Near-Field Multi-Focused Arrays Using Support Vector Regression

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***Abstract*—Support Vector Regression, a powerful and elegant framework in the field of Machine Learning, is proposed in a novel method for Near-Field Focusing using antenna arrays. It allows creating a model of an antenna array relating accurately the set of weights required in the elements of an array and the corresponding near-field distribution, maybe focused on one or more positions of interest. A previous learning process concentrates the computational cost so that the already trained system operates without relevant cost and it is fast enough for real applications where adaptation must be fast, for example because moving devices are involved. The learning capabilities of Support Vector Machines are increased with respect to other machine learning tools, allowing the use of a reduced number of training samples that may be generated with an adaptive system or any full-wave electromagnetic analysis tool, so that realistic effects such as coupling or non-uniformities can be accounted for. Illustrative examples are also presented to test the performance of the proposed approach.**

***Index Terms*—near field, antennas, focusing, multifocusing, support vector regression.**

I. Introduction

Near-Field Focusing (NFF) [1], [2], [3], [4], [5] is a state-of-the-art topic in the field of antenna design that is gaining more attention in the recent years as it is one of the most succesful approaches to emerging technologies such as RFID [2], [4], medical hyperthermia [6], Wireless Power (and Information) Transfer (WPT, WPIT) [7], [8], [9] that support applications such as Internet of Things (IoT) or 5G mobile telephony. It is key in scenarios where wireless links between devices located at short distances are involved. NFF allows concentrating the radiated field on an assigned position or spot in the Near-Field (NF) region of an antenna, typically (but not exclusively) an array. By doing so, the energy directed to positions of the space where it is not necessary is minimized. The Conjugate-Phase (CP) method [2], [10], [11] has been proven to be an excellent technique for calculating the phase-shift that must be applied to each array element so that all their contributions arrive in-phase to the so-called focal point, creating a constructive interference that results in an increased field level. This is a robust and simple idea that suffices for a wide range of applications. However, the extended requirements in some of the emerging applications that make use of NFF, require more sophisticated methods able to solve situations where the CP is limited. For example, if the field must be focused on more than one focal point or on an arbitrary volume, Near-Field Multi-Focusing (NFMF) has arised as an excellent alternative [12], [13].

NFMF is based on the minimization of a properly designed cost function that accounts for all the requirements, using optimization techniques, to obtain the set of weights on the elements of an array corresponding to a field distribution fulfilling the specifications. The resulting method is flexible and efficient, but it is time consuming due to the iterative nature of the optimization techniques. For example, fast adaptation in case of moving devices is not affordable and another approach is necessary.

In this work we propose the use of Support Vector Regression (SVR), a powerful Machine Learning (ML) method based on Support Vector Machines (SVM) [18] to obtain the weights so that an antenna array is able to focus on some assigned points or to generate a shaped Near Field (NF) footprint. SVR requires a strongly reduced set of training patterns with respect to other ML techniques (for example, Neural Networls [14]), so that obtaining them becomes much more affordable. Most of the computing time is concentrated in the previous training step, being its computational cost high, but providing solutions almost in real time once trained. The training or learning of a SVR consists on presenting known pairs of inputs and outputs (field distributions and the corresponding weights applied to the array elements to achieve such distributions, in array synthesis problems), called *training patterns*, so that the machine is able to extract the relationship between them and provide outputs to new inputs. Once trained, SVR is able to operate fast enough for real time applications without relevant computational cost. Moreover, if the training patterns are obtained through measurements or simulations of realistic models using full-wave electromagnetic analysis tools, able to account for coupling effects or any non-idealities of the array, so will do the SVR trained making use of those patterns.

II. Support Vector Regression for Inverse Modeling and Synthesis

The field radiated at a certain position $\vec{r}$ by an antenna array with N elements, independently of its geometry, is obtained as the superposition of the field radiated by its elements at such position. A simple formulation able to represent the superposition is given by

$$E(\vec{r}) = \sum_{n=1}^{N} \omega_n \cdot g_n(\vec{r}) = \mathbf{g}^T(\vec{r}) \cdot \mathbf{w} \qquad (1)$$

where $E(\vec{r})$ is the value of the considered component of the field at $\vec{r}$, ω_n is the feeding weight applied to the n-th element of the array, and $g_n(\vec{r})$ is the contribution of the n-th element to the field at $\vec{r}$. These values are also represented in vector form as $\mathbf{w} = [\omega_1, \omega_2, \ldots, \omega_N]^T$ and $\mathbf{g}(\vec{r}) = [g_1(\vec{r}), g_2(\vec{r}), \ldots, g_N(\vec{r})]^T$, with $(.)^T$ representing the transpose operator. If M positions of the near-field region of the antenna are sampled, $\vec{r}_m, m = 1 \ldots M$, equation (1) may be reformulated as

$$\mathbf{e} = \mathbf{G} \cdot \mathbf{w} \tag{2}$$

where $\mathbf{e} = [E(\vec{r}_1), E(\vec{r}_2), \ldots, E(\vec{r}_M)]^T$ and the matrix model $\mathbf{G} = [\mathbf{g}(\vec{r}_1), \mathbf{g}(\vec{r}_2), \ldots, \mathbf{g}(\vec{r}_M)]^T$. Determining $\mathbf{G}$ represents a proper modeling of the array. However, for synthesis purposes an inverse model may be more adequate.

In the usual case of $M > N$ (i.e., the number of positions where the field is evaluated is greater than the number of elements in the array) equation (2) is an overdetermined system of linear equations that can be solved using a least-squares criterion as

$$\underset{\mathbf{w}}{\text{minimize}} \; ||\mathbf{e} - \mathbf{G} \cdot \mathbf{w}||^2 \tag{3}$$

where $||.||$ stands for the Euclidean norm. The solution to (3) is well known [20] and given by

$$\mathbf{w} = (\mathbf{G}^H \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^H \cdot \mathbf{e} = \mathbf{G}^+ \cdot \mathbf{e} \tag{4}$$

where $\mathbf{G}^+$ is the pseudoinverse matrix for the overdetermined problem, and represents an inverse model of the antenna array. Equation (4) shows that a linear (inverse) model $\mathbf{A} = \mathbf{G}^+$ may be obtained to calculate the weights that must be applied to an antenna array so that it radiates according to a given field distribution specified by its samples, $\mathbf{w} = \mathbf{A} \cdot \mathbf{e}$.

The powerful SVR framework is proposed to calculate $\mathbf{A}$ from known training samples. Let us consider that a set of P pairs $\{\mathbf{w}^{(p)}, \mathbf{e}^{(p)}\}, p = 1 \ldots P$ are available. Each training pair (or training pattern) consists on a set of weights applied to the elements of the array and the corresponding field distribution (for example, power density) represented by its samples. These patterns may have been obtained by measurements or simulations, but it is important to recall that they may contain realistic data (e.g. coupling effects) as far as it is accounted for in the process of obtaining them.

The structural risk minimization (SRM) principle establishes that the n-th row of $\mathbf{A}$, $\mathbf{a}_n$, can be obtained through regression by minimizing a cost function given by [18]:

$$J(\mathbf{a}_n) = \frac{1}{2}||\mathbf{a}_n||^2 + C \sum_{p=1}^{P} |\omega_n^{(p)} - \mathbf{a}_n \cdot \mathbf{e}^{(p)}|_\varepsilon \tag{5}$$

where

$$|\omega_n^{(p)} - \mathbf{a}_n \cdot \mathbf{e}^{(p)}|_\varepsilon = \max\left(0, |\omega_n^{(p)} - \mathbf{a}_n \cdot \mathbf{e}^{(p)}| - \varepsilon\right) \tag{6}$$

is Vapnik's ε-insensitive loss function, and $C > 0$ is a trade-off parameter used to balance the model complexity (controlled by the first term of (5)) and the cost of deviations larger than ε. This minimization problem may be solved introducing a set of positive slack variables ξ_p and $\tilde{\xi}_p$ so that (5) becomes a constrained minimization problem given by

$$\underset{\mathbf{a}_n}{\text{minimize}} \; \frac{1}{2}||\mathbf{a}_n||^2 + C \sum_{p=1}^{P} (\xi_p + \tilde{\xi}_p) \tag{7}$$

subject to

$$\begin{aligned} \mathbf{a}_n \cdot \mathbf{e}^{(p)} - \omega_n^{(p)} &\leq \varepsilon + \xi_p, & \forall p = 1 \ldots P \\ \omega_n^{(p)} - \mathbf{a}_n \cdot \mathbf{e}^{(p)} &\leq \varepsilon + \tilde{\xi}_p, & \forall p = 1 \ldots P \\ \xi_p, \tilde{\xi}_p &\geq 0, & \forall p = 1 \ldots P \end{aligned}$$

This problem is typically solved using the Lagrange multiplier technique [18], which leads to a dual problem consisting on the maximization of:

$$\begin{aligned} W(\tilde{\alpha}, \alpha) = &-\sum_{p=1}^{P} \varepsilon(\tilde{\alpha}_p + \alpha_p) + \sum_{p=1}^{P} \omega_n^{(p)}(\tilde{\alpha}_p + \alpha_p) \ldots \\ &\ldots - \frac{1}{2} \sum_{p,q=1}^{P} (\tilde{\alpha}_p + \alpha_p)(\tilde{\alpha}_q + \alpha_q)\langle \mathbf{e}^{(p)}, \mathbf{e}^{(q)} \rangle \end{aligned} \tag{8}$$

subject to $0 \leq \tilde{\alpha}, \alpha \leq C$. The expression $\langle \mathbf{e}^{(p)}, \mathbf{e}^{(q)} \rangle$ stands for the inner product between pairs of field distribution training patterns. The solution of this dual quadratic programming (QP) problem can be efficiently found taking advantage of its convexity (for example, using [21], [22]). The resulting positive Lagrange multipliers $\tilde{\alpha}_p$ and α_p are then applied to:

$$\hat{\mathbf{a}}_n = \sum_{p=1}^{P} (\tilde{\alpha}_p - \alpha_p)\mathbf{e}^{(p)} \tag{9}$$

which is a function of the field distribution training patterns, linearly combined using $\tilde{\alpha}_p$ and α_p.

The support vector theory states that only a limited set of training patterns contribute to (9) with non-null Lagrange multipliers. Those training patterns with non-null multipliers are referred to as *support vectors*.

The weight ω_n that must be applied to the n-th element of the array is calculated using (9). Repeating this regression for $n = 1 \ldots N$ leads to the N rows of the estimated model $\hat{\mathbf{A}}$. Hence, the total set of weights to be applied to the elements of the array so that a given field distribution is radiated can be estimated as

$$\hat{\mathbf{w}} = \hat{\mathbf{A}} \cdot \mathbf{e} \tag{10}$$

The number of training patterns P used to perform the regression is a critical choice for the accuracy of the model. Although other machine learning methods may be affected by the problem of *overfitting* (for example neural networks [15]), the SRM principle guarantees both improved learning capabilities (i.e. a reduced required number of training patterns for similar performance) and robustness against overfitting when using Support Vector Regression.

Some previous methods proposed for Near-Field Focusing aplications are based on the use of models representing the behavior of the antenna array. For example, the method presented in [13] makes use of a typical formulation based on a near-field array factor represented in matrix form. In order to specify the NF requirements, a target field distribution is defined as a unitary value for the samples corresponding to the focal points and null values for the samples corresponding to any other position. The same procedure for the specification of the NF was used in [14], where a trained Neural Network outputs the set of feeding weights to be applied to the array. It is a very flexible specification procedure as far as it allows assigning multiple spots, arbitrary focusing volumes, shaped field distributions, etc. It is obvious that such artificial target distribution with only 1 and 0 values is not physically feasible, but it has been shown to be quite effective for NFF purposes.

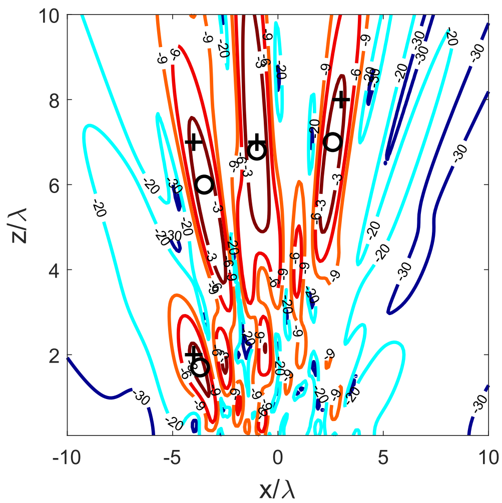
In the synthesis method proposed in this paper, the SVR-based inverse model receives as input a target NF distribution designed according to the NFF requirements: the required values for the focal points are assigned to the samples corresponding to their locations, and null values are assigned to any other sample. Furthermore, more complicated field distributions may be specified by assigning the required values to each location of the NF region. For NFF, the resulting target distribution is built according to the following formulation:

$$E(\vec{r}_m) = \begin{cases} C_m, & \text{if } \vec{r}_m \text{ is a focal point} \\ 0, & \text{if } \vec{r}_m \text{ is not a focal point} \end{cases} \quad (11)$$

where C_m is the field value required at the position $\vec{r}_m$ provided that it is a point where non-vanishing field is requested. Notice that C_m could be any value at any location if a shaped field distribution is specified, although it is related to the total radiated power as it represents the assigned amplitude of the field distribution. Once the target field distribution has been determined, synthesis is straightforward using (10).

III. Experiments and Results

Some experiments have been carried out to evaluate the performance of the proposed method. An antenna consisting on a regular planar array with 12×12 elements is considered. The aperture is placed in the plane $z = 0$. The interelement distance is 0.6λ. The chosen individual radiating elements are hemispherical dielectric resonator antennas (DRA) [23], already used in [24], [19], [14] for testing due to their low losses but relevant mutual coupling effects when included in an array. The radius of each hemisphere is 12.5 mm, and the relative permitivity is $\epsilon_r = 9.8$. A metallic pin with radius 0.63 mm, height 6.5 mm and offset 6.5 mm is used to feed the DRA. The working frequency is 3.6 GHz, hence exciting the TE_{11} mode. The range of variation of S_{11} for the elements is between -10.17 and -13.72 dB, being the maximum of the rest of S parameters -12.35 dB, so that the coupling effects are relevant in the resulting field distribution. The Method of Moments (MoM) [25] has been chosen to obtain the training patterns from random sets of weights. A tool based on MoM is able to analyze the array accounting for its real properties when calculating the resulting NF distribution. The NF region has been sampled within limits given by $x \in [-5\lambda, 5\lambda]$, $y \in [-5\lambda, 5\lambda]$ and $z \in [0.5\lambda, 10\lambda]$, with a sampling period $\lambda/2$ resulting in 8820 samples of the field distribution. A set



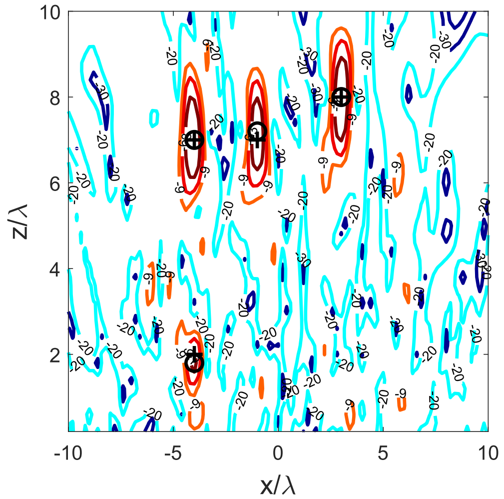
(a)

Fig. 1. Example #1. Normalized Near Field power density at $y = 0$ for the SVR method using a 12×12 DRA elements array. The symbols + and ∘ represent the focal and synthesized maximum points respectively.

of 120 training patterns has been generated. The SVR is used to obtain an inverse model of the array able to relate field samples and weights applied to the array, setting the trade-off parameter $C = 1$ and with $\varepsilon = 0.001$.

Once the model of the antenna array is obtained, it is used to calculate the weights that must be applied to the array in order to obtain a simultaneous focus on four focal points located at positions given by their Cartesian components as $\{x, y, z\} = \{3\lambda, 0, 8\lambda\}$, $\{-\lambda, 0, 7\lambda\}$, $\{-4\lambda, 0, 2\lambda\}$ and $\{-4\lambda, 0, 7\lambda\}$. All the focal points have been chosen at the plane $y = 0$ to facilitate the representation of the results. A target distribution is built using (11), with $C_m = \{10, 10, 3, 10\}$ respectively for the assigned focal points so that the difference in distance to the array is compensated by the required level, and $C_m = 0$ for any other location, and applied to (10) resulting in a set of weights used in MoM to analyze the resulting NF distribution. Fig. 1 shows the normalized NF power density in the plane $y = 0$ where all the focal points have been specified for easier representation.

The proposed method has been able to generate a distribution with -3dB focal spots containing the four focal points, and spending only 0.02s. The calculations have been carried out in a conventional PC with an Intel Core i5-7500 CPU, 3.4 GHz and 8 GB RAM, using MatLab R2019a as programming tool, and averaging 20 simulations. The assigned focal points are shown in Fig. 1 along with the resulting power density, the synthesized spots and the points where the radiated field power density is actually maximum. The resulting maximum values of radiated field power density are located at the positions $\{2.8\lambda, 0, 7.2\lambda\}$, $\{-\lambda, 0, 6.8\lambda\}$, $\{-3.8\lambda, 0, 1.81\lambda\}$ and $\{-3.6\lambda, 0, 6.1\lambda\}$, what represents a very reasonable accuracy for an array with 12×12 elements and four focal points, and considering that the resulting distribution has been obtained accounting for coupling effects between the elements of the array, as they are implicit in the generation of the patterns



(a)

Fig. 2. Example #2. Normalized Near Field power density at $y = 0$ for the SVR method using a 32×32 DRA element array. The symbols + and ∘ represent the focal and synthesized maximum points respectively.

without requiring complicated formulations.

The experiment has been modified to check the performance of the method when dealing with larger arrays. The number of elements has been set to 32×132 elements and the interelement distance has been set to 0.7λ, so that the resulting aperture is larger than the first structure, increasing the number of degrees of freedom and hence its focusing capabilities. All the other properties of the array and its elements remain unchanged, as well as the focusing requirements. The higher number of degrees of freedom should lead to better focusing performance. The number of training patterns used to obtain the model is now 170 (a larger number is required due to the higher complexity of the structure), and MoM has also been used to calculate them. The hyper-parameters of the regression are set to $C = 1$ and $\varepsilon = 0.001$ again. The resulting NF power density at the plane $y = 0$ is plotted in Fig.2 along with the assigned focal points and the resulting maximum-power density points. It can be noticed that the accuracy in the location of the maximum power density points is much higher than using the smaller structure due to the effect of the increased degrees of freedom. The positions where the maximum power density is found are $\{3\lambda, 0, 8\lambda\}$, $\{-\lambda, 0, 7.1\lambda\}$, $\{-4\lambda, 0, 1.8\lambda\}$ and $\{-4\lambda, 0, 7\lambda\}$, all of them very close to the positions where the focal points are assigned. All the assigned focal points lay into -3dB spots where the power density is much more concentrated than using a smaller array.

In example #3, a coverage area is requested in front of the antenna at a distance $z = 6\lambda$, in the plane $y = 0$. To specify it, a set of focal points representing *samples* of the coverage area has been selected at $x = [3\lambda, 2\lambda, \lambda, 0, -\lambda, -2\lambda, -3\lambda, -4\lambda]$. In all the cases the corresponding value C_m has been set to 1. A regular array with 16×16 DRA elements separated 0.6λ has been considered, operating again at 3.6 GHz. A set of 140 training patterns has been obtained using MoM as analysis tool. The proposed method is used to obtain a set of weights corresponding to the field distribution shown in Fig. 3a. It can be observed how a wide region is obtained over -3dB, with all the assigned focal points on it. To check a more complicated case, an additional focal point has been assigned for an isolated device at $\{2\lambda, 0, 2\lambda\}$, and using the same structure and methodology leads to the NF power density plotted in Fig. 3b, where the coverage area is still visible and a -3dB spot contains the new focal point.

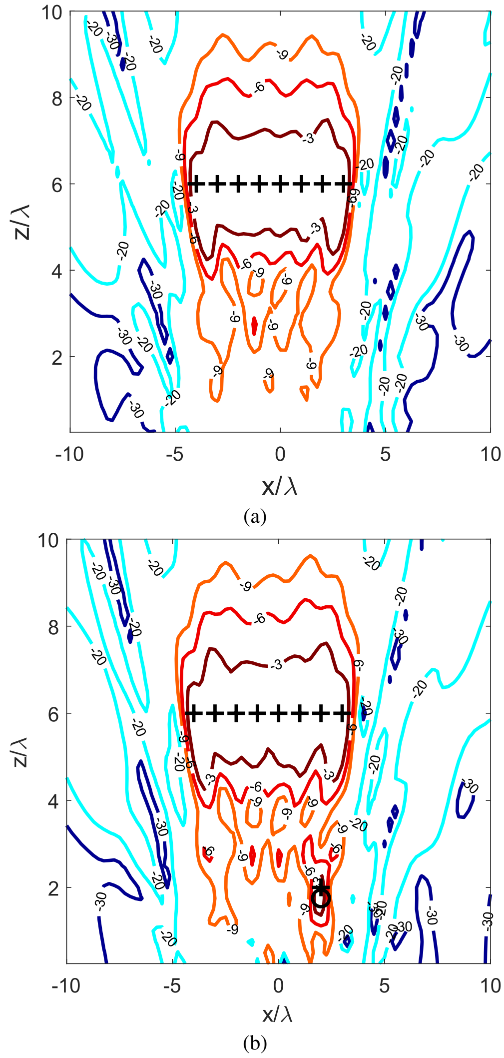


Fig. 3. Example #3. Normalized Near Field power density at $y = 0$ for the SVR method considering a coverage area defined by the assigned focal points represented by the symbol + (a), and with an additional focal point (b). The symbols + and ∘ represent the focal and synthesized maximum points respectively.

IV. Conclusion

A Machine Learning approach to Near Field Focusing based on the powerful and elegant Support Vector Machines framework is presented. Support Vector Regression is performed to develop an accurate model of a given antenna array from a set of training patterns consisting on known pairs of weights applied to the elements of the array and the corresponding

NF distribution. These patterns may be obtained through measurements or simulation, requiring a much more reduced set of patterns that other ML alternatives such as Neural Networks, due the increased learning capabilities of SVM techniques.

Once the model has been obtained, it can be used for synthesis of focused distributions by following a simple strategy, and without relevant computational cost or time, as synthesis becomes a simple matrix-vector product. The resulting method is suitable for applications requiring fast synthesis to operate in scenarios where real-time calculations are required, for example where moving devices in a near environment are involved (e.g. 5G femtocells, Internet of Things, etc.). Regarding existing alternative, although the most popular approach to NFF, Conjugate-Phase, is very fast, simple and accurate, it cannot deal with multiple specifications; an optimization method, able to account for multifocus requirements, is not fast enough to be used in real-time applications due to its iterative nature; the NN approach is able to be accurate, fast and deal with multifocus, but requires thousands of training pairs, what might overflow many applications and make impossible the use of measured training data. The proposed SVR method overcomes all these difficulties by reducing the number of required training patterns, fast operation and ability to deal with NF distribution specified in different ways, even a shaped distribution. Additionally, it is able to account for the real properties of the array (realistic effects, coupling effects, individual radiation patterns, non-uniformities, etc.) provided that the method for generating the training patterns accounts for all of them.

As a future work to be addressed in future developments, more sophisticated specifications such as a mask or template, phase-only distributions, or additional constraints to be considered, may be included in the synthesis scheme so that the resulting method becomes much more flexible and powerful. Meanwhile, SVR represents an interesting step for applications with different devices involved placed in the near-field regions, even when some of them are moving.

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