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XVI Congreso de Matemática Aplicada

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June 14-18, 2021

CEDYA CMA 2020 GIJÓN

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Foreword

It is with great pleasure that we present the Proceedings of the 26th Congress of Differential Equations and Applications / 16th Congress of Applied Mathematics (XXVI CEDYA / XVI CMA), the biennial congress of the Spanish Society of Applied Mathematics S$\vec{e}$MA, which is held in Gijón, Spain from June 14 to June 18, 2021.

In this volume we gather the short papers sent by some of the almost three hundred and twenty communications presented in the conference. Abstracts of all those communications can be found in the abstract book of the congress. Moreover, full papers by invited lecturers will shortly appear in a special issue of the S$\vec{e}$MA Journal.

The first CEDYA was celebrated in 1978 in Madrid, and the first joint CEDYA / CMA took place in Málaga in 1989. Our congress focuses on different fields of applied mathematics: Dynamical Systems and Ordinary Differential Equations, Partial Differential Equations, Numerical Analysis and Simulation, Numerical Linear Algebra, Optimal Control and Inverse Problems and Applications of Mathematics to Industry, Social Sciences, and Biology. Communications in other related topics such as Scientific Computation, Approximation Theory, Discrete Mathematics and Mathematical Education are also common.

For the last few editions, the congress has been structured in mini-symposia. In Gijón, we will have eighteen minis-symposia, proposed by different researchers and groups, and also five thematic sessions organized by the local organizing committee to distribute the individual contributions. We will also have a poster session and ten invited lectures. Among all the mini-symposia, we want to highlight the one dedicated to the memory of our colleague Francisco Javier "Pancho" Sayas, which gathers two plenary lectures, thirty-six talks, and more than forty invited people that have expressed their wish to pay tribute to his figure and work.

This edition has been deeply marked by the COVID-19 pandemic. First scheduled for June 2020, we had to postpone it one year, and move to a hybrid format. Roughly half of the participants attended the conference online, while the other half came to Gijón. Taking a normal conference and moving to a hybrid format in one year has meant a lot of efforts from all the parties involved. Not only did we, as organizing committee, see how much of the work already done had to be undone and redone in a different way, but also the administration staff, the scientific committee, the mini-symposia organizers, and many of the contributors had to work overtime for the change.

Just to name a few of the problems that all of us faced: some of the already accepted mini-symposia and contributed talks had to be withdrawn for different reasons (mainly because of the lack of flexibility of the funding agencies); it became quite clear since the very first moment that, no matter how well things evolved, it would be nearly impossible for most international participants to come to Gijón; reservations with the hotels and contracts with the suppliers had to be cancelled; and there was a lot of uncertainty, and even anxiety could be said, until we were able to confirm that the face-to-face part of the congress could take place as planned.

On the other hand, in the new open call for scientific proposals, we had a nice surprise: many people that would have not been able to participate in the original congress were sending new ideas for mini-symposia, individual contributions and posters. This meant that the total number of communications was about twenty percent greater than the original one, with most of the new contributions sent by students.

There were almost one hundred and twenty students registered for this CEDYA / CMA. The hybrid format allows students to participate at very low expense for their funding agencies, and this gives them the opportunity to attend different conferences and get more merits. But this, which can be seen as an advantage, makes it harder for them to obtain a full conference experience. Alfréd Rényi said: "a mathematician is a device for turning coffee into theorems". Experience has taught us that a congress is the best place for a mathematician to have a lot of coffee. And coffee cannot be served online.

In Gijón, June 4, 2021

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Asymptotic aspects of the logistic equation under diffusion

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Abstract

This talk is devoted to describe the nontrivial solutions to

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u - |u|^{q-2}u & x \in \Omega \\ u = 0 & x \in \partial\Omega. \end{cases}$$

Exponents satisfy $1 < p < q$ while $\lambda > 0$ is a bifurcation parameter. We are confining ourselves to the case where Ω is a ball and solutions are radial. More importantly, we are discussing the asymptotic behavior of these solutions as $p \to 1+$. We are further stating not only the existence of such limits but even introducing the limit problem which such limits solve.

1. Introduction

This talk is firstly devoted to describe the nontrivial solutions to the nonlinear eigenvalue problem:

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u - |u|^{q-2}u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, ν is outer unit normal, λ is a positive (bifurcation) parameter and $\Delta_p u = \operatorname{div}\ (|\nabla u|^{p-2}\nabla u)$ is the p–Laplacian operator. The exponents p, q are assumed to satisfy,

$$1 < p < q. \tag{1.2}$$

The case $p = 2$ is the logistic problem, a well–known model in population dynamics (see [17], [6], also [8] for related applications). As for the nonlinear diffusion regime $p \neq 2$, a detailed discussion of its positive solutions has been performed in [10–12], [15] and [9], the latter specially concerned with the one–dimensional case. Regarding the problem (1.1) observed in a N–dimensional domain Ω, see [13] for existence results on a closely related problem.

A further feature we are going to address is the analysis of the *limit perturbation* of problem (1.1) as $p \to 1$. Namely,

$$\begin{cases} -\Delta_1 u = \lambda \dfrac{u}{|u|} - |u|^{q-2}u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.3}$$

where $\Delta_1 u = \operatorname{div}\ \left(\dfrac{\nabla u}{|\nabla u|}\right)$ is the one–Laplacian operator. Such operator finds its natural applications in a broader class of fields ranging from image processing ([4], [18]) to torsion theory ([16]).

Due to the fact that the N–dimensional versions of problems (1.1) and (1.3) are plagued of technical obstacles, main emphasis here will be put on their radial versions. In such case, $\Omega = B(0,R) \subset \mathbb{R}^N$ is a N–dimensional ball with $N \geq 2$. It should be remarked that the one–dimensional versions of (1.1) and (1.3),

$$\begin{cases} -(|u_x|^{p-2}u_x)_x = \lambda |u|^{p-2}u - |u|^{q-2}u, & 0 < x < R, \\ u(0) = u(R) = 0, \end{cases} \tag{1.4}$$

and

$$\begin{cases} -\left(\dfrac{u_x}{|u_x|}\right)_x = \lambda \dfrac{u}{|u|} - |u|^{q-2}u, & 0 < x < R, \\ u(0) = u(R) = 0, \end{cases} \tag{1.5}$$

have been recently studied in [21] (problem (1.4) goes back to [14]).

This note is organized as follows. Basic results, specially those concerning the limit problem (1.3) are reviewed in Section 2. A global description of the set of nontrivial solutions to (1.1) in a ball is presented in Section 3. The features on the limit behavior of solutions to (1.1) as $p \to 1+$ are described in Section 4.

2. Background results

By a (weak) solution $u \in W_0^{1,p}(\Omega) \cap L^q(\Omega)$ to (1.1) it is understood that equality

$$\int_\Omega |\nabla u|^{p-2}\nabla u \nabla v = \lambda \int_\Omega |u|^{p-2}uv - \int_\Omega |u|^{q-2}uv,$$

holds for every test function $v \in C_0^1(\Omega)$. In fact it can be checked that such test functions can be allowed to belong to $W_0^{1,p}(\Omega)$ ([22]).

Analysis of (1.1) in a ball $B(0,R)$ is closely linked to the radial eigenvalues to

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u, & x \in B(0,R), \\ u = 0, & x \in \partial B(0,R), \end{cases} \tag{2.1}$$

which will be designated as,

$$0 < \lambda_{1,p} < \lambda_{2,p} < \dots .$$

We refer to [7], [23] and [20] for a detalied account (also [2] for an early source). Eigenvalues in the unit ball $B(0,1)$ are more conveniently expressed as $\lambda_{n,p} = \omega_n^p$ for certain positive numbers ω_n. Thus, eigenvalues in the ball $B(0,R)$ turn out to be $\lambda_{n,p} = R^{-p}\omega_n^p$.

Following the nowadays well settled down approach in [3] and [4] we introduce the concept of a solution to (1.3). Framework space is

$$BV(\Omega) = \{u \in L^1(\Omega) : Du \in C_0(\Omega, \mathbb{R}^N)'\},$$

that is, the space of functions in $L^1(\Omega)$ whose gradient Du is a vectorial zero order distribution, whose components define finite Radon measures $D_i u$, $1 \le i \le N$ (see [1] for a comprehensive source on this space).

To introduce the concept of weak solution to (1.3), the problematic term $\frac{Du}{|Du|}$ must be conveniently replaced with a suitable field $\mathbf{z} \in L^\infty(\Omega, \mathbb{R}^N)$. On the other hand, the formulation of a Green identity is required in order to test with functions $v \in BV(\Omega)$. Anzellotti's theory is instrumental for these purposes. A featured result in [5] is the identity,

$$\int_\Omega (\mathbf{z}, Dv) + \int_\Omega v \operatorname{div} \mathbf{z} = \int_{\partial\Omega} v[\mathbf{z}, \nu]\, ds, \tag{2.2}$$

which holds for every $\mathbf{z} \in L^\infty_{q'}(\Omega, \mathbb{R}^N) := \{\mathbf{z} \in L^\infty(\Omega, \mathbb{R}^N) : \operatorname{div} \mathbf{z} \in L^{q'}(\Omega)\}$ and $v \in BV_q(\Omega) := BV(\Omega) \cap L^q(\Omega)$. To account for every term in (2.2) it is shown in [5] that the normal component $[\mathbf{z}, \nu]$ has a well–defined trace on $\partial\Omega$ which belongs to $L^\infty(\partial\Omega)$. In addition, the scalar product $\mathbf{z} \cdot Du$ is extended as a bilinear mapping $(\mathbf{z}, Du)$, from $C^1(\overline{\Omega}, \mathbb{R}^N) \times W^{1,1}(\Omega)$ to $L^\infty_{q'}(\Omega, \mathbb{R}^N) \times BV_q(\Omega)$ in the following distributional way:

$$\langle (\mathbf{z}, Du), \varphi \rangle = -\int_\Omega u \operatorname{div}(\varphi \mathbf{z}), \qquad \varphi \in C_0^\infty(\Omega).$$

It is shown in [5] that $(\mathbf{z}, Du)$ defines a finite Radon measure in Ω such that

$$|(\mathbf{z}, Du)(B)| \le \|\mathbf{z}\|_\infty |Du|(B),$$

$B \subset \Omega$ being a Borelian and $|Du|$ standing for the total variation of Du.

We are now ready for the next definition.

Definition 2.1 A function $u \in BV_q(\Omega)$ defines a (weak) solution to (1.3) provided that there exist $\mathbf{z} \in L^\infty_{q'}(\Omega, \mathbb{R}^N)$, $\|\mathbf{z}\|_\infty \le 1$, $\beta \in L^\infty(\Omega)$, $\|\beta\|_\infty \le 1$ such that,

i) $-\operatorname{div} \mathbf{z} = \lambda\beta - |u|^{q-2}u$, in $\mathcal{D}'(\Omega)$,

ii) $\beta u = |u|$ and $(\mathbf{z}, Du) = |Du|$, in $\mathcal{D}'(\Omega)$,

iii) $[z, \nu]u = -|u|$ on $L^1(\partial\Omega)$, (boundary condition).

Remark 2.2 Boundary condition in iii) is suggested by two features. First one, the fact that the weak–$*$ limit $u \in BV(\Omega)$ of a sequence $u_n \in W_0^{1,1}(\Omega)$ could eventually exhibits a nonzero trace on the boundary. Second one, that solutions of (1.3) could be approximated as $p \to 1$ by corresponding solutions to (1.1).

3. Radial solutions

A general view on the nontrivial solutions to (1.1) in a ball is contained in the next statement.

Theorem 3.1 *Assume* $1 < p \le 2$. *Then, problem*

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u - |u|^{q-2}u, & x \in B(0,R), \\ u = 0, & x \in \partial B(0,R), \end{cases} \tag{3.1}$$

exhibits the following features.

i) [Range and amplitude] *Nontrivial solutions are only possible when* $\lambda > \lambda_{1,p}$ *while the normalized amplitude*

$$\alpha := \lambda^{-\frac{1}{q-p}} \|u\|_\infty,$$

satisfies $\alpha < 1$.

ii) [Positive solutions] *There exists a* unique *positive* (*radial*) *solution* $u_{\lambda,1}$ *for all* $\lambda > \lambda_{1,p}$, *bifurcating from* $u = 0$ *at* $\lambda = \lambda_{1,p}$ *while*:

$$\lambda^{-\frac{1}{q-p}} \|u_{\lambda,1}\|_\infty \to 1 \quad \text{as } \lambda \to \infty.$$

iii) [Existence of branches] *For all* $n \ge 2$, *a symmetric family* $\pm u_{\lambda,n}(r)$ *of nontrivial radial solutions,* exactly defined *for all* $\lambda > \lambda_{n,p}$, *bifurcates from* $u = 0$ *at* $\lambda_{n,p}$ *and,*

$$\lambda^{-\frac{1}{q-p}} \|u_{\lambda,n}\|_\infty \to 1 \quad \text{as } \lambda \to \infty.$$

iv) [Nodal properties] *Every* $\pm u_{\lambda,n}(r)$ *vanishes exactly at* $n-1$ *values* $r_k \in (0,R)$.

v) [Continuity of the branches] *Bifurcated branches* $\pm u_{\lambda,n}$ *define a* continuous *curve* C_n *when parameterized by the normalized amplitude* $\alpha = \lambda^{-\frac{1}{q-p}} \|u\|_\infty$, $0 < \alpha < 1$. *More precisely, there exist continuous mappings* $\alpha \mapsto \lambda_n(\alpha)$, $\alpha \mapsto u_n(\alpha) \in W_0^{1,p}(B(0,R))$, $0 < \alpha < 1$, *such that,*

$$\pm u_{\lambda,n} = \pm u_n(\alpha), \qquad \lambda = \lambda_n(\alpha).$$

Proof (Sketch) The scaling $u(r) = \lambda^{\frac{1}{q-p}} v(t)$, $t = \lambda^{\frac{1}{p}} r$, transforms (3.1) into,

$$\begin{cases} -(|v_t|^{p-2}v_t)_t - \dfrac{N-1}{t}|v_t|^{p-2}v_t = |v|^{p-2}v - |v|^{q-2}v, & 0 < t < \lambda^{\frac{1}{p}} R, \\ v(0) = \alpha, \quad v_t(0) = 0, \end{cases} \tag{3.2}$$

where:

$$\max v = \alpha, \qquad 0 < \alpha < 1,$$

and v must satisfies the boundary condition:

$$v(\lambda^{\frac{1}{p}} R) = 0.$$

The initial value problem (3.2) admits a unique C^2 solution $v = v(\cdot,\alpha)$ which is defined in $[0,\infty)$ and satisfies $\lim_{t\to\infty}(v(t), v_t(t)) = (0,0)$. Moreover, v exhibits infinitely many simple zeros,

$$0 < \theta_1(\alpha) < \theta_2(\alpha) < \cdots < \theta_n(\alpha) < \cdots, \qquad \theta_n \to \infty.$$

Functions $\theta_n(\alpha)$ are shown to be continuous in $\alpha \in (0,1)$ and,

$$\lim_{\alpha\to 0+} \theta_n(\alpha) = \omega_n, \qquad \lim_{\alpha\to 1-} \theta_n(\alpha) = \infty,$$

where $\omega_n = \lambda_{n,p}(B(0,1))^{\frac{1}{p}}$.

To solve (3.1) amounts to:

$$\lambda^{\frac{1}{p}} R = \theta_n(\alpha) \qquad \Leftrightarrow \qquad \lambda = R^{-p}\theta_n(\alpha)^p.$$

By setting this value of λ in the expression for u:

$$u(r) = \lambda^{\frac{1}{q-p}} v(\lambda^{\frac{1}{p}} r, \alpha), \qquad 0 \le r \le R,$$

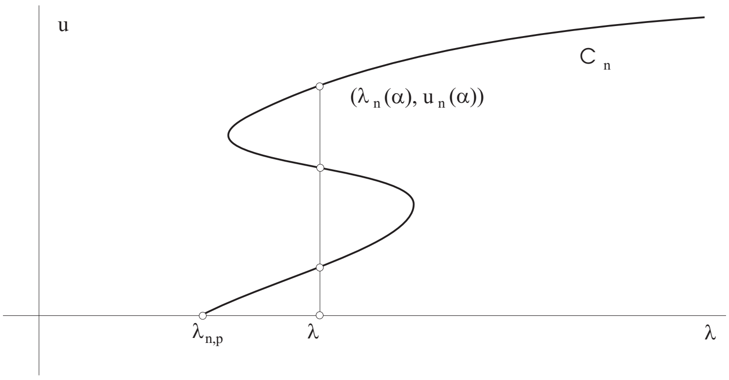

Fig. 1 Family C_n of nontrivial solutions bifurcated from $u = 0$ at $\lambda = \lambda_{n,p}$. Only a half of C_n has been depicted. That one corresponding to $u(0) > 0$. It is stressed that its exact range of existence is $[\lambda_{n,p}, \infty)$.

the family $u_{n,\lambda}$ is obtained. Moreover by defining:

$$\lambda_n(\alpha) = R^{-p}\theta_n(\alpha)^p, \qquad u_n(r,\alpha) = \lambda_n^{\frac{1}{q-p}} v(\lambda_n^{\frac{1}{p}} r, \alpha),$$

$\{u_{\lambda,n}\}$ is alternatively represented as a continuous curve $(\lambda_n(\alpha), u_n(\alpha))$ in $\mathbb{R} \times W_0^{1,p}(B(0,R))$. It should be also observed that $u_n(\cdot,\alpha)$ vanishes at the points,

$$r_k = R\frac{\theta_k(\alpha)}{\theta_n(\alpha)}, \qquad k = 1, \ldots, n.$$

Assertion concerning the existence of the family $u_{\lambda,n}$ exactly at the interval $[\lambda_{n,p}, \infty)$ is a consequence of the estimate:

$$\theta_n(\alpha) > \omega_n, \quad 0 < \alpha < 1.$$

The proof of this fact deserves a delicate proof and it is also omitted (see [22]). □

Remark 3.2 The existence of a global continuum C_n^* bifurcating from zero at $\lambda = \lambda_{n,p}$ was stated in [12] (see also [19]). Theorem 3.1 improves these results in two regards. Firstly, family of solutions $u_{\lambda,n}$ is shown to exists exactly at the range $\lambda > \lambda_{n,p}$. Secondly, ours is not a mere continuum C_n^* but rather a global continuous curve C_n.

4. Limit behavior

The sequence,

$$0 < \bar\lambda_1 < \bar\lambda_2 < \cdots$$

of radial eigenvalues to $-\Delta_1$,

$$\begin{cases} -\Delta_1 u = \lambda \dfrac{u}{|u|}, & x \in B(0,R), \\ u = 0, & x \in \partial B(0,R), \end{cases} \tag{4.1}$$

has been recently studied in [20]. Among other featured properties it is shown there that,

$$\lim_{p\to 1} \lambda_{n,p} = \bar\lambda_n, \qquad \text{for every } n \in \mathbb{N}.$$

Our next result describes a set of *distinguished* nontrivial radial solutions to (1.3). Those ones obtained as the limit of solutions to (1.1) as $p \to 1$. In addition this precise feature of the solutions is characterized by a suitable energy condition. In the forthcoming statement, the reference zeros θ_n introduced in the proof of Theorem 3.1 are involved. It should be remarked that they also depends on $p > 1$ and an important fact to be reported is the existence of their limits $\bar\theta_n$ as $p \to 1+$ (see ii) below). Figure 2 depicts this dependence through a simulation.

Theorem 4.1 *The structure of the set of* radial *nontrivial solutions to*

$$\begin{cases} -\operatorname{div}\left(\dfrac{\nabla u}{|\nabla u|}\right) = \lambda \dfrac{u}{|u|} - |u|^{q-2}u, & x \in B(0,R), \\ u = 0, & x \in \partial B(0,R), \end{cases}$$

can be described as follows.

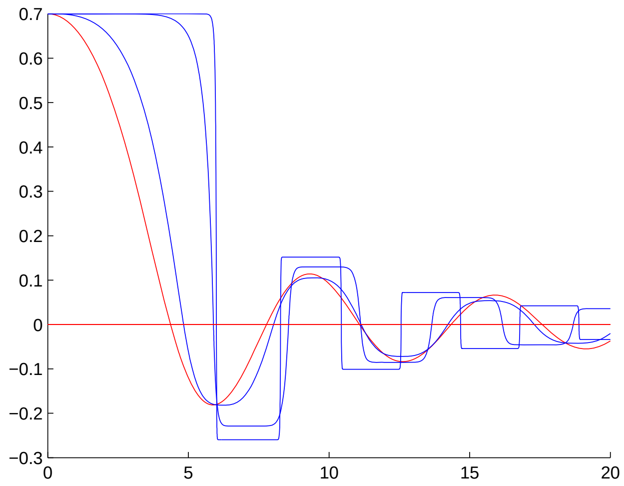

Fig. 2 Profiles of $v(t)$ and corresponding zeros θ_n for varying values of $p > 1$. Simulation has been performed for $N = 3$, $q = 3$, $\alpha = 0.7$. Then, chosen values of p are $p = 2$, $p = 1.5$, $p = 1.1$ and $p = 1.01$. Plots become steeper as p decays to unity.

i) [Normalized amplitude estimate] *Nontrivial solutions are only possible if* $\lambda > \bar{\lambda}_1$. *Moreover, the normalized amplitude* $\alpha := \lambda^{-\frac{1}{q-1}}\|u\|_\infty$ *of such solutions satisfies,*

$$0 < \alpha < 1.$$

ii) [Limits of zeros] *There exists a family of smooth functions* $\bar{\theta}_n(\alpha)$,

$$0 < \bar{\theta}_1(\alpha) < \bar{\theta}_n(\alpha) < \cdots,$$

such that,

$$\lim_{p\to 1} \theta_n(\alpha) = \bar{\theta}_n(\alpha), \qquad 0 < \alpha < 1.$$

iii) [Existence] *To every radial eigenvalue* $\bar{\lambda}_n$ *there corresponds a symmetric family* $\pm\bar{u}_{\lambda,n}$ *of nontrivial solutions which bifurcates from* $u = 0$ *at* $\bar{\lambda}_n$. *In addition, such family is defined for each* $\lambda > \bar{\lambda}_n$ *while the normalized amplitude of its members satisfies,*

$$\lim_{\lambda\to\infty} \lambda^{-\frac{1}{q-1}}\|\bar{u}_{\lambda,n}\|_\infty = 1.$$

iii) [Smoothness] *Family* $\pm\bar{u}_{\lambda,n}$ *constitutes a smooth curve* $\overline{C}_n$ *in* $\mathbb{R}\times BV(B(0,R))$ *when parameterized by the normalized amplitude* $0 < \alpha < 1$. *More precisely, a decreasing family of smooth positive functions* $\alpha \mapsto \bar{\alpha}_n(\alpha)$ *exists such that by setting,*

$$\bar{\lambda}_n(\alpha) = R^{-1}\bar{\theta}_n(\alpha), \qquad \bar{u}_n(\cdot,\alpha) = \bar{\lambda}_n^{\frac{1}{q-1}} \sum_{k=1}^{n} (-1)^{k-1}\bar{\alpha}_{k-1}\chi_{I_k},$$

χ_{I_k} *being the characteristic function of the interval* $I_k = \left(R\dfrac{\bar{\theta}_{k-1}(\alpha)}{\bar{\theta}_n(\alpha)}, R\dfrac{\bar{\theta}_k(\alpha)}{\bar{\theta}_n(\alpha)}\right)$, *then*

$$\pm\bar{u}_{\lambda,n} = \bar{u}_n(\alpha) \qquad \text{for } \lambda = \bar{\lambda}_n(\alpha).$$

iv) [Convergence of branches] *Let* C_n *be the n–th curve of nontrivial solutions introduced in Theorem* 3.1. *Then*

$$C_n \to \overline{C}_n \qquad \text{as } p \to 1+,$$

in the sense that,

$$\lim_{p\to 1} (\lambda_n(\alpha), u_n(\alpha)) = (\bar{\lambda}_n(\alpha), \bar{u}_n(\alpha)) \qquad \text{in } \mathbb{R}\times BV(B(0,R)),$$

for every $0 < \alpha < 1$.

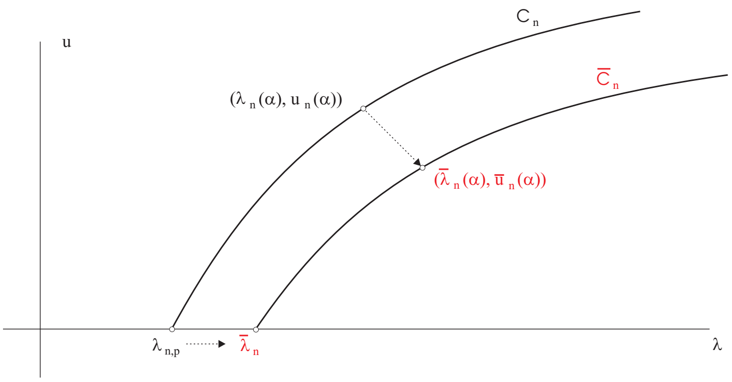

Fig. 3 Convergence of branches as $p \to 1+$.

v) [Uniqueness] Every *nontrivial solution u to* (4.1) *fulfilling the 'energy' condition,*

$$\frac{d}{dr}\left(\lambda|u| - \frac{|u|^q}{q}\right) = -\frac{N-1}{r}|u_r| \qquad in\ \mathcal{D}(0,R)'. \tag{4.2}$$

necessarily belongs to some of the previous families $\overline{C}_n = \{\pm\bar{u}_{\lambda,n}\}$.

Proof (Sketch) A first step of compactness nature is the following (subindex p refers to dependence on p). Family $v_p(\cdot,\alpha)$ of solutions to (3.2) admits a subfamily, still denoted v_p, while a function $v_1 \in BV_{loc}(0,\infty)$ exists so that,

$$v_p \rightharpoonup v_1 \qquad \text{weakly in } L^s(0,b;t^{N-1}\,dt) \text{ as } p \to 1,$$

for every $b > 0$ and $1 \le s < \infty$.

A second step consists in proving that $v = v_1(t)$ solves in the sense of Definition 2.1 the initial value problem,

$$\begin{cases} -\left(\dfrac{v_t}{|v_t|}\right)_t - \dfrac{N-1}{t}\dfrac{v_t}{|v_t|} = \dfrac{v}{|v|} - |v|^{q-2}v, & t > 0, \\ v(0+) = \alpha, \qquad v_t(0) = 0, \end{cases} \tag{4.3}$$

together with the energy condition,

$$\left(|v| - \frac{|v|^q}{q}\right)_t = -\frac{N-1}{t}|v_t| \qquad \text{in } \mathcal{D}(0,R)'. \tag{4.4}$$

A third and crucial step is showing that problem (4.3) constrained with condition (4.4) exhibits a unique solution. Moreover, such solution can be expressed in the exact form,

$$v_1(t) = \sum_{n=1}^{\infty}(-1)^{n-1}\bar{\alpha}_{n-1}\chi_{(\bar{\theta}_{n-1},\bar{\theta}_n)}(t),$$

for a precisely computed pair $\bar{\lambda}_n$, $\bar{\theta}_n$, of monotone sequences of positive numbers satisfying $\bar{\lambda}_n \to 0$ and $\bar{\theta}_n \to \infty$.

Final step is checking that family $\bar{u}_{\lambda,n}$ can be defined as,

$$\bar{u}_{\lambda,n}(r) = \lambda^{\frac{1}{q-1}}v_1(\lambda r), \qquad \text{where } \lambda = R^{-1}\bar{\theta}_n.$$

To this purpose suitable candidates for $\mathbf{z}$ and β in Definition 2.1 must be furnished.

A detailed account of the (lengthy) proofs of all these assertions is contained in [22]. □

Remark 4.2

a) Functions $\bar{\theta}_n(\alpha)$ and $\bar{\alpha}_n(\alpha)$ can be recursively computed starting at $n = 0$ with values $\bar{\theta}_0(\alpha) = 0$, $\bar{\alpha}_0(\alpha) = \alpha$.

b) Further families of nontrivial solutions to (4.1) not satisfying the energy condition (4.2) can be found. A characteristic property of such solutions is that they vanish in nonempty interior regions.

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