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Foreword

It is with great pleasure that we present the Proceedings of the 26$^{\text{th}}$ Congress of Differential Equations and Applications / 16$^{\text{th}}$ Congress of Applied Mathematics (XXVI CEDYA / XVI CMA), the biennial congress of the Spanish Society of Applied Mathematics S$\vec{e}$MA, which is held in Gijón, Spain from June 14 to June 18, 2021.

In this volume we gather the short papers sent by some of the almost three hundred and twenty communications presented in the conference. Abstracts of all those communications can be found in the abstract book of the congress. Moreover, full papers by invited lecturers will shortly appear in a special issue of the S$\vec{e}$MA Journal.

The first CEDYA was celebrated in 1978 in Madrid, and the first joint CEDYA / CMA took place in Málaga in 1989. Our congress focuses on different fields of applied mathematics: Dynamical Systems and Ordinary Differential Equations, Partial Differential Equations, Numerical Analysis and Simulation, Numerical Linear Algebra, Optimal Control and Inverse Problems and Applications of Mathematics to Industry, Social Sciences, and Biology. Communications in other related topics such as Scientific Computation, Approximation Theory, Discrete Mathematics and Mathematical Education are also common.

For the last few editions, the congress has been structured in mini-symposia. In Gijón, we will have eighteen minis-symposia, proposed by different researchers and groups, and also five thematic sessions organized by the local organizing committee to distribute the individual contributions. We will also have a poster session and ten invited lectures. Among all the mini-symposia, we want to highlight the one dedicated to the memory of our colleague Francisco Javier "Pancho" Sayas, which gathers two plenary lectures, thirty-six talks, and more than forty invited people that have expressed their wish to pay tribute to his figure and work.

This edition has been deeply marked by the COVID-19 pandemic. First scheduled for June 2020, we had to postpone it one year, and move to a hybrid format. Roughly half of the participants attended the conference online, while the other half came to Gijón. Taking a normal conference and moving to a hybrid format in one year has meant a lot of efforts from all the parties involved. Not only did we, as organizing committee, see how much of the work already done had to be undone and redone in a different way, but also the administration staff, the scientific committee, the mini-symposia organizers, and many of the contributors had to work overtime for the change.

Just to name a few of the problems that all of us faced: some of the already accepted mini-symposia and contributed talks had to be withdrawn for different reasons (mainly because of the lack of flexibility of the funding agencies); it became quite clear since the very first moment that, no matter how well things evolved, it would be nearly impossible for most international participants to come to Gijón; reservations with the hotels and contracts with the suppliers had to be cancelled; and there was a lot of uncertainty, and even anxiety could be said, until we were able to confirm that the face-to-face part of the congress could take place as planned.

On the other hand, in the new open call for scientific proposals, we had a nice surprise: many people that would have not been able to participate in the original congress were sending new ideas for mini-symposia, individual contributions and posters. This meant that the total number of communications was about twenty percent greater than the original one, with most of the new contributions sent by students.

There were almost one hundred and twenty students registered for this CEDYA / CMA. The hybrid format allows students to participate at very low expense for their funding agencies, and this gives them the opportunity to attend different conferences and get more merits. But this, which can be seen as an advantage, makes it harder for them to obtain a full conference experience. Alfréd Rényi said: "a mathematician is a device for turning coffee into theorems". Experience has taught us that a congress is the best place for a mathematician to have a lot of coffee. And coffee cannot be served online.

In Gijón, June 4, 2021

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New contributions to the control of PDEs and their applications

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Abstract

This paper deals with some recent achievements in control theory. Specifically, we will consider the null controllability problem for a quasi-linear parabolic PDE. We present some theoretical and numerical results. We also exhibit the results of a numerical experiment.

1. Introduction

Let $\Omega \subset \mathbb{R}^N$ be an open bounded regular domain ($N \leq 3$) and let $T > 0$ be given. We will mainly consider the system

$$\begin{cases} y_t - \nabla \cdot (a(y)\nabla y) = v\tilde{1}_\omega, \quad (x,t) \in Q := \Omega \times (0,T), \\ y = 0, \quad (x,t) \in \Sigma := \partial\Omega \times (0,T), \\ y(x,0) = y_0(x), \quad x \in \Omega. \end{cases} \tag{1.1}$$

Here, we assume that $\omega \subset\subset \Omega$ is a nonempty open set (the control domain), $\tilde{1}_\omega \in C_0^\infty(\Omega)$ satisfies $0 < \tilde{1}_\omega \leq 1$ in ω and $\tilde{1}_\omega = 0$ outside ω and $a \in C^3(\mathbb{R})$ possesses bounded derivatives of order ≤ 3 and satisfies

$$0 < m \leq a(r) \leq M \quad \forall r \in \mathbb{R}.$$

Obviously, we can interpret the control $v = v(x,t)$ as a heat source term and the state $y = y(x,t)$ as the associated temperature distribution in Q.

We will be concerned with the theoretical and numerical local null controllability of (1.1). Specifically, we will establish the existence of null controls when the initial state y_0 is small, we will present a related iterative algorithm and we will describe a numerical method for their computation.

The ideas and results that follow have been taken from [6] and [3]. They can be adapted to the solution of many other control problems; see for instance [2, 4] and the references therein. In particular, they serve to compute null controls for the Navier-Stokes and other similar equations, see [5].

2. The existence of null controls

The first main result in this contribution is the following:

Theorem 2.1 *Under the previous assumptions on the coefficient a, there exists $\varepsilon > 0$ such that, if $y_0 \in H_0^1(\Omega) \cap L^\infty(\Omega)$ and $\|y_0\|_{H^1} + \|y_0\|_{L^\infty} \leq \varepsilon$, there exists a control $v \in L^2(\omega \times (0,T))$ and an associated solution to the nonlinear system* (1.1) *satisfying*

$$y(x,T) = 0 \ \ in \ \ \Omega. \tag{2.1}$$

In order to prove Theorem 2.1, we can employ a technique relying on the so called *Liusternik's Inverse Function Theorem,* see [1].

Thus, in view of the regularizing effect, we can assume that $y_0 \in H^3(\Omega) \cap H_0^1(\Omega)$ and is small in this space. Then, we consider the linearized system at zero

$$\begin{cases} y_t - a(0)\Delta y = v\tilde{1}_\omega + h(x,t), \quad (x,t) \in Q, \\ y = 0, \quad (x,t) \in \Sigma, \\ y(x,0) = y_0(x), \quad x \in \Omega. \end{cases} \tag{2.2}$$

It is well known that, under some appropriate assumptions on h, (2.2) is null-controllable. More precisely, the adjoint of (2.2) is given by

$$\begin{cases} -\varphi_t - a(0)\Delta\varphi = F(x,t), \quad (x,t) \in Q, \\ \varphi = 0, \quad (x,t) \in \Sigma, \\ \varphi(x,T) = \varphi_T(x), \quad x \in \Omega, \end{cases} \tag{2.3}$$

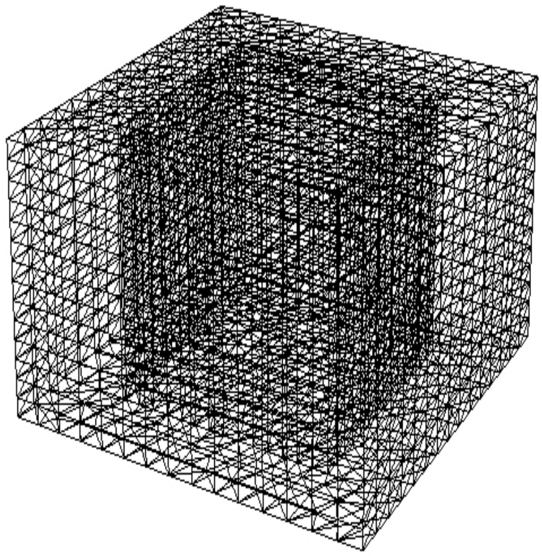


Fig. 1 The mesh. Number of vertices: 7425. Number of tetrahedrons: 38976.

where $\varphi_T \in L^2(\Omega)$; the announced null controllability property is implied by a well known Carleman inequality that can be established for any solution to a system of the form (2.3).

In a second step, we rewrite the null controllability problem for (1.1) as an equation in a well chosen space of “admissible" state-control pairs:

$$\mathcal{H}(y, v) = (0, y_0), \quad (y, v) \in Y. \tag{2.4}$$

Here, Y is a space of couples (y, v) satisfying, among other things, the following properties

$$\iint_Q \rho^2 |y|^2 + \iint_{\omega\times(0,T)} \rho_0^2 |v|^2 < +\infty$$

and

$$\iint_Q \hat{\rho}^2 |y_t - a(0)\Delta y - v\tilde{1}_\omega|^2 < +\infty,$$

where ρ, ρ_0 and $\hat{\rho}$ are appropriate weight functions that blow up to $+\infty$ as $t \to T$. Formally, the definition of $\mathcal{H}$ is the following:

$$\mathcal{H}(y, v) := (y_t - \nabla \cdot (a(y)\nabla y) - v\tilde{1}_\omega, y(\cdot\,, 0)) \quad \forall (y, v) \in Y.$$

Then, we apply Liusternik’s Theorem and we deduce the (local) desired result. To this purpose, we previously have to establish some nontrivial estimates for the null controls and the associated states of (2.2).

3. A convergent algorithm

The computation of a null control of (1.1) is not a simple task; here, we will argue as in [2, 6], taking advantage of the surjectivity of $\mathcal{H}'(0, 0)$.

Thus, let Y be the Hilbert space where we can find a solution (y, v) to (2.4). We introduce the following iterative algorithm:

ALG 1:

1. Choose $(y^0, v^0) \in Y$.
2. Then, for given $n \geq 0$ and $(y^n, v^n) \in Y$, compute

$$(y^{n+1}, v^{n+1}) = (y^n, v^n) - \mathcal{H}'(0, 0)^{-1}(\mathcal{H}(y^n, v^n) - (0, y_0)). \tag{3.1}$$

In these iterates, we use $\mathcal{H}'(0, 0)^{-1}$, which is by definition an inverse to the left of $\mathcal{H}'(0, 0)$.

Note that **ALG 1** is an elementary quasi-Newton method and consequently has the following interesting property: the finite dimensional approximations of the iterates lead to a set of algebraic systems whose coefficient matrices are always the same.

In our second main result, we prove the convergence of **ALG 1** and we furnish some estimates:

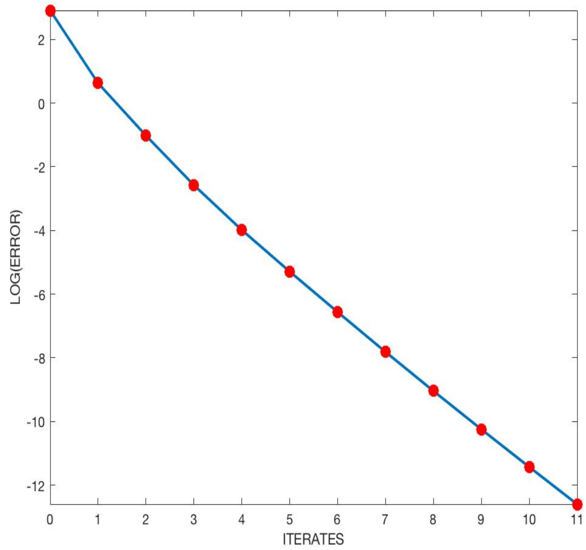


Fig. 2 Evolution of the error at logarithmic scale.

Theorem 3.1 *Let* $y_0 \in H_0^1(\Omega) \cap L^\infty(\Omega)$ *be given with* $\|y_0\|_{H_0^1} + \|y_0\|_{L^\infty} \le \varepsilon$ *(*ε *is furnished by Theorem* 2.1*). There exists* $\kappa \in (0,1)$ *such that, if* $(y^0, v^0) \in Y$ *and*

$$\|(y^0, v^0) - (y, v)\|_Y \le \kappa,$$

then the (y^n, v^n) *converge to* (y, v) *and satisfy*

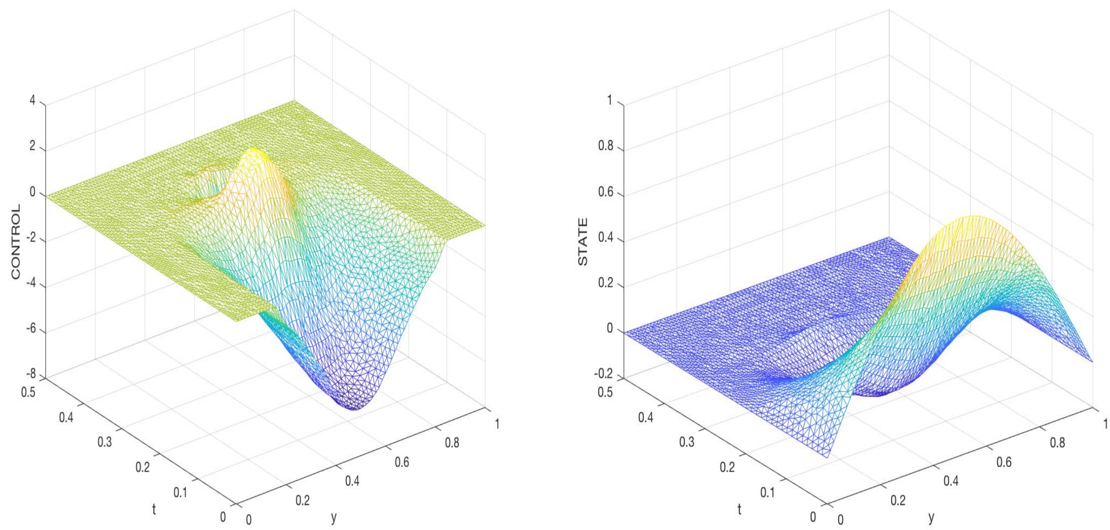
$$\|(y^{n+1}, v^{n+1}) - (y, v)\|_Y \le \theta \|(y^n, v^n) - (y, v)\|_Y \tag{3.2}$$

for all $n \ge 0$ *for some* $\theta \in (0,1)$.

Remark 3.2 A natural question is whether Theorems 2.1 and 3.1 also hold for similar systems with PDEs of the form

$$y_t - \nabla \cdot (a(x,t;y)\nabla y) = v \tilde{1}_\omega \ \text{ and/or } \ y_t - \nabla \cdot (a(x,t;\nabla y)\nabla y) = v \tilde{1}_\omega,$$

that is, with nonlinear diffusion coefficients nonhomogeneous in space and time and eventually depending on the gradient. In both cases, the answer is yes, provided they are regular enough, see [4]. ☐



(a) Computed control **(b)** Associated state

Fig. 3 The computed control and the associated state at $x_1 = 0.68$.

4. A numerical method for the solution of (3.1)

The computation of (y^{n+1}, v^{n+1}) in (3.1) can be achieved following the Fursikov-Imanuvilov method [7].

The strategy is to take

$$y = \rho^{-2} L^* p, \quad v = -\rho_0^2 \, \tilde{1}_\omega p\big|_{\omega\times(0,T)}, \tag{4.1}$$

where $L^* p := -p_t - a(0)\Delta p$ and p is the unique solution to the Lax-Milgram problem

$$\begin{cases} \iint_Q \left(\rho^{-2} L^* p L^* p' + \tilde{1}_\omega p p'\right) = \iint_Q h p' + \int_\Omega y_0(x) p'(x,0)\, dx \\ \forall p' \in P, \quad p \in P \end{cases} \tag{4.2}$$

(P is an appropriate Hilbert space of functions p with $L^* p \in L^2_{loc}(Q)$). Accordingly, we set $\mathcal{H}'(0,0)^{-1}(h, y_0) = (y, v)$, with y and v respectively given by (4.1) and (4.2).

Note that (4.2) is the weak formulation of a boundary-value problem for p that is second-order in time and fourth-order in space.

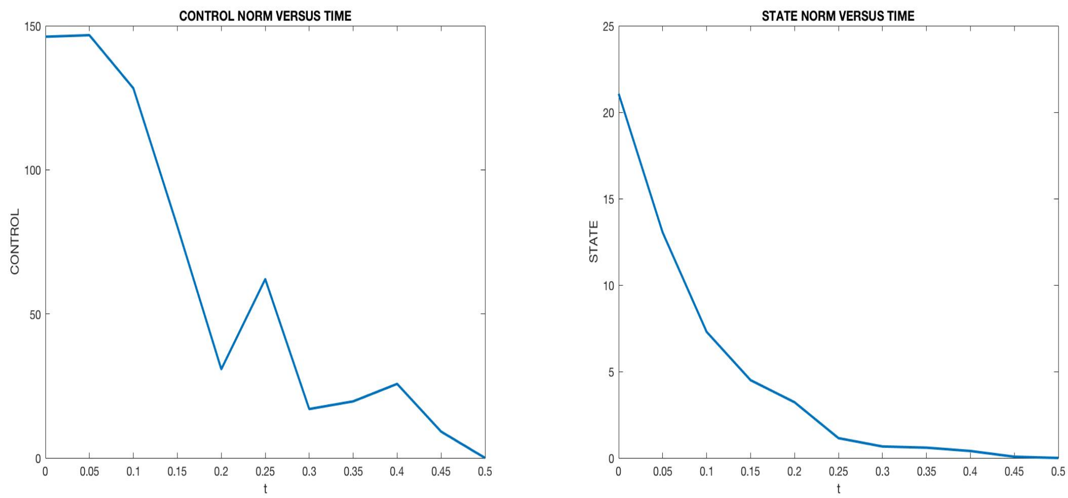
Unfortunately, it is not easy to construct and handle finite dimensional spaces $P_h \subset P$ (except in the particular case $N = 1$). Thus, it is convenient to introduce a mixed formulation (as in [5, 6]) and then reduce to finite dimension. This leads to numerical approximations with P_ℓ-piecewise continuous functions that furnish good results; see the details in [3].

5. A numerical experiment

The quasi-Newton method has been applied to the solution to the null controllability problem for (1.1) with the following data:

- $N = 2, \Omega = (0,1)\times(0,1), \omega = (0.2, 0.8)\times(0.2, 0.8), T = 0.5$.
- $y_0(x_1, x_2) = \sin(\pi x_1)\sin(\pi x_2)$.
- $a(s) = \exp(-2\exp(-0.3s))$.

The computations have been performed with the FreeFem++ package; see `http://www.freefem.org//ff++`. The stopping criterion for **ALG 1** has been $\|y^{n+1} - y^n\|_{L^2}/\|y^{n+1}\|_{L^2} \le \varepsilon_0$, where y^n is the computed state and $\varepsilon_0 = 10^{-5}$. The mesh, the error evolution, the computed control and state and their spatial L^2 norms are displayed in Fig. 1–4.



(a) Computed control **(b)** Associated state

Fig. 4 Evolution in time of the L^2 norms of the control and the state.

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