Wide Voltage-Regulation Range Tap-changing Transformer Model for Power System Studies

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***Abstract*—Tap-changing transformer models used in power system studies tend to neglect the change in the short-circuit impedance of the device at the different tap positions. However, the variation of the short-circuit impedance can be significant in transformers with a wide voltage-regulation range. Fortunately, in those cases in which the voltage variation of $\pm 5\%$ is exceeded, the manufacturer is obliged to test and provide data for the short-circuit impedance at terminal taps. The present contribution provides an update of a recently proposed model of the tap-changing transformer so that it can take advantage of the additional information available in the aforementioned cases. The increased accuracy of the resulting model has the potential to improve the quality of the results from power system studies with embedded tap-changing transformers.**

***Index Terms*—power transformers, tap changers, transformer models, power system studies**

I. Introduction

Tap-changing transformers are a key element in achieving voltage regulation at the different parts of the power system. Indeed, either in the form of off-load or on-load tap changers, their presence is ubiquitous both in the transmission and distribution system. Thus, a proper modeling of the tap-changing transformer is essential to conduct accurate power system studies, either in the operating or planning environment. Power flow analysis, economic dispatch or state estimation are just a sample of the tools which require this type of model in order to provide reliable results.

Recently, a consensus model of the tap-changing transformer, [1], was proposed by the authors to reconcile the significant difference arising from the use of traditional versions, widely spread in literature, [2], and software packages [3], [4]. The consensus model introduces a new parameter, which stands for the ratio between the impedances at both sides of the device. An educated guess of this parameter is enough to provide better results than the traditional models, which are based on extreme assumptions. However, the authors have recently proposed the use of parameter estimation techniques based on historical data in order to obtain accurate values of the aforementioned parameters, since they are not typically specified by the manufacturer [5]. Only in this way can the consensus model be fully exploited in its capabilities to provide enhanced results.

Neither the consensus tap-changing transformer model nor the traditional versions include the effect of the change of tap in the variation of short-circuit impedance that takes place in the tapped winding due to the modification of the series resistance and leakage inductance. In fact, this effect is completely neglected in transformers with a low voltage regulation range, to the extent that international standards do not require manufacturers to provide data on the short-circuit impedance of these devices out of the principal tap. However, according to [6], those tap-changing transformers with a wide voltage-regulation range, defined here as those exceeding a voltage variation of $\pm 5\%$ from the principal tap, must additionally provide the short-circuit impedance at the terminal taps. Indeed, this wide voltage variation range may imply a significant variation on the short-circuit impedance of the transformer at the different taps, which should be considered in the model in order to avoid an unwanted deterioration of its accuracy.

The present proposal contributes with an improved version of the consensus tap-changing transformer model which allows to include information on the short-circuit impedance of the transformer at different taps. Though the new model can be universally adopted to improve the accuracy of the results if detailed information of the device is available, it is of especial relevance for tap-changing transformers with a wide voltage regulation range, in which the accuracy can be seriously compromised when using other approaches. Thus, for the benefit of the reader, section II presents the consensus tap-changing transformer model. Section III introduces the update of the consensus model which allows for the inclusion of a varying short-circuit impedance at different taps. Two case studies are depicted in section IV to illustrate the benefits of the proposal and highlight the expected improvement in accuracy when compared with conventional implementations. Finally, the most important conclusions of this contribution are drawn in the last section.

II. Consensus Tap-changing Transformer Model

A new model of the tap-changing transformer was proposed in [1] with the aim of putting an end to the discrepancies

This work was supported by the Spanish Government Innovation Development and Research Office (MEC) under research Grant DPI2017-89186-R.

occurring when using two different traditional versions of the model of this device widely spread in literature and different software packages. The aforementioned contribution demonstrates that those differences are caused by an extreme assumption adopted by traditional models, which allocate all the short-circuit impedance of the device to either the off-nominal or nominal side of the transformer. The consensus model introduces a new parameter, k, which stands for the ratio between the short-circuit impedance at the nominal side, z_n, and the one at the off-nominal side, z_o. According to Fig. 1, in a transformer with an off-nominal turns ratio $a_t : 1$, where a_t is related with the tap, t, expressed as the voltage regulation (in percentage) through,

$$a_t = \frac{1}{1 + t/100}, \tag{1}$$

the short-circuit admittance of the device, as seen from the off-nominal side, can be formulated as

$$y_{sc}^{off} = \frac{1}{z_o + a_t^2 z_n} = \frac{1+k}{1+a_t^2 k} y_{sc}, \tag{2}$$

where y_{sc} stands for the p.u. short-circuit admittance. This is a value obtained through the short-circuit test, and always provided by the manufacturer as nameplate data in the form of the short-circuit impedance, z_{sc} at the principal tap.

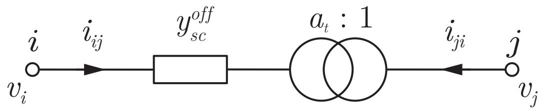

Fig. 1. Consensus model of the tap-changing transformer with the short-circuit admittance represented at the off-nominal side.

The application of Kirchhoff Laws, together with the well-known relationships that apply to ideal transformers, allows to express the terms of the bus admittance matrix of the nodal equations of the device as

$$Y_{ii} = \frac{1+k}{1+a_t^2 k} y_{sc}, \tag{3}$$

$$Y_{ij} = Y_{ji} = -\frac{a_t(1+k)}{1+a_t^2 k} y_{sc}, \tag{4}$$

$$Y_{jj} = \frac{a_t^2(1+k)}{1+a_t^2 k} y_{sc}. \tag{5}$$

Due to the symmetrical nature of the bus admittance matrix of the tap-changing transformer, it is possible to obtain a π-equivalent model of the device, as the one shown in Fig. 2. Straightforward calculations let us express the value of both the series and shunt branches of this equivalent as

$$y_{ij} = -Y_{ij} = \frac{a_t(1+k)}{1+a_t^2 k} y_{sc}, \tag{6}$$

$$y_{si} = Y_{ii} + Y_{ij} = \frac{1 - a_t + k(a_t - 1)}{1+a_t^2 k} y_{sc}, \tag{7}$$

$$y_{sj} = Y_{jj} + Y_{ij} = \frac{a_t(a_t - 1)(1+k)}{1+a_t^2 k} y_{sc}, \tag{8}$$

where y_{si} stands for the shunt branch at the off-nominal turns side and y_{sj} stands for the shunt branch at the nominal turns side.

As it is demonstrated in [1], [7], traditional models correspond to extreme values of k, such as 0 and ∞, which equal to allocating all the short-circuit impedance to the off-nominal or nominal windings, respectively. In the absence of further information, selecting $k = 1$, i.e. assuming a balanced contribution of both sides to the short-circuit impedance, minimizes the maximum expected error. In any case, [5] provides the tools for an accurate identification of this parameter in the context of a real grid.

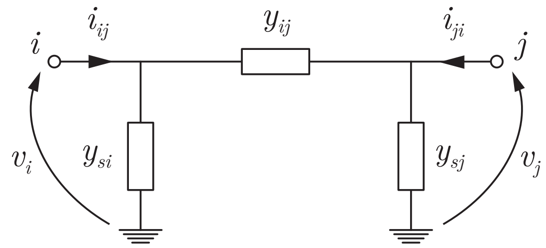

Fig. 2. π-equivalent model of the tap-changing transformer.

III. Including the Effect of Short-Circuit Impedance Variations

As it was stated in section I, neither the consensus model nor the traditional ones include the effect of the inherent short-circuit impedance variation that takes place when a tap change occurs. There are good reasons to support this approach in tap-changing transformers with a reduced voltage regulation range: (a) the influence of this variation may be not very significant in this case due to the small part of the winding affected, and (b), this data is not typically available, as manufacturers do not need to specify this variation provided that the voltage regulation range remains within $\pm 5\%$, [6]. On the contrary, a significant effect could be expected in transformers with a wider voltage regulation range, and [6] guarantees the availability of further data in this case. According to this standard, the short-circuit impedance at extreme tap positions should be "referred to the rated tapping voltage (at that tapping) and the rated power of the transformer".

Let z_{sc_0} and z_{sc_t} be the short-circuit impedance of the transformer at the principal tap, 0, and at a different tap, t. Let k_0 be the transformer impedance ratio at the principal tap, whether obtained from a deep knowledge of the constructive characteristics of the machine, as an educated guess (typically $k = 1$) or through the application of a parameter estimation technique based on historical data. As the contribution to the short-circuit impedance of the nominal turns side is not affected by the tap position, it can be stated that

$$z_{sc_0} = z_{o_0} + z_n, \tag{9}$$

$$z_{sc_t} = z_{o_t} + z_n, \quad (10)$$

where z_{o_0} and z_{o_t} stand for the short-circuit impedance provided by the tapped winding at taps 0 and t, respectively. Considering (9) and the definition of k_0, it is possible to calculate z_{o_0} from the given values, i.e.

$$z_{o_0} = z_{sc_0} - z_n = z_{sc_0} - k_0 z_{o_0} \rightarrow z_{o_0} = \frac{z_{sc_0}}{1 + k_0}. \quad (11)$$

Thus, applying (10) and (11), the value of the short-circuit impedance ratio for tap t can be determined as

$$k_t = \frac{z_n}{z_{o_t}} = \frac{1}{\frac{z_{sc_t}}{k_0 z_{o_0}} - 1} \rightarrow k_t = \frac{1}{\frac{(1+k_0) z_{sc_t}}{k_0 z_{sc_0}} - 1}. \quad (12)$$

Equation (12) can be expressed in terms of the corresponding short-circuit admittances at both taps, y_{sc_0} and y_{sc_t}, which is typically preferred in power system studies. Thus,

$$k_t = \frac{1}{\frac{(1+k_0) y_{sc_0}}{k_0 y_{sc_t}} - 1}. \quad (13)$$

According to this development, the model of the consensus tap changing transformer can now be reformulated. Thus, in the same sense as it was conveyed by (2), the short-circuit impedance of the transformer as seen from the off-nominal side for each particular tap t can now be expressed as

$$y_{sc_t}^{off} = \frac{1}{z_{o_t} + a_t^2 z_n} = \frac{1 + k_t}{1 + a_t^2 k_t} y_{sc_t}. \quad (14)$$

As a result, the terms of the bus admittance matrix of the nodal equations of the transformer turn to be not only dependant on the tap position through a_t, but also through the impact of the short-circuit impedance variation. Thus, (3)–(5) can now be expressed as

$$Y_{ii_t} = \frac{1 + k_t}{1 + a_t^2 k_t} y_{sc_t}, \quad (15)$$

$$Y_{ij_t} = Y_{ji_t} = -\frac{a_t (1 + k_t)}{1 + a_t^2 k_t} y_{sc_t}, \quad (16)$$

$$Y_{jj_t} = \frac{a_t^2 (1 + k_t)}{1 + a_t^2 k_t} y_{sc_t}. \quad (17)$$

Accordingly, the π-equivalent model of the device is now a function of the short-circuit impedance of the transformer at each particular tap position, as shown in Fig. 3, and the values of the admittances in this model can be expressed as

$$y_{ij_t} = -Y_{ij_t} = \frac{a_t (1 + k_t)}{1 + a_t^2 k_t} y_{sc_t}, \quad (18)$$

$$y_{si_t} = Y_{ii_t} + Y_{ij_t} = \frac{1 - a_t + k_t (a_t - 1)}{1 + a_t^2 k_t} y_{sc_t}, \quad (19)$$

$$y_{sj_t} = Y_{jj_t} + Y_{ij_t} = \frac{a_t (a_t - 1)(1 + k_t)}{1 + a_t^2 k_t} y_{sc_t}. \quad (20)$$

Notice that the different components of the π-equivalent model are not only affected by the short-circuit impedance measured at each particular tap, y_{sc_t}, but also by the variation of the impedance ratio of the contribution of each winding through, k_t, caused by the change of tap.

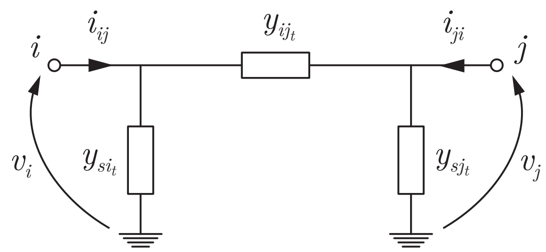

Fig. 3. π-equivalent model of the tap-changing transformer considering the effect of the short-circuit impedance variation caused by the tapped winding.

Although the practising engineering will typically be provided with only the short-circuit impedance at three taps (the principal and terminal ones), using a simple interpolation, which accounts for the rated voltage of each tap position, allows to determine a custom model for each tap based on sensible assumptions. Thus, considering y_{sc_0}, y_{sc_T} the short-circuit impedances at the principal and an extreme tap referred to the same bases, and T the voltage regulation percentage of the extreme tap, the short-circuit impedance of any intermediate tap, y_{sc_t} with a voltage regulation percentage of t should be calculated as

$$y_{sc_t} = y_{sc_0} + \frac{t}{T} (y_{sc_T} - y_{sc_0}). \quad (21)$$

IV. Case Studies

A. Case Study I

In order to highlight the importance of adopting the model proposed in this contribution for wide voltage-regulation range tap-changing transformers, a standard device, similar to the one previously analyzed in [1] is considered in this case study. Thus, the performance of an 80 MVA, 50 Hz, 230/132 kV $\pm 10\%$ transformer is studied in the following. The manufacturer provides data of the short-circuit impedance of the device at the principal tap, z_{sc_0}, which amounts for $0.01 + 0.12j$ p.u. The tap changer, which is located at the high voltage side of the transformer, has 21 positions, with a voltage regulation step of 1%. As the voltage regulation range exceeds $\pm 5\%$, and in order to comply with regulations, [6], the manufacturer provides the value of the short-circuit impedance of the device at terminal taps, $z_{sc_{10}}$ and $z_{sc_{-10}}$, which amounts for $0.0092 + 0.1104j$ p.u. and $0.0109 + 0.1308j$ p.u., respectively. Notice that, according to [6], "if the impedance (at non-principal taps) is expressed in percentage (or p.u. values), it shall be referred to the rated tapping voltage". For the sake of simplicity, short-circuit impedances at terminal taps have been already referred here to the rated voltage of the transformer at the principal tap.

According to (21), the use of interpolation allows for the calculation of a sensible estimate of the short-circuit admittance of the machine at the different tap positions. Notice that T=10 and $y_{sc_{10}}$ are used for taps, t, in the positive voltage regulation range and $T{=}-10$ and $y_{sc_{-10}}$ are considered for those in the negative range. In [1], [7], the authors demonstrated that, in the absence of detailed transformer construction data (which is the most common case), assigning a value of one to the transformer impedance ratio of the machine at the principal tap, $k_0{=}1$, is a prudent decision which minimizes the maximum expected error. This criterion is adopted in the following. Thus, all the information needed to calculate a custom value of the transformer impedance ratio at each tap position, k_t, by applying (13) is now available. Finally, (18)–(20), lead to the model of the tap-changing transformer shown in Fig. 3, which is the subject of this contribution.

Fig. 4 compares the results obtained from the consensus model described in section II with those derived from the new model proposed in section III, which includes the effect of impedance variations on the tapped winding. Thus, the voltage at the nominal side of the transformer at each tap is shown in both cases. At each operating point, the transformer is fed at rated voltage and current at the off-nominal side. Furthermore, three extreme power factors are considered, by varying the angle of the off-nominal side current with respect to the off-nominal side voltage, θ: unity (0°), pure capacitive (90°) and pure inductive (−90°).

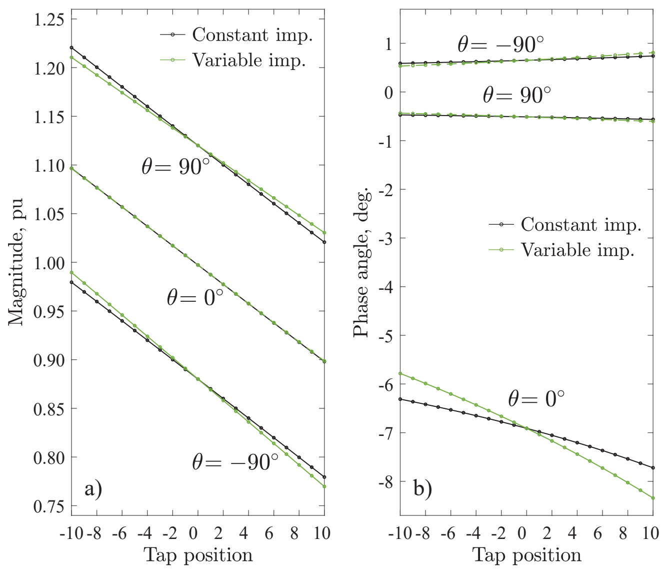

Fig. 4. Nominal turns side voltage of the transformer at the different tap positions for the constant impedance model and the proposed variable impedance model: a) voltage magnitude, and b) voltage phase angle. The transformer is operated at rated voltage and current from the off-nominal side with different power factors: unity (θ = 0), pure capacitive (θ = 90) and pure inductive (θ = −90).

As expected, the effect of the impedance variation of the tapped winding is magnified at terminal taps. According to Fig. 4.a), the maximum difference among both models in terms of voltage magnitude appears with pure capacitive or pure inductive power factors, reaching values of 1.01% at $t{=}-10$ and 0.97% at $t{=}10$ in both cases. Conversely, the effect of impedance variation on voltage magnitude at high power factors is practically negligible. From Fig. 4.b), it is clear that the nominal turns side voltage phase angle is hardly affected at any tap, provided that the transformer is operated at poor power factors. Indeed, the maximum difference between both models appears now at unity power factor, when discrepancies of 0.53 deg. and 0.62 deg. are confirmed at $t{=}-10$ and $t{=}10$, respectively.

Thus, this case study confirms that neglecting the effect of the impedance variation on the tapped winding can lead to significant errors in the results obtained from the transformer model, which may appear as voltage magnitude or phase angle errors depending on the operating point of the device. Specifically, according to Fig. 4.a), the voltage regulation range of the transformer can be overestimated at the lower taps and underestimated at the higher ones if this impedance variation is not considered.

B. Case Study II

A standard test grid has been used in this case study in order to highlight the improvements in the quality of the results that can be derived from the use of the tap-changing transformer model proposed in this contribution. Thus, the IEEE 57-bus system [8], which represents an approximation of the American Electric Power system in the U.S. Midwest as it was in the early 1960s, was selected for this case study due to the large amount of tap-changing transformers embedded in it. The IEEE 57-bus system, shown in Fig. 5, comprises 57 buses, 7 generators, 42 loads and 17 transformers. It is important to note that 15 of these transformers are set out of the principal tap at the operating point defined by the test case. This fact makes the system especially suitable to test the proposed model and compare it with other alternatives which do not consider the impedance variation.

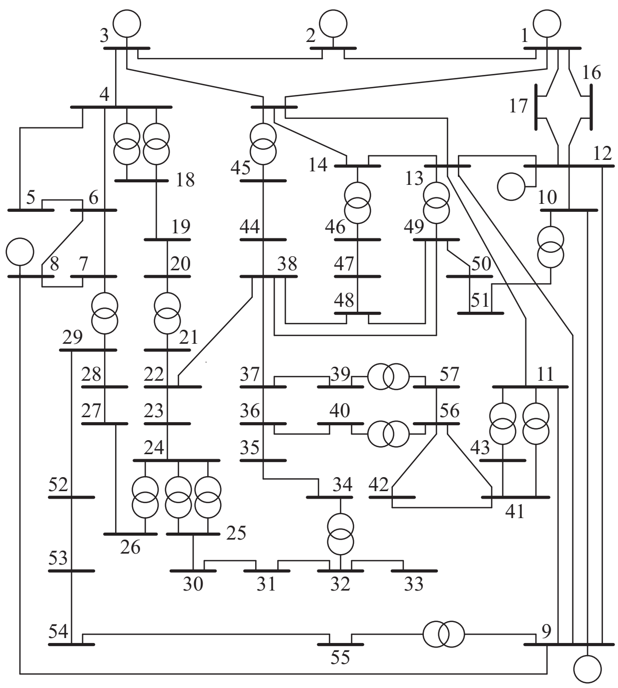

Fig. 5. IEEE 57-bus system.

Table I shows the parameters and set-up of the transformers in the IEEE 57-bus system as described in the test data files. This information suffices to run a power flow analysis of the grid in the case of traditional tap-changing transformer models. Thus, the bus voltages for a set of specific buses using the models which assume that all the short-circuit impedance is provided exclusively by the off-nominal turns side ($k_t = 0$) or by the nominal turns side ($k = \infty$) are shown in Table II. In the same way, this table shows the results derived from the use of the consensus model previously proposed by the authors [1]. Due to the lack of detailed information on the tap-changing transformers, an equal sharing of the short-circuit impedance between the off-nominal and nominal side is assumed, i.e. $k_t = 1$. Notice that in this case, the influence of the impedance variation on k_t is not taken into account.

TABLE I
TRANSFORMER DATA IN THE IEEE 57-BUS SYSTEM

From bus	**To bus**	$\boldsymbol{R}$, p.u	$\boldsymbol{X}$, p.u	**Tap,** $\boldsymbol{a_t}$
4	18	0	0.5550	0.970
4	18	0	0.4300	0.978
21	20	0	0.7767	1.043
24	25	0	1.1820	1.000
24	25	0	1.2300	1.000
24	26	0	0.0473	1.043
7	29	0	0.0648	0.967
34	32	0	0.9530	0.975
11	41	0	0.7490	0.955
11	45	0	0.1042	0.955
14	46	0	0.0735	0.900
10	51	0	0.0712	0.930
13	49	0	0.1910	0.895
11	43	0	0.1530	0.958
40	56	0	1.1950	0.958
39	57	0	1.3550	0.980
9	55	0	0.1205	0.940

TABLE II
BUS VOLTAGES SHOWING THE MAXIMUM DISCREPACIES

Voltage magnitude					
Bus	$k_t = 0$	$k_t = \infty$	$k_t = 1$	k_t variable	MAE (%)
49	1.029	1.036	1.032	1.030	0.196
56	0.963	0.968	0.966	0.964	0.152
57	0.959	0.965	0.962	0.961	0.147
50	1.017	1.023	1.020	1.019	0.143
Voltage phase angle					
Bus	$k_t = 0$	$k_t = \infty$	$k_t = 1$	k_t variable	MAE (deg.)
57	-16.972	-16.584	-16.780	-16.939	0.159
56	-16.430	-16.065	-16.249	-16.407	0.158
42	-15.875	-15.533	-15.705	-15.852	0.147
33	-19.081	-18.552	-18.819	-18.964	0.145

According to Table I, the most extreme positions of the tap-changing transformers correspond to the one between buses 13 and 49, with a positive voltage regulation, t, of 11.73%, and those between buses 21 and 20 and 24 and 26, with a negative voltage regulation, t, of -4.12%. To complete the information provided by the test case according to [6], a maximum voltage regulation range, T, of $\pm15\%$ was selected for all the transformers. Furthermore, the p.u. impedances of the transformers shown in Table I were used to determine the admittances at the central tap, y_{sc_0}, while these admittances were increased or decreased in a 15% to estimate the values at extreme tap positions, $y_{sc_{+15}}$ and y_{sc-15}. As in the previous case, an equal contribution of both windings to the short-circuit impedance is assumed, but now this corresponds exclusively to the central tap position, i.e. $k_0 = 1$. With those assumptions the power flow analysis was repeated for the tap-changing transformer model proposed in the present contribution, and the results are shown in Table II. The maximum absolute error, MAE, showing the discrepancies between the consensus model, $k_t = 1$, and the one considering the variable nature of the short-circuit impedance, is provided in the last column of this table. In fact, those buses showing the greatest differences, both in voltage magnitude or phase angle, were selected to highlight the benefits of the proposal. Note that even though the transformers in the study were not configured in extreme tap positions, the results can be significantly improved by considering the influence of the impedance variation on the tapped winding.

V. CONCLUSION

The variation of the short-circuit impedance of a tap-changing transformer at different tap positions can have a significant impact on the accuracy of the models used to represent this crucial equipment in power system studies. This is especially important in the case of tap-changers with a wide voltage regulation range. The present proposal introduce the modifications needed to adapt the recently proposed consensus model of the tap-changing transformer in order to include this important effect. The new version of the model relies on the additional information provided by manufacturers on the short-circuit impedance of the machine at terminal tap positions, which is required by international standards. The impact of the new model in the accurate identification of the impedance ratio of the transformer at the principal tap by using parameter estimation techniques, which was previously tackled by the authors for the standard consensus model, is left here for further investigation.

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