



The implications of batching in the bullwhip effect and customer service of closed-loop supply chains

Borja Ponte^{a,*}, Roberto Dominguez^b, Salvatore Cannella^c, Jose M. Framinan^d

^a Department of Business Administration, University of Oviedo, Spain

^b Industrial Management and Business Administration Department, University of Seville, Spain

^c Department of Civil Engineering and Architecture (DICAR), University of Catania, Italy

^d Industrial Management/Laboratory of Engineering for Environmental Sustainability, University of Seville, Spain

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ABSTRACT

Batching is a well-known cause of the bullwhip effect. Despite being very common in many industries to leverage economies of scale, it has been little explored due to its nonlinear complexity. This work examines how order-quantity batching affects the performance of closed-loop supply chains, which are gaining importance as a result of their environmental and economic value. Specifically, we analyse a hybrid system with both manufacturing and remanufacturing operations. We observe that, when an order-up-to policy is used in the serviceable inventory, bullwhip is always an increasing function of the batch size. Nevertheless, when a smoothing replenishment rule is used, the closed-loop supply chain behaves differently for low and high volumes of returns due to the different degrees of uncertainty they convey. In the high-volume case, batches should also be as small as possible. In contrast, in the low-volume case, bullwhip can be mitigated by setting the batch size to a divisor of the mean production rate. However, this may lower the customer service level achieved. We also find that reducing manufacturing batch sizes should be prioritised over remanufacturing ones when both are large. In the light of our results, we finally provide professionals with specific suggestions on how to better manage closed-loop supply chains where goods are produced and delivered in batches.

1. Introduction

Order batching consists of clustering items for purchasing, manufacturing, or transportation purposes (Hussain and Drake, 2011). This is a common practice in the supply chains of many industries, which aims to leverage economies of scale in the inventory, production, and/or distribution systems (Glock et al., 2019; van Gils et al., 2019; Aerts et al., 2021). In this sense, batching allows organisations to increase the efficiency of their processes, which may lead to a reduction in some types of costs, such as handling, manufacturing, or transportation costs.

However, order batching has negative consequences on the wider supply chain: the bullwhip effect (Lee et al., 1997). This phenomenon,

which refers to the amplification of the fluctuations of orders as they move up a supply chain, is probably the most important source of waste in supply chains of nearly all industries¹ (Cannella and Ciancimino, 2010; Isaksson and Seifert, 2016; Wang and Disney, 2016). Therefore, batching decisions need to be aware that this usual practice may induce chaotic dynamics in the wider supply chain.

Despite the fact that batching was identified in the seminal work by Lee et al. (1997) as one of the major causes of the bullwhip effect, only a few later works explored its actual implications in supply chains. In general terms, these works have shown that bullwhip increases as the batch size grows (Holland and Sodhi, 2004; Hassanzadeh et al., 2014; Pastore et al., 2019). Nevertheless, a particularly interesting result was

* Corresponding author. Polytechnic School of Engineering - East Building (Univ. of Oviedo), 33204, Gijon, Spain.

E-mail addresses: ponteborja@uniovi.es (B. Ponte), rdc@us.es (R. Dominguez), cannella@unicat.it (S. Cannella), framinan@us.es (J.M. Framinan).

¹ Ample evidence has shown that the bullwhip effect can be a very costly phenomenon in supply chains. It makes the operations of the supply chain actors much more variable, which has negative implications in terms of production, inventory and transportation costs, among others (see e.g. Disney and Lambrecht, 2008).

reported by [Potter and Disney \(2006\)](#), who showed that setting the batch size to a divisor of the mean demand enables a significant bullwhip reduction. This finding, subsequently buttressed by other studies ([Hussain and Drake, 2011](#); [Hussain and Saber, 2012](#)), may help managers realise economies of scale through batching while attenuating its impact on bullwhip. [Potter and Disney \(2006\)](#) used a proportional order-up-to (POUT) inventory policy ([Cannella et al., 2021](#)). This is a smoothing replenishment rule that is able to provide a good trade-off between the bullwhip effect and the customer service level, two important supply chain problems that are strongly interrelated ([Disney et al., 2006](#)).²

These prior works, whose findings we review in more detail in the next section, have focused on understanding the dynamics induced by batching in traditional supply chains. However, the literature has not addressed so far the batching effects in closed-loop supply chains (CLSCs), which are gaining importance in modern societies due to the environmental benefits and economic opportunities they offer. Unlike traditional supply chains, which are based on a forward flow of materials (i.e., extract, make, use, dispose; an open-loop structure), CLSCs hold the principles of the circular economy. They do this by incorporating the collection of products after their use and the processes for keeping the resources in use for as long as possible—including recycling, remanufacturing, and/or refurbishing—which results in a reverse materials flow (e.g. [Guide et al., 2003](#); [Souza, 2013](#); [Shekarian, 2020](#)).

Not only are CLSCs structurally more complex than traditional supply chains, but they also need to deal with a wider scenario of uncertainties ([Goltsos et al., 2019b](#)). Importantly, we acknowledge the uncertainty related to the quantity and quality of the returned products, as the returns channels tend to be far more uncertain than those for getting raw materials (e.g. [Zeballos et al., 2012](#)). Consequently, relevant differences may emerge between the implications of batching in traditional and CLSCs. Understanding these differences would facilitate the effective design of alternative supply chain structures, such as CLSCs, that build solid bridges between the economy and the environment ([Genovese et al., 2017](#)). This is essential to increase the circularity of our economic systems.³

Motivated by (i) the relevance of batching in real-world supply chains, (ii) the importance of CLSCs in modern societies' pursuit of more circular economies, and (iii) the lack of studies addressing how batching influences CLSC dynamics, this work systematically analyses the impact of batch sizes on the bullwhip effect and the customer service of CLSCs.

To this end, we investigate a CLSC based on a hybrid manufacturing-remanufacturing system (HMRS). In this CLSC, the demand of customers is satisfied from a serviceable inventory that brings together new (manufactured) and remanufactured products. Thus, HMRSs are common in practice when both products are perfect substitutes, such as the printing industry ([van der Laan et al., 1999](#)) and the spare parts industry

([Souza, 2013](#)). We implement a POUT policy to issue manufacturing orders, while used products are remanufactured when they are available. Moreover, we assume that batching occurs in both the manufacturing and remanufacturing operations. This allows us to analyse the main and interaction effects of both batch sizes on widely-used performance metrics (specifically, order and inventory variance ratios), as well as the combined effects of batching and smoothing in CLSCs. To better understand the impact of batching in these systems, we also consider the influence of the volume of returns.

The results of this analysis show that batching has large implications on the performance of CLSCs. In general terms, we find that it does not only amplify the bullwhip effect of these systems but also makes them less efficient when meeting the demand of customers. Nonetheless, interestingly, we also observe that bullwhip can be mitigated in CLSCs by setting the batch size to a divisor of the mean production rate. However, we reveal that: (i) this only occurs when a smoothing replenishment rule is used, i.e., this does not occur for the classic order-up-to (OUT) policy; (ii) the bullwhip benefits are reduced when the return rate grows, due to the higher uncertainty provoked by the reverse flow of materials; and (iii) this strategy is problematic in terms of customer service, as it may result in frequent stock-outs. In addition, we find that, when both batch sizes are relatively large, reducing manufacturing batch sizes yields higher benefits; however, if they are moderate, it may be more profitable to reduce remanufacturing batch sizes.

In this sense, we also observe in our CLSC the somewhat counterintuitive relationship between bullwhip and batch sizes reported by [Potter and Disney \(2006\)](#); however, this finding is subject to further considerations in these settings. Overall, our results complement those of previous investigations, and help CLSC professionals find a good trade-off between the positives (economies of scale) and negatives (bullwhip effect and customer service) of batching. This perspective leads to meaningful implications for supply chain managers.

The rest of this paper is structured as follows. Section 2 reviews the most relevant literature in the domain of this study. Section 3 presents the mathematical model of the CLSC as a set of difference equations that are built on empirically validated assumptions as well as the performance metrics used to measure bullwhip and customer service. Section 4 reports the experimental approach, which is based on four two-factor, full-factorial designs through which we analyse various real-world scenarios. Section 5 discusses the batching effects and the relevant interplays with the smoothing level of the POUT policy in CLSCs characterised by different return rates. Section 6 studies the combined effects of manufacturing and remanufacturing batches. Section 7 summarises the key findings and managerial implications of our work. Finally, Section 8 concludes and proposes interesting research opportunities.

2. Review of the literature

This section examines the background of this study by reviewing two research streams that are closely related to our work. We also identify some important knowledge gaps that impede the more effective design and operation of CLSCs in practice. First, we focus on those papers that have investigated the effects of batching in supply chains. Second, we consider those papers that have analysed the dynamic behaviour of CLSCs.

² It is important to highlight that the bullwhip problem is closely related to that of demand satisfaction, as orders are intrinsically connected to inventories in supply chains. Indeed, both perspectives (i.e., bullwhip and demand satisfaction) need to be simultaneously considered when designing replenishment rules for supply chains ([Disney and Lambrecht, 2008](#)).

³ This paper is built on the belief that increasing the circularity of production and consumption systems is one of the most serious challenges that modern societies currently face. This is evidenced by the number of programmes, plans, and policies launched in the recent years (e.g. [European Commission, 2020](#); [Government of the United Kingdom, 2020](#); [Ellen MacArthur Foundation, 2021](#)). These may be interpreted as strategies to reduce pressure on natural resources, minimise the waste generated and, in sum, set the foundations for the sustainable development of future generations ([Circle Economy, 2021](#)).

2.1. The impact of batching on supply chain performance

There are two common forms of batching in industrial practice: periodic and order-quantity batching (Ketzenberg et al., 2007). Periodic batching occurs when orders are placed less frequently than they are received. For example, retailers constantly receive customer orders (i.e., demand), but place orders to the logistic centres with less frequency to make the best possible use of the warehouse and the transport fleet (Potter and Disney, 2006). In contrast, order-quantity batching happens when the number of items requested (in purchase, manufacturing, or transportation orders) has to be a multiple of a predefined batch size. Capacity constraints often lead to the application of this type of batching⁴ (Potter and Disney, 2006; Hussain and Drake, 2011; Sodhi and Tang, 2011).

In both forms, batching entails cost advantages derived from economies of scale (Glock et al., 2019; van Gils et al., 2019; Aerts et al., 2021). Nevertheless, at the same time, batching contributes to distorting the information transmitted through the supply chain, desynchronising the supply from the demand (Hussain and Drake, 2011). This intensifies the bullwhip effect, leading to serious inefficiencies in the supply chain actors, as discussed by Lee et al. (1997). In other words, when orders are grouped in batches, the upstream members of supply chains may misinterpret the actual demand of products, which makes them place erratic orders that accentuate the bullwhip problem. This does not only decrease the smoothness of their operations but may also affect the availability of products in the downstream echelons of the supply chain (Hassanzadeh et al., 2014); in this fashion, batching impacts the operations of all supply chain members⁵ (Vicente et al., 2018).

From this perspective, supply chain experts generally advocate the reduction of batch sizes as much as possible (e.g. Wang and Disney, 2016). They aim for the ultimate goal of a ‘batch of one’ in order to mitigate the negative impact of batching on the supply chain, which is well-aligned with the lean production philosophy that emphasizes the ‘one-piece-flow’ (see Protzman et al., 2017). However, there are circumstances where this is not technically or economically possible (Potter and Disney, 2006).

This fact motivated some authors to explore the link between the size of batches and the bullwhip effect in supply chains. Via simulation, Holland and Sodhi (2004) found that the bullwhip increase is proportional to the square of the batch size. This means that small batch sizes provoke a relatively small increase in bullwhip, while large batches greatly damage the supply chain operations. This endorses the long-held view that bullwhip reduces as the batch size decreases (see e.g. Burbidge, 1981).

Nevertheless, Potter and Disney (2006) came to a remarkable finding, as discussed in the introduction of this work. They found that establishing the batch size (say, δ) as a divisor of the mean demand (say, μ) may allow managers to minimise bullwhip. This finding has relevant implications: when the ideal $\delta = 1$ is not possible, managers may use $\delta = \mu/n$, with $n \in \{1, 2, 3, \dots\}$, to mitigate the bullwhip pressure in the supply chain. Between these points (characterised by $\delta = \mu/n$), they

⁴ In some capacitated production settings, it is common to define batch sizes to reduce unit costs (Potter and Disney, 2006). For instance, consider an oven that can only process a certain number of units per cycle, which involves a considerable amount of fixed costs. Also, manufacturers often define a minimum order quantity (i.e., a minimum batch size) to lower the unit production cost by decreasing the costs of setup and/or changeover (Hussain and Drake, 2011). Something similar occurs in the constraints of packaging or transportation systems (Lee et al., 1997; Sodhi and Tang 2011). In this case, the products are usually shipped in quantities that fit the size of a container or a road trailer.

⁵ It is convenient to note that batching does not only increase the bullwhip effect of decentralised supply chains, as evidenced by the real data of Pastore et al. (2019). The same also applies in centralised systems, as shown by Hassanzadeh et al. (2014) and Devika et al. (2016).

observed a waveform with peaks halfway between the minima, which are higher as δ increases. Also, Potter and Disney (2006) noticed that the bullwhip ratio is an increasing function of the batch size when $\delta > \mu$. Hussain and Drake (2011) and Hussain and Saber (2012) also spotted the waveforms.

These prior works prove that some research efforts have been conducted to understand the supply chain effects of batching.⁶ However, all studies referred to in the previous paragraphs have analysed batching in traditional supply chains. To the best of our knowledge, no study has yet focused on understanding how batching affects the bullwhip effect of CLSCs. This is turning into an important research gap given the increasing significance of these supply chains in the transition towards more circular economies. In addition, we highlight that these previous studies have considered the bullwhip consequences of batching, but they have mostly ignored its impact on customer service.⁷ This is also an important research gap as both perspectives have important cost implications and are strongly connected. In this work, we address both research gaps by investigating the impact of order-quantity batching in the bullwhip behaviour and customer service performance of CLSCs.

2.2. Bullwhip and customer service in closed-loop supply chains

CLSCs have gained considerable attention in the scientific literature over the last decade. A growing body of this discipline has investigated how the bullwhip effect impacts them. As this phenomenon may be an important barrier to the successful operation of CLSCs, these papers have focused on understanding how it emerges and how it can be attenuated in these systems. This is essential to efficiently and resiliently integrate the forward and reverse flows of materials in circular economies (Adenso-Diaz et al., 2012; Govindan et al., 2015; Bhatia et al., 2020).

The paper by Tang and Naim (2004) is often referred to as the first study of bullwhip in CLSCs (e.g. Goltos et al., 2019b). They modelled a HMRS and observed that the reverse flow of materials helps to mitigate the bullwhip effect in CLSCs if the manufacturing orders use information about the state of the remanufacturing pipeline. This interesting finding—specifically, the reduction of bullwhip as the return rate grows—was corroborated by subsequent works, such as Zhou and Disney (2006), Turrisi et al. (2013), and Cannella et al. (2016). However, some recent works showed that the bullwhip reduction in CLSCs only occurs on the condition that the uncertainty in the reverse flow of materials is low; when this does not apply, CLSCs may experience (much) higher bullwhip levels than traditional supply chains (Hosoda et al., 2015; Hosoda and Disney, 2018; Ponte et al., 2019).

Nonetheless, the bullwhip effect of CLSCs does not only depend on the volume and the uncertainty of the reverse flow. Prior works also considered the impact of other factors that characterise the operation of these supply chains in the real world, including the mean and variability of lead times (Ponte et al., 2020a; Dominguez et al., 2020), capacity constraints (Dominguez et al., 2019), and the number of echelons in the supply chain (Zhou et al., 2017).

⁶ Nonetheless, we believe that the modelling of batches in supply chain studies lags clearly behind their practical relevance. Indeed, the vast majority of the bullwhip effect literature, reviewed by Wang and Disney (2016), assume that the supply chain operations are not constrained by batch sizes. We argue that this is mainly due to the fact that modelling batches introduces a nonlinearity in the mathematical model, which makes the analysis much more complex (see e.g. Disney et al., 2020). However, nonlinearities also have strong implications on the behaviour and performance of production and distribution systems (see e.g. Wang et al., 2014). This implies that many implications of batching in supply chains may still be unknown.

⁷ It is interesting to note that prior works have not studied how the solution proposed by Potter and Disney (2006) affects the capacity of the supply chain to efficiently satisfy customers. Indeed, they identified this as a key direction for future research: ‘it is important to understand the link [of batching] with inventory holding costs’ (Potter and Disney, 2006, p. 416).

Due to the previously highlighted interplays between the bullwhip effect and the service level achieved, some of these works also studied the ability of the CLSC to satisfy customers in a cost-effective manner. Interestingly, Zhou and Disney (2006) and Cannella et al. (2016) found that CLSCs are better able to efficiently satisfy the demand of customers as the return rate grows. However, Turrisi et al. (2013) and Ponte et al. (2020a) observed the opposite impact of the return rate, while Dominguez et al. (2021) revealed that the relationship between customer service and the volume of returns depends on the structure of the CLSC. The different results suggest that the inventory performance of CLSCs (like their bullwhip effect) significantly depends on the modelling assumptions.

All in all, while some works have provided valuable insights into the dynamics of CLSCs, these systems have still received much less attention than traditional ones in what is termed as the supply chain dynamics literature (Braz et al., 2018; Goltsov et al., 2019b; Framinan, 2022). Under these circumstances, new studies should contribute to better understanding the bullwhip effect and customer service of CLSCs, which is essential to accelerate the transition to more circular economies. These studies need to evaluate some realistic characteristics of CLSCs that have not been (or have scarcely been) considered so far, and thus their implications on the dynamics of CLSCs are totally (or mostly) unknown. One of them, which is addressed in this paper, is order batching.

Indeed, to the best of our knowledge, only the work by Adenso-Díaz et al. (2012) modelled batches in the CLSC dynamics literature. They studied simultaneously the impact of 11 factors on the bullwhip effect of these systems. One of them was related to the lot-sizing policy. They analysed a lot-for-lot policy (in which the quantity ordered matches the net requirements) and a minimum-lot-size policy (in which the order is only placed if it is higher than 40% of the mean demand). They found no difference in the bullwhip behaviour of their CLSC⁸ with both policies. Our paper differs from this work in several ways. Most importantly, we focus on the effects of batching on CLSCs, while in their paper the batching policy was only one of the many factors considered. Also, we investigate multiple batch sizes, while they only used one batch size. Moreover, we do not only study the bullwhip effect but also the service level of the CLSC. In this sense, we provide a much deeper understanding of the effects of batching in CLSCs.

3. The closed-loop supply chain with order-quantity batching

This work investigates a CLSC whose backbone is a HMRS. In this system, the manufactured and remanufactured products are perfect substitutes (Souza, 2013). We thus implicitly assume that remanufactured products, produced from ‘cores’ (i.e., used products collected from the customer), meet the same quality standards and have the same price as newly manufactured ones, which are produced from raw materials (Ponte et al., 2021). In addition to the industrial relevance of HMRSs, which we discussed in the introduction of this paper, we selected this CLSC structure in a consistent manner with most of the CLSC dynamics literature; see Goltsov et al. (2019b). This will facilitate the comparison of our results against well-established models, such as that by Tang and Naim (2004). The study of other CLSC structures would also be of value to complement our findings, but is beyond the scope of this work.

Fig. 1 provides a schematic representation of the CLSC under consideration. It shows the key inventories (namely, serviceable and recoverable) and processes (that is, manufacturing, remanufacturing, consumption, and return or disposal, given that not all used products come back to the supply chain), as well as the relevant flows (i.e., forward materials, reverse materials, and orders). Next, we explain how the

CLSC operates over time.

3.1. Mathematical model with key assumptions

We assume that the inventory of the HMRS is managed according to periodic-review policies, which are typically less expensive to implement and operate than continuous-review policies (Axsäter, 2003). From this perspective, we model the operation of the CLSC on a discrete-time basis through a set of difference equations. These equations, which represent the mathematical model of the supply chain, are compiled in Table 1, which also includes the key performance indicators.

Following standard practice in the CLSC dynamics literature (e.g. Hosoda et al., 2015; Hosoda and Disney, 2018; Ponte et al., 2020b), we consider a sequence of events that can be structured across three stages per period (we use t to denote a generic period, which is defined according to the review period of the HMRS; e.g. a day, a week, or a month). These are described below, together with the relevant equations and the underlying assumptions, which we briefly justify.

3.1.1. Reception stage

At the beginning of t , the serviceable inventory receives batches of manufactured and remanufactured products. The batches of manufactured, new products correspond to the production orders issued $T_m + 1$ periods ago (Eq. (1)). The batches of remanufactured, as-good-as-new products correspond to those orders issued $T_r + 1$ periods ago (Eq. (2)). Note: T_m and T_r are the fixed manufacturing and remanufacturing lead times, respectively; we use $T_m + 1$ and $T_r + 1$, given that the orders are issued at the end of each period. The assumption of constant production lead times seems reasonable when the inventory review period is large (e.g. a week or a month) and the variability of the operations is relatively small (e.g., a few hours or days); see e.g. Goltsov et al. (2019a).

The initial position of the serviceable stock in t then can be calculated as the sum of the final position of the serviceable stock in $t-1$ and the manufacturing and remanufacturing batches that have just been received in t (Eq. (3)). This sum provides the products that are available for meeting the demand of customers in t . At this point, we highlight another important assumption of our work: we consider a backlogging inventory system. This represents well the operations of most make-to-order environments and B2B relationships (Disney et al., 2020); thus, it may be well aligned with the (re)manufacturing of laptops for final customers and engines for industrial customers, for example. In this sense, when customer demand cannot be fully met at present (t), it will be satisfied as soon as possible —ideally, at the start of the next period ($t+1$), as long as it is allowed by the new batches received.

3.1.2. Serving stage

During t , customers request products and they are served. The overall demand of each period is considered to be uncertain. It is modelled through a set of independent and identically distributed (i.i.d.) random variables, x_t , that are normally distributed with mean μ and standard deviation σ . Here, $CV_d = \sigma/\mu$ is the demand’s coefficient of variation. These variables are truncated to non-negative values (Eq. (4)). We adopt the i.i.d. and normality assumptions as they capture the behaviour of certain real-world demand patterns (e.g. Disney et al., 2016). Specifically, they are often reasonable when the demand originates from many independent customers (Disney et al., 2020).

The demand of customers is satisfied from the serviceable stock. Therefore, the final position of this inventory each period is the difference between the initial position and the demand of the period (Eq. (5)). In line with the backlogging assumption, the end-of-period position of the serviceable inventory represents a net stock (Disney and Lambrecht, 2008). In this regard, positive values ($ns_t > 0$) indicate that all customers needs have been met and there are some excess products in the inventory (which can be used to satisfy next period’s demand). Meanwhile, negative values ($ns_t < 0$) reveal that stock-outs have occurred,

⁸ A four-echelon, serial supply chain (i.e., glass factory, bottler, wholesaler, and retailer), similar to that in the well-known Beer Game, together with a recycler that receives the used products from the customer and delivers them to the bottler.

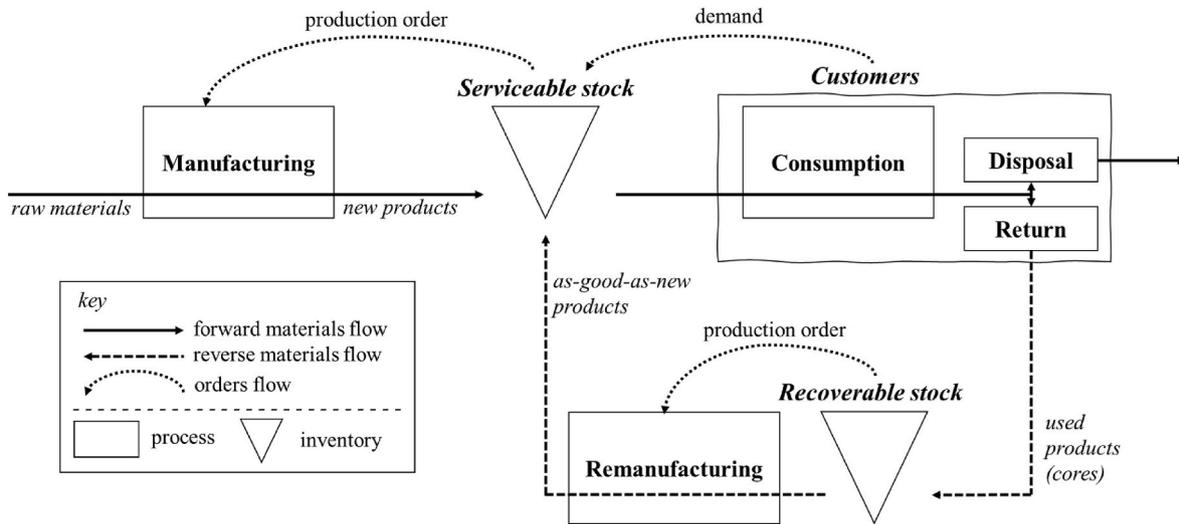


Fig. 1. Illustration of the main processes, inventories, and flows in the CLSC.

and there are pending orders that need to be met as soon as possible.

During t , the cores are also collected at the recoverable inventory. The return rate, which represents the percentage of products that return to the production facilities after their use, is also considered to be uncertain, which is common in the industrial practice of most CLSCs (Goltsos et al., 2019b). In this fashion, our model considers a twofold uncertainty (i.e., demand + returns uncertainty) in the CLSC.

The return rate is modelled with a set of i.i.d. random variables, y_t , that follow a beta distribution with the shape parameters α_y and β_y . This distribution is a reasonable modelling assumption, as it is defined on the interval $[0,1]$ and can take different shapes depending on α_y and β_y . The mean, μ_y , and standard deviation, σ_y , of y_t are functions of the shape parameters as follows: $\mu_y = \frac{\alpha_y}{\alpha_y + \beta_y}$, $\sigma_y = \frac{\sqrt{\alpha_y \beta_y}}{(\alpha_y + \beta_y) \sqrt{\alpha_y + \beta_y + 1}}$. Hence, the return rate's coefficient of variation is $CV_y = \frac{\sigma_y}{\mu_y} = \frac{\beta_y}{\alpha_y(\alpha_y + \beta_y + 1)}$. In this way, the returns in t are obtained as the product of the return rate in t and the demand T_c periods before, where T_c is the consumption or usage lead time (Eq. (6)). For instance, this relationship between returns and past demand fits well with the operations of certain CLSCs where returns are associated to leasing practices, which are becoming increasingly common in many industries.⁹

The used products collected in t increase the availability of cores for the remanufacturing operations. In this sense, the position of the recoverable inventory in t becomes the position of this inventory at the end of the previous period ($t-1$), i.e., the excess returns, plus the returns collected in t (Eq. (7)).

Finally, the position of the work-in-progress can also be updated. This includes the products that have been ordered to the manufacturing and remanufacturing lines but are not yet available to satisfy customer demand. In this fashion, the work-in-progress is the accumulated difference between the production orders and the receipts. Therefore, the work-in-progress in t becomes the work-in-progress in $t-1$ plus the manufacturing and remanufacturing orders of the previous period ($t-1$)

⁹ For example, leasing is now becoming a common practice in the smartphone industry (Rousseau, 2020). In leasing contracts, customers have two main options at the end of the lease term: they can either return the product (and maybe lease a different smartphone) or keep it (they can purchase the smartphone that they have been using, or sign a new lease for it). Under these circumstances, it seems reasonable to assume a random return rate (the percentage of customers that return the product may significantly vary over time), but a constant usage lead time (most of the products that come back return after the lease term).

minus the completion rates in t (Eq. (8)).

3.1.3. Sourcing stage

At the end of t , the position of the serviceable inventory triggers the placement of new production orders. As previously mentioned, we consider that the inventory is controlled in the CLSC according to periodic-review policies. Also, we assume that remanufacturing operations are prioritised due to environmental and/or economic reasons. Indeed, remanufacturing is not only more environmentally friendly than traditional manufacturing, but it is also generally a cheaper alternative (see Parker et al., 2015).

To implement the prioritisation rule, we consider that the recoverable stock is managed according to a push policy, such as in Tang and Naim (2004), Hosoda et al. (2015), and Cannella et al. (2021). According to this policy, the remanufacturer processes as many cores as available each period. Nonetheless, in our case (unlike those previous works), the push policy is limited by the remanufacturing batch size, denoted by δ_r . Thus, the remanufacturing order is calculated by rounding the recoverable stock to the immediately lower multiple of the batch size (Eq. (9), where $\tau(\cdot)$ is the truncation function). This makes that the excess returns in t are the difference between the position of the recoverable stock in t and the remanufacturing order that has been placed (Eq. (10)).

In contrast, the manufacturing orders are periodically placed according to a POUT model. These orders aim to meet the portion of customer demand that cannot be satisfied by remanufacturing cores. The POUT model is a generalisation of the widely-used OUT model that is able to provide a better trade-off between bullwhip and service level. For this reason, it has been successfully applied to several real-world supply chains, such as in Lexmark (Disney et al., 2013), and it has been widely used in recent works (e.g. Zhou et al., 2017; Ponte et al., 2019; Cannella et al., 2021). Again, it is necessary to consider the manufacturing batch size, denoted by δ_m . In this sense, the manufacturing order is calculated through a two-step procedure:

- First, we determine the manufacturing requirements by using the POUT model. The requirements are the sum of: (i) the difference between the demand forecast and the remanufacturing completion rate, representing a net demand; (ii) a portion of the gap between the target (a safety stock) and the actual end-of-period position of the serviceable inventory; and (iii) a portion of the gap between the target and the actual position of the work-in-progress; see Tang and Naim's (2004) type-3 system. The sum is truncated to prevent negative orders from being issued (Eq. (11)). Here, we highlight the

Table 1
Mathematical formalisation of the operation of the CLSC and key performance indicators.

Variable	Equation No.
1. Reception stage	
Manufacturing completion rate, mc_t	$mc_t = mo_{t-(T_m+1)}$ (1)
Remanufacturing completion rate, rc_t	$rc_t = ro_{t-(T_r+1)}$ (2)
Initial stock (serviceable inventory), is_t	$is_t = ns_{t-1} + mc_t + rc_t$ (3)
2. Serving stage	
Customer demand, d_t	$d_t = \max\{x_t, 0\}, x_t \rightarrow N(\mu, \sigma^2)$ (4)
Net stock (serviceable inventory), ns_t	$ns_t = is_t - d_t$ (5)
Customer returns, r_t	$r_t = y_t \times d_{t-T_r}, y_t \rightarrow B(\alpha_y, \beta_y)$ (6)
Recoverable stock, rs_t	$rs_t = er_{t-1} + r_t$ (7)
Work-in-progress, w_t	$w_t = w_{t-1} + (mo_{t-1} - mc_t) + (ro_{t-1} - rc_t)$ (8)
3. Sourcing stage	
Remanufacturing order, ro_t	$ro_t = \tau \left(\frac{rs_t}{\delta_r} \right) \times \delta_r$ (9)
Excess returns, er_t	$er_t = rs_t - ro_t$ (10)
Manufacturing requirements, mr_t	$mr_t = \max \left\{ \widehat{d}_t - rc_t + \frac{1}{T_i} (ss_t - ns_t) + \frac{1}{T_w} (tw_t - w_t), 0 \right\}$ (11)
Manufacturing order, mo_t	$mo_t = \rho \left(\frac{mr_t}{\delta_m} \right) \times \delta_m$ (12)
Demand forecast, \widehat{d}_t	$\widehat{d}_t = ad_t + (1 - \alpha) \widehat{d}_{t-1}$ (13)
Safety stock, ss_t	$ss_t = e \widehat{d}_t$ (14)
Target work-in-progress, tw_t	$tw_t = T_p \widehat{d}_t$ (15)
Key performance indicator	
Manufacturing Order Variance Ratio, <i>MOVR</i>	$MOVR = \frac{var(mo_t)}{var(d_t)}$ (16)
Remanufacturing Order Variance Ratio, <i>ROVR</i>	$ROVR = \frac{var(ro_t)}{var(d_t)}$ (17)

(continued on next page)

Table 1 (continued)

Variable	Equation No.
Inventory Variance Ratio, <i>IVR</i>	$IVR = \frac{\text{var}(ns_t)}{\text{var}(d_t)} \quad (18)$

role of the time constants of the controllers (T_i for the serviceable inventory, T_w for the work-in-progress). These are two key decision parameters of the POUT policy that allow managers to appropriately control the supply chain dynamics.

- Second, the actual manufacturing order is calculated by rounding the manufacturing requirements to the nearest multiple of the batch size (Eq. (12), where $\rho(\cdot)$ is the rounding function). Notice the difference with the remanufacturing line, where the position of the recoverable inventory is always rounded to the immediately lower multiple of the batch size to ensure that the excess returns are non-negative.

The implementation of the POUT policy requires the estimation of customer demand, the determination of the safety stock, and the establishment of a target work-in-progress. In our case, we use the industrially popular exponential smoothing to forecast demand, see Petropoulos et al. (2014). This technique projects demand in t as the weighted average, according to the smoothing factor α , of demand in t and the forecast in $t-1$ (Eq. (13)). In addition, the safety stock is calculated as the demand forecast times the safety stock factor, ε (Eq. (14)). This is also a popular option in practice, see Disney et al. (2013). Last, the target work-in-progress is obtained as the product of the demand forecast and the estimate of the pipeline lead time, T_p (Eq. (15)), see Lin et al. (2017). Last, we highlight that α , ε , and T_p are three important decision parameters: α impacts the sensitivity of the forecast to changes in demand; ε determines the number of periods that are protected against stock-outs by the safety stock; and T_p significantly affects the trade-off between the customer service level and the inventory holding cost.

3.2. Key performance indicators

To evaluate the performance of the CLSC, we consider three well-known metrics: the Manufacturing Order Variance Ratio, *MOVR*; the Remanufacturing Order Variance Ratio, *ROVR*; and the Inventory Variance Ratio, *IVR*. These metrics, which are defined in the last rows of Table 1, offer a rich perspective for understanding the effectiveness and efficiency of the CLSC. Specifically, they provide information about the bullwhip effect and the customer service level that can be achieved, two important supply chain problems that are strongly interrelated as discussed before.

First, *MOVR* evaluates the variance of manufacturing orders in relative terms to that of customer demand (Eq. (16)). In traditional supply chains (and often also in CLSCs), this is usually named the bullwhip ratio (e.g. Disney et al., 2020), given that it quantifies the bullwhip effect in the system. High *MOVR* is indicative of costly inefficiencies in a production system or supply chain, including overproduction and waiting time¹⁰.

We also analyse *ROVR*, which compares the variance of remanufacturing orders to that of demand (Eq. (17)). Following a similar line of

reasoning, high levels of *ROVR* would be symptomatic of harmful inefficiencies in the remanufacturing operations due to the high variability in the use of these resources (Ponte et al., 2019).

Finally, *IVR* is the ratio of the variance in the net stock to that of customer demand (Eq. (18)), which is sometimes referred to as the net stock amplification ratio (e.g. Disney et al., 2020). It is well known that supply chains that operate with low values of *IVR* are better able to achieve high levels of customer service while maintaining low inventory investments. In contrast, when *IVR* is large, customer service levels tend to be significantly lower. In these cases, high customer satisfaction can only be achieved at the expense of excessive inventory holding costs.

For a given demand pattern, the goal of the HMRS is to operate with the lowest possible levels of *MOVR*, *ROVR*, and *IVR*. However, some of these metrics can be minimised at the expense of the others. Therefore, it is fundamental to find an appropriate trade-off between them, so that the CLSC satisfies most customers at a reasonably low operational cost (which includes taming the bullwhip effect); see Disney and Lambrecht (2008).

4. Design of the simulation experiments

The general objective of this work is to gain a thorough understanding of the effects induced by order-quantity batching on the dynamic behaviour and operational performance of CLSCs. The strong nonlinear nature of our mathematical model makes the analytical study very difficult (see Wang et al., 2014; Ponte et al., 2017; Disney et al., 2020), so we resort to simulation and experimental techniques to meet our objective.

To perform the simulations, the discrete-time CLSC model detailed in the previous section has been implemented in MATLAB R2019b. This computing environment allows us to rapidly conduct statistically significant, long-horizon simulations and facilitates the programming of exhaustive experimental designs. The simulation system has been verified and validated through common practices, including the analysis of intermediate simulation outputs (tracing) and the comparison of the outputs of the system against well-known results in the literature.

In addition, we have analysed the stability of the system (i.e., its ability to provide bounded outputs in response to bounded inputs). To this end, we have simulated the response of the CLSC against 100,000 step inputs in the demands and returns for random, but reasonable,¹¹ values of the system parameters. We have assessed the step response of the net stock, manufacturing order, and remanufacturing order, and we have checked that in all cases the CLSCs produces a bounded output (i.e., it is not unstable). Nonetheless, it is interesting to note that in most of the

¹⁰ The importance of this metric can be interpreted from the lens of Lean production, which refers to overproduction and waiting as two of the most important sources of waste in organisations and supply chains. Indeed, Lean usually refers to overproduction as the most grievous form of waste, as it hides and generates other forms of waste (Bahri, 2009).

¹¹ This refers to positive lead times ($T_m, T_r, T_c > 0$), non-negative mean and standard deviations of the demand and return rate ($\mu, CV_d, \mu_y, CV_y \geq 0$), smoothing factor (α) in the common interval $[0,1]$, non-negative safety stock factor ($\varepsilon \geq 0$), optimal regulation of the lead-time estimate ($T_p = (1 - \mu_y) \times T_m + \mu_y \times T_r$, see Tang and Naim, 2004), positive time constants of the controllers ($T_i, T_w > 0$), and production batches equal to or higher than 1 ($\delta_m, \delta_r \geq 1$). We have not checked the stability of the CLSC outside these ranges. Indeed, prior works suggested that while CLSCs are generally stable for reasonable values of the parameters, they may become unstable outside these ranges (see e.g. Ponte et al., 2021).

simulations the responses form a repeating cycle. This is due to the impact of batching, as explained by Potter and Disney (2006). In this sense, batching may make the CLSC behave like a marginally stable system.

4.1. Experimental approach

In line with the findings obtained by Potter and Disney (2006) for traditional supply chains, we need to investigate multiple levels of the batch sizes to capture the batching effects in enough detail—otherwise, the waveforms may be overlooked. For this reason, we study a wide range of levels of the manufacturing batch size, δ_m ; specifically, multiples of 5 up to $\delta_m = 1.25 \times \mu$ (when the batch size is significantly higher than the mean demand), as well as the ideal scenario defined by $\delta_m = 1$ (where the manufacturing system is able to process units of product independently, i.e., one-piece-flow). Similarly, we study a wide range of levels of the remanufacturing batch size, δ_r . In this case, we use multiples of 10 up to $\delta_r = 1.25 \times \mu_y \times \mu$ (where $\mu_y \times \mu$ represents the average number of products that return to the CLSC to be remanufactured), also including $\delta_r = 1$.

It is particularly interesting to consider the interactions between the batching effects and the smoothing effects of the POUT policy, as this would yield relevant managerial implications about the appropriate control of CLSC dynamics. In this sense, we consider five levels of the time constant of the inventory controller, T_i . In particular, we study $T_i = 2^k$, from $k = 0$ to $k = 4$. Notice that for $k = 0$ ($T_i = 1$), the POUT policy simplifies to the classic OUT model; while $k = 4$ ($T_i = 16$) has a large smoothing effect on the manufacturing orders. It is also important to note that we explore the so-called Deziel-Eilon setting (Deziel and Eilon, 1967), characterised by $T_w = T_i$. This is the most common configuration of the POUT policy in practice (see e.g. the implementation of the POUT policy in Tesco, one of the largest retailers in the world, in Potter and Disney, 2010).

In addition, we aim to analyse the interactions between the batching effects and the volume of the reverse flow of materials. This allows us to study how the level of circularity in the CLSC modifies the consequences of batching. To this end, we consider the return rate, μ_y . We investigate three levels: $\mu_y = 0$, which results in a traditional supply chain (no reverse flow); $\mu_y = 1/3$, which represents a CLSC with a moderate volume of returns; and $\mu_y = 2/3$, which leads to a highly circular supply chain, where most demand can be met by remanufacturing cores.

Given that considering multiple levels per factor and several factors greatly complicates the design of experiments and its analysis, we opt to perform four two-factor, full-factorial designs, with the following specific objectives:

- *Design 1* ($\delta_m, T_i; \mu_y = 0$) – This aims to capture the interactions between the (manufacturing) batch size and the proportional controller in traditional supply chains, which defines a good starting point for the analysis of the CLSC. We aim to contrast our results against relevant findings in the literature.
- *Design 2* ($\delta_m, T_i; \mu_y = 1/3; \delta_r = 1$) – This aims to analyse the same interactions in CLSCs. By comparing the results of this design to those of *design 1*, we attempt to understand the differences between the batching and smoothing effects in both supply chain settings.
- *Design 3* ($\delta_m, T_i; \mu_y = 2/3; \delta_r = 1$) – This aims to explore the impact of the volume of returns on the batching and smoothing effects in CLSCs, through detailed comparison of the results of this design to those of *designs 1 and 2*.
- *Design 4* ($\delta_m, \delta_r; \mu_y = 1/3; T_i = 4$) – This aims to investigate the interactions between the manufacturing and remanufacturing batch sizes in CLSCs based on HMRSs.

4.2. Supply chain scenario and simulation setup

With the aim of offering prescriptive recommendations for practitioners in Section 7, we define a realistic supply chain scenario that is characterised by fixed values for the other parameters of the system. The rationale that we have used for the selection of these values is justified as follows.

First, we focus on the lead times. In this regard, we employ a manufacturing lead time of two periods, $T_m = 2$. This is a common value in supply chain dynamics studies (e.g. Costantino et al., 2013). Regarding the remanufacturing process, we follow the suggestion made by Hosoda and Disney (2018) to make both lead times equal; hence, $T_r = 2$. This allows CLSC managers to increase the sustainability of their operations without sacrificing economic performance, given that shorter remanufacturing lead times are often problematic in practice; see the ‘lead-time paradox’ in Hosoda and Disney (2018). Finally, as it is common in CLSC settings, we assume that the consumption lead time is the longest one. Specifically, we use $T_c = 16$. In our study, we will not focus on the lead-time effects, which have been widely studied in both traditional supply chains (Michna et al., 2020, among many others) and CLSCs (Cannella et al., 2016, among many others).

Now we consider the stochastic parameters. The customer demand is described by a mean of $\mu = 120$ and a coefficient of variation of $CV_d = 25\%$ (i.e., $\sigma = 30$). This degree of variability is in line with the observations by Dejonckheere et al. (2003), who reported that the typical coefficients of variations of real-world demand time series are in the range between 15% and 50%. In relation to the return rate, we also use a coefficient of variation of $CV_r = 25\%$ (for the mean values of this parameter that have been highlighted in the previous subsection) to make both sources of uncertainty of relatively similar importance.

As regards the decision parameters of the replenishment model, we use a smoothing factor of $\alpha = 0.1$, given that forecasting experts generally recommend $0.05 \leq \alpha \leq 0.2$ (Teunter et al., 2011). We employ a safety stock factor of $\varepsilon = 1$. That is, the serviceable inventory is protected against an extra period to reduce the stock-out risk, such as in Cannella et al. (2021). For the lead-time estimate, we use the configuration determined by Tang and Naim (2004), i.e., $T_p = T_p^{TN} = (1 - \mu_y) \times T_m + \mu_y \times T_r$.¹² Otherwise, an inventory offset emerges that would damage the service level achieved (Disney and Towill, 2005).

To sum up, Table 2 describes the specific set of parameter values used for each design to facilitate the replicability of our study. Finally, we note that, given the low experimental effort of our numerical approach, we have run simulations of 20,000 periods and each experimental point has been explored 8 times (i.e., 8 replications) to ensure the consistency of our results and the reliability of our findings. Thus, our experimental approach leads to a total of 1,240 simulation runs (31 levels of $\delta_m \times 5$ levels of $T_i \times 8$ replications) for each of *designs 1, 2 and 3* and 1,488 runs (31 levels of $\delta_m \times 6$ levels of $\delta_r \times 8$ replications) for *design 4*; This makes a total of 5,208 runs. The stability of the CLSC in all the simulation runs has also been verified *ex post* through the analysis of their results.

5. The interactions of batching and smoothing in closed-loop supply chains

Given that meaningful interactions emerge between the batching and smoothing effects in supply chains, this section studies the impact of batching for various replenishment decisions that differ on the smoothing level. Moreover, we consider the degree of circularity in the CLSC, which also affects the impact of batching. This refers to *designs 1, 2, and 3*. Although we have previously defined three metrics, we only

¹² In the traditional supply chain (characterised by $\mu_y=0$), the previous equation results in the typical configuration of this parameter in manufacturing systems, i.e., $T_p=T_m$ (Lin et al., 2017).

Table 2
Parameter values in the four designs.

Parameter	Design 1	Design 2	Design 3	Design 4
<i>Lead-time parameters</i>				
Manufacturing lead time	$T_m = 2$	$T_m = 2$	$T_m = 2$	$T_m = 2$
Remanufacturing lead time	$T_r = \text{NR}^1$	$T_r = 2$	$T_r = 2$	$T_r = 2$
Consumption lead time	$T_c = \text{NR}^1$	$T_c = 16$	$T_c = 16$	$T_c = 16$
<i>Stochastic parameters</i>				
Demand: Mean	$\mu = 120$	$\mu = 120$	$\mu = 120$	$\mu = 120$
Demand: Coefficient of variation	$CV_d = 25\%$	$CV_d = 25\%$	$CV_d = 25\%$	$CV_d = 25\%$
Return rate: Mean	$\mu_y = 0$	$\mu_y = 1/3$	$\mu_y = 2/3$	$\mu_y = 1/3$
Return rate: Coefficient of variation ^a	$CV_y = \text{NR}^1$	$CV_y \approx 25\%$	$CV_y \approx 25\%$	$CV_y \approx 25\%$
<i>Decision parameters of the replenishment model</i>				
Smoothing factor	$\alpha = 0.1$	$\alpha = 0.1$	$\alpha = 0.1$	$\alpha = 0.1$
Safety stock factor	$\epsilon = 1$	$\epsilon = 1$	$\epsilon = 1$	$\epsilon = 1$
Lead-time estimate	$T_p = T_m$	$T_p = T_p^{TN}$	$T_p = T_p^{TN}$	$T_p = T_p^{TN}$
Inventory controller: Time constant	$T_i = \{1, 2, 4, 8, 16\}$	$T_i = \{1, 2, 4, 8, 16\}$	$T_i = \{1, 2, 4, 8, 16\}$	$T_i = 4$
Work-in-progress controller: Time constant	$T_w = T_i$	$T_w = T_i$	$T_w = T_i$	$T_w = T_i$
<i>Batching parameters</i>				
Manufacturing batch size	$\delta_m = \{1, 5, 10, \dots, 150\}$			
Remanufacturing batch size	$\delta_r = \text{NR}^1$	$\delta_r = 1$	$\delta_r = 1$	$\delta_r = \{1, 10, \dots, 50\}$

Notes: ¹NR stands for ‘not relevant’. In the traditional supply chain (design 1), the impact of the parameters related to the reverse flow is null, so there is no need to select levels for them.

^a In all cases, the shape parameters of the beta distribution (i.e., α_y and β_y) have been adjusted to ensure that μ_y and CV_y are as planned. Specifically: $\alpha_y = 0, \beta_y = 1$ results in $\mu_y = 0$ (and $\sigma_y = 0$); $\alpha_y = 10, \beta_y = 20$ results in $\mu_y = 1/3$ and $CV_y \approx 25\%$; and $\alpha_y = 4.5, \beta_y = 2.25$ results in $\mu_y = 2/3$ and $CV_y \approx 25\%$.

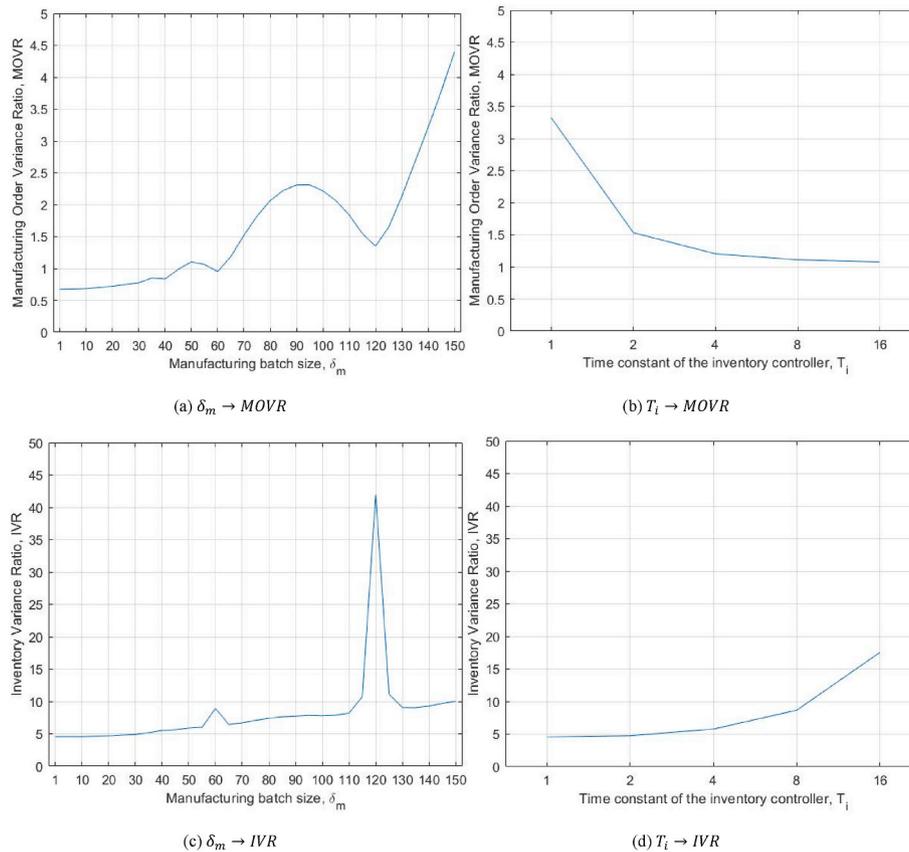


Fig. 2. Main effects plots in the traditional supply chain.

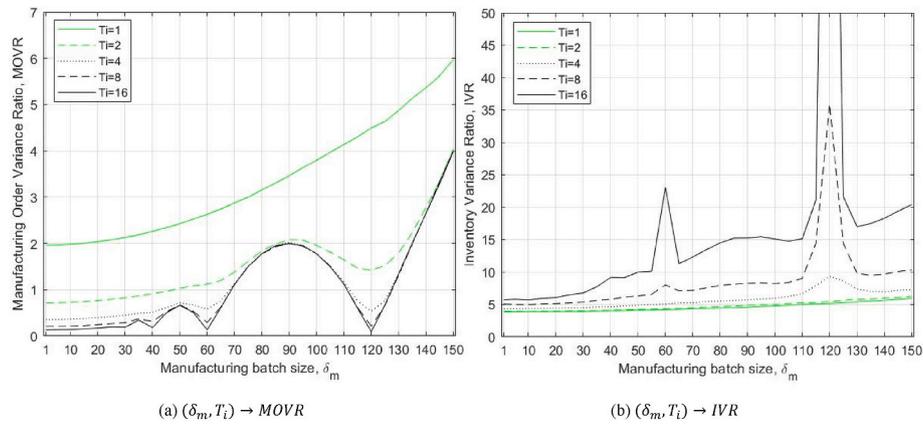


Fig. 3. Interactions plots in the traditional supply chain.

consider here the manufacturing order variance ratio (MOVR) and the inventory variance ratio (IVR). We do not study the remanufacturing order variance ratio (ROVR) given that, in our HMRS, the manufacturing batch size (δ_m) and the time constant of the inventory controller (T_i) do not affect the push operations of the remanufacturing line.

Before looking in detail into the implications of the reverse flow of materials, we first consider the particular case in which the return rate is null (i.e., $\mu_y = 0, \sigma_y = 0$). Under such circumstances, the used products do not come back to the supply chain, and consequently there are no remanufacturing operations. In this sense, the CLSC under study turns into a traditional, open-loop supply chain. Understanding this system represents a good initial position for our work: it will allow us to compare our results against those of prior works in the supply chain literature and, from this perspective, we provide insights into the impact of batching in CLSCs in the subsequent subsections.

5.1. The baseline traditional supply chain

Fig. 2 displays the main effects plots obtained from the results of the 1,240 runs conducted in *design 1*. It shows the mean response values, both for MOVR and IVR, at each level of the parameters δ_m and T_i . Thus, these curves provide insights into the general impact of these parameters in the traditional supply chain. The statistical significance of the impact of δ_m and T_i has been confirmed following standard practices, which we do not detail here for the sake of brevity.

Although the novelty of our paper arises from the study of batching, we first look at the impact of the POUT controller. Fig. 2b reveals that increasing T_i allows for a significant reduction in the bullwhip effect (i.e., MOVR decreases). However, Fig. 2d shows that this improvement comes at the expense of worsening the inventory performance (i.e., IVR increases). These effects of the POUT controller are well known in the specialised literature in both traditional (e.g. Disney and Lambrecht, 2008) and CLSCs (e.g. Cannella et al., 2021). In this sense, it becomes essential for managers to find an appropriate trade-off between the positive effects (on production stability) and the negative effects (on customer service) of increasing T_i . For instance, in this setting it may be reasonable to use $T_i = 4$; given that this allows for a significant reduction in MOVR (approx. from 3.3 to 1.2, see Fig. 2b) without suffering from a severe increase in IVR (approx. from 4.75 to 5.75, see Fig. 2d). Nonetheless, the optimal configuration of T_i would always depend on the cost structure of the specific supply chain under study.

Now we address the batch size. In terms of bullwhip, Fig. 2a shows that the best solution is to use $\delta_m = 1$. In this case, the supply chain

attenuates the variability of the demand signal ($MOVR < 1$), while an amplification effect emerges ($MOVR > 1$) for larger values of δ_m . However, it is not always feasible to use $\delta_m = 1$. In such cases, it should not simply be assumed that the batches must be as small as possible to reduce bullwhip. Notice in Fig. 2a that the bullwhip effect is more pronounced for $\delta_m = 90$ than for $\delta_m = 120$. In this sense, we perceive traces of the waveforms reported by Potter and Disney (2006). Specifically, we can see local minima of MOVR at $\delta_m = \mu = 120, \delta_m = \mu/2 = 60$, and $\delta_m = \mu/3 = 40$. Nonetheless, in the work by Potter and Disney (2006), the values of MOVR at the local minima ($\delta_m = \mu/n$, with $n \in \{1, 2, 3, \dots\}$) are close to that of MOVR when $\delta_m = 1$, which does not happen in Fig. 2a. To understand why, Fig. 3 plots the effects of the interaction between δ_m and T_i .

Fig. 3a reveals that MOVR displays an articulated waveform curve when T_i is very large, such as $T_i = 16$. In this case, the manufacturing requirements are strongly smoothed by T_i (Eq. (11)), showing a small variability around the mean. As a result, when δ_m is a divisor of μ (Eq. (12)), most orders equal μ ; MOVR is thus greatly reduced. For other δ_m , the orders tend to form repeating cycles with two order sizes: the closest (upper and lower) multiples of δ_m . The occurrences of both order sizes in the cycles are in such a way that the mean order is μ ; therefore, local maxima of MOVR appear halfway between the points characterised by $\delta_m = \mu/n$.¹³ In contrast, the waveform is not perceived when $T_i = 1$ (i.e., the traditional OUT policy). In this case, Fig. 3a shows that MOVR increases as δ_m grows. This occurs because $T_i = 1$ makes the production requirements much more variable over time. Consequently, the variability of orders cannot be minimised for certain values of δ_m ; in this way, orders do not (nearly) always equal μ when $\delta_m = \mu/n$.¹⁴ Looking at the intermediate values of T_i , the waveform structure fades as T_i decreases. For $T_i = 16$, we see local minima at $\delta_m = \{40, 60, 120\}$; for $T_i = 2$, we only see a local minimum at $\delta_m = 120$.

¹³ The emergence of the waveform is described in analytical detail by Potter and Disney (2006) when the demand is constant. To illustrate our explanations, we provide some results of the simulations in *design 1* for $T_i=16$ ($\mu=120$). In this case, $f(mo_t=\mu)=96.47\%$ for $\delta_m=60$, and $f(mo_t=\mu)=99.42\%$ for $\delta_m=120$ (we use $f(mo_t=X)$ to denote the percentage of manufacturing orders in which $mo_t=X$). This enormously decreases MOVR. In contrast, when $\delta_m=90$, $f(mo_t=\delta_m)=66.28\%$ and $f(mo_t=2\delta_m)=33.32\%$. Therefore, MOVR is $T_p = T_p^{TN}$ significantly higher than for $\delta_m=\mu/2$.

¹⁴ To facilitate the comparison of $T_i = 16$ and $T_i = 1$, we now provide the simulation results of $T_i = 1$. In this case, for $\delta_m = 60$, $f(mo_t = \mu) = 48.87\%$. Note that $f(mo_t = \mu)$ is much lower than when $T_i = 16$. At the same time, $f(mo_t = 0) = 2.38\%$; $f(mo_t = \delta_m) = 23.20\%$; $f(mo_t = 3\delta_m) = 23.13\%$; $f(mo_t = 4\delta_m) = 2.36\%$; $f(mo_t > 4\delta_m) = 0.03\%$. Under these circumstances, MOVR is much higher than for $T_i = 16$, and the bullwhip benefits of $\delta_m = \mu/n$ are nullified.

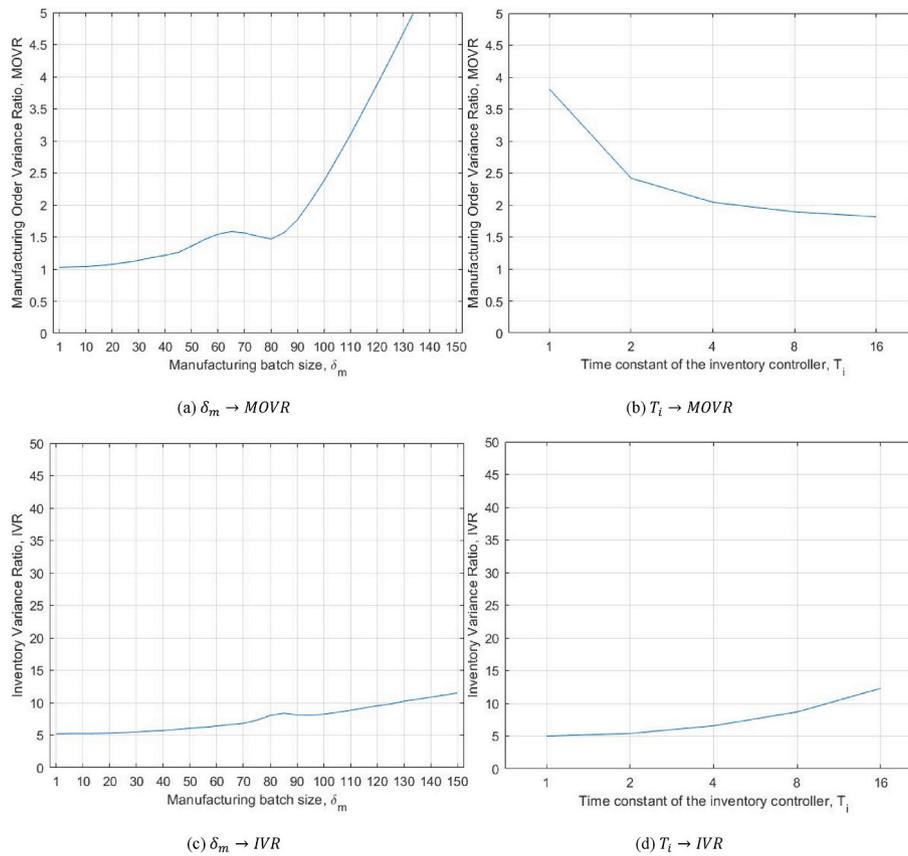


Fig. 4. Main effects plots in the CLSC with $\mu_y = 1/3$.

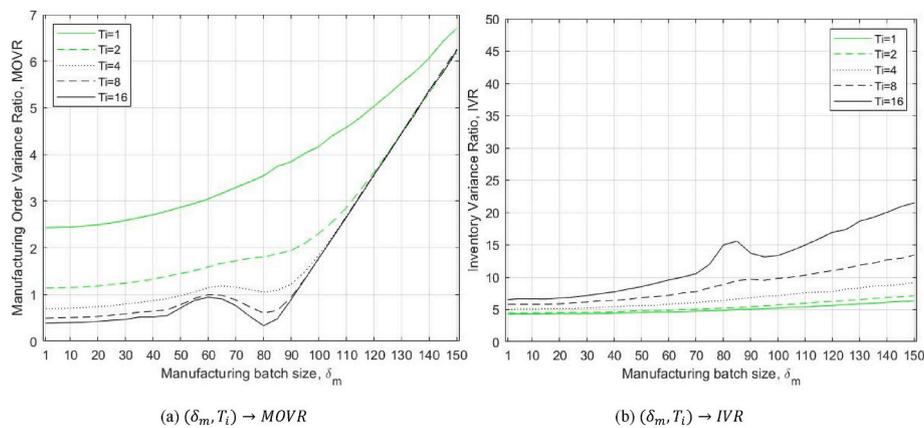


Fig. 5. Interactions plots in the CLSC with $\mu_y = 1/3$.

Finally, we investigate the effects of δ_m on the capability of the supply chain to efficiently satisfy the demand of customers. In this regard, Fig. 2c shows that, in general terms, IVR grows as δ_m increases. However, there are local maxima for $\delta_m = \mu = 120$ and $\delta_m = \mu/2 = 60$, which suggests that selecting a batch size that is a divisor of the mean demand to reduce bullwhip may be problematic in terms of customer satisfaction. This is a particularly interesting finding, given that the previous works that observed the waveforms (Potter and Disney, 2006;

Hussain and Drake, 2011; Hussain and Saber, 2012) had not considered their implications on inventory performance.

To better understand this finding, we look at the interactions plots. A joint analysis of Fig. 3a and b reveals that minimising MOVR through $\delta_m = \mu$ occurs at the expense of dramatically increasing IVR. In this sense, the operations of the production line are smoothed, but this deteriorates the efficiency in customer satisfaction. This occurs because, when T_i is high, $\delta_m = \mu$ results in a production levelling strategy (as

discussed before, $mo_t = \mu$ for most of the periods). This strategy ignores, to a large extent, the evolution of demand and the position of the serviceable inventory when issuing production orders, and thus becomes ineffective if demand varies considerably over time.¹⁵ Fig. 3a and b shows that, when $\delta_m = \mu/2$ and $\delta_m = \mu/3$ are used, the same rationale applies, although the peak in *IVR* is smaller as δ_m decreases. This is due to the replenishment system being more sensitive to the state of the inventory and the evolution of demand for smaller δ_m . All in all, we claim that setting the batch size to minimise bullwhip should not be done without considering the inventory implications of this decision.

5.2. The implications of closing the loop

Figs. 4 and 5 show the main effects and interactions plots, respectively, for the simulations of *design 2*. The only difference in the parameter setting with respect to the previous subsection is that now $\mu_y = 1/3$, i.e., one out of three sold products (on average) return to the supply chain after consumption. This has important implications, as it activates the reverse flow of materials that includes the remanufacturing line. The limits of the y-axis in Figs. 4 and 5 are the same as those in Figs. 2 and 3 to facilitate the comparison between both scenarios.

Fig. 4b and d indicate how T_i broadly affects *MOVR* and *IVR*. The effects of the POUT controller in CLSCs are similar to those observed for traditional ones: rising T_i reduces *MOVR* but increases *IVR*. This in line with previous literature in the CLSC dynamics discipline (e.g. Cannella et al., 2021). Therefore, it also becomes necessary to consider the trade-off between *MOVR* and *IVR* when adjusting POUT controllers in CLSCs.

Now we focus on the effects of batching, where the main novelty of our study lies. First, Fig. 4a proves that the bullwhip effect is again minimised when $\delta_m = 1$. Nonetheless, we can also observe traces of Potter and Disney's (2006) waveform, given that a local minimum exists for $\delta_m = 80$. At this point, it is convenient to highlight that, when the remanufacturing line is incorporated, the mean manufacturing rate does not match the mean demand of customers (μ). Rather, a portion of this demand is met by remanufacturing used products; therefore, the mean manufacturing order would now be $(1 - \mu_y) \times \mu = (2/3) \times 120 = 80$. This explains the local minimum at $\delta_m = 80$.

The detailed comparison of Figs. 2a and 4a reveals that the waveform structure is smoother in the CLSC than in the traditional supply chain. This is an important theoretical insight with meaningful implications for supply chain practice. While the bullwhip effect of traditional systems may significantly benefit from strategically setting the manufacturing batch size to divisors of the mean demand, in CLSCs that are built on HMRSs it should be considered that: (i) the batch size needs to be set to divisors of the mean manufacturing rate (i.e., $(1 - \mu_y) \times \mu$), rather than the mean demand, to benefit from a bullwhip reduction; and (ii) the dynamic improvement derived from this batching decision is significantly smaller than in traditional supply chains.

The interactions plot, shown in Fig. 5a, explains these findings. For high T_i (e.g. $T_i = 16$), the relationship between *MOVR* and δ_m defines a waveform with a local minimum at $\delta_m = (1 - \mu_y) \times \mu$. For intermediate T_i (e.g. $T_i = 4$), we can see the minimum but the waveform structure is less clear. For low T_i (e.g. $T_i = 1$), rising δ_m always result in an increase of *MOVR*. While the same observations hold in both traditional and CLSC settings, the waveform found by Potter and Disney (2006) is more evident, and thus has greater implications, in traditional supply chains.

¹⁵ Interestingly, we note that the underlying mechanism that explains why *MOVR* decreases significantly and *IVR* increases considerably at $\delta_m = \mu$ is the same as the one that makes *MOVR* reduce and *IVR* grow when T_i increases in POUT inventory systems (see e.g. Disney et al., 2006).

in the CLSC (Fig. 5a), the bullwhip improvement derived from using $\delta_m = (1 - \mu_y) \times \mu$ is relatively small. Overall, this batching strategy aimed at mitigating the bullwhip effect yields lower benefits in CLSCs, as these systems are subject to a wider scenario of uncertainties. Specifically, this may be interpreted as a consequence of the impact of the uncertainty in the volume of returns on the POUT policy (see Eq. (11)).¹⁶

Last, we analyse the effects of order-quantity batching on *IVR*. Like in the traditional supply chain, Fig. 4a and c show that a local minimum in the *MOVR* curve generates a local maximum in the *IVR* curve. The same can be concluded from the inspection of Fig. 5a and b. In this fashion, while it may be advantageous to set δ_m to divisors of the mean manufacturing rate from a bullwhip perspective, this batching decision tends to have negative consequences on customer service due to those reasons discussed in the previous subsection. Nonetheless, the comparison of Figs. 5b to 3b reveals that the maximum in *IVR* is of a smaller magnitude in CLSCs than in traditional supply chains. Given that setting δ_m to minimise bullwhip is in general terms less effective in CLSCs, the downsides of this decision on customer satisfaction are also less severe.

5.3. The impact of the volume of returns

Finally, we investigate the interactions of δ_m and T_i in a CLSC with $\mu_y = 2/3$. In this new scenario, the level of circularity is significantly higher than in the previous one. Indeed, most demand (approx. 67%) can be satisfied by remanufacturing products collected from the market. The mean manufacturing rate reduces to $(1 - \mu_y) \times \mu = (1/3) \times 120 = 40$. In this sense, the analysis of the results of *design 3* allows us to understand in more detail how the volume of returns affects the implications of batching in CLSCs. To facilitate the analysis, Figs. 6 and 7 provide the main effects and interactions plots, respectively, using the same limits of the y-axis as before.

The detailed comparison of the main effects of T_i in this scenario (Fig. 6b and d) to the equivalent curves in the previous subsections (Fig. 2b, d, 4b, and 4d) reveals interesting patterns. First, we consider the bullwhip curves (Figs. 2b, 4b and 6b). We note that: (i) for low T_i , there is an inverted U-shaped relationship between *MOVR* and μ_y , in a such a way that *MOVR* is maximised for $\mu_y = 1/3$; while (ii) for high T_i , *MOVR* is an increasing function of μ_y . Second, we explore the inventory curves (Figs. 2d, 4d and 6d). It should be acknowledged that: (i) for low T_i , *IVR* grows as μ_y increases; while (ii) for high T_i , *MOVR* displays a U-shaped relationship with μ_y , in such a way that *MOVR* is minimised for $\mu_y = 1/3$.

In view of these considerations, we underline that, while the general effects of the POUT controller in CLSCs are the same as those in traditional supply chains (i.e., increasing T_i reduces *MOVR* but increases *IVR*), CLSCs exhibit substantially more complex behaviours. These create multifactor relationships between the performance metrics (*MOVR* and *IVR*) and the mean return rate, which are heavily influenced by other CLSC parameters. Overall, we conclude that the variability in the operations of CLSCs generally increases as μ_y grows (which applies both for *MOVR* and *IVR*) as they are subject to a higher uncertainty (demand and returns). This is aligned with prior works, such as Hosoda and Disney (2018) and Ponte et al. (2019). At the same time, it is important to highlight that more complex (U- and inverse U-shaped) relationships appear for some levels of T_i . These relationships, which have also been

¹⁶ The traditional system is only subject to demand uncertainty. However, the CLSC is also subject to returns uncertainty. To illustrate the impact of this uncertainty, we include some results of the simulations in *design 2* for $T_i = 16$. In this case, $f(mo_t = (1 - \mu_y) \times \mu) = 71.55\%$ for $\delta_m = 40$, and $f(mo_t = (1 - \mu_y) \times \mu) = 95.37\%$ for $\delta_m = 80$. Note that these values are significantly lower than those obtained in *design 1* for $T_i = 16$ (see footnote 13). Consequently, the bullwhip benefits derived from setting δ_m to divisors of $(1 - \mu_y) \times \mu$ are mitigated, and thus the waveform structure is less clear.

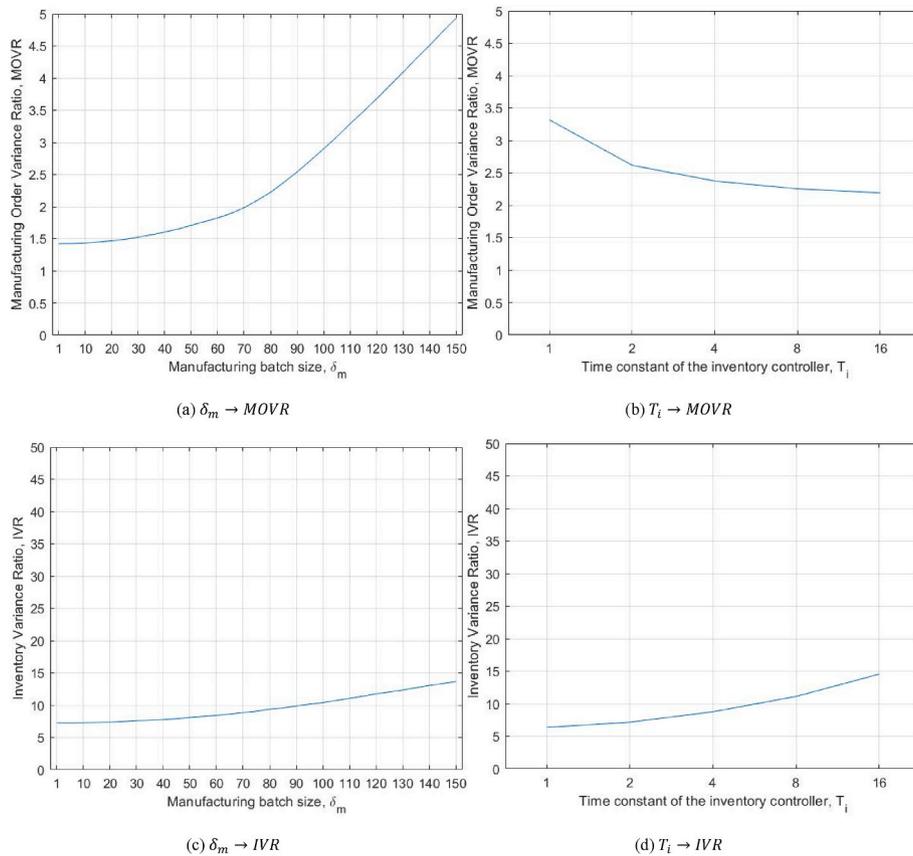


Fig. 6. Main effects plots in the CLSC with $\mu_y = 2/3$.

identified in previous works (see e.g. Goltsov et al., 2019b and Ponte et al., 2020a), should also be recognised by CLSC managers to optimise the performance of their supply chains.

When analysing the main effects of δ_m on *MOVR* in Fig. 6a, we cannot observe the waveform. This buttresses the findings reported in the previous subsection: as the level of circularity grows in CLSCs, the benefits of setting the manufacturing batch size to divisors of the mean order rate are considerably reduced due to the impact of the uncertainty in the returns.¹⁷ Indeed, for $\mu_y = 2/3$ we observe no benefits. Therefore, the local maximum at $\delta_m = (1 - \mu_y) \times \mu$ does not appear now in the *IVR* curve, see Fig. 6b. In this case, reducing δ_m as much as possible facilitates the operations of the CLSC from the perspective of both the advocated production smoothing (i.e., *MOVR*; Fig. 6b) and the capacity of the system to efficiently satisfy customer demand (i.e., *IVR*; Fig. 6d).

The interactions plots in Fig. 7a and b lead us to the same conclusions. Not even for high values of T_i does a local minimum exist in the *MOVR* curve (Fig. 7a). Consequently, the local maximum does not appear in the *IVR* curve either (Fig. 7b). The size of batches should then be as low as possible. Nonetheless, the increase of the metrics is relatively low for small batches, as the slope of *MOVR* and *IVR* grows as δ_m increases. This is aligned with the findings of Holland and Sodhi (2004). In these circumstances, it may be reasonable to use small batches to leverage economies of scale, as this would not entail a significant

deterioration of the CLSC dynamics. Notice that, even when $\delta_m = (1 - \mu_y) \times \mu = (1/3) \times 120 = 40$, the increase in *MOVR* and *IVR* is relatively small.

In this regard, it is also interesting to note that the sensitivity of *MOVR* and *IVR* to changes in the manufacturing batch size is smaller when T_i is low (e.g. in the traditional OUT policy) than when it is high. That is, using high values of T_i makes the CLSC more vulnerable to high batch sizes. In this sense, the benefits of using high T_i are attenuated when the manufacturing system operates with high δ_m . Thus, high values of T_i are unproductive (they barely reduce *MOVR*, but they considerably increase *IVR*) when δ_m is large. This perspective highlights that the tuning of POUT controllers in CLSCs should carefully take the size of batches into consideration.

6. The interactions of manufacturing and remanufacturing batches

In this section, we investigate how the remanufacturing batch size (δ_r) impacts the dynamics of the CLSC. We thus consider the results of design 4, which aims to explore the interactions between both batch sizes, i.e., δ_m and δ_r . To this end, we analyse the CLSC with $\mu_y = 1/3$; thus, the mean manufacturing order is $(1 - \mu_y) \times \mu = 80$ and the mean remanufacturing completion rate is $\mu_y \times \mu = 40$. Also, we use $T_i = 4$ in the POUT policy, which offers a reasonable trade-off between *MOVR* and *IVR* (see Fig. 4b and d). We now also measure the remanufacturing order variance ratio, *ROVR*, to analyse how batching affects not only the operations of the forward flow of materials (described by *MOVR* and *IVR*) but also those of the reverse flow.

Fig. 8 shows the main effects of the two batch sizes. An important first observation is that the remanufacturing batch size (δ_r) impacts the three performance metrics; however, the manufacturing batch size (δ_m) only affects *MOVR* and *IVR*. That is, δ_m does not impact *ROVR*, which is

¹⁷ As discussed before, the local minima of the waveforms emerge when most production orders are constant. To understand why they do not appear in this CLSC, we offer some simulation results of design 3. For $T_i = 16$, $f(mo_t = (1 - \mu_y) \times \mu) = 23.64\% \ll 100\%$ when $\delta_m = 20$, and $f(mo_t = (1 - \mu_y) \times \mu) = 45.36\% \ll 100\%$ when $\delta_m = 40$. These values are insufficient to create the waveforms; they are much lower than those of designs 1 and 2 reported in footnotes 13 and 16.

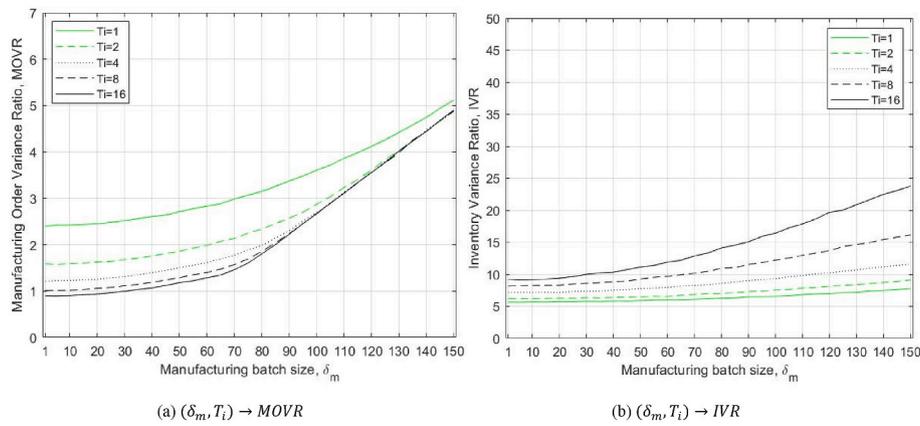


Fig. 7. Interactions plots in the CLSC with $\mu_y = 2/3$.

consistent with previous discussions; as remanufacturing has been prioritised for economic and/or environmental reasons, its control does not depend on the manufacturing configuration. Although the statistical analysis is not included here for the sake of brevity, the impact of δ_m and δ_r on *MOVR*, *ROVR*, and *IVR* has also been evaluated through standard techniques. In all cases, the impact of the parameter on the metric is statistically significant, except for the impact of δ_m on *ROVR* (Fig. 8c).

We do not analyse here the impact of δ_m on *MOVR* and *IVR* (in Fig. 8a and e), which we considered in the previous section. Instead, we examine the main effects of δ_r . In this regard, Fig. 8d shows that the variability of the remanufacturing schedule significantly depends on the remanufacturing batch size. Specifically, *ROVR* grows considerably as δ_r increases. The curve suggests the existence of a nonlinear relationship: the slope of the curve grows as δ_r increases. In this sense, reductions of the batch size when δ_r is moderately small (e.g. $\delta_r \leq 10$) do not have significant implications on the stability of the remanufacturing line; however, small reductions of the batch size when δ_r is high (e.g. $\delta_r \geq 30$) significantly smooth the remanufacturing operations.

As discussed before, although the remanufacturing operations do not depend on the manufacturing batch size, the manufacturing line is affected by the remanufacturing batch size when the POUT policy is used in the serviceable inventory. Fig. 8b illustrates how δ_r impacts *MOVR*: the metric grows as the parameter increases. Again, we can see a nonlinear relationship with an increasing slope as δ_r increases. While the improvement in *MOVR* derived from decreasing δ_r may seem relatively small (in comparison to that derived from decreasing δ_m), it is interesting to highlight that this measure makes the manufacturing schedule more stable, particularly when δ_r is high.

Last, detailed inspection of Fig. 8f provides evidence that *IVR* increases as δ_r grows (*MOVR* ≈ 6.5 for $\delta_r = 1$; *MOVR* ≈ 7 for $\delta_r = 50$). In this sense, reducing the batch size of the remanufacturing process facilitates the efficient satisfaction of the demand of customers, even though again the impact of δ_m on this metric is significantly stronger.

In this fashion, reducing δ_r has positive implications on the three operational metrics under study. That is, decreasing δ_r does not only have positive consequences on the stability of the operations in the reverse flow of materials of the CLSC, but it also smooths the operations in the forward flow and improves the efficient management of the serviceable inventory. It is convenient to highlight that we cannot observe any waveform structure to be considered when making decisions on the remanufacturing batch size in our CLSC built on a HMRS.

Finally, we analyse the interactions plots, which are represented in Fig. 9. Given that δ_m does not impact *ROVR*, there are no relevant interactions to be considered in terms of the variability of the remanufacturing orders. In this fashion, the small deviations in Fig. 9b are only due to the randomness of the simulations. In contrast, meaningful interplays between δ_m and δ_r emerge in terms of the variability of the

manufacturing line. Fig. 9a shows that, when δ_m is small or moderate (approx. up to $\delta_m = 90$), *MOVR* is significantly affected by δ_r ; however, for high values of δ_m (approx. from $\delta_m = 110$), δ_r does not have significant implications on *MOVR*. It is also interesting to note that we see traces of the waveforms identified by Potter and Disney (2006) for low values of δ_m , but they cannot be observed when δ_r becomes higher. Similarly, Fig. 9c reveals that reducing δ_r when δ_m is small or moderate has positive implications on customer service (i.e., *IVR* decreases); nevertheless, reducing δ_r when δ_m is high does not necessarily decrease the variability of the net stock.

Overall, we conclude that, when both batch sizes are large, reducing the manufacturing one yields higher benefits for the CLSC. Indeed, the benefits of reducing δ_r are marginal when δ_m is very high. However, once δ_m is moderate (or low), reducing δ_r has positive implications from all perspectives considered in this work.

7. Findings and implications

Since there are numerous insights in this paper, we now summarise the main findings that contribute to the advancement of the supply chain literature by increasing our understanding of the impact of order-quantity batching. These are categorised into three levels. First, we present general findings on the batching effects that apply to all kinds of supply chains (1-3). Second, we discuss those findings that are exclusive to traditional supply chains (4). And third, we consider the findings that apply to the increasingly important CLSCs (5-7). In a later subsection, we evaluate the main implications of these findings for managerial decision making.

7.1. Main findings

7.1.1. General findings on the impact of batching

- (1) Ignoring batch sizes generally overestimates the dynamic performance of supply chains. In this way, these systems behave significantly worse (both the bullwhip effect increases and the customer service level decreases) when goods are produced and delivered in batches to leverage economies of scale, as this desynchronises the supply from the demand.
- (2) When the popular OUT replenishment policy is used (i.e., $T_i = 1$), the bullwhip effect increases in the batch size according to a nonlinear relationship (the slope of the bullwhip curve increases as the size grows). In addition, the capacity of the supply chain to efficiently satisfy customer demand decreases as the batch size grows.
- (3) Using a POUT replenishment policy (i.e., $T_i > 1$) always allows for a reduction in the bullwhip effect of the supply chain at the

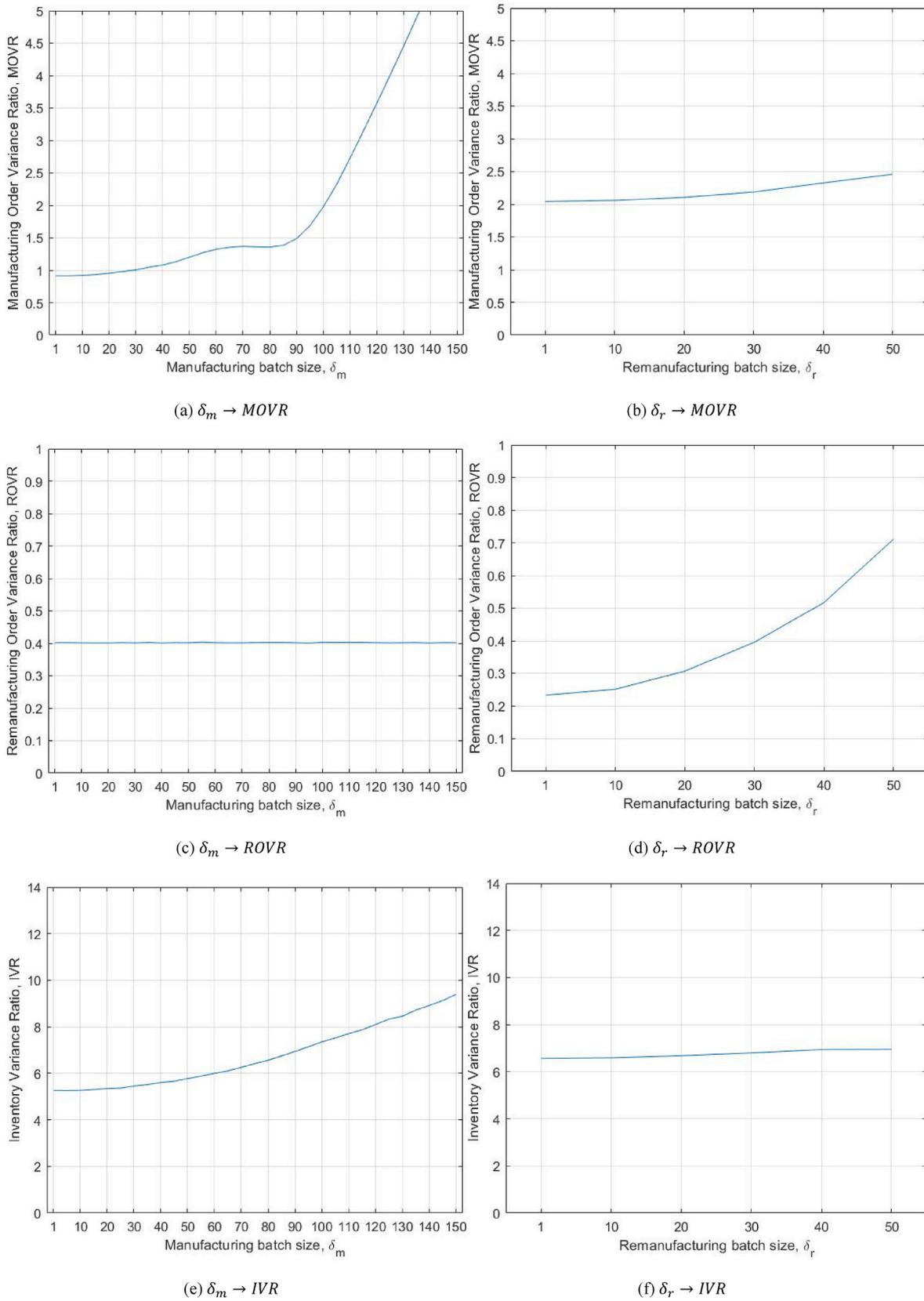


Fig. 8. Main effects plots in the CLSC with different remanufacturing batch sizes.

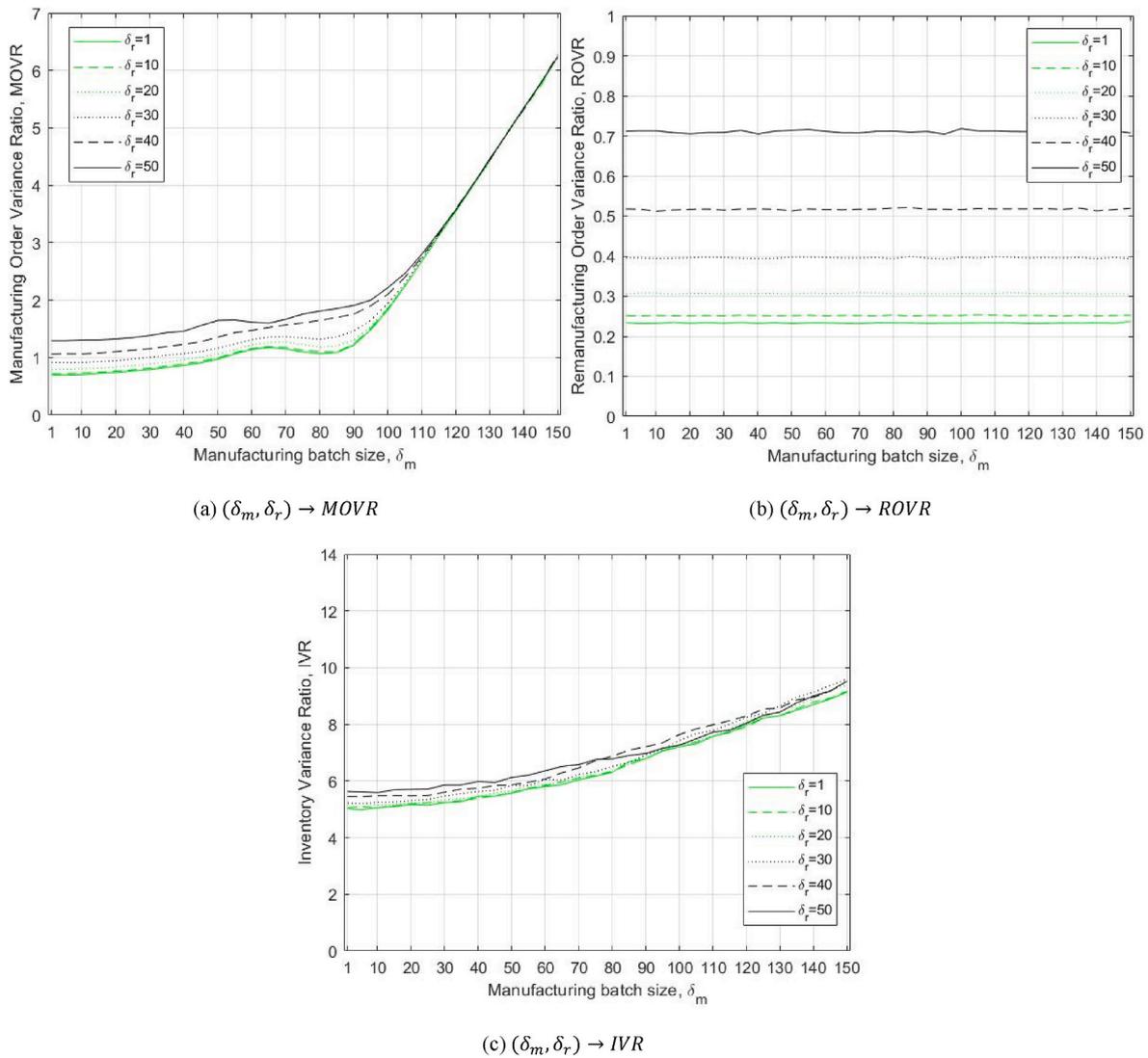


Fig. 9. Interactions plots in the CLSC with different remanufacturing batch sizes.

expense of an increase in inventory costs (i.e., the sum of inventory holding and stock-out costs), regardless of the batch size. This implies that this policy is also a useful mechanism to optimise the overall performance of supply chains constrained by batching requirements.

7.1.2. Specific findings for traditional supply chains

- (4) When the POUT replenishment policy is used (i.e., $T_i > 1$) in traditional supply chains, the bullwhip effect draws a waveform curve as a function of the manufacturing batch size. In this sense, bullwhip can be reduced by adopting a batch size that is a divisor of the mean demand, as highlighted by Potter and Disney (2006). However, it should be acknowledged that:
- The bullwhip effect reduction occurs at the expense of decreasing the customer service level and/or increasing inventory holding costs; that is, setting the size of batches to a divisor of the mean demand may hinder the efficient satisfaction of customers;
 - The waveform is more clearly defined as T_i grows; hence, the benefits of using divisors of the mean demand are lower when T_i is moderately small (e.g. $T_i = 2$);

- Batch sizes that are halfway between the divisors of the mean demand should be avoided, as this diminishes the bullwhip benefits of using the POUT policy;
- Batch sizes over the mean demand should always be avoided, as this results in a dramatic reduction of the efficiency of supply chains (it increases both the bullwhip effect and inventory costs).

7.1.3. Specific findings for closed-loop supply chains

- (5) When the POUT replenishment policy is used (i.e., $T_i > 1$) in CLSCs, the waveform structure of the bullwhip effect can only be seen when both the volume of returns is low (in our case, $\mu_y \leq 1/3$) and the level of smoothing is high (in our case, $T_i \geq 4$). The waveform structure tends to disappear in CLSCs mainly due to the additional uncertainty (in the returns) faced by these systems. Under these circumstances:
- When this occurs (μ_y is low and T_i is high), the bullwhip effect can be mitigated by setting the batch size to a divisor of the mean production rate, but this strategy would also have a negative impact on inventory performance;
 - Otherwise (μ_y is high and/or T_i is low) batches should be as small as possible, as order and inventory variabilities are increasing functions of the batch size;

Table 3
The different dynamics induced by the batch size: Summary of main insights.

	Closed-loop supply chain		
	Traditional supply chain	Low volume of returns	High volume of returns
<i>Bullwhip effect</i>	<ul style="list-style-type: none"> - Increases as δ_m grows for low T_i - Waveform function of δ_m for medium and high T_i (up to $\delta_m = \mu$) → Bullwhip can be minimised by $\delta_m = \mu/n$ 	<ul style="list-style-type: none"> - Increases as δ_m grows for low and medium T_i - Waveform function of δ_m for high T_i (up to $\delta_m = (1 - \mu_y) \times \mu$) → Bullwhip can be minimised by $\delta_m = (1 - \mu_y) \times \mu/n$ - Increases as δ_r grows for low and medium δ_m - Is not affected by δ_r for high δ_m 	<ul style="list-style-type: none"> - Increases as δ_m grows for all T_i
<i>Inventory performance^a</i>	<ul style="list-style-type: none"> - Decreases as δ_m grows for all T_i, with large valleys of performance for $\delta_m = \mu$ when T_i is large 	<ul style="list-style-type: none"> - Decreases as δ_m grows for all T_i, with valleys of performance for $\delta_m = (1 - \mu_y) \times \mu$ when T_i is very large - Decreases as δ_r grows for low and medium δ_m - Is slightly affected by δ_r for high δ_m 	<ul style="list-style-type: none"> - Decreases as δ_m grows for all T_i

Notes.

^a Inventory performance refers to the trade-off between customer service and inventory holding costs.

- In any case, batch sizes that are over the mean production rate should always be avoided, as they have strong adverse implications on the efficiency of CLSCs and diminish the bullwhip benefits of using the POUT policy;
- (6) An increase in the volume of returns generally raises the overall degree of variability in the operations of CLSCs, also due to the impact of the uncertainty related to the collection channel. In this sense, an increase in the level of circularity of CLSCs entails environmental benefits and economic opportunities, but this increase:
- Tends to worsen the dynamic behaviour of CLSCs (with some exceptions noted in Section 5.3);
 - Reduces, or even nullifies, the bullwhip mitigation derived from strategically setting the size of batches to a divisor of the mean production rate (in line with the previous finding).
- (7) In HMRSs in which the reverse flow of materials is controlled according to a push policy to prioritise the remanufacturing of used products when satisfying the demand of customers, it needs to be considered that:
- The manufacturing batch size affects the variability of the manufacturing orders and the performance of the serviceable inventory, but does not have an impact on the remanufacturing operations;
 - The remanufacturing batch size does not only affect the variability of the remanufacturing line, but also the performance of the serviceable inventory and the variability of the manufacturing operations. In this regard, we found that:
 - o The three relevant metrics improve as the remanufacturing batch size decreases (although the impact of this batch size on the serviceable inventory is considerably smaller than that of the manufacturing batch size);
 - o When the manufacturing batch size is significantly higher than the mean manufacturing rate, the impact of the remanufacturing batch size on the manufacturing operations is marginal.

To sum up, Table 3 comprises the main insights on the different dynamics induced by order-quantity batching in traditional and CLSCs, also considering the effect of the volume of returns.

7.2. Implications for professionals

Now we discuss the key implications of this set of findings for managerial practice. We aim to provide supply chain professionals with helpful suggestions on how to appropriately manage both traditional and closed-loop production and distribution systems in which the goods are produced and delivered in batches.

An important result of our study is that there are strong interplays

between the effects of smoothing replenishment rules and those of order-quantity batching in both supply chain structures. In this sense, smoothing and batching decisions should never be made independently because of the significant inefficiencies that may emerge. For instance, inadequate batch sizes (e.g. those halfway between the divisors of the mean production rate) may markedly diminish the bullwhip benefits resulting from POUT policies with high levels of T_i . Similarly, when inventory controllers are tuned independently of the batch size (e.g. to intermediate values, such as $T_i = 4$), setting this size to a divisor of the mean production rate may not be an effective solution to mitigate the bullwhip effect.

In practice, managers operate in two different scenarios, depending on their capacity to decide the size of batches. In some cases, managers have a limited scope of decision making in the short run regarding the size of batches. For example, this would occur when the size of batches is highly conditioned by the size of a container or a road trailer. In such cases—in which the batch size may be interpreted as an uncontrollable parameter—, managers should take the batch size into consideration when adjusting POUT replenishment rules.

In the event that the batch size is considerably higher than the mean production rate, we recommend managers use low values of T_i (e.g. $T_i = 1$, i.e., the classic OUT policy), given that $T_i \gg 1$ barely reduces bullwhip and has important negative effects on customer service. However, using $T_i \gg 1$ may be productive when the size of batches is smaller. In this case, we establish a difference between: (i) traditional supply chains and CLSCs with a low volume of returns, which are mainly affected by demand uncertainty; and (ii) CLSCs with a medium or high volume of returns, which also need to face considerable uncertainty in the reverse flow of materials.

In the former, we suggest organisations first measure the distance between the batch size and the closest divisor of the mean production rate. When this distance is relatively high, it is reasonable to use moderately low values of T_i , such as $T_i = 2$. This is due to the fact that $T_i = 2$ generates substantially less bullwhip than $T_i = 1$ at a very small inventory cost, while setting high T_i may not yield the expected benefits. In contrast, when the distance is low, the regulation of T_i needs to be mainly based on the cost structure of the supply chain. Specifically, as production variability becomes more costly in comparison to the inventory-related costs, managers should opt for higher values of T_i (see Cannella et al., 2021). In the latter (CLSCs with a medium or high volume of returns), the regulation of T_i should also primarily consider the cost structure of the system.

In other cases, managers can decide on the size of batches, which may then be analysed as a decision parameter. When this occurs, managers should always try to avoid establishing batch sizes that are higher than the mean production rate, since this dramatically increases the variability of orders (even with high regulations of T_i ; that is, the use of $T_i \gg 1$ to smooth the operations of the supply chain becomes ineffective)

and inventories.

If a classic OUT policy is used to replenish the serviceable stock with the aim of optimising customer service, the batches should simply be as small as possible. This occurs as the OUT policy makes that the bullwhip effect and the inventory costs are increasing functions of the batch size in both traditional and CLSCs. Therefore, the ideal would be to have a 'batch of one'; nonetheless, the performance of the supply chain is robust to a moderate increase in the batch size. In this way, small batches (approx. up to a quarter of the mean production rate in traditional supply chains; even larger in CLSCs) would provide acceptable performance, at the same time as they leverage some economies of scale.

When a POUT policy is used to control the serviceable inventory with the aim of smoothing the supply chain operations, the tuning of T_i should be done in conjunction with the determination of the batch size. In general terms, we also recommend managers use batches of one unit when it is possible. Otherwise, the volume of returns plays a key role in determining the optimal batch size. Specifically, when the return rate is null (i.e., traditional supply chain) or low (i.e., CLSC with a low volume of returns), it may be convenient to set the batch size to half of the mean production rate (although other divisors may also work) and to use medium values of T_i , such as $T_i = 4$. This combination would allow for a significant reduction in the variability of orders while having a small impact on customer service. We do not recommend using the mean production rate as the batch size and high values of T_i unless bullwhip minimisation is the clear priority of the management team, given that this would have a detrimental effect on inventory performance. In contrast, when the return rate in the CLSC is high, the batches should again be as small as possible. This occurs given that the high uncertainty faced by these CLSCs diminishes the bullwhip reduction derived from setting the batch size to divisors of the mean production rate.

Finally, we highlight that decision-makers of HMRSs should consider the batches of both the manufacturing and remanufacturing processes to improve the performance of their CLSCs. When the manufacturing orders are placed by using a POUT model and the remanufacturing line operates according to a push policy, we suggest that they prioritise the reduction of manufacturing batch sizes when both are large (higher than their mean production rates). This is because reducing remanufacturing batches is not very effective while manufacturing batch sizes are that large. However, once manufacturing batch sizes have been reduced to reasonable values, managers should focus on reducing remanufacturing batch sizes given that it would yield higher benefits. Specifically, reducing remanufacturing batch sizes would improve not only the stability of the remanufacturing line but also that of the manufacturing line and the efficiency in customer service.

8. Conclusions

Order batching is very common in the supply chains of many industries. However, most of the supply chain literature is built on the assumption that goods can be produced and delivered in the exact quantities needed. We observed that this simplification overestimates the performance of both traditional supply chains and CLSCs for two main reasons. First, batching generally amplifies the bullwhip effect. Second, it makes the supply chain less efficient when satisfying the demand of customers.

Accordingly, supply chain experts have generally advocated that batch sizes have to be as small as possible. Nevertheless, [Potter and Disney \(2006\)](#) noticed that the relationship between bullwhip and the size of batches is more nuanced. Specifically, they showed that bullwhip can be reduced in traditional supply chains by setting the batch size to a divisor of the mean demand when smoothing replenishment rules are used. While this is undoubtedly an interesting and valuable result, we found that this decision may be problematic in terms of customer service.

In CLSCs, we revealed that managers can mitigate the bullwhip effect by using batch sizes that are a divisor of the mean production rate.

Nonetheless, this solution is less effective than in traditional supply chains due to the higher uncertainty that CLSCs face. Indeed, if the return rate is high, the bullwhip benefits of this strategic adjustment of the batch size are nullified. When this occurs, batches should simply be as small as possible to minimise bullwhip and optimise customer service. Having noted that, we observed that batch sizes up to 40% of the mean production rate may allow managers to leverage some economies of scale with a small impact on CLSC performance.

In HMRSs, we concluded that reducing the size of manufacturing batches yields higher benefits (than reducing the size of remanufacturing ones) when both are higher than the mean production rate. Once manufacturing batch sizes are reasonable, reducing remanufacturing batch sizes usually becomes more profitable. This decision would increase the stability of the operations in the forward and reverse flow of materials, and it would also allow for a more efficient satisfaction of customers.

Under these circumstances, we demonstrated that the adjustment of smoothing replenishment rules should be well-coordinated with the determination of the size of batches in both traditional and CLSCs. We found that large supply chain inefficiencies may result when both decisions are made independently. For instance, implementing high levels of smoothing becomes ineffective in terms of bullwhip (and costly in terms of inventory) when batch sizes are large.

Managers of CLSCs that understand the results of our paper will likely find a better trade-off between the bullwhip effect in their supply chains, their efficiency in customer satisfaction, and the economic benefits (economies of scale) of batching. Nonetheless, in our work, we focused on a HMRS. Future research may focus on providing specific insights into the implications of batching in other supply chain structures that also hold the principles of a circular economy, such as pure remanufacturing systems, recycling systems, or industrial symbiosis networks. In addition, we studied the behaviour of the CLSC when a push policy manages the reverse flow of materials. Analysing the impact of batching when pull policies are employed in the recoverable inventory is an avenue of research worth pursuing. Finally, we investigated the response of the CLSC for i.i.d. random demands. Studying how batching affects other demand types, such as autocorrelated and seasonal demand patterns, would also lead to valuable insights for CLSC professionals.

Declaration of competing interest

We wish to confirm that there are no known conflicts of interest associated with this research work.

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