Incorporating preferential weights as a benchmark into a Sequential Reference Point Method

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Abstract

In multi-objective optimization models, it is common that the decision maker expresses the relative importance of objectives through a weighting scheme. However, many solving techniques do not assure that the corresponding solution fits the preferential weights. It could be the case that an objective with a very low weight achieves a good value, whereas another with a high weight yields a very poor achievement. In order to overcome the aforementioned drawback, this paper proposes a new resolution method based on the well-known Reference Point Method. The methodology consists in generating a sequence of Reference Point Method models which share the same reference point fixed at the vector of preferential weights. In the iterative process, the projection direction on the Pareto frontier changes in each iteration according to the deviations between the preferential weights and the current normalised objective values. In this way, a sequence of Pareto-efficient solutions is generated which converges towards a solution that best fits the decision maker's preferential weights. The proposed method is illustrated by means of a numerical example. In order to show its feasibility and usefulness, the methodology is applied to a portfolio selection problem where the corporate sustainability performance of each firm is taken into account.

Keywords: Multi-objective optimisation; Pareto optimality; Preferential weights; Reference point method.

1. Introduction

Decision making usually involves multiple conflicting criteria. This conflict could involve the non-existence of a solution unanimously accepted by any stakeholder. By an accepted solution we refer to a Pareto optimal solution which the decision maker (DM) considers to be her/his best option. We focus on multi-objective programming (MOP) problems, i.e. decision problems formulated by a set of objective functions of the decision variables that have to be simultaneously optimised over a feasible set defined by constraint functions (Miettinen, 2008). An important task

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that should be frequently addressed when solving MOP problems is to introduce DM preferential information. Miettinen (1999) and Ruiz et al. (2009) classify MOP methods according to the information flow between the DM and the modeller. *No information* methods, involve no information flows at all. *A priori* information methods exist where the DM is asked to provide some information about her/his preferences before solving the problem with a view to finding the solution that best fits these preferences. A *posteriori* information method involves the DM choosing one solution from a set of possible solutions, depending on her/his preferences. Finally, *interactive* methods are characterised by a continuous information flow between the DM and the modeller. In MOP problems, we highlight three common ways of incorporating a priori preferential information. The DM can assign importance weights to the objectives (Saaty, 1987, Ruiz et al., 2009, 2010, Jones & Tamiz, 2010) which are used by the modeller for building the MOP model. In some situations, the DM could establish the so-called aspiration or reference levels (Romero et al., 1998, Wierzbicki, 1977) which are values that the DM wishes to achieve for each objective. Lastly, it is possible to use the lexicographic ordering in which each objective is set at a predefined level (Romero, 2003, van Haveren et al., 2017). Each of these approaches can be used both separately and jointly. Moreover, all of them present several drawbacks to which we will refer to below. The aim of this paper is focused on proposing a new MOP model into which the DM's preferential weights can be incorporated.

The existing literature regarding the essential role of weighting in multiple-criteria decision making (MCDM) is really extensive. Pairwise comparison methods (Saaty, 1980, Pamučar et al, 2018), in which the DM supplies information regarding the pairwise relative importance of the objectives, are a very classical tool to determine a preferential set of weights. Ruiz et al. (2009) analyse and classify several weighting schemes for the general reference point (a vector formed by the reference levels) interactive procedure. They distinguish pure normalisation schemes from others where the weights have a preferential meaning. Hunt et al. (2010) present a cone-based preference framework for modelling the relative importance of the criteria. The DM's perception of the relative importance is quantified by an allowable trade-off between two objectives representing the maximum allowable amount of deterioration of a less important objective per one unit of improvement of a more important objective. Nevertheless, real situations exist in which it is very difficult to assign a precise value to the relative importance of objectives. In order to overcome this circumstance, a widespread approach is to apply fuzzy logic to classical procedures (van Laahovden and Pedriyzc, 1983, Cvetkovic & Parmee, 2002, Mikhailov, 2004, Bilbao et al., 2014, Chan et al., 2019)

Using a goal programming framework, Jones and Tamiz (2010), develop an algorithm for the analysis of the space of weight, where the term 'weight' has a preferential meaning. From an initial solution (weights equal or specified by the DM), the authors explore the entire weight space to produce a set of solutions that are presented to the DM for his/her consideration. However, the generation of such solutions involves an unnecessary computational effort and is unsuitable, since a large number of generated solutions are not in accord with the DM's preferences (Jones, 2011, p. 239). In order to improve these issues, Jones (2011) proposes a weight space exploration algorithm that allows the DM to give additional preference information in order to more effectively guide the bounds of the search and produce a set of solutions that are more in accord with her/his preferences. Jones's weight sensitivity algorithm can be used to investigate a portion of weight space resulting of interest to the DM. The author presents the type of information regarding the initial estimate of weights and additional preference information as well as its impact on the weighting space. This information (absolute information about the relative importance of a single weight and/or a set of weights and/or pairwise ordinal/cardinal information regarding weights) is modelled by linear constraints and it is used to restrict weight space exploration. A set of lexicographic goal programs is constructed to find the maximum values of each weight direction for a single weight change. The aim of this paper is, therefore, to offer a methodology for carrying out a post-optimum analysis on the weighting scheme. A proposal presented by Jones and Jiménez (2013) incorporates an additional meta-objective, the aim of which is to reduce the discrepancy between the preferences of the DM and the preferences shown by the solution.

Between the proposed methods to solve MCDM problems the interactive procedures using reference points (Wierzbicki, 1982, Mietinen, 2010, p. 131) have proved one of the most utilised. The most usual way for finding a Pareto-efficient solution, close to the reference point, is by optimising an achievement scalarising function (Wierzbicki, 1977, 1980, 1982). In this way the reference point is projected over the Pareto frontier. By changing the reference point, in an interactive way, the DM is able to derive a subset of the Pareto frontier and from this subset can select the most preferred solution. In most of these methods, the weights of the achievement function are kept unchanged their purpose mainly being to normalise the different objectives. That is, the reference point changes but the projection direction over the set of Pareto solution does not. Luque et al. (2009) and Miettinen et al. (2009) introduce new ways of utilizing preference information specified by the DM in interactive RPM. The authors take into account the desires of the DM when projecting the reference point into the Pareto frontier. In this way, they find the most satisfactory solutions faster. Wierzbicki et al. (2000) and Cabello et al. (2014) generalise the classic RPM using a double reference point. Namely, the DM is asked to give, for each criterion, a level under which the values of the function are not regarded as acceptable (reservation level) and a desirable value for the criterion (aspiration level).

Another interesting extension of RPM is presented by Miettinen et al. (2010). They propose an interactive RPM, Nautilus, where the weights and reference points are changed for each iteration in accordance with the information provided by the DM. Therefore, Nautilus requires intervention from DM for each step with the exception of the first one. It does not use an initial preferential reference point but instead starts with the nadir point. The reason argued by the authors for this choice is to avoid the a priori elimination of zones belonging to the Pareto frontier.

Incorporating a priori preferential information could prove to be a difficult task. When the DM sets preferences through importance weights assigned to the objectives, she/he could face an undesired situation because the results shown in the objective space could deviate from those expressed preferences: an objective with a low weight can reach a very high value, and vice versa, an objective with a large weight can present a very low value (assuming maximisation). In addition, sensitivity analysis often shows that small changes in the weights can lead to large changes in the solution and large changes in the weights may not produce any change whatsoever in the solution (Jones & Tamiz, 2010, Jones, 2011, Jones & Jiménez, 2013). In summary, there is no assurance that the preference expressed by the weights is reflected in the obtained solution. As mentioned above, setting aspiration levels is another way of incorporating preferential information. But, there are many contexts in which the lack of information makes the aspiration level amount difficult to estimate, this leading to the proposal of either over optimistic or over pessimistic levels. Obviously, in these circumstances, the achieved solution could be wrong. Similar issues can be applied to the classical RPM, which guarantee the efficiency of the solution, but distort the balanced nature of the solution, that is, non-adjustment to the relative preferences proposed by the DM (Romero et al., 1998). Besides, the RPM convergence may not prove fast enough because the method does not help the DM to find improved solutions. (Miettinen, 1999, p.170).

On the other hand, using a lexicographic order for the set of objectives gives rise pre-emptive priorities. This involves infinite trade-offs among objectives placed at different levels of priority, leading to a high level of achievement for the objectives placed at the higher priority levels and a very low level of achievement for those situated at secondary priority levels, a scenario that may lead to unsatisfactory results.

In order to overcome the aforementioned drawbacks, we propose a method that handles the preferential weights of the DM as prior information. Furthermore, the establishment of aspiration levels for the objectives is not required. We integrate the preferential information proposed by the DM as a reference point or benchmark in a sequential RPM. Unlike classical RPM methods, this reference point remains unaltered, while the coefficients in the achievement scalarising function are modified in each iteration. That is, the projection directions on the Pareto frontier change according to the distance between the initial preferential weights and the current normalised objective values. The algorithm runs until it finds the solution that best approximates the reference point, i.e. the preference weights. Our proposal is framed within the *a priori* information methods. Therefore, the DM intervention is restricted to the starting point -providing the weighting system- and the algorithm can run autonomously until the end. To the best of our knowledge, no method exists for solving a MOP in which weights are integrated as aspiration levels of the model.

In the current literature, MCDM techniques are reported in a wide range of real-world problems. For green transporting models, see, e.g. Demir et al. 2014, Jabir et al. 2015 and Sawik et al. 2017. For forest planning, see, e.g. Diaz-Balteiro & Romero, 2007, 2008, Jiménez et al., 2012, Bilbao-Terol et al., 2014, Diaz-Balterio & Romero, 2016 and references therein. For humanitarian logistic operations, see, e.g. Ferrer et al. 2018 and Mejia-Argueta et al., 2018. For portfolio selection Sawik, 2008, Aouni et al. 2018, Bilbao-Terol et al. 2018 and, amongst others. We apply our proposal to a portfolio selection problem where the Corporate Sustainability (CS) performance of each firm is taken into account. Thus, the firms are assessed by both financial and CS criteria. We have CS valuations of the firms from a corporate sustainability rating agency (Vigeo) and the financial measures are gathered from the financial agencies (Morningstar Direct and YCharts). Vigeo is a European extra-financial rating agency that measures companies' CS performance by 6 domains. We group these domains under three objectives: environment, social and governance (ESG) and also consider the three financial ratios which assess the financial performance of the companies. We have worked with 117 firms and assume that the investor reveals her/his preferences by assigning importance weights for the six objectives. The proposed model is sensitive to the particular preferences of the investor with respect to the importance granted to each objective considered. The preferences may change from one investor to another as they depend on diverse factors such as the personal values and beliefs, religion, country, amongst others. This gives rise to different investor profiles: sustainable, environmental, social, financial, balanced, etc.

The rest of the paper is structured as follows. Section 2 presents the Sequential Weighting Reference Point Method (SWRPM). In Section 3, we use a numerical example to illustrate the proposed methodology. In Section 4, the feasibility of the SWRPM is demonstrated for a portfolio selection problem where corporate sustainability and financial criteria are considered simultaneously. Finally Section 5 draws conclusions.

2. The Sequential Weighting Reference Point Method (SWRPM)

Let us consider the following MOP problem:

$$\begin{cases} \text{optimise } \mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}) \right) \\ \text{subject to } \mathbf{x} \in S \end{cases} \tag{1}$$

involving k conflicting objective scalar functions f_i. The decision variables $\mathbf{x} = \left(x_1, x_2, \ldots, x_n \right)$ belong to the nonempty feasible region $S \subseteq R^n$. Objective vectors in objective space R^k consist of objective values $\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}) \right)$ and the image of the feasible region is called the feasible objective region $Z = \mathbf{f}(S) \subseteq R^k$.

Solving model (1) means looking for a Pareto-efficient solution which satisfies the DM preferences.

In an *a priori* information framework based on preferential weights, the difficulty for the DM lies in setting values in the weight space that correspond with the results in the objective and decision space. Furthermore, the relation between the changes in weight space and the changes in decision and objective space could be abrupt. Equally, seemingly large changes in weight space can lead to no change in decision (and hence objective) space.

We assume that the weighting system is known and it represents the relative importance of the objectives for the DM.

As mentioned, the aim of this paper is to propose a model for determining a Pareto-efficient solution to model (1) when the DM' preferences are expressed by importance weights and no aspiration level has been set. An original MOP model has been built that improves the earlier referred issues.

2.1. *Normalisation of the objectives*

In MOP, the ideal and nadir points, provide an upper and a lower bound for the objective values. We calculate the ideal point $\mathbf{f}^* = \left(f_1^*, f_2^*, ..., f_k^* \right) \in R^k$ by optimising each objective function individually in the feasible region, that is,

$$f_r^* = \underset{\mathbf{x}\in S}{opt}\ f_r\ (\mathbf{x})\text{ , for all } r = 1, 2, ..., k \tag{2}$$

The nadir point $\mathbf{f}_* = \left(f_{1*}, f_{2*}, ..., f_{k*} \right) \in R^k$ represents the vector of worst values for each objective in the set of Pareto optimal solutions. It is not always easy to obtain and several ways to approximate it have been suggested (Miettinen, 1999; Ehrgott & Tenfelde-Podehl, 2003; Ruiz et al., 2009). Here, we consider the anti-ideal point, defined as the worst element of each column of the pay-off matrix (Ballestero and Romero, 1998), as a proxy of the nadir point.

In order to carry out the necessary normalising process of the objective values, we use the relative L_1 distance to the ideal point. Thus, for each objective function f_i we build the individual achievement function F_i as

$$F_i\left(\mathbf{x}\right) = \frac{f_i(\mathbf{x}) - f_{i*}}{f_i^* - f_{i*}} \tag{3}$$

An achievement value equal to 1 means that the objective has reached its best value $\left(f_i^* \right)$ and a value equals to 0 is reached when the objective is at its worst value. This way, all objectives are measured on a 0-1 scale.

2.2. *Preferential information*

It is assumed that it is possible to know a preferential weighting system

$\mathbf{w} = \left(w_1, w_2, \ldots, w_k \right)$ being $\sum_{i=1}^{k} w_i = 1$,

that could has been obtained by applying some suitable methodology adjusted to the problem to be solved (e.g. AHP approach, MACBETH, entropy technique, fuzzy sets among others, see, Saaty, 1980, Bana e Costa & Vansnick, 1994, Cvetkovic & Parmee, 2002, Mikhailov, 2004, Zhi-hong, 2006, Kumar et al., 2017, Pamučar et al, 2018, for further details).

In this way, the DM would be satisfied by finding a solution, $\mathbf{x}$, which verifies as much as possible the following relationships:

$$F_i(\mathbf{x}) = w_i^{scl} \quad \text{with} \quad 1 \le i \le k \qquad \ldots\ldots\ldots \qquad (4)$$

being $w_i^{scl} = \dfrac{w_i}{max\, w_i}$ scalarised weights moving in the same scale as the normalised F_i and keeping the original priority ratios: $r_{ij} = \dfrac{w_i}{w_j} = \dfrac{w_i^{scl}}{w_j^{scl}}$.

Hence, it is necessary to build a multi-objective model with an aggregating objective function that measures the distance between the objective vector $\mathbf{F} = \left(F_1(\mathbf{x}), F_2(\mathbf{x}), \ldots, F_k(\mathbf{x}) \right)$ and the aspiration vector $\mathbf{w^{scl}} = \left(w_1^{scl}, w_2^{scl}, \ldots, w_k^{scl} \right)$. This aggregating objective function should be minimised. Many multi-objective models fall into this framework. In this research, we use the RPM for several reasons. It is easy to introduce the relationships (4) into a RPM by fixing the aspiration levels at the desired weights and it is also easy to design an interactive method for determining what weights should be used in the corresponding aggregation function in order to improve the current solution. Another good feature of the RPM is that the Pareto-efficiency of the obtained solution is assured (Miettinen, 1999).

2.3. *Reference Point Method*

The Reference Point Method (RPM) proposed by Wierzbicki (1980), is a well-known multi-criteria decision making methodology framed within distance function methods (Romero et al., 1998). This method is based on the so-called augmented (or regularised) min-max aggregation. Thus, the worst individual achievement is essentially maximised but the optimisation process is additionally regularised with the term representing the average achievement. The min-max aggregation guarantees fair treatment of all individual achievements by implementing an approximation to the Rawlsian principle of justice. The min-max aggregation is crucial for allowing the RPM to generate Pareto-efficient solutions. On the other hand, the regularisation term is necessary to guarantee that only Pareto-efficient solutions are generated.

The RPM assumes that the DM specifies reference levels for each objective reflecting values considered as desirable ones by the DM.

In a situation where all objectives are maximised (that is, the case 'more is better') the classic RPM formulation is given as[2]:

$$\min_{x\in S}\max_{i=1,\dots,k}\left[\mu_i\left(b_i-f_i(x)\right)\right]-\varepsilon\sum_{i=1}^{k}\mu_i f_i(x) \tag{5}$$

where $\mathbf{b}=\left(b_1,b_2,\dots,b_k\right)$ is the reference point, $\boldsymbol{\mu}=\left(\mu_1,\mu_2,\dots,\mu_k\right)$ is the direction of projection of $\mathbf{b}$ to the Pareto frontier and ε is an arbitrary small positive number (for example $\varepsilon=10^{-6}$) required to avoid generating weakly non-dominated points. In most of the RPM interactive methods, while the reference point is changed at each iteration, coefficients $\mu_i,\ i=1,2,...,k$ are kept unaltered during the whole process.

The application of model (5) to our proposal implies the following RPM model:

$$\min_{x\in S}\max_{i=1,\dots,k}\left[\mu_i(w_i^{scl}-F_i(\mathbf{x})\right]-\varepsilon\sum_{i=1}^{k}\mu_i F_i(\mathbf{x}) \tag{6}$$

Our proposal is to solve a sequence of models (6) where all elements are kept unaltered, with the exception of the coefficients μ_i, that vary according to the results of the current iteration. This iterative process has a stop criterion determined by a measure of goodness defined as the L_1-distance between the reached priority ratios, $\frac{F_i}{F_j}$, and the desired ones, $\frac{w_i^{scl}}{w_j^{scl}}$, so

$$D=\sum_{\substack{i,j=1\\ i\neq j}}^{k}\left|\frac{F_i}{F_j}-\frac{w_i^{scl}}{w_j^{scl}}\right| \tag{7}$$

The coefficients μ_i, for each iteration, are determined by taking into account which normalised objectives have been left below and which ones above their corresponding level of aspiration in the current solution.

2.4. *Algorithm*

Initialisation process

1. *Normalising*. Given problem (1), obtain (estimates of) the ideal objective vector $\mathbf{f}^*$ and the nadir objective vector $\mathbf{f}_*$. Calculate normalised objective vector $\mathbf{F}$ using formula (3).
2. *Preference information*. Preferential weights are supposed to be known, $w_1,\dots,w_k$, that express the relative importance of reaching each individual ideal value.

[2] It is also possible to eliminate the coefficients μ_i in the regularisation term (Miettinen et al., 2010).

3. *Scalarising weights*. Normalise the raw weights using $\max_i w_i$ for obtaining w_i^{scl}, $i = 1, \ldots, k$.

Iterative process

Step 1. *Starting*. Fix: $h = 1$, $D^0 = 10^8$, equal coefficient, i.e. $\mu_i^h = 1/k$, a small number called tolerance error Tol (e.g. 10^{-6}) and the maximum number of iterations M.

Step 2. *New solution*. Solve problem (6) with $\mu_i = \mu_i^h$. Let $\mathbf{x_h}$ be the optimal solution, $\mathbf{F^h}$ the corresponding normalised objective vector and D^h the matching measure. The set of objectives is partitioned in those objectives reaching a value below their aspiration level, I_1, i.e. objective $i \in I_1$ if $F_i < w_i^{scl}$, and those reaching a value greater or equal to their aspiration level, I_2, i.e. objective $i \in I_2$ if $F_i \geq w_i^{scl}$.

Step 3. *Stop Criterion*. Calculate $D^{h-1} - D^h$, if this difference is less than Tol stop the process and $\mathbf{x_h}$ is the obtained solution.

If the number of iterations, M, has not been surpassed and the difference $D^{h-1} - D^h$ is greater than Tol then go to Step 4. Alternatively, it is possible to ask the DM whether she/he agrees with the current solution and, then the process can stop.

Step 4. *New weights*. Calculate the normalised deviations

$$dev_i = \frac{w_i^{scl} - F_i^h}{w_i^{scl}} \tag{8}$$

and new weights according to:

i) Calculate

$$\mu_i^{h+1} = \mu_i^h + h \times dev_i \tag{9}$$

If μ_i^{h+1} is lesser than 0 then fix $\mu_i^{h+1} = 10^{-p*k}$ being p a factor lesser or equal than 1.

If μ_i^{h+1} is greater than 1 then fix $\mu_i^{h+1} = 1 - \rho$, being ρ an arbitrary small positive number.

Note that $i \in I_1$ gives rise $dev_i > 0$ and therefore the updated coefficient μ_i^{h+1} is greater than the current coefficient, μ_i^h. Otherwise, the updated coefficient is lesser or equal than the current coefficient.

ii) Normalise the weights μ_i^{h+1} according to

$$\hat{\mu}_i^{h+1} = \frac{\mu_i^{h+1}}{\sum_{i=1}^{k} \mu_i^{h+1}} \tag{10}$$

Fix $\mu_i^{h+1} = \hat{\mu}_i^{h+1}$, $h = h+1$ and go to Step 2.

2.5 The features of our proposal:

- Necessary information. The method is demanding of a priori preferential information. Therefore, it is applicable when the DM is able of expressing the relative importance between the objectives directly (Miettinen et al., 2010) or applying some methodology adapted to the problem considered. Pairwise comparison methods have been used to determine a set of weights (Saaty, 1980, Gass, 1986, Bana e Costa & Vansnick, 1994, Cvetkovic & Parmee, 2002, Mikhailov, 2004, Wey and Wu, 2007, Kahraman and Büyüközkan, 2008, Li et al., 2009, Kou et al., 2016, Pamučar et al, 2018). Besides, penalty structures (Jones & Tamiz, 1995, Chang & Lin, 2009) and methodology from the field of multi-criteria decision analysis such as the Promethee method (Martel & Aouni, 1990) are used to give weighting schemes. When it is easy to communicate and interpret percentages of achievement of the best values for each objective our proposal could be useful.
- The weights work as reference levels for the normalised objective functions. This seems more suitable for fitting DM preferences with obtained solution. Our aim is to be able to produce a solution that is more satisfactory to the DM than the ones produced with standard approaches.
- Much more iterative than interactive. Once the weights have been set, little information and interaction is required from the DM. The algorithm knows how to go to the following iteration without interacting with the DM, current results providing all the necessary information. This feature can be considered as a good characteristic because information such as the trade-off between objectives is difficult for the DM. Only the DM joins in to express her/his agreement with the current solution or alternatively, her/his wish to continue iterating.
- Classical RPM method does not help the DM to find improved solution, so there is no clear strategy to find the final solution (see, Miettinen et al., 2010, p.170). However, with our proposal the coefficients are modified in order to achieve a solution close to the preferred one by the DM.

- The non-compensatory character of the aggregation min-max helps to the convergence of the algorithm, large weights acting on positive deviations are reflected on the solution and, thus weighting is not lost.

In order to illustrate the proposed model and evaluate its performance, SWRPM is tested on a numerical example in the next section.

3. Numerical example

In this section, we illustrate the behaviour of the method introduced in Section 2 with a linear multi-objective optimization problem involving three objective functions of the form

$$\begin{aligned}
&\text{Max } f_1(x_1,x_2,x_3,x_4) = 3x_1 + 7x_2 + 3x_3 + 5x_4 \\
&\text{Max } f_2(x_1,x_2,x_3,x_4) = x_1 + 4x_2 + 6x_3 + 2x_4 \\
&\text{Min } f_3(x_1,x_2,x_3,x_4) = 4x_1 + 6x_2 + 0.5x_3 + x_4 \\
&\text{subject to} \\
&X = \begin{cases} 7x_1 + 6x_2 + 8x_3 + 6x_4 \leq 110 \\ 2x_1 + 3x_2 + 2x_3 + 5x_4 \geq 50 \\ 3x_1 + 4x_2 + 7x_3 + 6x_4 \leq 80 \\ x_i \geq 0 \end{cases}
\end{aligned} \tag{11}$$

For this problem, we have the following ideal and anti-ideal objective vectors $f^* = (128.33, 75, 10)$ and $f_* = (50, 20, 110)$, respectively. The objectives are normalised using (3)

$$\begin{aligned}
F_1(\mathbf{x}) &= \frac{3x_1 + 7x_2 + 3x_3 + 5x_4 - 50}{78.33} \\
F_2(\mathbf{x}) &= \frac{x_1 + 4x_2 + 6x_3 + 2x_4 - 20}{55} \\
F_3(\mathbf{x}) &= \frac{110 - 4x_1 - 6x_2 - 0.5x_3 - x_4}{100}
\end{aligned}$$

From these expressions the following RPM model is formulated as:

$$\min_{x \in X} \max_{i=1,2,3} \left[\mu_i (w_i^{scl} - F_i(\mathbf{x}) \right] - \varepsilon \sum_{i=1}^{3} \mu_i F_i(\mathbf{x}) \tag{12}$$

Let us suppose that the DM assigns the following relative importance to the objectives $w = (0.2, 0.6, 0.2)$. Referring w_i to the range 0-1, we obtain the reference point, $w^{scl} = (1/3, 1, 1/3)$ that will remain unchanged throughout the whole process.

We have set the maximum number of iterations, M, at 150, the stop criterion number, Tol, at 10^{-6} and $\varepsilon = 10^{-6}$. Then the corresponding model (12) that we have to solve in the first iteration is

$$\min_{x\in X} \max\left[\frac{1}{3}\left(\frac{1}{3} - F_1(\mathbf{x})\right), \frac{1}{3}(1 - F_2(\mathbf{x})), \frac{1}{3}\left(\frac{1}{3} - F_3(\mathbf{x})\right)\right] - 10^{-6}\sum_{i=1}^{3}\frac{1}{3}F_i(\mathbf{x})$$

The solution is shown in Table 1. Observe that F_1 reaches values bigger than the aspiration level $(1/3)$, whereas F_2 and F_3 reach values lower than the respective aspiration levels (1 and $1/3$). For this solution we obtain the matching measure defined as (7), $D^1 = 2$. If we apply the stop criterion for this first solution, we have $D^0 - D^1 = 10^8 - 2 > 10^{-3}$ and therefore, we must continue iterating. To do this, we calculate the normalised deviations and the new weights according to (8) and (9) fixing $p = 1$ in Step 4-i):

$$dev_1 = \frac{1/3 - 0.703}{1/3} = -1.108;\ \ dev_2 = \frac{1 - 0.964}{0.967} = 0.036;\ \ dev_3 = \frac{1/3 - 0.297}{0.297} = 0.108$$

$$\mu_1^2 = 0.001;\ \ \mu_2^2 = \frac{1}{3} + 0.036 = 0.369;\ \ \mu_3^2 = \frac{1}{3} + 0.108 = 0.441$$

Normalising the above values, we obtain the second iteration weights as (10):

$$(\hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3) = (0.001, 0.455, 0.544)$$

that we introduce in model (12):

$$\min_{x\in X} \max\left[0.001\left(\frac{1}{3} - F_1(\mathbf{x})\right), 0.455(1 - F_2(\mathbf{x})), 0.544\left(\frac{1}{3} - F_3(\mathbf{x})\right)\right] -$$
$$-10^{-6}\left[0.001F_1(\mathbf{x}) + 0.455F_2(\mathbf{x}) + 0.544F_3(\mathbf{x})\right]$$

The process stops after 150 iterations. The supplied solution could be considered as the Pareto-efficient one that best fits the preferences of the DM.

Table 1

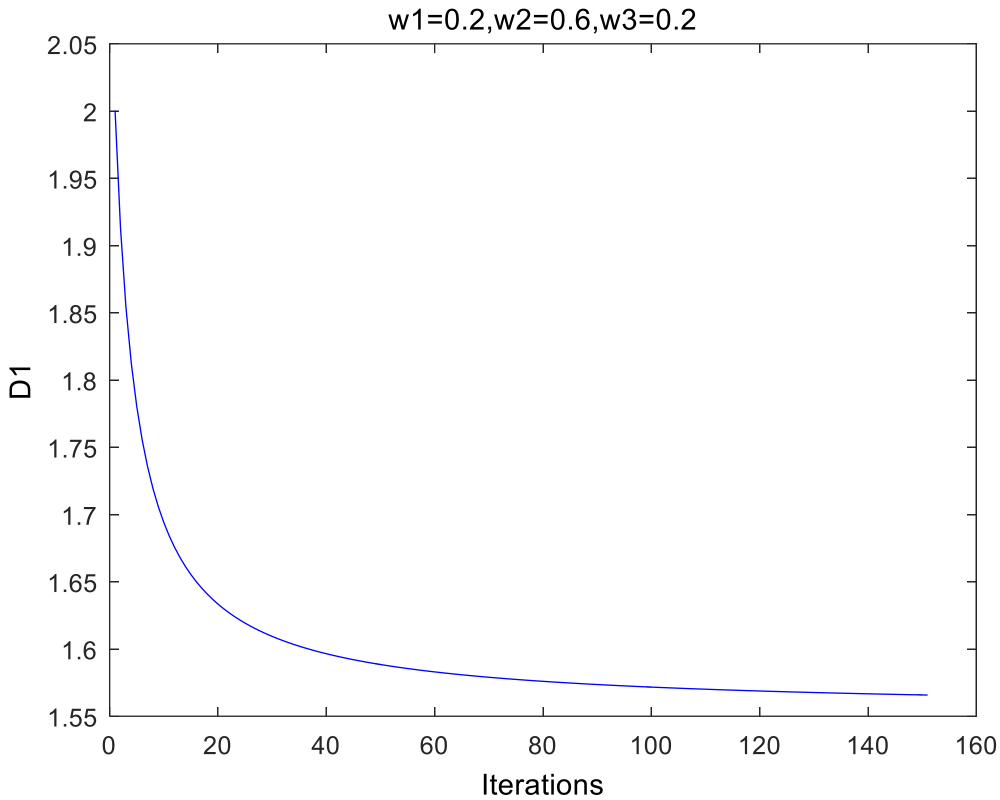
Pareto-efficient solutions: the objective and the normalised objective functions.

ITERATION	f_1 F_1	f_2 F_2	f_3 F_3	D
1	105.033 0.703	73.020 0.964	80.267 0.297	2
2	104.776 0.699	72.865 0.961	79.913 0.301	1.913

⋮	⋮	⋮	⋮	⋮
150	103.656 0.685	72.195 0.949	78.400 0.316	1.566

Figure 1 shows how at each iteration the solution improves the previous one, converging to the one that best fits the DM preferences. We have realised an analysis of sensitivity changing the vector of weights at $w=(0.25,0.5,0.25)$, therefore $w^{scl}=(1/2,1,1/2)$, the results obtained are, in this situation, $F_1=0.61, F_2=0.89, F_3=0.39$. Comparison with the last row in Table 1 shows the performance of the algorithm.

Figure 1. Evolution of the measure of goodness, D, obtained at each iteration.



In order to prove the goodness of our model we have compared our solution with the solutions that were obtained if we use compromise programming (Zeleny, 1973) and RPM with the initial weights set by the DM. Table 2 shows the solutions obtained by the compromise programming with the distances L_1 , L_∞ and RPM.

Table 2

Compromise and RPM solutions: the objective and the normalised objective functions.

	f_1	f_2	f_3	D

	F_1	F_2	F_3	
L_1	112.5 0.798	75 1	91.25 0.187	6.053
L_∞	92.246 0.539	65.348 0.825	62.65 0.474	1.718
RPM	85.42 0.4522	60.69 0.7399	52.91 0.5709	2.19

The goodness measures D corresponding to the L_1 and RPM solutions are higher than that of our first solution (iteration 1) while for the L_∞ distance the solution would correspond roughly to the one obtained in the iteration 7. Therefore, our procedure allows us to improve the compromise and RPM solutions.

The presented methodology is applied to a portfolio selection problem being the firms (see Table 11 in the Appendix) assessed by both financial and corporate sustainability (CS) criteria. We have CS valuations of the firms from corporate sustainability rating agencies and the financial measures are gathered from the financial rating agencies. We assume that the investor reveals her/his preferences assigning importance weights for the criteria. In this case, the sustainability and financial performance of each of the firms invested is taken into account.

4. Application: selecting firms based on both Corporate Sustainability and Financial Criteria

Corporate Sustainability (CS) is a mainstream element of the business in the 21st century, where corporations address the positive and negative impacts of its corporative actions. A first consequence of the CS concerns is the necessity on the part of organizations to keep all stakeholder groups well informed. The CS reports are the key tool used by the firms although the self-declaration aspect is criticised. CS rating agencies (e.g. VigeoEiris, Covalence, MSCI ESG STATS, ASSET4 database and Sustainable Investment Research Institute-SIRIS) have arisen with the aim of providing external and reliable information about business behaviour. Each one of these agencies has its own methodology and information sources. We consider that the analysis of a set of firms, based on both sustainability and financial criteria provides an interesting field for the application of our methodology.

In this paper, data for firms' CS performance evaluation come from Vigeo (VigeoEiris, created in 2015 from the merger of two leaders in their historical markets). Vigeo is a rating and research agency that measures the integration of environmental, social and governance (ESG) factors into corporate strategies, operations and management with a focus on promoting economic performance, responsible investment and sustainable value creation. Vigeo offers an ESG rating system based on 38 precise sustainability criteria grouped into 6 domains of analysis: environment, human rights, human resources, community involvement, business behaviour and corporate governance. A description of these domains is presented in Table 3. We group these domains into three objectives: Environmental **(E)**, Social **(S)** and Corporate Governance **(G)**.

Table 3.

Vigeo evaluation domains.

Objective	Domain	Description
Environment **(E)**	Environment (ENV)	Integration of environmental issues into corporate policy, product manufacturing, distribution, use and disposal.
Social **(S)**	Human Rights (HRts)	Proactive human resources corporate policy, including career development, continuous improvement of labour relations, quality of working conditions.
	Human Resources (HR)	Constant improvement of professional and labour relations, as well as working conditions.
	Community Involvement (CIN)	Integration of the firm's impacts on local communities and responsible societal behaviour.
	Business Behaviour (C&S)	Sustainable and transparent relationships with customers and suppliers.
Corporate Governance **(G)**	Corporate Governance (CG)	Balanced power within the board of directors, respect of shareholders' rights, executive remuneration, audit and internal controls.

Source: http://www.vigeo-eiris.com

Through a series of questions, Vigeo's analysis focuses on how each company addresses each criterion in terms of leadership, implementation and results. Each of these questions is scored on a scale from 0 to 100, representing the level of the firm's CSR engagement and management of associated risks. Detailed description of the Vigeo methodology is given in Bilbao-Terol et al. (2017, 2019) and Liern & Perez-Gladish (2018).

We also have considered three financial ratios of the companies in order to assess their financial performance: Tobin´s Q, Return on Equity (ROE) and Market Value´s Growth. According to the financial literature these ratios are appropriated measures for estimating financial performance. The financial data are gathered from Morningstar Direct and YCharts databases.

- **TOBIN's Q** (Tobin, 1969; Chung and Puitt, 1994; Chung et al., 2005). We consider an approximation of Tobin's Q, based on the research by Chung and Pruitt (1994). These authors noted that an approximation of Tobin's Q could be made as an "approximate Q" where this is made with market capital, preferred stock, short term liabilities without short term debt, and total assets:

$$TOBIN's\ Q = \frac{Market\ Cap + Pref\ \ Stocks + Short\ T\ \ Liab - Short\ T\ \ Debt}{Total\ assets}$$

- Return on Equity **(ROE)** (Penman, 1991). It is the return on equity (net profit divided by stockholders equity).

$$ROE = \frac{Net\ Profit}{Stockholder\ Equity}$$

- Market Value´s Growth **(GROWTH)** (Ramirez-Orellana et al., 2017). It is the level of growth of the market value.

$$GROWTH = \frac{Market\ Val(t+1) - Market\ Val(t)}{Market\ Val(t)}$$

A total of 117 firms constitute our set of investment options (see Table 11 in the Appendix), $\mathbf{x} = \left(x_{1,} x_2, \ldots, x_j, \ldots, x_{117}\right)$ while the criteria are the sustainability and financial objectives defined above, $f_E(\mathbf{x}), f_S(\mathbf{x}), f_G(\mathbf{x}),\ f_Q(\mathbf{x}), f_R(\mathbf{x}), f_{GR}(\mathbf{x})$. Our model includes the usual constraints, the budget constraint and the no-short-sale constraint. Thus, the formulation of our initial problem is as follows:

$$Maximise\left(f_E(\mathbf{x}), f_S(\mathbf{x}), f_G(\mathbf{x}), f_Q(\mathbf{x}), f_R(\mathbf{x}), f_{GR}(\mathbf{x})\right)$$

$$s.t.\quad \sum_{j=1}^{117} x_j = 1, x_j \geq 0$$

In order to calculate the ideal and the anti-ideal point it is necessary to obtain the individual optimum of each objective (see Table 4).

Table 4.

The pay-off matrix.

	f_E	f_S	f_G	f_Q	f_R	f_{GR}
f_E	**75**	52	53	**0.1313**	10.24	0.0342
f_S	63	**69.5**	49	2.532	14.05	0.1777
f_G	48	**34.25**	**89**	0.5914	**-2.88**	**0.027**
f_Q	**40**	47.75	76	**3.725**	46.52	0.3973
f_R	66	40.25	68	1.62	**92.38**	0.5525
f_{GR}	60	51.75	**30**	0.3492	10.76	**2.2566**

The ideal point appears on the main diagonal, while the worst values per column correspond to the anti-ideal point. Then, we have the following ideal and anti-ideal objective vectors**:**

$$f^* = (75, 69.5, 89, 3.725, 92.38, 2.2566)$$

$$f_* = (40, 34.25, 30, 0.1313, -2.88, 0.027).$$

Thus, for the sustainability and financial objectives we built the individual achievement function according to (3), $F_E(\mathbf{x}), F_S(\mathbf{x}), F_G(\mathbf{x}), F_Q(\mathbf{x}), F_R(\mathbf{x}), F_{GR}(\mathbf{x})$.

One key point in the modelling based on our proposal is the consideration of the importance of the objectives for the DM. The DM preferences are included in the model through positive normalised weights noted by w_i^{scl}, $i \in \{E, S, CG, Q, R, GR\}$. The application of model (6) to our selection portfolio problem is as follows:

$$\min \max_{i \in \{E,S,G,Q,R,GR\}} \left[\mu_i (w_i^{scl} - F_i(\mathbf{x}) \right] - \varepsilon \sum_{i \in \{E,S,G,Q,R,GR\}} \mu_i F_i(\mathbf{x})$$

$$s.t. \quad \sum_{j=1}^{117} x_j = 1, \; x_j \geq 0$$

We consider different investor profiles. Each profile is determined by the DM weighting system that we integrate into the RPM model as the reference point (see Table 5).

Table 5.

Investor profile and weights.

Profile	w_E (w_E^{scl})	w_S (w_S^{scl})	w_G (w_G^{scl})	w_Q (w_Q^{scl})	w_R (w_R^{scl})	w_{GR} (w_{GR}^{scl})
Balanced	1/6 (1)	1/6 (1)	1/6 (1)	1/6 (1)	1/6 (1)	1/6 (1)
Environmental	0.5 (1)	0.1 (0.2)	0.1 (0.2)	0.1 (0.2)	0.1 (0.2)	0.1 (0.2)
Social	0.1 (0.2)	0.5 (1)	0.1 (0.2)	0.1 (0.2)	0.1 (0.2)	0.1 (0.2)
ESG	0.7/3 (1)	0.7/3 (1)	0.7/3 (1)	0.1 (0.4286)	0.1 (0.4286)	0.1 (0.4286)
Financial	0.1 (0.4286)	0.1 (0.4286)	0.1 (0.4286)	0.7/3 (1)	0.7/3 (1)	0.7/3 (1)

We apply the SWRPM model for each profile. Table 6 collects the results obtained in the first iteration of our sequential process considering all the equal coefficients, $\mu_i = \frac{1}{6}$, $i \in \{E, S, G, Q, R, GR\}$.

Table 6.

First iteration: The objective and the normalised objective functions.

Profile	f_E F_E	f_S F_S	f_G F_G	f_Q F_Q	f_R F_R	f_{GR} F_{GR}
Balanced	57.1209 0.48917	50.4246 0.45886	57.0725 0.45886	1.7803 0.45886	40.8305 0.45886	1.0501 0.45886
Environmental	72.2669 0.92191	48.6762 0.40925	56.9031 0.45598	0.5694 0.12191	16.2173 0.20048	0.2988 0.12191
Social	61.2415 0.60690	67.9858 0.95704	50.3677 0.34522	2.2033 0.57657	13.7038 0.17409	0.3771 0.15704
ESG	65.5433 0.72981	59.9757 0.72981	73.0587 0.72981	1.6003 0.40877	37.7215 0.42622	0.3801 0.15838
Financial	54.0904 0.40258	47.3706 0.37222	56.6336 0.45142	1.9216 0.49818	44.5765 0.49818	1.1377 0.49818
	D	***Decision variables***				
Balanced	0.3303	$x_{22}=0.19$	$x_{23}=0.13$	$x_{49}=0.26$	$x_{56}=0.06$	$x_{80}=0.36$
Environmental	24.9004	$x_{45}=0.14$	$x_{66}=0.69$	$x_{79}=0.17$		
Social	21.4291	$x_{56}=0.80$	$x_{73}=0.20$			
ESG	13.6439	$x_{23}=0.76$	$x_{56}=0.09$	$x_{73}=0.13$	$x_{80}=0.02$	
Financial	3.8923	$x_{22}=0.25$	$x_{49}=0.37$	$x_{80}=0.38$		

The convergence speed of the algorithm changes depending on the investor profile. In this sense, we can emphasize that for a balanced or financial investor convergence is reached at the first iteration whereas for an Environmental, Social or ESG profile the maximum fixed number of iterations (100) is reached (see Figures 2-4). Table 7 shows the optimal solution obtained in the final iteration of our algorithm.

Table 7.

The optimal final solution: the objective and the normalised objective functions.

Profile	f_E F_E	f_S F_S	f_G F_G	f_Q F_Q	f_R F_R	f_{GR} F_{GR}
Balanced	57.1209 0.48917	50.4246 0.45886	57.0725 0.45886	1.7803 0.45886	40.8305 0.45886	1.0501 0.45886
Environmental	71.0172 0.88620	47.1537 0.36606	58.6974 0.48640	0.7669 0.17685	18.9077 0.22872	0.4213 0.17685
Social	62.0594 0.63027	67.2962 0.93748	47.7712 0.30121	2.2071 0.57763	14.1467 0.17874	0.4442 0.18711
ESG	63.7730 0.67923	58.1928 0.67923	70.0744 0.67923	1.2917 0.32291	32.5249 0.37167	0.6750 0.29061
Financial	54.0904	47.3706	56.6336	1.9216	44.5765	1.1377

	0.40258	0.37222	0.45142	0.49818	0.49818	0.49818
	D	***Decision variables***				
Balanced	0.3303	$x_{22} = 0.19$	$x_{23} = 0.13$	$x_{49} = 0.26$	$x_{56} = 0.06$	$x_{80} = 0.36$
Environmental	15.0391	$x_{45} = 0.20$	$x_{66} = 0.55$	$x_{79} = 0.25$		
Social	18.2073	$x_{56} = 0.84$	$x_{73} = 0.07$	$x_{80} = 0.09$		
ESG	2.7399	$x_{23} = 0.64$	$x_{45} = 0.04$	$x_{73} = 0.25$	$x_{80} = 0.07$	
Financial	3.8923	$x_{22} = 0.25$	$x_{49} = 0.37$	$x_{80} = 0.38$		

For the environmental profile, the portfolio obtained at the first iteration presents poor performance (0.12191) in two financial criteria, namely Q-Tobin and Growth. After iterative process the found portfolio presents features more balanced and Q-Tobin and Growth rate reach values (0.17685) close to the fixed aspiration levels (0.2). In this case the distance, between the reached ratios, $\frac{F_i}{F_j}$, and the desired ones, $\frac{w_i^{scl}}{w_j^{scl}}$, improves greatly from 24.9004 at the first iteration to 15.0391 at the last one (see Figure 2).

We note that the Balanced portfolio consist of firms that also appear in the ESG and Financial portfolios. For Environmental and Social profile, the firms with the highest proportion in the portfolio are those that reach the ideal values in the environmental and social objectives, respectively. In the ESG portfolio, the firm with the highest proportion in the portfolio has scores above the mean in the ESG criteria. Finally, the Financial portfolio consists of three firms that reach one of the three financial ideals (see Tables 8 and 9).

Table 8.

Scores of the final optimal firms.

Firms	E	S	G	Tobin- Q	ROE	Growth
F22	66	40.25	68	1.620	**92.38**	0.5525
F23	68	58.75	80	1.653	46.26	0.2129
F45	65	38.25	73	0.015	6.64	1.8931
F49	40	47.75	76	**3.725**	46.52	0.3973
F56	63	**69.5**	49	2.532	15.05	0.1777
F66	**75**	52	53	0.131	10.24	0.0342
F73	54	61.75	56	0.850	8.16	1.1983
F79	67	43.50	60	2.744	47.44	0.1197
F80	60	51.75	30	0.349	10.76	**2.2566**

Source: Financial data obtained from Morningstar Direct and YCharts at 2015. ESG data obtained from Vigeo agency

Table 9.

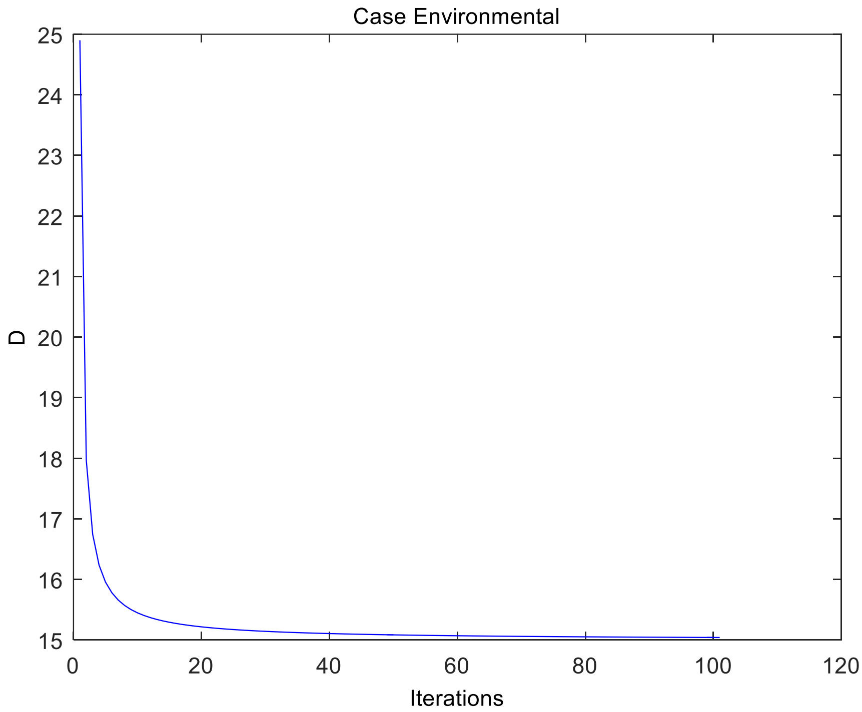
Descriptive statistics for ESG and financial measures.

Statistics	E	S	G	Tobin- Q	ROE	Growth

Mean	48.761	46.494	57.128	0.895	11.120	0.076
St.Dev.	12.303	10.349	13.383	0.714	15.287	0.438
Variance	151.373	107.097	179.095	0.509	233.685	0.192
Kurstosis	1.128	-0.112	-0.751	2.710	7.291	10.900
Asymmetry	-0.719	-0.401	-0.058	1.450	1.150	0.767
Min	1	16.75	30	0.0044	-38.18	-1.934
Max	75	69.5	89	3.725	92.38	2.2566

Source: Financial data obtained from Morningstar Direct and YCharts at 2015. ESG data obtained from Vigeo agency

Figure 2. Evolution of the distance D for the environmental profile.



For the ESG profile, the portfolio obtained at the first iteration reaches a low value of the Growth objective (its value is 0.15838 in the first solution). In order to reach a higher value (0.29061) the algorithm runs 100 iterations and, of course, decreases the achieved outcomes on other objectives (see Table 7 and Figure 3). A similar behavior can be observed in the social case (see Figure 4). In the other cases (balanced and financial profile) the best solution is reached in the first iteration of our algorithm.

Figure 3. Evolution of the distance D for the ESG profile.

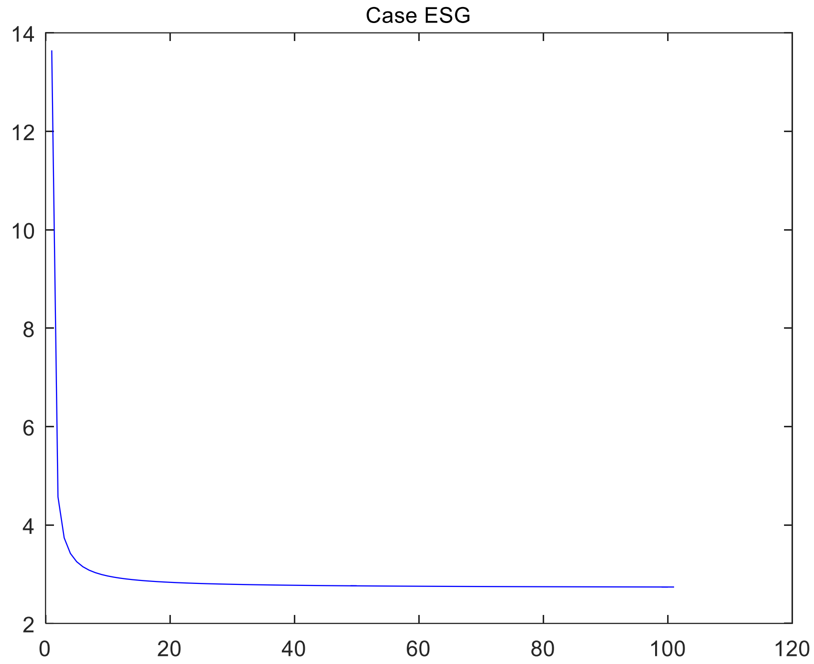
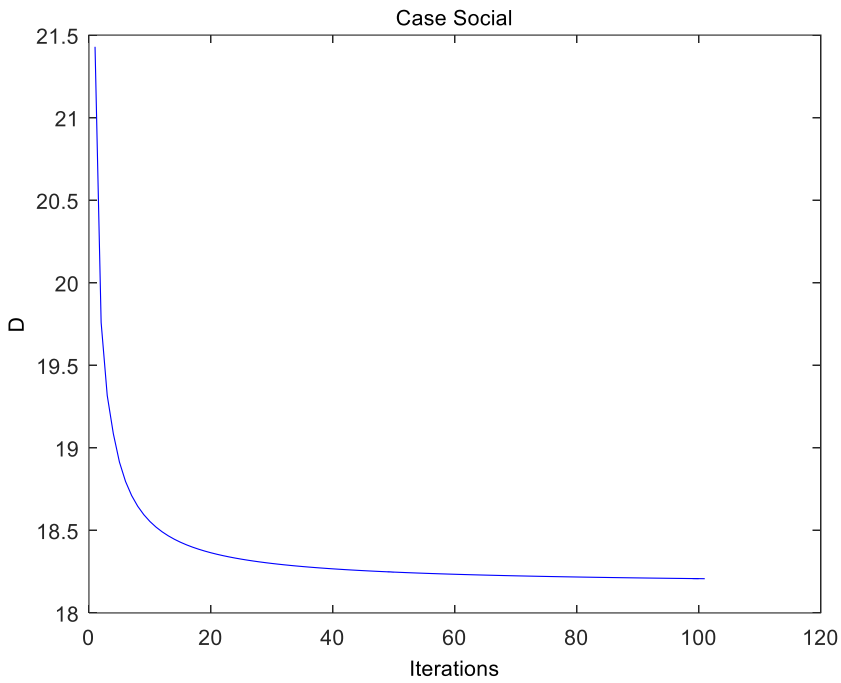


Figure 4. Evolution of the distance D for the Social profile.



The above numerical and graphical results are obtained using Matlab R2018b software.

Conclusions

In many real decision situations, it may be suitable to express preferential information in terms of relative importance weights between objectives. For example, it can be easier for the DM to explain these weights to stakeholders than other types of preferential information such as aspiration levels or pre-emptive priorities for the objectives. However, the possibility of non-correspondence between the proposed weights and the solution obtained in some multi-objective programming models can be a barrier to the use of such models. This work tries to overcome this difficulty by proposing a model that can force to fulfil these weights. Based on the classical RPM approach, a sequential algorithm has been developed.

We have proposed a model where the preferential weights work as the reference point of a RPM. It should be noted that in our method, the reference point in each iteration remains unaltered throughout the whole process, however the scaling function coefficients are modified in order to achieve a solution that best suits DM's wishes. Unlike other methods in this work, the DM intervention is restricted to the starting point and the algorithm can run autonomously until the end, i.e. our proposal is framed within *a priori* information methods. In each iteration, the algorithm minimises the distance between the DM preferences and those achieved by the current solution. To do this, it uses information from the previous iteration updating the technical set of coefficients of the SWRPM. We use a measure of goodness defined by the L_1-distance. The algorithm converges to a Pareto-efficient solution that can be considered as the most preferred by the DM.

In summary, our contribution circumvents several issues of the weighting scheme that often appear in multi-objective programming models. In our framework, it is available a priori preferential information defined in the way of the relative importance between the objectives. This type of information could be easy to communicate to non-specialised stakeholders and also is easy to interpret percentages of achievement of the best values for each objective.

Moreover, the method presented is more iterative than interactive because, once the weights have been set, little technical information is required from the DM. Our algorithm knows how to go to the following iteration without interacting with the DM, current results are providing all the necessary information. Only, the DM joins in to express her/his agreement with the current solution or alternatively, prefers to continue iterating. The non-compensatory character of the aggregation min-max helps the convergence of the algorithm, large weights are acting on positive deviations and are reflected on the solution and, thus weighting scheme is not lost.

A numerical example is used to compare our results with those obtained by close traditional methods (Compromise Programming and RPM). Our procedure allows us to improve these traditional solutions. Finally, a real selection portfolio model demonstrates the performance of the proposal. For an investor concerned by social, environmental and governance issues it would be easy to express the features of the desired portfolio by pairwise comparisons. Several investor profiles have been modelled. The results show that the matching between proposed relative importance weights and weights achieved in the obtained portfolios.

Acknowledgments

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Appendix

Table 10.

Notations.

Notation	***Description***
$f_i(\mathbf{x}),\ i=1,...,k$	*k* conflicting objective scalar functions
$\mathbf{f}(\mathbf{x})=\left(f_1(\mathbf{x}),\ f_2(\mathbf{x}),\dots,\ f_k(\mathbf{x})\right)$	Objective vector
x_m	Decision variable
$S \subseteq IR^n$	Nonempty feasible region
$Z=\mathbf{f}(S)\subseteq R^k$	Feasible objective region
$\mathbf{f}^*=\left(f_1^*,\ f_2^*,...,\ f_k^*\right)$	Ideal point
f_r^*	Optimum of the *r*-th objective in S
$\mathbf{f}_*=\left(f_{1*},\ f_{2*},...,\ f_{k*}\right)$	Anti-ideal point
f_{i*}	The worst element of the *i*-th column of the pay-off matrix
$\mathbf{F}=\left(F_1(\mathbf{x}),F_2(\mathbf{x}),\dots,F_k(\mathbf{x})\right)$	Achievement vector
F_i	Individual achievement function
$\mathbf{w}=\left(w_1,w_2,\dots,w_k\right)$	Weighting system of the criteria
w_i	Weight assigned to the *i*-criterion
$r_{ij}=\dfrac{w_i}{w_j}$	Original priority ratios
$w_i^{scl}=\dfrac{w_i}{max\, w_i}$	Scalarised weight
$\mathbf{w}^{\mathbf{scl}}=\left(w_1^{scl},w_2^{scl},\dots,w_k^{scl}\right)$	Scalarised aspiration vector
μ_i	Coefficient assigned to f_i in the classic RPM formulation
$\mathbf{b}=\left(b_1,b_2,\dots,b_k\right)$	Reference point
$\boldsymbol{\mu}=\left(\mu_1,\mu_2,\dots,\mu_k\right)$	Direction of projection of the reference point to the Pareto frontier
ε	Arbitrary small positive number
D	Measure of goodness
Tol	Tolerance error
M	Maximum number of iterations
$dev_i=\dfrac{w_i^{scl}-F_i}{w_i^{scl}}$	*i*-th normalised deviations

Table 11.

Firms' database.

Firm	Name	Firm	Name
F1	ABB Ltd	F60	Lloyds Banking Group PLC
F2	Accor SA	F61	LM Ericsson Telephone Co B
F3	adidas AG	F62	LVMH Moet Hennessy Louis Vuitton SE
F4	Ageas NV	F63	Man Group PLC
F5	Akzo Nobel NV	F64	Marks & Spencer Group PLC
F6	Allianz SE	F65	Mediaset SpA
F7	Anglo American PLC	F66	Munich Re
F8	Anheuser-Busch Inbev SA	F67	National Grid PLC
F9	ArcelorMittal SA	F68	Nestle SA
F10	Assa Abloy AB B	F69	Nokia Oyj
F11	Assicurazioni Generali	F70	Norsk Hydro ASA
F12	AstraZeneca PLC	F71	Novartis AG
F13	Aviva PLC	F72	Greek Organisation of Football Prognostics SA
F14	AXA SA	F73	Orange SA
F15	BAE Systems PLC	F74	Orkla ASA
F16	Banco Bilbao Vizcaya Argentaria SA	F75	Pearson PLC
F17	Banco Santander SA	F76	Pernod Ricard SA
F18	Barclays PLC	F77	Prudential PLC
F19	Basf SE	F78	Reckitt Benckiser Group PLC
F20	BHP Billiton PLC	F79	Reed Elsevier PLC (RELX PLC)
F21	BP PLC	F80	Renault SA
F22	Sky PLC	F81	Repsol SA
F23	BT Group PLC	F82	Rio Tinto PLC
F24	Carrefour	F83	Roche Holding AG Dividend Right Cert.
F25	Centrica PLC	F84	Koninklijke DSM NV
F26	Cie Generale des Etablissements Michelin SA	F85	Royal Dutch Shell PLC Class A
F27	Continental AG	F86	RWE AG
F28	Credit Suisse Group AG	F87	Ryanair Holdings PLC
F29	CRH PLC	F88	SABMiller PLC
F30	Daimler AG	F89	Compagnie de Saint-Gobain SA-CODYY
F31	Danone SA	F90	Sanofi SA
F32	Deutsche Bank AG	F91	SAP SE
F33	Deutsche Boerse AG	F92	Schneider Electric SE
F34	Deutsche Telekom AG	F93	SES SA DR
F35	Diageo PLC	F94	Siemens AG
F36	Electricite de France SA	F95	Sodexo
F37	Enel SpA	F96	SSE PLC
F38	Eni SpA	F97	Statoil ASA
F39	Ferrovial SA	F98	STMicroelectronics NV
F40	Fortum Oyj	F99	Stora Enso Oyj R
F41	GDFSUEZ (Engie SA)	F100	Swiss Re AG
F42	GKN PLC	F101	Syngenta AG
F43	Glencore PLC	F102	Telecom Italia SpA
F44	Heineken NV	F103	Telefonica SA
F45	HSBC Holdings PLC	F104	Teliasonera AB (Telia Company AB)
F46	Iberdrola SA	F105	Tenaris SA
F47	Imperial Tobacco GRP (Imperial Brands PLC)	F106	Tesco PLC
F48	ING Groep NV	F107	Linde AG
F49	ITV PLC	F108	Total SA
F50	Sainsbury (J) PLC	F109	Valeo SA
F51	Johnson Matthey PLC	F110	Veolia Environnement SA
F52	PPR (Kering)	F111	Vinci SA
F53	Kingfisher PLC	F112	Vivendi SA
F54	Royal Philips NV	F113	Volkswagen AG
F55	Air Liquide SA	F114	Volvo AB B
F56	L'Oreal SA	F115	Wolters Kluwer NV
F57	Lagardere SCA	F116	Yara International ASA
F58	Land Securities Group PLC	F117	Zurich Insurance Group AG
F59	Legal & General Group PLC		

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