# Calculating ultimate pit limits and determining pushbacks in open-pit mining projects

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#### Abstract

A very important part of any open-pit mining project is determining the ultimate pit limits and optimizing the pushbacks. Currently, block models are the most widely used for calculating mineral resources and reserves. Block models can define the ultimate pit limits through algorithms like floating cone, Lerchs-Grossmann, PseudoFlow, etc. However, only through an economic study will the best pits be selected, from most to least economically valuable, and the different possible pushbacks can then be defined.

Based on pre-determined operating costs, all possible scenarios should be studied while varying the selling price in fixed increments between minimum and maximum values that cover all possible future prices. The starting selling price should be very low, and consequently, the only phase that will be economical is the one with adequate grade and a low stripping ratio. As the selling price increases, the ultimate pit limits do not vary or vary only slightly until the price reaches the point at which another of the ore bodies in the deposit becomes economical. This process is repeated as the price increases until the maximum price considered is reached; then, the project reaches the last phase that is economical if economic conditions are very favourable.

The data obtained in each phase can be used to simulate different scenarios, since the different phases will not change because operating costs or economic conditions change. The ore is not going to move; what changes is the phase up to which mining will be economical and what profits will be obtained in each phase.

#### Keywords

Mining, Open-pit, Floating cone, Ultimate pit limit, Pushbacks, Block model, Cut-off grade, Copper.

## 1. INTRODUCTION

A critical component to any feasibility study of an open-pit mining project is determining the ultimate pit limits and the optimal mining phases (pushbacks).

Today, block models are the most commonly used technique for calculating mineral resources and reserves (Krzemień et al., 2016; Riesgo García et al., 2019; Sterba et al., 2020).

A block model is essentially a database in which each record represents a block of rock and in which the fields define the block's location and properties, such as density, lithology and grade.

Block models are used to divide the subsurface into parallelepipeds so that each represents a database record with specific characteristics (Figure 1).



Figure 1. Block models

Block models for calculating mineral resources and reserves have been used for a long time, though early on there were database size and processing time limitations. However, these limitations have been virtually eliminated due to the speed of modern computers and the ease of managing large databases.

Block models are used not only to calculate resources and reserves but also for planning, grade control, calculating volumes and dilutions and other activities (Lesage et al., 2018).

Block models, combined with different algorithms, can be used to determine the possible ultimate pit limits under different market conditions. However, optimizing the ultimate pit limits and designing and planning the different pushbacks can only be determined by including an economic analysis.

The process for determining the pushbacks is based on the calculation parameters, operating costs, processing recovery, geotechnical angles and selling prices, as well the equations that relate them when calculating the ultimate pit limits, and is discussed below.

All the calculations and images presented were obtained using RecMin (free version) and RecMin Professional software (Castañón Fernández, 2019). RecMin and a wealth of information and helpful tutorials are available free of charge on the Internet.

## 2. CALCULATION PARAMETERS AND CUT-OFF GRADES

The following parameters are needed to calculate the ultimate pit limits:

- The mining cost of waste rock includes the costs of drilling, blasting, loading, hauling and dumping material in the waste rock dump; the cost is usually given per ton or per cubic metre.
- The mining cost of ore includes the costs of drilling, blasting, loading, and hauling to the processing plant and grade control; this cost is usually given per ton or cubic metre. It tends to be somewhat higher than that of waste rock due to grade control costs and because the mined benches are generally smaller.
- The ore processing cost includes all operating costs in the processing plant, including crushing, milling, processing, handling of concentrates and administrative costs. This cost is usually given per ton of ore processed. We have also added administrative and other costs here, which are generally calculated per ton of ore processed.
- The average processing recovery is the percentage by weight of metal or ore that is recovered in the processing plant.
- The selling price is the final selling price of the metal or ore. It is equal to the market price minus the costs of transport, freight, fines, smelting and royalties. Although there are many types of purchase contracts for concentrates produced at the mine, some are complex; the actual price paid is related not only to the metal content of the concentrate but also to the content of other metals. The standard practice is to calculate the real price that is charged minus all these costs.
- Geotechnical angles represent the maximum allowable slope angles that, depending on the characteristics of the material, ensure a stable mining operation. These angles usually vary according to lithology, degree of alteration and location, and are critical for determining the ultimate pit limits since they considerably impact the amount of waste that needs to be extracted.

Once the calculation parameters are defined, the break-even cut-off grade that the ore must have to be profitable can be determined. This grade is equal to the operating and processing costs divided by the revenue (Githiria & Musingwini, 2019; Khan, Waqar, & Asad, 2019).

$$Cut - off \ grade_{break-even} = \frac{Cost_{ore} + Cost_{processing}}{Price \cdot Rec} \tag{1}$$

where

*Costore* = Cost of mining ore per ton

*Cost*<sub>processing</sub> = Cost of treating ore per ton

*Price* = Final selling price per grade unit

*Rec* = Recovery average at plant

The ore that does not meet the cut-off grade and is within the mined pit would be considered waste. However, knowing this waste must be mined and trucked to the waste rock dump with a certain cost, *Cost<sub>waste</sub>* (cost of mining waste per ton), if we eliminate this necessary cost in the cut-off grade calculation, the internal cut-off grade can be defined and is equal to the costs of mining the ore and its processing minus the costs of waste divided by the revenue.

$$Cut - off \ grade_{internal} = \frac{Cost_{ore} + Cost_{processing} - Cost_{waste}}{Price \cdot Rec}$$
(1)

## **3. FLOATING CONE METHOD**

The ultimate pit limits can be determined manually based on sections or as an automated process (Kennedy, 1990). Considering how easy it is to access computerised resources, using automated systems would seem the obvious choice, but a manual review of the calculated sections is always useful.

Several algorithms have been developed to calculate ultimate pit limits. Lerchs and Grossman (1965) and Giannini and Caccetta (1988) adopted a graph-theoretic approach; Picard (1976) used the network flow approach; Kakaie and Yousefi (2011) used the floating cone method, as was later used by RecMin (Castañón Fernández, 2019); and Muir (2004) used PseudoFlow methods as implemented in MiningMath (MiningMath Software, 2020), previously known as SimSched software, that classifies the blocks during the process as waste or ore to achieve the maximum net present value (NPV) at each analysed point (Pell et al., 2019).

Regardless of the method used, the ultimate pit limits should be the same or similar for the same calculation parameters, with the main difference being computation speed since traditional methods are usually quite slow; therefore, automated algorithms such as PseudoFlow have advantages in this regard.

In addition to the algorithms, some programs consider the time factor in the calculation, applying an annual interest and differentiating the initial negative cash flow required for hauling waste in the profitability calculation, which can be calculated using products such as GEOVIA Whittle (Dassault Systemes, 2019) and Minesight (Hexagon Mining, 2019).

To calculate the ultimate pit limits, it is necessary to have a database of blocks that includes all the ore and waste blocks in the entire mining area with their properties, size, location, density and grades.

The floating cone or inverted cone method examines each ore block, in descending order, to determine which blocks in the cone would have to be removed to mine that block. If the sum of the revenue exceeds the costs when that cone is mined, then that cone is removed from the block database (Figure 2).

		45 47	45 47	51 67	51 67	49 53	49 53	36 22	36 22	41 17	5464 3943	8088 4381	8088 4381	9090 4666	9090 4666	8124 4104	8124 4104	4927 3088	4927 3088	1005 2297	210 147	174 125	174 125	162 101	162 101	141 131	4 4 4
43	43	57	57	78	78	93	93	103	103	109	6472	9902	9902	10847	10847	8214	8214	4214	4214	1432	1432	267	267	245	245	192	4
28	28	52	52	93	93	126	126	81	81	29	3611	4470	4470	5110	5110	3740	3740	3033	3033	2946	2946	215	215	153	153	124	
37	37	67	67	124	124	185	185	173	173	186	2932	9266	9266	8489	8489	6617	6617	3203	3203	1483	1483	333	332	296	296	3980	1111
22	22	38	38	90	90	176	176	203	203	92	946	3484	3484	4211	4211	3774	3774	3585	3585	2836	2836	356	256	288	288	3386	
16	16	60	60	119	119	200	200	285	285	315	2942	7051	7051	6210	6210	5243	5243	2194	2194	1531	1531	378	378	337	337	4359	4 13
17	17	28	28	60	60	132	132	200	200	198	902	3506	3506	3507	3507	3146	3146	3269	3269	3224	3224	505	505	405	405	3389	
30	30	59	59	118	118	194	194	220	220	257	2994	3959	3959	3707	3707	3003	3003	1828	1828	1576	1576	344	344	318	3611	4161	4 4 4
17	17	24	24	47	47	85	85	126	126	153	947	3436	3436	3324	3324	3221	3221	2922	2922	2891	2891	477	477	531	2443	3354	
26	26	54	54	83	83	144	144	204	204	256	3340	3598	3598	3426	3426	3237	3237	2539	2539	2732	2732	4060	386	365	3314	3788	1111
88	88	26	26	63	63	110	110	85	88	91	1176	2859	2859	2673	2673	3231	3231	3055	3055	2009	2909	2161	534	561	2302	3326	
97	97	158	158	159	159	170	170	207	207	234	234	3440	3440	3273	3273	3258	3258	2567	2567	2280	2280	2780	335	309	2576	3540	
123	123	161	161	123	123	93	93	112	112	74	74	2731	2731	2545	2545	2722	2722	2790	2790	2812	2812	1991	412	415	2075	2990	
131	131	165	165	169	169	160	160	165	165	201	201	3441	3441	3022	3022	3048	3048	2714	2714	1902	1902	3153	295	2585	2585	3427	
135	135	177	177	203	203	123	123	82	82	87	87	2651	2651	2616	2616	2870	2870	2636	2836	2578	2578	2170	322	2249	2249	3032	
134	134	178	178	182	182	168	168	184	184	232	232	3570	3570	3417	3417	4309	4300	3316	3316	1942	1942	3961	3961	2978	2978	4381	4 14
158	158	171	171	172	172	177	177	153	153	80	80	2982	2982	3046	3046	2863	2363	2867	2867	2757	2757	2386	2386	2357	2357	2774	
149	149	176	176	189	189	153	153	135	135	182	182	3988	3988	4074	4074	4347	4341	3394	3394	1993	1993	3706	3706	3604	3604	4923	4 14
138	138	196	196	149	149	119	119	111	111	100	100	3453	3453	3176	3176	3126	3126	3234	3234	3005	3005	2449	2449	2367	2367	2351	
188	188	203	203	171	171	122	122	97	97	107	107	3375	3375	4040	4040	4516	4516	3894	3894	2052	2052	3086	3086	3440	3440	4682	4
124	124	173	173	157	157	99	99	58	58	69	69	1970	1970	2757	2757	3128	3128	3242	3242	2881	2881	2075	2075	2416	241°	2237	
175	175	182	182	155	155	135	135	82	82	81	81	3694	3694	3868	3868	3576	3576	3340	3340	2163	2163	3395	3395	4279	4279	4663	4
114	114	156	156	141	141	92	92	40	40	57	57	2123	2123	2576	2576	2801	2801	3168	3168	2873	2873	2353	2353	2623	2623	2064	

#### Figure 2. Central section of a block's cone

The process is repeated from top to bottom, checking all the ore blocks repeatedly until a cycle is completed without a new economic cone appearing. An ore block may not be economical in one cycle but may be in the next if a subsequent ore block removes part of the associated waste rock.

The process must be rigorously performed from top to bottom because if done from bottom to top, it is possible that a block in the lower zone may be considered economic when it is not, since we would be paying for the losses in this lower zone with the gains generated from the higher zone, thus overestimating the economic value of the ore body.

The same process is repeated with groups of ore blocks at each level to avoid excluding ore blocks that by themselves are not economical but are in fact economical when considered with nearby blocks.

The ore between the break-even cut-off grade and the internal cut-off grade is the marginal ore.

The study of each cone considering the internal cut-off grade is based on the parameters and equations shown in Table 1.

TYPE OF BLOCK	WASTE	MARGINAL ORE	ORE
Condition	Cut-off grade < internal cut-off grade	internal cut-off grade <= cut-off grade < break-even cut-off grade	Cut-off grade >= break-even cut-off grade
Totals	Waste tonnage: t <sub>waste</sub>	Marginal ore tonnage: t <sub>marg</sub> Marginal ore average grade: Grade <sub>marg</sub>	Ore tonnage: t <sub>ore</sub> Ore average grade: Grade <sub>ore</sub>
Costs	Waste mining cost: $t_{waste} \times Cost_{waste}$	Marginal waste mining cost: $t_{marg} \times Cost_{ore}$ Marginal ore processing cost: $= t_{marg} \times Cost_{processing}$	$\begin{array}{l} \text{Ore mining cost:} \\ t_{ore} \times Cost_{ore} \\ \text{Ore processing cost:} \\ t_{ore} \times Cost_{processing} \end{array}$
Revenue	0	$t_{marg} \times Grade_{marg} \times Rec \times Price$	$t_{ore} \times Grade_{ore} \times Rec \times Price$

Table 1. Optimization parameters of the cone of an ore block considering the internal cut-off grade

Based on the definitions in Table 1, the expected profit from the mining of each ore block is obtained with the following equation:

$$Profit = (t_{ore} \times Grade_{ore} + t_{marg} \times Grade_{marg}) \times Rec * Price - t_{waste} \times Cost_{waste} - (t_{ore} + t_{marg}) \times (Cost_{ore} + Cost_{processing})$$
(2)

Table 2 presents the optimization parameters of the cone of an ore block considering the break-even cut-off grade, summa-

rizing the results for each cone using the cut-off grade, without considering the internal cut-off grade.

Table 2. Optimization parameters of the cone of an ore block considering the break-even cut-off grade

TYPE OF BLOCK	WASTE	ORE					
Condition	Cut-off grade < break-even cut-off grade	Cut-off grade >= break-even cut-off grade					
		Ore tonnage:					
Totals	Waste tonnage: $t_{waste}$	t <sub>ore</sub> Ore average grade: <i>Grade<sub>ore</sub></i>					
Costs		Ore mining cost:					
	Waste mining cost:	$t_{ore} \times Cost_{ore}$					
	$t_{waste} \times Cost_{waste}$	Ore processing cost:					
		$t_{ore} \times Cost_{processing}$					

Based on the definitions in Table 2, the expected profit from mining each block is obtained with the following equation:

 $Profit = t_{ore} \times Grade_{ore} \times Rec \times Price - t_{waste} \times Cost_{waste} - t_{ore} \times (Cost_{ore} \times Cost_{processing})$ (3)

Strictly speaking, the internal cut-off grade should be considered in the calculation, but from a conservative standpoint, it may be more prudent to use the break-even cut-off grade for the following reasons:

- Except in cases where the processing cost is very low, the difference between the break-even cut-off grade and the internal cut-off grade is generally very small. This small difference implies that in the mining operation and in grade control, separating the ore of that grade is very difficult if not impossible.
- Additionally, ore in that grade range (marginal ore) is very low-grade ore and will never be higher than the break-even grade; thus, the operational profit derived from it is very small.
- If marginal ore is introduced to the plant, then the plant is operating with low-profit ore and a higher-grade ore cannot be introduced because it would reduce profit. This is critical, especially in the first few years, when it is of utmost importance to recover the investment as soon as possible.
- Many mining projects will plan to process the ore that is above the break-even grade and to stockpile the marginal ore for use when economic conditions allow, when there is not enough ore extracted to feed the plant, or when the mine is at the end of its life. Stockpiling can be risky if market prices drop and this marginal ore is never processed; however, if prices rise, it can be a very profitable resource, both during and especially at the end of the life of the project, when the mine has difficulties supplying the plant with the tonnage it requires for processing, which has happened in all mines recently.

Whether the internal cut-off grade is considered or not, the following parameters influence the profits:

- The costs of mining waste and ore are both determined precisely. In the feasibility study phase, these costs are usually the result of a detailed study of the characteristics of the materials, the costs of drilling, blasting and loading and also the hauling distances to the destination, either the waste rock dump or the plant. These costs do not vary much, except in special cases. When mining begins, these costs should stay within +/-10%, which is the range of conceptual precision in feasibility studies according to the AusIMM Guidelines for Techno-Economic Evaluations (2012).
- Costs of ore processing and recovery, although in some cases are difficult to determine, should not vary more than +/10% if the pilot-scale processing test work was performed with a quantity of ore that is sufficient and representative of
  the entire life of the mining project.
- In terms of the selling price, little can be done to limit the variability of this parameter, especially in metallic mining, since the variation in metal prices, in addition to being unpredictable, can increase dramatically over a very short time

(Sánchez Lasheras et al., 2015; Krzemień et al., 2015; Riesgo García et al., 2018; Matyjaszek et al., 2019; Matyjaszek et al., 2020).

A +/-20% variation in selling price in the short term and greater than 50% in the medium term is typical in metals, such as Cu, Au, Zn and Ni.

Any study of ultimate pit limits must consider this selling price variation and determine the possible scenarios, both at the beginning and in the successive production phases.

To simplify the profit equation, we can consider the stripping ratio as follows:

$$ratio = t_{waste} / t_{ore} \Rightarrow t_{waste} = ratio \times t_{ore}$$
(4)

Substituting this variable into the profit equation, we have the following:

$$Profit = t_{ore} \times (Grade_{ore} \times Rec \times Price - ratio \times Cost_{waste} - Cost_{ore} - Cost_{processing})$$
(5)

Additionally, the profit per ton of ore  $b_t$  can be calculated as follows:

$$b_t = Profit / t_{ore} = Grade_{ore} \times Rec \times Price - ratio \times Cost_{waste} - Cost_{ore} - Cost_{processing}$$
(6)

If we study profit variance obtained with a pit  $\Delta b_t$  for a percent variance of each of the parameters, for example, if the processing cost varies  $\Delta$ % percentage, we would have the following:

$$\Delta b_t (Cost_{processing}) = (Grade_{ore} \times Rec \times Price - ratio \times Cost_{waste} - Cost_{ore} - Cost_{processing}) - (Grade_{ore} \times Rec \times Price - ratio \times Cost_{waste} - Cost_{ore} - (1 + \Delta \%) \times Cost_{processing})$$

$$(7)$$

Simplifying this equation yields:

$$\Delta b_t(Cost_{processing}) = -\Delta \% \times Cost_{processing} \tag{8}$$

If we repeat this operation with the remaining parameters, the following equations are obtained:

$$\Delta b_t(Cost_{ore}) = -\Delta \% \times Cost_{ore} \tag{9}$$

$$\Delta b_t(Cost_{waste}) = -\Delta \% \times ratio \times Cost_{waste}$$
(10)

$$\Delta b_t(Rec) = \Delta \% \times Grade_{ore} \times Rec \times Price \tag{11}$$

$$\Delta b_t(Price) = \Delta \% \times Grade_{ore} \times Rec \times Price$$
(12)

Note that the profit variances are the same for the same percent variance in selling price or recovery.

We can exemplify this using typical values for the following hypothetical case:

 $Cost_{waste} = Mining \text{ cost of waste per ton} = 2 \notin/t$ 

 $Cost_{ore}$  = Mining cost of ore per ton = 2.2  $\notin$ /t (Ore density is usually higher than that of waste; therefore, mining ore is usually costlier.)

 $Cost_{processing} = Cost of processing ore per ton = 12 \notin/t$ 

- *Price* = Final selling price = 5000 €/t
- Rec = Average recovery at the plant = 82%
- *ratio* = Stripping ratio (waste/ore ratio) = 2
- $Grade_{ore} = Average ore grade = 1\%$

Calculating all the cut-off grades, we obtain the following:

$$Cut - off \ grade_{break-even} = \frac{Cost_{ore} + Cost_{processing}}{Price * Rec} = \frac{2.2 + 12}{5000 * 0.82} = 0.0035 = 0.35\%$$
(1413)

$$Cut - off \ grade_{internal} = \frac{Cost_{ore} + Cost_{processing} - Cost_{waste}}{Price * Rec} = \frac{2.2 + 12 - 2}{5000 * 0.82} = 0.0030 = 0.30\%$$
(14)

Additionally, calculating the profit variance based on the variance of each parameter that influences the calculation leads to:

$$\Delta b_t (Cost_{processing}) = -\Delta \% \times Cost_{processing} = -\Delta \% \times 12$$
(15)

$$\Delta b_t(Cost_{ore}) = -\Delta \% \times Cost_{ore} = -\Delta \% \times 2.2 \tag{16}$$

$$\Delta b_t(Cost_{waste}) = -\Delta \% \times ratio \times Cost_{waste} = -\Delta \% \times 4 \tag{17}$$

$$\Delta b_t(Rec) = \Delta \% \times Grade_{ore} \times Rec \times Price = \Delta \% \times 0.01 \times 0.82 \times 5,000 = \Delta \% \times 41$$
(18)

$$\Delta b_t(Cost_{ore}) = \Delta \% \times Grade_{ore} \times Rec \times Price = \Delta \% \times 41$$
(19)

If equations (16) and (20) are compared, we can see that the change in processing cost must be 3.41 times greater (and of opposite sign) than the change in price to yield the same profit change.

$$\frac{\Delta b_t(Price)}{\Delta b_t(Cost_{processing})} = \frac{\Delta \% \times Grade_{ore} \times Rec \times Price}{-\Delta \% \times Cost_{processing}} = \frac{\Delta \% \times 41}{-\Delta \% \times 12} = -3.41$$
(20)

Therefore, a 10% increase in processing cost is the same as a 10/3.41 = 2.93% decrease in selling price.

If the same is done by comparing equations (17) and (18) with equation (20), we can conclude that the change in the mining cost of ore must be 18.6 times higher (and of opposite sign) than the change in selling price to yield the same profit change. In addition, the change in the mining cost of waste must be 10.25 times higher (and of opposite sign) than the change in selling price to yield the same profit change.

Therefore, a 10% increase in mining costs of ore is the same as a 10/18.6 = 0.54% decrease in the selling price. Additionally, a 10% increase in the mining cost of waste is equivalent to a 10/10.25 = 0.98% decrease in selling price.

Therefore, the profit variance is identical for equal changes in recovery and in selling prices.

$$\frac{\Delta b_t(Price)}{\Delta b_t(Rec)} = \frac{\Delta \% \times Grade_{ore} \times Rec \times Price}{\Delta \% \times Grade_{ore} \times Rec \times Price} = 1$$
(21)

Figure 3 shows a spider graph from the sensitivity analysis of the profit per ton of ore vs. changes in *Costprocessing*, *Costore Price* and *Rec*. As shown by the auxiliary green lines, a 10% increase in the mining cost (the maximum inaccuracy previously determined for a feasibility study) will yield the same profit as a selling price decrease of just -0.54%. Again, an extreme 10% variation in the concentration cost, which means that no pilot-scale testing was performed or that there is lack of confidence or experience concentrating the ore, will be equivalent to just a -2.9% change in the metal price. Thus, the metal price definition will be the key element that will define the feasibility of the study and plays a much greater role than the cost factors.



Figure 3. Sensitivity analysis of the profit per ton of ore (y-axis) vs. variation in the costs of ore processing and mining (x-axis) Since the formulas are linear, the effects of each variation in the parameters in the previous equations can be summed. Thus, a simultaneous x% increase in the costs of mining waste, costs of mining ore and the costs of processing would reduce the profit per ton of ore as follows:

$$\Delta b_t (Cost_{waste}, Cost_{ore}, Cost_{processing}) = -x\% \times ratio \times Cost_{waste} - x\% \times Cost_{ore} - x\% \times Cost_{processing}$$
(22)  
= -x% × (12 + 2.2 + 4) = -18.2 × x%

If we wanted to determine how much the selling price would have to vary ( $\Delta$ %) such that the profit changes in parallel with these three operating variables, making the previous equation equal, we obtain the following:

$$\Delta b_t(Price) = \Delta \% \times Grade_{ore} \times Rec \times Price = \Delta \% \times 41 = -18.2 \times x\%$$
(23)

Therefore:

$$\Delta \% = -0.44 \times x\% \tag{24}$$

Thus, the selling price has a much greater impact on profits than the remaining operating variables, except for plant recovery, which has the same impact on profit variance as the selling price.

Selling price is the parameter that most impacts the study of ultimate pit limits, and its percent variance is usually much larger than those of the other parameters.

The recovery of material in the processing plant has the same impact as selling price on profit variance, but its possible variability is much more unlikely since laboratory tests must be performed in suitable pilot test-plants to prevent significant variations in this critical value, which could negatively influence the viability of the project.

## 4. VARIABILITY IN METAL PRICES

Based on the data provided by the World Bank on the prices of three important metals, namely, copper, gold and zinc, we perform a comparative analysis of their price evolution in recent years.

Copper is a basic metal for industry and is essential to machinery and electrical wiring. Its main deposits are in South America, the USA and Southeast Asia, and it has global markets with growing consumption in China and other countries with strong industrial growth. An increase in the price of copper has traditionally been linked to increased economic growth. Just as oil is a source of economic strength, increased economic growth implies greater energy needs, which lead to greater copper consumption and a subsequent increase in its price (Jaunky, 2013).

Figure 4 shows the evolution of copper prices between January 2013 and November 2019 (World Bank, 2019a). Between January 2013 and January 2016, prices dropped significantly, with subsequent growth during the following two years (between January 2016 and January 2018), followed by another decrease, losing approximately 25% of its value since January 2013.



Figure 4. Copper (LME), grade A, minimum 99.9935% purity, cathodes and wire bar shapes, settlement prices in USD/t, between January 2013 and September 2020 (World Bank, 2020a), with half-year and annual variations

Second, we analyse gold, which represents the world's monetary standard and has multiple technical applications, in addition to its purely ornamental value. Gold deposits are scattered throughout the world and are present in world markets. The growth of financial market indices and macroeconomic indicators negatively impact its value, while the volume of gold reserves and energy prices positively impact it (Lili & Chengmei, 2013). Figure 5 shows its evolution for the same period as copper.



Figure 5. Gold (UK), 99.5% fine, London afternoon fixing, average of daily rates in \$/oz t (Troy ounce), between January 2013 and September 2020 (World Bank, 2020b), with half-year and annual variations

As shown in Figure 5, the global variability in the price of gold during the period has been lower than that of copper, dropping by 12% between January 2013 and September 2019.

Third, Zn is a basic metal for industry (in which it is used for galvanization, as pigment in paints and to form alloys with nickel or copper), and mines are present in more than 50 countries. The USA, Peru, Australia, Canada and China are the largest producers of zinc.

Figure 6 shows the evolution of zinc prices for the same period as copper and gold.



Figure 6. Zinc (LME), high grade, minimum 99.95% purity, settlement price beginning April 1990; previously special high grade, minimum 99.995%, cash prices in \$/t, between January 2013 and September 2020 (World Bank, 2020c), with half-year and annual variations
We can see that during the period analysed, the price of zinc increased by 25%; therefore, its behaviour has been radically different from that of copper and gold.

For the three metals, semi-annual variations in absolute terms of up to 47% can be seen. It is a very important change that, following the reasoning in the previous section, will absorb any change in the remaining variables when optimizing the ultimate pit limit. Below is an example for copper.

The maximum biannual price variation of copper for the period considered was +26%, as shown along the horizontal axis in Figure 7. The point on the grey dotted line (which coincides with the yellow line) vertically above this point on the horizontal axis corresponds to a value of approximately 47% on the vertical axis. Put differently, this variability in selling price would cause a 47% increase in the relative profit per ton, which would also have been achieved with a 90% reduction in processing cost. This

can be confirmed in the figure by moving vertically down from the point on the blue line, which represents the variance of processing costs, that corresponds to a value of 47% on the vertical axis.



Figure 7. Relationship between the increase in copper price and decrease in processing cost

Figure 8 shows how a relative price increase of 13.33% would be equivalent, at a fixed profit, to either a 20% reduction in processing cost or a 100% reduction in the mining cost of ore.



Figure 8. Relationships between an increase in selling price, a decrease in processing costs and a decrease in the mining cost of ore Both alternatives, i.e., lowering either the processing cost or ore mining cost, are highly unrealistic scenarios. Processing costs are pre-determined prior to the mining operation using comparative studies with similar ore deposits, detailed laboratory tests and, if the project is profitable, testing in small-scale pilot plants with multiple processing options to better understand the optimal process flow for developing for the concentration plant.

The ore mining cost is also a parameter with low variability that depends fundamentally on economies of scale in the mining operation and the degree of isolation (e.g., access to roads, presence of workshops, availability of skilled personnel and expatriation costs of experts) of the area where the operation is developed. Bozorgebrahimi et al. (2003) show that ore hauling costs vary from \$0.65/t to \$1.20/t depending on the size of the truck used, which depends on the capacity of the loading equipment that will be used, which in turn depends on the annual production target.

Therefore, selecting the size of the hauling equipment, although important, becomes secondary in the overall study of a mining project. Regardless of the exact characteristics of the final pit selected, the variability in ore selling prices will be much more important than the selection of the loading or hauling equipment.

#### 5. CALCULATION OF THE ULTIMATE PIT LIMITS

The calculation of the ultimate pit limits is based on the block model that was used to calculate the mineral resources contained in a deposit and in which the block sizes, the categories of the resources, weights and grades of each block are defined.

When calculating which mineral resources can be mined, only resources under the categories Measured and Indicated should be considered and Inferred resources should not be used, since the latter lack sufficient research to be considered. This is in accordance with international codes, such as the Australian JORC Code (2012), the European PERC Reporting Standard (2017) and the Canadian National Instrument 43-101 (2011).

Blocks are needed not only in ore areas but also in waste areas that may fall within some part of the ultimate pit limits, therefore, the block model needs to be filled with new waste blocks in all areas that could be part of the pit all the way to the surface.

Starting from known operating costs, all possible scenarios will be studied by varying the selling price in fixed increments and between minimum and maximum values that include all future price possibilities. As stated above, studying the variations in ultimate pit limits by varying the selling price percentage is also the same as studying percent variance for the remainder of the parameters, because of their linear relationships.

In metallic mining, grade concentrations tend to vary significantly in different parts of the deposit, as seen in the crosssections of the two deposits shown in Figure 9; therefore, it is critical to determine the different mining phases (pushbacks) of the deposits.



Figure 9. Grade variations in different metallic mining deposits

To calculate the different pushbacks, a very low initial selling price is used, and therefore, only the phase with sufficient grade and a low stripping ratio will be economically valuable at that initial price. This is considered phase 1, as shown in the example in Figure 10.



Figure 10. Different pushbacks, with blocks coloured according to grade, with darker colours indicating higher grades As the selling price increases, the ultimate pit limits do not vary or vary very little until a price is reached at which another of the ore areas in the deposit becomes economical, as is the case of phase 2 in Figure 10.

This process is repeated as the price increases, until the maximum price considered is reached. This maximum price represents the last phase; this phase will be economical if economic conditions are very favourable.

During the study, there will be selling prices for which there is a noticeable increase in the tonnage to be mined. These are the prices we want to identify to define the different phases of the deposit. These pushbacks are numbered 1 through 7 in the example in Figure 10.

Unless there are changes that are different than those considered here, in the ultimate pit limit analysis, it is the pushbacks, not the pits, that are calculated, since the pits do not vary if any of the other parameters change.

Examples of changes that are different than those considered include finding other metals, such as arsenic or bismuth, that penalize the selling of concentrates, or finding too much zinc, which makes copper less buoyant and increases processing costs. In the latter case, it would be necessary to determine the zones with excessive zinc and recalculate all parameters. Subsequently, the data obtained in each phase can be used to simulate the different scenarios. Put differently, changes in the calculation parameters or economic conditions do not necessarily mean the different phases will change. The ore is not going to move; what changes is how economical each phase is and what profits can be gained from each phase.

The various ultimate pit limits will be used to study the base case and analyse the outcome of the different scenarios after an increase or decrease in the selling price.

The Río Tinto mining area (Huelva, Spain), in the southwest portion of the Iberian Peninsula, is one of the eight enormous deposits of massive sulphides in the Iberian Pyrite Belt and perhaps the largest concentration of massive sulphides on land globally, with more than 400 Mt of massive sulphides and 2000 Mt of low-grade stockwork. It has been exploited from the 8th century B.C. until the present date. It is, in simple terms, a volcanogenic massive sulphide (VMS) deposit (Martin-Izard et al., 2015), in which the massive sulphides appear as lenses composed of pyrite with accessory amounts of chalcopyrite, sphalerite, galena and traces of other sulphides. The main element mined is copper, with reserves of 153E6 tons with an average grade of 0.45% Cu. The mining operation's cut-off grade is 0.20%, and the stripping ratio is 1.9.

Figure 11 shows the calculation for the optimal ultimate pit in the area called "Cerro Colorado", with an area of 2.5 x 1.1 km<sup>2</sup> and a maximum depth of 300 m. Therefore a volume of 0.825 km<sup>3</sup> is filled with 10 m x 10 m x 10 m blocks for running the optimization procedure, which yields 825,000 blocks that are easily computed by the software RecMin (Castañón Fernández, 2019).



Figure 11. Optimal ultimate pit, plan and cross-section (Cerro Colorado, Rio Tinto)

Additionally, the Touro copper mine (La Coruña, Spain), which is in its administrative phase, is in the northwestern portion of the Iberian Peninsula. It encompasses the Arinteiro, Bama, Fornás and Manoca deposits, which have been characterized as ophiolitic metamorphic VMS deposits (Badham & Williams, 1981; Williams, 1983). More recent studies suggest an origin related to syn-metamorphic metasomatism along fault zones (Castiñeiras et al., 2002). This area has been mined since the 1970s, and its Measured and Indicated resources (CIM Definition Standards, 2014) are greater than 129 Mt (Noble, 2018) with an average copper grade of 0.39%. This mine and the Río Tinto project are currently two of the largest open-pit copper mining projects in Spain.

The optimization study is established in an area of 2.5 x 2 km<sup>2</sup> to depths of up to 210 m. This involves modelling a volume of 1.31 km<sup>3</sup> using 10 m x 10 m x 10 m blocks, yielding 1.3 M blocks, which are easily managed by the RecMin optimization algorithm. Figure 12 shows the optimization results for Touro West.



Figure 12. Optimal ultimate pit in plan view and cross-section (Touro West)

## 6. PROFITABILITY STUDY

Figures 13 through 16 show four scenarios for different selling prices and the profitability of each pit, as well as the profits and cumulative tonnage of ore, which will help us when deciding the most profitable ultimate pit and the different mining phases (pushbacks). The figures show cumulative tonnage of ore, cumulative operating profit and the percentage of operating profit relative to the total operation expenditure. The last term conveys the percentage earned for every \$100 spent in operation; this is an important indicator that presents the risk in each phase and if each phase's profitability is sufficient. This operating profit must pay for costs such as investments and loans. It is not the project's profit that must cover those costs, since for that an economic assessment would need to be performed to determine cash flows, NPV, internal rate of return (IRR) and other financial indicators.



Figure 13. Price of copper: \$4,409/t and internal cut-off grade: 0.25% Cu



Figure 14. Price of copper: \$5,512/t and internal cut-off grade: 0.2% Cu





Figure 15. Price of copper: \$6,614/t and internal cut-off grade: 0.17% Cu

Figure 16. Price of copper: \$7,716/t and internal cut-off grade: 0.14% Cu

The results are used to conduct a profitability study of each phase for the base case, namely, with the operating costs and selling price that will be considered in the feasibility study. In the example, this would correspond to a selling price of \$3/lb and an internal cut-off grade of 0.17% Cu, as shown in Figure 15. In this case, phase 7 would generate a 16.4% profit, and phase 8 would produce only a 6.4% profit. To determine how many phases can be included in the feasibility study, sensitivity to changes in operating costs, recovery and selling price must be analysed. The selling price, as we've seen, must especially be analysed because it varies significantly over time and because the variance in operating costs is studied along with it due to their linear relationship.

Here, we study the effect of a 16.7% drop in selling price to \$2.5/lb (Figure 14). As shown in the figure, phase 8 would no longer be profitable since the loss would be 14.9%, and neither would phase 7 since the loss would be 6.2%. Phase 6 would be profitable, with an operating profit of 8.1%.

In this example, if the project were to be profitable at a price of \$2.5/lb, phase 6 would be considered the last mining phase, and we can say that this is a robust project since it would support a selling price drop of 16.7%.

Similarly, if an even more unfavourable case is considered, in which the selling price drops 33.3% to \$2/lb (Figure 13), only phase 4 would be reached since the subsequent phases would no longer be profitable. In the feasibility study, with a selling price

of only \$2/lb and mining only until phase 4 to stay profitable, the project can be considered very robust since it would be profitable even under worst-case conditions.

Determining what happens if the selling price increases also provides valuable information on the possible increase in profits of the project if economic and market conditions are favourable, as shown in Figure 16, which corresponds to a selling price of \$3.5/lb. In this case, phase 9 could be reached and would have an operating profit of 12.5%. However, given the low operating profit, the best option would be to reach phase 8, with an operating profit of 25.8%.

Other important conclusions can be drawn from our project. For example, up to phase 5, there is a significant increase in the volume mined, as illustrated by the cumulative tonnage of ore curve. After phase 5, the curve flattens and the tonnage increments in each phase are smaller. A similar pattern is seen in the cumulative profit.

#### 7. CONCLUSIONS

In most mining projects, and mainly in metallic mining projects, the parameter that most impacts profitability studies is the selling price of the metals or minerals, not only because this parameter cannot be influenced to prevent its fluctuation but because its percent variance can be much higher than those of other parameters.

In addition, the possible variances in the remaining parameters are very limited during the feasibility study. The waste and ore mining costs are determined very precisely and do not generally vary by more than +/-10%. Additionally, the cost of ore processing and recovery at the plant, despite being difficult to determine in some cases, should not vary by more than +/-10% if processing tests are performed in pilot plants with a sufficient amount of ore representative of the entire life of the mine.

Since there is a linear relationship between the parameters impacting the pit's profitability, it is possible to simulate all scenarios by studying only what occurs if the selling price changes, since a percentage change in the selling price is equivalent to a much smaller percentage change in any of the other parameters.

The process to calculate the different pushbacks is as follows. Based on pre-determined operating costs, all possible scenarios are studied while varying the selling price in fixed increments between minimum and maximum values that cover all possible future prices. The starting selling price will be very low, and consequently, the only phase that will be economical is the one with adequate grade and a low stripping ratio.

As the selling price increases, the ultimate pit limits do not vary or vary only slightly until the price reaches the point at which another of the ore bodies in the deposit becomes economical. This process is repeated as the price increases until the maximum price considered is reached; then, the project reaches the last phase that is economical if economic conditions are very favourable.

During the study, there will be selling prices for which there is a noticeable increase in the tonnage to be mined. These are the prices that need to be identified to define the different phases of the deposit in the analysis. Unless there are changes other than those considered in the analysis, such as the appearance of metals that penalize the value of the concentrate, the ultimate pit limits it will be the pushbacks that are calculated, since the pits will not change when one of the parameters changes. Subsequently, the data obtained in each phase can be used to simulate different scenarios, since the different phases will not

change because operating costs or economic conditions change. The ore is not going to move; what changes is the phase up to

which mining will be economical and what profits will be obtained in each phase.

#### References

- AusIMM Guidelines for Techno-Economic Evaluations. (2012). Guidelines for Technical Economic Evaluation of Minerals Industry Projects. The Australasian Institute of Mining and Metallurgy. https://ausimm.com/wp
  - content/uploads/2018/10/guidelines\_tech\_economic\_evaluation2012.pdf [Accessed 26 Oct. 2020]
- Badham, J., & Williams, P. (1981). Genetic and Exploration Models for Sulfide Ores in Metaophiolites, Northwest Spain. Economic Geology, 76(8), 2118–2127. https://doi.org/http://dx.doi.org/10.2113/gsecongeo.76.8.2118
- Bozorgebrahimi, E., Hall, R., & Blackwell, G. (2003). Sizing equipment for open pit mining a review of critical parameters. Transactions of The Institutions of Mining and Metallurgy. Section A-Mining Technology, 112(Dec), 171–179. https://doi.org/10.1179/037178403225003591
- Castañón Fernández, C. (2019). RECMIN & RECMIN PRO. https://recmin.com/WP/
- Castiñeiras, P., Gómez Barreiro, J., Martínez Catalán, J., & Arenas, R. (2002). Nueva interpretación petrológica y tectónica de las anfibolitas pobres en calcio del antiforme de Arinteiro (NO del Macizo Ibérico). I: Descripción de las anfibolitas pobres en calcio y rocas asociadas. Geogaceta, 32, 83–85. http://hdl.handle.net/10272/9361
- CIM Definition Standards. (2014). The CIM Definition Standards on Mineral Resources and Reserves. Canadian Institute of Mining, Metallurgy and Petroleum Standing Committee on Reserve Definitions. https://mrmr.cim.org/media/1128/cim-definitionstandards\_2014.pdf

Dassault Systemes. (2019). GEOVIA Whittle. https://www.3ds.com/products-services/geovia/products/whittle/

- Dyas, K. E. (2009). Project Evaluation. In: SME Mining Reference Handbook. Lowrie, R.L. (ed.). Society for Mining, Metallurgy and Exploration, Inc., Colorado, USA.
- Giannini, L. M., & Caccetta, L. (1988). An Application of Discrete Mathematics in the Design of an Open Pit Mine. Discrete Applied Mathematics, 21, 1–19.
- Githiria, J., & Musingwini, C. (2019). A stochastic cut-off grade optimization model to incorporate uncertainty for improved project value. The Journal of The Southern African Institute of Mining and Metallurgy, 119, 217–228.
- Hexagon Mining. (2019). MineSight. https://hexagonmining.com/solutions-old/by-function/planning
- Jaunky, V. C. (2013). A cointegration and causality analysis of copper consumption and economic growth in rich countries. Resources Policy, 38(4), 628–639. https://doi.org/10.1016/j.resourpol.2013.10.001
- JORC Code. (2012). Australasian Code for Reporting of Exploration Results, Mineral Resources and Ore Reserves. Joint Ore Reserves Committee of The Australasian Institute of Mining and Metallurgy, Australian Institute of Geoscientists and Minerals Council of Australia. http://www.jorc.org/docs/JORC code 2012.pdf
- Kakaie, R., & Yousefi, A. (2011). A new algorithm for optimum open pit design: Floating cone method III. Journal of Mining & Enviroment, 2(2), 118–125.
- Kennedy, B. A. (1990). Surface Mining (2nd Edition). Society for Mining, Metallurgy and Exploration (SME). Englewood, Colorado.
- Khan, A., Waqar, M., & Asad, A. (2019). A method for optimal cut-off grade policy in open pit mining operations under uncertain supply. Resources Policy, 60(December 2018), 178–184. https://doi.org/10.1016/j.resourpol.2018.12.003
- Krzemień, A., Riesgo Fernández, P., Suárez Sánchez, A., & Diego Álvarez, I. (2016). Beyond the pan-european standard for reporting of exploration results, mineral resources and reserves. Resources Policy, 49, 81–91. https://doi.org/10.1016/j.resourpol.2016.04.008
- Krzemień, A., Riesgo Fernández, P., Suárez Sánchez, A., & Sánchez Lasheras, F. (2015). Forecasting European thermal coal spot prices. Journal of Sustainable Mining, 14, 203–210. https://doi.org/10.1016/j.jsm.2016.04.002
- Lerchs, H., & Grossman, F. (1965). Optimum Design of Open-Pit Mines. Transaction CIM, 58, 47-54.
- Lesage, M., Juliani, C., & Ellefmo, S. L. (2018). Economic Block Model Development for Mining Seafloor Massive Sulfides. Minerals, 8, 468. https://doi.org/10.3390/min8100468
- Lili, L., & Chengmei, D. (2013). Research of the influence of macro-economic factors on the price of gold. Procedia Computer Science, 17, 737–743. https://doi.org/10.1016/j.procs.2013.05.095
- Martin-Izard, A., Arias, D., Arias, M., Gumiel, P., Sanderson, D. J., Castañon, C., Lavandeira, A., & Sanchez, J. (2015). A new 3D geological model and interpretation of structural evolution of the world-class Rio Tinto VMS deposit, Iberian Pyrite Belt (Spain). Ore Geology Reviews, 71, 457–476. https://doi.org/10.1016/j.oregeorev.2015.06.006
- Matyjaszek, M., Fidalgo Valverde, G., Krzemien, A., Wodarski, K., & Riesgo, P. (2020). Optimizing Predictor Variables in Artificial Neural Networks when Forecasting Raw Material Prices for Energy Production. Energies, 13(2017), 15. https://doi.org/10.3390/en13082017
- Matyjaszek, M., Riesgo Fernández, P., Krzemień, A., Wodarski, K., & Fidalgo Valverde, G. (2019). Forecasting coking coal prices by means of ARIMA models and neural networks, considering the transgenic time series theory. Resources Policy, 61(December 2018), 283– 292. https://doi.org/10.1016/j.resourpol.2019.02.017

MiningMath Software (2020). MiningMath. https://miningmath.com.

- Muir, D. C. W. (2004). Pseudoflow, New Life for Lerchs-Grossmann Pit Optimisation. Orebody Modelling and Strategic Mine Planning, Spectrum Series Volume 14., 97–104.
- National Instrument 43-101. (2011). Standards of Disclosure for Mineral Projects. Ontario Securities Commission.
- https://www.osc.gov.on.ca/documents/en/Securities-Category4/ni\_20160509\_43-101\_unofficial-consolidation.pdf
- Noble, A. (2018). Technical Report on the Mineral Resources and Reserves of the Touro Copper Project. https://ww4.fscwire.com/sites/default/files/news\_release\_pdf/atalaya05212018.pdf
- Pell, R., Tijsseling, L., Palmer, L. W., Glass, H. J., Yan, X., Wall, F., Zeng, X., & Li, J. (2019). Environmental optimisation of mine scheduling through life cycle assessment integration. Resources, Conservation & Recycling, 142, 267–276.

https://doi.org/10.1016/j.resconrec.2018.11.022

- PERC Reporting Standard. (2017). Pan-European Standard for Reporting of Exploration Results, Mineral Resources and Mineral Reserves. Pan-European Reserves and Resources Reporting Committee. http://percstandard.org/documents/PERC REPORTING STANDARD 2017.pdf
- Picard, J. (1976). Maximal Closure of a Graph and Applications to Combinatorial Problems. Management Science, 22(11), 1268–1272.
- Riesgo García, M. V., Krzemień, A., Manzanedo del Campo, M. Á., Escanciano García-Miranda, C., & Sánchez Lasheras, F. (2018). Rare earth elements price forecasting by means of transgenic time series developed with ARIMA models. Resources Policy, February. https://doi.org/10.1016/j.resourpol.2018.06.003
- Riesgo García, M. V., Krzemień, A., Sáiz Bárcena, L. C., Diego Álvarez, I., & Castañón Fernández, C. (2019). Scoping studies of rare earth mining investments: Deciding on further project developments. Resources Policy, 64(August). https://doi.org/10.1016/j.resourpol.2019.101525
- Sánchez Lasheras, F., de Cos Juez, F. J., Suárez Sánchez, A., Krzemień, A., & Riesgo Fernández, P. (2015). Forecasting the COMEX copper spot price by means of neural networks and ARIMA models. Resources Policy, 45, 37–43. https://doi.org/10.1016/j.resourpol.2015.03.004
- Sterba, J., Krzemień, A., Fidalgo Valverde, G., Diego Álvarez, I., & Castañón Fernández, C. (2020). Energy-sustainable industrialized growth in the Czech Republic: The Cínovec lithium mining project. Resources Policy, 68(May). https://doi.org/10.1016/j.resourpol.2020.101707
- Williams, P. (1983). The genesis and metamorphism of the Arinteiro-Bama Cu deposits, Santiago de Compostela, northwestern Spain. Economic Geology, 78(8), 1689–1700. https://doi.org/https://doi.org/10.2113/gsecongeo.78.8.1689
- World Bank. (2020a). Copper (LME), grade A, minimum 99.9935% purity, cathodes and wire bar shapes, settlement price in \$/t. http://pubdocs.worldbank.org/en/561011486076393416/CMO-Historical-Data-Monthly.xlsx
- World Bank. (2020b). Gold (UK), 99.5% fine, London afternoon fixing, average of daily rates in \$/oz t (Troy ounce). http://pubdocs.worldbank.org/en/561011486076393416/CMO-Historical-Data-Monthly.xlsx
- World Bank. (2020c). Zinc (LME), high grade, minimum 99.95% purity, settlement price beginning April 1990; previously special high grade, minimum 99.995%, cash prices in \$/t. http://pubdocs.worldbank.org/en/561011486076393416/CMO-Historical-Data-Monthly.xlsx

# **Figure Caption:**

Figure 1. Block models

- Figure 2. Central section of a block's cone
- Figure 3. Sensitivity analysis of the profit per ton of ore (y-axis) vs. variation in the costs of ore processing and mining (x-axis)
- Figure 4. Copper (LME), grade A, minimum 99.9935% purity, cathodes and wire bar shapes, settlement prices in USD/t, between January 2013 and September 2020 (World Bank, 2020a), with half-year and annual variations
- Figure 5. Gold (UK), 99.5% fine, London afternoon fixing, average of daily rates in \$/oz t (Troy ounce), between January 2013 and September 2020 (World Bank, 2020b), with half-year and annual variations
- Figure 6. Zinc (LME), high grade, minimum 99.95% purity, settlement price beginning April 1990; previously special high grade, minimum 99.995%, cash prices in \$/t, between January 2013 and September 2020 (World Bank, 2020c), with half-year and annual variations
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- Figure 16. Price of copper: \$7,716/t and internal cut-off grade: 0.14% Cu

# **Table Caption:**

Table 1. Optimization parameters of the cone of an ore block considering the internal cut-off grade

Table 2. Optimization parameters of the cone of an ore block considering the break-even grade