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Electrothermal Model for Power LEDs Based on the Equivalent Resistance Concept for LED Driver Design

Heriberto Adamas-Pérez, Mario Ponce-Silva, René Osorio-Sánchez and J. Marcos Alonso

***Abstract*—An electrothermal LED model for design and simulation of LED drivers based on the concept of the equivalent resistance is proposed in this paper. The main contribution of this work is the integration of the temperature variables as the encapsulation temperature, heatsink temperature and the ambient temperature in an electrical model based on the equivalent resistance of a power LEDs. Therefore, an improved model is achieved by including these temperatures. This is because the LED temperatures are critical variables in the LED performance that could reduce the useful life of the LEDs or may even destroy them. Therefore, these temperatures must be considered in the design of LED drivers. The "equivalent resistance" concept has been previously used in the electrical modelling of power LEDs without considering the temperature variables, and this model was used to achieve fast simulations in LEDs-driver systems. The proposed model is oriented to the area of electronic engineering, and it is recommended for the design and simulation of LED-converter systems Experimental and simulation results are obtained that demonstrate the performance of the proposed model.**

***Index Terms*—Light emitting diodes (LEDs), lighting, modelling and thermal resistance.**

I. Introduction

Nowadays, power LEDs are a good alternative for many lighting applications because they have a high luminous efficiency and a long useful life[1]. Power LEDs needs a dc-dc converter (LED Driver) to work properly. An LED model facilitates the design and simulation of LED-converter system [2].

It is well known that temperature is a critical variable of power LEDs, since the lifetime of LEDs depends on temperature. In extreme cases, LEDs are destroyed when the junction temperature reaches a critical value. Therefore, an electrothermal model of LEDs would be considered as a necessary tool for LED driver designers[3-5].

Some electrothermal models have previously been proposed but they are quite complex [6-9]. An electrothermal model of LEDs usually requires extensive knowledge of semiconductor physics and thermodynamics to obtain an adequate model [4, 7, 10]. In addition, parameter estimation in some complex models is very complicated and specialized laboratory equipment is required [11].

Currently, there are some models proposed in the literature [7, 8, 12, 13] but they are difficult to use, as they have several optical and thermal parameters to extract. The simulations of these systems must be carried out with a reliable LED model, so that the complete system behaviour can be investigated [4, 14]. For this reason, a LED model that represents the electrothermal behaviour of the power LED with acceptable accuracy and with a simple parameter estimation process will be a useful tool for electronic engineers to perform proper analysis and simulation of LED systems. In this paper, a LED model that incorporates the temperature performance (electrothermal model) as improvement of the electrical model shown in [9] is presented. The proposed electrothermal model is based on the concept of "equivalent resistance" shown in [9]. This concept was used in [9] to model only an electrical performance of power LEDs. The LED parameters are evaluated by direct measurements of temperature, current and voltage with the help of conventional instruments such as thermocouple, multimeter and oscilloscope. These instruments are available in a conventional electronics laboratory. However, a temperature chamber is needed to characterize the LEDs, which is not a common laboratory equipment. This model is suitable to achieve fast simulations of LED converter systems and is obtained with thermal and electrical measurements.

This paper is structured as follows: Section II presents the behavior and the proposed electrothermal model for power LEDs. The electrical measurements of the LEDs are presented in Section III. Experimental and simulation results are presented in Section IV. Finally, Section V presents the conclusions of this work.

II. Equivalent resistance of power LEDs

Power LEDs exhibit a steady-state *VI* characteristic curve, where each point on the curve is measured when the voltage and current in the LED have reached steady state, similarly to a conventional diode.

A. LED Modelling

Based on [15-17], the "equivalent resistance" of the LED resistance R_D can be expressed as function of its power p_D. is expressed as a function of time, which is shown in equation (1).

$$R_D(t) = f\left(p_{Dx}(t)\right) = B_1 p_{Dx}(t)^{B_2} + B_3 \qquad (1)$$

Where: p_{dx} is a variable related to the LED power, which can be observed in Fig. 4. The LED time constant is related to the dynamic behaviour of the power LED and shows how fast the

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dynamic response of the LED is. This time constant can be measured when a current step is applied to the LED. It has been observed that power LEDs have a very fast dynamic response, and a very small time constant (of the order of nanoseconds).

An approximation of this time constant is made for the LEDs *(LMT-P12Y-77-N)* of the Siled company which gives 17 ns. The time constant was calculated as the time needed to reach 100% voltage from the point of 100% current. This is in response to a current step [15].

The time constant of the LED is modelled by using an RC network. However, there is another important variable whose effects must be considered in the model presented in [19]. This is the ambient temperature, which affects de VI curve of the LED.

B. Thermal modelling of Power LEDs

The heating process in LEDs is caused by the joule effect [18]. A linear electrical analogy model is proposed to model the thermal behavior of the power LEDs. The proposed thermal circuit is shown in the Fig. 1

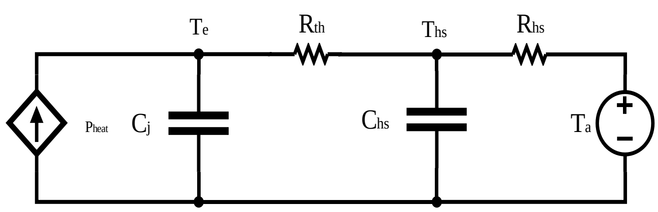

Fig. 1. Proposed thermal circuit

A high percentage of the LED power is converted to heat, between 70% to 80%. The thermal model of the LED shown in Figure 7 uses a general thermal resistor (R_{th}) that goes from the p-n junction to the surface of the package where it contacts the heat sink represented by another thermal resistor (R_{hs}) similar to those estimated in [18] in order to achieve a better estimation of the dynamic behavior of the LED equivalent resistance.

When the LED is connected to a power source, the electrical power in the LED which is dissipated as heat is represented by the current source (p_d). The case temperature (T_e) together with the heatsink temperature (T_{hs}) will start to increase from an initial temperature, in this case the ambient temperature (T_a), until a final steady state temperature is reached. Thermal resistance calculations are based on JEDEC standard for LEDs as reviewed in some articles [19-21]. This time is given by the thermal time constants expressed by the following equations.

$$\tau_j = C_j R_{th} \tag{2}$$

$$\tau_{hs} = C_{hs} R_{hs} \tag{3}$$

When the electrical power is disconnected, the thermal capacitances start to discharge. Therefore, the package and heatsink temperature will start to decay until reaching the ambient temperature [22]. An analysis of the proposed thermal model yields the following differential equations.

$$p_{heat} = \frac{T_e - T_{hs}}{R_{th}} + C_j \frac{dT_e}{dt} \tag{4}$$

$$\frac{T_e - T_{hs}}{R_{th}} = C_{hs} \frac{dT_{hs}}{dt} + \frac{T_a + T_{hs}}{R_{hs}} \tag{5}$$

Solving the differential equations (6) and (7). The following expressions are obtained. These equations correspond to the encapsulation and heatsink temperature.

$$T_e = \left(R_{th} p_{heat}\right)\left(1 - e^{\frac{-t}{R_{th}C_j}}\right) + T_{hs} \quad t > 0 \tag{6}$$

$$T_{hs} = \left(R_{hs} p_{heat}\right)\left(1 - e^{\frac{-t}{R_{hs}C_{hs}}}\right) + T_a \quad t > 0 \tag{7}$$

In equation 6 when the time t approaches infinity, the exponential element will become zero and the junction temperature will reach steady state. Extracting the data from the experimental thermal measurements. It is observed that the difference between T_e-T_{hs} remains practically constant in the steady state for different dissipation powers. Giving as a result T_e-T_{hs} *=10.49868°C,* taking as a reference an electrical power of 1 W, approximately 80% of this power is converted into heat. This is the heating factor k_h [23]. Therefore, the calculation of the thermal resistance of the encapsulation is as follows.

$$R_{th} = \frac{T_e - T_{hs}}{p_d k_h} = 13.12335°C/W \tag{8}$$

To calculate the thermal resistance of a series array, divide the thermal resistance obtained in equation (11) by the number of LEDs to be modelled [24], in this case 12.

$$R_{th} = \frac{\frac{T_e - T_{hs}}{p_d k_h}}{12} = 1.0936°C/W \tag{9}$$

The temperature of the heatsink is close to the ambient temperature and remains practically constant at different power dissipation in the LED, so it can be expressed in the following equation T_{hs}-T_a*=0.8542°C*. Taking as reference a power dissipation of 1 W, the calculation of the thermal resistance of the heatsink is obtained as follows.

Because the LEDs are all connected on the same heat sink, the case thermal resistance remains the same for both one LED and n LEDs. If they are connected on the same package, the thermal resistance remains the same for both one LED and *N* LEDs [24].

$$R_{hs} = \frac{T_{hs} - T_a}{p_d k_h} = 1.06775°C/W \tag{10}$$

The values of the thermal resistances are not universal. The junction thermal resistance depends on the internal properties of the LED, while the thermal resistance of the heatsink depends on its dimensions and physical structure. In the case of this model, a 75 mm comb-type aluminum heatsink with 8 separate fins is used. These heatsinks have a typical thermal resistance of 1°C/W.

C. Electrical Model of LED Panel (12 LEDs)

Connecting a series array of 12 LEDs (LMT-P12Y-77-N) of the Siled company. By making experimental measurements at an ambient temperature in the CENIDET lighting laboratory of (Ta = 25 °C), the following graph of equivalent resistance as a function of LED power was obtained.

A parametric curve fitting was done to estimate the values of the constants ***B₁*, *B₂* y *B₃*** by using the "curve fitting toolbox" of MATLAB. The value of which is **992, -0.982 y 49.52** respectively.

E. Electrothermal model

LED operating temperatures cause a variation in their I-V curves. The variations lie mainly in the displacement of the threshold voltages as a function of the ambient temperature at which the power LEDs are operating.

Changes in the threshold voltages are considered from the time when there starts to be a shift in the VI curves. Since the modelling presented in [15] implicitly includes the opening of the curves at higher current, this opening is considered with the calculation of the equivalent resistance as a function of power.

The data were obtained from the electrothermal characterization shown in section III. This mathematical function is obtained using the "Curve Fitting Tool" of MATLAB. This function is shown in (13).

$$\Delta Vf = f\left(T_a\right) = \left(0.09518e^{\left(-0.02007T_a\right)} - 0.07474\right) \quad (11)$$

Combining equation (11) with equation (12) yields a general expression for the LED voltage. This equation includes the concept of the dynamic resistance of the LED and the effects of ambient temperature on the LED threshold voltage.

$$V_D\left(t\right) = I_D\left(t\right) R_D\left(t\right) + \Delta v_f \quad (12)$$

$$V_D\left(t\right) = I_D\left(t\right)\left(B_1 p_{Dx}\left(t\right)^{B_2} + B_3\right) + \left(0.09518e^{\left(-0.02007T_a\right)} - 0.07474\right) \quad (13)$$

Where ΔV_f is the change of the LED threshold voltage, T_a is the ambient temperature value at which the LED is operating. For this model, the ambient temperature values range from -30 °C to 60 °C. This equation gives the displacement of the I-V curves as a function of ambient temperature ΔV_f vs T_a .

F. LED analysis based on photo-electro-thermal theory (PET).

The presented electrothermal model can be evaluated with PET. This analysis is presented in [23, 25, 26]. In LED systems, the luminous efficacy (E) is the ratio of the total luminous flux to the total input power multiplied by the number of connected LEDs (N). It can be expressed as:

$$E = \frac{\phi_v}{NP_d} \quad (14)$$

The luminous efficacy can be approximated as a linear function dependent on the junction temperature.

$$E = E_0\left[1 + k_e(T_j - T_0)\right] \quad (15)$$

Where T_0 is normally defined as 25°C in LED datasheets, E_0 is the luminous efficacy at 25°C and k_e is a negative coefficient representing the rate of reduction of the luminous efficacy with the junction temperature. However, not all of the LED power ends up as light. A portion of the input electric power is dissipated as heat. The amount of electric power dissipated as heat is:

$$P_{heat} = k_h P_d \quad (16)$$

Where k_h is the heat dissipation coefficient, which represents the portion of input power dissipated as heat.

The luminous efficacy depends on the ambient temperature, the thermal resistances, the coefficients, the number of LEDs and the electrical power. It can be calculated with the following expression.

$$E = E_0\left[1 + k_e(T_a - T_0)\right] + k_e k_h (R_{th} + NR_{hs}) P_d \quad (17)$$

Substituting the luminous efficacy equation into the luminous flux equation. A luminous flux equation is obtained that incorporates the interactions of heat, light and power.

$$\phi_v = NE_0\left\{\left[1 + k_e(T_a - T_0)\right]P_d + k_e k_h (R_{th} + NR_{hs}) P_d^{\,2}\right\} \quad (18)$$

Where ϕ_v is the luminous flux, N the number of LEDs connected, k_e and k_h the luminance and heat coefficients, R_{th} and R_{hs} are the thermal resistances, and P_d is the electrical power consumed by the LEDs. The values of the thermal resistances were previously calculated using the proposed thermal analysis. The nominal luminous efficacy value at 25°C was obtained from the manufacturer's data sheet. The coefficient values are approximated values for power LEDs. Table I shows the different values.

The values of the thermal resistances were previously calculated using the proposed thermal analysis. The nominal luminous efficacy value at 25°C was obtained from the manufacturer's datasheet. The coefficient values are approximate and typical values for power LEDs. TABLE 1 shows the optical and thermal parameters of the power LED module used in this work.

TABLE 1. THERMAL AND OPTICAL PARAMETERS OF THE LED MODULE *LMT-P12Y-77-N from Siled Company.*

k_e	k_h	T_0	E_0	N	R_{hs}	R_{th}
-0.005	0.8	25°C	7.9861 lm/W	12	0.8542 °C/W	0.87489 °C/W

The Fig. 2 shows the theoretical and simulated luminous flux curves at different operating temperatures as a function of the electrical power of the LED panel (*LMT-P12Y-77-N from Siled Company*). It is observed that the luminous flux degrades with increasing ambient temperature.

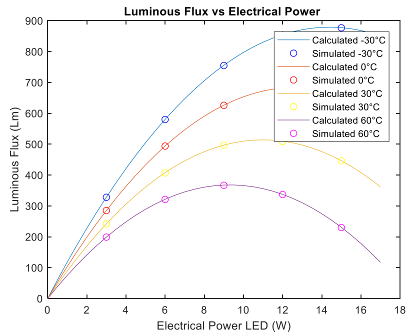

Fig. 2. Theoretical and simulated curves of electric flux versus electrical power of LEDs.

III. Electrical Experimental Tests

The schematic diagram of the test circuit used to extract the steady state LED parameters is shown in Fig. 3.

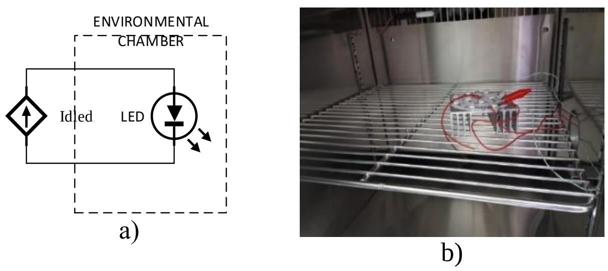

Fig. 3. LED module inside environmental chamber: a) Schematic diagram, b) physical implementation.

A test under steady state conditions was carried out to obtain the parameters needed in equation (8).

The experimental results for one LED of the panel (*LMT-P12Y-77-N from Siled Company*) are shown in Table 2.

TABLE 2. EXPERIMENTAL DATA FOR ONE LED FROM THE LED PANEL (LMT-P12Y-77-N FROM SILED COMPANY) FOR DIFFERENT POWER LEVELS, AT STEADY STATE AND 20° C.

Number of samples	$V_D(V)$	$I_D(A)$	p_D or p_{Dx} (W)	$RD(\Omega)$
1	2.5	0.003	0.0075	833.333333
2	2.6	0.018	0.0468	144.444444
3	2.7	0.042	0.1134	64.2857143
4	2.8	0.071	0.1988	39.4366197
5	2.9	0.104	0.3016	27.8846154
6	3	0.139	0.417	21.5827338
7	3.1	0.175	0.5425	17.7142857
8	3.2	0.213	0.6816	15.0234742
9	3.3	0.252	0.8316	13.0952381
10	3.36	0.277	0.93072	12.1299639

Using a curve fitting method (nonlinear least squares) $\boldsymbol{B_1}$, $\boldsymbol{B_2}$ and $\boldsymbol{B_3}$ values are obtained as ***7.986***, ***-0.9607*** and ***2.828***, respectively. The curve fitting process was realized with the help of the curve fitting toolbox of *MATLAB* software. Equation (8) is plotted in Figure 8(a) along with experimental results. Both curves (theoretical and experimental) match very well, showing a *Root Mean Square Error* (*RMSE*) of only *0.5784 Ω*.

IV. Experimental and Simulation Results

A. Thermal Simulation

The proposed thermal model was implemented in Simulink. The diagram is shown in Fig. 5. This model is a linear electrical analogy model. The thermal resistances are calculated at the thermal steady state where the temperatures of the heatsink and the encapsulation reach the steady state.

With the thermal resistances extracted from the experimental thermal measurements the values of the thermal capacitances were calculated. The physical definition of the thermal time constants is the time where the trigger and encapsulation temperatures reach 63% of their final value. The 63% value comes from the transient behaviour of RC circuits. Once the thermal time constants are obtained, the thermal capacitances of the encapsulation and heatsink are calculated with the following equations. The time constant in the encapsulate remains practically constant for different temperatures. The time constant in the potting is equal to 0.0944874 s. So, the calculation of the potting thermal capacitance is shown in the following equation. Considering that 1/s is equivalent to 1 W.

$$C_j = \frac{\tau_j}{R_{th}} = \frac{0.0944s}{10.4986°C/W} = 0.009J/°C \qquad (19)$$

To calculate the thermal capacitances of an array of LEDs connected in series is divided by the number of LEDs to be modelled, in this work 12.

$$C_j = \frac{\frac{\tau_j}{R_{th}}}{12} = \frac{\frac{0.0944s}{10.4986°C/W}}{12} = 0.00075J/°C \qquad (20)$$

Extracting the data from the thermal measurements it is observed that to reach 63% of the steady state temperature at the heatsink, the time constant is equal to 0.7687s. Therefore, the calculation of the thermal capacitance of the heatsink is shown in the following equation. Considering that 1/s is equivalent to 1 W. As the LEDs are mounted on the same heatsink, the value of the thermal capacitance is the same for n LEDs [24].

$$C_{hs} = \frac{\frac{\tau_j}{R_{th}}}{12} = \frac{\frac{0.0944s}{13.12335°C/W}}{12} = 0.000599440J/°C \qquad (21)$$

B. Electrothermal Experimental Results

The LED was subjected to a controlled environment through a Cincinnati Sub-Zero *MCH-3-.33-.33-H/AC* environmental chamber. The temperature range to which the LED was subjected is from *-30 °C* to *60 °C*.

Fig. 4. Shows the results of this electrothermal characterization.

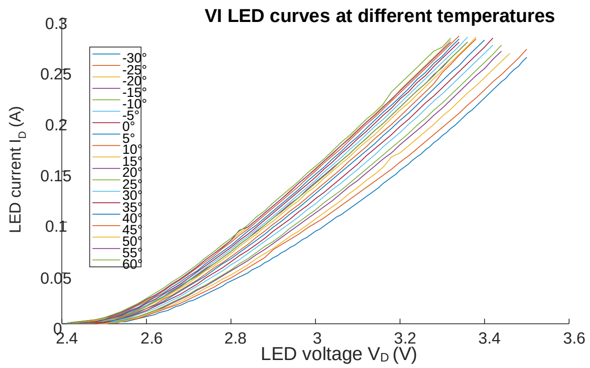

Fig. 4. Electrothermal characterization of the LED.

C. Thermal Experimental Results

The LED was polarized at three different powers, (0.25 W, 0.5 W and 1 W). Under an operating range from -30°C to +60°C. The electrothermal model consists of two parts, on the left side is the electrical model and on the right side is the thermal model. The output of the electrical model is the power dissipated and in turn is the input of the thermal model which represents the heat dissipated by the LED. The electrothermal model was simulated in Simulink. This model is shown in Fig. 5.

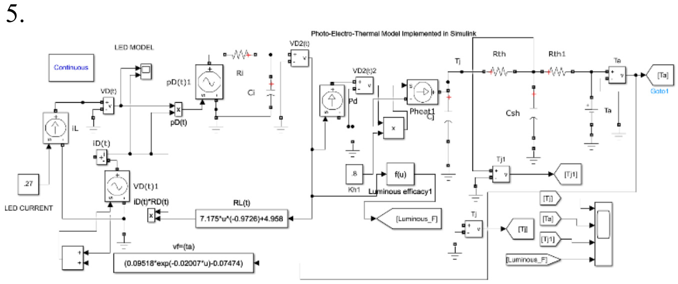

Fig. 5. Photo-Electro-Thermal Model Implemented in Simulink.

The experimental and simulation results of the electrothermal model. Are shown in Fig. 6

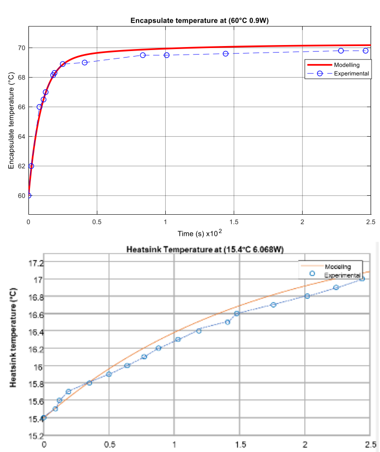

Fig. 6. Experimental and modelled thermal results.

V. Cooling temperature calculation and model comparison

When the LED power is turned off (P_{heat}=0). The electrothermal model enters a cooling condition, which starts with an initial temperature $T_j(0)$ and $T_{hs}(0)$ which are the temperatures reached in the steady state. The thermal capacitances will start to discharge, the behavior of the LED temperatures is a natural response. The dynamic cooling equation can be expressed by the following equation.

$$T_e = \left[T_e(0) - T_{hs}(0)\right] e^{\frac{-t}{R_{th}C_j}} + T_{hs} \tag{22}$$

$$T_{hs} = \left[T_{hs}(0) - T_a\right] e^{\frac{-t}{R_{hs}C_{hs}}} + T_a \tag{23}$$

Calculations and temperature measurements were performed. With the LED operating in a wide current range which gradually increases. The ambient temperature undergoes an abrupt change achieving with this a cooling condition for this period, the calculation of the encapsulation temperature was performed obtaining accurate results. This is shown in the Fig. 7.

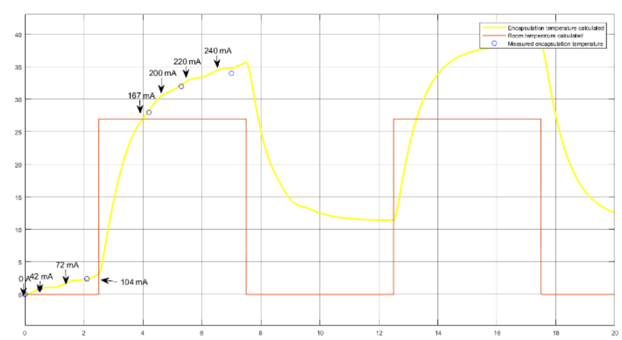

Fig. 7. Calculated and measured potting temperatures under cooling conditions.

A comparison was made between the model shown in [27] and the model presented in this work. The main differences between the models are the following: The simulation programs are different in [27] SPICE is used while in this work Simulink is used.

In the electrical model presented in [27] they use three controlled current sources; they use a mathematical expression derived from the Shockley equation to represent the series resistance of the LED while the model presented in this work uses a new concept called "Equivalent Resistance" which contains simplified mathematical expressions. The electrical results of the I-V curve have the same curved appearance of the Shockley equation with those shown in this work, but they cannot be compared since in [27] they use higher power LEDs.

In the optical model in [27] they use two controlled voltage sources which represent the radiation density, the optical power and the luminous flux. They use different mathematical expressions than those presented in this work. But the behavior of the luminous flux curve is similar to the one presented in this work, pointing out that [27] higher power LEDs are used.

The thermal model presented in [27] is very similar to the one presented in this work. Both models coincide in using RC networks: thermal resistances and thermal capacitances. Although the obtaining of the values occupies different mathematical expressions. The results of the waveform under the cooling condition coincide with the waveform presented in Fig. 7, clarifying that in [27] a much higher current is used in the LED .

VI. Conclusions

In this work, a simplified electrothermal model for power LEDs has been proposed using the concept of "equivalent LED resistance". This model is based on electrical variables such as voltage, current, resistance and power in LEDs. The model also considers the time constant of the LEDs, which is related to the dynamic behaviour of the LEDs. The model can be implemented in simulation software such as Simulink, Pspice and PSIM. The parameters of the proposed model are obtained from experimental measurements of the current and voltage in the LED. The results obtained were satisfactory. The proposed model is recommended to simulate lighting systems based on power LEDs.

This model has only two input variables: LED power and ambient temperature. With this model it is possible to predict the behavior of a power LED as well as an entire LED module in both series and parallel connection. With this model it is possible to estimate quite accurately: LED current, LED voltage, LED power, threshold voltage LED, encapsulation temperature and heatsink temperature.

The main contribution of this electrothermal model is he incorporation of the temperature variables to the existing electrical model based on the LED equivalent resistance. The proposed model is useful in a lighting systems application.

References

[1] P. Mottier, *LED for lighting applications* vol. 134: John Wiley & Sons, 2010.

[2] M. Arias, A. Vázquez, and J. Sebastián, "An overview of the AC-DC and DC-DC converters for LED lighting applications," *automatika*, vol. 53, pp. 156-172, 2012.

[3] J. M. A. D. Gacio, J. Garcia, M. S. Perdigao, E. S. Saraiva and F. E. Bisogno, "Effects of the Junction Temperature on the Dynamic Resistance of White LEDs," *IEEE Transactions on Industry Applications*, vol. 49, pp. 750-760, March-April 2013.

[4] J. P. a. C. C. Lee, "An electrical model with junction temperature for light-emitting diodes and the impact on conversion efficienc," *IEEE Electron Device Letters*, vol. 26, pp. 308-310, may 2005.

[5] M. Ponce-Silva, D. Salazar-Pérez, O. M. Rodríguez-Benítez, L. G. Vela-Valdés, A. Claudio-Sánchez, D. León-Aldaco*, et al.*, "Flyback Converter for Solid-State Lighting Applications with Partial Energy Processing," *Electronics*, vol. 10, p. 60, 2021.

[6] M. Juárez, J. Martínez, G. Vázquez, J. Sosa, P. Martínez, I. Villanueva*, et al.*, "A model for electrical characteristics of high power UV LED," in *2016 13th International Conference on Power Electronics (CIEP)*, 2016, pp. 110-115.

[7] P. Baureis, "Compact modeling of electrical, thermal and optical LED behavior," *Proceedings of 35th European Solid-State Device Research Conference*, pp. 145-148, 2005.

[8] K. Górecki, "Modelling mutual thermal interactions between power LEDs in SPICE," *Microelectronics Reliability*, vol. 55, pp. 389-395, 2014.

[9] Y. G. Lalith Jayasinghe, & Nadarajah Narendran, "Characterization of Thermal Resistance Coeffient Of High-power LEDs," *Sixth International Conference on solid state Lighting*, pp. 1-11, 2006.

[10] D. Salazar-Pérez, M. Ponce-Silva, J. A. Aqui-Tapia, J. García-Guzmán, and J. H. Pérez-Cruz, "Effects of the LED modelling on the output capacitance of power converters," *IET Power Electronics*, vol. 13, pp. 3467-3474, 2020.

[11] D. Marcuse and I. Kaminow, "Computer model of a superluminescent LED with lateral confinement," *IEEE Journal of Quantum Electronics*, vol. 17, pp. 1234-1244, 1981.

[12] W. C. H. C. S. Y. R. H. Huang-Ting Chen, "Characterization, Modeling, and Analysis of Organic Light-Emitting Diodes With Different Structures," *IEEE Transactions on Power Electronics*, vol. 31, pp. 581-592, Jan. 2016.

[13] A. Poppe, "Multi-domain compact modeling of LEDs: An overview of models and experimental data," *Microelectronics Journal*, vol. 46, pp. 1138-1151, 2015.

[14] V. C. B. P. S. Almeida, H. A. C. Braga, M. A. Dalla Costa, T. B. Marchesan and J. M. Alonso, "Static and Dynamic Photoelectrothermal Modeling of LED Lamps Including Low-Frequency Current Ripple Effects," *IEEE Transactions on Power Electronics*, vol. 30, pp. 3841-3851, July 2015.

[15] J. M. A. R. Osorio, S.E Pinto, G Martínez, N Vázquez, M Ponce-Silva, A.J. Martínez., "Simplified electrical modelling of power LEDs for DC–DC converter analysis and simulation," *International Journal of Circuit Theory and Applications*, 2017.

[16] R. Osorio, N. Vazquez, S. Pinto, G. Martínez, M. Ponce, A. Padilla*, et al.*, "Stationary state error reduction on the electrical modelling of high pressure sodium lamps," *IET electric power applications*, vol. 5, pp. 350-358, 2011.

[17] N. V. Rene Osorio Sanchez, Claudia Hernandez, Elias Rodriguez, Sergio Pinto and Mario Juarez, "Electric Dynamic Modeling of HID Lamps for Electronic Ballast Design," *IEEE Transactions on Industrial Electronics*, vol. 57, pp. 1655-1662, May 2010.

[18] A. Efremov, N. Bochkareva, R. Gorbunov, D. Lavrinovich, Y. T. Rebane, D. Tarkhin*, et al.*, "Effect of the joule heating on the quantum efficiency and choice of thermal conditions for high-power blue InGaN/GaN LEDs," *Semiconductors*, vol. 40, pp. 605-610, 2006.

[19] J. Standard, "Implementation of the Electrical Test Method for the Measurement of Real Thermal Resistance and Impedance of Light-Emitting Diodes with Exposed Cooling," *JESD51-51*.

[20] H. Zou, L. Lu, J. Wang, B. Shieh, and S. R. Lee, "Thermal characterization of multi-chip light emitting diodes with thermal resistance matrix," in *2017 14th China International Forum on Solid State Lighting: International Forum on Wide Bandgap Semiconductors China (SSLChina: IFWS)*, 2017, pp. 32-37.

[21] B. Sun, J. Fan, X. Fan, and G. Zhang, "A SPICE-based Transient Thermal-Electronic Model for LEDs," in *2019 20th International Conference on Thermal, Mechanical and Multi-Physics Simulation and Experiments in Microelectronics and Microsystems (EuroSimE)*, 2019, pp. 1-5.

[22] X. Tao and D. Zhang, "Thermal parameter extraction method for light-emitting diode (LED) systems," *IEEE Transactions on Electron Devices*, vol. 60, pp. 1931-1937, 2013.

[23] R. Hui, *Photo-electro-thermal Theory for LED Systems: Basic Theory and Applications*: Cambridge University Press, 2017.

[24] S. Y. H. a. Y. X. Qin, "A General Photo-Electro-Thermal Theory for Light Emitting Diode (LED) Systems," *IEEE Transactions on Power Electronics*, vol. 24, pp. 1967-1976, 2009.

[25] S. Hui and Y. Qin, "A general photo-electro-thermal theory for light emitting diode (LED) systems," *IEEE Transactions on Power electronics*, vol. 24, pp. 1967-1976, 2009.

[26] X. Tao and S. Hui, "A general photo-electro-thermo-temporal theory for light-emitting diode (LED) systems," in *2010 IEEE Energy Conversion Congress and Exposition*, 2010, pp. 184-191.

[27] K. Górecki and P. Ptak, "New dynamic electro-thermo-optical model of power LEDs," *Microelectronics Reliability*, vol. 91, pp. 1-7, 2018.