Consensus Phase Shifting Transformer Model

Md Rejwanur R. Mojumdar
Dept. Energy & Petroleum Eng.
University of Stavanger
Stavanger, Norway
md.r.mojumdar@uis.no

Jose M. Cano
Dept. Electrical Eng.
University of Oviedo
Gijon, Spain
jmcano@uniovi.es

Mohsen Assadi
Dept. Energy & Petroleum Eng.
University of Stavanger
Stavanger, Norway
mohsen.assadi@uis.no

Gonzalo A. Orcajo
Dept. Electrical Eng.
University of Oviedo
Gijon, Spain
gonzalo@uniovi.es

***Abstract*—The model of regulating transformers used in classic power system studies, such as load flow analysis or state estimation, is still debatable. A recent publication has demonstrated that the two alternative tap-changing transformer models usually found in the literature and power system simulation software packages may lead to important discrepancies, especially at extreme tap positions. This work is an extension of the aforementioned publication, written by the same authors, with the aim of reconciling those models for the case of the phase shifting transformer (PST). This work demonstrates that prevailing formulations of PST, particularly of the asymmetrical type, may also lead to important discrepancies when operating far from the nominal tap, with a different impact depending on the power factor of the power flowing through the device. Furthermore, a general model for the PST is proposed in this contribution. The new model uses the same data available in traditional formulations leading to improved results and avoiding any ambiguity.**

***Index Terms*—power transformers, phase shifters, transformer models**

I. Introduction

With the deregulation of the electricity market, control of power flows over transmission lines and tie lines has become an important concern. Moreover, uneven loading of parallel transmission lines is a recurring problem to solve during the transmission of energy. The use of phase shifting transformers (PST) is a well-established solution to provide control of real power flows through transmission lines. Several PSTs, as in the case of asymmetrical types, offer also some control over the magnitude of output voltage, thus providing regulation of the reactive power flows up to a certain limit [1].

The quality of the results of classic power system studies such as power flow and optimal power flow analysis, state estimation, etc. are highly dependent on the accuracy of the models used to describe system components. However, simplified models are typically used in these algorithms, due to the complexity and magnitude of the problems and also because of the scarce information generally available for the engineering practitioner in charge of these tasks. The nameplate of the transformer is usually the exclusive data source used in PST modeling.

In literature and practical implementations, two alternative voltage-magnitude regulating transformer models can be found [2]–[5]. The differences in these models arise from the fact that they consider the short-circuit impedance either provided exclusively by the nominal or off-nominal turns side winding of the device. These extreme assumptions cause that both models yield different results and thus, the users can be misled trying to validate their results with different tools. This fact was originally observed in [6], but the authors chose one of those models and focused their efforts on the manipulations needed on the other model to reach the same results. However, in [7] those discrepancies were fully explained and a reconciled solution was proposed. Furthermore, [7] demonstrates that even if the differences between the two models are trivial at the principal tap, significant mismatches take place at distant tap positions. Similarly, this lack of consistency also exists in the representation of PST through the two traditional models typically used in power system studies [8]–[10]. The aim of this contribution is to addressed this problem precisely by extending the applicability of [7] to PSTs.

Now, in this work, the authors' objective is to demonstrate that the two available models also yield different results for asymmetrical PSTs, which is misleading. Though these discrepancies from two models can be considered trivial at the principal tap, the inconsistency can lead to huge differences at distant tappings.

In this work, a new model of the PST is proposed in Section II in order to explain the causes of the discrepancies between the two existing models and with the aim of reaching a reconciled solution free of any ambiguity. Furthermore, the new model opens the door to a more accurate description of the device. Section III describes the traditional models and demonstrates that they are degenerate cases of the new proposal. Section IV presents a theoretical assessment of the discrepancies caused by traditional models and relates them with the solution offered by the new model. A case study is presented in Section V in order to highlight the importance of the new proposal. Finally, the conclusions of this study are drawn in section VI.

II. Description of the New Model

Neglecting the shunt admittances of the detailed model of PSTs (i.e. those responsible for the magnetizing current and core losses) is a common practice in power system studies. This fact, simplifies the analysis, as the internal bus of the detailed model is removed from the problem. If the

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PST operates at nominal turns ratios, no further assumption is needed, as the specific contribution of each transformer winding to the short-circuit impedance is irrelevant in that case. However, as is demonstrated in the following, this is far from being true when the PST works at an off-nominal tap position. Let us consider a PST with off-nominal turns ratio $|a|e^{j\theta}$ as depicted in Fig. 1.

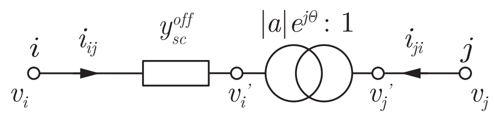

Fig. 1. Model of the phase shifting transformer with short-circuit impedance at the off-nominal turns side

The fundamental equations of a PST can be formulated as

$$\frac{v_i^{'}}{v_j^{'}} = a = |a|e^{j\theta}, \tag{1}$$

$$\frac{i_{ij}}{i_{ji}} = -\frac{1}{a^*} = -\frac{1}{|a|e^{-j\theta}}. \tag{2}$$

Then, though these parameters are not normally known by the user, let k be the ratio between the per p.u. impedance in the nominal winding, z_2 and tapped winding, z_1 (for the sake of simplicity, the same ratio is considered for resistance and leakage reactance). So, from (1) and (2), z_2 can be referred to the off-nominal turns side as

$$z_2^{off} = \frac{v_i^{'}}{v_j^{'}}\frac{-i_{ji}}{i_{ij}}z_2 = aa^* z_2 = |a|^2 z_2. \tag{3}$$

Therefore, considering the new ratio k together with (3), the series transformer admittance, as seen from the off-nominal side, can be calculated as

$$y_{sc}^{off} = \frac{1}{z_1 + |a|^2 z_2} = \frac{1}{z_1\left(1 + |a|^2 k\right)}. \tag{4}$$

Typically, the data provided to the engineering practitioner in order to model the PST is the short-circuit impedance of the transformer, z_{sc}, which is also available at the nameplate of the device. This data is obtained from the short-circuit test, which is conducted, at least, at the nominal tap (i.e. $|a| = 1$). Thus, the rated short-circuit admittance of the PST, y_{sc}, can be expressed as

$$y_{sc} = \frac{1}{z_1 + z_2}. \tag{5}$$

From (5) and the definition of k, the contribution of the winding at the off-nominal side to the short-circuit impedance, z_1, can be calculated as

$$z_1 = \frac{1}{y_{sc}\left(1 + k\right)}, \tag{6}$$

Using this value in (4) yields

$$y_{sc}^{off} = \frac{1 + k}{1 + k|a|^2} y_{sc}. \tag{7}$$

Considering KVL, the nodal equations of the PST can now be written as

$$\begin{bmatrix} i_{ij} \\ i_{ji} \end{bmatrix} = \begin{bmatrix} Y_{ii} & Y_{ij} \\ Y_{ji} & Y_{jj} \end{bmatrix} \begin{bmatrix} v_i \\ v_j \end{bmatrix}, \tag{8}$$

where

$$Y_{ii} = y_{sc}^{off} = \frac{1 + k}{1 + k|a|^2} y_{sc}, \tag{9}$$

$$Y_{ij} = -a y_{sc}^{off} = -\frac{a\left(1 + k\right)}{1 + k|a|^2} y_{sc}, \tag{10}$$

$$Y_{ji} = -a^* y_{sc}^{off} = -\frac{a^*\left(1 + k\right)}{1 + k|a|^2} y_{sc}, \tag{11}$$

$$Y_{jj} = |a|^2 y_{sc}^{off} = \frac{|a|^2\left(1 + k\right)}{1 + k|a|^2} y_{sc}. \tag{12}$$

It is important to note that the Y-bus matrix of the nodal equations for PST is not symmetrical as $Y_{ij} \neq Y_{ji}$. Therefore forming a π-equivalent model for PST is not straightforward; rather the model will have two different branch admittances depending on the current under consideration (i_{ij} or i_{ji}). Keeping this fact in mind, the parameters of a pseudo π-equivalent model for the PST, which has been depicted in Fig. 2, can be derived from (9)–(12), as

$$y_{ij} = -Y_{ij} = \frac{a\left(1 + k\right)}{1 + k|a|^2} y_{sc}, \tag{13}$$

$$y_{ji} = -Y_{ji} = \frac{a^*\left(1 + k\right)}{1 + k|a|^2} y_{sc}, \tag{14}$$

$$y_{si} = Y_{ii} + Y_{ij} = \frac{\left(1 - a\right)\left(1 + k\right)}{1 + k|a|^2} y_{sc}, \tag{15}$$

$$y_{sj} = Y_{jj} + Y_{ji} = \frac{\left(|a|^2 - a^*\right)\left(1 + k\right)}{1 + k|a|^2} y_{sc}. \tag{16}$$

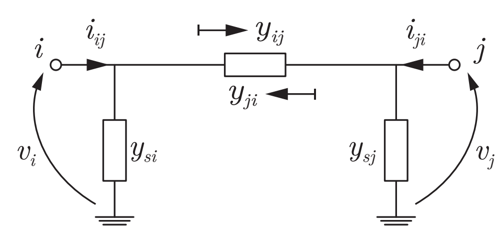

Fig. 2. A pseudo π-equivalent model of PST

III. Reconciliation of Previous Models

It can be easily derived that the two PST models most extensively used in the literature and practical implementations, correspond to the particular cases of making the parameter k equal to 0 and ∞ in (7), and thus in (9)–(16). The version with $k = 0$ corresponds to the assumption that all the short-circuit impedance of the PST is provided by the winding at the off-nominal turns side. In this case, the off-nominal admittance of the PST, y_{sc}^{off}, turns to be the same as the rated short-circuit admittance, y_{sc}, and the parameters of the pseudo π-equivalent circuit shown in Fig. 3(a) are obtained. On the other hand, considering $k = \infty$, corresponds to the assumption that all the short-circuit impedance of the PST is provided by the winding at the nominal turns side. Thus, y_{sc}^{off} turns to be $y_{sc}/|a|^2$, and the set of parameters of the pseudo π-equivalent circuit shown in Fig. 3(b) is reached.

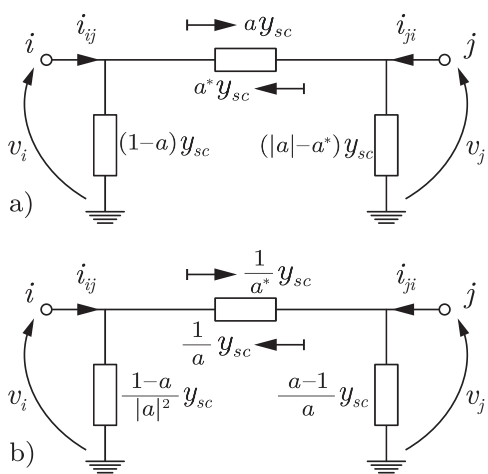

Fig. 3. π-equivalent traditional models of the phase shifting transformer. (a) $k = 0$, and (b) $k = \infty$

It is interesting to note that the lack of consistency between the results offered by the two traditional alternatives of the PST models was initially identified in [6]. However, this publication focused on the adjustments needed to make them yield the same results and, thus, appropriately pointed out that the model in Fig. 3(a) turns to be the same as the one in Fig 3(b) if all the admittances are divided by $|a|^2$. The problem of taking this approach lies on the fact that all the short-circuit impedance of the device is still being assigned to one specific side of the transformer which, as is demonstrated in this contribution, can lead to important errors. Conversely, the new model is capable of describing the cause of the discrepancies and allows for a description of the device free of any ambiguity, provided that the specific value of k used in the analysis is reported.

It is important to highlight that the new PST model presented here, opens the door to obtain accurate results if k is known (e.g. being provided by the manufacturer or estimated from off-line field measurements). But, even if this is not the case, much more realistic estimates can be obtained if k is set to 1, which stands for an equal contribution of each winding of the transformer to the short-circuit impedance. In fact, this is a common engineering practice, typically used when a detailed model of the transformer is to be used [10], [11]. While the benefits of using $k = 1$ in the case of the tap-changing transformer model was previously discussed in [7], this work analyzes the advantage of making the same assumption for the case of PSTs.

IV. Error Assessment

By using the new PST model proposed in Section II, the voltage at the nominal turns side of the PST, v_j, can be determined if the variables at the off-nominal turns side, v_i and i_{ij}, are provided. Indeed, using the nodal equation of the PST displayed in (8), v_j can be calculated as

$$v_j = \frac{i_{ij} - Y_{ii} v_i}{Y_{ij}}. \tag{17}$$

Let us designate the voltage at the nominal turns side for a generic value of k as v_j^k. Thus, the discrepancies between the traditional models and the consensus model, can be assessed, by just considering the values obtained for $k = 0$ and $k = \infty$, i.e. v_j^0 and v_j^∞. Indeed, using (17) according to the values of Y_{ii} and Y_{ij} in (9) and (10), yields,

$$\Delta v_j^0 = v_j^0 - v_j^k = \frac{k\left(|a|^2 - 1\right) i_{ij}}{a\,(1 + k)\, y_{sc}}, \tag{18}$$

$$\Delta v_j^\infty = v_j^\infty - v_j^k = \frac{\left(1 - |a|^2\right) i_{ij}}{a\,(1 + k)\, y_{sc}}. \tag{19}$$

As it is immediately derived from (18) and (19), the discrepancies grow with the load level of the transformer as well as with the value of the rated short-circuit impedance. Moreover, those equations imply that the mismatch does not take place when $|a|{=}1$, which is the case of symmetrical PSTs (i.e. those causing a pure phase-angle shift but with no effect on voltage magnitudes). In this specific case, the proposed model cannot contribute to provide better results. In fact, both the traditional and new models turn to be the same under that particular circumstances.

However, many asymmetrical PSTs are also used in power system applications. The proposed model can highly contribute to their modeling and simulation. According to [12], there exist three types of asymmetrical PSTs. For the widely-used quadrature booster, shown in Fig. 4.(a), the regulating winding is connected at ± 90 deg., whereas for other asymmetric PSTs, as the one shown in Fig. 4.(b), the regulating winding can be connected at different angles, $0{<}\delta{<}180$. For the asymmetrical PST with in-phase transformer, as in Fig. 4.(c), voltages on both primary and secondary side can be boosted with a common ratio r while the regulating winding remains connected with same angle, δ. As $|a|{\neq}1$ in those cases, traditional models cause inconsistent results and the new model may effectively solve this problem.

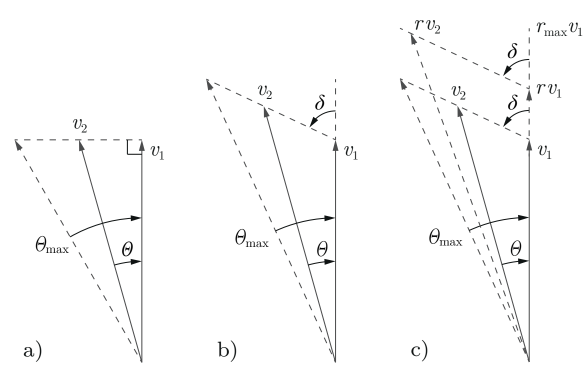

Fig. 4. Phasor diagram of PSTs. (a) Quadrature booster (b) asymmetrical PST, (c) in-phase transformer and asymmetrical PST

Notice that the new model is also useful to account for voltage magnitude tap-changing transformers provided that their vector group causes a non-zero phase-shift. In those cases, the discrepancies can be exacerbated by extreme tap positions, due to the high differences between $|a|$ and 1 that can be found in this type of devices.

V. Case Study

A case study is presented in this section in order to point out the inconsistencies implied by the use of the traditional PST models shown in Fig. 3. Furthermore, this case study demonstrates that the new PST model, proposed in this contribution and depicted in Fig. 2, can solve this problem assuring certainty in reporting results.

From Fig. 4, it can be easily seen that, for any of these asymmetric PSTs, there are general relations between the tap position, n, neutral tap position, n_0, phase shift, θ, magnitude of the off-nominal p.u. turns ratio, $|a|$, regulating winding connection angle, δ, and p.u. voltage step increment per tap change of the regulating winding, du. The general relations, including the effect of r, are well documented in [12], [13]. For the particular case of the asymmetrical PST, which is considered in the present case study, those relations, according to Fig. 4.(b), can be expressed as

$$\theta = -\arctan\left(\frac{(n-n_0)\,du\;sin\delta}{1+(n-n_0)\,du\;cos\delta}\right), \tag{20}$$

$$|a| = \frac{1}{\sqrt{\left((n-n_0)\,du\;sin\delta\right)^2 + \left(1+(n-n_0)\,du\;cos\delta\right)^2}}. \tag{21}$$

Even if the manufacturer can provide different short-circuit impedance values for different tap positions, this fact is omitted in the following, not to obscure the core of the proposal.

Let us consider an 80 MVA, 50 Hz, 220/132 kV, asymmetric PST, with a nameplate short-circuit impedance, z_{sc}, of $0.01 + 0.12j$ and a maximum no-load phase shift, θ_{max}, of -4.715 deg. The regulating winding connection angle, δ, is at 60 deg. and the tap changer, located on the higher voltage side, has 11 positions (from the neutral tap, n_0=0, to n=10) and a voltage step increment per tap change, du, of 1%.

A. Analysis of the nominal turns side voltage deviations for different tap positions and power factors

The effect of the tap position in the deviations caused by traditional models is studied in this case. The voltage at the off-nominal turns side of the PST, v_i is fixed at 1 p.u. as well as the current at the same side, i_{ij}, which is also forced to supply the rated current of the transformer. Two different power factors are used in this analysis: (a) a unity power factor, φ=0 deg., i.e. i_{ij} is in-phase with v_i, and (b) a pure capacitive case, φ=90 deg. in which i_{ij} leads v_i in this amount. Thus, the voltage at the nominal turns side of the transformer, v_j, can be calculated using the different models. Fig. 5 shows the results for the traditional versions, i.e. k=0 and k=∞, together with those obtained using the new model and assuming an equal contribution of both windings to the short-circuit impedance, i.e. k=1. Although the setting of k in this way is not necessarily exact, it is according to well-accepted engineering practices, and is a more sensible estimation than the one derived from the extreme assumptions made in the traditional models.

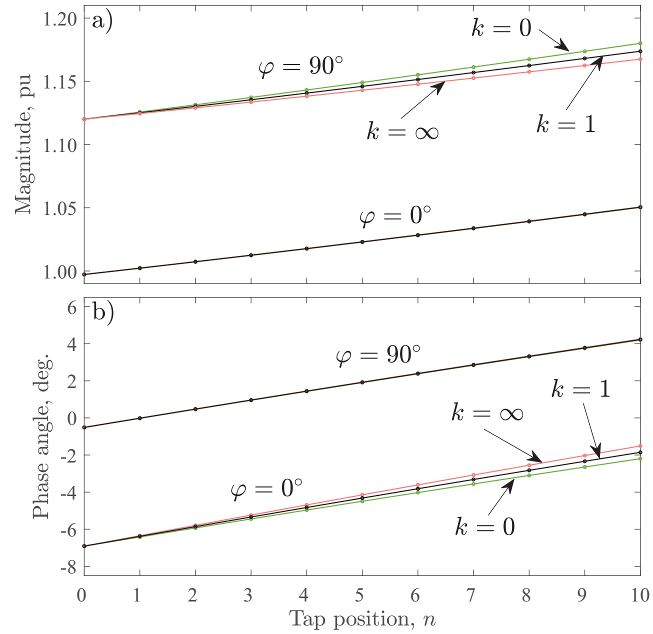

Fig. 5. Nominal-turns side voltage at different tap positions for the different PST models. The PST is operated at rated values at the off-nominal turns side at two different power factors: unity (φ = 0 deg.) and pure capacitive (φ = 90 deg.). (a) Voltage magnitude, and (b) voltage phase angle

The discrepancies in the magnitude of voltage at the nominal turns side of the transformer can be observed in Fig. 5.a. While they can be practically neglected at high power factors, the inconsistency is exacerbated in the capacitive case. Furthermore, and in agreement with (18) and (19), the mismatch grows when moving to distant positions from the neutral tap. In the same vein, the phase angle of the voltage at the nominal turns side (the voltage at the off-nominal side is taking as a reference) is depicted in Fig. 5.b. Unlike in the previous case, the discrepancies appear now magnified at high power factors and tend to be negligible with pure capacitive loads. As it is concluded from Fig. 5, the model proposed in this contribution offers a consensus estimate even if k is not accurately known and, more importantly, it removes any ambiguity from the results if the value of k used in the analysis is provided.

B. Maximum deviations in the calculation of the nominal-turns side voltage

In order to obtain the maximum deviations taking place in using the different models of the PST under study, the equivalent circuits of the traditional (k=0 and k=∞) and new model (setup with k=1) were used to calculate the nominal turns side voltage at every tap position, n, and with every possible power factor (i.e. letting φ vary in the full range, which includes reverse power flow) while operating the transformer at rated values on the off-nominal side. The results obtained with the new model, v_j^1, were taken as a reference. Thus, Fig. 6.(a) represents the differences in voltage magnitude between the traditional models and the present proposal, i.e. $|v_j^0| - |v_j^1|$ and $|v_j^\infty| - |v_j^1|$. The maximum difference reaches a value of 0.63% which is, in fact, a significant discrepancy. Notice that the mismatch between the traditional models doubles the previous result, being as high as 1.26%. The same differences are depicted in Fig. 6.(b) for the case of the phase angle of the nominal turns side voltage. The mismatch reaches in this case 0.69 deg. between the traditional models, being reduced to 0.35 deg. when compared with the new model. Noticeably, these inconsistencies in the calculation of voltage phase angle can have a deep impact in the regulation of power flows by means of PST in real grids. The same results can be directly obtained from (18) and (19).

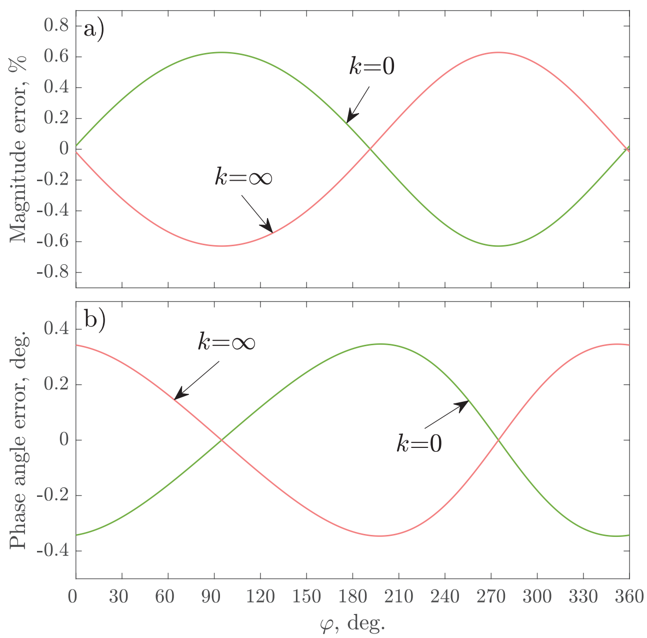

Fig. 6. Maximum deviation in the calculation of the nominal-turns side voltage at rated current. (a) Voltage magnitude, (b) voltage phase angle

VI. Conclusion

The use of simplified single-phase models of PSTs is standard practice in the execution of steady-state balanced power system studies. Although the specific contribution of each of the transformer windings to the short-circuit impedance can be completely neglected in untapped devices or when the operation takes place at the nominal tap, the same does not hold true at different tap positions. Traditional models of voltage-magnitude regulating transformers and PSTs are based on the assumption that all the short-circuit impedance is fully provided either by the winding at the nominal or off-nominal side, leading to two alternative models that yield different results. This may have strong implications, not only in the accuracy but also on the consistency of the outcomes from different tools. Although this problem does not appear in symmetrical PSTs, it can be a serious issue in asymmetrical PSTs or in voltage-magnitude regulating transformers with a non-zero vector group. Indeed, this work demonstrates that, in those cases, the mismatch of the results from those traditional models may be relevant, especially at extreme tap positions. These discrepancies appear both in voltage phase and voltage magnitude, depending on the power factor of the power flow handled by the device. Furthermore, this contribution proposes a consensus model of the PST which fully explains the aforementioned differences. The new model includes a new parameter that takes into account the contribution of each transformer winding to the short-circuit impedance. The use of this model gets rid of any ambiguity provided that the value of this parameter is reported within the data of the study. Moreover, the new model can boost the accuracy of the results if good estimates for the new parameter are available. Even if this is not the case, a sensible setup, as the one derived from the assumption of an equal contribution of each transformer winding to the short-circuit impedance, provides a more reliable outcome than those obtained from the extreme assumptions of the traditional models. Thus, the inclusion of the proposed PST model in power system software packages has the potential to significantly improve the consistency of power system studies with embedded PSTs.

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