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Foreword

It is with great pleasure that we present the Proceedings of the 26th Congress of Differential Equations and Applications / 16th Congress of Applied Mathematics (XXVI CEDYA / XVI CMA), the biennial congress of the Spanish Society of Applied Mathematics S$\vec{e}$MA, which is held in Gijón, Spain from June 14 to June 18, 2021.

In this volume we gather the short papers sent by some of the almost three hundred and twenty communications presented in the conference. Abstracts of all those communications can be found in the abstract book of the congress. Moreover, full papers by invited lecturers will shortly appear in a special issue of the S$\vec{e}$MA Journal.

The first CEDYA was celebrated in 1978 in Madrid, and the first joint CEDYA / CMA took place in Málaga in 1989. Our congress focuses on different fields of applied mathematics: Dynamical Systems and Ordinary Differential Equations, Partial Differential Equations, Numerical Analysis and Simulation, Numerical Linear Algebra, Optimal Control and Inverse Problems and Applications of Mathematics to Industry, Social Sciences, and Biology. Communications in other related topics such as Scientific Computation, Approximation Theory, Discrete Mathematics and Mathematical Education are also common.

For the last few editions, the congress has been structured in mini-symposia. In Gijón, we will have eighteen minis-symposia, proposed by different researchers and groups, and also five thematic sessions organized by the local organizing committee to distribute the individual contributions. We will also have a poster session and ten invited lectures. Among all the mini-symposia, we want to highlight the one dedicated to the memory of our colleague Francisco Javier "Pancho" Sayas, which gathers two plenary lectures, thirty-six talks, and more than forty invited people that have expressed their wish to pay tribute to his figure and work.

This edition has been deeply marked by the COVID-19 pandemic. First scheduled for June 2020, we had to postpone it one year, and move to a hybrid format. Roughly half of the participants attended the conference online, while the other half came to Gijón. Taking a normal conference and moving to a hybrid format in one year has meant a lot of efforts from all the parties involved. Not only did we, as organizing committee, see how much of the work already done had to be undone and redone in a different way, but also the administration staff, the scientific committee, the mini-symposia organizers, and many of the contributors had to work overtime for the change.

Just to name a few of the problems that all of us faced: some of the already accepted mini-symposia and contributed talks had to be withdrawn for different reasons (mainly because of the lack of flexibility of the funding agencies); it became quite clear since the very first moment that, no matter how well things evolved, it would be nearly impossible for most international participants to come to Gijón; reservations with the hotels and contracts with the suppliers had to be cancelled; and there was a lot of uncertainty, and even anxiety could be said, until we were able to confirm that the face-to-face part of the congress could take place as planned.

On the other hand, in the new open call for scientific proposals, we had a nice surprise: many people that would have not been able to participate in the original congress were sending new ideas for mini-symposia, individual contributions and posters. This meant that the total number of communications was about twenty percent greater than the original one, with most of the new contributions sent by students.

There were almost one hundred and twenty students registered for this CEDYA / CMA. The hybrid format allows students to participate at very low expense for their funding agencies, and this gives them the opportunity to attend different conferences and get more merits. But this, which can be seen as an advantage, makes it harder for them to obtain a full conference experience. Alfréd Rényi said: "a mathematician is a device for turning coffee into theorems". Experience has taught us that a congress is the best place for a mathematician to have a lot of coffee. And coffee cannot be served online.

In Gijón, June 4, 2021

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SEIRD model with nonlocal diffusion

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Abstract

Some non-local diffussion processes may be described by an integral operator such as $\int K(x,y)(u(y)-u(x))dy$. In the compartmental models in epidemilogy, the diffusion terms may be replaced by an integral term, similar to above, to describe the dispersion of population. Classical numerical approximations, like finite element methods or finite difference methods, lead to systems of equations with dense matrices. Wavelets are a kind of functions which have some properties that make them a very useful tool for discretizing the convolution operators getting sparse matrices, thus saving computing time and storage memory.

1. Introduction

From the classical theoretical works on epidemic models by Kermack and McKendrick (1927), a big effort has been made to apply mathematics to the spread and control of infectious diseases (see, for instance, [5]). In particular, the called compartmental models distribute population in compartments and explain the mechanism through which individuals move from one compartment to another.

The spatial spread of the disease is incorporated in the models through diffusion terms that describe the local movement of population. The nonlocal diffusion models are more realistic, but when we discretize these models with classical methods as finite element or difference methods led to algebraic systems with large and dense matrices. We can circunvect this problem using a specific kind of functions, wavelets, to discretize the integral operators. Due to some of the properties of these functions, if the integral operators show certain properties, most of the matrix elements that we get from the discretization process are very close to zero and may be neglected, with the consequent saving in time and computer storage.

2. Multiresolution Analysis and wavelets

We give a brief review of Multiresolution Analysis, which is the starting point for constructing wavelets, and we show some relevant properties that explain why wavelets play a definitive role in the discretization of some kind of problems. For details, see, for instance, [2, 3].

Let us consider a sequence of spaces $\{V_j\}_{j\in\mathbb{Z}}$, that we will use to approximate functions with a level of resolution j, so we need a basis of functions to generate those spaces.

Definition 2.1 A sequence of spaces $\{V_j\}_{j\in\mathbb{Z}}$ of spaces $V_j \subset L_2(\mathbb{R})$ is called a Multiresolution Analysis, if

a) The spaces are nested, i.e. $V_j \subset V_{j+1}$, so the information contained in some level of resolution is also contained in the finer.

b) Any function may be approximated with arbitrary precission, i.e. $\lim_{j\to\infty} \|f - P_j f\|_{L^2} = 0$

c) The only common function is 0, i.e. $\lim_{j\to-\infty} \|P_j f\|_{L^2} = 0$

d) The spaces arise by scaling with factor 2: $f(\cdot) \in S_j \Leftrightarrow f(2\cdot) \in S_{j+1} \Leftrightarrow f(2^{-j}\cdot) \in S_0$

e) The spaces are shift-invariant: $f(\cdot) \in S_0 \Leftrightarrow f(\cdot - k) \in S_0$

f) $\exists\,\varphi$ s.t. $\Phi_j := \{\varphi_{j,k} : k \in \mathbb{Z}\}$ is a uniformly stable basis for V_j, $j \in \mathbb{Z}$, being $\varphi_{j,k}$ the dilated and traslated version of some function φ to be determined:

$$\varphi_{j,k}(x) = 2^{j/2}\varphi(2^j x - k) \tag{2.1}$$

From the condition a) and f) in definition 2.1, the function that is going to be the basis for the approximation space V_j have to verify the following recursion formula

$$\varphi(x) = \sum_k a_k \varphi(2x - k) \tag{2.2}$$

The function $\varphi(x)$ is called a scaling function or refinement function and its properties are determined by the refinement coefficientes $a_k \in \mathbb{R}, k \in \mathbb{Z}$. The approximation of functions is carried out projecting the function onto the space V_j, such that

$$P_j : L_2(\mathbb{R}) \to V_j, \quad P_j f = \sum_k c_{j,k}\varphi_{jk}(x). \tag{2.3}$$

The difference between two consecutive levels of resolution is encoded by functions $\psi_{j,k}(x)$, which *complete* the information in V_j to achive the information in V_{j+1}. These functions are called wavelets and the space W_j generated by

$$\psi_{j,k} = 2^{j/2}\psi(2^j x - k) \tag{2.4}$$

is the complement of V_j in V_{j+1}, i.e. $V_{j+1} = V_j \oplus W_j$. As $W_0 \subset V_1$, wavelets also verify a recursion formula similar to (2.2),

$$\psi(x) = \sum_k b_k \varphi(2x - k) \tag{2.5}$$

where the coefficients b_k are determined by the coefficients a_k.

So, we have spaces V_j and spaces W_j such that

$$V_j = \text{span}\,\{\varphi_{j,\lambda}\}, \quad W_j = \text{span}\,\{\psi_{j,\lambda}\}, \tag{2.6}$$

and we can approximate f in V_J or we can carry out an initial approximation V_{J_0} and add *details* to achive J, using spaces W_j, projecting the function on

$$V_J = V_{J-1} \oplus W_{J-1} = V_{J-2} \oplus W_{J-2} \oplus W_{J-1} = \cdots = V_{J_0} \oplus \bigoplus_{j=J_0}^{J-1} W_j \tag{2.7}$$

therefore we approximate f with a coarse level J_0 and we add finer resolution j:

$$f_J = \sum_\lambda c_\lambda^{J_0}\varphi_{J_0,k} + \sum_{j=J_0}^{J-1}\sum_\lambda d_k^j \psi_{j,k}. \tag{2.8}$$

The next two results (proofs and details can be seen in [2, 3] and references there) about wavelets are the key for compression and for the potential of wavelets in numerical calculations:

Proposition 2.2 *If the scaling functions reproduce exactly a polynomial of order d, the associated wavelets have d vanishing moments*

$$M_r(\psi_{j,k}) = \int_\Omega x^r \psi_{j,k}(x)dx = 0 \quad 0 \le r < d \tag{2.9}$$

Proposition 2.3 *If ψ have d vanishing moments, then*

$$|d_k^j| \lesssim 2^{-js}\|f\|_{s;supp\ \psi_{j,k}} \tag{2.10}$$

where d_k^j are the coefficients in (2.8). As a consequence of (2.10), many of the coefficientes of wavelets are small in the case of functions that are *regular enough* and, therefore, may be neglected.

3. Nonlocal models

The classical approach to diffusion problems implies local effects; nonlocal evolution equations of the form

$$\partial_t u(x,t) = \int_\Omega K(x.y)u(y,t)dy - u(x,t) \tag{3.1}$$

take into account long range effects and have been used to model nonlocal diffusion processes, replacing the local term

$$\partial_t u(x,t) = \nabla(\nu\nabla u(x,t)) \tag{3.2}$$

by its nonlocal version (3.1), [1].

Discretization in time of (3.1) leads to

$$\frac{u^{n+1}(x) - u^n(x)}{\delta t} = \int_\Omega K(x-y)u^{n+1}(y) - u^{n+1}(x) \tag{3.3}$$

Let us consider the variational formulation of (3.3), writing functions $u^n(x)$ in wavelet basis

$$u^n(x) = \sum_\lambda d_\lambda^n \psi_\lambda(x) \tag{3.4}$$

and using as test function $\psi_\mu(x)$ (to simplify notation, we don'n distinguish here between scaling functions and wavelets, and include the level of resolution j in the index λ, which should be (j, λ))

$$\begin{aligned} &\sum_\lambda d^{n+1} \int_\Omega \psi_\lambda(x)\psi_\mu(x)dx - \sum_\lambda d^n \int_\Omega \psi_\lambda(x)\psi_\mu(x)dx \\ = \delta t \sum_\lambda d^{n+1} &\int_\Omega \int_\Omega \psi_\lambda(y)\psi_\mu(x)dxdy - \delta t \sum_\lambda d^{n+1} \int_\Omega \psi_\lambda(x)\psi_\mu(x)dx \end{aligned} \tag{3.5}$$

and in matrix form

$$[(1+\delta t)\mathbf{B} - \delta \mathbf{K}]\, d^{n+1} = \mathbf{B} d^n \tag{3.6}$$

where the matrix elements for $\mathbf{B}$ and $\mathbf{K}$ are respectively

$$B_{\lambda\mu} = \int_\Omega \psi_\lambda(x)\psi_\mu(x)dx \tag{3.7}$$

$$K_{\lambda\mu} = \int_\Omega \int_\Omega K(x-y)\psi_\lambda(y)\psi_\mu(x)dxdy \tag{3.8}$$

If the scaling functions (and so wavelets) have compact support, most of elements in (3.7) are equal to zero, whereas the matrix $\mathbf{K}$ is a dense matrix, what involves a long computing time and storage. Anyway, if the distributional kernel verifies that

$$|\partial_x^\alpha \partial_y^\beta K(x,y)| \lesssim \mathrm{dist}(x,y)^{n+\alpha+\beta} \tag{3.9}$$

it is shown that many of the $|K_{\lambda,\mu}|$ in (3.8) are neglected and the matrix $\mathbf{K}$ has a sparse pattern similar to the local operators

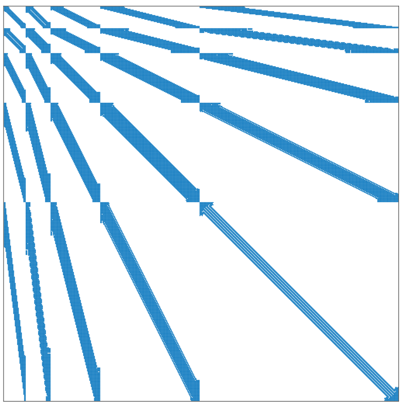


Fig. 1 Sparse pattern for non local operator

This fact, together with (2.10), makes wavelets a powerfull tool to design numerical algorithms to solve integro-differential equations wich come from the nonlocal diffusion models.

4. Epidemiological models

The starting point to describe transmission of diseases are the compartmental models by Kermack and McKendrick, in the final 20's and early 30's. These and later models share the idea of compartmental models, in which the population is distributed in compartments and may change between them under some conditions. In [6,7] a SEIRD (susceptible, exposed, infected, recovery and deceased) model is proposed, where the spatial spread of the infection is included trough a local diffusion term

$$\nabla \cdot (\nu_u \nabla u) \tag{4.1}$$

where u represents the population in any of the compartments before and ν_u is the diffusion coefficient for that population group. For a more realistic description of dispersion of the different groups of populations we can replace (4.1) by its non local version, as in (3.1)

$$\int_\Omega K(x-y)\,(u(y)-u(x))\,dy \tag{4.2}$$

where $K(x-y)$ represents the probability for the population to move from a location y to a location x. If the integral kernel $K(x-y)$ is normalized we can rewrite (4.2) by

$$\int_\Omega K(x-y)u(y)dy - u(x) = K*u - u \tag{4.3}$$

where $K*u$ is a convolution product. So, the equations for the nonlocal SEIRD model are

$$\partial_t S = K*S - S - \beta_i SI - \beta_e SE \tag{4.4}$$
$$\partial_t E = K*E - E + \beta_i SI + \beta_e SE - \sigma E - \phi_e E \tag{4.5}$$
$$\partial_t I = K*I - I + \sigma E - \phi_r I - \phi_d I \tag{4.6}$$
$$\partial_t R = K*R - R + \phi_e E + \phi_r I \tag{4.7}$$
$$\partial_t D = \phi_d I \tag{4.8}$$

where the $\{\beta_e, \beta_i\}$ are the contact rates of susceptible with exposed and infected, σ is the rate of exposed people who develops synthoms, $\{\phi_e, \phi_r\}$ are the rates of recovery for exposed and infected and ϕ_d is the rate of deceased people. The discretization in time leads to

$$\begin{aligned}\frac{S^{n+1}(x)-S^n(x)}{\delta t}&\\ &= \int K(x-y)S^{n+1}(y)dy - S^{n+1}(x) - (\beta_i I^n(x) + \beta_e E^n(x))\,S^{n+1}(x)\end{aligned} \tag{4.9}$$

$$\begin{aligned}\frac{E^{n+1}(x)-E^n(x)}{\delta t}&\\ &= \int K(x-y)E^{n+1}(y)dy - E^{n+1}(x) + (\beta_i I^n(x) + \beta_e E^n(x))\,S^{n+1}(x)\\ &- (\sigma+\phi_e)E^{n+1}(x)\end{aligned} \tag{4.10}$$

$$\begin{aligned}\frac{I^{n+1}(x)-I^n(x)}{\delta t}&\\ &= \int K(x-y)I^{n+1}(y)dy - I^{n+1}(x) + \sigma E^{n+1}(x) - (\phi_r+\phi_d)I^{n+1}(x)\end{aligned} \tag{4.11}$$

$$\begin{aligned}\frac{R^{n+1}(x)-R^n(x)}{\delta t}&\\ &= \int K(x-y)R^{n+1}(y)dy - R^{n+1}(x) + \phi_e E^{n+1}(x) + \phi_r I^{n+1}(x)\end{aligned} \tag{4.12}$$

$$\frac{D^{n+1}(x)-D^n(x)}{\delta t} = \phi_d I^{n+1}(x) \tag{4.13}$$

Let us consider the weak formulation for (4.9)-(4.12) (once we know I we get D from (4.13)) and write the functions $\{S^n, E^n, I^n, R^n\}$ in terms of wavelet basis,

$$S^n(x) = \sum_\lambda s^n_\lambda \psi_\lambda(x), \quad E^n(x) = \sum_\lambda e^n_\lambda \psi_\lambda(x), \tag{4.14}$$
$$I^n(x) = \sum_\lambda i^n_\lambda \psi_\lambda(x), \quad R^n(x) = \sum_\lambda r^n_\lambda \psi_\lambda(x) \tag{4.15}$$

we get

$$[(1+\delta t)\,\mathbf{B} - \delta t\mathbf{K}]\,s^{n+1} = \mathbf{B}s^n - \delta t\mathbf{F} \tag{4.16}$$
$$[(1+\delta t(1+\sigma+\phi_e)\,\mathbf{B} - \delta t\mathbf{K}]\,e^{n+1} = \mathbf{B}e^n + \delta t\mathbf{F} \tag{4.17}$$
$$[(1+\delta t(1+\phi_r+\phi_d)\,\mathbf{B} - \delta t\mathbf{K}]\,i^{n+1} = \mathbf{B}i^n + \delta t\sigma\mathbf{B}e^{n+1} \tag{4.18}$$
$$[(1+\delta t)\,\mathbf{B} - \delta t\mathbf{K}]\,r^{n+1} = \mathbf{B}r^n + \delta t\mathbf{B}(\phi_e e^{n+1} + \phi_r i^{n+1}) \tag{4.19}$$

where $\mathbf{F}$ in (4.16) and (4.17) is

$$\mathbf{F}_\mu = \int_\Omega p^{n+1} \psi_\mu(x) dx \tag{4.20}$$

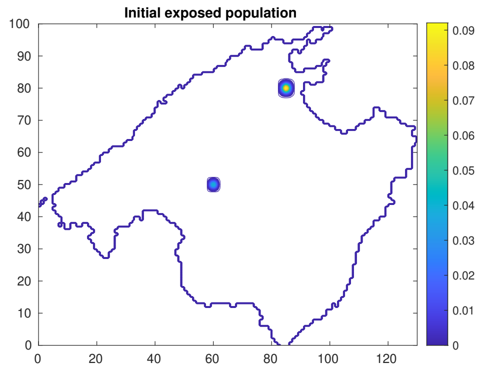
for the coefficients p^{n+1} for the expansion of

$$(\beta_i I^n(x) + \beta_e E^n(x))\, S^{n+1}(x) = \sum_\lambda p^{n+1} \psi_\lambda(x) \tag{4.21}$$

(this last term is treated with an implicit scheme for (4.16) and implicit in (4.17)).

5. Numerical test

To test the numerical algorithms, we have solved the nonlocal SEIRD model in Mallorca Island (Spain), to compare with the local model solved in [4]. We have considered two focus of asynthomatic people concentrated in a short distance, modeled by gaussian functions, as in the figure below,



and the dispersal kernel

$$K(x.y) = e^{-\|x-y\|^2/d^2}, \tag{5.1}$$

being d a kind of mean distance that individuals from different compartments *travel*.

Figures below show the evolution of infected people in different moments, comparing the local SEIRD model with the nonlocal with different values of d; as it is seen, longer is d, faster the epidemic spreads.

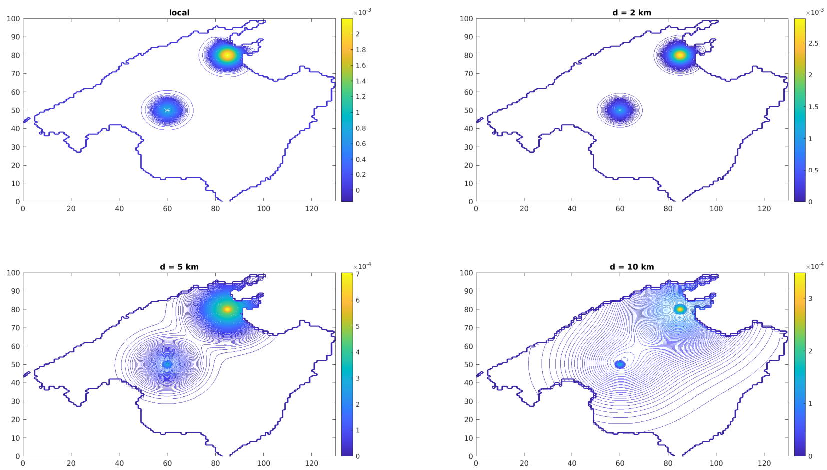


Fig. 2 infected population after 5 days

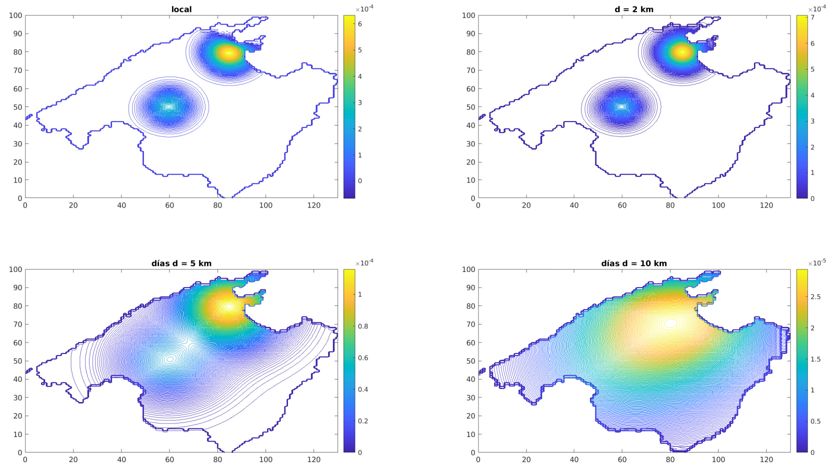


Fig. 3 infected population after 15 days

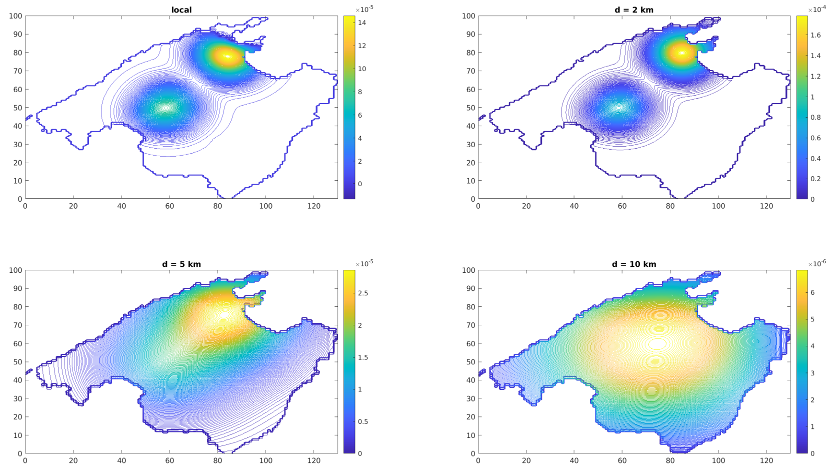


Fig. 4 infected population after 30 days

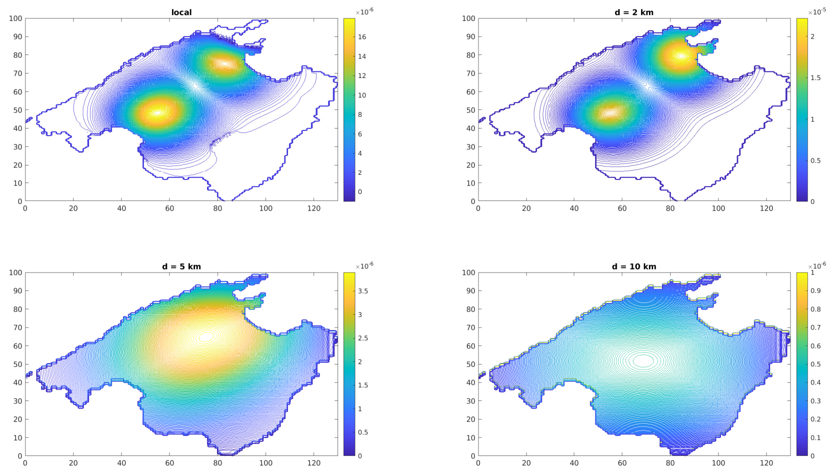


Fig. 5 infected population after 60 days

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