) We consider $x_i^*, i = 1, \ldots, D$, which are the points at which the reflection coefficients are to be obtained.
3) A binary search is used in each dimension i to find index k_i such that:

$$x_{i,k_i} \leq x_i^* \leq x_{i,k_i+1},\ i = 1, \ldots, D. \tag{3}$$

4) Finally, the index to access the reflection coefficient closest to the origin belonging to the D-dimensional hyper-rectangle enclosing x_i^* is:

$$k = k_1 + \sum_{i=2}^{D} (k_i - 1) \prod_{h=1}^{i-1} N_{g_h}. \tag{4}$$

C. N-Linear Interpolation

To perform an N-linear interpolation we have to access 2^D entries in the LUT. Indeed, as previously mentioned, the point x_i^* is surrounded by 2^D points in a regular grid, as illustrated in Fig. 1. However, given the condition in (3), and since the grid is regular, we only need to identify the point closest to the origin. The rest of the vertices are easily found by adding one to the relevant k_i from (4).

The formula of the N-linear interpolation (in this case, $N = D$) is [10]:

$$f(\vec{x}) = \sum_{u=0}^{2^D - 1} R_{(k_1 + b_1(u), \ldots, k_D + b_D(u))} \prod_{i=1}^{D} W_i^{b_i(u)}, \tag{5}$$

where $\vec{x} = (x_1^*, \ldots, x_D^*)$ are the coordinates of the desired interpolated reflection coefficient and they correspond to the physical lengths of the geometrical features of the unit cell; $b_i(u)$ is a function that gives the i-th bit of the integer number u; R is the selected reflection coefficient for a given u and indices k_i; and W_i is the weighting vector

$$W_i = (1 - w_i(x_i^*), w_i(x_i^*)), \tag{6}$$

where

$$w_i(x_i^*) = \frac{x_i^* - x_{i,k_i}}{x_{i,k_i+1} - x_{i,k_i}}. \tag{7}$$

Regarding the notation of $W_i^{b_i(u)}$ in (5) the sub-script indicates the current dimension, while the super-script refers to the indization of the array in (6), which has been indexed such as the first component has index zero. Since the function $b_i(u)$ returns a bit (0 or 1), either $1 - w_i(x_i^*)$ or $w_i(x_i^*)$ are selected.

The reflection coefficient R in (5) is indexed with $(k_1 + b_1(u),\ k_2 + b_2(u),\ \ldots,\ k_D + b_D(u))$, where k_i are the indices, found by binary search for each dimension i, that correspond to the coordinates that bound the desired coordinate x_i^* where the reflection coefficient will be interpolated. The D-dimensional hyper-rectangle has of 2^D vertices and the indices of the vertex closest to the origin is $(k_1,\ k_2,\ \ldots,\ k_D)$ (corresponding to $u = 0$). The rest of the vertices are obtained in a lexicographical order by considering the binary pattern of a number $u = 0, 1, \ldots, 2^D - 1$ comprised of D bits. Then, with the indization of R we can easily find the reflection coefficient at each vertex to apply the interpolation in (5).

III. Results

A. Testing Conditions

To test the proposed database, the same large rectangular reflectarray of [11] is employed for a fair comparison with other techniques. In addition, the same unit cell is employed, which consists in eight coplanar and parallel dipoles, four for each polarization. For the sake of comparison with other works, we reduce the number of DoF to two by imposing a scaling between parallel dipoles as in [11], thus having $N = D = 2$.

In order to compare the accuracy and computing performance of the database, the SVR-based analysis technique described in [12] is employed. The MoM-LP employed as baseline and used to populate the database and generate the training samples of the SVR is fully described in [13]. The total number of samples for the database and the SVR are the same, 380 000 ($N_f = 1$, $N_a = 152$, $N_{g_1} = N_{g_2} = 50$, $N_g = N_{g_1} N_{g_2}$).

To assess the computing efficiency, an Intel i9-9900 CPU working at 3.1 GHz and with 32 GB of memory has been employed. All computations have been parallelized employing the maximum number of threads allowed by this CPU.

B. Reflection coefficients

The first step to assess the accuracy of the LUT is to compare its output, i.e., the interpolated reflection coefficients,

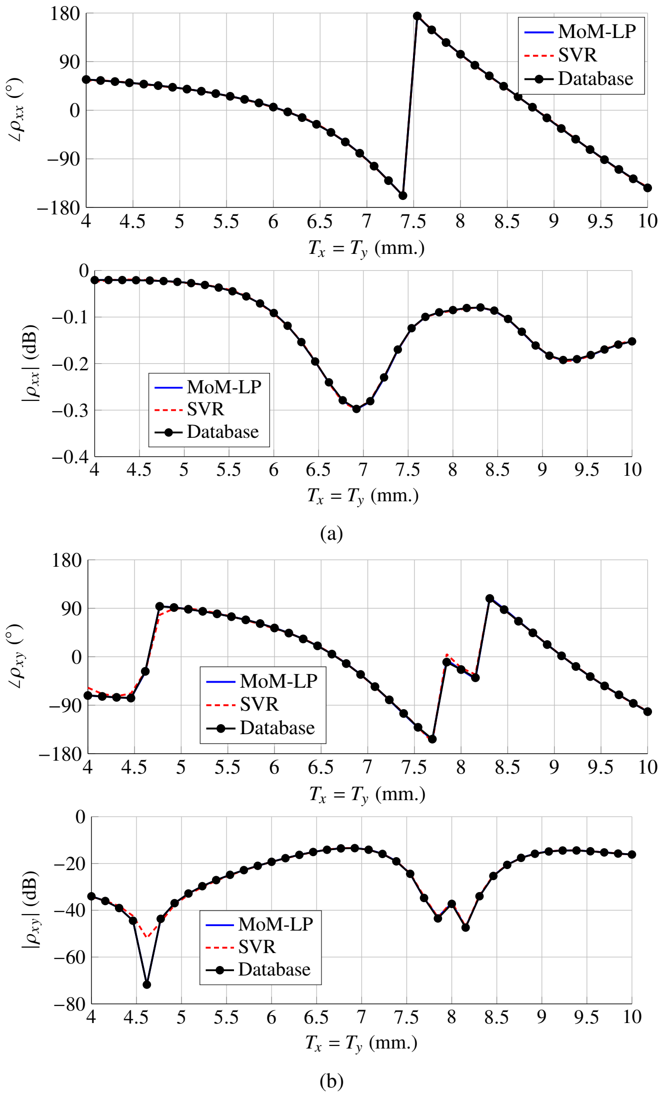


Fig. 2. Comparison in magnitude and phase of the (a) direct reflection coefficient ρ_{xx} and (b) cross-coefficient ρ_{xy} for oblique incidence at $(\theta, \varphi) = (29°, 35°)$. When not visible, the curves are superimposed on each other.

to that of the reference MoM-LP tool. Fig. 2 shows a comparison for a direct reflection coefficient (ρ_{xx}) and a cross-coefficient (ρ_{xy}) in magnitude and phase for oblique incidence with $(\theta, \varphi) = (29°, 35°)$. The results of the SVR models are also included. As can be seen, all curves are superimposed on each other, showing the high degree of accuracy achieved by the LUT. Similar results were obtained for other coefficients and angles of incidence.

C. Layout Design

The high accuracy in the prediction of the reflection coefficients shown in Fig. 2, and in particular of the phase of the direct coefficients, should translate into an accurate layout design. For that task, a phase-only synthesis is carried out to obtain a phase-shift distribution from which a layout will be obtained by following the procedure detailed in [14]. A layout was obtained with the three tools and the computing performance can be seen in TABLE I. As can be seen, both the LUT and the SVR tool are substantially faster than the MoM-LP, but the LUT is an order of magnitude faster than the SVR for this task.

TABLE I
PERFORMANCE OF THE THREE TOOLS FOR LAYOUT DESIGN.

Tool	Time (s)	Speed-up
MoM-LP	1 572.55	1
SVR	1.11	1 417
LUT	0.03	52 418

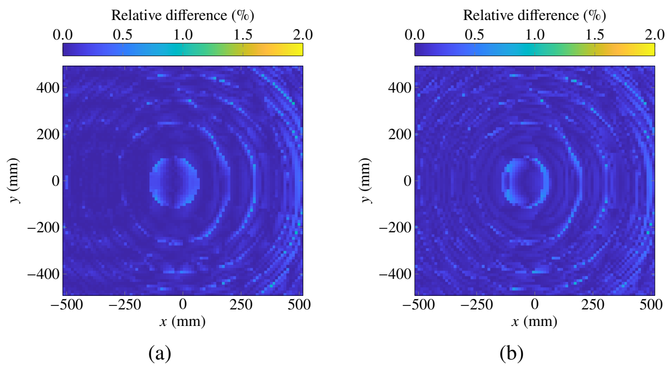


Fig. 3. For the reflectarray layout design, relative error of the layout for polarization X with regard to the design carried out with MoM-LP of the (a) LUT and (b) SVR.

Fig. 3 shows the relative error in the obtained design when using the LUT and the SVR tool compared to the design obtained with MoM-LP. As can be seen, the relative error for both tools is very low and of the same order, confirming the accuracy of the proposed LUT.

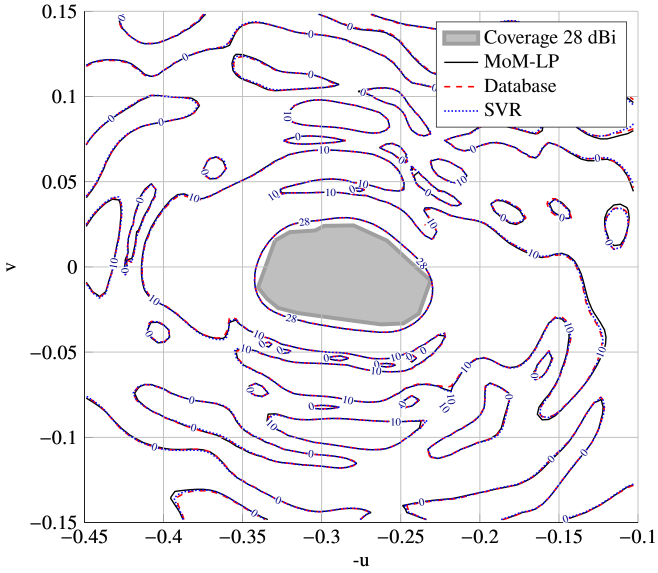
D. Reflectarray Analysis

In order to assess the performance of the LUT for reflectarray analysis, the layout obtained with the LUT in the previous subsection is selected and simulated with the three tools to compare three figures of merit in the coverage zone: the minimum gain ($\mathrm{CP_{min}}$), the minimum crosspolar discrimination ($\mathrm{XPD_{min}}$) and the crosspolar isolation (XPI). These results are summarized in TABLE II. There are some discrepancies among the figures, especially for the cross-polarization figures of merit, although they are very small. The largest difference is for $\mathrm{XPD_{min}}$ for polarization X, with a difference of 0.17 dB between the simulations using the LUT and MoM-LP. Nevertheless, comparing the LUT and the SVR simulations, the figures of merit are very similar. This is due to the error in the radiation pattern produced by the discretization of the angles of incidence, which has been thoroughly studied in [9].

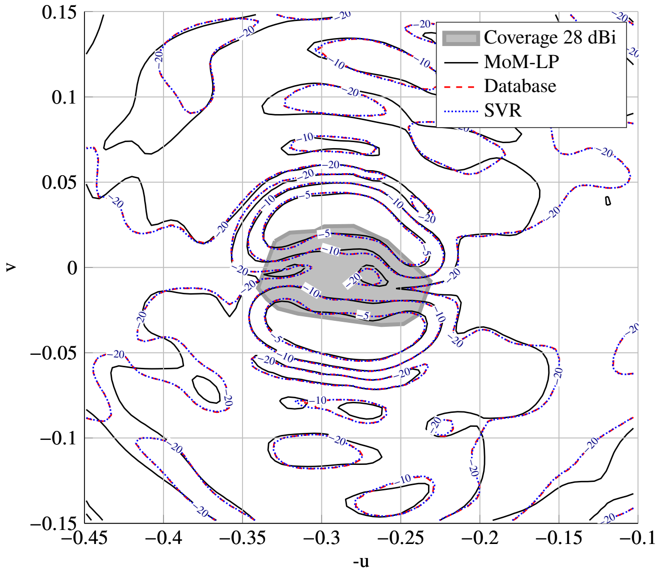
Fig. 4 shows a visual comparison of the radiation pattern obtained by the analyses carried out with MoM-LP, the LUT and SVR. This radiation pattern is the same pattern used to collect the data in TABLE II. As it can be seen, the simulations with the LUT and SVR offer virtually the same results, and both simulations match with a high degree of accuracy that of the MoM-LP.

IV. CONCLUSIONS

In this work, we have presented a simple and efficient LUT for general reflectarray design. The LUT performs a



(a)



(b)

Fig. 4. Comparison of the simulations with MoM-LP, database and SVR for the (a) copolar and (b) crosspolar patterns for polarization X of a reflectarray with European coverage using a layout obtained with the database.

TABLE II
FIGURES OF MERIT OF A EUROPEAN-COVERAGE PATTERN WHEN THE DESIGN IS CARRIED OUT USING THE LUT AND SIMULATED WITH DIFFERENT TOOLS. CP_{MIN} IS IN DBI, XPD_{MIN} AND XPI ARE IN DB.

	Pol. X			Pol. Y		
Analysis tool	**CP_{min}**	**XPD_{min}**	**XPI**	**CP_{min}**	**XPD_{min}**	**XPI**
MoM-LP	30.03	32.97	32.92	30.01	32.95	32.90
Database	29.97	32.81	32.59	29.94	32.85	32.81
SVR	30.03	32.80	32.58	30.02	32.79	32.76

N-linear interpolation and an efficient access to the reflection coefficients stored in memory for a very fast analysis of the unit cell. Performance was assessed in terms of computational efficiency and accuracy at the unit cell and radiation pattern levels, showing a high degree of accuracy with regard to the MoM-LP tool while considerably improving computing time. Compared to machine learning techniques such as SVR, it avoids the training phase that can be time consuming [11].

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