On the Consistency of Tap-Changing Transformer Models in Power System Studies

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***Abstract*—The tap-changing transformer models used in steady-state power system studies have been recognized as somehow controversial for a long period. Indeed, discrepant versions arise depending on slightly different underlying assumptions. As a consequence, two alternative models are traditionally implemented in power system simulation software packages. A new model, recently proposed by the authors, has reconciled those versions, leading the way in removing the ensuing ambiguity. In this work, several case studies are introduced in order to highlight the important inconsistencies which can be drawn from the use of the traditional versions. Furthermore, this contribution presents a framework for the correct configuration of the new model in the usual scenario of imperfect information on transformer construction data. This work demonstrates that the adoption of the new model solves the aforementioned ambiguity, thus being a valuable tool to provide consistent results in power system studies on grids with embedded tap-changing transformers.**

***Index Terms*—power transformers, tap changers, transformer models**

I. Introduction

Tap-changing transformers are a key asset in the regulation of voltage in power systems. Thus, models of these devices are intensively used in the different fields of electric energy systems analysis and operation. Nonetheless, the models of the tap-changing transformer traditionally used in steady-state balanced studies, such as the ones conducted during power flow calculations or voltage stability analyses, have been burdened with a long-standing controversy [1]. Indeed, two alternative tap-changing transformer models can be found in the description of these devices in different books and simulation software packages [2]–[5]. Under specific operating conditions, using one model or the other can lead to results with significant differences, which produces a serious lack of consistency in reporting the outcome of the analysis of electric grids with embedded tap-changing transformers.

In [6], the authors of the present work proposed a consensus model with the aim of solving the aforementioned controversy. In this recent publication, the theoretical background that explains the differences caused by traditional models is presented. Moreover, by introducing an additional parameter, the new model allows to produce consistent results free of any ambiguity. The present contribution tries to highlight the importance of adopting the new model by state-of-the-art software packages. With this aim, the discrepancies between traditional models are theoretically assessed and the benefits of the consensus model are clearly displayed. Furthermore, a couple of case studies, based on a classical IEEE test bus system are introduced, in order to demonstrate that the differences in the outcomes offered by the traditional models cannot be neglected even in normal operating conditions.

The structure of the contribution is as follows. In Section II, the classical and newly proposed tap-changing transformer models are briefly described for the benefit of the reader. A relevant discussion on the set-up of the new model by using the limited data typically available for this type of transformers is provided in Section III, together with an assessment of the discrepancies between the different models. Section IV presents a classical power flow analysis and a voltage stability study conducted in the IEEE 57-bus system [7]. Both cases clearly highlight the importance of using the new model to provide consistency in reporting the results of power system studies.

II. Tap-Changing Transformer Models

A. Traditional Models

The most widely used traditional tap-changing transformer models are derived from two different (and not easy to justify) alternatives: either considering that all the short-circuit impedance of the transformer, z_{sc}, is provided by the winding at the off-nominal side (Type 1) or by the one at the nominal side (Type 2). Both extreme assumptions are shown in Fig. 1 for a transformer with an off-nominal turns ratio $a : 1$, where y_{sc} is the short-circuit admittance of the transformer (typically provided by the manufacturer as an impedance and shown in the nameplate of the device). In this figure, the short-circuit admittance has been referred in both cases to the off-nominal side, and thus designated as y_{sc}^{off}.

The well-known relations that apply to ideal transformers, together with Kirchhoff's Laws yield

$$v_i = \frac{i_{ij}}{y_{sc}^{off}} + a v_j, \tag{1}$$

$$i_{ij} = -\frac{i_{ji}}{a}, \tag{2}$$

This work was supported by the Spanish Government Innovation Development and Research Office (MEC) under research Grant DPI2017-89186-R.

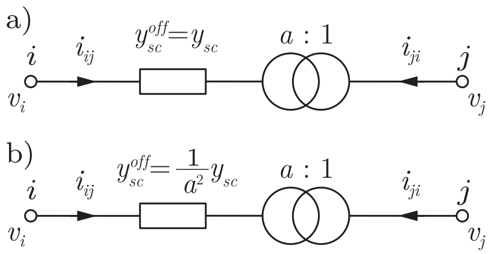

Fig. 1. Alternative assumptions made in traditional tap-changing transformer models. (a) Type 1, and (b) Type 2

and thus, the nodal equations of the device can be written in a compact form as

$$\begin{bmatrix} i_{ij} \\ i_{ji} \end{bmatrix} = \begin{bmatrix} Y_{ii} & Y_{ij} \\ Y_{ji} & Y_{jj} \end{bmatrix} \begin{bmatrix} v_i \\ v_j \end{bmatrix}. \tag{3}$$

The elements of the bus-admittance matrix, Y_{bus}, are shown in Table I for the Type 1 and Type 2 models, according to the value of y_{sc}^{off} used in each case. From those values, the parameters of the π-equivalent circuit of both transformer models can be straightforwardly derived. They have been explicitly shown in Fig. 2.

TABLE I
Y_{bus} MATRIX FOR THE DIFFERENT TAP-CHANGING TRANSFORMER MODELS

$\boldsymbol{Y_{bus}}$	**Type 1**	**Type 2**	**Type 3**	**Type 3** ($k=1$)
Y_{ii}	y_{sc}	$\frac{1}{a^2}y_{sc}$	$\frac{1+k}{1+ka^2}y_{sc}$	$\frac{2}{1+a^2}y_{sc}$
$Y_{ij}=Y_{ji}$	$-ay_{sc}$	$-\frac{1}{a}y_{sc}$	$-\frac{a(1+k)}{1+ka^2}y_{sc}$	$-\frac{2a}{1+a^2}y_{sc}$
Y_{jj}	a^2y_{sc}	y_{sc}	$\frac{a^2(1+k)}{1+ka^2}y_{sc}$	$\frac{2a^2}{1+a^2}y_{sc}$

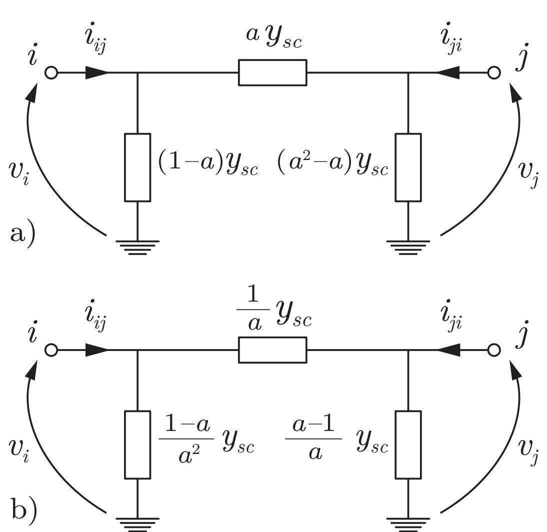

Fig. 2. π-equivalent circuit of traditional tap-changing transformer models. (a) Type 1, and (b) Type 2

B. Consensus Model

A new model, presented in [6], was proposed in order to remove any ambiguity from the results of power system studies with embedded tap-changing transformers. This model, designated in the following as Type 3, includes a new parameter in order to account for the contribution of each of the transformer windings to the short-circuit impedance. This parameter, k, is defined as the ratio between the p.u. impedance of the winding at the nominal turns side, z_j and the p.u. impedance of the tapped winding (i.e. the one at the off-nominal turns side), z_i. Thus, the short-circuit admittance referred to the off-nominal side can be calculated as

$$y_{sc}^{off} = \frac{1}{z_i + a^2 z_j} = \frac{1+k}{1+ka^2} y_{sc}. \tag{4}$$

By applying (1) and (2) to the new value of y_{sc}^{off}, the parameters of the Y_{bus} matrix in (3) can be immediately determined for the Type 3 model. They have been shown in Table I. From those values, the different admittances of the corresponding π-equivalent circuit, as depicted in Fig. 3(a), can be directly obtained as

$$y_{ij} = -Y_{ij} = \frac{a\,(1+k)}{1+a^2k} y_{sc}, \tag{5}$$

$$y_{si} = Y_{ii} + Y_{ij} = \frac{1-a+k\,(1-a)}{1+a^2k} y_{sc}, \tag{6}$$

$$y_{sj} = Y_{jj} + Y_{ij} = \frac{a\,(a-1)\,(1+k)}{1+a^2k} y_{sc}. \tag{7}$$

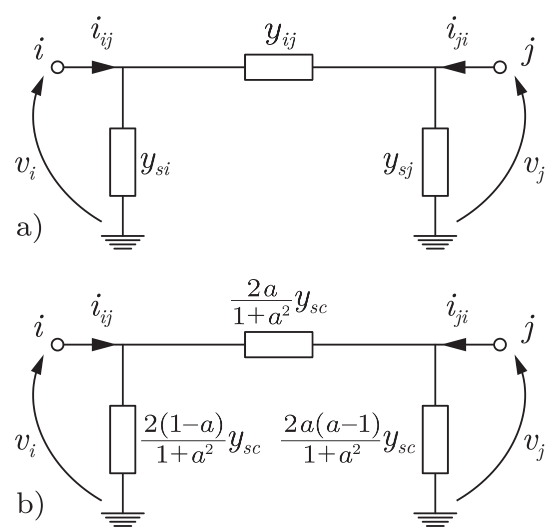

Fig. 3. Consensus model of the tap-changing transformer. (a) General model, and (b) recommended set-up

It is important to notice that the traditional models, Type 1 and Type 2, are just particular cases of the proposed general model, Type 3. Indeed, assigning the values 0 and ∞ to parameter k in (5)–(7) yields to the well-known values already presented in Fig. 2.

III. Set-up of the Consensus Model

The practical use of the new model should take into account that, usually, the data provided to the engineering practitioner in order to model the different components of the power system, and specifically, tap-changing transformers, is quite limited. The particular contribution of each transformer winding to the short-circuit impedance is not a nameplate value and, since this specification is not required by standards [8], it is seldom provided by the manufacturer. Thus, setting up the value of the new parameter, k, turns to be challenging.

As stated in [6], the discrepancy of traditional models with respect to the consensus model, assessed using the differences that arise in the voltage of the nominal winding, v_j, when the off-nominal turns side is fed at a fixed voltage, v_i, and a fixed current, i_{ij}, can be obtained, as a function of k, according to

$$\Delta v_j^0 = v_j^0 - v_j^k = \frac{k\left(a^2-1\right)}{a\left(1+k\right)}\frac{i_{ij}}{y_{sc}}, \tag{8}$$

$$\Delta v_j^\infty = v_j^\infty - v_j^k = \frac{1-a^2}{a\left(1+k\right)}\frac{i_{ij}}{y_{sc}}, \tag{9}$$

where v_j^0, v_j^∞ and v_j^k stand for the values obtained with Type 1, Type 2 and Type 3 models, respectively.

From (8) and (9), it can be concluded that the discrepancies in v_j between the different models grow with the loading of the transformer as well as with the tap position (extreme tap positions, i.e. those further from the central tap, exacerbate the differences). Due to the mainly inductive behavior of transformer short-circuit impedances, the effect of those discrepancies mostly affects the magnitude of voltage when feeding reactive loads (e.g., in that case, Δv_j^0 is close to aligned with v_j^0 and v_j^k). Conversely, resistive loads tend to magnify the differences in voltage phase angle. A detailed analysis of these facts can be found in [6].

Another interesting conclusion that can be drawn from (8) and (9) is that the particular case $k = 1$ is the midpoint between the extreme assumptions implied by traditional models. Certainly, for $k = 1$, it can be followed that $\Delta v_j^0 = -\Delta v_j^\infty$. Thus, using $k = 1$ guarantees the minimization of the maximum error caused by the lack of precise knowledge of the contribution of each transformer winding to the short-circuit impedance. The Y_{bus} elements of this recommended set-up are shown in Table I and the corresponding π-equivalent circuit is depicted in Fig. 3(b). Specifically, using $k = 1$ assures that the error is limited to

$$\Delta v_j^{max} = \pm\frac{a^2-1}{2a}\frac{i_{ij}}{y_{sc}}. \tag{10}$$

The use of $k = 1$ implies the assumption of an equal contribution of both transformer windings (expressed in per unit values) to the short-circuit impedance. In fact, this is a traditional engineering practice used in detailed transformer modeling [9]–[11], which reinforces the recommendation to use this set-up when no further data is available.

Undoubtedly, the use of parameter estimation techniques may allow to improve the quality of the set-up of tap-changing transformer models in real scenarios by obtaining accurate values of k for each specific device from the off-line analysis of field measurements. The authors are currently working in this field. However, it should be highlighted that, regardless of the accuracy of the model, which is dependent on the quality of the estimation of k, the consensus model puts an end to the lack of consistency in communicating the results of power system studies, provided that the set-up of this parameter is included in the data set.

IV. Case Studies

The discrepancies in the results provided by the traditional tap-changing transformer models are not trivial. A set of case studies are provided in this section in order to highlight this fact, and thus, to urge the adoption of the new model by power system software packages. As it is demonstrated in this section, the use of the new model can guarantee the consistency of the results obtained in such common power system studies as power flow or voltage stability analysis.

The IEEE 57-bus system, which is shown in Fig. 4, has been adopted as a test case [7]. It represents a simple approximation of the American Electric power system in the U.S. Midwest as it was in the early 1960s which has been extensively used as a test system by the power community. The IEEE 57-bus system comprises 57 buses, 7 generators, 42 loads and 17 transformers. It is important to note that 15 of these transformers are set out of the principal tap at the operating point defined by the test case. This fact makes the system especially suitable to test the new tap-changing transformer model. Table II shows the parameters and set-up of those transformers as described in the IEEE 57-bus system data files.

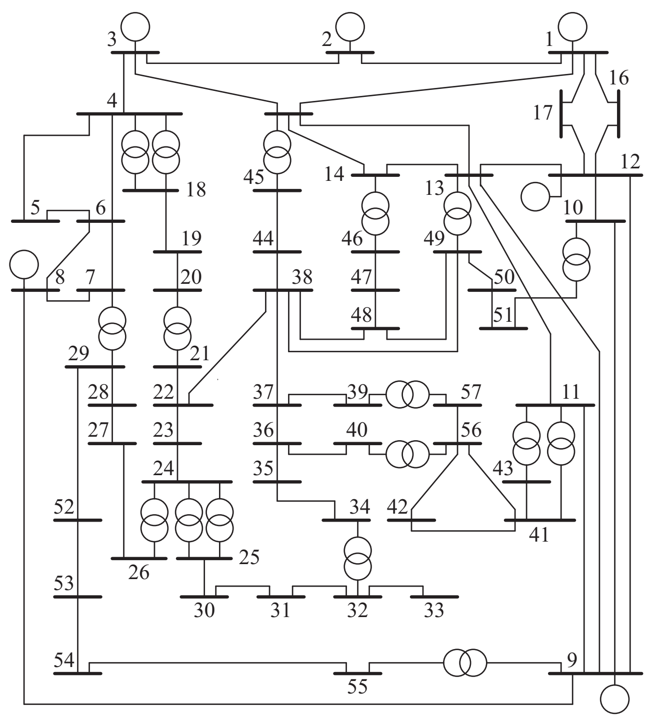

Fig. 4. IEEE 57-bus system.

TABLE II
TRANSFORMERS SET OUT OF THE PRINCIPAL TAP IN THE IEEE 57-BUS SYSTEM

From bus	To bus	R, p.u	X, p.u	Tap, a
4	18	0	0.5550	0.970
4	18	0	0.4300	0.978
21	20	0	0.7767	1.043
24	26	0	0.0473	1.043
7	29	0	0.0648	0.967
34	32	0	0.9530	0.975
11	41	0	0.7490	0.955
11	45	0	0.1042	0.955
14	46	0	0.0735	0.900
10	51	0	0.0712	0.930
13	49	0	0.1910	0.895
11	43	0	0.1530	0.958
40	56	0	1.1950	0.958
39	57	0	1.3550	0.980
9	55	0	0.1205	0.940

A. Power Flow Analysis

The state variables of the IEEE 57-bus system have been calculated by using a Newton-based power flow method for the different tap-changing transformer models under evaluation. MATPOWER [12] was used to conduct this implementation. It is important to note that this open-source electric power system simulation tool assumes one of the traditional tap-changing transformer models commented in the paper. Specifically, MATPOWER considers all the short-circuit impedance of the tap-changing transformer as being provided by the winding at the nominal turns side (i.e. $k = \infty$) [13]. The modification of those functions of the software devoted to the construction of the bus admittance matrix, has allowed the authors to test also the other alternatives (i.e, the traditional model which considers all the short-circuit impedance as provided by the off-nominal turns side, $k = 0$, and the new proposal, in which a balanced contribution is considered, $k = 1$). Those alternative transformer models can be directly implemented in MATPOWER by considering equations (5)–(7). Fig. 5 shows the resulting voltage profile of the IEEE 57-bus system according to the different tap-changing transformer models. The detailed results, for those buses showing the highest inconsistencies, are reported in Table III.

As it is shown in Table III and is also highlighted in Fig. 5, the maximum discrepancy in the calculation of voltage magnitudes between the two traditional models (i.e. $k = \infty$ and $k = 0$) takes place at bus 49. This discrepancy reaches a value of 7.63×10^{-3} p.u. (i.e. 0.763%), which can be considered a significant amount even though the transformers of the IEEE 57-bus system are not set at particularly extreme tap positions. In the same vein, the maximum discrepancy in the calculation of voltage phase angle arises at bus 33 with a value of 0.529 deg. Obviously, those differences in the state variables spread to the post-calculation of other magnitudes with an important impact on active and reactive power flows, currents, etc.

This test clearly reinforces the conclusion of the present contribution. Indeed, it confirms that the use of different tap-changing transformer models, such as the widely adopted

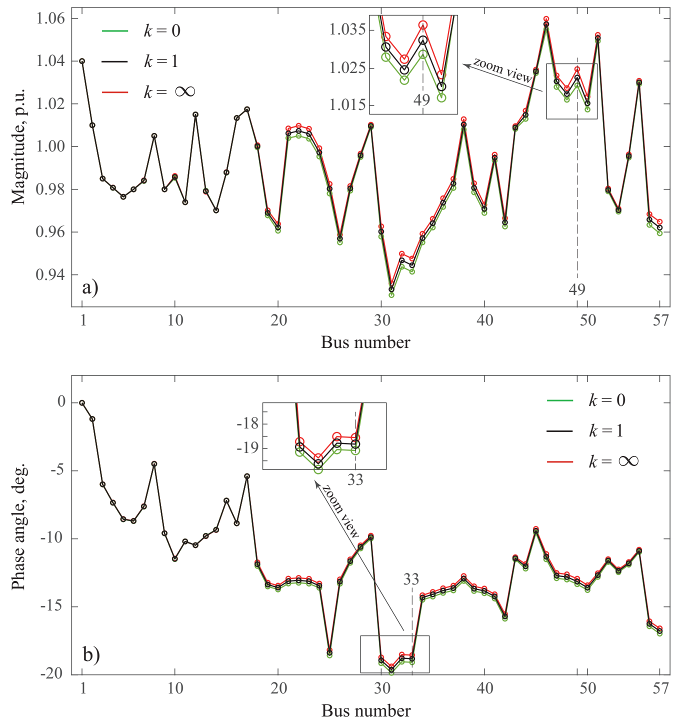

Fig. 5. Voltage profile of the IEEE 57-bus system using the traditional and new tap-changing transformer models. (a) Voltage magnitude and (b) Phase angle.

$k = 0$ (Type 1) and $k = \infty$ (Type 2) versions, leads to different and thus, inconsistent results. Conversely, the adoption of the new model by software packages for power system analysis can solve the problem, just by allowing the user to fix and report the specific value of k utilized in the study. If further information is not available to precisely determine this parameter, selecting it as $k = 1$ provides a sensible estimation that leads to results that lie between the two traditional solutions and, what is more, minimizes the maximum expected error.

B. Voltage Stability Analysis

Taking again the IEEE 57-bus system as a basis, voltage stability has been tested by gradually increasing the active power demand at bus 49. Notice that this bus was selected for the study in view of the results of the power flow analysis shown in subsection IV-A. Indeed, these results demonstrate that the voltage magnitude at bus 49 show the highest discrepancy when calculated using different traditional tap-changing transformer models.

According to [7], the active power demand at bus 49 in the IEEE 57-bus system is 18 MW. This active power was increased in steps of 1 MW and, in each case, the power flow analysis of the system was repeated for the traditional tap-changing transformer models and for the new proposed model with the recommended set-up of $k = 1$. The results, in the form of the power-voltage curve (also known as “nose” curve or P-V curve), are depicted in Fig. 6.

Notice that voltage collapse is reached at quite different values of the active power demand at bus 49. While stability

TABLE III
BUS VOLTAGES SHOWING THE HIGHEST DISCREPANCIES

	Tap-changing transformer model					
	Type 1 ($k = 0$)		**Type 2** ($k = \infty$)		**Type 3** ($k = 1$)	
Bus	**Magnitude, p.u.**	**Phase angle, deg.**	**Magnitude, p.u.**	**Phase angle, deg.**	**Magnitude, p.u.**	**Phase angle, deg.**
33	0.941	−19.081	0.948	−18.552	0.944	−18.819
49	1.029	−13.336	1.036	−12.936	1.032	−13.141

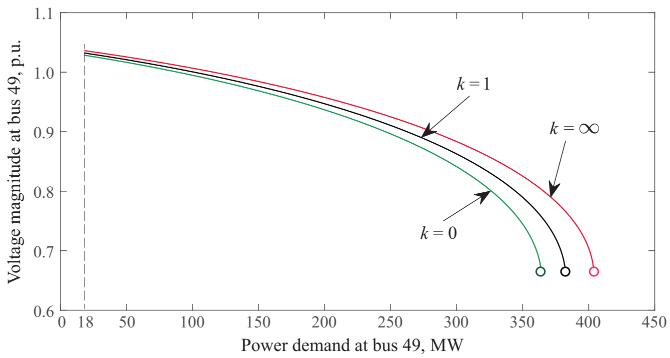

Fig. 6. Power-Voltage curves at bus 49 for the traditional tap-changing transformer models ($k = \infty$ and $k = 0$) and for the new proposed model ($k = 1$).

is lost at 364 MW in the case of $k = 0$, collapse is not reached until 404 MW if the model with $k = \infty$ is considered. The first case may be a too conservative approach while the second is certainly underestimating the voltage drop. On the contrary, the use of the new model with $k = 1$ estimates that the voltage collapse would take place at 382 MW which is certainly a sensible compromise.

Once more, this voltage stability analysis comes to emphasize the important differences that may arise from the use of the different versions of traditional tap-changing transformers. Security constraints may be compromised by using simplified assumptions, such as the ones implied in traditional models. What is more, even if $k = 1$ is a sensible estimation in a context of scarce data, a precise knowledge of this parameter could add certainty to the results obtained in this type of power system studies.

V. CONCLUSION

For decades, the different software simulation packages devoted to conduct power system studies have included one of two alternative traditional models of the tap-changing transformer. Those different models arise from the assumption of considering all the short-circuit impedance as provided by the nominal or off-nominal turns side of the transformer. Though the differences in the results offered by those models may be small at close-to-central tap positions, they can be remarkably large at extreme tap positions. Thus, the consistency in reporting results from power system studies can be compromised. The present contribution demonstrates that those discrepancies can be significant even in the case of a well-known standard grid, which is illustrated by a power flow analysis and a stability analysis. The use of a recently proposed consensus model is introduced as an ideal solution to the aforementioned problem. The new model is free from ambiguity and, whereas an additional parameter has to be provided, a sensible selection of its value can be done if accurate manufacturing data is not available.

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