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# Intuitionistic fuzzy set and Fuzzy Mathematical Morphology applied to color Leukocytes segmentation

Agustina Bouchet · Susana Montes · Virginia Ballarin · Irene Díaz

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**Abstract** This work presents a new algorithm based on Atanassov's intuitionistic fuzzy sets and Fuzzy Mathematical Morphology to Leukocytes segmentation in color images. The main idea is based on modeling a color image as an Atanassov's intuitionistic fuzzy set using the hue component in the HSV color space. Then, a pixel labelled as leukocyte is selected and compared to the whole image with a similarity measure. Thus, the leukocyte is segmented and separated from the rest of the image. The experimental results show that the algorithm have a good performance, reaching a value of 99.41% for the correct classification of Leukocytes and a 99.23% for the correct classification of the background. Other metrics such as accuracy, precision and recall have been calculated obtaining 99.32%, 99.41% and 99.24%, respectively. The algorithm presents two important characteristics: It works directly over the color images without the need of converting the image in gray scale and it does not produce false colors because as Fuzzy Morphological operators guarantees it.

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Agustina Bouchet  
ICYTE, CONICET - UNMDP, Universidad Nacional de Mar del Plata, Argentina.  
E-mail: abouchet@fi.mdp.edu.ar

Susana Montes  
Department of Statistics and O.R., University of Oviedo, Spain.  
E-mail: montes@uniovi.es

Virginia Ballarin  
ICYTE, CONICET - UNMDP, Universidad Nacional de Mar del Plata, Argentina.  
E-mail: vballari@fi.mdp.edu.ar

Irene Díaz  
Department of Computer Science, University of Oviedo, Spain.  
E-mail: sirene@uniovi.es

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## 1 Introduction

Color medical images provide much more information than gray level ones. As consequence of the increasing computer power, storage capacities and low-cost systems for capturing images, color images are extensively analyzed in Medicine. Thus, new techniques for Digital Image Processing for correcting, segmenting, measuring and/or quantifying the structures present in color images represent a challenge as clinical information about the anatomic structures could provide a more accurate diagnosis. In this way, the theoretical and technological developments that allow the biomedical image interpretation are fundamental for making better diagnoses. , in order to get the most out of the information they contain.

The analysis of blood cell images is an important task in the pathological laboratories for the detection of diseases such as infections, hemorrhage, leukemia, anemia, inflammation, among others. Shape and number of white blood cells (WBC), also called leukocytes, in the cell images are a very important characteristic for the detection of leukemia. Also, there are immature red blood cells when there is a major demand of red blood cells released by the bone marrow. The abnormal WBC, i.e. immature red blood cells, are surrounded with red blood cells and platelets. Therefore, the abnormal WBC are very difficult to identify. In a slide of dyed normal blood cells, it is possible to find red blood cells, WBC, platelets and plasma as a background. The separation of WBC, immature red blood cells or any other ab-

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normal cell is a critical task. The texture, the color or the shape of the kernel are the features that differentiate them. In smear blood images, there are more red blood cells than WBC. Thus, it is necessary a precise cell segmentation that gives the quantitative diagnosis information. Pathologists used to manually count them by putting the portaobjects with the sample blood under the microscope. This is a very tedious task and can sometimes give inexact results. Nowadays, using the automated techniques, the analysis is done more quickly and it gives more precise results. Blood cell segmentation is an essential step for separating the WBC of the kernel and the plasma region. [12]

Atanassov's intuitionistic fuzzy sets give a flexible mathematical framework to manipulate the uncertainty and imprecision in the images. These sets consider not only the membership degree but also the uncertainty that the membership degree called hesitance measure imply. We propose to study the color characteristics, expressed in the color space HSV, as images descriptors through the imprecision and vagueness of the pixels color values. Hue and value are two characteristics color that allow to construct fuzzy intuitionistic sets.

For all that has been discussed, in this paper we propose the use of Atanassov's intuitionistic fuzzy sets for modeling blood cell color images and develop a new segmentation method for this kind of images.

The paper is organized as follow. Section 2 presents a brief of the related works. Section 3 provides all the theoretical concepts necessities for the develop of the method. In section 4 the propose method is described. Section 5 provides the different metrics used to validate the results. Section 6 shows the experimental results. Finally, conclusions are discussed in section 7.

## 2 Related works

Atanassov's intuitionistic fuzzy sets [4](A-IFS) have been extensively used in different domains related to image processing, such as thresholding, edge detection, clustering, image retrieval, segmentation, among others.

A-IFS have been applied in some works [15,22,23] for image thresholding. In particular, in [15] A-IFS represent the uncertainty in the assignment of pixels to the different regions in the proposed multilevel color thresholding method. The method is applied separately over each RGB component and then the results of the three components are aggregated. In [22], A-IFS are employed to represent the hesitance of the expert in determining whether the pixel belongs to the background or to the object for a grayscale image. Mushrif *et al.* [23] uses the concept of A-IFS histon for multithresholding.

With regard to edge detection, in [10] it is developed an edge detector based on A-IFS for gray level images. In addition, it is proposed a new distance measure, called intuitionistic fuzzy divergence that considers the membership degree, the non-membership degree and the hesitation degree. The results obtained using the new measure outperforms previous approaches. In [6] several Interval-Valued Fuzzy Sets construction methods for detecting edges in a gray scale image are proposed. These construction methods are based on considering information related to the neighborhood of each point. In [26] the sinogram of low-dose X-ray computed tomography is regarded as an IFS. Then, the intuitionistic fuzzy entropy of the sinogram is calculated in order to distinguish edges and flat areas.

Image clustering is other topic where A-IFS are valuable. In [11], a novel intuitionistic fuzzy C means clustering based on intuitionistic fuzzy sets is proposed. This new algorithm considers the hesitation degree to define the membership function that allows cluster centers to converge faster than the cluster centers obtained by the traditional fuzzy C means algorithm. This method is applied in gray scale CT scan brain image to identify the abnormalities present in the brain. In [12] a color cell image clustering algorithm using intuitionistic fuzzy sets and different color models was proposed. In [14] a novel method for medical image segmentation using kernel based Atanassov's intuitionistic fuzzy clustering was proposed to overcome the limitations of the euclidean distance. The results show that tumors and lesion images were clustered accurately, especially in a noisy environment.

In [1] a color image retrieval was developed. The image represented in HSV space was partitioned and then the intuitionistic fuzzy joint of the features were calculated. The color image contents were used as the image descriptors through the introduction of fuzziness, which naturally arises from the imprecision of the pixel color values and human perception. In addition, fuzzy quantities as similarity measure for two intuitionistic fuzzy sets were used instead of a just real number.

As it can be observed, A-IFS are extremely important for processing leukocyte images. In [24] a review on leukocyte image segmentation, their advantages and flaws are presented. In this work different approaches are revised, such as methods based on iterative thresholding, adaptative thresholding and the use of OTSU threshold. Also, methods based on clustering algorithms (K-means and C-means) or on mathematical morphology were revised. Finally, several fuzzy divergence and membership functions have been developed with promising results on gray scale image leukocyte segmentation. The conclusions of this review are that

1 there is not a standard automatic leukocyte segmen-  
2 tation method and fuzzy logic may lead to more promising  
3 results. In [18] it is proposed a robust approach to auto-  
4 matic segmentation of leukocyte's nucleus from micro-  
5 scopic blood smear image under normal as well as noisy  
6 environment by employing a new exponential intuitionis-  
7 tic fuzzy divergence based on thresholding technique.  
8 The methodology involves transforming the color image  
9 into grayscale and then minimization of intuitionistic  
10 fuzzy divergence between the current image and the  
11 ideal threshold image. Moreover, a neighborhood based  
12 membership function has been formulated to cope up  
13 with noisy images. In [2] a method to segment leuko-  
14 cytes in blood smear images using interval-valued in-  
15 tuitionistic fuzzy sets is presented. In the method pro-  
16 posed two thresholds are determined to segment the  
17 image. Generally, these values are not presented in the  
18 image. Also, the use of an ideally segmented image is  
19 necessary. The results are compared to other existing  
20 IFS and T2FS based methods [13], K-means (KM) [30]  
21 and GVF [19]. In [13] an automatic leukocyte segmen-  
22 tation is proposed using intuitionistic fuzzy and interval  
23 Type II fuzzy set theory. The author declares that as  
24 the images are considered fuzzy due to imprecise gray  
25 levels, advanced fuzzy set theories may be expected to  
26 give better result. In [30] a color space decomposition  
27 and k-means clustering were combined for WBC seg-  
28 mentation. Finally, In [19], a WBC image segmentation  
29 method using stepwise merging rules based on mean-  
30 shift clustering and boundary removal rules with a gra-  
31 dient vector flow (GVF) snake is proposed.

32 Recently, deep learning has been applied to segmen-  
33 tation and classification of leukocytes. Some of the most  
34 recent methods are mentioned below. In [16] an end-to-  
35 end leukocyte localization and segmentation method is  
36 proposed. Pixel-level prior information is used for su-  
37 pervisor training of a deep convolutional neural network  
38 which is then employed to locate the region of interests  
39 (ROI) of leukocyte and finally segmentation mask of  
40 leukocyte is obtained based in the extracted ROI by  
41 forward propagation of the network. In this work, au-  
42 thors compare the results of their technique with oth-  
43 ers three most recent methods described in [3,25,27]  
44 using the first one watershed and the others two deep  
45 learning, respectively. In [3] an unsupervised approach  
46 is developed. Authors propose to model color and shape  
47 characteristics of WBC by defining two transformations  
48 and introduce an efficient use of these transformations  
49 in a marker-controlled watershed algorithm. In [27] a  
50 fully convolutional network (FCN) that take input of  
51 arbitrary size and produce correspondingly-sized out-  
52 put with efficient inference and learning is built. A skip  
53 architecture that combines semantic information from

a deep, coarse layer with appearance information from  
a shallow, fine layer to produce accurate and detailed  
segmentations is defined. Last, in [25] a U-Net is de-  
veloped based on FCN. This architecture is modified  
and extended such that it works with very few training  
images and yields more precise segmentations.

In this work a new method to leukocyte segmen-  
tation is proposed. The method is based on a similar-  
ity measure used to segment the objects of interest in  
an image. The model developed here overcomes sev-  
eral drawbacks of other approaches. In particular, our  
method is able to evaluate the images automatically  
using just the information of the image itself, without  
having to create a threshold, which surely is not a value  
belonging to the original image. In addition, the method  
developed here works directly over the image, without  
considering the associated gray-scale image.

### 3 Preliminaries

#### 3.1 Color model

A color model is a mathematical model describing the  
way colors can be represented as tuples of numbers.  
There are different types of color models such as RGB,  
HSV, CIELab. RGB is the most common representa-  
tion of color image due to its simple representation. It  
is device dependent and it is not a perceptual model  
[12]. However, due to the high correlation between the  
three components red, green and blue, it is considered  
more beneficial working in the HSV space because it is a  
more intuitive and user oriented color space [15,17,21].  
Therefore, the use of the HSV space is convenient when  
color characterization by only one dimension is desired.  
In this approach, the HSV color space is selected due  
to these facts.

#### 3.2 Atanassov's intuitionistic fuzzy sets

Intuitionistic fuzzy sets introduce uncertainty in the  
membership degree (known as the hesitance degree).  
It is well known that traditional fuzzy set theory as-  
signs a membership degree to each element and the non-  
membership degree is automatically computed as one  
minus the membership degree. However, human think-  
ing often involves uncertainty or imprecision and some-  
times the standard fuzzy sets are not enough because  
the presence of hesitance or uncertainty or the unknowl-  
edge in the membership definition [10].

Taking into account this fact, Atanassov [4] intro-  
duced the concept of intuitionistic fuzzy set, which aims  
to reflect the fact that the degree of non-membership is

not always equal to one minus the membership degree due to the presence of some hesitation.

Given a referential set  $X$ , an intuitionistic fuzzy set  $A$  [4] is defined as:

$$A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\} \quad (1)$$

where  $\mu_A(x) \rightarrow [0, 1]$  and  $\nu_A(x) \rightarrow [0, 1]$  are the membership degree and the non-membership degree of an element  $x \in X$ , satisfying:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \quad (2)$$

An intuitionistic or hesitation degree is also introduced by Atanassov ( $\pi_A(x)$ ), which arises due to the lack of knowledge about the membership degree. The hesitation degree  $\pi_A(x)$  is given by:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x). \quad (3)$$

Atanassov's intuitionistic fuzzy index represents the ignorance or intuition in the construction of the fuzzy sets [15]. Atanassov's intuitionistic index in the context of image segmentation indicates the knowledge/ignorance of the expert when assigning a pixel either to the background or to the object. The Atanassov's intuitionistic index value associated with a pixel has the value zero when the expert is absolutely sure that the pixel belongs either to the background or to the object. The Atanassov's intuitionistic index value increases with respect to the ignorance/intuition of the expert as to whether the pixel belongs to the background or to the object. If the expert does not know if a pixel belongs to the background or to the object, its membership to both must be represented with the value 0.5 (the greatest ignorance/intuition) resulting in an Atanassov's intuitionistic fuzzy index maximum value. However, the ignorance/intuition should have the least possible influence on the choice of the membership degree [15].

### 3.3 Intuitionistic fuzzy generator

In order to construct Atanassov's intuitionistic fuzzy sets from fuzzy sets, intuitionistic fuzzy generators are used. According to Bustince *et al.* [9], an intuitionistic fuzzy generator is a function  $\varphi : [0, 1] \rightarrow [0, 1]$  satisfying:

$$\varphi(x) \leq 1 - x, \forall x \in [0, 1] \quad (4)$$

Therefore,  $\varphi(0) \leq 1$  and  $\varphi(1) = 0$ .

Intuitionistic fuzzy generators have been deeply studied. For example, Sugeno [29] employs a generating function to create the intuitionistic fuzzy generator or fuzzy complement. The fuzzy complement functional is defined as:

$$N(\mu(x)) = g^{-1}(g(1) - g(\mu(x))) \quad (5)$$

where  $g : [0, 1] \rightarrow [0, 1]$  is an increasing function.

Sugeno class can be generated by using the following function:

$$g(\mu(x)) = \frac{1}{\lambda} \log(1 + \lambda\mu(x)) \quad (6)$$

and the Sugeno type intuitionistic fuzzy generator is written as:

$$N(\mu(x)) = \frac{1 - \mu(x)}{1 + \lambda\mu(x)}, \lambda > 0 \quad (7)$$

where  $N(0) = 1$  and  $N(1) = 0$ .

As a consequence, using the Sugeno type intuitionistic fuzzy generator, the fuzzy intuitionistic set is given by:

$$A_\lambda^{IFS} = \{(x, \mu_A(x), \frac{1 - \mu_A(x)}{1 + \lambda\mu_A(x)}) / x \in X\} \quad (8)$$

The hesitation degree or intuitionistic fuzzy index is:

$$\pi_A(x) = 1 - \mu_A(x) - \frac{1 - \mu_A(x)}{1 + \lambda\mu_A(x)} \quad (9)$$

Since the denominator  $1 + \lambda\mu_A(x)$  is greater than 1, so  $\frac{1 - \mu_A(x)}{1 + \lambda\mu_A(x)}$  is less than 1. When this hesitancy factor is zero, the intuitionistic fuzzy set become a normal fuzzy set in which  $\mu_A(x) + \nu_A(x) = 1$ .

Sugeno type intuitionistic fuzzy generator is used in this work to create an intuitionistic fuzzy image for modeling a color image.

### 3.4 Intuitionistic fuzzy divergence

Li and Cheng [20] propose a new similarity measure for intuitionistic fuzzy sets. Although many measures of similarity between fuzzy sets have been proposed in the literature, those measures cannot deal with the similarity measures between intuitionistic fuzzy sets. In this section, some similarity measures for intuitionistic fuzzy sets are presented.

Let  $IFS(X)$  be the set of all intuitionistic fuzzy sets over a referential set  $X$  and  $S : IFS(X) \times IFS(X) \rightarrow [0, 1]$  a mapping.  $S(A, B)$  is said to be the degree of similarity between  $A \in IFS(X)$  and  $B \in IFS(X)$  if  $S(A, B)$  satisfies the following conditions:

1.  $S(A, B) \in [0, 1]$
2.  $S(A, B) = 1$  if  $A = B$
3.  $S(A, B) = S(B, A)$
4.  $S(A, C) \leq S(A, B)$  and  $S(A, C) \leq S(B, C)$  if  $A \subset B \subset C$ ,  $C \in IFS(X)$

The intuitionistic fuzzy divergence (IFD) between two intuitionistic fuzzy sets measures to what extent the two sets are different one to each other [18].

Let  $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$  and  $B = \{(x, \mu_B(x), \nu_B(x)) / x \in X\}$  two intuitionistic fuzzy sets. Taking into account the

hesitation degree, the membership degree range of two intuitionistic fuzzy sets can be represented by

$$(\mu_A(x), \mu_A(x) + \pi_A(x)) \quad (\mu_B(x), \mu_B(x) + \pi_B(x)),$$

where  $\mu_A(x)$  and  $\mu_B(x)$  are the membership degree and  $\pi_A(x)$  and  $\pi_B(x)$  are the hesitation. The interval is due to the hesitation or lack of knowledge in the assignment of the membership values.

Hence, the intuitionistic fuzzy divergence (IFD) between two intuitionistic fuzzy sets A and B that takes into account this hesitation is defined as [14]:

$$\begin{aligned} IFD(A,B) = & \sum_i 2 - [1 - \mu_{AB}(x_i)]e^{\mu_{AB}(x_i)} - \\ & - [1 + \mu_{AB}(x_i)]e^{-\mu_{AB}(x_i)} + \\ & + 2 - [1 - \mu_{AB}(x_i) - \pi_{AB}(x_i)]e^{\mu_{AB}(x_i) - \pi_{AB}(x_i)} \\ & - [1 + \pi_{AB}(x_i) + \mu_{AB}(x_i)]e^{-\pi_{AB}(x_i) - \mu_{AB}(x_i)} \end{aligned}$$

being  $\mu_{AB}(x_i) = \mu_A(x_i) - \mu_B(x_i)$  and  $\pi_{AB}(x_i) = \pi_A(x_i) - \pi_B(x_i)$

### 3.5 Fuzzy mathematical morphology

Fuzzy Mathematical Morphology (FMM) is a different approach for the extension of the Mathematical Morphology (MM) as binary operators to gray level images, by redefining the set operations as fuzzy set operations, based on Fuzzy set theory [5,8]. FMM is based on a solid theoretical framework and it is proved to be able to solve several problems in image processing and segmentation problems with high texture content. Although these techniques have been widely studied and applied to gray scale images, currently the use of color medical imaging is growing, due to the additional information they present and the increased computing power available.

However, the extension to color or multispectral images is not simple, due to the nature of the data vector and the difficulty of finding a proper order for it. In Bouchet et.al. [7] the authors presented the basic operators of the FMM for color images from the definition of a fuzzy order based on fuzzy preference relations. In this approach a color image is considered as a function  $f : D_f \subseteq \mathbb{R}^2 \rightarrow \tau \subseteq \mathbb{R}^3$ , where  $D_f$  is the domain of the function and  $\tau$  is a color space. The basic operators of the MM for a color image are the dilation and erosion. After, combining these operators another more complex can be defined. The basic operators are defined as follows.

Let  $\tau \subseteq \mathbb{R}^3$  be a color space with a structure of complete lattice provided by a total order  $\leq_\tau$ . Let  $B$  be a structuring element, the basic operators, dilation

( $\delta_B^{\leq_\tau}$ ) and erosion ( $\varepsilon_B^{\leq_\tau}$ ), associated to a color image  $f$  are defined as:

$$\delta_B^{\leq_\tau}(f) = \sup_{x \in B} \{f \circ T_{-x}\} \quad \varepsilon_B^{\leq_\tau}(f) = \inf_{x \in B} \{f \circ T_x\} \quad (10)$$

being  $T_x : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  the translation function by the element  $x \in \mathbb{R}^2$ , that is,  $T_x(s) = s + x$ . Note that for any color image  $f$ , any dilation  $\delta_B^{\leq_\tau}$  and any erosion  $\varepsilon_B^{\leq_\tau}$  are new color image. In this work, FMM have been used to make automatic the proposed method.

## 4 Hue intuitionistic based method

In this section the proposed method is developed. This method first considers the hue component of a HSV image and model this component as an A-IFS. Then select a pixel from the image as reference to compute the similarity and compute the IFD between each element and the pixel selected as reference. If the IFD for a point is less or equal than a fixed value, the point is conserved, otherwise discarded. Figure 1 show a block diagram of the proposed algorithm.

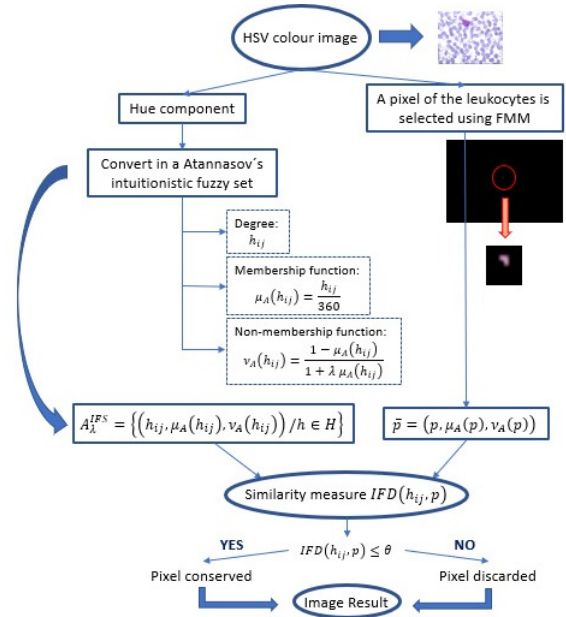


Fig. 1 Block diagram of the proposed method.

The method to segment the leukocytes can be summarized in the following six steps:

**Step 1:** HSV color model is used for representing the color image. Let  $f$  be a color image, three components are extracted according to the hue ( $H$ ), saturation ( $S$ ) and value ( $V$ ) components.

**Step 2:** In this step the image is modeled with A-IFS. Let  $H$  the hue component of the image with size

$M \times N$ . The component  $H$  is defined as an intuitionistic fuzzy set as follows:

1. The first coordinate represents the degree:  $h_{ij} \in [0^\circ, 360^\circ], 0 \leq i \leq M, 0 \leq j \leq N$ .
2. The second coordinate is the normalized value of  $h$ . It is obtained applying the membership function:

$$\mu_A(h_{ij}) = \frac{h_{ij}}{360^\circ} \quad (11)$$

3. The non-membership function is applied over the value obtaining in the above step. The non-membership function is computed by using the Sugeno type intuitionistic fuzzy generator as follows:

$$\nu_A(h_{ij}) = \frac{1 - \mu_A(h_{ij})}{1 + \lambda \mu_A(h_{ij})} \quad (12)$$

Therefore, the intuitionistic fuzzy set for the hue component is defined as:

$$A_\lambda^{IFS} = \{(h_{ij}, \mu_A(h_{ij}), \frac{1 - \mu_A(h_{ij})}{1 + \lambda \mu_A(h_{ij})}) / h_{ij} \in H\} \quad (13)$$

In this paper, we have  $\lambda = 0.5$ .

**Step 3:** The hesitance degree is calculated for each pixel of the image:

$$\pi_A(h_{ij}) = 1 - \mu_A(h_{ij}) - \frac{1 - \mu_A(h_{ij})}{1 + \lambda \mu_A(h_{ij})} \quad (14)$$

**Step 4:** For leukocytes segmentation is necessary one point as a reference to measure the similarity between this point and the image to analyse. Therefore, a fixed pixel corresponding to the object to segment is selected. This selection is made using FMM. For this, a successive fuzzy color erosion given by equation 10 is applied over the original image with the goal of obtaining the fixed point, called  $\bar{p}$ . As the image is eroded the objects in the image are disappearing until there is only one point belonging to one of the leukocytes. This point is represented by their intuitionistic coordinates and their hesitance degree as

$$\bar{p} = (p, \mu_A(p), \frac{1 - \mu_A(p)}{1 + \lambda \mu_A(p)}).$$

**Step 5:** The similarity measure between the fixed pixel and each pixel in the image is calculated in the following way. Let  $A_\lambda^{IFS}$  the intuitionistic fuzzy set, given by equation 13, corresponding to the hue component  $H$  of the image, which their dimension is  $M \times N$ . The IFD between each element of  $H$  and  $\bar{p}$ , using equation 10, is given by:

$$\begin{aligned} IFD(h_{ij}, p) &= 2 - [1 - \mu_{A_{h_{ij}, p}}]e^{\mu_{A_{h_{ij}, p}}} - [1 + \mu_{A_{h_{ij}, p}}]e^{-\mu_{A_{h_{ij}, p}}} \\ &+ 2 - [1 - \mu_{A_{h_{ij}, p}} + \pi_{A_{h_{ij}, p}}]e^{\mu_{A_{h_{ij}, p}} - \pi_{A_{h_{ij}, p}}} \\ &- [1 - \pi_{A_{h_{ij}, p}} + \mu_{A_{h_{ij}, p}}]e^{\pi_{A_{h_{ij}, p}} - \mu_{A_{h_{ij}, p}}} \end{aligned}$$

where  $h_{ij}$  represented each pixel in  $H$  with  $1 \leq i \leq M$  and  $1 \leq j \leq N$ ,  $\mu_{A_{h_{ij}, p}} = \mu_A(h_{ij}) - \mu_A(p)$  and  $\pi_{A_{h_{ij}, p}} = \pi_A(p) - \pi_A(h_{ij})$

Therefore, IFD is a matrix of size  $M \times N$  in which each number represents the similarity between  $h_{ij}$  and the fixed point  $\bar{p}$ .

**Step 6:** In this step a decision is taken. If the similarity measure  $IFD(h_{ij}, p)$  is less or equal than a fixed value, denoted by  $\theta$ , then the point  $h_{ij}$  is conserved; else discarded. As a consequence, the segmented leukocytes image is obtained, i.e., for all  $i$  and  $j$ ,  $0 \leq i \leq M$ ,  $0 \leq j \leq N$ :

$$\text{If } IFD(h_{ij}, p) \leq \theta \Rightarrow \text{Result}(i, j) = h_{ij}$$

$$\text{If } IFD(h_{ij}, p) > \theta \Rightarrow \text{Result}(i, j) = 0$$

where  $\text{Result}(i, j)$  represents the result image to pixel  $(i, j)$ .

## 5 Validation

To evaluate the performance of the proposed algorithm, some well known metrics in classification and information retrieval have been used [28]. These metrics are Accuracy, Precision, Recall, Dice and Jaccard coefficients. Accuracy is a metric normally used for computing the overall classification grade. Precision computes the percentage of positive prediction made by the classifier that is correct while Recall computes the percentage of positive patterns that are correctly detected by the classifier. On the other hand, the Dice coefficient is a statistic used for comparing the similarity of two samples and the Jaccard coefficient measure the similarity degree between two sets.

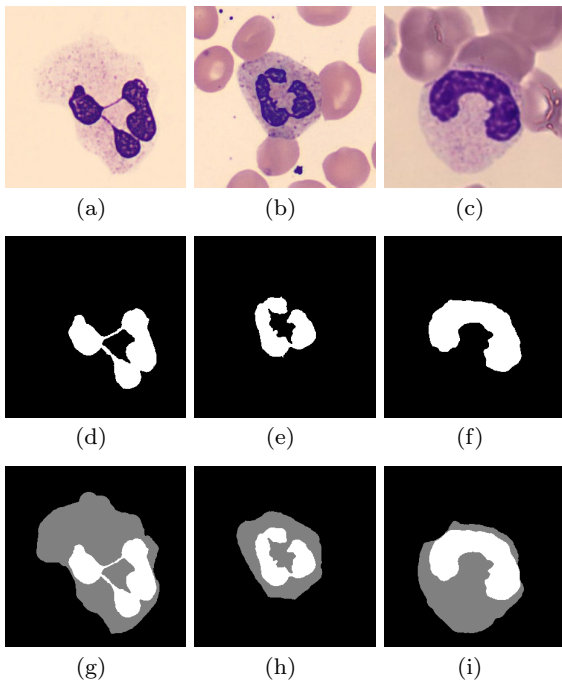
## 6 Experimental results and discussion

In this section some experiments are conducted to show the results of the application of the proposed method. The dataset consists of one hundred  $300 \times 300$  color images, which were collected from the CellaVision blog<sup>1</sup>. The cell images are generally purple and may contain many red blood cells around the white blood cells. Also, the area related to the nuclei and cytoplasm were manually segmented by an expert.

As an example, Figure 2 shows the results of the proposed method applied to three images of the dataset. Original images are showed from (a) to (c), their segmentations are showed from (d) to (f) and finally the expert's segmentation are showed from (g) to (i).

Table 1 shows the average classification error as a confusion matrix [28], for the proposed method applied

<sup>1</sup> <http://blog.cellavision.com/>



**Fig. 2** Results of the proposed method. (a)-(c) Original Images. (d)-(f) Segmentation obtaining by the proposed method. (g)-(i) Segmentation given by the experts.

**Table 1** Confusion matrix.

% of pixels	Leukocytes	Background
Classified as leukocytes	99.41	0.59
Classified as background	0.77	99.23

**Table 2** Mean and standard deviation (SD) of the metrics using for the validation.

	Acc	P	R	Dice	Jaccard
Mean	0.9932	0.9941	0.9924	0.9966	0.9933
SD	0.0047	0.0041	0.0093	0.0021	0.0042

to the dataset. We can see that true positives and true negatives are both high values. These values are 99.41% and 99.23%, respectively. Also, we can see that higher values are obtained for false negatives than for false positives, 0.77% and 0.59%, respectively.

Table 2 shows the mean and the standard deviation of the accuracy, precision, recall, Dice coefficient and Jaccard coefficient for the images processed. It can be seen that the values obtained for the means are high, while the values obtained for the standard deviation are low, showing the good performance of the proposed method.

Besides the quantitative validation made before, a description and comparison against the state-of-the-art methods in the literature is developed. There are two

**Table 3** Accuracy rate (1st row) and Dice coefficient average (2nd row) of IFS, T2FS, K-means, GVF, J-M, A-M and the proposed method (P-M) for the images of the dataset.

IFS	T2FS	KM	GVF	J-M	A-M	P-M
0.9451	0.9641	0.9672	0.9727	0.8319	0.9872	0.9932
0.7395	0.8895	0.8833	0.9555	0.3217	0.9717	0.9966

**Table 4** Precision rate (1st row) and Dice coefficient average (2nd row) of W, FCN, U-Net, LM and the proposed method (P-M) for the images of the dataset.

W	FCN	U-Net	LM	P-M
0.962	0.9461	0.9608	0.9779	0.9997
0.663	0.9388	0.9559	0.9615	0.9931

important works mentioned in the introduction which used intuitionistic fuzzy sets as our proposed method. This is the fact that we compare these techniques. In the first one [18], Jati method (J-M) is applied on a ROI but not on the complete RGB image, working efficiently. The main contribution of our proposal is that it does not need the definition of a ROI as it works on the whole image. Furthermore, the result of our method on a ROIs has a better performance than Jati method. In [2] a improve segmentation accuracy of Jati method is proposed. The basic idea of Ananthi method (A-M) is to find a threshold for which the similarity between ideally segmented image and the segmented image using the determined threshold is maximum. Therefore, this method requires an ideally segmented image but it is not always available. Table 3 shows the accuracy rate and the Dice coefficient average for our proposal, Jati and Ananthi methods for the images of the dataset. The table includes the values obtaining with the techniques mentioned in section 2: IFS and T2FS based methods [13], K-means (KM) [30] and GVF [19].

Also, the segmentation performances of our proposed method are compared with the most recent methods mentioned in section 2: LeukocyteMask without data augmentation (LM) [16], watershed-based method (W) [3] and two deep learning-based methods FCN [27] and U-Net [25]. Table 4 shows the precision rate and the average value of Dice coefficient between all these methods, considering the complete cell instead of only the nuclei.

## 7 Conclusions

In this work it is proposed a new method for color image segmentation of leukocytes based on Atanassov's intuitionistic fuzzy sets and Fuzzy Mathematical Morphology. A similarity measure is used as tool to segment



the objects of interest in an image. The algorithm works directly on color images without the need to transform them to gray scale images. Thus it avoids the associated loss of information produced by this transformation. Another advantage of this approach is that, because of the use of the fuzzy color image operators in the detection of the reference point, no false colors are created in the process, and the resulting point contains exactly the same color present in the original image. As we can see in the results, the performance of the proposed algorithm is high.

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