Universidad de Oviedo
Facultad de Formación del Profesorado y Educación

ESTUDIO SOBRE LA PROBABILIDAD EN EL AULA DE PRIMARIA A TRAVÉS DE EXPERIMENTOS CON PROBABILIDAD SUBJETIVA

A STUDY ON PROBABILITY IN THE PRIMARY CLASSROOM THROUGH EXPERIMENTS WITH SUBJECTIVE PROBABILITY

TRABAJO FIN DE GRADO

GRADO EN MAESTRO EN EDUCACIÓN PRIMARIA BILINGÜE

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Julio 2022

RESUMEN

Hay carencias en la enseñanza de la probabilidad en Educación Primaria principalmente por la reciente incorporación explícita de este campo en el contexto educativo. Es el significado subjetivo de probabilidad el que todavía no está presente en las aulas; no obstante, el alumnado de esta etapa tiene ciertas nociones sobre ello y ha adquirido lenguaje y términos probabilísticos que le permiten hacer frente a situaciones-problema que emplean este significado de probabilidad, fundamentalmente porque es el que más usamos en nuestra vida cotidiana. El problema es la falta de enseñanza formal acerca de la probabilidad subjetiva que conduce a errores, mal uso de los términos y a malas interpretaciones de la probabilidad, es decir, una baja alfabetización probabilística en su sentido global. Desde este enfoque, se expone la importancia de conocer qué nociones tiene el alumnado para poder comprender las lagunas en la enseñanza de la probabilidad. Se concluye que el alumnado tiende a expresar la probabilidad de forma cuantitativa por la fuerte presencia de la probabilidad clásica, que hay una gran confusión y un mal uso de los términos y que existen grandes dificultades en el razonamiento de sus respuestas, así como en la interpretación de nueva información.

PALABRAS CLAVE

Probabilidad subjetiva, Educación Primaria, Razonamiento probabilístico, Expresiones de probabilidad, Influencia de nuevos datos

ABSTRACT

There are shortcomings in the teaching of probability in Elementary school mainly due to the recent explicit incorporation of this field in the educational context. It is the subjective meaning of probability that is not yet present in the classroom; nevertheless, pupils at this stage have certain notions about it and have acquired language and probabilistic terms that enable them to deal with situation-problems that use this meaning of probability, fundamentally because it is the one we use the most in our daily lives. The problem is the lack of formal teaching about subjective probability that leads to errors, misuse of terms and misinterpretations of probability, i.e., low probabilistic literacy in its global sense. From this approach, the importance of knowing what notions students have in order to understand the gaps in the teaching of probability, especially subjective probability, is discussed. It is concluded that students tend to express probability in a quantitative way due to the strong presence of classical probability, that there is a high misconception and misuse of terms, and that there are great difficulties in the reasoning of their answers, as well as in the interpretation of new information.

KEYWORDS

Subjective probability, Elementary school, Probabilistic reasoning, Probability expressions, Influence of new data

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1. INTRODUCCIÓN

La percepción y aplicación de la probabilidad en el contexto escolar está fuertemente influenciada por la trayectoria histórica de esta área en la educación, por la variedad de sus significados que generan debate en su interpretación y por la insuficiente formación del profesorado en este campo, tanto en contenidos como en didáctica (Alsina *et al.*, 2020). Según señaló Alsina (2016), en el currículo de Educación Primaria se incorpora la probabilidad como área de contenido por primera vez de forma explícita hace solamente 16 años, por lo que la necesidad de investigar cómo se implementa, cómo está condicionada y qué limitaciones comprende es evidente para poder impulsar su mejora. Un aspecto que origina controversia es la presencia de los significados de probabilidad clásico o *laplaciano*, frecuencial, intuitivo y axiomático en el contexto académico, pero no el subjetivo, siendo este último el que más utilizamos en nuestra vida cotidiana. Indagar en el estudio de qué nociones tiene el alumnado de Primaria sobre probabilidad subjetiva y cómo se ha de modificar la didáctica para fomentar su alfabetización probabilística es, por tanto, objeto de análisis.

Este trabajo presenta una justificación, objetivos generales y específicos y metodología empleada para llevar a cabo la investigación; contenidos que serán desarrollados en inglés más adelante. En cuanto al marco teórico, se expone la importancia y necesidad de la alfabetización probabilística, especialmente la subjetiva, así como las limitaciones o condicionantes presentes en el paradigma de la didáctica de la matemática actual. También se lleva a cabo una explicación de los distintos significados de la probabilidad mencionados anteriormente y de las distintas expresiones de lenguaje probabilístico. A continuación, se desarrolla la intervención educativa que consiste en dos actividades sobre probabilidad subjetiva y que ha sido llevada a cabo en 5º de Primaria. Se describen la organización, los materiales, la base teórica y el desarrollo de las actividades. Para finalizar, se presentan los resultados recogidos orales y escritos, su análisis y conclusiones finales.

2. JUSTIFICACIÓN

Los contenidos de estadística y probabilidad se han introducido en el currículo español no hace mucho tiempo. La asignatura de matemáticas incluía los números, las operaciones, la geometría y la medida, pero no se mencionaba la probabilidad ni la estadística. Como señaló Alsina (2016), fue en 2006 con la Ley Orgánica de Educación (LOE) cuando se introdujo un bloque de contenidos sobre azar y probabilidad. La siguiente ley (LOMCE) de 2013 modificó estos contenidos e introdujo la 'Estadística y la Probabilidad' como un nuevo bloque. Un hito anterior fue la incorporación de 'datos y aleatoriedad' como área en los estándares curriculares y de evaluación para las Matemáticas Escolares del Consejo Nacional de Profesores de Matemáticas (NCTM, 1989). Esta nueva orientación curricular promovió el desarrollo de diferentes fases para enseñar nociones probabilísticas. Este currículo americano y el español comparten conocimientos y contenidos probabilísticos similares que se basan principalmente en las acepciones intuitiva, clásica y frecuencial de la probabilidad (Alsina, 2016). Al no contemplar la probabilidad subjetiva, habría que realizar experimentos para observar si el alumnado de Educación Primaria tiene habilidades para resolver problemas que utilicen este tipo de probabilidad, reforzando la necesidad de enseñar un abanico más amplio de nociones sobre la probabilidad ya que la asimilación e incorporación de datos puede modificar las respuestas previas, por ejemplo.

Los estándares curriculares y de evaluación establecidos por el NCTM (1989) también fomentan el uso de metodologías que representen situaciones de la vida real y actividades contextualizadas basadas en la experimentación y en los conocimientos previos del alumnado para que adquiera una comprensión más holística de la probabilidad. Por tanto, es de gran importancia, especialmente en Educación Primaria, enseñar la probabilidad con un enfoque flexible, que incluya juegos y simulaciones, que sean capaces de desarrollar la intuición y la capacidad del alumnado para captar los conceptos generales desde el principio. Según Fischbein (1975) y Baroody (1993), los juegos y las simulaciones son adecuados para el desarrollo de las nociones intuitivas de los alumnos sobre la probabilidad.

Se ha demostrado que la enseñanza de la probabilidad en la escuela es crucial para el aprendizaje de los niños, no solo en un contexto matemático, sino para que se conviertan en futuros ciudadanos capaces de enfrentarse a situaciones de la vida real (Gal, 2005). Dado que los fenómenos aleatorios y el azar son intrínsecos a nuestra sociedad, (previsiones meteorológicas, juegos, datos médicos, riesgos...) es de gran importancia que el alumnado desarrolle las herramientas necesarias para enfrentarse a este tipo de problemas más adelante, por no mencionar que la construcción de estos conceptos les permitirá comprender mejor los conceptos estadísticos más adelante en el sistema escolar (Batanero *et al.*, 2005).

3. OBJETIVOS

El objetivo general a cumplir es:

- Realizar dos secuencias diferentes de actividades para observar y analizar las nociones y la aplicación de la probabilidad subjetiva del alumnado de 5º de Educación Primaria.

Los objetivos específicos son:

- Promover el razonamiento matemático respecto a los problemas de probabilidad subjetiva.
- Abordar el razonamiento de la probabilidad subjetiva a través de situaciones contextualizadas.
- Fomentar en el alumnado el manejo del lenguaje y las nociones probabilísticas, así como de los datos estadísticos.

4. METODOLOGÍA

A continuación, se describe la metodología seguida, así como los participantes, el contexto y la recogida de datos.

4.1 PARTICIPANTES Y CONTEXTO

La población participante en el experimento se dividió en dos grupos, cada uno de los cuales realizó una actividad diferente. Dentro de cada grupo hay alumnos de dos colegios públicos, uno en Oviedo y otro en las afueras de la misma ciudad. El primero es un colegio grande del que participaron 34 alumnas y alumnos (18 y 16 en las actividades de Masterchef y viaje de fin de curso respectivamente); el segundo es un pequeño colegio rural con sólo 15 alumnas y alumnos (11 y 4 en las actividades de Masterchef y viaje de fin de curso respectivamente). El alumnado es de 5º de Educación Primaria, por lo que tiene entre 10 y 11 años. Pertenecen, por tanto, al tercer estadio de desarrollo cognitivo de Piaget, que va de los 7 a los 11 años. Sin embargo, Cañizares (1997) señaló la necesidad de rechazar la concepción lineal del razonamiento probabilístico de Piaget, ya que está demostrado que las niñas y niños son capaces de aplicar nociones y conceptos pertenecientes a diferentes estadios independientemente de su edad, con ciertos límites. Según Inhelder y Piaget (1955), la comprensión de la probabilidad se adquiere durante el cuarto estadio, antes del cual las niñas y niños no son capaces de resolver problemas o situaciones que impliquen conocimientos probabilísticos. Sin embargo, Fischbein sostuvo que los menores de 7 años ya tienen lo que él llama *conocimiento intuitivo primario* sobre el azar y son capaces de distinguir entre fenómenos aleatorios y deterministas (Batanero, 2013). Además, Alsina *et al*. (2021) expusieron que el alumnado, cuando se le pedía que hiciese predicciones y comparaciones de probabilidad a partir de una información dada, no solo analizaban los sucesos favorables que se pedían, sino también los desfavorables. Se espera que las estimaciones y predicciones con un razonamiento intuitivo sean habilidades que el estudiantado de esta edad haya adquirido (Fischbein, 1975). Cabe destacar que el alumnado tenía muy pocos conocimientos sobre probabilidad y estadística. No habían explorado estas áreas durante los años anteriores de Educación Primaria porque los libros de texto utilizados para enseñar probabilidad no incluían estos temas. Por lo tanto, su alfabetización probabilística es limitada y las diferencias entre el alumnado son notorias.

Se espera que el alumnado utilice el lenguaje probabilístico durante el desarrollo de este experimento. Dadas las diferentes expresiones, verbales, numéricas, etc. se han identificado dificultades en el uso de estas representaciones. La falta de conocimiento probabilístico presente en la escuela puede ser un factor importante que afecta a las concepciones erróneas y a las dificultades en conceptos matemáticos interrelacionados como la probabilidad, los porcentajes y los gráficos. En esta línea, también se produce un mal uso de términos como "cierto" y "seguro". Además, la interpretación y evaluación de datos estadísticos para emitir juicios y opiniones, se ha identificado como un aspecto importante para desarrollar la alfabetización estadística que presenta una dificultad para el estudiantado (Muñiz-Rodríguez *et al.*, 2020).

4.2 RECOGIDA DE DATOS

Los datos se recogieron de dos formas: respuestas escritas y debate oral. La primera es la principal fuente de información para el análisis de los resultados, sin embargo, la segunda jugó un papel importante que influyó en las posteriores respuestas escritas y, por tanto, es relevante a la hora de analizar los razonamientos del alumnado. Se entregó una hoja de papel con la información y las preguntas de cada actividad. Escribieron en este papel y fue recogido posteriormente. Por consiguiente, las respuestas escritas son una fuente primaria de datos, ya que proceden de los propios autores. Los resultados del debate oral realizado al principio se recogieron de dos maneras: grabaciones de algunas alumnas y alumnos justificando sus respuestas y notas tomadas a lo largo de este debate. Las grabaciones son una fuente primaria y las notas tomadas son una fuente secundaria porque no proceden de los propios autores, sino que corresponden a la transcripción e interpretación de una tercera persona. Un hándicap de la grabación es que no todo el alumnado pudo explicar sus respuestas, por lo que no tenemos un conjunto completo de este tipo de datos, y no se muestran las interacciones entre ellas y ellos. Las respuestas escritas tampoco muestran este intercambio de ideas, pero hay muchas más respuestas escritas que en la discusión oral porque el debate solo se hizo después de la primera pregunta de cada actividad. Los resultados se clasificaron posteriormente y se representaron con tablas.

Cabe mencionar que, teniendo en cuenta que la primera lengua del estudiantado es el español y no el inglés, ambas actividades se realizaron en su lengua materna. Aunque los colegios en los que están matriculados siguen un programa bilingüe y la mayoría de ellas y ellos saben inglés básico, la mayoría tiene un nivel de comprensión muy deficiente y, en general, las habilidades de producción y output son todavía muy limitados. Como aclararon Runnqvist *et al.* (2011) el alumnado de segunda lengua (L2) “son más lentos y menos precisos a la hora de recordar los nombres de los objetos, les cuesta más articular palabras y frases completas” (pág. 1). Por este motivo, tanto la fase oral como la escrita se realizaron en español. Esto permitiría al alumnado sentirse más seguro a la hora de dar razones y participar activamente en clase. Teniendo en cuenta lo complicado que puede resultar justificar sus respuestas, el uso de su primera lengua dará mejores resultados para el posterior análisis.

4.3 METODOLOGÍA DE ANÁLISIS

Teniendo en cuenta la importancia del razonamiento en este trabajo, la metodología que se utilizará es el análisis de contenido cualitativo. Las hojas de trabajo recogidas son el principal objeto de estudio, por lo que se examinarán las respuestas escritas del alumnado (especialmente sus justificaciones) mediante un análisis interpretativo que posteriormente servirá para clasificar y estructurar la información recogida según su diferente naturaleza. La inferencia de los significados expuestos por los estudiantes responde a lo expuesto por Krippendorff (2018) sobre el análisis de contenido. Durante la evaluación inicial de los resultados se utilizó un enfoque de muestreo teórico porque las respuestas eran bastante variadas, cortas o carecían de una justificación elaborada. Los datos se recogieron, luego se codificaron y analizaron y finalmente se definieron para construir teorías y extraer resultados.

Existe un componente subjetivo a la hora de interpretar los resultados porque es el investigador el que filtra y clasifica la información; para que el análisis sea lo más objetivo posible, se llevará a cabo una *subjetividad disciplinada*, tal y como describe López-

Noguero (2002) al explicar las características del análisis de contenido cualitativo de las que el investigador debe ser consciente. Rodríguez-Suárez (2021) muestra y resume otras características explicadas por López-Noguero sobre el análisis cualitativo que también fueron consideradas al realizar este trabajo.

5. INTRODUCTION

The perception and application of probability in the school context is strongly influenced by the historical trajectory of this area in education, by the variety of its meanings that generate debate in its interpretation and by the insufficient teacher training in this field, both in content and didactics (Alsina et al., 2020). As pointed out by Alsina (2016), probability was first explicitly incorporated as a content area in the Elementary curriculum only 16 years ago, so the need to investigate how it is implemented, how it is conditioned and what limitations it comprises is evident in order to promote its improvement. One aspect that causes controversy is the presence of the classical or Laplacian, frequentist, intuitive and axiomatic meanings of probability in the academic context, but not the subjective one, the latter being the one we use most in our daily lives. Inquiring into the study of what notions Elementary students have about subjective probability and how didactics should be modified to foster their probabilistic literacy is, therefore, the object of analysis.

This essay presents a justification, general and specific objectives and methodology used to carry out the research. In terms of the theoretical framework, the importance and necessity of probabilistic literacy, especially subjective literacy, as well as the limitations or conditioning factors present in the current paradigm of the didactics of mathematics, are presented. An explanation is also given of the different meanings of probability mentioned above and of the different expressions of probabilistic language. This is followed by the educational intervention consisting of two activities on subjective probability, which has been carried out in the 5th grade Elementary school. The organisation, materials, theoretical basis and development of the activities are described. Finally, the oral and written results collected, their analysis and final conclusions are exposed.

6. RATIONALE

Statistics and probability contents have been introduced in the Spanish curriculum not very long ago. Mathematics as a subject included numbers, operations, geometry and measure, but there was no mention of probability or statistics. As Alsina (2016) pointed out, it was in 2006 with the Organic Law on Education (LOE) that a content block on randomness and probability was introduced. The next law (LOMCE) in 2013 changed these contents and introduced ‘Statistics and Probability’ as a new block. A previous milestone was the incorporation of ‘data and randomness’ as an area in the *Curriculum and Evaluation Standard for School Mathematics* by the National Council of Teachers of Mathematics (NCTM, 1989). This new curricular direction promoted the development of different phases in order to teach probabilistic notions. This American curriculum and the Spanish one share similar probabilistic knowledge and contents which are mainly based on the intuitive, classic and frequentist meanings of probability (Alsina, 2016). As subjective probability is not considered, experiments should be carried out to observe if students in Elementary school have the abilities to solve problems that use this type of probability, reinforcing the need to teach a wider range of notions regarding probability as data assimilation and incorporation can modify previous answers, for example.

The curricular and evaluation standards stated by NCTM (1989) also encourage the use of methodologies that portray real-life situations and contextualised activities based on experimentation and students' previous knowledge to enable them to acquire a more holistic understanding of probability. So, it is of great importance, especially in Elementary schools, to teach probability with a flexible approach, involving games and simulations, which are able to develop students' intuition and ability to capture the general concepts from the beginning. According to Fischbein (1975) and Baroody (1993), games and simulations are suitable for the development of students' intuitive notions about probability.

Teaching probability in school has been proved to be crucial for children's learning, not only in a mathematical context, but to become future citizens able to face real-life situations (Gal, 2005). Given random phenomena and chance are intrinsic to our society, (weather forecasts, games, medical data, risks…) it is of great importance that students develop the necessary tools to deal with these types of problems later on, not to mention that building on these concepts will allow them to better understand statistical concepts higher up in the school system (Batanero et al. 2005).

7. OBJECTIVES

The general objective to be met is:

- To perform two different sequences of activities to observe and analyse 5th grade Elementary school students' notions and application of subjective probability.

The specific objectives are:

- To promote mathematical reasoning regarding subjective probability problems.
- To approach subjective probability reasoning through contextualised situations.
- To encourage students to handle probabilistic language and notions as well as statistical data.

8. THEORETICAL FRAMEWORK

Probability and chance are fields of knowledge that are significantly present in day-to-day situations, Gal (2005) even argued that "the learning of probability is essential to help prepare students for life" (p. 39), not only appreciating the notorious presence probability has on our society but acknowledging its importance in people's cognitive development for their further correct functioning in society (Alsina et al., 2020). Batanero and Godino (2002) identified four main fields where random phenomena appear and where statistics can be applied in order to study them. The fields are: the biological world, the physical world, the social world and the political world. This demonstrates the close link and inherent nature between probabilistic and statistical notions and the human being. Many explain that acquiring probabilistic literacy is the first key step in order to develop further statistical understanding (Batanero et al., 2005). The main problem arises when judgements on probability are not reasoned due to the lack of probabilistic literacy taught in school resulting in biased arguments that lead to incorrect answers (Batanero, 2015). The reason behind this insufficient level in probability notions is explained by the poor training of Elementary school teachers on these contents. Many of them are aware of their need to improve their mathematical knowledge, especially regarding statistics and probability, as well as their pedagogical knowledge on mathematics didactics. Muñiz-Rodríguez et al. (2020) defended the importance of training maths teachers on these two areas using contextualised situations that portray real-life contexts to approach the teaching of mathematics. The overuse of textbooks in Spanish schools does not help meet this objective, and this is another main obstacle when trying to overcome the aforementioned problems regarding mathematical teaching. Torres et al. (2013) described the little presence of probability and statistics in Elementary school textbooks, being verbal representations the rarest probability expressions in them. There is a focus on teaching rather formal notions regarding probability, mainly the classical and frequentist meanings, instead of the intuitive and subjective ones, and this is supported by how textbooks treat probability. However, intuitive probability is the first one students begin developing, they even have probabilistic notions and terms before they start the Elementary stage (Fischbein, 1975). If this is the case, then failing to approach or simply not approaching at all probabilistic teaching, will hinder students' further understanding of statistical and probability language in higher courses.

It has been mentioned that there are different meanings of probability: intuitive, frequentist, classic, subjective and axiomatic. These will be explained below:

Intuitive meaning: it is based on the degree of belief about an event occurring. It is of qualitative nature and it is the first one children acquire and are able to express. Terms used range from 'impossible' to 'certain' and can be later on translated into a number line from 0 to 1. Fischbein and Gazit (1984) described how children have misconceptions about the use of these terms and many times argue something is certain if they think it is very likely to happen. Same happens with 'impossible', students refer to very unlikely events as impossible because they have not assimilated that impossible means an event will never occur. It is fundamental these concepts are clarified and worked on in order to build on more complex knowledge and ensure the correct acquisition of these concepts. Random and deterministic experiments should be distinguished when carrying out these experiences. In deterministic experiments the outcome can be predicted, for example, how long it will take for an object to fall to the floor after dropping it. Random experiments have different outcomes but the result cannot be predicted (Alsina et al., 2020). Fischbein (1975) gave a significant importance to intuition whilst exploring probabilistic reasoning arguing that a subjective and global assessment of the likelihood of an event occurring is involved in the intuition of probability. He defined a *primary intuition* as the ability children under seven have to distinguish between deterministic and random experiments.

Classic meaning: it is established on Laplace's formula. In an experiment, all events have the same probability to occur and this formula is used to calculate the probability of combined events. It is expressed with a fraction between 0 and 1. It is linked to percentages because fractions and percentages are usually taught alongside each other and students have a prepared mindset to understand to link these concepts whenever one appears. As a result, many answers regarding probability, even if it is not about its classical meaning, are given as a percentage. This meaning of probability is introduced to 10–12-year-old students who will very likely wrongly apply it to subjective situations or to situations where there is an infinite number of possible events or when events are not equiprobable (Alsina et al., 2020). This incorrect use of Laplace formula may be directly linked to the overfocus of this meaning of probability in the school context without ensuring prior concepts and other meanings of probability are experienced and understood.

Frequentist meaning: this type of probability can be experienced throughout the whole Elementary stage and it is based on the *Law of Large Numbers* (Batanero et al., 2013). The probabilities are obtained after a high number of repetitions of an experiment, and the more repetitions, the closer the result will be to the theoretical probability of that event occurring (Alsina & Vásquez, 2016).

Subjective meaning: also called Bayesian, can be introduced to students aged nine and above. As its own name states, this type of probability is based on students' previous knowledge, beliefs, likes and convictions. There are not usually correct or incorrect answers, but better or worse justifications in terms of the knowledge or data used to make estimates. Experiments that use this type of meaning are affected by many factors and cannot be repeated under equal conditions. The probability is also influenced by the amount of data that the student has been provided, thus, when more data is given, the probability is subject to change. As it is the case of the intuitive meaning, verbal language plays an important role when expressing subjective probability, however, many students will answer with numerical values such as percentages or on a scale from 0 to 1 because verbal language has limitations in terms of precision (Batanero & Godino, 2002). The didactic experiment on subjective probability done by Rodríguez-Muñiz et al. (2022c) proved that "students are able to quite naturally handle probability in informal and even numerical terms" but that in order to acquire a global probabilistic literacy, more contextualised problems including subjective probability must be carried out in school to ensure the correct understanding and use of probability. Moreover, students also use combinatorics when solving subjective probability exercises.

Axiomatic meaning: this type of probability is present in Secondary Education students, given it has a high abstraction component for Elementary school students to fully understand, it will not be used in the development of this essay.

Regarding how probability is represented and expressed, Gómez et al. (2013) identified five groups, each one is briefly described below:

Verbal expressions: refer to the terms used to talk about probabilistic concepts, properties (mainly adjectives) or procedures (mainly verbs). These expressions mainly appear in expressions about intuitive and classical meanings of probability.

Numerical language: it is very frequent and refers to whole numbers, fractions and decimals. Probability of an event or its frequency are normally expressed numerically.

Symbolic language: it uses symbols that have a shared and accepted meaning when expressing probability. Examples are the subtraction or the equal signs. As this language is closely linked to the classical meaning of probability, it does not appear until the higher grades of Elementary school.

Tabular language: it is associated with the frequentist meaning of probability as it uses tables to represent data. Therefore, this language is explicitly linked to probability at the end of Elementary school.

Graphic Language: it is the use of bar charts, pie charts, tree diagrams, histograms, etc. to organise and represent data, each one being suitable to express probabilities with different nature variables. It is also linked to the frequentist meaning of probability.

9. METHODOLOGY

The methodology followed as well as the participants, context and data collection are described hereunder.

9.1 PARTICIPANTS AND CONTEXT

The population involved in the experiment was split into two groups, each one performed a different activity. Within each group there are students from two public schools, one in Oviedo and the other in the outskirts of the same town. The former is a large school from which 34 students participated (18 and 16 in the MasterChef and End-of-year trip activities respectively); the latter is a small rural school with only 15 students (11 and four in the MasterChef and End-of-year trip activities respectively). They all are 5^{th} grade Elementary school students; they are between 10 and 11 years old. These students therefore belong to Piaget’s third stage of cognitive development that ranges from 7- to 11-year-olds. However, Cañizares (1997) pointed out the need to reject Piaget’s linear understanding of probabilistic reasoning as there is proof that children are able to apply notions and concepts belonging to different stages independently of their age, with certain limits. According to Inhelder and Piaget (1955), probability comprehension is acquired during the fourth stage, before which children are not able to solve problems or situations involving probabilistic knowledge. Nevertheless, Fischbein argued that children under seven years old already have what he calls *primary intuition* knowledge about randomness and are able to distinguish between random and deterministic phenomena (Batanero, 2013). Furthermore, Alsina et al. (2021) exposed that students, when asked to make probability predictions and comparisons based on given information, would not only analyse those favourable events they question might be asking, but unfavourable events too. Estimates and predictions with an intuitive reasoning are expected to be abilities students this age have acquired (Fischbein, 1975). It is worth pointing out that students from both schools had very little knowledge on probability and statistics. They had not explored these areas during the previous years of Elementary school because the textbooks used to teach probability didn’t include these topics. Their probabilistic literacy is therefore limited and differences between students are notorious.

Students are expected to use probabilistic language throughout the development of this experiment. Given the different expressions, verbal, numerical, etc. difficulties have been identified regarding the use of these representations. The lack of probabilistic knowledge present in school can be an important factor affecting misconceptions and difficulties in interrelated mathematical concepts such as probability, percentages and graphs. In these terms, there is also a misuse of terms like ‘certain’ and ‘sure’. Moreover, statistical data interpretation and evaluation in order to make judgements and express opinions, has been identified as an important aspect to develop statistical literacy that presents a difficulty for students (Muñiz-Rodríguez et al., 2020).

9.2 DATA COLLECTION

The data was collected in two forms: written answers and oral discussion. The former is the main source of information for the analysis of the results; however, the latter played an important role that influenced the subsequent written answers and is therefore relevant when analysing students’ reasonings. Students were handed a sheet of paper with the information and the questions of each activity. They wrote on this piece of paper and was later collected. Consequently, the written answers are a primary source of data as they come from the same students. The results of the oral discussion made at the beginning were collected in two ways: recordings of some students justifying their answers and notes taken throughout this debate. The recordings are a primary source and the notes taken are a secondary source because they did not come from the authors themselves, they correspond to the transcription and interpretation of a third person. A handicap regarding the recording is that not every student was able to explain their answers, therefore we do not have a complete set of this type of data, and interactions between students are not shown. Written answers do not show this exchange of ideas either, but there are many more answers than to the oral discussion because the debate was only done after the first question of each activity. Results were later classified and represented with tables.

It is worth mentioning that considering students' first language is Spanish and not English, both activities were carried out in their mother tongue. Even if the schools they are enrolled in follow a bilingual programme and most of them know basic English, there are many that have a very deficient level of comprehension, and in general, production skills and output are still very limited. As clarified by Runnqvist et al. (2011) second language (L2) learners “are slower and less accurate in retrieving object-names, it takes them longer to articulate complete words and phrases” (p. 1). For this reason, both the oral and the written phase were performed in Spanish. This would allow students to feel more confident when giving reasons and actively participate in class. Taking into account the potentially complicated nature of justifying their answers, using their first language will give better results for further analysis.

9.3 ANALYSIS METHODOLOGY

Considering the reasoning importance of this work, the methodology used was qualitative content analysis. The worksheets collected were the main object of study, thus, students’ written answers (especially their justifications) were examined using an interpretative analysis that was later used to classify and structure the information gathered according to their different nature. Inferring the meanings exposed by students responds to what Krippendorff (2018) displayed on content analysis. A grounded theory approach was used during the initial assessment of the results because answers were quite varied, short or lacked an elaborate justification. The data was collected, then coded and analysed and finally defined in order to build theories and extract results.

There is a subjective component when interpreting the results because it is the researcher who filters and classifies information; to ensure the analysis was as objective as possible, a *disciplined subjectivity* was carried out, as described by López-Noguero (2002) when explaining the characteristics of qualitative content analysis for which the researcher must be self-conscious. Rodríguez-Suárez (2021) displayed and summarised other characteristics explained by López-Noguero about the qualitative analysis that were also considered when carrying out this work.

10. DEVELOPMENT OF DIDACTIC EXPERIMENT

In order to analyse children's notions about probability and specially about how they handle the subjective meaning of probability to real life situations, two activities have been developed with two different groups of 5th grade students. The first one is based on a famous TV show they probably know and it uses fictional characters the students' age. The second one presents an end-of-year trip for which students need good weather for it to take place. Both activities involve contextualised situations where children play an important role. This brings students closer to the mathematical problems proposed, encouraging them to take part and increasing their intrinsic motivation, defined as "the inherent tendency to seek novelty and challenge, to extend and exercise one's abilities, to explore, and to learn" (p.70) by Deci & Ryan (2000), towards the task. These tasks accomplish the objectives previously stated.

10.1 MASTERCHEF

This first activity was separately undertaken with 29 students from two different schools (18 and 11 respectively) and it focused on subjective probability. The task was about the final test in the Masterchef Junior contest where three participants could win. Given the context of the situation and further information about the participants, students made predictions about the probability of one of them winning. The organisation of the activity, the materials used and the theoretical basis and the development are explained below.

10.1.1 Organisation

This activity was mainly developed individually. Students wrote their answers on a sheet of paper with a set of questions. However, an initial debate was also a key part of this task, so that students were able to share their ideas and opinions and discuss. This task consisted of 3 stages, each one had at least one question and a justification was needed for every stage. More information about the contest and the participants was given before each stage of the task. When discussing as a whole class, students were expected to take turns to speak and respect each other's speaking time by actively listening to each other.

10.1.2 Materials and resources

The main material needed for this activity is the sheet with the information and questions that students must answer (see Annex 1). They also need a pen, pencil and a rubber. The blackboard was used to clarify any words, concepts and help students in case they did not understand any key parts of the questions. The projector was used to project the sheet so that the questions were visible to everyone whilst facing front. Finally, a presentation was projected at the beginning of the activity for supporting the initial explanation highlighting the main ideas (see Annex 2).

10.1.3 Theoretical basis

Students were asked to express the probability of an event using the information provided, their own beliefs with respect to this information and any previous knowledge they had on the topic. Children might already know something about cooking or they might be familiar with the TV programme. As there were many variables and factors affecting the outcome, it was not possible to use Laplace's formula to calculate the probability of this event, therefore, the classical meaning could not be applied here. Furthermore, it was an experiment that cannot be repeated under the same conditions and there was no previous reference that can be mathematically taken to calculate a probability, therefore neither the experimental meaning of probability was applicable. Consequently, there were no correct or incorrect answers, but there were better and worse justifications that should expose students' thinking process when answering the question.

All answers are subjective and conditioned by students' conceptions, likes and interpretation of the data provided. Adding new data modified opinions about the context based on their previous knowledge about the situation (Alsina et al., 2020). Willingness for the event to occur may affect subsequent answers when more information is added if students have a special interest for the event to take place. This interest might be driven by an empathic involvement in the problem, eased by the fact that participants are a similar age to them; this will allow students to feel more identified and closer to the context presented. There is also a motivational aspect in this problem that will encourage students to pay attention and try to answer 'correctly'. Taking their interests into account will help accomplish this task. Expressions of probabilities are not limited to verbal language (typical expression of subjective probability), mainly because of the numerical emphasis in the teaching of probability and students' notions on the topic. Any representations are valid and positive to assess students' conceptions and misconceptions about probability.

10.1.4 Development of the activity

To begin this activity, the teacher made a general review on what students know about probability. Terms and notions were recalled for students to refresh their memory on the topic. No theoretical concepts were deeply explained to try and ensure their answers were not biased. This initial debate might also be understood as a general brainstorm on probabilistic notions and its representations. Then, the teacher described the context of the task and explained what the students had to do and what materials could be used for each question, if any. The teacher also ensured every student understood the activity and answered any doubts throughout the development of the task.

The task was then introduced explaining the initial statement on the sheet and the MasterChef situation. Despite this being written down, a clear oral description allowed the teacher to further explain anything if needed. Many students were already familiarised with and knew something about this TV programme given it has a ‘junior’ version (MasterChef Junior).

Students were handed the sheet and asked to read the first stage, then had to think about it and discuss the answers in pairs or small groups. Once they had done this, they shared their opinions to the rest of the class and then they individually wrote their answers on their sheet. For these first questions, information about the elimination tests each participant had taken part in and information about the MasterChef Summer Camp was given. Students had to rank these three participants and give a probability to each one of winning. They were expected to graphically represent the three probabilities on the pie chart, but when answering the last question of this stage they should use another representation method, be it graphic, written, numerical or a picture. Inés’ probability of winning was the main question throughout the rest of the stages which students will have to answer after reading further information about the contest to see how this probability changes and why.

For the second stage, students answered the questions individually. The information given referred to the votes given to Inés on a digital survey done before the final. The main idea was to see how students incorporated this information, if they thought it was valid or not in order to make a probabilistic judgement and to think about how their initial answer might change when taking this information into account. It is important to highlight that viewers’ votes were also an opinion in terms of subjective probability. Students had to decide if they thought that Inés being chosen the ‘favourite contestant’ with 50% of the votes is reliable information to change their previous probability.

The last stage incorporated more information, this time related to the type of food the participants will have to cook during the final test. Students had to decide how this new information affected their judgement on what they knew up to then and if (and how) they would modify the probability of Inés winning. The new information given could be understood from different perspectives so that it did not have to be potentially positive or negative for each participant. For example, Inés being a vegetarian could have meant she did not have a clue about how to cook a fish or it could not be relevant enough: she cooks fish and meat but doesn’t eat them. Information about if the participants had to cook animals before during the contest was not clarified, therefore, some students assumed it was not the first time Inés cooked fish and the information given in the statement of this third stage regarding Inés was not useful and did not say much. This critical thinking could have been done for every question and statement; however, we did not expect every student to be able to accomplish this; we were looking for different approaches and perspectives when assimilating and incorporating new information to modify the previous probability.

10.2 END-OF-YEAR TRIP

The next activity was done separately undertaken with 20 students from two different schools (16 and four respectively) and it focused on subjective probability. This task was about an end-of-year trip to Llanes for which good weather was needed. Students were given information about rainfall and temperature over the past five years as well as the weather forecast for the days of the trip in different steps and made predictions about the probability of having good weather and going on the trip. The organisation of the activity, the materials used, the theoretical basis and the development are explained below.

10.2.1 Organisation

This activity was mainly developed individually. Students wrote their answers on a sheet of paper with a set of questions. However, an initial debate was also a key part of this task, so that students were able to share their ideas and opinions and discuss. This task consisted of five sets of questions; each one required a justification. More information about the weather conditions was given before each set of questions. When discussing as a whole class, students were expected to take turns to speak and respect each other's speaking time by actively listening to each other.

10.2.2 Materials and resources

The main material is a sheet of paper that includes the initial contextualisation of the premise and the questions (see Annex 3). Students need a pen, a pencil and a rubber. Some of the questions required the use of graphical data which was provided in digital format given every student had a Chromebook and could easily use it to view the information needed and the graphs. In this case, the data was in pdf format and was posted on a Teams channel. It could have been printed, but given the resources the school had available and that students were very familiar with the use of Chromebooks, it was preferable to save paper and ink. These data were also projected on the board to ensure everyone saw properly when facing front. The statistical data on the weather in Llanes was taken from the AEMET weather station in Llanes (Asturias) from the site *Meteosolana* (https://es.meteosolana.net/). Finally, a presentation was projected at the beginning of the activity for supporting the initial explanation highlighting the main ideas (see Annex 4).

10.2.3 Theoretical basis

Students were asked to express the probability of an event using the information provided, their own beliefs with respect to this information and any previous knowledge they had on the topic. Children might already be familiar with weather patterns throughout the year and statistical data on the weather. There were many random variables and factors affecting the weather phenomena (Batanero & Godino, 2002). It was not possible to use Laplace's formula to calculate the probability of this event, therefore, the classical meaning could not be applied here. Furthermore, it was an experiment that cannot be repeated under the same conditions and there was no previous reference that can be mathematically taken to calculate a probability, therefore neither the experimental meaning of probability was applicable. Consequently, there were no correct or incorrect answers, but there were better and worse justifications that should expose students' thinking process when answering the question.

All answers were subjective and conditioned by students' conceptions, likes and interpretation of the data provided. Adding new data modified opinions about the context based on their previous knowledge (Alsina et al., 2020). Willingness for the event to occur may affect subsequent answers when more information is added if students have a special interest for the event to take place. This interest might have been driven by an empathic involvement in the problem if students felt they really want to go on the trip, which could lead to different results if the emotional component did not exist. Interpreting statistical data and graphs may be a challenge if they are not used to handling such data. Difficulties that arose with graph comprehension that handicapped the interpretation of data and therefore, the answer regarding subjective probability. There can be a significant variability in the acquisition of the four levels of graphical comprehension: literal reading, data interpretation, inference making and data analysis stated by Batanero and Godino (2002) within the students which can lead to major differences when analysing the graphs and answering the questions. There was also a motivational aspect in this problem that could have encouraged students to pay attention and try to answer 'correctly'. Taking their interests into account will help accomplish this task. Expressions of probabilities are not limited to verbal language (typical expression of subjective probability), mainly because of the numerical emphasis in the teaching of probability and students' notions on the topic. Any representations are valid and positive to assess students' conceptions and misconceptions about probability.

10.2.4 Development of the activity

To begin this activity, the teacher made a general review on what students knew about probability. Terms and notions were recalled for students to refresh their memory on the topic. No theoretical concepts were deeply explained to try and ensure their answers were not biased. This initial debate might also be understood as a general brainstorm on probabilistic notions and its representations. Moreover, weather concepts such as amount of rainfall and the units it is measured in were reviewed because it was necessary to understand the data given. Students struggled understanding these concepts, so the teacher made comparisons and put some examples. The teacher also ensured every student understood the activity and answered any doubts throughout the development of the task.

The context of the activity was described; students will be going on an end-of-year trip to Llanes in June and they will be doing several activities for which they need good weather. This information was projected on the board to ensure they were all paying attention when facing front. The term 'good weather' was discussed as it can be very relative. As a class, they got to the conclusion that the ideal weather conditions are: sunny with no or little rain and a nice temperature, not too cold but not too hot, comparisons with the weather and temperature that day were made to ensure they understood what we were looking for.

The first set of questions was read and a discussion was started about what they thought. Once they had shared general ideas, they began writing their answers to the first set individually.

Graphs and statistical data about the weather in Llanes in June over the last five years was uploaded to Microsoft Teams. They looked through this information (mean temperature, mean rainfall and total days of rain) freely when answering the second set of questions. New data was provided which could modify their previous answer, they had to explain why and which data they were looking at when they made a new prediction.

The third set of questions was very similar to the previous one, the difference was that new data was uploaded to the Teams channel and this data was specific to the days the trip would be done: 8, 9 and 10th of June, again throughout the past five years. The same question about probability was asked.

The fourth question introduced more information, this time it was an official weather forecast prediction for June 2022. Students had to think if they would change their previous answer and why. They could use data from previous questions if they wanted to.

Finally, the 5th question gave information about the weather in Llanes the previous day of the trip. Students answered if they would modify the previously predicted probability explaining their reasons why.

11. RESULTS

The main piece of evidence was the sheet with the written answers on, but given an initial discussion was carried out, these answers were also gathered. Regarding the latter, they might have influenced subsequent answers which were all reflected on the sheets. The results were therefore organised into two groups, oral discussion and written answers.

11.1 ORAL DISCUSSION

An initial debate after the first question of each task was done.

To begin with, the MasterChef experiment generated different rankings in the first question in each school: in one most students agreed that the winning order was Diego, Inés, Samuel, and in the other school, they decided it would be Diego, Samuel, Inés. In both cases, they all agreed that Diego had a higher probability of winning because he was older and had more experience cooking, the latter is an assumption they all made regarding Diego's cooking knowledge. Predictions mainly used verbal instead numerical language when expressing probability. However, when percentages were used as an answer, they seemed not following a pattern. They even used decimals like 50.5% but when asked to justify this choice, they could not. Diego being the favourite in both schools could have conditioned the subsequent answers because students showed a particular willingness for Diego to win at the beginning. Arguments during this oral discussion were richer than the written answers, some students thought about how different information could be interpreted in different ways. For example, a student said that the more elimination tests the participant had gone to, the less chance he or she had to win because those who were the worst went to the elimination tests. Other students replied to this answer saying that maybe that participant was unlucky because his/her teammates were not the best at cooking and was conditioned by them. And another student pointed out that the more elimination tests the participant had gone to, the more practice he/she had at cooking alone under stressful conditions. These counter arguments expressed in the debate were not later reflected in the written answers, each student wrote his/her opinion without taking into account other students' beliefs.

In the End-of-year trip activity the debate was done after the first two questions of the first set were read. Most of them agreed that there will be good weather in June because there is normally nice weather this month. This justification was based on their previous knowledge; however, it was not included in every student's answer. There was also a motivational factor that affected their initial ideas and prediction; they were very excited because they identified and emphasised with the students going on the trip as if they were them. Obviously, they wanted the trip to take place, so the initial general prospect was favourable for them. Nonetheless, this emphasis was not so well captured in their written answers. Regarding the probability expression, most of them used a percentage, which is reflected later on during the writing phase. However, they did not justify the choice for that percentage, especially when values such as 74% and 52% were given, they said they 'simply knew'.

11.2 WRITTEN ANSWERS

This section contains the results written by the students on the sheets given. The results are organised in two groups, one for each activity: first the MasterChef activity and then, the End-of-year trip activity. Given the small number of students in one of the schools, separating their answers into a different table would turn out rather poor and meaningless. For this reason, the data collected has been classified and organised into tables in which the results from both schools have been put together and no distinction has been made. As there are many questions and tables, some analysis will be done throughout the explanation of the results inferring reasons for determined answers and possible patterns.

11.2.1 Masterchef

In the first question of stage 1 students had to rank the three participants according to the place they thought they were going to get in the final test and justify their answer. The results have been classified in compliance with this order and the nature of the different reasons given (Table 1). The names of the participants have been shortened to their first letter: D for Diego, I for Inés and S for Samuel.

Table 1: Number of students according to their answers to question 1 of stage 1.

Order	**DIS**			**DSI**			**ISD**		
	18			7			4		
Justification	**Data**	**Beliefs**	**No justification**	**Data**	**Beliefs**	**No justification**	**Data**	**Beliefs**	**No justification**
	15	2	1	5	2	0	2	0	2

At a first glance we see that there are only three possibilities out of the six possible ones. The majority thought Diego would win because he had more experience and knowledge. This can also be related to his age. The interesting data here was the reasoning they made when justifying their answers. The majority, 17 students, based their answers on the data provided and re-wrote some of the statements given. Some students combined different data to rank the participants; most of them focused on their age and their 'experience cooking' arguing that Diego had more experience because he had gone to the Summer Camp, was older and had participated in more elimination tests. However, many of them focused only on part of the information to rank them, one student who ordered them as DSI wrote: "it is ranked by the number of elimination tests", so the more experience in these tests, the higher the probability of winning. Other students used this same classification rule but the other way round interpreting elimination tests as something that affected them negatively when it came to winning. Moreover, these students that used the data provided made many assumptions about the three participants and the contest. They assumed things such as "Diego has more cooking knowledge" to validate their answers and they did not infer double meanings to the statements.

Those students who based their answers on their beliefs wrote arguments such as: "I don't think Inés cooks very well", "Inés is better than Samuel" or "Diego is first because he is more patient". The arguments usually started with "I think…" or "I believe…", yet, a large number of students wrote explanations such as "It is known that Samuel will be third" or "Diego will win".

In the second question of stage 1 they had to think about if they were certain that their prediction would be correct and why. Table 2 shows the number of students who answered and the different nature of the reasons given.

Table 2: Number of students according to their answers to question 2 of stage 1.

Will it come true for sure or not?	**Yes**				**No**		**Without answer**
	13				6		10
Justification	**Different percentages**	**Beliefs/ Willingness**	**Contradiction**	**No justification**	**Everyone has a chance**	**No justification**	
	1	1	7	4	5	1	

A large number of students did not answer this question, ten of them. For those who did write Yes or No, five of them did not justify their answers. Examples of answers were: "My prediction is correct" and "My prediction will be true for sure". Only one student wrote "I am sure this prediction is certain but they might have different probabilities of winning". She was sure of her answer but knew that they could have a different probability of winning even if the order remained unaltered. Another student wrote "I think it will be certain because if I put my mind to it, it will come true". She based her answer on her beliefs and her willingness for Diego to win. Thinking that wanting very hard something that you cannot control will be enough reveals a sort of fantastic and childish mindset. Four students answered 'Yes' but gave a contradictory answer: "Yes, it is certain. There is 90% probability of it becoming true". The others wrote different percentages. This misunderstanding is normal given the little knowledge they have on the topic.

Those students who answered a justified 'No' shared the same argument: "It is not certain because everyone has possibilities of winning". Some of them even went a little bit further and argued that "It is not certain because it will depend on the final test and they all have the same probability". This student was considering that other factors that have not been mentioned can affect the participants' performance and cooking and that you can't know for sure what will happen, you can't predict the future.

The next part of this stage consisted of using a pie chart to represent the probability each participant had of winning. Students knew they had to split it into three parts where the larger the sector, the higher the probability of winning. Even if this representation 'forced' them to understand that probabilities should add up to 1 or 100%, many did not extrapolate this concept when answering the next question as explained below. Some of them included percentages in their pie charts and were clearly well represented as shown in Figure 1. However, others were confusing and showed that students did not know how pie charts are used in statistics. Figures 2 and 3 show two pie charts that try to represent what was asked but that are not correctly done: Figure 2 shows a slight idea and comprehension but the student did not join the sector at the centre of the circle; Figure 3 corresponds to a student who had little or no notions on graphical representations used in statistics and probability.

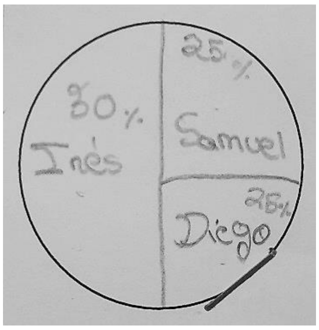

Figure 1: Student's pie chart representation of probability for stage 1.

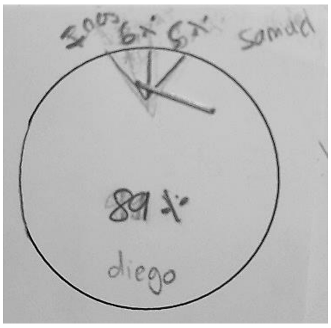

Figure 2: Student's pie chart representation of probability for stage 1.

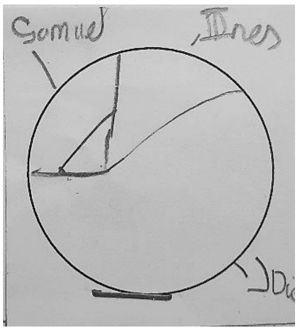

Figure 3: Student's pie chart representation of probability for stage 1.

In the next question, students wrote the probability of Inés winning. The results were classified according to the different representations of the answers (Table 3). They were not told to write their answers in any particular way; however, they could use the graph drawn previously to help them in this question.

Table 3: Number of students according to their answers to question 3 of stage 1.

Probability of Inés winning	**Percentage**	**Verbal language**	**Both**	**Without answer**
	25	2	1	1

The vast majority of students, 25 out of 29, decided to answer with a percentage even if they were not told explicitly that this is a way to express probability. Many students used their pie charts to write a percentage for each participant, in this case, the answer was counted as 'Percentage' because they had been asked to draw the pie chart previously, it was not something that came out from them. Most students not only gave a probability for Inés winning, but gave a probability to Diego and Samuel too. The percentages used were very diverse, some of them were round numbers such as 30% or 40%, and others seemed more haphazard such as 6%, 34% or 26%. An interesting aspect was that some students who wrote percentages for the three participants did not make them add up to 100%. For example, an answer was "Inés has 89%, Samuel 30% and Diego 40%". Even though many students wrote the percentages on the pie chart, some of them did not realise that the sum of all the percentages should add up to 100%. One student initially stated that even if he thought that the ranking could be DSI, the three of them had the same probability of winning. He separated what he thought about the ranking from what he thought the 'real' probability could be. As the data given could have favoured the three participants and there are too many factors to take into account when carrying out a valuation, he could not give a probability of winning to each one. However, he then wrote that Inés had 50% probability of winning when asked in this third question of stage 1. The only answer that showed a combined representation, both verbal language and a percentage was a boy who wrote: "The probability is very low. It is 12%".

For the next stage, new information was provided: results to an online survey showed that Inés was voted as the favourite contestant with 50% of the votes. Students were asked to think if they would change their previous answer regarding Inés' probability of winning justifying it. The arguments given were classified into different groups and shown in the table below (Table 4). The main idea was to analyse how students interpret new information and incorporate it to their previous opinions to see how the answers were modified or not and why.

Table 4: Number of students according to their answers to the question of stage 2.

Would you change your answer?	**Yes**	**No**					**Without answer**
	5	22					2
Justification	**Data influence**	**Assumptions/ Beliefs**	**Validates their answer**	**Data is irrelevant**	**Other answer**	**No justification**	
	5	5	2	8	4	3	

The most popular response was that they would not change their answer. As to the ones who decided to modify their answer, they all gave the same argument: they thought this new data was valid and reliable, hence, their previous answer was inadequate and incorrect. A repeated answer was: “I trust the audience and they think Inés is going to win”. These students assumed that Inés’ probability of winning was now 50% even if being voted as favourite and winning the final test could mean different things. A student wrote: “Inés has 50% of the victory assured”.

Regarding those who did not modify the probability from the previous stage, various reasons were given to justify their answers. To begin with, two of them gave very similar reasoning to the ones who modified their answer, they trusted the audience and thought Inés had 50% probability of winning. The reason for this was that they both had given Inés a probability of 50%, therefore, this new data validated their previous answer and reinforced it, so they automatically interpreted it as correct and valid in order to answer this question. Other students made assumptions and justified their answer basing themselves on beliefs and made-up inferences. These five students did not change their answer and the repeated reasoning was that they thought the survey was not valid because the votes had been bought and the public had been bribed. One student also wrote that the audience felt pity for Inés because it was her first elimination test and wanted her to win. And another one stated that he did not “trust Inés will make a good dish”. The most popular reasoning within this group made reference to the new data provided but argued that you cannot trust others’ opinions, the data is irrelevant. These students wrote statements such as: “I don’t change my answer because what other people vote is not relevant, you don’t change your opinion just because other people say so”, “My answer is my own answer and other people’s answer is their answer, so I don’t change my opinion” and “I don’t care what other people say, my opinion cannot be changed by anyone”. These students seemed to be on the defensive and these arguments might tell us more about their personalities than anything else. Finally, there were other reasons worth mentioning. A student wrote that winning does not depend on the audience, it depends on her performance and the judges. Other two students wrote they would not change their answer but did not mention the new information given to justify this answer, they repeated the same arguments as for stage 1 using previous data. Another student wrote: “I think Inés still has the same probability of winning because people wanting her to win and being a better cook than Diego and Samuel are two different things”.

The next and last stage also introduced more information, this time about the final test and some context about each participant. Students were asked again to think if they would change their answer and explain why. Table 5 shows the results where the justification was classified according to its different nature and general groupings. Those who decided to modify their answer had to say what probability they thought Inés had of winning now.

Table 5: Number of students according to their answers to the question of stage 3.

<table>
<tr><th>Would you change your answer?</th><th colspan="3">Yes</th><th colspan="2">No</th><th>Without answer</th></tr>
<tr><td></td><td colspan="3">16</td><td colspan="2">9</td><td>4</td></tr>
<tr><th>Justification</th><th colspan="2">Data about:</th><th rowspan="2">No justification</th><th>Data is irrelevant</th><th>Other answer</th><td></td></tr>
<tr><td></td><th>Inés</th><th>Everyone</th><td>4</td><td>5</td><td></td></tr>
<tr><td></td><td>8</td><td>4</td><td>4</td><td></td><td></td><td></td></tr>
</table>

New probability of Inés winning	Percentage	Percentage and graph	Verbal language	No answer
	6	8	1	1

Comparing this table with the previous one, now the majority of students decided to modify their answers, 16 in comparison to nine who did not change their probability. Students who changed their response argued that the new data provided was important and meaningful to make a prediction and change their answer. The new data influenced their opinion but not all data was taken into account when justifying themselves. Two thirds of them, focused on what they thought was more important: Inés being a vegetarian. They thought she would not be able to cook fish because either she did not know how or she would not want to: “I changed my answer to 15% because if she is a vegetarian, it is likely she hasn’t cooked fish before”, “Inés will not win because she has never cooked fish before” and “Inés will be disgusted by cooking fish”. However, other students valued all the new information given and their answers reflected Diego’s and Samuel’s context too. Modifications could have been made taking into account the boys’ information and deciding they have more probability of winning, therefore, Inés probability will be lowered, instead of only focusing on Inés’ data in order to decrease her probability. This argumentation was not thought of by any of the students. Those students who included all the new data wrote responses such as:

> “This test is harder for Inés because she is a vegetarian, but she still has probabilities of winning. Diego’s grandfather is a fishmonger and could have taught him about fish. Samuel knows how to cook fish well because he returned to the contest thanks to a fish dish, so he has more probabilities of winning than before”.

With respect to the children who did not change their answers, four of them argued that the data was not relevant, Inés being a vegetarian does not mean she does not know how to cook fish: “It's nothing to do with her being a vegetarian, she can cook fish just as well”. The other five students had different arguments. One of them believed that Diego would win and was convinced of it. Other two students did not use this new information to justify their answers, they used previous data given at the beginning. Another answer was: “Inés had many possibilities of winning, but now they are lower because she is a vegetarian, Diego is, for me, the best cook and Samuel is the worst”. She based her answer on her likes and beliefs, she preferred Diego over the rest but did not explain the reason for this. Finally, another student stated: “I don’t change my answer because they all have the same probability of winning”.

Those students who decided to modify the probability of Inés winning had to give a new probability. Only one student used verbal language to express this probability, he had used verbal language before in other questions instead of percentages or graphical representations: “Inés has now little chance of winning”. Six students used a percentage to express probability, they all had used percentages in previous answers. Others used both percentage and graphs. This might have been influenced by the pie chart drawn on stage 1. Some of them drew a new pie chart with the new percentages. Some of these percentages were round numbers like 40% or 15%, but others were decimal numbers such as 0.2%, 0.9% and 3.1%. No reasons were given as to why a decimal number instead of a whole number. One student decided to draw a bar chart (Figure 4), it was correctly done although not very accurate as she did not have squared paper. Other representations consisted of a rectangle divided into different parts, one for each participant. It is difficult to tell if the divisions are correct because it is difficult to split a rectangle to represent percentages accurately. This was a girl’s idea trying to innovate and a boy who did not have much idea and decided to copy her (Figure 5).

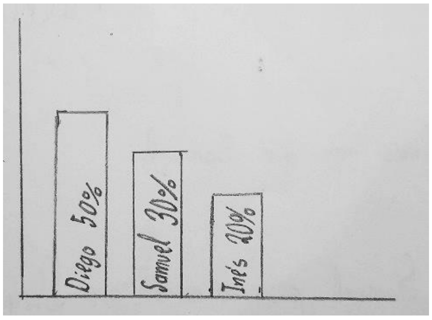

Figure 4: Student’s bar chart representation for percentage distribution for the question of stage 3.

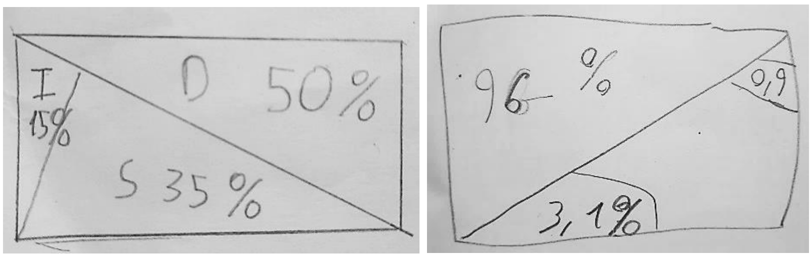

Figure 5: Students’ graphical representation for percentage distribution for the question of stage 3.

11.2.2 End-of-year trip

The first question of set 1 asked students to predict a probability of good weather in June. Table 6 shows the different probability expressions given when answering this question. Students were free to write their answers in any form, nevertheless, the vast majority used percentages.

Table 6: Number of students according to their answers to question 1 of first set.

Probability of good weather	Percentage	Percentage and graph	Verbal language
	15	4	1

The graphs were pie charts where probabilities had been added, the other students drew a bar chart. All answers except 1, expressed probability in numerical form, particularly with a percentage. These percentages were generally round numbers like 60%, 90% or 75%. Many answers were 50%, and the general argument was that "it can be sunny or it can rain", a student added to this: "just like a coin landing on heads or tails". A repeated pattern was that several students wrote a probability for good weather but also a probability for raining (they understood raining as bad weather), but some answers did not add up to 100% (Figure 6) and led to a confusing interpretation of the other probabilities expressed. The only verbal answer was: "Good weather in June 2022 is very likely", although some other students tried to include a verbal answer and wrote things like: "good weather is positive" when explaining that they thought it was more likely good weather than rain.

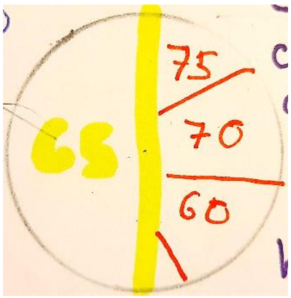

Figure 6: Student's graphical representation for percentage distribution for question 1 of first set.

In this first set of questions they had to justify their answer explaining what they based their arguments on. The reasons were classified into their different nature after interpreting the results (Table 7).

Table 7: Number of students according to their answers to question 2 of first set.

Justification	Observation	Previous knowledge	Both	Without answer
	5	7	5	4

There were repeated explanations, mainly due to the previous whole class discussion and answers were quite equally distributed. Some of them observed the weather that week or that month and made a prediction based on that observed information, for example: "I base my answer on the weather in April because that could be a great approach to the weather in June". Others used their previous knowledge to answer: "It is summer and in summer there is good weather". These students thought of June as summer because it is the month they begin their Summer holidays even though the dates we are talking about (8, 9 and 10) are still spring. There were also more scientific answers: "Climate change is getting worse and June will be a very hot month". Those students who used their previous knowledge and observations argued that if this month's weather is quite good, then June will be much better because it is summer.

The last question of this first set asked students to express how reliable they thought their answer was, many of them expressed this in terms of certainty and confidence and used different representations of probability. No justification was given to this question by any student. Table 8 shows the answers of students classified according to how they expressed the probability of their answer being true.

Table 8: Number of students according to their answers to question 3 of first set.

Reliability of their estimate	**Percentage**	**Verbal language**	**Vague answer**	**Without answer**
	5	5	3	7

There was a high number of students who did not answer this question. When comparing these results to the ones from the first question, we see that there is a more balanced expression of probability, only five students used a percentage again to express their degree of confidence in their answer. Examples are: "I am 70% certain of my answer" and "My estimate is 60% reliable". However, more students decided to express it with verbal language, something they had not done before. Some answers were: "I think it is very likely that it is reliable", "I am pretty sure of my answer" and "I am almost certain of the reliability of my answer". Other answers were vague and proved the misunderstanding of the question: "I am an insecure person" and "I am not very sure if it will rain or if it will be sunny".

The following questions, 2 and 3, incorporated new information. Students had to look at some graphs and statistical data on the weather in June for the last five years. They could use the data they want to, but answers had to be justified. Question 3 incorporated more precise data that corresponded to 3 days of June. Answers to questions 2 and 3 were very similar and have been organised in two groups, one according to the type of probability representation (Table 9), and the other according to the data used in the justification (Table 10).

Table 9: Number of students according to their answers to questions of sets 2 and 3.

	Percentage	**Verbal language**	**Without answer**
2	13	2	5
3	11	3	6
Changed probability from 2 to 3	**Yes**	**No**	**Without answer**
	7	6	7

Again, the majority of answers were percentages. A student wrote in several questions: "my 'precise' percentage is 67%" when she thought she was giving an 'exact' estimate instead of a rounded-up answer as could be 70% for example. Examples of the use of verbal language were: "It is more likely that it rains than that it is sunny" and "There is little probability that there is bad weather". This last question represented something that many students did throughout the task: they gave a probability of bad weather instead of good weather when the question specifically said ‘probability of good weather’. Seven students decided to change their answer to question 2 when answering question 3, no justification was given for this change or vague arguments were exposed.

Table 10: Number of students according to the data used in questions of sets 2 and 3.

	Rain	Temperature	Both	Not specified	Without answer
2	4	2	6	2	6

Years		
1	**Between 2 and 4**	**5**
1	10	3

	Rain	Temperature	Both	Not specified	Without answer
3	6	3	2	3	6

Years		
1	**Between 2 and 4**	**5**
2	8	4

Table 10 shows the results according to the data used to justify answers to questions 2 and 3. Most students used data about temperature and rainfall in question 2, however, most students in question 3 only used data about rain. Temperature was quite stable throughout the month, and once children saw that the differences in temperature between one day and the next were small or non-existent, they decided to use rainfall as the main factor to take into account. Differences in rainfall are much more evident and notorious, this made students use it as the main source of information to take into account when justifying their answers. Moreover, they decided what years to look at, in both questions most of them used between two and four years. There were a few students who used the data from the five years provided. Within the students who did not look at the five years, most of them only looked at two, especially the last two years (2020 and 2021). Others made comparisons between 2017 and 2021 to see the evolution and then made an estimate for 2022: “In 2017 there was a mean temperature of 17.8ºC and in 2021 of 18.2ºC, this means there will be better weather in 2022”. Year 2021 was used in almost every answer because it is the year right before 2022 and could provide more reliable information; they thought 2022 would be more similar to the year before than to 2017.

Table 11: Number of students according to the data used in questions of sets 2 and 3 (simplified).

	Some data	All data
2	13	1
3	11	3

Table 11 shows simplified results of the data used to answer questions 2 and 3. All data referred to those answers which included information about both the temperature and the rainfall and mentioned the five years. Most students decided to use some of the data provided instead of all, this was an individual decision.

Question 4 incorporated new information, this time an actual weather forecast for June 2022. Students had to decide if they would change their answer after seeing what the experts say about the weather. The results were organised according to their answer, Yes or No, and the nature of the justification given (Table 12).

Table 12: Number of students according to their answers to question 4.

Would you change your answer?	**Yes**			**No**				**Without answer**
	6			9				5
Justification	**Data**	**Beliefs**	**No justification**	**Data**	**Beliefs**	**Other answer**	**No justification**	
	4	1	1	5	2	1	1	

Those who decided to change their answer mainly used the data provided to justify their response. They all interpreted the data as worse than expected: "It rains more than I thought it would and it will be colder than what I thought". Another student made a very similar argument but she added that "it will rain 1 of the 3 days of the trip", she used the data for the whole month and applied it to the days of the trip. There was a rainfall prediction of 10 days and the month has 30 days; she simplified the fraction from 10/30 to ⅓ in order to compare it to the days of the trip. However, she then wrote: "90% likely to rain and 10% likely to be sunny". The percentage given did not prove the logic used previously in her answer. Only one student justified her answer without using the data provided in this question and based it on her beliefs, she also wrote: "I am not sure because I am not a fortune-teller". Most students who modified the probability of good weather with respect to the previous question also gave a new probability, the majority were expressed as percentages although there were a couple of them who used verbal language.

As for the students who did not modify their answer, most of them used the data provided when justifying themselves. Answers were similar to those who modified their answers and used the data, but this time the data about temperature and rainfall matched their expectations of good weather and the prediction made in the previous answer. This contrast showed how different perceptions prove relativity and subjectivity of information. Examples of these answers were: "I thought there would be between 16ºC and 18ºC and the forecast says 16.4ºC" and "10 days of rain out of 31 are not so many and 16.4ºC is a good temperature". Students who based their answer on their beliefs argued they were certain their probability was correct and it would not rain. Finally, another student wrote: "Like any normal person would think, as those three days have not yet passed, we cannot assure with 100% certainty that there will be 16.4ºC".

The last question also provided new information, this time about the day before the trip as if they had fast forwarded to that date. They were asked again if they would change their answer now that they have more information. Results were organised into Yes or No answers and the reasons classified according to their nature (Table 13).

Table 13: Number of students according to their answers to question 5.

Would you change your answer?	**Yes**			**No**		**Without answer**
	5			10		5
Justification	**Data**	**Beliefs**	**No justification**	**Data**	**Beliefs**	
	2	1	2	5	5	

More students decided to maintain their previous answer. The main difference between these answers and the ones to question 4 was that in this question there was a higher number of justifications based on beliefs instead of data. Those who used the data in the reasoning considered that the weather during the trip would be the same as the day before the trip thinking that it would rain in the morning and be sunny in the afternoon. They made their predictions taking into account that half of the day it rained and the other half it was sunny and therefore, the probability of good weather would be 50%. Students who did not change their answer and who did not use the data argued that it could rain and it could be sunny. A student wrote "I still trust my probability" and another one used his previous knowledge about Asturias being a variable microclimate to explain his reasons.

12. ANALYSIS OF THE RESULTS

Results will be now analysed in general terms taking into account those considerations mentioned throughout the previous section. The first analysis will be that of the MasterChef activity followed by the End-of-year trip activity.

- **MasterChef**

In the first question of stage 1, students sitting in the same group or pair came to a consensus about the ranking, this affected the written response but individual justifications were quite varied. The arguments given to support their answer were mainly based on the data provided about the three participants, however, these justifications used the same statements written at the beginning, sometimes rephrasing them, but no further analysis or inference was done. The data was either assessed as positive or negative for the participant, depending on each child's interpretation, without seeing the double-sidedness of data or making assumptions. For example, almost everyone thought that Inés had less practice cooking alone in the competition and they value this as a negative thinking she has been lucky in the team tests. They value this luck but not Samuel's luck in having cooked a good dish in the repechage, if it was a matter of luck. No one ranked Samuel first, this might have been because they thought age was a significant factor to take into account and because he had been eliminated before and that meant he was worse than the others (no one thought that he could have had a bad day the day he was eliminated).

Answers to the question on certainty presented a challenge for many, ten of them did not answer. The reason for this might be the lack of comprehension or the little importance given to the question. This contradiction also showed a misunderstanding of probabilistic terms when students answered "it is 90% certain". They interpreted this question as if they thought it would come true and what degree of confidence they had in their prediction. An event cannot be certain and have a 70% probability of occurring. An event being certain means it will happen, its probability is therefore 100%, however, students used it wrongly when expressing their level of confidence. Terms that they should have used respond to the intuitive meaning of probability and are for example: very likely or likely.

The pie charts drawn are generally okay, they show students understood what they were asked to do and most of them included percentages too. Even if they were not exact, you can interpret what the child was trying to represent. However, there were a few answers that made no sense where the divisions made seem completely random or do not follow the rules on how pie charts are drawn. This might be explained by the lack of experience making graphs, especially pie charts.

In the last question of this stage students had to express the probability of Inés winning. The representation of probabilities using pie charts is generally taught alongside percentages and students seem to have associated both representations. This is one of the reasons why they chose percentages to express probability after drawing a pie chart. The other main reason is that probability and percentage are terms that appear to be taught strictly together as it is the classical meaning of probability and Laplace's rule, the type of probability that is mostly seen in school at this stage. A proof of this is shown in many answers where students exchanged both terms (percentage and probability) as if they meant the same thing: a student wrote, "my percentage is 60%" when she was expressing the probability of Inés winning. Some answers exhibit students' need to give a 'precise' percentage, which for them is not a round number as they might interpret round numbers as approximations or estimates. Another important aspect to point out is that percentages written for each one did not add up to 100%. This demonstrates a misunderstanding of how percentages are calculated and what they show. Finally, there were answers where the % symbol was not written, students wrote the number believing it is clear it represents a percentage, this shows they understand percentages are the only way of expressing probability. They do apply the subjective meaning of probability, it is something they are used to doing all the time, but not in the school context, therefore, they have problems when linking it to the academic mathematical knowledge they have.

The next stage introduced information on a survey and results show this information was not important enough to change their opinion. The most repeated reasoning was that the data is irrelevant. This might be explained because students do not feel their opinion should be modified just because of what other people believe. They seem to have strong convictions not easily changed by other people's influence. Some arguments used this same idea but went a little bit further and made assumptions or invented theories to explain why they thought the data should not be taken into account: the audience had been bribed.

In the third stage, more students decided to change their answer. Comparing this result with the one of stage 2 where most decided to keep their answers might indicate that students validate objective information (the data given about the participants and the contest in this stage) more than subjective information (the votes from the survey). Even if the number of votes is an objective data, the reasons behind the choice of vote is not. When justifying their answers in this third stage they gave more importance to the new information provided to the previous information given at the beginning of the task which shaped their initial opinion. No justification included all the input stated throughout the activity. This shows that students tend to give more importance to the new information presented in order to make a statement and maybe the order in which facts are introduced could alter the results. Further analysis of this stage shows that most students focused only on Inés' characteristics (being a vegetarian) in order to answer. This might be because the question focused on Inés probability to win, however, modifying the other two participants' probabilities by only taking into account the data about them will also have an effect on Inés probability to win and will be modified; if the probability of the boys increases, then that of Inés will decrease. When justifying their answers statements were rephrased but were not deeply interpreted. Students who changed their answer used percentages with decimals, it looked like students were trying to be more 'accurate' with their answers now that they knew more. Graphical representations were also present, but many of them were attempts to express probability that do not make sense. This shows the lack of graphical representation notions that they have as they either repeat something that has already been given to them, pie charts, or they draw something that proves they do not have the tools to express probability in any other graphical way.

- **End-of-year trip**

In the first question of the first set most answers were given in numerical form, specifically with a percentage. Once again, students associate probability with percentage because it is the most present probability representation they have been taught in school. Furthermore, the percentages used are random, like 63% or 74% as if they were trying to give 'exact' answers instead of approximations or estimates. Those students who also drew a graph were sitting next to each other and made very similar pie charts which did not make sense as the percentages written on them did not add up to 100%. These same students also talked about climate change in their answers, one of them must have used this argument and the others could have thought he or she was right. The justifications to this question were mainly based on their previous knowledge. The use of previous knowledge clearly shows they are able to understand and apply the subjective meaning of probability as their judgements are based on facts and concepts they already know, even if this is done unconsciously. The question about reliability seemed to have been a challenge as many did not answer or justified it very poorly. However, there was a higher representation of probability with verbal language when compared to the first question. They might associate reliability with level of confidence in their answer showing an intuitive meaning of probability as they are expressing the "degrees of belief for the occurrence of events, based on a qualitative scale ranging from certain to impossible" (Alsina et al., 2020). Some of those who wrote percentages confused the probability of good weather with the probability of their prediction coming true and used percentages incorrectly to express their answer.

Students who decided to change their answer in question 3 could have compared the data from those days to the mean data of the month to see if those days had better weather than the whole month. If there was a pattern it could be used to extrapolate it to their predictions for 2022. However, no justification was given in this question or vague arguments were exposed. There is a clear difficulty when using the data provided, this might be laziness in looking at all the data and deciding just to use some of it but with no logical reasoning. Answers also show the probability of bad weather or rain instead of what has been asked: good weather. As rainfall statistics have been provided, students might have focused on them when making a prediction without properly reading the question. Misreading the information or misinterpreting it is a common mistake students make that should be tackled as it is crucial to solve mathematical problems. Sometimes it is not lack of knowledge, but lack of reading comprehension abilities that affect students' results. There was also an absence of justification for the probabilities given and what data they have used to make the predictions. Many were based on the last year or took a general look at all graphs without explaining their choices. Almost everyone included 2021 in their answers because they thought 2022 would have similar results to the year before rather than to 2017. Overall, handling data and graphs turned out to be a complicated task, students are not used to these types of activities and therefore, have difficulties in selecting and identifying important information justifying their choices.

In question 4 most students decided to maintain their answer, however, the interesting analysis of the results is that many of them used the same data to justify their answers, whether they modified it or not. This proves the relativity of data interpretation and how the same data is perceived differently depending on students’ expectations and knowledge. Information interpretation is done subjectively, affecting students’ answers.

Regarding question 5, the main observation was that most students decided to maintain their previous answer and that many of them assumed that the data from the day before the trip would be the same the next day; this inference was used to reason their answers. There was also a misuse of the classic meaning of probability when students argued the probability of good weather was 50% because it rained for half of the day (morning) and was sunny during the second half (evening). These students were treating the events as equiprobable in a subjective context showing the tendency to use Laplace’s formula whenever possible as it is the most settled idea they have of what probability means or when it is implemented. As concluded by Lecoutre (1992), students tend to apply equiprobability to all random events. A higher number of belief-based arguments were given and the overall answers had a favourable weather prediction. Setting aside the data and believing the weather would be good in order to go on the trip can be explained because students might empathise with the situation and might be willing to go on the trip, therefore, they want the weather to be okay. This more personal approach may be completely unconscious but shows that probability predictions are affected by how involved students are in the situation-problem.

13. CONCLUSIONS

The main conclusion we can draw from this work is based on the lack of notions and applications of probability among Elementary school pupils. In addition, the new Organic Law on Education (LOMLOE) of 2020 includes content on probability as a subjective measure of uncertainty: "Recognition of uncertainty in everyday life situations and by conducting experiments" in block E of basic knowledge in the second cycle. Therefore, the need to carry out activities such as the ones presented in this paper is even more evident. The implementation of two activities based on real situations whose protagonists were girls and boys of the same age as the pupils increases attractiveness, motivation and, as a result, generates more meaningful connections with the mathematical content to be addressed. Students have been able to experience and make explicit the application of probability beyond the academic context, which fosters the acquisition and internalisation of knowledge about probability as well as the improvement of probabilistic literacy, as Gal (2005) explained.

The use of subjective probability to solve the problems posed has generated conflicts when it comes to arguing the answers. The notorious presence of percentages as practically the only method of representing probability and the misuse of probabilistic terms demonstrates the importance of allowing students to experience numerous problem-situations where more senses of probability are explored beyond the classical and frequential. For example, when students applied equiprobability to subjective contexts. With this, we will be able to lay more solid foundations that will allow students to develop a greater statistical understanding (Batanero et al., 2005). Furthermore, it is the intuitive and subjective meanings of probability that are most present in our daily lives, so it is essential to carry out activities that put these concepts into practice. Fischbein (1975) pointed out that students already have notions of probability before they are introduced to the 'formal' study of this concept, so we point out the need to work on these two types of probability from an earlier age. The incorporation of new information throughout the development of the activities demonstrated the relativity of the interpretations of this information. It would be useful to study in depth the interpretation and assimilation of new data in subjective probability problems and how they modify their answers. The strength of initial convictions and the refusal to change one's mind if new information given is not interpreted as objective data has been observed. Finally, in order to achieve probabilistic literacy among students, it will be essential to train teachers in mathematical knowledge of probability and statistics and in didactic and pedagogical knowledge (Muñiz-Rodríguez et al., 2020).

To conclude, it is necessary to highlight the limitations of the intervention and how they have conditioned the results. Given that the sample size is very small, and on several occasions a large number of students did not answer some questions, we cannot extrapolate and generalise the results. In addition, the pupils had almost no prior ideas about probability, or at least these had not been formed in the academic environment, as the textbooks used throughout the Elementary school stage at the school do not include the subject of statistics and probability.

14. CONCLUSIONES

La principal conclusión que podemos extraer de este trabajo se basa en las carencias en cuanto a nociones y aplicaciones de la probabilidad del alumnado de Educación Primaria. Además, la nueva Ley Orgánica de Educación (LOMLOE) de 2020 incluye contenidos sobre probabilidad como medida subjetiva de incertidumbre: “Reconocimiento de la incertidumbre en situaciones de la vida cotidiana y mediante la realización de experimentos” en el bloque E de saberes básicos de segundo ciclo. Por ello, la necesidad de llevar a cabo actividades como las que se presentan en este trabajo es aún más evidente. La implementación de dos actividades basadas en situaciones reales cuyos protagonistas eran niñas y niños de la misma edad que el alumnado aumenta el atractivo, la motivación, y como resultado, genera conexiones más significativas con los contenidos matemáticos a abordar. El alumnado ha podido experimentar y explicitar la aplicación de la probabilidad más allá del contexto académico, lo que fomenta la adquisición e interiorización de conocimientos sobre probabilidad, así como la mejora de la alfabetización probabilística como expuso Gal (2005).

El uso de la probabilidad subjetiva para resolver los problemas planteados ha generado conflictos a la hora de argumentar las respuestas. La notoria presencia de porcentajes como prácticamente único método de representación de probabilidad y el mal uso de términos probabilísticos, demuestra la importancia de permitir al alumnado experimentar numerosas situaciones-problema donde se exploren más sentidos de la probabilidad más allá del clásico y frecuencial. Por ejemplo, cuando el alumnado aplicaba la equiprobabilidad a contextos subjetivos. Con esto conseguiremos asentar bases más sólidas que permitirán al alumnado desarrollar una mayor comprensión estadística (Batanero *et al.*, 2005). Asimismo, son los significados intuitivo y subjetivo de probabilidad los que están más presentes en nuestra vida cotidiana, por lo que resulta esencial realizar actividades que pongan en práctica estos conceptos. Fischbein (1975) señaló que el alumnado ya tiene nociones de probabilidad antes de iniciarse en el estudio 'formal' de este concepto, por ello, apuntamos la necesidad de trabajar estos dos tipos de probabilidad desde una edad anterior. La incorporación de nueva información a lo largo del desarrollo de las actividades demostró la relatividad en las interpretaciones de dicha información. Sería conveniente profundizar en el estudio de la interpretación y asimilación de nuevos datos en problemas de probabilidad subjetiva y cómo modifican sus respuestas. Se ha observado la fuerza de las convicciones iniciales y el rechazo a cambiar de opinión si la nueva información dada no es interpretada como datos objetivos. Por último, para lograr la alfabetización probabilística del alumnado será primordial formar al profesorado en conocimientos matemáticos sobre probabilidad y estadística y en conocimientos didácticos y pedagógicos (Muñiz-Rodríguez *et al.*, 2020).

Para concluir, es necesario subrayar las limitaciones de la intervención y cómo han condicionado los resultados. Dado que el tamaño de la muestra es muy pequeño, y en varias ocasiones un gran número de alumnas y alumnos no respondieron a alguna pregunta, no podemos extrapolar y generalizar los resultados. Además, el alumnado no tenía casi ideas previas sobre probabilidad, o al menos estas no habían sido formadas en el entorno académico ya que los libros de texto utilizados a lo largo de la etapa de Primaria en el centro, no incorporan el temario sobre estadística y probabilidad.

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16. ANNEXES

Annex 1: MasterChef worksheet

MASTERCHEF

El concurso Masterchef consiste en una competición en la que 10 aspirantes compiten en 3 pruebas cada semana. La primera prueba es individual, los 2 mejores concursantes serán los capitanes de la siguiente prueba. La segunda prueba es por equipos en la que los dos mejores de la prueba anterior son los 2 capitanes y el equipo perdedor se juega la eliminación en la siguiente prueba. La última prueba es de eliminación en la que compiten individualmente y el peor es eliminado. En el sexto programa ha habido una repesca en la que un concursante ha vuelto a participar en el concurso. Han pasado 8 programas y solo quedan 3 aspirantes en la final, de los cuales uno será el ganador. En esta última prueba competirán de forma individual.

Los aspirantes son:
Inés (10 años)
Samuel (9 años)
Diego (12 años)

FASE 1

1: Inés nunca ha competido en una prueba de eliminación, mientras que Samuel ha estado en 4 y Diego en 7. Samuel fue eliminado en el programa 4 y repescado en el programa 6. En los dos últimos veranos Diego participó en los campamentos Masterchef.

Ordena a los 3 concursantes según el puesto que crees que tendrán en la final (1º, 2º o 3º) y explica por qué has elegido ese orden.

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____

____

2. ¿Crees que esta predicción se cumplirá seguro o no?

____

MASTERCHEF

Ahora representa en un gráfico de sectores la probabilidad que crees que tiene cada uno de ganar.

3. ¿Qué probabilidad crees que tiene Inés de ganar?

____

FASE 2

Se ha hecho una encuesta en RRSS antes de la última prueba en la que los seguidores del programa han votado al concursante que creen que va a ganar. Votaron 500.000 personas e Inés salió como la favorita del público con un 50% de los votos.

Sabiendo esta información, ¿cambiarías tu respuesta anterior? ¿Por qué? ¿Si la has cambiado, qué probabilidad crees que tiene Inés de ganar ahora?

____

____

____

FASE 3

La prueba final consiste en elaborar un plato con bacalao. Es la primera vez que cocinan este pescado. El abuelo de Diego es pescadero; Samuel regresó al programa gracias a que cocinó un exquisito plato de merluza en la repesca; e Inés es vegetariana y no come ni carne ni pescado.

¿Cambiarías tu respuesta anterior? ¿Por qué? ¿Si la has cambiado, qué probabilidad crees que tiene Inés de ganar ahora?

____

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Annex 2: Presentation of the MasterChef activity

Annex 3: End-of-year trip worksheet

Viaje fin de curso

Nos vamos de viaje de fin de curso a Llanes la segunda semana de junio, los días 8, 9 y 10. Las actividades van a ser:

- ir a la playa
- hacer una ruta
- ir al parque de aventuras Selva asturiana

Necesitamos buena temperatura y que no llueva para realizar el viaje.

SET 1

1. ¿Podéis hacer una estimación ahora en abril de la probabilidad de que haga buen tiempo la segunda semana de junio?

2. ¿En qué os basáis?

3. ¿Cómo de fiable creéis que es esta estimación

SET 2

Estos son los datos de Tª media del mes de junio y los días que ha llovido de este mes de los últimos 5 años. ¿Qué probabilidad crees que hay de que haga buen tiempo en junio y se realice el viaje? Justifica tu respuesta.

Viaje fin de curso

SET 3

Centrándonos en los días 8, 9 y 10 de junio de los últimos 5 años, estos son los datos de los días que ha llovido y la Tª media. ¿Qué probabilidad crees que hay de que haga buen tiempo los días 8, 9 y 10 de junio y se realice el viaje? Justifica tu respuesta.

SET 4

La previsión para el mes de junio es que haga una Tª media de 16.4°C y precipitaciones durante 10 días de 94mm. ¿Cambiarías tu respuesta anterior? ¿Por qué?

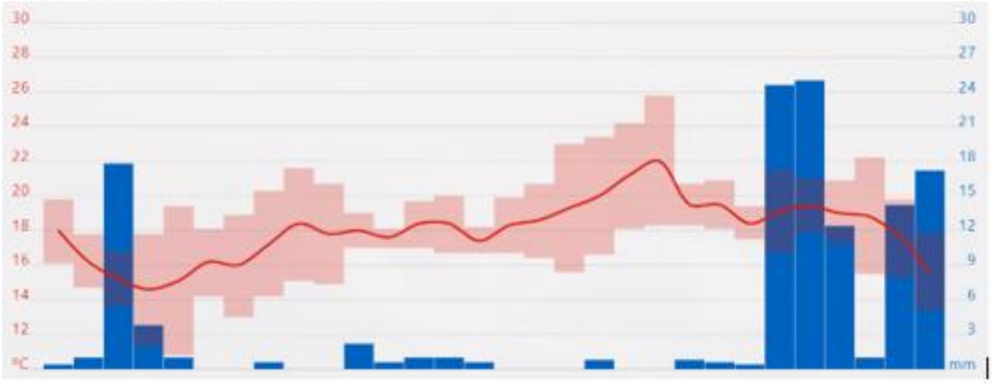
SET 5

El día antes del viaje amanece muy nublado y llueve durante toda la mañana en Llanes aunque por la tarde despeja y sale el sol. ¿Cambiarías tu respuesta anterior? ¿Por qué?

Data for question 2:

2017:

- Temperatura media del mes: 18ºC
- Llovió 21 días
- Precipitaciones del mes: 127mm

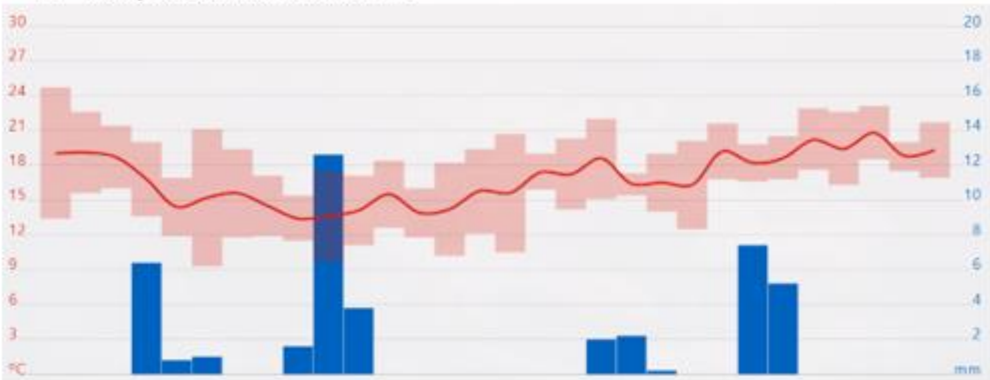


2018:

- Temperatura media del mes - 17.8ºC
- Llovió 17 días
- Precipitaciones del mes: 86mm

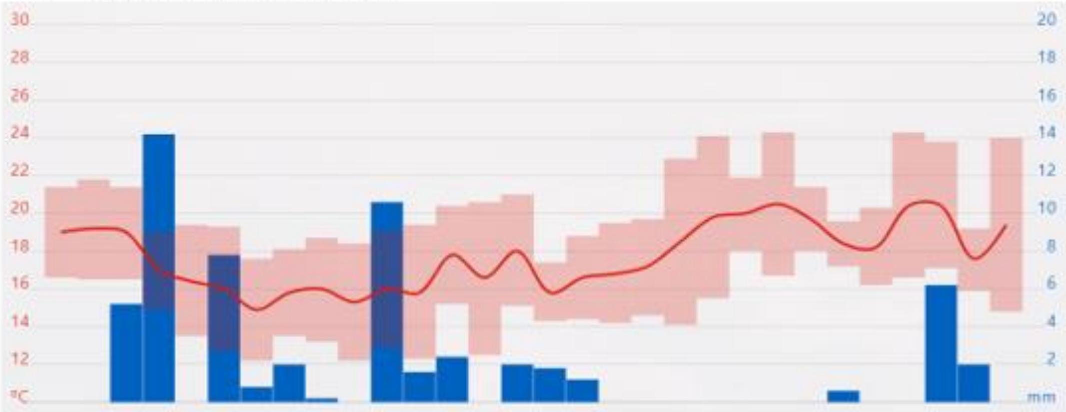
2019:

- Temperatura media del mes: 16.9ºC
- Llovió 11 días
- Precipitaciones del mes: 43.2mm



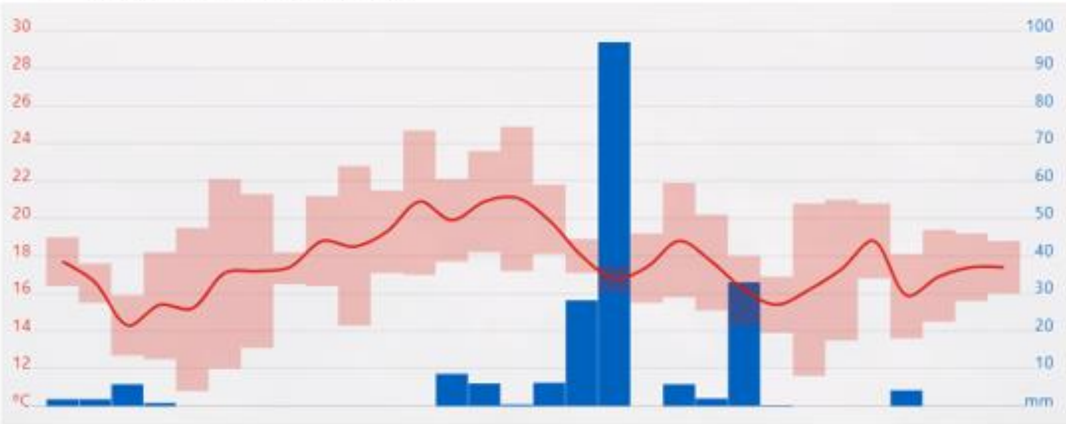
2020:

- Temperatura media del mes - 17.7ºC
- Llovió 15 días
- Precipitaciones del mes: 58.6mm



2021:

- Temperatura media del mes: 17.7ºC
- Llovió 15 días
- Precipitaciones del mes: 201.8mm



Data for question 3:

2017:

- Tª media de los días 8, 9, 10: 17.8ºC
- Llovió 1 de estos días
- Precipitaciones de los días 8, 9, 10: 0.6mm

2018:

- Tª media de los días 8, 9, 10: 17.8ºC
- Llovió 3 de estos días
- Precipitaciones de los días 8, 9, 10: 23.6mm

2019:

- Tª media de los días 8, 9, 10: 13.8ºC
- Llovió 2 de estos días
- Precipitaciones de los días 8, 9, 10: 14.2mm

2020:

- Tª media de los días 8, 9, 10: 15.7ºC
- Llovió 2 de estos días
- Precipitaciones de los días 8, 9, 10: 2.2mm

2021:

- Tª media de los días 8, 9, 10: 18.2ºC
- No llovió ninguno de estos días
- Precipitaciones de los días 8, 9, 10 - 0mm

Annex 4: Presentation of the End-of-year trip activity