Quantifying the Bullwhip Effect in closed-loop supply chains: The interplay of information transparencies, return rates, and lead times

Borja Ponte[a], Jose M. Framinan[b], Salvatore Cannella[c,*], Roberto Dominguez[b]

[a] *Department of Business Administration, University of Oviedo, Gijon, Spain*
[b] *Departamento de Organización Industrial y Gestión de Empresas I, Universidad de Sevilla, Sevilla, Spain*
[c] *Dipartimento di Ingegneria Civile e Architettura, Università degli Studi di Catania, Catania, Italy*

Abstract

The Bullwhip Effect manifests itself in the form of an increased order and inventory variability in the uppermost nodes of the supply chain. This dynamic phenomenon is not yet well understood in closed-loop supply chain settings, despite their growing importance in modern societies pursuing circular economy opportunities. Indeed, the problem-specific literature has provided somewhat conflicting findings. To better understand the Bullwhip Effect in closed-loop systems, we obtain expressions for the order and inventory variance amplification in four archetypes that differ in the structure of information transparency. Interestingly, we observe that the impact of return rates and lead times on the system performance strongly depend on the degree of supply chain visibility. This perspective allows us to revisit discrepancies in prior works. We later move the study from the operational to the economic prism. Here we prove the existence of an optimal return rate, and we derive its expression in the four closed-loop supply chain archetypes. We show that the optimal rate is dependent on the node's cost structure, the lead times, and the variability of demand. Properties of the different closed-loop systems and relevant managerial implications are also discussed in our work.

Keywords: Supply chain management; Bullwhip Effect; closed-loop supply chain; information sharing; remanufacturing.

This is the postprint version of the paper accepted by the International Journal of Production Economics and published in the vol. 230 (December 2020). https://doi.org/10.1016/j.ijpe.2020.107798.

1. Introduction

The Bullwhip Effect, through which the supply chain amplifies the variability of consumer orders as they get closer to manufacturing echelons, is a popular phenomenon in the operations management discipline due to its costly implications in production and distribution systems. The celebrated articles by Lee et al. (1997a,b), Metters (1997), Chen et al. (2000a,b), and Dejonckheere et al. (2003, 2004), among

*Corresponding author
Email addresses: ponteborja@uniovi.es (Borja Ponte), framinan@us.es (Jose M. Framinan), cannella@unict.it (Salvatore Cannella), rdc@us.es (Roberto Dominguez)

others, widely increased the attention of academics and practitioners on this logistically-grounded form of butterfly effect in the late 1990s and early 2000s. Since then, it has become a consolidated field of operational research. We refer the interested reader to Wang & Disney (2016) for a recent review of the Bullwhip literature, while the empirical study by Isaksson & Seifert (2016) may be consulted for a contemporary analysis of this complex and dynamic phenomenon in different industries.

However, only a relatively small number of studies have investigated the Bullwhip Effect in the increasingly important closed-loop supply chain setting. This refers to an emerging, circular economy-based, supply chain paradigm that includes the collection and recovery processes of used products, bringing both environmental benefits to modern societies and economic opportunities for businesses; e.g. see Guide et al. (2003), Govindan et al. (2015), Genovese et al. (2017) and Reimann et al. (2019). In this sense, Goltsos et al. (2019a) identify a ratio of approximately one Bullwhip study in closed-loop supply chain settings per 26 studies in traditional, or open-loop, settings. In a similar vein, the literature review by Wang & Disney (2016) highlights the investigation of the Bullwhip phenomenon in reverse logistics systems as a necessary avenue for future research. From this perspective, understanding the dynamic behaviour of closed-loop systems may be interpreted as a stepping stone towards the deployment of sustainable supply chains in practice (Braz et al., 2018), allowing practitioners to efficiently implement production and distribution systems inspired by circular economy principles.

Having noted that this is an underexplored area, some relevant efforts of Bullwhip research in closed-loop supply chains have been conducted. From the first exploration generally attributed to Tang & Naim (2004) to the recent studies by Zhou et al. (2017), Hosoda & Disney (2018), and Ponte et al. (2019), meaningful insights on the dynamics of such systems have been provided. Interestingly, as underlined by Adenso-Diaz et al. (2012), Cannella et al. (2016), and Goltsos et al. (2019a), among others, they have reached somewhat conflicting conclusions. In light of this, we argue that the impact of the reverse flow of materials on the dynamics of closed-loop supply chains is not yet completely understood. Below we discuss findings from the extant literature, highlighting relevant discrepancies that we will account for in the present paper.

1.1. The Bullwhip Effect in closed-loop supply chains

Like in traditional supply chains, papers looking at the Bullwhip problem in closed-loop settings generally measure the variability of orders and inventories, as both are symptomatic of poor dynamic behaviour and high operational costs, see e.g. Disney & Lambrecht (2008) and Cannella et al. (2013). As per Goltsos et al. (2019a), the Bullwhip literature exploring the closed-loop field concentrates on three main domains: (i) the impact of the *return rate*, which allows one to compare the performance of closed-loop systems against traditional systems; (ii) the influence of *lead times*, both in the forward and in the reverse flow of materials, which led to the discovery of the so-called lead-time paradox that we discuss below; and (iii) the value of *information transparency* for improving the dynamics of these supply chains, which underscores the need for re-thinking and adapting traditional inventory models.

- *On the impact of the return rate.* Most of the literature is united in the positive consequences of increasing return rates on closed-loop supply chain dynamics through a reduced order and/or inventory variability; see e.g. Tang & Naim (2004), Zhou & Disney (2006), Cannella et al. (2016), and Dev et al. (2017). Other works add interesting considerations. Adenso-Diaz et al. (2012) show that, while increasing low return rates contributes to mitigate Bullwhip, rising high rates may be detrimental. This suggests a U-shaped relationship between the Bullwhip Effect and the return rate. Zhou et al. (2017) conclude in a three-echelon system that supply chain dynamics often, but not always, benefit from the reverse flow of materials. In this regard, they observe that closing the loop magnifies the Bullwhip phenomenon under specific circumstances.

  The above may lead us to conclude that closed-loop systems generally benefit from improved dynamics in comparison with traditional ones. However, recent works challenge this finding. Indeed, Hosoda et al. (2015) claim that closed-loop supply chains are more likely to experience Bullwhip, as they face a wider scenario of uncertainties. This may be explained from the perspective of Ponte et al. (2019), who observe that the independent and perfectly correlated components of the returns series (with respect to the demand) have opposite effects on system dynamics. This is aligned with the recent studies by Hosoda & Disney (2018) and Dominguez et al. (2019a,b), who observe that order and inventory variabilities in closed-loop settings are very sensitive to the uncertainty on future returns, resulting in a decreased operational performance of such systems. All in all, the divergence in the conclusions of prior works suggests that the impact of return rates on the dynamics of closed-loop systems is very sensitive to the modelling assumptions.

  Finally, it is worth to mention that returns have also been studied in traditional supply chain settings. Indeed, linear models of these production and distribution systems are often built on the assumption that excess stock can be returned to the upstream nodes at no extra cost (Ponte et al., 2017). Chatfield & Pritchard (2013) explore the implications of these returns (of non-used products) and observe that they deteriorate the operational dynamics of serial supply chains. Dominguez et al. (2015) show that this also occurs in divergent supply chains, even though these are generally more robust to the return assumption. Although the nature of these returns and the supply chain structure are different than those considered in closed-loop systems, some interesting similarities may emerge from the dynamics induced by the return process.

- *On the influence of lead times.* In closed-loop settings, the effect of lead times located in the forward flow of materials has been found to be consistent with the conventional impact of this parameter (i.e. the lower, the better). For example, Zhou et al. (2006) show the benefits derived from reducing manufacturing lead times in terms of order and inventory variabilities. That is, shortening these lead times contributes to both smooth production requirements and satisfy customer needs in a more cost-effective manner. Nevertheless, the study of the lead-time effects of those in the reverse flow of materials, such as remanufacturing lead times, has led to different insights. First, we note that some works report operational benefits derived from reducing remanufacturing lead times (Zhou et al., 2006; Cannella et al., 2016; Zhou et al., 2017).

Nevertheless, other studies have discovered a counterintuitive relation (labelled the lead-time paradox), which refers to settings where reducing remanufacturing lead times has detrimental consequences, as previously found in other areas of closed-loop research, e.g. see Inderfurth & van der Laan (2001). In terms of Bullwhip, Tang & Naim (2004) observe it for the inventory variability metric in one of the three models they explore, more specifically that with the highest information transparency. Hosoda et al. (2015) also find that cutting remanufacturing times may result in worsened inventory dynamics, while Hosoda & Disney (2018) show that it can also occur for order variability metrics. After investigating the paradox in detail, the latter authors reveal that it generally appears when remanufacturing requires less time than manufacturing, since the ordering policy is not able to make the best use of the information of the reverse flow. Dominguez et al. (2019a) study a closed-loop system under both deterministic and stochastic remanufacturing lead times. They detect the paradox in the former case, but not in the latter. The different findings on lead-time effects in closed-loop systems highlight again the fundamental role of assumptions in such studies, which may even reverse the impact of key variables.

- *On the value of information transparency.* Previous works propose different information sharing strategies for closed-loop supply chains, quantifying the value of information transparencies. Tang & Naim (2004) develop three supply chain models with different degrees of information sharing, and show that the one with the highest one greatly outperforms the others. A similar conclusion was reached by Cannella et al. (2016), who consider two inventory models differing in the information available. Hosoda et al. (2015) discuss the benefits of an advance notice return scheme, revealing that sharing data on the remanufacturer's work-in-progress allows for a dramatic improvement in the management of the serviceable inventory. Considering the decrease in the value of information as the returns pipeline becomes more uncertain, Ponte et al. (2019) suggest a control mechanism for the reverse flow based on regulating the returns inventory.

  Overall, there is little doubt about the value of closed-loop supply chain visibility from a Bullwhip perspective. Nonetheless, as evidenced by those prior studies, the benefits of information transparencies strongly rely on the scenario under consideration. Here, interactions between the relevant factors play a crucial role in determining the dynamic behaviour and operational performance of the system. However, how such visibility modifies the return-rate and lead-time effects in closed-loop supply chains has not been comprehensively investigated yet. And, in light of previous considerations (see also Goltsos et al., 2019a), this may be interpreted as a key research direction to better understand the emergence of the Bullwhip Effect in real closed-loop systems.

1.2. Contribution and structure of our work

As per the above discussion, it is not clear how key operational factors of closed-loop supply chains and the practices of information sharing in such systems interact to determine their Bullwhip behaviour. To fill this important gap, we derive analytical expressions of the Bullwhip ratio (Bw) and the Net Stock Amplification ratio ($NSAmp$) in closed-loop supply chains, depending on the return rate and the lead times, under different structures of information transparency. Specifically, we define four archetypes of hybrid manufacturing-remanufacturing systems according to two dimensions of visibility: (1) remanufacturing pipeline (i.e. visibility on the returns that have been collected by the remanufacturer but not yet received at the on-hand inventory site); and (2) market pipeline (i.e. visibility on sold products that will come back to the supply chain via the remanufacturer). By doing so, we identify seven properties that define the performance effects of the return rate, lead times, and information transparency degree, and their interactions, in closed-loop supply chains. Our main findings are:

- Increasing the volume of returns in closed-loop supply chains may improve or deteriorate the dynamics of the system, depending on the degree of information transparency.
- Improving the visibility on the remanufacturing pipeline may allow for smoothing (but not always smooths) the operation of the manufacturing line in closed-loop supply chains.
- Closed-loop supply chain settings generally suffer from a decreased inventory performance in comparison with traditional settings, even when the system is highly transparent.
- Visibility on the remanufacturer's work-in-progress reduces the variability of net stocks, unless the remanufacturing lead time is much longer than the manufacturing lead time.
- The lead-time paradox only appears, but not always, in closed-loop supply chains with information transparencies; reducing the inventory performance for low remanufacturing lead times.
- Visibility on the market contributes to reducing the variability of orders in closed-loop supply chains, but may have adverse effects in terms of inventory performance.

Later, we analyse the economics of the Bw-$NSAmp$ trade-off in closed-loop settings and investigate how to optimise the supply chain by targeting the appropriate return rates and lead times; thus providing a comprehensive understanding of the Bullwhip phenomenon in circular economic systems.

The rest of our article has been structured in the following manner. Section 2 demarcates the generic closed-loop supply chain that we consider, across with the underlying assumptions, and describes the four closed-loop archetypes differing on the supply chain visibility. Section 3 derives the expressions of Bw and $NSAmp$ in the four cases. Section 4 discusses the main insights and findings. Section 5 extends the analysis to the economic field, investigating the existence of an optimal return rate in the four archetypes. Finally, Section 6 concludes and reflects on avenues for future closed-loop research.

2. Closed-loop supply chain structure and the four information sharing archetypes

Closed-loop supply chains display different structures in practice. For benchmarking purposes with most prior research in this discipline (including Tang & Naim, 2004; Zhou et al., 2006; Hosoda et al., 2015; Cannella et al., 2016; Hosoda & Disney, 2018; Dominguez et al., 2019b; Ponte et al., 2019), we investigate a hybrid manufacturing-remanufacturing system. In these closed-loop systems, the inventory of serviceable products is formed by manufactured, new products and remanufactured, as-good-as-new products; both being able to satisfy consumers' demand. Therefore, they are prevalent in practice where the assumption of perfect substitution (between new and remanufactured products) holds, such as the spare parts industry; see Souza (2013) and Goltsos et al. (2019a).

Figure 1 represents the basic structure of the hybrid system, highlights the flow of materials, and introduces the notation of the four main parameters: the return rate, α, and the relevant lead times, L, L_R, and L_C. The rate α defines the percentage of used products that return to the supply chain, while L, L_R, and L_C represent the lead times of the manufacturing (of new products from virgin resources), remanufacturing (of as-good-as-new products from used ones), and consumption processes, respectively. In our analysis, we assume that these parameters are deterministic, as in most of previous closed-loop supply chain dynamics studies (see e.g. Adenso-Diaz et al. 2012; Cannella et al. 2016; Zhou et al. 2017). This allows us to provide closed analytical equations of the key performance indicators under different operational scenarios, enabling an in-depth understanding of the system dynamics and the later economic study. In this manner, we clarify the effect of information transparencies on closed-loop systems by addressing a best-case scenario in which closed-loop uncertainties are minimised.

Assume the serviceable inventory needs to satisfy every period t the demand of a group of consumers, d_t. A portion of the demand is met through remanufactured products, r_t. These are used products,

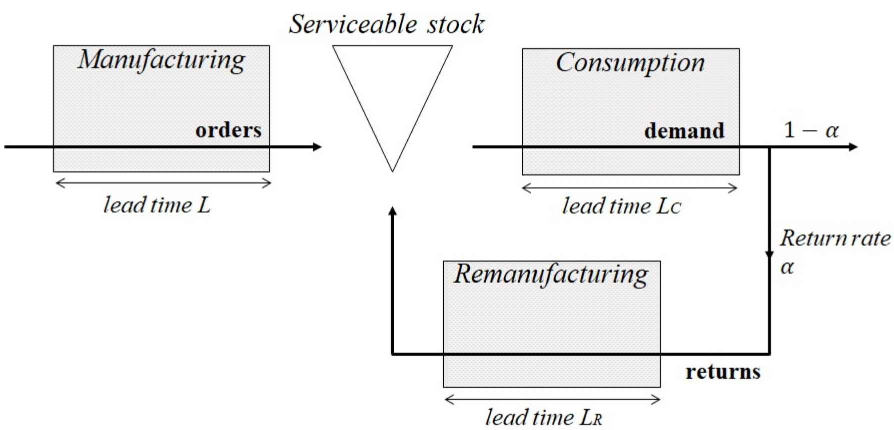


Figure 1: Flow of materials in the closed-loop supply chain.

whose quantity is determined by α, that come back to the supply chain after a lead time L_C. Then, they are reprocessed in the remanufacturing facilities up to a fully operational state, with a lead time L_R. The remanufacturer uses a push policy, according to which all the products collected are directly reprocessed. This is a common assumption in the field (Goltsos et al., 2019a), as it fits well with the ethics of sustainability (Hosoda & Disney, 2018). These considerations lead to the relationship,

$$r_t = \alpha d_{t-L_C-L_R}. \tag{1}$$

The remaining demand has to be met by manufacturing new products. We assume the industrially popular order-up-to replenishment policy is employed to manage the serviceable inventory, see Disney & Lambrecht (2008). Then, consider that the manufacturer at the end of each period t places a production order for new products, o_t, to fill the gap between the desired and the actual position of the inventory. The former is defined by the order-up to point, S_t. The latter considers both inventory on-hand, i.e. the serviceable inventory, i_t, and on-order, i.e. the work-in-progress, w_t. That is,

$$o_t = S_t - i_t - w_t. \tag{2}$$

The manufactured products, m_t, respond to these production orders. They will be received in the serviceable inventory after a fixed manufacturing lead time, L; therefore,

$$m_t = o_{t-L}. \tag{3}$$

As it occurs in many practical settings, we assume in our study that the consumption lead time is significantly longer than the manufacturing and remanufacturing lead times, that is, $L_C >> L, L_R$ (e.g. Tang & Naim 2004; Cannella et al. 2016; Ponte et al. 2019).

Note, Eqs. (1) and (3) assume unconstrained production capacities in the manufacturing and remanufacturing facilities. In addition, we consider that both the new and as-good-as-new products are available at the beginning of each period. This makes that the serviceable inventory, i_t, is the accumulated difference between the receipts from both production processes and the demand, by

$$i_t = i_{t-1} + m_t + r_t - d_t \tag{4}$$

Following common notation, $i_t > 0$ represents excess inventory held at the end of the period (thus, entailing higher storage costs), while $i_t < 0$ represents backlog, i.e. demand that has not been fulfilled on time and must be satisfied as soon as inventory becomes available.

Consumer demand is considered to be an independent and identically distributed (i.i.d.) random variable with a mean of μ and a standard deviation of σ (with $\mu >> \sigma$, so that the probability of negative demand is negligible). The study is then valid for, but is not limited to, the case of normally distributed demand; a reasonable assumption when demand stems from many independent customers as per the central limit theorem (Hedenstierna et al., 2019). For the later economic study, we define

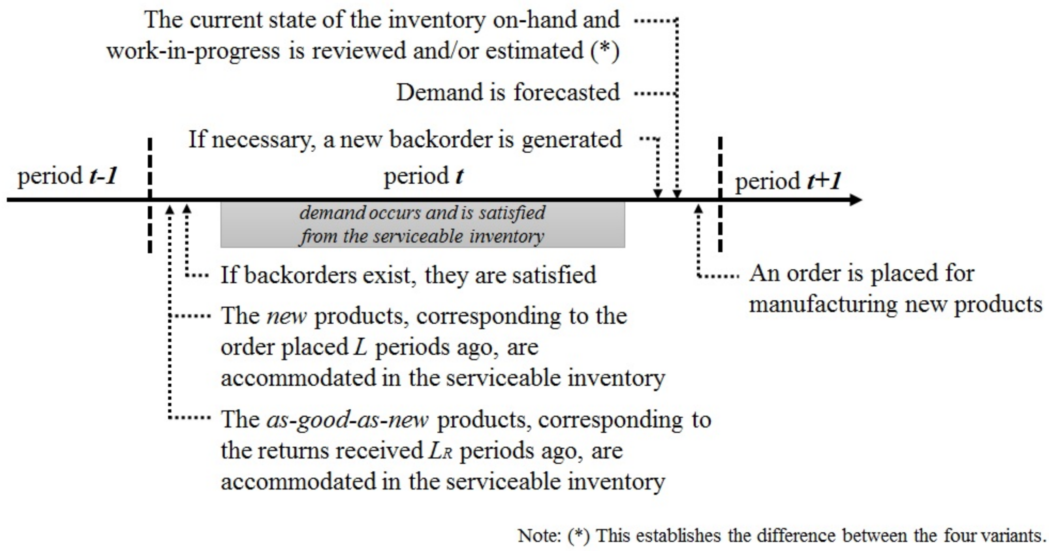


Figure 2: Sequence of events in the closed-loop supply chain.

$\lambda = \sigma^2/\mu$ as the demand's variance-to-mean ratio, also known as the index of dispersion. We assume the manufacturer is able to perceive the i.i.d. nature of the demand series and to accurately estimate the population mean. Under these circumstances, it is reasonable to use fixed forecasts, $f_t = \eta$, with $\eta = \mu$, as this results in minimum mean square error (MMSE) forecasts; see e.g. Hosoda & Disney (2009). Importantly, the i.i.d. demand together with the MMSE forecasts can be reasonably assumed to result in the use of a time-invariant order-up-to point, S_t; like e.g. in Ponte et al. (2017).

To sum up, Figure 2 describes the sequence of events in the management of the serviceable inventory of the closed-loop supply chain. The operation is mathematically described by Eqs. (1) to (4), which define the fundamental relationships. These apply to the four archetypes we describe next.

2.1. Work-in-progress models in the hybrid manufacturing-remanufacturing system

The concept of work-in-progress is generally straightforward in open-loop supply chains, covering the product that has been ordered but not yet received at the on-hand inventory site, see Lin et al. (2017). However, a look at the studies reported in Section 1.1 reveals that the work-in-progress notion may be subject to several interpretations in closed-loop supply chain settings. Building on this literature, we below present four computations of the work-in-progress. They are defined according to the type of information available. The four work-in-progress models result in four variants, which we label as archetypes, of the same hybrid system with different degrees of supply chain visibility.

These models can be categorised according to two dimensions of visibility, see Figure 3. First, we consider the *remanufacturing pipeline visibility*, which refers to the returns that have been collected by the remanufacturer but not yet received (after their processing) at the on-hand inventory site. Second, we consider the *market pipeline visibility*, which refers to the sold products that will return to the supply chain via the remanufacturer but have not yet done so. In both dimensions, we establish a difference between transparency and opaqueness depending on whether or not the manufacturer has

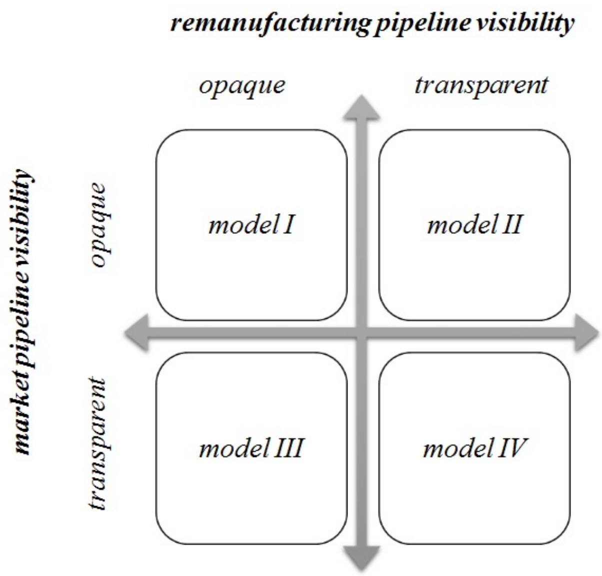


Figure 3: Four work-in-progress policies for the manufacturer.

access to the relevant information. We assume the manufacturing pipeline is transparent in all cases.

The four models displayed in Figure 3, which are detailed below, represent different real-world settings. Note that in practice the remanufacturing pipeline may or may not be observable by the manufacturer; for instance, depending on whether this node internalises or externalises the remanufacturing operations and/or whether information sharing mechanisms have been implemented in the closed-loop supply chain. We refer the interested reader to the typology of remanufacturing systems by Abbey & Guide Jr (2018), which emphasises the role of third-party remanufacturers in the industry. On the other hand, in most cases the state of the products in the market can be hardly known. However, under some specific circumstances the trajectory of products can be tracked, such as for highly expensive and multiple lifecycle products or those that require an exhaustive control. For instance, Goltsos et al. (2019b) studies the case of a (re)manufacturer of optical products that operates in a military setting, keeping a strict control of the products in the market due to contract requirements.

- *Model I.* This is the most basic work-in-progress configuration, where the manufacturer only considers its own information. Therefore, the work-in-progress, $w_t = w_t^m$, is the accumulated sum of the previous orders and the current manufacturing receipts, which is the same as the sum of the orders issued between $t-1$ and $t-(L-1)$, by

$$w_t = w_t^m = w_{t-1} + o_{t-1} - m_t = \sum_{i=1}^{L-1} o_{t-i}. \quad (5)$$

  Several authors, e.g. Tang & Naim (2004), in their type-1 system, and Zhou & Disney (2006) analyse the performance of this work-in-progress model, which does not have access to (or makes use of) the remanufacturer and market pipelines information, in closed-loop supply chain settings.

- *Model II.* This model also considers the remanufacturer's work-in-progress, which may (should) enable an improvement in the dynamics of the hybrid system. As per the previous assumptions, the work-in-progress in the remanufacturing pipeline, w_t^r, can be expressed as the products collected by the remanufacturer between $t-(L_R-1)$ and t. This in turn represents a portion, defined by the return rate α, of the demand before the consumption lead time L_C. Then, the work-in-progress, $w_t = w_t^m + w_t^r$, here results in

$$w_t = w_t^m + w_t^r = \left(\sum_{i=1}^{L-1} o_{t-i}\right) + \left(\alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i}\right). \tag{6}$$

The potential of this information for improving closed-loop supply chain performance has been explored by Tang & Naim (2004), in their type-2 and -3 systems, and Cannella et al. (2016).

- *Model III.* The same idea can be applied to capture data on the products in the market that will return to the supply chain after their use. This work-in-progress, w_t^c, can be estimated as the product of the return rate α and the last L_C demands. In this sense, we now consider the case in which the remanufacturer's work-in-progress is not accounted for, but the market pipeline is considered together with the manufacturer's one. Then, the work-in-progress, $w_t = w_t^m + w_t^c$, is

$$w_t = w_t^m + w_t^c = \left(\sum_{i=1}^{L-1} o_{t-i}\right) + \left(\alpha \sum_{i=0}^{L_C-1} d_{t-i}\right) \tag{7}$$

While this may be the least common case in practice, it allows us to complete the closed-loop picture. Therefore, revealing its dynamic properties would allow for a deeper understanding of the Bullwhip behaviour of our supply chain. Note that *models II* and *III* represent two different intermediate steps between the null visibility of *model I* and the full transparency of *model IV*.

- *Model IV.* Finally, we consider the case of full visibility in the supply chain. In this case, the work-in-progress model includes both the pipeline of remanufactured products, w_t^r, and the pipeline of future returns, w_t^c. The overall work-in-progress, $w_t = w_t^m + w_t^r + w_t^c$, then becomes

$$\begin{aligned} w_t = w_t^m + w_t^r + w_t^c &= \left(\sum_{i=1}^{L-1} o_{t-i}\right) + \left(\alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i}\right) + \left(\alpha \sum_{i=0}^{L_C-1} d_{t-i}\right) \\ &= \left(\sum_{i=1}^{L-1} o_{t-i}\right) + \left(\alpha \sum_{i=0}^{L_C+L_R-1} d_{t-i}\right). \end{aligned} \tag{8}$$

Both Hosoda et al. (2015) and Hosoda & Disney (2018) develop estimations of the pipeline of future returns within their inventory models for hybrid manufacturing-remanufacturing systems, and show how these lead to an improvement of the supply chain's operational performance.

The different work-in-progress models result in four variants of the order-up-to policy that will be analysed in depth in this article. These represent a set of replenishment approaches that can be implemented in real-world closed-loop supply chain settings depending on the information available. Note, *model I* defines the base system, in which the 'classic' order-up-to policy is used. This adjusts itself to the closed-loop scenario due to the effects of returns on the serviceable inventory. At the other end of the conceptual matrix shown in Figure 3, the inventory policy derived from *model IV* represents the managerial attempt to track the full circle defined by the product in the closed-loop supply chain.

3. Bullwhip and Net Stock Amplification in the four closed-loop supply chain archetypes

To analyse the dynamic behaviour of the closed-loop supply chain with the different information transparency structures, we consider two common indicators in the Bullwhip literature that provide a comprehensive overview of the supply chain performance under a predefined replenishment policy; see Disney & Lambrecht (2008), Cannella et al. (2013), and Zhou et al. (2017). First, the Bullwhip ratio (Bw) measures the variance of the orders issued against that of the demand,

$$Bw = \frac{\text{var}(o_t)}{\text{var}(d_t)}. \tag{9}$$

This metric reports on the stability in the manufacturing line, which is an essential contributor to production efficiency. In light of this, reducing Bw enables to reduce the capacity-related production costs in the supply chain. Second, the Net Stock Amplification ratio ($NSAmp$) compares the variance of the serviceable inventory at the end of the period to the variance of demand,

$$NSAmp = \frac{\text{var}(i_t)}{\text{var}(d_t)}. \tag{10}$$

In this sense, this metric provides information on the capacity of the system to satisfy customer demand in a cost-effective manner. Therefore, reducing $NSAmp$ allows supply chain managers to improve the trade-off between the investment in inventories and the customer service level.

In Appendix A, we derive the expressions for Bw and $NSAmp$ in the four closed-loop supply chain archetypes discussed before. In short, Table 1 compiles the expressions of the Bw and $NSAmp$ metrics as functions of the return yield, α, and the lead times, L, L_R, and L_C, in the four archetypes.

Table 1: Bw and $NSAmp$ ratios in the four closed-loop supply chain archetypes.

Archetype	Bw	Eq.	$NSAmp$	Eq.
model I	$1+\alpha^2$	(A.5)	$L+\alpha^2 L$	(A.7)
model II	$1+\alpha^2$	(A.9)	$L+\alpha^2\|L-L_R\|$	(A.14)
model III	$(1-\alpha)^2+2\alpha^2$	(A.16)	$L+\alpha^2\left(L_C+L_R-\|L-L_R\|\right)$	(A.20)
model IV	$(1-\alpha)^2$	(A.22)	$L+\alpha^2(L_C+L_R-L)$	(A.24)

4. Discussion of the results: The different values of information transparency

In this section, we formalise relevant properties that apply to the closed-loop supply chain archetypes investigated. First, we focus on the smoothness of the closed-loop supply chain operations through the

Bw indicator (*properties A, B, C*). Later, we look at the balance between holding requirements and stock-out occurrence in the hybrid system through the $NSAmp$ metric (*properties D, E, F, G*). In the next section, we will consider in detail the trade-off between both metrics.

To derive the properties, we quantify the value of information transparency, VIT, in the remanufacturing and market pipelines from both the Bw and $NSAmp$ perspectives. We use VIT_b^a; a refers to the metric, Bw or $NSAmp$, and b refers to the pipeline, Rem (Remanufacturing) or Mk (Market).

In supply chain studies, VIT is generally measured as the (absolute or relative) difference in the performance of the system with and without transparency (Lee et al., 2000; Raghunathan, 2001; Teunter et al., 2018). We adopt a similar approach by looking at the Bw and $NSAmp$ curves as functions of α. In light of this, we quantify VIT as the difference in area between both curves, i.e. with and without information transparency, in the entire range $\alpha \in [0, 1]$. Note, covering the whole range of α provides a wider perspective on the scope for improvement derived from information sharing policies, as the return rate often varies over time in the different stages of product lifecycles (Ostlin et al., 2009). Also, using the unit interval $\alpha \in [0, 1]$ makes that VIT_b^a informs on the expected decrease ($VIT > 0$) or increase ($VIT < 0$) in a derived from adopting the information sharing policy on b.

Property A. *The Bullwhip ratio in closed-loop supply chains may be an increasing or decreasing function of the return rate, or even display a U-shaped relation, depending on the information transparency.*

Inspection of Table 1 reveals that Bw is not influenced by the lead times of the hybrid system in any of the four archetypes; the demand's variance-to-mean ratio is not altered either. It should be noted that the same occurs in traditional supply chains under the conditions of i.i.d. demand and MMSE forecasting, where the order-up-to policy results in a pass-on-orders strategy, and hence $Bw = 1$ (see e.g. Ponte et al. 2017). This perspective explains why, in the four models, $Bw = 1$ for $\alpha = 0$; thus verifying that the closed-loop archetypes appropriately encompass the traditional supply chain.

However, the Bullwhip ratio, Bw, is here strongly influenced by the return rate, α. Figure 4 displays the relationship between Bw and α in the four archetypes. For *models I* and *II*, Bw increases as α grows; $Bw^I = Bw^{II}$. This implies the closed-loop supply chain suffers from a worsened dynamics in comparison with traditional systems. In contrast, in *model IV*, Bw decreases as α grows, converging asymptotically to $Bw = 0$. This closed-loop model thus benefits from an improved dynamics over traditional systems, especially for high volumes of returns. Finally, in *model III*, Bw depicts a U-shaped relationship with α. Therefore, a slight increase in low return rates reduces Bw, while increasing high rates has the opposite effect. Importantly, $\alpha = 1/3$ results in the lowest order variability, $Bw^{III} = 2/3$.

Overall, we see that the return-rate effects on the dynamics of closed-loop supply chains may be opposite in different scenarios of information transparency. This finding allows us to better understand prior discrepancies in the literature; see e.g. Zhou et al. (2006) for the positive impact of α, Hosoda & Disney (2018) for the negative impact, and Adenso-Diaz et al. (2012) for the U-shaped relationship.

Property B. *Increasing the visibility on the remanufacturer's work-in-progress may help to smooth,*

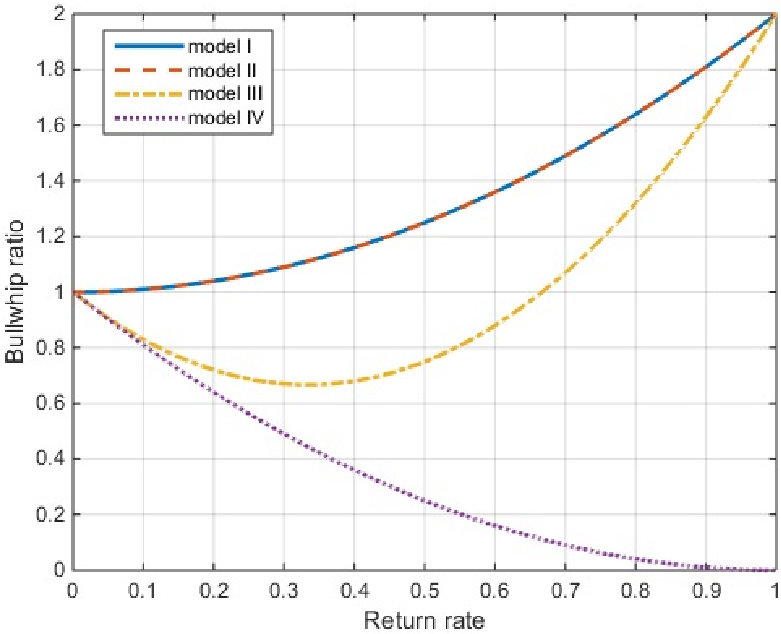


Figure 4: Bw ratio in the closed-loop supply chain.

but not necessarily smooths, the operation of the manufacturing line in closed-loop supply chains.

As per Figure 3, enhancing the transparency on the remanufacturing pipeline refers to moving from *model I* to *II* (when the market pipeline is opaque) or from *model III* to *IV* (when the market pipeline is transparent). Table 1 shows that the demand variability amplification is the same in *models I* and *II*. That is, even though the manufacturer has visibility on the remanufacturing process and uses this information within its ordering rule (in *model II*), this does not translate into a decreased Bw (against *model I*). Accordingly, the value of the remanufacturing pipeline's information transparency in terms of the Bullwhip ratio, VIT_{Rem}^{Bw}, for the case of an opaque market is null,

$$VIT_{Rem}^{Bw} = \int_{\alpha=0}^{\alpha=1} \left(Bw^{I} - Bw^{II}\right) d\alpha = \int_{\alpha=0}^{\alpha=1} \left((1+\alpha^2) - (1+\alpha^2)\right) d\alpha = 0. \tag{11}$$

Nonetheless, we will later discuss how this additional information improves the dynamics of the serviceable stock, enabling a more cost-effective fulfillment of customer requirements.

However, as long as the manufacturer is able to track the products in the market, the visibility on the remanufacturing pipeline always results in a Bullwhip reduction. This can be observed by comparing the Bw curves of *models III* and *IV* in Figure 4. This reduction is relatively low for $\alpha < \frac{1}{3}$ (i.e. the minimum of the convex Bw curve in *model III*), but the difference strongly grows from there. Now, with a transparent market pipeline, the value of the remanufacturing pipeline's information is

$$\begin{aligned} VIT_{Rem}^{Bw} &= \int_{\alpha=0}^{\alpha=1} \left(Bw^{III} - Bw^{IV}\right) d\alpha = \int_{\alpha=0}^{\alpha=1} \left((1-\alpha)^2 + 2\alpha^2 - (1-\alpha)^2\right) d\alpha \\ &= \int_{\alpha=0}^{\alpha=1} 2\alpha^2 d\alpha = \left(\frac{2\alpha^3}{3}\right)_{\alpha=0}^{\alpha=1} = \frac{2}{3}. \end{aligned} \tag{12}$$

This confirms that a significant average reduction in the Bw ratio can be expected when enabling remanufacturing visibility in contexts in which market information can be accessed. To sum up, we may conclude that the value of the remanufacturer's information for smoothing the operation of closed-loop supply chains heavily relies on whether or not the manufacturer can also access market data.

Property C. *Visibility on the market always allows for a reduction in the Bullwhip ratio in closed-loop settings; this reduction becomes more pronounced with transparency in the remanufacturing pipeline.*

Let's now consider two different scenarios depending on the visibility on remanufacturing operations; see Figure 3. First, an opaque scenario where the manufacturer does not know the manufacturer's work-in-progress (i.e. *models I* and *III*). Second, a transparent scenario in which the manufacturer is able to see the state of the remanufacturers's pipeline (i.e. *models II* and *IV*). Figure 4 reveals that visibility on the market reduces Bw in both scenarios. Nonetheless, the value of market's information transparency is significantly higher if information on the remanufacturing pipeline is also available.

Note that, when the market pipeline transparency is not enabled (*models I* and *II*), the demand amplification phenomenon always occurs in the closed-loop supply chain, i.e. $Bw^{I} = Bw^{II} > 1\ \forall\alpha > 0$. In the opaque scenario of no access to the remanfacturer's pipeline, incorporating the market information into the ordering rule eliminates the Bullwhip phenomenon unless the return rate is very high; $Bw^{III} > 1\ \forall\alpha > 2/3$ and $Bw^{III} < 1\ \forall\alpha < 2/3$. The value of information transparency then is

$$\begin{aligned} VIT_{Mk}^{Bw} &= \int_{\alpha=0}^{\alpha=1} \left(Bw^{I} - Bw^{III}\right) d\alpha = \int_{\alpha=0}^{\alpha=1} \left((1+\alpha^2) - (1-\alpha)^2 - 2\alpha^2\right) d\alpha \\ &= \int_{\alpha=0}^{\alpha=1} (2\alpha - 2\alpha^2) d\alpha = \left(\alpha^2 - \frac{2\alpha^3}{3}\right)_{\alpha=0}^{\alpha=1} = \frac{1}{3}. \end{aligned} \tag{13}$$

In contrast, in the transparent scenario, accessing to the market information always removes the Bullwhip Effect from the closed-loop supply chain; $Bw^{IV} < 1\ \forall\alpha > 0$. Now we obtain

$$\begin{aligned} VIT_{Mk}^{Bw} &= \int_{\alpha=0}^{\alpha=1} \left(Bw^{II} - Bw^{IV}\right) d\alpha = \int_{\alpha=0}^{\alpha=1} \left((1+\alpha^2) - (1-\alpha)^2\right) d\alpha \\ &= \int_{\alpha=0}^{\alpha=1} 2\alpha d\alpha = \left(\alpha^2\right)_{\alpha=0}^{\alpha=1} = 1. \end{aligned} \tag{14}$$

Our results indicate that the value of market visibility is higher when there is information transparency in the remanufacturing site. Therefore, considering also the previous study, we conclude that the Bullwhip-related benefits of sharing a specific type of information increase as the rest of the supply chain is more transparent. All in all, it is clear that the impact of information sharing heavily relies on the specific setting studied, see Tang & Naim (2004), Hosoda et al. (2015), and Cannella et al. (2016).

Property D. *The closed-loop supply chain always experience a higher net stock variability than the traditional supply chain; thus, closed-loop settings suffer from a decreased inventory performance.*

With positive lead times positive (i.e. $L, L_R, L_C > 0$), and under our assumption that the consumption lead time is the longest (i.e. $L_C >> L, L_R$), the four $NSAmp$ functions in Table 1 are convex curves whose minimum occurs for $\alpha = 0$, that is, the open-loop system. This can be demonstrated applying basic properties of functions; their first derivative with respect to α equates 0 for $\alpha = 0$ in all cases (except if $L = L_R$ in $model_{II}$, where $NSAmp^{II} = L$), and their second derivative is positive in all cases (except if $L >> L_C, L_R$ in *models III* and *IV*, which does not fit with our lead-time assumption).

For $\alpha = 0$, $NSAmp = L$. This explains the known behavior of the traditional system under an order-up-to policy, i.i.d. demand, and MMSE forecasting; see e.g. Ponte et al. (2017). From this point, $NSAmp$ in the four closed-loop supply chain archetypes increases as the return rate grows in the interval $\alpha \in [0, 1]$. This leads us to conclude that the closed-loop supply chain always generates a higher $NSAmp$ than the benchmark open-loop system. It means that a higher amount of safety stock is required to achieve a predefined customer service level or, equivalently, the same safety stock results in a lower service level. In this sense, while closed-loop supply chain contexts may enable a reduction in the Bw ratio, this comes at the expense of an increase in the $NSAmp$ metric. This finding buttresses the results of some prior studies in this discipline, e.g. Turrisi et al. (2013). Having said that, we note that we have not observed in the context of our analysis the positive impact of increasing the volume of returns in terms of net stock dynamics that was reported by other authors, e.g. Cannella et al. (2016).

Finally, it may be interesting to underline that while Bw only depends on α in the four models, $NSAmp$ also depends on the lead times; see Table 1. Specifically, $NSAmp$ depends on L, L_R, and L_C in *models III* and *IV*, while it only depends on L in *model I* and on both L and L_R in *model II*.

Property E. *Visibility on the remanufacturing pipeline always provokes a reduction in the variability of net stocks, except if the remanufacturing lead time is much longer than the manufacturing lead time.*

We consider the same case as in *property B*, comparing first *model II* against *model I* and second *model IV* against *model III*, but now from the perspective of $NSAmp$. Here, and looking at the equations in Table 1, we can see that the reduction in $NSAmp$ provoked by enabling access to the remanufacturer's work-in-progress is the same when the market pipeline is opaque as when it is transparent. That is, $NSAmp^I - NSAmp^{II} = NSAmp^{III} - NSAmp^{IV} = \alpha^2 (L - |L - L_R|)$.

We observe that, if $L \geq L_R$, sharing the remanufacturing pipeline information always reduces $NSAmp$ (and hence inventory-related costs) in closed-loop supply chains. As per Zhou et al. (2017), $L \geq L_R$ represents the most common case in real-world closed-loop settings. In this case, $NSAmp^I - NSAmp^{II} = NSAmp^{III} - NSAmp^{IV} = \alpha^2 L_R$. The higher L_R is, the stronger the reduction in $NSAmp$. However, the analysis is different for $L < L_R$. Here, $NSAmp^I - NSAmp^{II} = NSAmp^{III} - NSAmp^{IV} = \alpha^2(2L - L_R)$. Hence, enabling transparency in the remanufacturing pipeline only reduces $NSAmp$ as long as $L < L_R < 2L$. In the case that $L_R > 2L$, the impact of information transparency on inventory performance turns to be negative. If $L_R = 2L$, there are neither benefits nor losses. For a given L, the highest reduction is achieved for $L_R = L$, which is in line with the recommendations put forward by Hosoda & Disney (2018) on the benefits derived from making both lead times equal.

Now, we quantify the value of transparency in the remanufacturing pipeline in terms of $NSAmp$, VIT_{Rem}^{NSAmp}, by looking at the difference between the curves. As seen before, the inventory benefits of using such information in the ordering rule do not depend on the access to market data. We obtain

$$\begin{aligned} VIT_{Rem}^{NSAmp} &= \int_{\alpha=0}^{\alpha=1} \left(NSAmp^{I} - NSAmp^{II}\right) d\alpha = \int_{\alpha=0}^{\alpha=1} \left(NSAmp^{III} - NSAmp^{IV}\right) d\alpha \\ &= \int_{\alpha=0}^{\alpha=1} \alpha^2 \left(L - \lvert L - L_R \rvert\right) d\alpha = \left(\frac{\alpha^3}{3}\left(L - \lvert L - L_R \rvert\right)\right)_{\alpha=0}^{\alpha=1} = \frac{L}{3} - \frac{\lvert L - L_R \rvert}{3}. \end{aligned} \tag{15}$$

For $L \geq L_R$, Eq. (15) results in $VIT_{Rem}^{NSAmp} = \frac{L_R}{3}$; for $L < L_R$, Eq. (15) leads to $VIT_{Rem}^{NSAmp} = \frac{2L - L_R}{3}$. Note that, in line with the previous discussion, VIT_{Rem}^{NSAmp} becomes negative for $L_R > 2L$.

By way of illustration, Figure 5a displays the relationship between $NSAmp$ and α for $L = 4,\ L_R = 2,\ L_C = 20$. In this case, $L > L_R$, and the system benefit equally from remanufacturing pipeline visibility when the market pipeline is opaque and when it is transparent. Figure 5b considers $L = 4,\ L_R = 4,\ L_C = 20$. In line with the previous discussion, we see that making the remanufacturing lead time equal to the manufacturing lead time maximises the value of information sharing. Finally, Figure 5c shows the latter case, in which $L = 4,\ L_R = 10,\ L_C = 20$. As $L_R < 2L$, the value of information transparency turns into negative in both cases; note that $NSAmp$ increases.

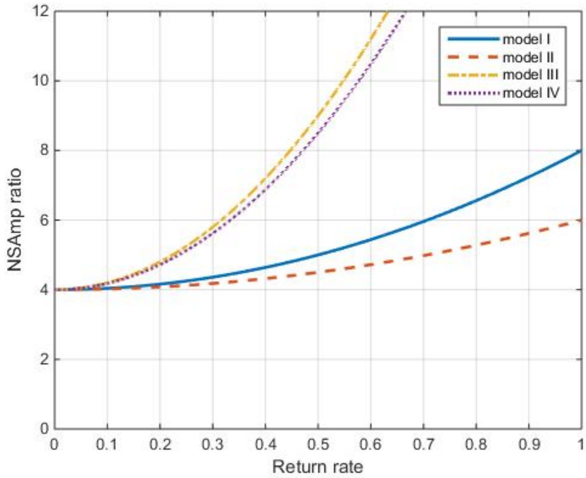
Property F. *Capturing and using the market pipeline information increases the net stock variance, thus having a negative impact on the trade-off between service level and holding costs.*

In *property C*, we discussed that information transparency in the market pipeline is very valuable in terms of Bw. Indeed, in *model IV*, Bw is close to 0 for high return rates, which undoubtedly has an economic value. However, the dynamic improvement in terms of production smoothing is achieved at the expense of a decrease in the performance of the supply chain inventory. This can be seen across the different representations of Figure 5. In the three lead-time scenarios, *models III* and *IV* provide a $NSAmp$ significantly higher than that of *models I* and *II*. Looking at the expressions in Table 1, the main cause of the $NSAmp$ increase is the influence of the long consumption lead time.

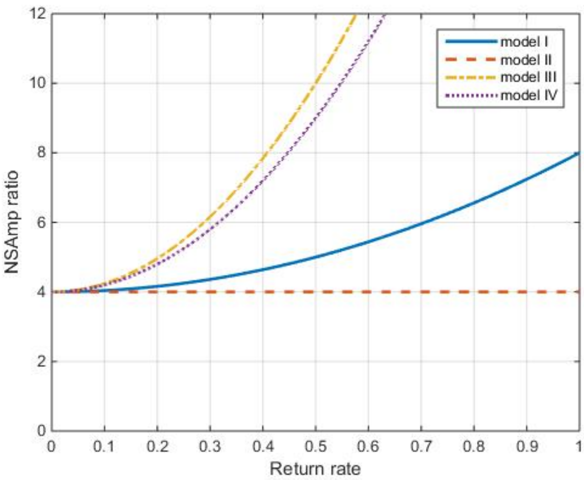
Importantly, the impact of enabling access to the market data on the $NSAmp$ is the same regardless of the remanufacturing pipeline visibility. That is, $NSAmp^{I} - NSAmp^{III} = NSAmp^{II} - NSAmp^{IV} = \alpha^2 \left(L + \lvert L - L_R \rvert - L_C - L_R\right)$. Therefore, while the value of a specific dimension of information transparency depends on the other in terms of Bw (see Eqs. 11-14), it is independent of the other dimension in terms of $NSAmp$ (see also Eq. 15). Taking this into consideration, we quantify the value of information transparency in the market pipeline from the perspective of $NSAmp$, VIT_{Mk}^{NSAmp}, as follows,

$$\begin{aligned} VIT_{Mk}^{NSAmp} &= \int_{\alpha=0}^{\alpha=1} \left(NSAmp^{I} - NSAmp^{III}\right) d\alpha = \int_{\alpha=0}^{\alpha=1} \left(NSAmp^{II} - NSAmp^{IV}\right) d\alpha \\ &= \int_{\alpha=0}^{\alpha=1} \alpha^2 \left(L + \lvert L - L_R \rvert - L_C - L_R\right) d\alpha = \left(\frac{\alpha^3}{3}\left(L + \lvert L - L_R \rvert - L_C - L_R\right)\right)_{\alpha=0}^{\alpha=1} \\ &= \frac{L + \lvert L - L_R \rvert - L_C - L_R}{3}. \end{aligned} \tag{16}$$

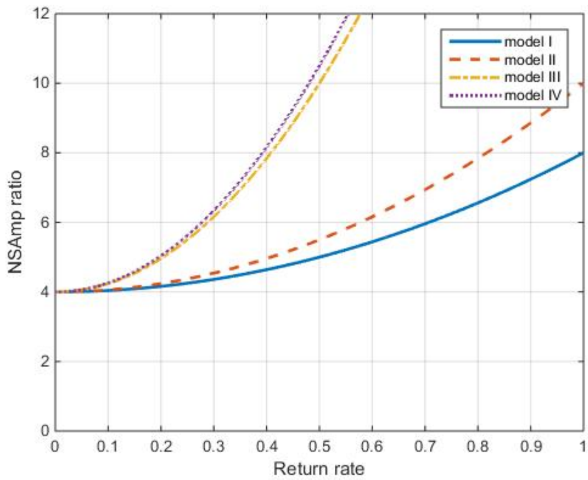
For $L \leq L_R$, Eq. (16) simplifies to $VIT_{Mk}^{NSAmp} = (-L_C)/3$. Due to $L_C > 0$, VIT_{Mk}^{NSAmp} will always be negative. On the other hand, for $L > L_R$, Eq. (16) results in $VIT_{Mk}^{NSAmp} = (2(L - L_R) - L_C)/3$. Under the assumption $L_C >> L, L_R$, here VIT_{Mk}^{NSAmp} will also be negative.



(a) $L = 4, L_R = 2, L_C = 20$



(b) $L = 4, L_R = 4, L_C = 20$



(c) $L = 4, L_R = 10, L_C = 20$

Figure 5: $NSAmp$ ratio as a function of α in the closed-loop supply chain.

Under these circumstances, tracking the trajectory of the product in the market is valuable in terms of Bw, but the fact remains that this information is not necessary in terms of $NSAmp$. Note, when the closed-loop supply chain is fully transparent and this information is used in the ordering rule (i.e. *model IV*), the dynamic effects of the return rate—based on opposite effects on order and inventory variabilities—, are similar to others described in the literature, such as those of the inventory controller (Disney & Lambrecht, 2008) and the capacity constraints (Ponte et al., 2017). This highlights the need for finding an appropriate trade-off between the potential benefits of increasing the volume of returns in terms of Bw and its undesired effects on $NSAmp$, which we will explore in the next section.

Property G. *The lead-time paradox only manifests itself, but not always, when there is information transparency in the closed-loop supply chain, and it can only be seen in terms of inventory variability.*

Lead times are a well-known cause of inefficiencies in production and distribution systems. Investing in processes of lead-time reduction thus commonly becomes a key priority for businesses, e.g. see Ponte et al. (2018), which is strongly aligned with the principles of Lean operations (Womack & Jones, 1997).

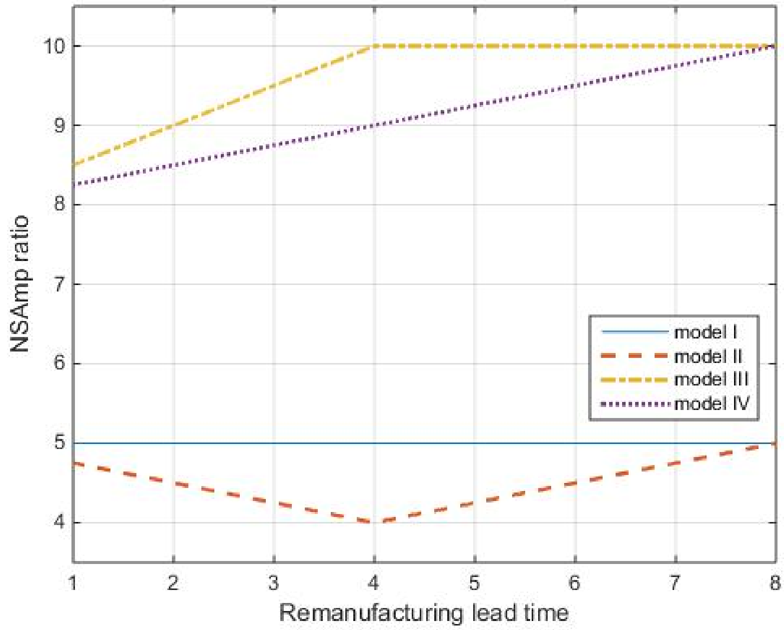


Figure 6: $NSAmp$ ratio as a function of L_R in the closed-loop supply chain..

In this sense, the discovery of a counterintuitive lead-time effect in closed-loop supply chains boosted the curiosity of researchers: the performance of hybrid systems sometimes benefits from an increase in the remanufacturing lead time (Inderfurth & van der Laan, 2001; Hosoda & Disney, 2018).

In our study, we have also seen this lead-time paradox, specifically from the perspective of the $NSAmp$ ratio. Although it is known that it can also occur from the perspective of Bw (Hosoda & Disney, 2018), literature shows that is more common in terms of inventory variability (Tang & Naim, 2004; Hosoda et al., 2015). As discussed in *property E*, $NSAmp$ is minimised for $L_R = L$ in *model II*. That is, if $L_R > L$, L_R should be reduced until it equates L_R; while if $L_R < L$, L needs to be increased until this is equal to L_R. This would nullify the second term of the $NSAmp$ expression in Table 1. Note that this is a clear manifestation of the lead-time paradox. On the other hand, neither L nor L_R impact on Bw. For the sake of clarity, Figure 6 represents the relationship between $NSAmp$ and L_R for *model II* when $\alpha = 0.5$ (the impact of this parameter was investigated before), $L = 4$, $L_C = 20$. It can be clearly seen how in this context reducing L_R should only be a priority when it is higher than L.

Figure 6 also show the relationship between $NSAmp$ and L_R for the other three models. In *model I*, as previously noted, $NSAmp$ does not depend on L_R. This means that the paradox does not emerge in absence of information transparency in the hybrid system. In *model IV*, $NSAmp$ is an increasing function of L_R. Therefore, in this case, reducing L_R always improves the dynamics of the system. Hence, when both pipelines, i.e. market and remanufacturing, are transparent, the paradox cannot be perceived either. Finally, reducing L_R also smooths inventory dynamics in *model III* as long as $L_R \leq L$. However, reducing L_R does not impact net stock variability in the case that $L_R > L$.

In short, while the effects of the manufacturing lead times on hybrid manufacturing-remanufacturing systems are straightforward (in the four models, reducing them turns to be an effective strategy for improving inventory performance), those of remanufacturing lead times may adopt different forms. This perspective underscores the importance of understanding the dynamics of such lead times on closed-loop supply chains before taking any action in this regard to avoid unnecessary investments.

5. The variability trade-off in closed-loop supply chains: An economic study

As per the previous discussion, increasing the volume of returns via α has the potential to reduce the variability in the manufacturing orders—and hence capacity-related costs—, but this would occur at the expense of a higher variability in the net stock—and hence inventory-related costs. This underlines the need for finding an appropriate balance between Bw and $NSAmp$ in closed-loop settings, in the same way as in open-loop ones. This has been labelled as the *variability trade-off* in production and distribution systems, see Disney & Lambrecht (2008). Interestingly, this perspective suggests the existence of an optimal return rate, α^*, in our supply chain, which will be explored in this section.

To model the variability trade-off, we assume that the capacity-related production costs, associated with unstable production schedules, are linearly related to the variance of orders (σ_o^2). The unit cost is denoted by k_o. Also, we assume that the inventory-related costs, i.e. the sum of holding and stock-out costs, are linearly related to the variance of inventories (σ_i^2). In this case, the unit cost is denoted by k_i. Note, $k_o, k_i \geq 0$, with k_o, k_i in mu/pu^2 (mu: monetary units, pu: product unit). This approach defines a common objective function in traditional supply chain contexts that enables academics and practitioners to analyse the key variability trade-off, $J_{ol} = k_o\sigma_o^2 + k_i\sigma_i^2$; see e.g. Disney et al. (2004). Using the metrics' definition in Eqs. (9) and (10), we obtain $\sigma_o^2 = \sigma^2 Bw$ and $\sigma_i^2 = \sigma^2 NSAmp$, which allows us to easily express our cost-based objective function depending on Bw and $NSAmp$.

We adapt this approach to closed-loop supply chain settings by considering that the cost of remanufacturing products is often significantly different, ideally lower, than the cost of manufacturing new ones. For instance, Giutini et al. (2003) estimate that remanufactured products typically incur costs that are 40-65% less than new products (as the former often require only a fraction of the material processing needed by the latter). We use the unit cost k_r, in mu/pu, to represent the difference between the manufacturing and the remanufacturing fixed costs of one product—in other words, the saving associated to each unit of remanufactured product (for $k_r > 0$) or the extra cost associated to each unit of remanufactured product (for $k_r < 0$). Therefore, we consider the objective function,

$$J_{cl} = k_o\sigma^2 Bw + k_i\sigma^2 NSAmp - k_r\alpha\mu; \tag{17}$$

where $\alpha\mu$ is the average number of remanufactured products. Note that the three components of our cost-based objective function, which can be expressed in mu, depend on the return rate, α.

In line with the above, $k_r > 0$ represents the most common scenario in real-world hybrid systems. However, the objective function also holds for $k_r < 0$. This could be due to an excessive cost of collecting used products from the market or disassembling them (Goltsos et al., 2019a). In this case, remanufacturing has no direct economic value (as it is the most costly alternative to satisfy customer demand), but remanufacturing may still have a key environmental role, see Giutini et al. (2003). Nonetheless, even in this case, it may still be profitable to remanufacture a small volume of products if that allowed for enhancing the supply chain dynamics, which we will explore later in this section.

Dividing Eq. (17) by the mean demand, μ, leads to $J_{cl}^d = \frac{J_{cl}}{\mu} = k_o\lambda Bw + k_i\lambda NSAmp - k_r\alpha$. Finally, we group the economic parameters k_o, k_i together with the index of dispersion $\lambda = \sigma^2/\mu$, as for the purposes of our study they can be interpreted as uncontrollable characteristics of the system. To this aim, we define $k_o^d = k_o\lambda$ and $k_i^d = k_i\lambda$, both in mu/pu, leading to the objective function

$$J_{cl}^d = k_o^d Bw + k_i^d NSAmp - k_r\alpha. \tag{18}$$

Importantly, J_{cl}^d captures three strongly interrelated sources of cost that depend on the return rate, α, given that Bw and $NSAmp$ are functions of this parameter (see Table 1). In light of prior discussions, this cost-based objective function includes critical concerns for closed-loop supply chain managers. Note, we do not consider the cost of information sharing in J_{cl}^d, as this cost does not necessarily depend on α. In the following subsections, Eq. (18) will be particularised for the four archetypes under study. We aim to get a deeper understanding of the economic performance of these systems, as well as to derive the optimal return rate, α^*, from the perspective of the convex metric J_{cl}^d.

5.1. Model I

Using the expressions in Table 1, Eq. (18) leads to $J_{cl}^d = k_o^d(1+\alpha^2) + k_i^d(L+\alpha^2 L) - k_r\alpha$, whose first derivative is $\frac{dJ_{cl}^d}{d\alpha} = 2\alpha k_o^d + 2\alpha L k_i^d - k_r$. Making the derivative equal to 0, J_{cl}^d is minimised for

$$\alpha_I^* = \frac{k_r}{2(k_o^d + Lk_i^d)}. \tag{19}$$

Eq.(19) provides the return rate that should be pursued from an economic perspective. Relevantly, it proves the existence of an optimal volume of returns in the closed-loop supply chain, α_I^*. Note that α_I^* strongly depends on the parameters k_o^d, k_i^d, and k_r; and consequently on the unit costs k_o, k_i, and k_r, as well as on the demand's variance-to-mean ratio λ. Consistently with their definition, k_r has an increasing impact on α_I^*, while k_i, k_o, and λ contribute to decrease α_I^*. This occurs given that both the Bw and the $NSAmp$ ratios are negatively impacted by an increase in α. In addition, α_I^* also depends on the manufacturing lead time, L, as it impacts on the net stock variability. The longer L, the smaller α_I^*. However, α_I^* is not affected by the remanufacturing and consumption lead times, L_R and L_C.

Assuming that α is constrained in practice to the interval [0,1] and considering the characteristics of the cost-based objective function, Eq. (19) allows us to conclude: (i) α should always be 0 if $k_r \leq 0$; (ii) α should be 1 if k_r is large enough to outweigh the weighted sum of the other costs, specifically if $k_r \geq 2(k_o^d + Lk_i^d)$; and (iii) the target α should be between 0 and 1 if $0 < k_r < 2(k_o^d + Lk_i^d)$.

5.2. Model II

By means of the Bw and $NSAmp$ equations shown in Table 1 for this model, Eq. (18) now results in $J^d_{cl} = k^d_o(1+\alpha^2)+k^d_i(L+\alpha^2|L-L_R|)-k_r\alpha$, whose first derivative is $\frac{dJ^d_{cl}}{d\alpha} = 2\alpha k^d_o+2\alpha|L-L_R|k^d_i-k_r$. Equating the first derivative to 0, J^d_{cl} can be optimised in our cost model for

$$\alpha^*_{II} = \frac{k_r}{2(k^d_o + |L - L_R|k^d_i)}. \tag{20}$$

This expression is relatively similar to that obtained for *model I*, with the difference that the effect of the manufacturing lead time, L, is now replaced by the effect of the absolute value of the difference between the manufacturing and remanufacturing lead times, $|L - L_R|$. Consequently, and as long as $L_R < 2L$, $\alpha^*_{II} > \alpha^*_I$. That is, visibility on the remanufaturing line should motivate the manufacturer to pursue a higher level of circularity in the hybrid system. Again, if the return rate is constrained to [0,1], α should always and only be 0 for negative values of k_r. This occurs because the Bw and $NSAmp$ ratios increase as α grows (which applies to this and the previous closed-loop supply chain archetype). However, even so, when kr_0, it is always economically efficient to remanufacture, i.e. $\alpha^*_{II} > 0$.

5.3. Model III

Here, the equations that can be seen in Table 1 turn Eq. 18 into $J^d_{cl} = k^d_o((1-\alpha)^2+2\alpha^2))+k^d_i(L+\alpha^2(L_C+L_R-|L-L_R|))-k_r\alpha$, whose first derivative is $\frac{dJ^d_{cl}}{d\alpha} = (-2(1-\alpha)+4\alpha)k^d_o+2\alpha(L_C+L_R-|L-L_R|)k^d_i-k_r$. Making it equal to 0, J^d_{cl} is minimised for

$$\alpha^*_{III} = \frac{k_r + 2k^d_o}{2\left(3k^d_o + (L_C + L_R - |L - L_R|)\, k^d_i\right)}. \tag{21}$$

Eq. (21) is more complex than those for the previous models and contains interesting properties. Importantly, in this case, even if k_r is negative, the optimal return rate, α^*_{III}, can be positive. This occurs as long as $|k_r| < 2k^d_o = 2k_o\lambda$. This is a consequence of the Bw metric being positively impacted by an increase in α for low values of the return rate. This remarkable insight reveals that the closed-loop supply chain may economically benefit from remanufacturing products collected from the market, even in the case that these were more expensive than original, manufactured products.

It can also be underlined that the consumption lead time, L_C, has a significant effect on α^*_{III}, is not observed in the previous models. The longer L_C is, the lower the optimal value of α is. In this scenario of information transparencies, the incentives for developing remanufacturing practices would be less strong in industries with long-lasting products. In addition, note that if $L > L_R$, increasing L contributes to an increase in α^*_{III} (given that $L_C >> L$) while increasing L_R contributes to a decrease in α^*_{III}; in contrast, if $L < L_R$, increasing L contributes to a decrease in α^*_{III}, which does not depend on L_R. Finally, we underline that the lead-time effects on α^*_{III} discussed become stronger as k^d_i grows.

5.4. Model IV

Lastly, we look at the model with full information transparency. In this case, Eq. (18) leads to $J_{cl}^d = k_o^d(1-\alpha)^2 + k_i^d\left(L + \alpha^2(L_C + L_R - L)\right) - k_r\alpha$, by using the equations in the last row of Table 1. The first derivative is $\frac{dJ_{cl}^d}{d\alpha} = -2(1-\alpha)k_o^d + 2\alpha(L_C + L_R - L)k_i^d - k_r$. Then, $\frac{dJ_{cl}^d}{d\alpha} = 0$ results in

$$\alpha_{IV}^* = \frac{k_r + 2k_o^d}{2\left(k_o^d + (L_C + L_R - L)k_i^d\right)}. \tag{22}$$

The structure of this expression is relatively similar to that obtained for *model III*. First, we note that it may be economically rational to remanufacture when $k_r < 0$, due to its smoothing effect on the manufacturing line. The condition for ensuring the economic viability of remanufacturing is the same as before, $|k_r| < 2k_o^d = 2k_o\lambda$, which differs to those for *models I* and *II*, represented by $k_r > 0$. Second, only if $k_r \geq 2(L_C + L_R - L)k_i^d$, the target from an economic perspective should be $\alpha = 1$.

Finally, we focus on the lead times in the closed-loop system. Given that $L_C >> L$, $(L_C + L_R - L) > 0$. Note that the manufacturing and remanufacturing lead times have opposite effects on the optimal return rate, α_{IV}^*. Increasing the former, L, contributes to increase α_{IV}^*; increasing the latter, L_R, contributes to reduce α_{IV}^*. The impact of the consumption lead time, L_C, is the same as that of L_R.

5.5. Numerical example

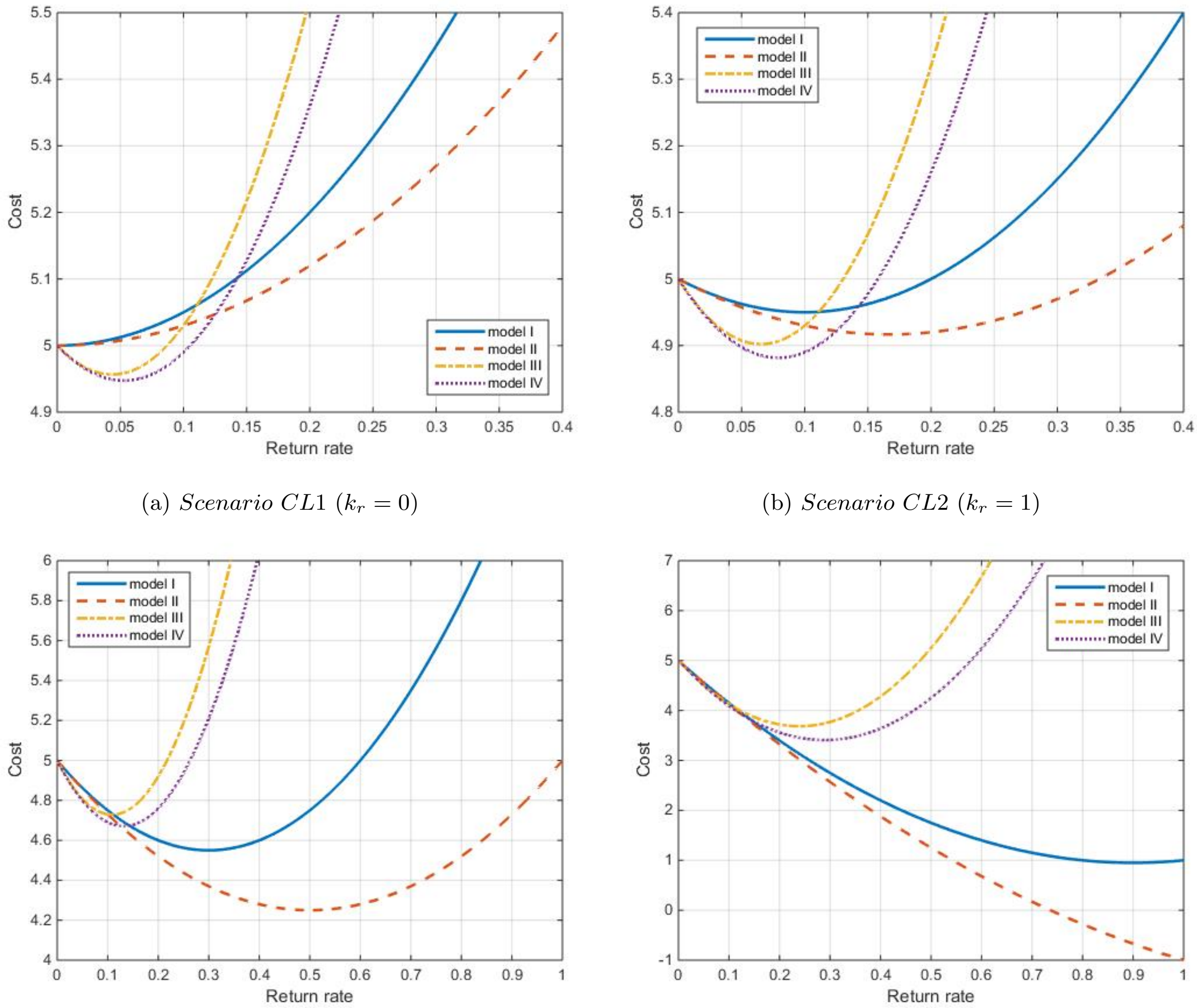
To illustrate the insights from the previous analysis, we conduct a numerical study. For the sake of simplicity, we assume $k_o^d = 1\ mu/pu$, $k_i^d = 1\ mu/pu$. Moreover, we use the values for the lead times considered in Figure 5a: $L = 4, L_R = 2, L_C = 20$. Figure 7 displays the metric J_{cl}^d, an indicator of the operational costs in the closed-loop supply chain, as a function of the return rate, α, in four different scenarios. These represent a set of real-world settings, and are defined by different values of k_r:

- *Scenario CL1*, representing the baseline scenario in which the cost of remanufacturing is similar to the cost of manufacturing. Hence, there is no direct economic benefit from remanufacturing used products when compared to traditional manufacturing. To illustrate it, we use $k_r = 0$.

- *Scenario CL2*, representing practical scenarios in which the cost of remanufacturing is significantly lower than the cost of manufacturing, but the difference between them is comparatively similar to the other costs in the closed-loop supply chain. In this case, we consider $k_r = 1$.

- *Scenario CL3*, representing a scenario in which the difference between the cost of remanufacturing and manufacturing is significantly higher than in the previous one. We now employ $k_r = 3$.

- *Scenario CL4*, representing a scenario in which the cost of remanufacturing is very small in comparison with the cost of manufacturing. Therefore, a great economic opportunity would stem from remanufacturing. We use $k_r = 9$ to model this practical closed-loop setting.



(a) *Scenario CL*1 ($k_r = 0$)

(b) *Scenario CL*2 ($k_r = 1$)

(c) *Scenario CL*3 ($k_r = 3$)

(d) *Scenario CL*4 ($k_r = 9$)

Figure 7: Minimising the cost of the closed-loop supply chain.

Several findings can be derived from the analysis of the results. First, Figure 7a presents the costs of the closed-loop supply chain for *scenario CL1*. In line with the previous discussion, the optimal value of the return rate for *models I* and *II* is 0. That is, there are no economic reasons for remanufacturing in this case, given that: (i) it does not entail direct savings in comparison with manufacturing; and (ii) it does not improve the dynamics of the closed-loop supply chain. However, in *models III* and *IV*, even though $k_r = 0$, the hybrid system slightly benefits from remanufacturing a small amount of products. For example, the optimal rate for *model IV* is $\alpha^*_{IV} = 5.26\%$. As we showed analytically, this may also happen in settings in which remanufacturing used products is slightly more expensive (i.e. $k_r < 0$), but only in *models III* and *IV*, where Bw benefit from an increase in α; see Figure 4.

Figure 7b provides the results for *scenario CL2*. In this case, the four curves present an optimal return rate within the interval (0,1). Interestingly, the return rate that minimises costs is the highest for *model II*, which is $\alpha^*_{II} = 16.67\%$, while for instance for *model IV* it is $\alpha^*_{IV} = 7.89\%$. Note that, in general terms, high returns rates are penalised by the strong increase in $NSAmp$; see Figure 5.

As can be expected, the optimal return rates for the four closed-loop supply chain archetypes increase as the economic parameter k_r grows. In this sense, when the savings associated to remanufacturing are larger, the differences between the archetypes are emphasised. Figure 7c provides the curves for *scenario CL3*. Following from the previous line of argument, the optimal return rate of *model II* is again the highest, i.e. $\alpha^*_{II} = 50.00\%$, while that of *model IV* has only increased to $\alpha^*_{IV} = 13.16\%$. Note that the increase in α^*_{III} and α^*_{IV} is strongly obstructed by the pronounced increase of $NSAmp$ for high values of the return rate in these models; see Figure 5. Moreover, it can be underlined that in this scenario the closed-loop supply chain may benefit from a strong cost reduction in comparison with traditional open-loop supply chains (represented in Figure 7 by $\alpha = 0$).

Finally, Figure 7d shows the supply chain cost for *scenario CL4*. It illustrates that only if $k_r >> k^d_o, k^d_i$ the optimal return rate is 1. Even so, this only happens here for *model III*. This model allows us to confirm that, while previously we saw that $\alpha^*_{IV} > \alpha^*_{III} > \alpha^*_{II} > \alpha^*_I$ for low values of k_r, $\alpha^*_{II} > \alpha^*_I > \alpha^*_{IV} > \alpha^*_{III}$ for moderate high values of k_r. In addition, we observe that *model IV* is capable of minimising closed-loop supply chain costs for low and moderate values of k_r, while *model II* provides the lower cost for high values of this economic parameter. Nonetheless, it is important to note that the comparisons of total costs among models should be carried out with caution given that, as we discussed before, we are not considering here the costs of information sharing.

6. Conclusions

The transition from a linear to a circular economy model needs to be built, among other important aspects, on the understanding of the dynamic characteristics of closed-loop supply chains. However, the interactions between the relevant parameters have not been explored in detail, and as such the complex behaviour of these systems in the real world is not yet well understood. Indeed, insights so far have led to somewhat conflicting conclusions, which may be misleading for closed-loop managers.

In this work, we investigate the dynamics and performance of closed-loop supply chains through the lens of the Bullwhip Effect. We thus look at the variability of orders and inventories via the Bw and $NSAmp$ metrics. We quantify them as functions of the return rate and three relevant lead times in four archetypes of hybrid manufacturing-remanufacturing systems that differ in the degree of information visibility. This allows us to gain a thorough understanding on the cost-efficiency of the production system and the trade-off between holding requirements and stock-out occurrence, as well as to revisit previous findings in the literature in a bid to delve into the discrepancies.

We demonstrate that the impact of increasing the volume of returns on the smoothness of the production process (i.e. order variability) may be positive or negative depending on the sources of information available in the closed-loop supply chain. In this sense, while in some circumstances supply chains may dynamically benefit from 'closing the loop', in other ocassions this worsens their dynamic behaviour. In contrast, we observe that closed-loop supply chains tend to suffer from a reduced inventory performance (i.e. increased inventory variability) in comparison with traditional systems.

Exploring the four archetypes, we show that increasing the transparency in the remanufacturing pipeline generally has a positive impact on both order and inventory variability—although some exceptions have been observed and discussed. However, enabling visibility in the market pipeline tends to decrease order variability at the expense of an increase in inventory variability. In addition, we reveal that the value of information transparency in the market pipeline is the same regardless of the visibility in the remanufacturing pipeline; however, the value of information transparency in the remanufacturing pipeline strongly depends on the information available on the market pipeline.

Under our assumptions, Bw does not depend on the lead times (like in traditional systems). However, the lead times significantly affect inventory variability. Having noted that, their impact strongly depend on the degree of information transparency. Interestingly, we observe that increasing the degree of transparency in the closed-loop supply chain makes it more sensitive to lead-time effects. In addition, our analysis finds that the previously documented paradox of the remanufacturing lead time in hybrid systems may or not emerge depending on the degree of information transparency available.

Considering the economic implications of our study, we reveal the existence of an optimal return rate. We obtain it in the four archetypes, depending on: (i) the cost structure of the supply chain; (ii) the lead times; and (iii) the variability of the consumer demand. The optimal return rate emerges from considering both the key variability trade-off—as increasing the return rate may have opposite effects on order and inventory variability—and the economic value of remanufacturing. Only in some cases (generally, when the savings derived from the reverse flow of materials are much more acute than the cost of production and inventory variability), the optimal return rate is 1. In addition, and interestingly, we show that the supply chain may benefit from remanufacturing products collected from the market even in the case that these are more expensive than manufacturing new ones.

Finally, we discuss the limitations of our study and suggest avenues for future work. First, we use the industrially popular order-up-to policy to manage the serviceable inventory, while the popular push model is employed for the recoverable inventory. The study of other policies is a research direction worth pursuing, as it may help closed-loop managers to enhance their control systems. In addition, the analysis is restricted to the case of i.i.d. demand and MMSE forecasting. Future research could be directed towards the understanding of other demand characteristics and forecasting methods. Also, it may be necessary to address other relationships between the demand and returns processes. In our study, we assume that the return rate and lead times are deterministic, as in most previous papers in the area. However, in several practical settings, return rates and lead times are highly uncertain. Exploring the trajectory of the Bullwhip curves in these cases would also yield valuable managerial insights. Besides, our work does not incorporate the costs of enabling information transparencies. Bringing them into the analysis would allow for managerially relevant studies of cost-benefit trade-offs. Finally, we here study a hybrid manufacturing-remanufacturing system for benchmarking purposes. Considering other closed-loop structures, like pure remanufacturing systems or divergent effects (e.g. multiple retailers), would also be necessary to support the effective implementation of real-world closed-loop systems.

7. Acknowledgements

Jose M. Framinan and Roberto Dominguez appreciate the financial support of the Spanish Ministry of Science and Innovation, through the project PROMISE (reference DPI201680750P), the Junta de Andalucia (I+D+i FEDER Andalucía 2014-2020) under the project EFECTOS (reference US-1264511), and the University of Seville, through the V/VI PPIT-US programme. Salvatore Cannella acknowledges the financial support provided by the Italian Ministry of Education, University, and Research, through the Rita Levi Montalcini fellowship programme.

References

Abbey, J. D., & Guide Jr, V. D. R. (2018). A typology of remanufacturing in closed-loop supply chains. *International Journal of Production Research*, *56*, 374–384.

Adenso-Diaz, B., Moreno, P., Gutierrez, E., & Lozano, S. (2012). An analysis of the main factors affecting bullwhip in reverse supply chains. *International Journal of Production Economics*, *135*, 917–928.

Braz, A. C., De Mello, A. M., de Vasconcelos Gomes, L. A., & de Souza Nascimento, P. T. (2018). The bullwhip effect in closed-loop supply chains: A systematic literature review. *Journal of Cleaner Production*, *202*, 376–389.

Cannella, S., Barbosa-Póvoa, A. P., Framinan, J. M., & Relvas, S. (2013). Metrics for bullwhip effect analysis. *Journal of the Operational Research Society*, *64*, 1–16.

Cannella, S., Bruccoleri, M., & Framinan, J. M. (2016). Closed-loop supply chains: What reverse logistics factors influence performance? *International Journal of Production Economics*, *175*, 35–49.

Chatfield, D. C., & Pritchard, A. M. (2013). Returns and the bullwhip effect. *Transportation Research Part E: Logistics and Transportation Review*, *49*, 159–175.

Chen, F., Drezner, Z., Ryan, J. K., & Simchi-Levi, D. (2000a). Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science*, *46*, 436–443.

Chen, F., Ryan, J. K., & Simchi-Levi, D. (2000b). The impact of exponential smoothing forecasts on the bullwhip effect. *Naval Research Logistics*, *47*, 269–286.

Dejonckheere, J., Disney, S. M., Lambrecht, M. R., & Towill, D. R. (2003). Measuring and avoiding the bullwhip effect: A control theoretic approach. *European Journal of Operational Research*, *147*, 567–590.

Dejonckheere, J., Disney, S. M., Lambrecht, M. R., & Towill, D. R. (2004). The impact of information enrichment on the bullwhip effect in supply chains: A control engineering perspective. *European Journal of Operational Research*, *153*, 727–750.

Dev, N. K., Shankar, R., & Choudhary, A. (2017). Strategic design for inventory and production planning in closed-loop hybrid systems. *International Journal of Production Economics*, *183*, 345–353.

Disney, S. M., & Lambrecht, M. R. (2008). On replenishment rules, forecasting, and the bullwhip effect in supply chains. *Foundations and Trends® in Technology, Information and Operations Management*, *2*, 1–80.

Disney, S. M., Towill, D. R., & Van de Velde, W. (2004). Variance amplification and the golden ratio in production and inventory control. *International Journal of Production Economics*, *90*, 295–309.

Dominguez, R., Cannella, S., & Framinan, J. M. (2015). On returns and network configuration in supply chain dynamics. *Transportation Research Part E: Logistics and Transportation Review*, *73*, 152–167.

Dominguez, R., Cannella, S., Ponte, B., & Framinan, J. M. (2019a). On the dynamics of closed-loop supply chains under remanufacturing lead time variability. *Omega*, *in press*, 102106.

Dominguez, R., Ponte, B., Cannella, S., & Framinan, J. M. (2019b). On the dynamics of closed-loop supply chains with capacity constraints. *Computers & Industrial Engineering*, *128*, 91–103.

Genovese, A., Acquaye, A. A., Figueroa, A., & Koh, S. L. (2017). Sustainable supply chain management and the transition towards a circular economy: Evidence and some applications. *Omega*, *66*, 344–357.

Giutini, R., Gaudette, K. et al. (2003). Remanufacturing: The next great opportunity for boosting us productivity. *Business Horizons*, *46*, 41–48.

Goltsos, T. E., Ponte, B., Wang, S., Liu, Y., Naim, M. M., & Syntetos, A. A. (2019a). The boomerang returns? accounting for the impact of uncertainties on the dynamics of remanufacturing systems. *International Journal of Production Research*, *57*, 7361–7394.

Goltsos, T. E., Syntetos, A. A., & van der Laan, E. (2019b). Forecasting for remanufacturing: The effects of serialization. *Journal of Operations Management*, *65*, 447–467.

Govindan, K., Soleimani, H., & Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research*, *240*, 603–626.

Guide, V. D. R., Harrison, T. P., & Van Wassenhove, L. N. (2003). The challenge of closed-loop supply chains. *Interfaces*, *33*, 3–6.

Hedenstierna, C. P. T., Disney, S. M., Eyers, D. R., Holmström, J., Syntetos, A. A., & Wang, X. (2019). Economies of collaboration in build-to-model operations. *Journal of Operations Management*, *65*, 753–773.

Hosoda, T., & Disney, S. M. (2009). Impact of market demand mis-specification on a two-level supply chain. *International Journal of Production Economics*, *121*, 739–751.

Hosoda, T., & Disney, S. M. (2018). A unified theory of the dynamics of closed-loop supply chains. *European Journal of Operational Research*, *269*, 313–326.

Hosoda, T., Disney, S. M., & Gavirneni, S. (2015). The impact of information sharing, random yield, correlation, and lead times in closed loop supply chains. *European Journal of Operational Research*, *246*, 827–836.

Inderfurth, K., & van der Laan, E. (2001). Leadtime effects and policy improvement for stochastic inventory control with remanufacturing. *International Journal of Production Economics*, *71*, 381–390.

Isaksson, O. H., & Seifert, R. W. (2016). Quantifying the bullwhip effect using two-echelon data: A cross-industry empirical investigation. *International Journal of Production Economics*, *171*, 311–320.

Lee, H. L., Padmanabhan, V., & Whang, S. (1997a). The bullwhip effect in supply chains. *Sloan Management Review*, *38*, 93–102.

Lee, H. L., Padmanabhan, V., & Whang, S. (1997b). Information distortion in a supply chain: The bullwhip effect. *Management Science*, *43*, 546–558.

Lee, H. L., So, K. C., & Tang, C. S. (2000). The value of information sharing in a two-level supply chain. *Management Science*, *46*, 626–643.

Lin, J., Naim, M. M., Purvis, L., & Gosling, J. (2017). The extension and exploitation of the inventory and order based production control system archetype from 1982 to 2015. *International Journal of Production Economics*, *194*, 135–152.

Metters, R. (1997). Quantifying the bullwhip effect in supply chains. *Journal of Operations Management*, *15*, 89–100.

Ostlin, J., Sundin, E., & Björkman, M. (2009). Product life-cycle implications for remanufacturing strategies. *Journal of Cleaner Production*, *17*, 999–1009.

Ponte, B., Costas, J., Puche, J., Pino, R., & de la Fuente, D. (2018). The value of lead time reduction and stabilization: A comparison between traditional and collaborative supply chains. *Transportation Research Part E: Logistics and Transportation Review*, *111*, 165–185.

Ponte, B., Naim, M. M., & Syntetos, A. A. (2019). The value of regulating returns for enhancing the dynamic behaviour of hybrid manufacturing-remanufacturing systems. *European Journal of Operational Research*, *278*, 629–645.

Ponte, B., Wang, X., de la Fuente, D., & Disney, S. M. (2017). Exploring nonlinear supply chains: the dynamics of capacity constraints. *International Journal of Production Research*, *55*, 4053–4067.

Raghunathan, S. (2001). Information sharing in a supply chain: A note on its value when demand is nonstationary. *Management Science*, *47*, 605–610.

Reimann, M., Xiong, Y., & Zhou, Y. (2019). Managing a closed-loop supply chain with process innovation for remanufacturing. *European Journal of Operational Research*, *276*, 510–518.

Souza, G. C. (2013). Closed-loop supply chains: A critical review, and future research. *Decision Sciences*, *44*, 7–38.

Tang, O., & Naim, M. M. (2004). The impact of information transparency on the dynamic behaviour of a hybrid manufacturing/remanufacturing system. *International Journal of Production Research*, *42*, 4135–4152.

Teunter, R. H., Babai, M. Z., Bokhorst, J. A., & Syntetos, A. A. (2018). Revisiting the value of information sharing in two-stage supply chains. *European Journal of Operational Research*, *270*, 1044–1052.

Turrisi, M., Bruccoleri, M., & Cannella, S. (2013). Impact of reverse logistics on supply chain performance. *International Journal of Physical Distribution & Logistics Management*, *43*, 564–585.

Wang, X., & Disney, S. M. (2016). The bullwhip effect: Progress, trends and directions. *European Journal of Operational Research*, *250*, 691–701.

Womack, J. P., & Jones, D. T. (1997). Lean thinking—banish waste and create wealth in your corporation. *Journal of the Operational Research Society*, *48*, 1148–1148.

Zhou, L., & Disney, S. M. (2006). Bullwhip and inventory variance in a closed loop supply chain. *Or Spectrum*, *28*, 127–149.

Zhou, L., Naim, M. M., & Disney, S. M. (2017). The impact of product returns and remanufacturing uncertainties on the dynamic performance of a multi-echelon closed-loop supply chain. *International Journal of Production Economics*, *183*, 487–502.

Zhou, L., Naim, M. M., Tang, O., & Towill, D. R. (2006). Dynamic performance of a hybrid inventory system with a kanban policy in remanufacturing process. *Omega*, *34*, 585–598.

Appendix A. Derivation of the Bullwhip and Net Stock Amplification ratios

In the following subsections, we obtain Bw and $NSAmp$ in the four archetypes. For Bw, we always start from the difference between orders issued in two consecutive periods, t y $t-1$. Applying Eq. (2),

$$o_t - o_{t-1} = (S_t - S_{t-1}) - (i_t - i_{t-1}) - (w_t - w_{t-1}). \tag{A.1}$$

The time-invariant condition of the order-up-to point makes $S_t - S_{t-1} = 0$. In addition, as per Eq. (4), $i_t - i_{t-1} = m_t + r_t - d_t$, where $m_t = o_{t-L}$ (Eq. 3) and $r_t = \alpha d_{t-L_C-L_R}$ (Eq. 1). Therefore,

$$\begin{aligned} o_t &= o_{t-1} - (o_{t-L} + \alpha d_{t-L_C-L_R} - d_t) - (w_t - w_{t-1}) \\ &= (d_t - \alpha d_{t-L_C-L_R}) + (o_{t-1} - o_{t-L}) - (w_t - w_{t-1}). \end{aligned} \tag{A.2}$$

To obtain $NSAmp$, we will start by isolating the on-hand inventory in Eq. (2),

$$i_t = S_t - o_t - w_t, \tag{A.3}$$

Appendix A.1. Model I

Here, Eq. (5) results in $w_t - w_{t-1} = o_{t-1} - o_{t-L}$. Thus, Eq. (A.2) simplifies to

$$o_t = d_t - \alpha d_{t-L_C-L_R} \tag{A.4}$$

As the demand is an i.i.d., non-autocorrelated, random variable, we easily obtain the Bullwhip ratio,

$$Bw^I = \frac{\text{var}(o_t)}{\text{var}(d_t)} = \frac{\sigma^2 + \alpha^2\sigma^2}{\sigma^2} = 1 + \alpha^2. \tag{A.5}$$

Now we focus on the Net Stock Amplification ratio. In *model I*, Eq. (A.3), by means of Eq. (5) and later Eq. (A.4), leads to the following inventory equation,

$$i_t = S_t - o_t - \sum_{i=1}^{L-1} o_{t-i}. = S_t - \sum_{i=0}^{L-1} o_{t-i} = S_t - \sum_{i=0}^{L-1} \left(d_{t-i} - \alpha d_{t-L_C-L_R-i}\right). \tag{A.6}$$

Under our assumption $L_C >> L$, there is no correlation between the relevant addends. That is, a specific demand (in t) and its associated returns (in $t + L_C + L_R$) will not be considered at the same time in the previous L-term sum. In addition, as S_t is fixed over time, we obtain the $NSAmp$ metric,

$$NSAmp^I = \frac{\text{var}(i_t)}{\text{var}(d_t)} = \frac{L\sigma^2 + \alpha^2 L\sigma^2}{\sigma^2} = L + \alpha^2 L. \tag{A.7}$$

Appendix A.2. Model II

In this case, the difference in the work-in-progress for two consecutive periods, by means of Eq. (6), is $w_t - w_{t-1} = (o_{t-1} - o_{t-L}) + \alpha(d_{t-L_C} - d_{t-L_C-L_R})$. Eq. (A.2) then becomes

$$o_t = (d_t - \alpha d_{t-L_C-L_R}) - \alpha(d_{t-L_C} - d_{t-L_C-L_R}) = d_t - \alpha d_{t-L_C}. \tag{A.8}$$

Considering the properties of the demand series, this results in the same Bw as in the previous model,

$$Bw^{II} = \frac{\text{var}(o_t)}{\text{var}(d_t)} = \frac{\sigma^2 + \alpha^2\sigma^2}{\sigma^2} = 1 + \alpha^2. \tag{A.9}$$

To calculate $NSAmp$, we start again from Eq. (A.3). Using first the work-in-progress equation of this model (Eq. 6) and second the relation between orders and demand (Eq. A.8), the inventory is

$$\begin{aligned} i_t &= S_t - o_t - \sum_{i=1}^{L-1} o_{t-i} - \alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i} = S_t - \sum_{i=0}^{L-1} o_{t-i} - \alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i} \\ &= S_t \underbrace{- \sum_{i=0}^{L-1} d_{t-i}}_{(1)} \underbrace{+\alpha \sum_{i=0}^{L-1} d_{t-L_C-i}}_{(2)} \underbrace{-\alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i}}_{(3)}. \end{aligned} \tag{A.10}$$

Note, term (1) covers the demand from period t to $t - (L - 1)$; term (2) spans from period $t - L_C$ to $t - (L_C + L - 1)$; and term (3) spans from period $t - L_C$ to $t - (L_C + L_R - 1)$. Due to $L_C >> L$, an overlap may only occur between terms (2) and (3). In this sense, three scenarios need to be studied, depending on the relationship between the manufacturing and remanufacturing lead times, L and L_R.

- $L = L_R$. When the lead times are equal, terms (2) and (3) are cancelled. Therefore, Eq. (A.10) simplifies to $i_t = S_t - \sum_{i=0}^{L-1} d_{t-i}$, providing a simplified version of the $NSAmp$,

$$NSAmp = \frac{\text{var}(i_t)}{\text{var}(d_t)} = \frac{L\sigma^2}{\sigma^2} = L. \tag{A.11}$$

- $L > L_R$. When the manufacturing lead time is higher, term (3) falls within term (2). Thus, Eq. (A.10) results in $i_t = S_t - \sum_{i=0}^{L-1} d_{t-i} + \alpha \sum_{i=L_R}^{L-1} d_{t-L_C-i}$, and $NSAmp$ becomes

$$NSAmp = \frac{\text{var}(i_t)}{\text{var}(d_t)} = \frac{L\sigma^2 + \alpha^2(L - L_R)\sigma^2}{\sigma^2} = L + \alpha^2(L - L_R). \tag{A.12}$$

.

- $L < L_R$. Inversely, when the remanufacturing lead time is higher, term (2) falls within term (3). In this case, Eq. (A.10) provides $i_t = S_t - \sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=L}^{L_R-1} d_{t-L_C-i}$, and $NSAmp$ is

$$NSAmp = \frac{\text{var}(i_t)}{\text{var}(d_t)} = \frac{L\sigma^2 + \alpha^2(L_R - L)\sigma^2}{\sigma^2} = L + \alpha^2(L_R - L). \tag{A.13}$$

Eqs. (A.11)-(A.13) can be generalised to the following expression of the $NSAmp$ ratio in *model II*,

$$NSAmp^{II} = L + \alpha^2|L - L_R|. \tag{A.14}$$

Appendix A.3. Model III

A similar procedure can be followed to obtain Bw and $NSAmp$ in this model, where $w_t - w_{t-1} = (o_{t-1} - o_{t-L}) + \alpha(d_t - d_{t-L_C})$, see Eq. (7). In this model, Eq. (A.2) simplifies to

$$o_t = (d_t - \alpha d_{t-L_C-L_R}) - \alpha(d_t - d_{t-L_C}) = (1-\alpha)d_t + \alpha d_{t-L_C} - \alpha d_{t-L_C-L_R}. \tag{A.15}$$

Accordingly, the Bullwhip ratio under i.i.d. demand is

$$Bw^{III} = \frac{\text{var}(o_t)}{\text{var}(d_t)} = \frac{(1-\alpha)^2\sigma^2 + \alpha^2\sigma^2 + \alpha^2\sigma^2}{\sigma^2} = (1-\alpha)^2 + 2\alpha^2. \tag{A.16}$$

We now look at the inventory equation to obtain $NSAmp$. Here, Eq. (A.3) provides the following inventory balance, by means of first Eq. (7) and second Eq. (A.15),

$$\begin{aligned} i_t &= S_t - o_t - \sum_{i=1}^{L-1} o_{t-i} - \alpha \sum_{i=0}^{L_C-1} d_{t-i} = S_t - \sum_{i=0}^{L-1} o_{t-i} - \alpha \sum_{i=0}^{L_C-1} d_{t-i} \\ &= S_t \underbrace{-(1-\alpha)\sum_{i=0}^{L-1} d_{t-i}}_{(1)} \underbrace{-\alpha \sum_{i=0}^{L-1} d_{t-L_C-i}}_{(2)} \underbrace{+\alpha \sum_{i=0}^{L-1} d_{t-L_C-L_R-i}}_{(3)} \underbrace{-\alpha \sum_{i=0}^{L_C-1} d_{t-i}}_{(4)}. \end{aligned} \tag{A.17}$$

Now the inventory equation has four terms. Term (1) covers the demand from period t to $t-(L-1)$; term (2) spans from period $t - L_C$ to $t - (L_C + L - 1)$; term (3) spans from period $t - (L_C + L_R)$ to $t-(L_C+L_R+L-1)$; and term (4) spans from period t to $t-(L_C-1)$. Importantly, given that $L_C >> L$, terms (2) and (3) do not overlap with terms (1) and (4). However, terms (1) and (4) overlap during L periods, and the sum can be simplified, $(1) + (4) = -\sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=L}^{L_C-1} d_{t-i}$, which removes the overlap. The sum of terms (2) and (3) can be expressed as follows, $(2) + (3) = -\alpha \sum_{i=0}^{L-1} d_{t-L_C-i} + \alpha \sum_{i=L_R}^{L+L_R-1} d_{t-L_C-i}$. In this regard, two scenarios may occur. If $L \leq L_R$, there is no overlap between both terms. Otherwise (i.e. if $L > L_R$), there is an overlap between both terms in $L - L_R$ periods; thus the following expression removes the overlap, $(2) + (3) = -\alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i} + \alpha \sum_{i=L}^{L+L_R-1} d_{t-L_C-i}$. Overall, the analysis leads to two different expressions of $NSAmp$.

- $L \leq L_R$. When the remanufacturing lead time is higher or both are equal, $i_t = S_t - \sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=L}^{L_C-1} d_{t-i} - \alpha \sum_{i=0}^{L-1} d_{t-L_C-i} + \alpha \sum_{i=L_R}^{L+L_R-1} d_{t-L_C-i}$. This results in

$$\begin{aligned} NSAmp = \frac{\text{var}(i_t)}{\text{var}(d_t)} &= \frac{L\sigma^2 + \alpha^2(L_C - L)\sigma^2 + \alpha^2 L\sigma^2 + \alpha^2 L\sigma^2}{\sigma^2} \\ &= (1-\alpha^2)L + \alpha^2(L_C + 2L) \end{aligned} \tag{A.18}$$

- $L > L_R$. When the manufacturing lead time is higher, $i_t = S_t - \sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=L}^{L_C-1} d_{t-i} - \alpha \sum_{i=0}^{L_R-1} d_{t-L_C-i} + \alpha \sum_{i=L}^{L+L_R-1} d_{t-L_C-i}$. In this case, the expression becomes

$$\begin{aligned} NSAmp = \frac{\text{var}(i_t)}{\text{var}(d_t)} &= \frac{L\sigma^2 + \alpha^2(L_C - L)\sigma^2 + \alpha^2 L_R \sigma^2 + \alpha^2 L_R \sigma^2}{\sigma^2} = \\ &= (1-\alpha^2)L + \alpha^2(L_C + 2L_R). \end{aligned} \tag{A.19}$$

Again, Eqs. (A.18) and (A.19) can be generalised to a joint expression, $NSAmp = (1-\alpha^2)L + \alpha^2(L_C + 2\min\{L_R, L\})$. Applying that $2\min\{L_R, L\} = (L + L_R) - |L - L_R|$, we obtain

$$NSAmp^{III} = L + \alpha^2(L_C + L_R - |L - L_R|). \tag{A.20}$$

Appendix A.4. Model IV

Finally, in the archetype in which the work-in-progress covers the manufacturing, remanufacturing, and market pipelines, via Eq. (8), $w_t - w_{t-1} = (o_{t-1} - o_{t-L}) + \alpha(d_t - d_{t-L_C-L_R})$. Eq. (A.2) leads to

$$o_t = (d_t - \alpha d_{t-L_C-L_R}) - \alpha(d_t - d_{t-L_C-L_R}) = (1-\alpha)d_t. \tag{A.21}$$

Therefore, the Bullwhip ratio is defined per the following expression,

$$Bw^{IV} = \frac{\text{var}(o_t)}{\text{var}(d_t)} = \frac{(1-\alpha)^2\sigma^2}{\sigma^2} = (1-\alpha)^2. \tag{A.22}$$

Finally Eq. (A.3) results in the following inventory balance, after considering Eqs. (8) and (A.21),

$$\begin{aligned} i_t &= S_t - \sum_{i=0}^{L-1} o_{t-i} - \alpha \sum_{i=0}^{L_C+L_R-1} d_{t-i} = S_t - (1-\alpha)\sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=0}^{L_C+L_R-1} d_{t-i} \\ &= S_t - \sum_{i=0}^{L-1} d_{t-i} + \alpha \sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=0}^{L_C+L_R-1} d_{t-i} = S_t - \sum_{i=0}^{L-1} d_{t-i} - \alpha \sum_{i=L}^{L_C+L_R-1} d_{t-i}. \end{aligned} \tag{A.23}$$

Under these circumstances, and due to S_t being fixed over time, $NSAmp$ becomes

$$NSAmp^{IV} = \frac{\text{var}(i_t)}{\text{var}(d_t)} = \frac{L\sigma^2 + \alpha^2(L_C + L_R - L)\sigma^2}{\sigma^2} = L + \alpha^2(L_C + L_R - L). \tag{A.24}$$