RETROEXTRAPOLATION OF CRACK GROWTH CURVES USING PHENOMENOLOGICAL MODELS BASED ON CUMULATIVE DISTRIBUTION FUNCTIONS OF THE GENERALIZED EXTREME VALUE FAMILY

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ABSTRACT

The evolution of crack lengths under fatigue load is interpreted as a damage process that, once normalized, allows a phenomenological fitting using cumulative distribution functions (cdfs) of the generalized extreme value family (GEV). In this work, the denoted "back-extrapolation" procedure is applied to determine the complete evolution of the a-N crack growth curve from the final fragment of the curve recorded during the test. The proposed methodology ensures excellent fitting of a-N curves, its conversion to fatigue crack growth rate curves (CGR) and as a result, the analysis of the transition from micro- to macrocracks in the material tested, depending on the type of transition and of the resulting parameters. It also allows a probabilistic analysis on the prediction of fatigue life originated from random initial sizes of cracks, smaller or larger than those induced in the experimental program to determine the a-N curve. In this way, a statistically reliable basis is provided for the application of the damage tolerance concept to the practical component design.

KEYWORDS: a-N curves, retroextrapolation, initiation-propagation model, damage tolerance design

NOMENCLATURE

a: generic crack size

a_c: critical crack size

$a_{i,t}$: pre-crack size for the test

a_0: crack size predicted from the model as the original maximal crack size for the un-cracked specimen

a_{ref}: reference crack size

a_{up}: asymptotic fatigue crack size

cdf: cumulative distribution function

p: probability in the cumulative distribution function

CGR: crack growth rate

GEV: generalized extreme value

da/dN: crack grow rate

N: generic number of cycles

N_g: number of cycles hypothetically applied for the crack to grow hypothetically from a_0 to a generic crack size a

N*: normalized number of cycles

N_{add}: predictive number of cycles for the crack to grow hypothetically from a_0 to pre-crack size ai,t

N_t: predictive number of cycles for the crack to grow hypothetically from a_0 to the critical crack size a_c

N_{test}: number of cycles applied in the real test for the crack to grow from $a_{i,t}$ to the critical crack size a_c

N_{up}: predictive number of cycles for the crack to grow hypothetically from a_0 to the asymptotic crack size a_{up}

ΔN_{up}: number of cycles applied in the real test for the crack to grow from a_c to the asymptotic crack size a_{up}

ΔK: stress intensity factor range

ΔK_{th}: threshold value of the stress intensity factor range. It is referred to the long-crack regimen in this work.

SIF: stress intensity factor

R: stress ratio

R^2: correlation coefficient

S-N field: Wöhler field

δ: scale parameter of the Weibull distribution

β: shape parameter of the Weibull distribution

INTRODUCTION AND MOTIVATION

Despite the apparently simple shape of the *a*-N curves that provide the evolution of crack growth under fatigue conditions as a function of the number of cycles [1-3], no general analytical model has been found that confirms satisfactory fitting of this damage phenomenon. The use of hyperbolic or potential laws neither guarantees accurate fitting of the *a*-N curves with a reasonably low number of parameters, nor confirms the potential applicability of this type of proposals [4,5]. In fact, the results of the experimental campaigns sometimes contradict the usually expected monotonically increasing growth of cracks, evidencing a changing trend, possibly of sigmoidal type, in the initial phase of the curve, which apparently corresponds to the growth of the microcrack phase (see [6-8]). This could point out that fatigue crack growth processes can exhibit potentially different evolution, depending on the relative weight represented by the initiation phase (microcracks) with respect to the propagation one (macrocracks) in these processes. On the contrary, to fit the experimental fatigue crack growth rate (CGR) curves, phenomenological models based on cumulative distribution functions (cdfs) of the generalized extreme value (GEV) family in particular Gumbel distributions, are proposed by Castillo et al. [9]. This allows the complete evolution of the CGR process, that is, the da/dN-ΔK curve, to be faithfully reproduced, without having to resort to subterfuges to achieve the asymptotic matching of both domains I and III of the CGR curve, such as those provided by NASGRO type models [10,11], which evidence dubious effectiveness and lack physical justification and interpretation. However, the expected correspondence in the conversion between both *a*-N and CGR curves related to crack growth, is unfortunately not confirmed, which again point out the need to find an alternative model for providing analytical solution of the *a*-N curve.

In this work a possible solution to the problem is proposed based on the application of phenomenological models, using cdfs of the GEV family to achieve the fit of the complete *a*-N curve including the prospective phase supposedly preceding the recorded *a*-N curve in the crack growth test. This succeeds by means the retro-extrapolation of registered *a*-N curve fragments to the initial phase of the crack growth process, whose experimental measurement is difficult or even impossible. A dual concept of the stochastic nature of damage is assumed. On the one side, this model provides realistic information on the crack growth, as damage evolution, which depends on the initial predictive crack size but also on the microstructure of the material. This evolution can be in certain sense normalized (according to the uniformity of the model parameters estimated in each sample). On the other side, the final fatigue life exhibits a statistical distribution which can be contemplated as the variability of the normalizing parameter.

The quality of fitting and the versatility of the approach proposed, proves the universality of application and its reliability when applied to the definition of the crack growth process. Furthermore, it contributes to explain and describe analytically the transition between the initiation and propagation processes and to establish the necessary correspondence between the *a*-N and the CGR curves.

Note that the assessment of *a*-N results including the corresponding lifetime estimation based on the recorded *a*-N curve during the test, does not provide information about the lifetime distribution, which would result from the real crack growth process originated from the original material, i.e. without pre-crack, see [7].

RETROEXTRAPOLATION MODEL OF THE *a*-N CRACK GROWTH CURVE

In trying to resolve the apparently contradictory situation in the correspondence between crack growth and crack growth rate models, the experimental results of the *a*-N crack growth curves under cyclic loading can be understood as a censored record that represents only one extract or window of a hypothetical complete *a*-N curve, similar to that of other damage processes of different nature observed in metals and other materials [13]. Regardless of the corresponding peculiarities of each phenomenon, crack growth would represent a "pre-determined" evolution of damage, that could be denoted genetic heritage, as established by the micromechanical characteristics of the material. This complex micromechanical state defines the succession of local failures as a phenomenon governed by a statistical law of extreme values, applicable to a large number of primary elements that make build the material bulk in the test specimen or component.

It is assumed that the advanced phase of the crack growth is in some way the result of the preceding former crack growth phase and as such deterministically defined. The recorded monotonically increasing *a*-N curve during the test should be simply understood as the final phase of the entire crack growth damage process. In this way, the complete crack growth evolution supposedly comprises also the potential description of that crack growth phase corresponding to the crack initiation whose record is difficult or even impossible. The recovering of that early information can be achieved by recursive extrapolation of the recorded test data, denoted “retroextrapolation”, assuming a predefined type of trial functions. Because the positive experience gained in the analysis of CGR curves, see [9, 13,14], and their similarity with the *a*-N curves as representing the same phenomenon, the use of trial functions as cdfs of the GEV family could be “a priori” adequate candidates. Their suitability should be subsequently and firmly confirmed by the corresponding fitting and parameter analysis. studied. As it will be seen later, the morphology of the complete *a*-N curves that result from the fit will be, or not, of sigmoidal type, depending on the corresponding parameter, β, estimated in each case.

Ultimately, the above considerations imply that the experimental *a*-N curve recorded during the test only reflects a censored final fragment of the whole, virtually retroextrapolated, crack growth *a*-N curve. In Fig. 1 one possible evolution of the *a*-N curve of the material characteristic for the applied loading conditions and initial retrospective crack size a_0 is shown. It highlights the disparity between the *a*-N curve recorded during the test and the predictive total crack growth process.

MODEL DERIVATION

The evolution of the *a*-N curves from different tests of the same material sample seems to obey a statistical law, not sufficiently understood until now. Some recent works confirm an alteration of the traditional shape in the initial phase of the *a*-N curves, while other authors point out a possible sigmoidal evolution of this accumulated damage phenomenon [7,8,15,16]. If the number of cycles divided by the fatigue life until failure, $N^* = N/N_F$, is entered as a normalized variable, its range of variation is limited between 0 and 1, which besides the monotonically increasing evolution of the *a*-N* curve, allows this to be identified, by definition, as a distribution function. If the crack growth phenomenon is understood as a process of successive local failures at the crack front implying a large number of primary elements with which the crack front can be identified along the crack growth process, it is amenable to propose cdfs belonging to the generalized extreme value (GEV) family, that is, Weibull, Gumbel or Fréchet distributions [17,18] to fit the *a*-N* curves.

The justification for identifying the problem as a case of maximum or minimum values is still under discussion. However, since the initial stage of the crack growth process seems to exert the most significant influence on fatigue life, the family of minima is initially proposed as the most suitable for this function. Among the three possible distributions of the GEV families, the necessary existence of a lower limit of the crack size, which in no way can be negative, supports minimal Weibull distribution as the suitable option. On the other hand, the Weibull distribution for minima advocates an unlimited domain of the analyzed variable (in this case, crack length) at the upper tail that could hypothetically reach an infinite value. In practice, the length of the crack can never exceed the dimensional limits of the specimen or component, so that the process is interrupted prematurely, anyway with a negligible impact in terms of the actual or theoretical number of cycles until failure.

The general Weibull equation, proposed to model the complete *a*-N curves, depends on the 3 Weibull parameters (location, a_0; scale, δ and shape, β, parameters, respectively) plus one additional normalizing parameter representing the virtual total number of cycles, N_t, applied for the crack to grow from the hypothetical original material state, characterized by the retrospective crack size, a_0, up to the asymptotic crack size a_{up}, according to Expr. (1), see Fig. 1:

$$N_{up} = N_{add} + N_{test} + \Delta N_{up} = N_t + \Delta N_{up} \qquad (1)$$

where N_{add} represents the predictive number of cycles required for the crack to grow virtually from the retrospective size, a_0, up to the pre-crack size, $a_{i,t}$; N_{test}, is the number of cycles associated with the crack growth from $a_{i,t}$ to the critical crack size at failure, a_c; and $\Delta N_{up}=N_{up}-N_t$, represents the number of cycles elapsed for the crack to grow hypothetically from a_c to the asymptotic (virtual) crack size value, a_{up}, in consonance with the asymptotic CGR model, see [13]. Note that N_{add} the same as ΔN_{up} are not verifiable from the test but are determined by fitting the test results to the proposed Weibull model. Note also that the *a*-N test curve, although provides only partial information of the crack growth evolution, allows the whole virtual crack growth to be determined.

According to above, the normalized number of cycles, N*, can be expressed as the Weibull cdf given by Eq. (2), which, once fitted, provides the total crack growth evolution:

$$N^* = \frac{N}{N_{up}} = 1 - exp\left\{-\left[\frac{(a-a_0)}{\delta}\right]^{\beta}\right\}, \qquad (2)$$

where N represents the virtual number of cycles applied to the original material when $N<N_{add}$, whereas it represents N_{add} plus the number of cycles applied at a certain test time where $N>N_{add}$.. From a dimensional viewpoint, the crack length is not necessarily to be converted in a non-dimensional variable as long as the crack length and the location and scale parameters are coherently measured in the same units as the crack length *a*, while the shape parameter remain non-dimensional.

In this way, the general model a-N, according to equation (2), depends on four physically interpretable parameters: a_0, δ, β and N_{add}, whereas N_t is found from Expr. (1).

Before the test, the specimens are pre-cracked with sufficient sharpness so that initial conditions, similar to those present in the real component exposed to a fatigue crack growth process, prevail at the crack front of the machined notch. Consequently, even if the initial number of cycles in the actual test is theoretically zero, a previous number of cycles must be considered as it would be required to promote crack progression from the retrospective original crack size of initial length, a_0 (see Fig. 1).

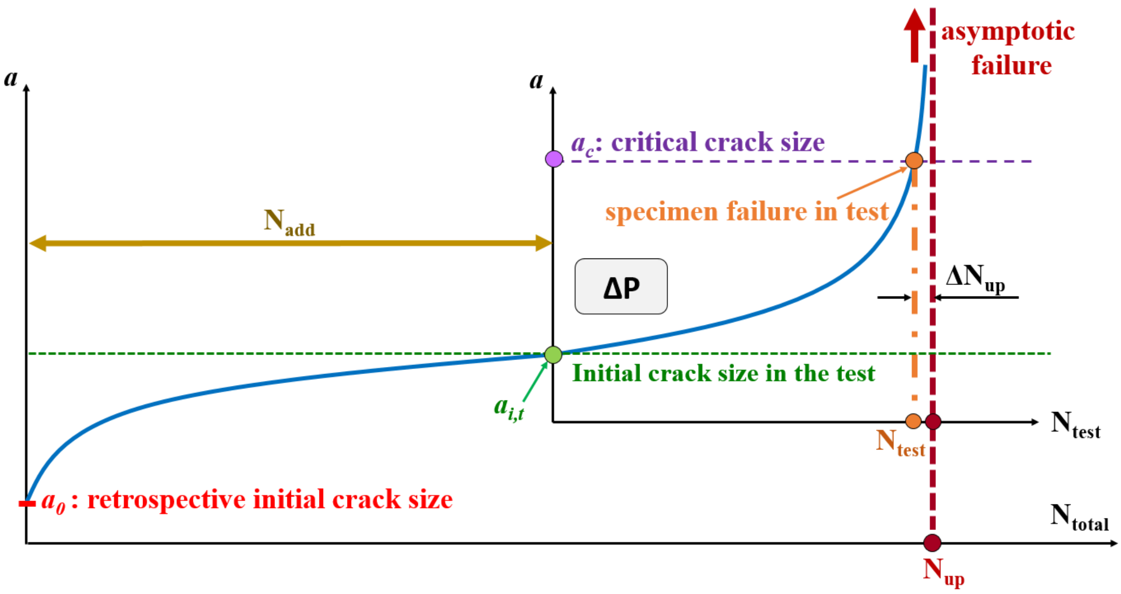


Figure 1. Schematic representation of the complete crack growth curve a-N defined from the initial retrospective crack size a_0 by retro-extrapolation

The feasibility of physical interpretation of the fitted model parameters represents a significant advance over any alternative based on empirical models, whose parameters cannot be identified as physical variables.

A conventional approach to the model would entail the following interpretations of the parametric variables:

- a_0 is the location parameter identified as the initially natural, not machined, "retrospective" crack size of the analyzed sample, which is not identifiable with the pre-crack generated into each specimen. Thus, the whole crack growth is represented by the extended a-N curve which starts from a certain initial crack size, a_0, under the same load conditions, ΔP, applied in the test. The existence of a lower limit of the crack size value is physically acceptable, since it is not possible to assign negative values to the crack size.
- δ is the scale factor related to the geometry and the sample size. It is also influenced by the load value according to the stress or deformation distribution.
- β is the shape parameter related to the crack growth mechanism. It depends on the type of the characteristic defect acting as dominant fatigue crack growth mechanism for the given load conditions. According to the proposed distribution in Eq. (1), the extended, i.e. retro-extrapolated, a-N curve adopt a sigmoidal, monotonic increasing with curvature change for $\beta > 1$, monotonic increasing without curvature change for $\beta < 1$, and exponential shape for $\beta = 1$.

EXAMPLE OF APPLICATION

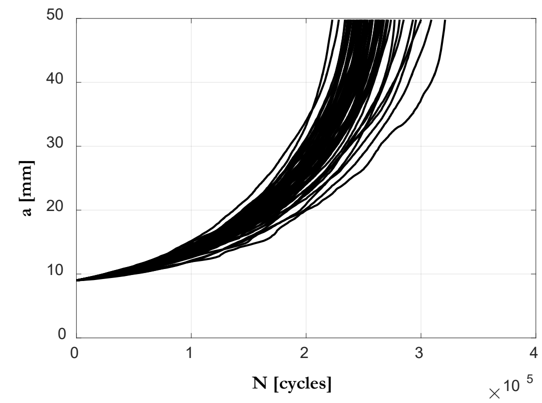
In this section, the proposed model is applied to the assessment of the results of extensive experimental campaigns carried out by Winkler et al. [1], Wu and Ni [2], and Ghonem and Dore [3] about crack growth carried out with different Aluminum alloys and specimen geometries, see Table 1.

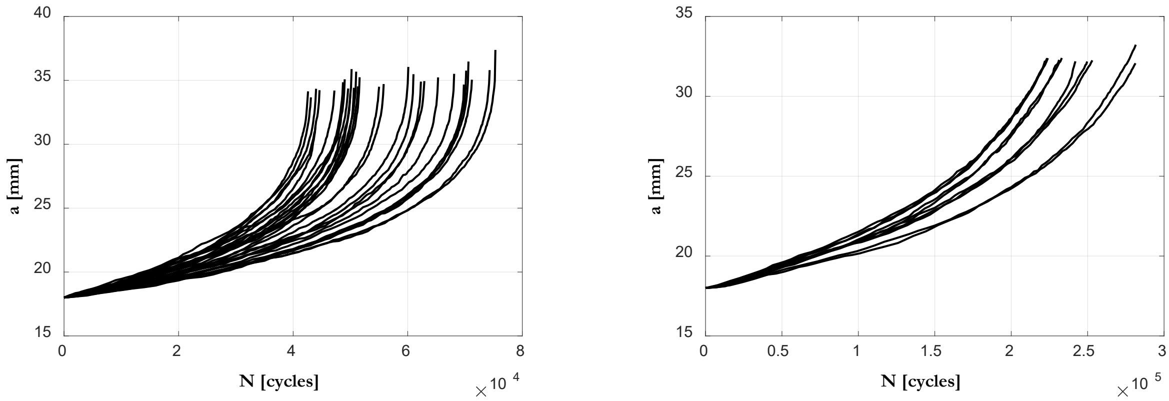
Table 1. Summary of technical data (material and specimen geometry) referred to the experimental test campaigns of Virkler [1], Wu and Ni [2] and Ghonem and Dore [3].

Research Program	**Material**	**Specimen geometry**	**W [mm]**	**a_0 [mm]**	**ΔP [kN]**	**$R = P_{min}/P_{max}$**
Virkler et al.	AA 2024-T3	MT	152.4	9	18.69	0.2
Wu-Ni	AA 2024-T351	CT	50.0	18	3.6	0.2
Wu-Ni	AA 2024-T351	CT	50.0	18	2.24	0.63
Ghonem-Dore	AA 7075-T6	MT	101.6	9	18.60	0.4
Ghonem-Dore	AA 7075-T6	MT	101.6	9	21.60	0.5
Ghonem-Dore	AA 7075-T6	MT	101.6	9	17.65	0.6

In the three experimental campaigns (Virkler et al. [1], Wu et al [2] and Ghonem et al. [3]), the same initial pre-crack size, $a_{i,t}$, was generated before the monitored fatigue test was carried out. The typical evolution of the crack growth curves under constant load conditions, as a function of time, are shown in Fig. 2, which summarize the results corresponding to those three extensive experimental works. Fig. 2 evidences the remarkable dispersion of the results of fatigue crack growth of the different test samples, despite the homogeneity of the material and the theoretically identical initial crack size, $a_{i,t}$, in each test.

a)





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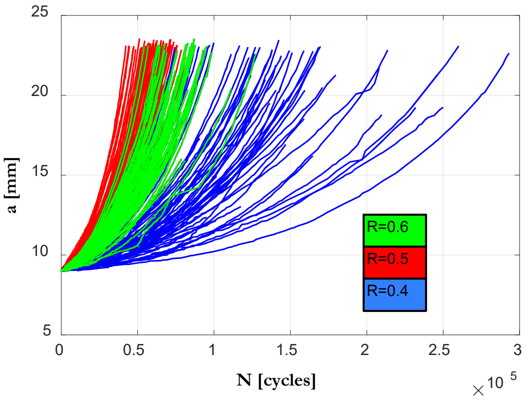


Figure 2. Experimental a-N curves from the research campaigns of a) Virkler et al. on AA 2024-T3 (from [1]); b) Wu and Ni on AA 2024-T351 (for R=0.2 -left- and 0.63 -right-) (from [2]));c) Ghonem and Dore on AA 7075-T6 (for R=0.4, 0.5 and 0.6) (from [3]).

Consequently, the variability of the fatigue lives due to crack growth for the specimens of the same homogeneously manufactured material sample, can be attributed to two factors: a) the retrospective initial crack size, a_0, deductible from the recorded fragment of the a-N curve in each test specimen starting with the initial crack size, $a_{i,t}$, presumably identical for each sample and b) the influence of the microstructural peculiarities of each specimen. Hence the interest to define the whole a-N curve, that is, the complete evolution of the crack from the original finishing state of the specimen, prior to pre-cracking, characterized by a_0, up to the critical crack size at failure, a_c, by retro-extrapolation based on application of the model given by Eq. (2) to the partial a-N record during the test. The specimen geometry and loading conditions are assumed to be constant.

According to the noticeable goodness of the fitting results for the complete a-N curves, Figs. 3-5, the suitability of the choice of the Weibull functions is confirmed as a predefined function law type, from the very beginning of the crack growth process. In the absence of a statistically sound justification, an alleged "microstructural genetic heritage" is invoked to explain this predetermined evolution of crack growth, from the retrospective initial size, a_0, for each sample analyzed. This "microstructural genetic heritage" would result from the probabilistic behavior at failure, given as a Weibull function of minima, of high number of “primary elements”, being determinant at successive crack fronts. Their failure strength properties are randomly distributed, as a result of the complex and unique state characterized by the microstructure of that specimen, as determined by grain size distribution, dislocations, inclusions, etc., see [19]. This failure behavior and statistical background, which is to be understood as a stochastic sample function (see [20-22]), is also applied to the fitting of the CGR curves, see [13,14], ensuring unchanged parameters throughout the crack growth process and explains the conceptual difference between a phenomenological approach and simply empirical fitting.

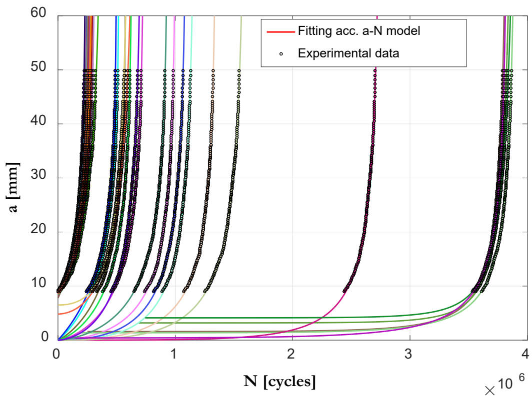
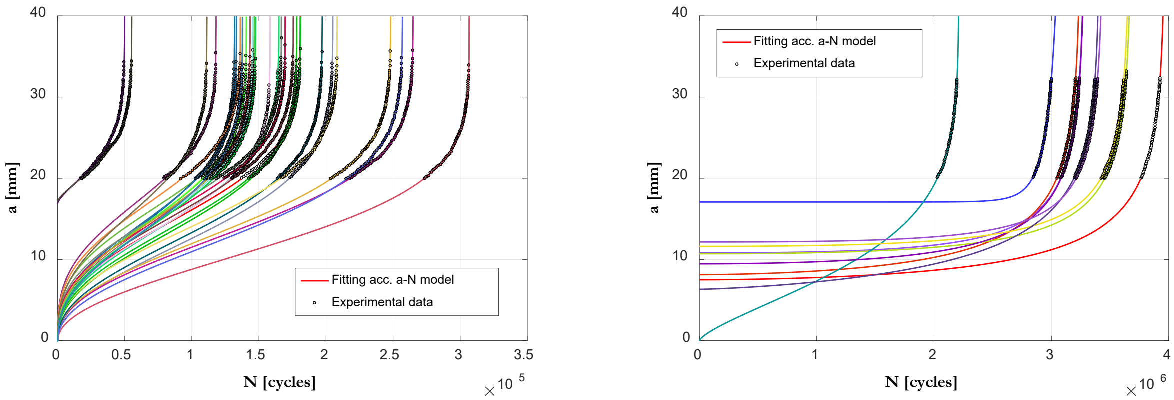


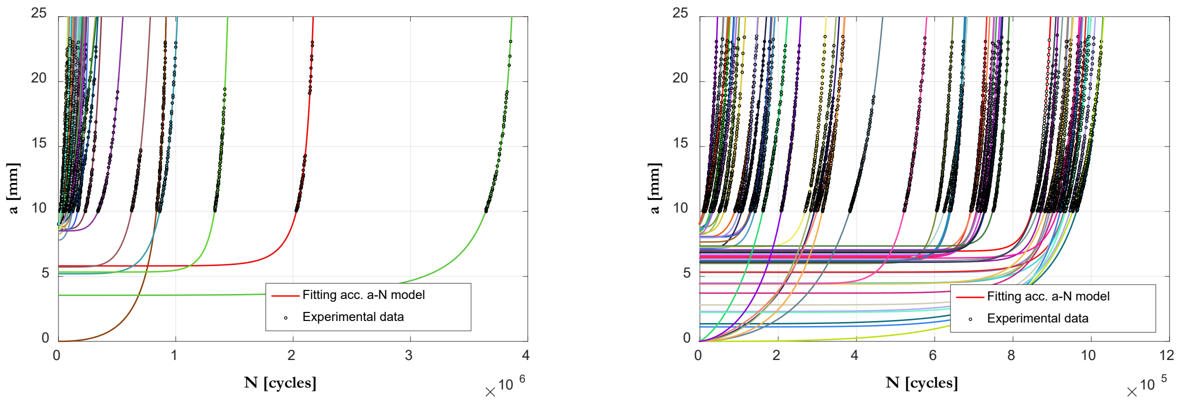
Figure 3. a-N curves fitted with the extrapolation approach from the research campaigns of a) Virkler et al. on AA 2024-T3 (from [1])



a)

b)

Figure 4. a-N curves fitted with the extrapolation approach from the research campaigns Wu and Ni on AA 2024-T351: a) for R=0.2; b) for R= 0.63) (from [2]))



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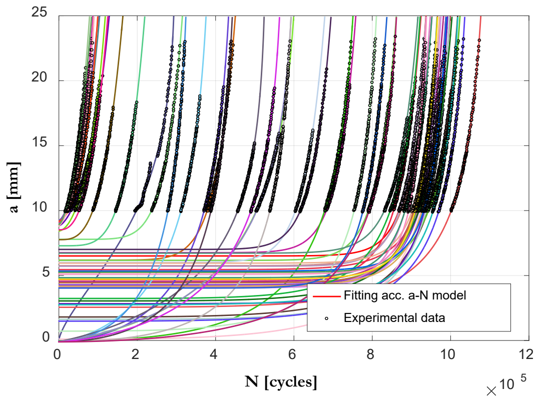


Figure 5. a-N curves fitted with the extrapolation approach from the research campaigns of Ghonem and Dore on AA 7075-T6: for a) R=0.4; b) for R= 0.5; c) for R=0.6) (from [3]).

Figures 6-8 summarize the values of the fitting parameters (a_0, δ, β) and the R^2 values corresponding to each sample. For the sake of highlighting the fitting goodness, the latter are represented on a scale very close to unity. The excellent results obtained when applied the retroextrapolation fitting (see Eq. 2) to the experimental data of Virkler et al. [1] (Fig. 6), Wu et al [2] (Fig. 7) and Ghonem et al. [3] (Fig. 8) confirm the suitability of the proposed model as a cdf of the GEV family to describe the damage process from the initial retrospective crack size, as regards of the quality of the fits practically for all the samples, as measured by the correlation coefficient R^2 (higher than 0.999 in most evaluations), the uniformity of the parameter values within each sample.

The fundamental role of the parameter denoted N_{add} is emphasized by the restoration of the whole crack growth process, that is, of the *a*-N curve built up from its origin a_0. When analyzing associated with each experimental curve, a moderate variation is observed in the values of the remaining parameters fitted in each sample, see Figs. 6-8, which confirms the consistency in the fitting proposal.

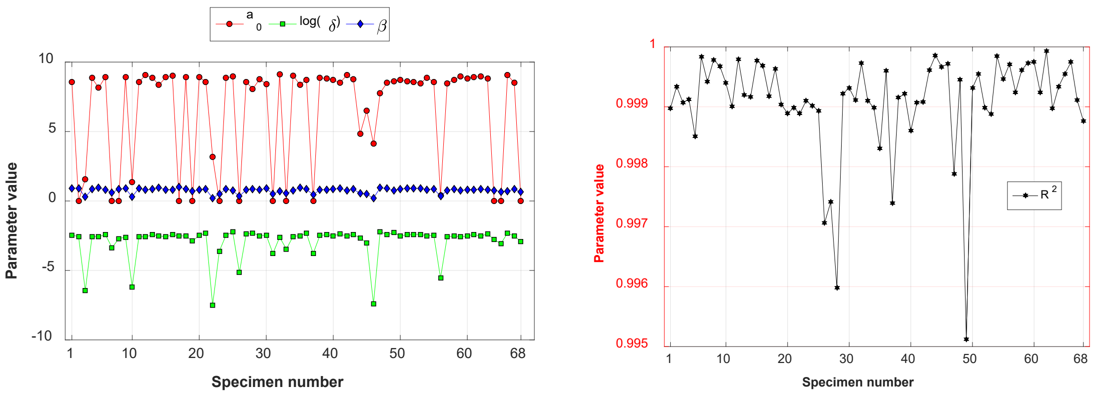
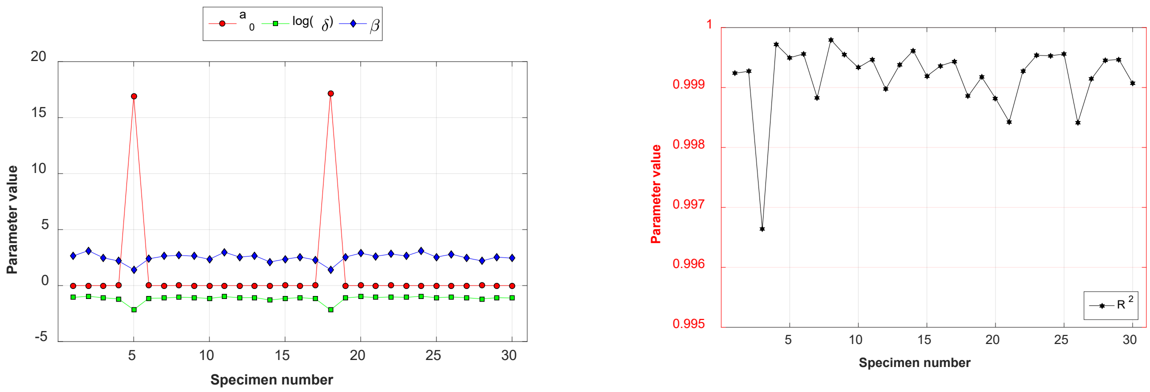


Figure 6. Fitting results of the Weibull parameters (left) and R^2 factor (right) using the retroextrapolation approach from from the research campaigns of Virkler et al. on AA 2024-T3 (from [1])



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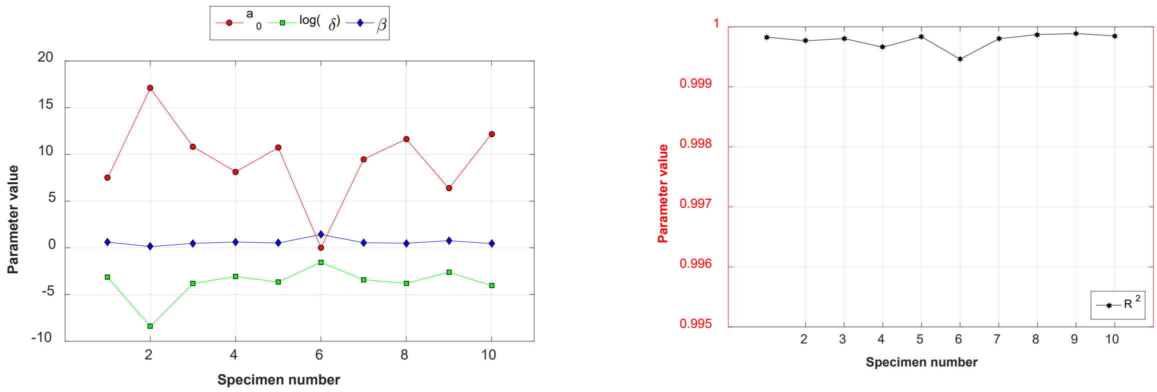
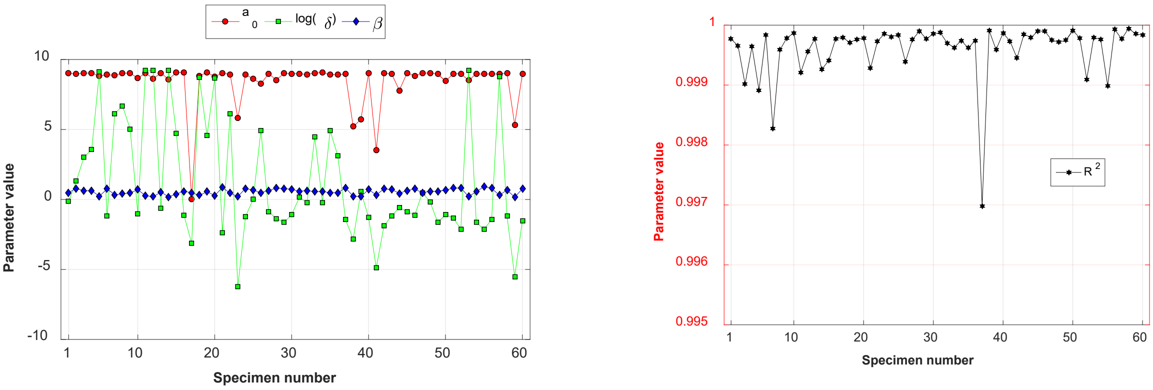


b)

Figure 7. Fitting results of the Weibull parameters (left) and R^2 factor (right) using the retroextrapolation approach from the research campaigns Wu and Ni on AA 2024-T351: a) for R=0.2; b) for R=0.6 (from [2]))



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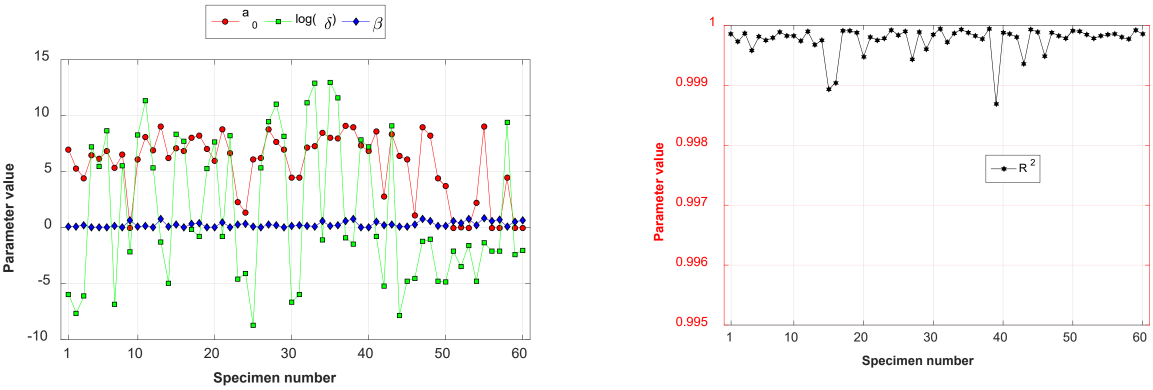

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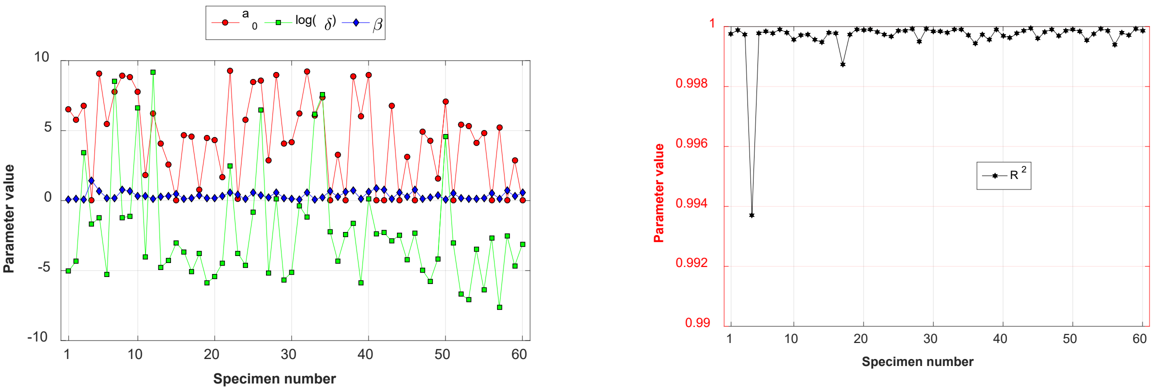


c)

Figure 8. Fitting results for the Weibull parameters (left) and R^2 factor (right) from the research campaigns of Ghonem and Dore on AA 7075-T6: a) for R=0.4; b) for R=0.5 and c) for R=0.6 (from [3])

Once all the *a*-N curves of a certain sample are fitted, the model allows the expected lifetime for each specimen to be estimated as well as the probabilistic distribution of lifetimes in a sample to be assessed in all three possible cases. a) for the hypothetical initial intrinsic crack size, a_0; b) for the actually pre-crack size used in the test, $a_{i,t}$ and c) for any test condition concerning any else initial limiting crack size, a_{lim}, smaller or larger than $a_{i,t}$. The latter case has immediate application in the damage tolerance design. On the other hand, once the results have been converted to the d*a*/dN-ΔK field, it is possible to proceed with the analysis of variability of the threshold value of the stress intensity factor (SIF) range, ΔK_{th} (respectively of $K_{max,th}$) from the values of a_0 and its distribution, for the load conditions applied.

The subjacent statistical background is corroborated by the narrow range of values exhibited by the model in the three campaigns perfomed by Wirkler [1], Wu and Ni [2] and Gohnem and Dore [3] The analysis of the experimental results obtained for the different samples confirms the validity of the model, specially, taking into account the generality of the model to quality to fit the experimental results despite the diversity of the samples tested in what concerns materials and specimen geometries. The possible interdependence between the anomalous values of the fitting parameters accentuated the model validity and suitability. The shape parameter, *β,* is accepted as a significant factor, which informs about the influence of the microstructural characteristics and the type of the particular defect responsible for the successive crack growth, i.e. failure of the subsequent specimen link at the crack front.

CORRESPONDENCE BETWEEN *a*-N AND CGR CURVES

The need for correspondence between the *a*-N and CGR curves, as representation of the same phenomenon from different perspectives, is evident. The analysis of this relationship could help to clarify the transition between micro- and macro-cracks and, consequently, between the initiation and propagation phases. Eq. (3), derived from (2), allows the crack size to be expressed as a function of the number of cycles, N, and its derivative with respect to N, leading to the definition of the crack growth rate law as a function of the number of cycles, see Eq. (4).

$$a = a_0 + \delta\left[-\log\left(1-\frac{\mathrm{N}}{N_{up}}\right)\right]^{1/\beta} \tag{3}$$

$$\frac{\mathrm{da}}{\mathrm{dN}} = \frac{\delta[-\log(1-\mathrm{N}^*)]^{\frac{1}{\beta}-1}}{\mathrm{N}_{up}\beta(1-\mathrm{N}^*)} \tag{4}$$

In parallel, each crack length value, *a*, can be analytically related to the SIF range, ΔK, by the well-known relation, ΔK = f (ΔP, *a*, g (*a*/W)) once the specimen geometry and the applied load, are known. In this way, the analytical formulation of the CGR is achieved from the proposed *a*-N model.

Following this approach, the CGR curves can be directly obtained from the retroextrapolated *a*-N curves evaluated in the previous section, see Figs. 9-11. Two alternatives in the model parameter estimation are envisaged in this work: a) the standard procedure assuming free estimation of the *β* parameter and b) assuming a prefixed representative value of the *β* parameter as resulting from a). According to this procedure, the crack rate behavior below the corresponding threshold for long-cracks regime, ΔK_{th}, can be retrospectively deduced. A reduction of growth rates for increasing values of ΔK, until reaching a minimum from which the trend reverses towards the long-cracks regimen, is observed in several cases. This is consistent with the typical evolution from short to long cracks regimes, and in line with the observations reported in several experiments on aluminum alloys, as in [23,24]. Thus, the proposed model opens a new perspective in order to connect the behavior of both micro- with that of macro-cracks based on a generic phenomenological model.

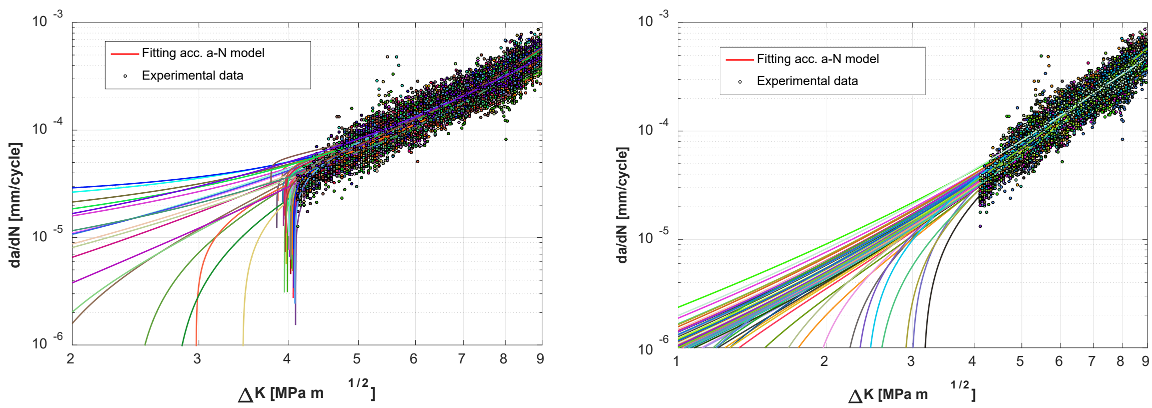
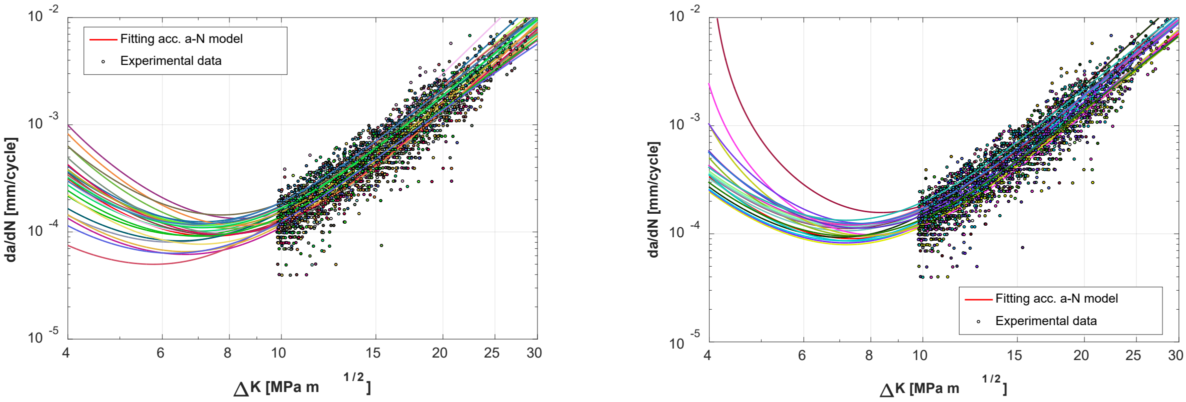
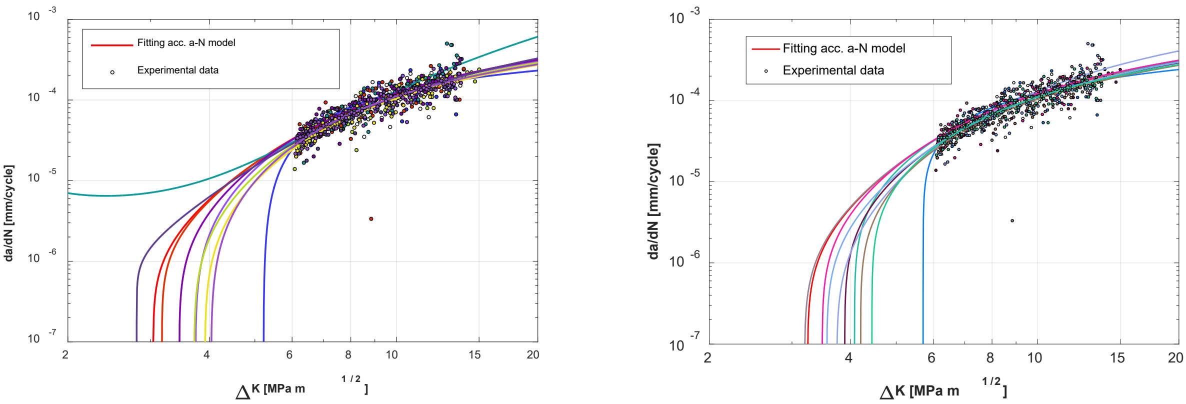


Figure 9. CGR curves derived from the retroextrapolated a-N curves from Virkler's research program [1] for AA 2024-T3; (left) for free β; (right) for fixed β

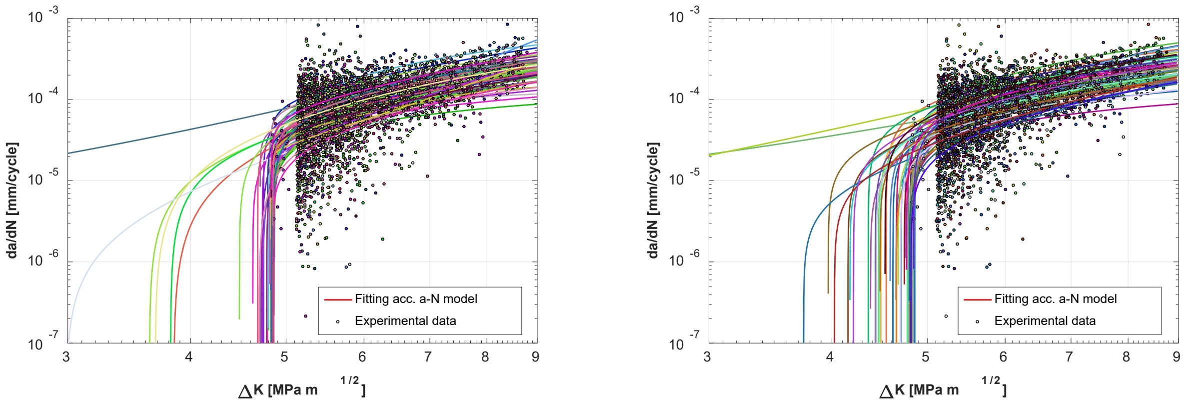


a)

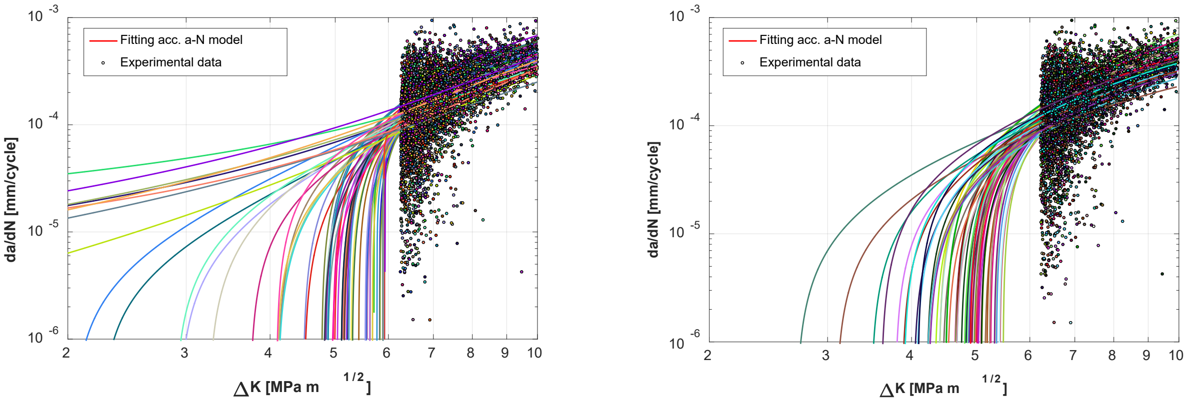


b)

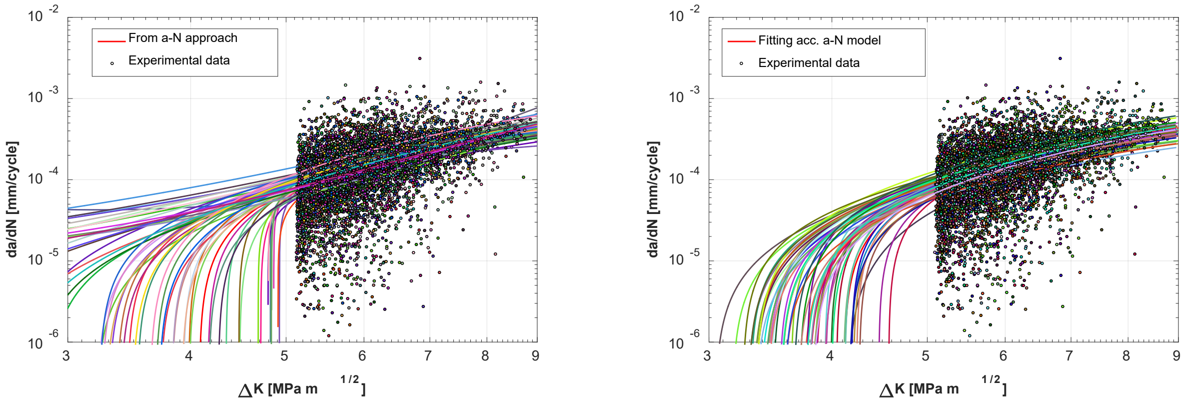
Figure 10. CGR curves derived from the retroextrapolated a-N curves from Wu and Ni research program [2] for AA 2024-T351: (left) for free β; (right) for fixed β; a) for R=0.2; b) for R=0.63)



a)



b)



c)

Figure 11. CGR curves derived from a-N curves from Ghonem and Dore research program [3] for AA 7075-T6: (left) for free β;(right) for fixed β; a) for R=0.4; b) for R= 0,5; c) for R= 0.6)

In this way, the fit of *a*-N curves with the model based on Eq. (2) allows the analytical definition of these curves to be achieved, the influence of the value of the shape parameter which decides the sigmoidal shape or not of the *a*-N curve, to be assessed, and the CGR curves to be derived. Depending on the type of crack growth evolution, it is possible to determine the ΔK_{th} value of the material tested when macrocrack growing can be assumed, or the initiation-propagation transition process in crack growth, otherwise.

The results shown in Figs. 9-11 proves that, even for the same sample, the dominant crack follows a micro- or macrocracking evolution mode according to the schematic types, 1 or 2, shown in Fig. 12, which depends on the particular specimen tested: retrospective initial crack size, a_0, specimen type, loading condition (in this case analyzed by the influence of R) and material microstructure features. Accordingly, the results of Wirkler exhibits mixed trend of the two modalities, in those of Wu and Ni depend clearly of the load ratio applied (type 1 for R=0.4 whereas type 2 for R=0.6) and in the Ghonem et al. modality trend 1 for R=0.4 and mixed trend for R=0.5 and 0.7). In any case, homogeneity within possible single exceptions is observed within the same sample, i.e. either only one predominant crack growth modality (Wu and Ni and Ghonem foor R=4) or balanced proportion of both types (Winkler and Ghonem for R=0.5 and 0.6).

Note that some anomalous CGR curves could possibly be assigned to the anomalous value of the shape parameter, evidencing that sometimes good fitting can be achieved with different sets of the three Weibull parameters particular related to the shape parameter value found, as well known in statistical assessment (see Figs. 6-8, where often the anomalous set of parameters proves the single parameters values to be correlated). Therefore, the homogeneity in the crack growth type increases if the shape parameter value is fixed as the general observed in the sample (see in particular Fig. 10a) and Fig. 11 b) and c)).

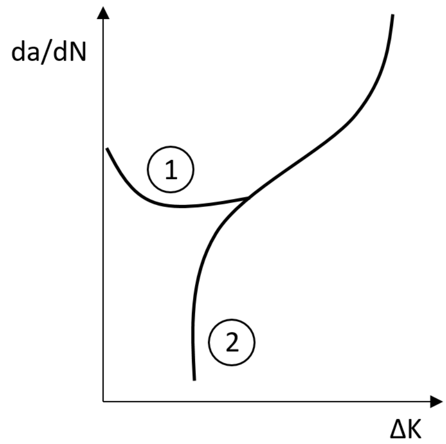


Figure 12. Schematic representation of the da/dN-K curves according to the microgrowing modalities resulting for the samples tested

APPLICATION TO EXPERIMENTAL PROGRAMS ON STRUCTURAL STEELS

In order to ensure validity of the proposed crack growth approach when applied to materials other than the aluminum alloys previously analyzed, three different structural steel types were analyzed in [13] the results of which are not included for the space sake. In the retroextrapolation assessment, only partial a-N records implying reference crack sizes, $a_{ref} > a_0$, over 6.5 mm were considered. High quality fits and satisfactory correspondence of the predicted CGR curves with the experimental ones are confirmed. Particularly, the experimental ΔK_{th} values associated with each test specimen are in good agreement with those obtained from the retroextrapolation of the a-N test record and the subsequent conversion to the CGR curves, even if the latter are derived from a relatively small fraction of the predictable or even recorded a-N crack growth curve. Unfortunately, in what concerns test number and parameter variety, the available tests results are not comparable with those provided by the extensive experimental campaigns of Wirkler [1], Wu and Ni [2] and Ghonem and Dore [3] on aluminum alloys, so that the predictable confidence and reliability of the model assessment when applied to these steel alloys are not comparable with those for the results obtained in the foregoing Sections. Accordingly, further evaluation of other test results has to be envisaged for the future.

CONCLUSIONS

The main conclusions derived from this work are the following:

- A phenomenological approach for the analytical definition of crack growth, a-N, curves is proposed. The methodology implies enlarging the censored crack growth data recorded in the test using back extrapolation, their normalization and subsequent fitting assuming trial functions as cdfs of the GEV family.
- The back extrapolation process allows the predictive original size of the dominant crack in the specimen to be estimated regardless of the size of the pre-cracked crack used in the test.
- The outstanding quality of the fittings to the data of three extensive experimental campaigns implying different aluminum alloys, test geometries and R ratios, the capability to fit the shape diversity of the crack growth a-N curves, the coherence and uniformity observed in the values for each of the extreme value parameters within any sample and low number of parameters implied, all them emphasizes the versatility and suitability of the proposed approach.
- All the exposed reasons, besides the congruent correspondence between a-N and CGR curves now afforded, prove the adequacy and usefulness of the approach proposed as an optimal way of interpreting and investigating the phenomenon of crack growth process from a very early micromechanical phase as a common example more of damage accumulation.
- The observation of the variety of CGR curves derived from the retro-extrapolated a-N crack growth curves, in particular concerning the early phase from micro- to macrocracks, support the interest of the model to analyze the transition between the initiation and propagation phases of cracks according to the sigmoidal shape of the conventional CGR curves or those shapes denoting preliminary decreasing crack growth followed by its approaching to the sigmoidal traditional shape.

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REFERENCES

[1] Virkler D.A., Hillberry B.M., Goel P.K., The statistical nature of fatigue crack propagation. Journal of Engineering Materials and Technology, Transactions of the ASME (1979) 101:148–53.

[2] Wu W.F., Ni C.C., Statistical aspects of some fatigue crack growth data. Engineering Fracture Mechanics (2007) 74:2952–63.

[3] Ghonem H., Dore S., Experimental study of the constant-probability crack growth curves under constant amplitude loading. Engineering Fracture Mechanics (1987); 27:1–25.

[4] Bogdanov S, Mikheevskiy S, Glinka G., Probabilistic analysis of the fatigue crack growth based on the application of the Monte-Carlo method to UniGrow model. Materials Performance and Characterization Vol. 3, No. 3, 214-231, 2014.

[5] Mikheevskiy, S., Bogdanov, S., Glinka, G., Statistical Analyses of Fatigue Crack Growth Based on the UniGrow Model, Proceedings of the ICAF Symposium, Montreal, Canada, June 1–3, 2011.

[6] Vasudevan A.K., Sadananda K., Louat N., Two critical stress intensities for threshold fatigue crack propagation, Scripta Metallurgica et Materiala, 28, 65–70, 1993.

[7] Stanzl-Tschegg S.E., When do small fatigue cracks propagate and when are they arrested?, Corros Review https://doi.org/10.1515/corrrev-2019-0023.

[8] Vallellano C., Vázquez J., Navarro A., Domínguez J., A micromechanical model for small fatigue crack growth: an approach based on two threshold conditions. Fatigue & Fracture of Engineering Materials & Structures, 32, 6, 6 2009.

[9] Castillo E., Fernández-Canteli A., Siegele D., Obtaining S-N curves from crack growth curves. An alternative to self-similarity, Int. J. of Fracture, 187(1), 159-172, 2014.

[10] NASGRO. Fatigue Crack Growth Computer Program NASGRO Version 3.0- Reference Manual 2000.

[11] NASGRO Fracture Mechanics and Fatigue Crack Growth Analysis Software, Reference Manual; Version 4.02. (2002). NASA and Southwest Research Institute, San Antonio, TX.

[12] FKM-Guideline. Fracture Mechanics Proof of Strength for Engineering Components 2009.

[13] Blasón S., Phenomenological approach to probabilistic models of damage accumulation application to the analysis and prediction of fatigue crack growth, PhD Thesis, Universidad de Oviedo, 2019.

[14] Pokorný P., Vojtek T., Náhlík L., Hutar P., Crack closure in near-threshold fatigue crack propagation in railway axle steel EA4T. Eng. Fracture Mechanics, 185, 2–19, 2017.

[15] Zerbst U., Madia M., Fracture mechanics based assessment of the fatigue strength: Approach for the determination of the initial crack size. Fatigue and Fracture of Engineering Materials and Structures, 38,1066–75, 2015.

[16] Zerbst U., Madia M., Vormwald M., Beier H.T., Fatigue strength and fracture mechanics - A general perspective. Eng Fract Mech, 2018;198:2–23.

[17] Castillo E., Extreme value theory in engineering, Acdemic Press, San Diego, Ca. 1988.

[18] Castillo E., Hadi A.S., Balakrishnan N., Sarabia J.M., Extreme value and related models with applications in engineering and science. New York: Wiley, 2005.

[19] Lucarini S., Micromechanics based fatigue modeling using Fast Fourier Transform based homogenization. PhD Thesis, Polytechnic University of Madrid, 2020.

[20] Bogdanoff J.L., Kozin F., Probabilistic models of cumulative damage, Wiley-Interscience, 1985.

[21] Bolotin V.V., Mechanics of Fatigue. CRC Press; 1998.

[22] Bolotin V.V., Wahrscheinlichkeitsmethoden zur Berechnng von Konstruktionen, VEB VErlag für Bauwesen, Berlin, 1981.

[23] Suresh S, Ritchie RO. Newman. International Metals Review 1984;16:445–76.

[24] Newman JC, Phillips EP, Everett RA. Fatigue Analyses Under Constant- and Variable-Amplitude Loading Using Small-Crack Theory. Technology 1999.