Geometric reconstruction and measurement of long steel products using 3D sensors in real-time

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***Abstract*—Calibration, registration, reconstruction and measurement are the fundamental tasks required for the inspection of dimensions in long steel products. Calibration is performed offline. The rest of the tasks are performed repeatedly while the long steel product is moved under 3D reconstruction sensors. This work proposes robust methods for the reconstruction of long steel products. In addition, measurement procedures for some representative dimensions are presented. Three different reconstruction procedures are proposed: reconstruction based on geometric primitives in the model of the product, reconstruction based on local fitting and reconstruction based on piecewise linear approximation. Tests on synthetic on real data indicate excellent performance in terms of computational costs and measurement accuracy. Conclusions also provide recommendations for the application of the proposed reconstruction procedures depending on whether a model is available or not, and the type of features that need to be calculated for the long steel products.**

***Index Terms*—Geometric measurement, Long steel product, Inspection, Quality control**

I. Introduction

Long steel products, including billets, blooms, rebars, wire rod, sections, or rails, are used in a wide variety of industries, from construction to energy and automotive. These manufactured long products in the steel industry need to be examined for compliance with quality criteria before they are used [1]. In addition to its physical and internal properties, the geometry of the long products is an important feature that requires measurement, monitoring and control. An accurate dimension of the long product ensures correct assembly in final production, resulting in a high-quality final product. Moreover, the early detection of potential deviations from the models results in lower production costs because they can be corrected in a timely manner.

3D sensors have displaced contact sensors for geometric measurements [2]. Contact sensors require contact with the product to provide a set of measurement points. These points then require interpolation to obtain approximated dimensional values. Contact sensors are not only slow, which reduces the flow of production, they are also affected by the wear of the contact device, requiring periodic maintenance. 3D optical sensors based on laser projection, on the other hand, can acquire geometric information about the entire product with high-resolution, enabling much more accurate measurements. Moreover, 3D sensors can perform measurements extremely fast, without affecting the manufacturing speed of the industrial line, which represents a significant advantage [3]. Therefore, this technology can be used to provide precise, fast, and robust measuring solutions for long steel products.

The most common configuration for optical 3D sensors applied to long steel products are the profilometers based on laser line projection. One or multiple laser lines are projected onto the product from different orientations. Then, high-resolution cameras acquire images of the deformed laser line projected onto the product. The deformation of the projected laser lines provides the required information to calculate the dimensions of the product. However, this procedure requires performing specific tasks in order to produce accurate geometric measurements from the information provided by the images. Each profilometer must be calibrated, so the transformation from pixels in the image where the laser line is projected to millimeters is possible. In addition, when multiple sensors are placed around the product to ensure full coverage of the 3D surface, a calibration task to combine the information obtained by all sensors is required. The combined profiles use a unique common reference system. During real-time measurement, vibrations affect the movement of long products. Thus, registration is required to align the acquired profile with the geometric model of the product being manufactured. After registration, two more tasks are required: reconstruction and measurement. Reconstruction is applied to transform the description of the long product, a profile with a set of points or point cloud, to geometric primitives. Measurement can be performed on the reconstructed geometric primitives according to specific standards, depending on the type of product and dimension, in order to perform quality control.

The calibration of 3D sensors based on laser projection is extensively studied in [4]. In [5] a framework is presented to calibrate the combination of profiles using multiples 3D sensors, and to perform robust alignment in real time. This work deals with the two other tasks in the quality control pipeline: reconstruction and measurement. The reconstruction procedure proposes different alternatives to transform the set of points, which provide information about the coordinates of the shape of the product, into a description that can be used to perform accurate measurements about the dimensions. Three different methods are proposed: reconstruction based on geometric primitives; reconstruction based on local fitting, including the centroid, and linear quadratic fitting; and reconstruction based on iterative piecewise linear approximation. The considered methods are compared and evaluated, includ-

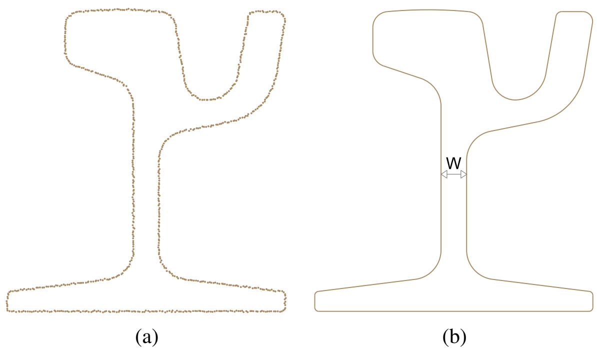


Fig. 1. Reconstruction and measurement from an aligned profile. (a) Profile consisting in a set of points describing the shape of the product. (b) Measurement of the width using the reconstruction geometrical model of the product being inspected.

ing a discussion of their applicability in different scenarios. The measurement task depends on the product being inspected, the analyzed dimension and the standard used, which also indicates maximum tolerances for each dimension. In this work, a straightforward approach is proposed to calculate any measurement using the reconstructed product shape.

This paper is organized as follows: Section II presents the proposed approach for the reconstruction and measurement of long steel products in real-time; Section III discusses the results obtained, and finally, Section IV reports conclusions.

II. Proposed Approach

The proposed approach assumes multiple 3D sensors are used to measure the coordinates around a cross-section of the long steel product. The resulting sets of points are combined into a single profile based on a unique common reference system. The resulting profile is aligned with the geometrical model of the product being inspected. The result is a profile similar to the profile represented in Figure 1a, which is a grooved rail used for tramways. The objective of the proposed approach is to transform this profile into a geometrical description similar to Figure 1b, where measurements about the dimensions of the product can be calculated, such as the width of the rail.

A. Reconstruction

1) Reconstruction based on geometric primitives: The geometrical model

A long steel product is usually described by the model of the cross-section. This model, in most cases, is defined in terms of basic geometric primitives: line segments and circular arcs. Figure 2a shows the model of the grooved rail used in the example. Different adjacent geometric primitives are represented with a different color. The objective of the reconstruction based on geometric primitives is to transform the aligned profile into a set of equivalent geometric primitives.

In order to reconstruct the model based on the aligned profile, it is necessary to determine which points are going to be used to calculate each geometric primitive. The approach

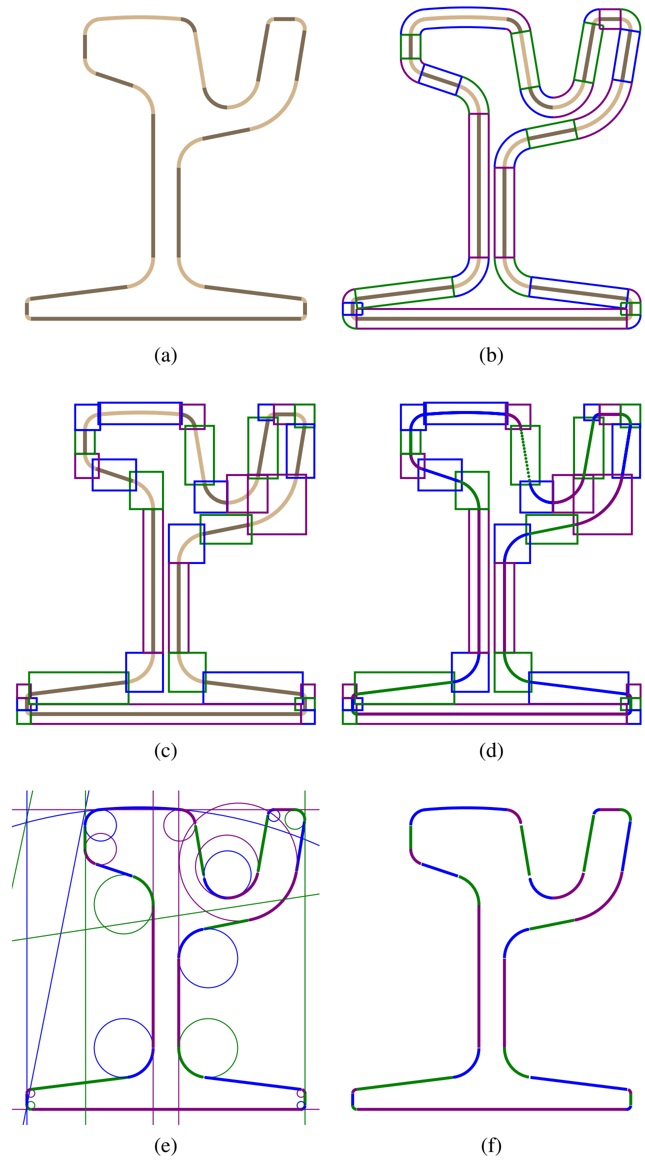


Fig. 2. Reconstruction of the model using the profile based on geometric primitives. (a) Model. (b) Envelope of geometric primitives using a distance threshold. (c) Bounding box of geometric primitives based on the envelopes regions. (d) Points for each geometric primitive. (e) Fitting to lines and circles. (f) Reconstruction of the model using circular arcs and line segments.

followed in this work is to assign a point to the closest geometric primitive in the model, but only if this point was considered an inlier during registration. A naive approach to compute the closest geometric primitive to each point is to compare the squared distances from all the geometric primitive in the model to each point in a loop. However, this approach is extremely inefficient, because it requires an excessive number of close point computations and comparisons. Only the close primitives of the points really need to be considered. The R-tree is an efficient spatial data structure that provides a solution to this problem, as it can be used to efficiently to determine the closest geometric primitives of a point [6].

In order to create the R-tree for the model, the following approach is proposed. First, a closed envelope region around every geometric primitive is created. These envelope regions are created so that only relevant points closer than a distance threshold to the primitive are included. The envelope around each geometric primitive can be seen in Figure 2b. The calculated regions around each primitive are good indications of which primitive is closer to a point, but this calculation is not computationally efficient. Thus, they are transformed into rectangles by calculating the bounding box of the envelope. The result can be seen in Figure 2c. The coordinates of these rectangular regions are the coordinates with which the R-Tree is created. Using the created R-Tree determining the points corresponding to each primitive is very fast and efficient. The result can be seen in Figure 2d.

When all the points in the profile are assigned to a geometric primitive in the model, the primitive can be reconstructed by fitting the points to the particular type of primitive. If the primitive is a line segment, a line is fitted, if the primitive is a circular arc, a circle is fitted. The result can be seen in Figure 2e. The final step is to determine the limits of these primitives considering the points. Thus, the primitives are finally transformed into circular arcs and line segments. The result can be seen in Figure 2f. Points in the profile have been transformed into geometric primitives where measurements can be performed easily.

2) Reconstruction based on local fitting: An alternative approach to the reconstruction of geometric primitives is local fitting. The reconstruction of geometric primitives is an ideal approach when major dimensional features need to be calculated, such the width or height. However, there are cases where the deviation from the reconstructed surface to the model needs to be calculated. In these cases, a local fitting approach provides more resolution, which is required to detect small defects such as cracks.

The proposed procedure in this work for local fitting is based on three methods: centroid, linear fitting and quadratic fitting. The first steps in all methods are the same. Reconstruction based on local fitting begins with a model and a profile of points, possibly with noise. An example can be seen in Figure 3a. The geometrical model of the inspected product is known, thus, the model can be sampled along the perimeter with a specific resolution. The normal to the surface of the model at each point is also calculated. Figure 3b shows the result of this calculation for one sampled point.

Local fitting requires many computations to determine the closest points in the profile to each sampled point in the model. Therefore, in order to improve the execution speed and reduce the required computation, a kd-tree is used. This data structure organizes points in space, which enables fast close point calculations [7]. Using the calculated kd-tree the computation of the closest points in the profile to each sampled point in the model is very efficient. Figure 3c shows an example with the closest points in the profile to a particular point in the model, given a radius distance.

The set of points closer to the considered distance are then used to fit a local function. Three methods are considered. The first method is the calculation of the centroid of the selected points. The result of this method can be seen in Figure 3c. The second method approximates the selected points to a line using the least squares method. The intersection of the approximated line with the normal to the model is calculated. The result of this method can be seen in Figure 3e. The third method approximates the selected points to a quadratic function. This approximation is more complex and requires a rotation of the selected points according to the normal of the model to avoid degenerate solutions. Then, the intersection of the approximated parabola with the normal to the model is calculated. The result of this method is shown in Figure 3f.

Figure 3f show a comparison of the three considered methods for local fitting. The centroid and the linear method provide a very similar result. In the curved area of the model,

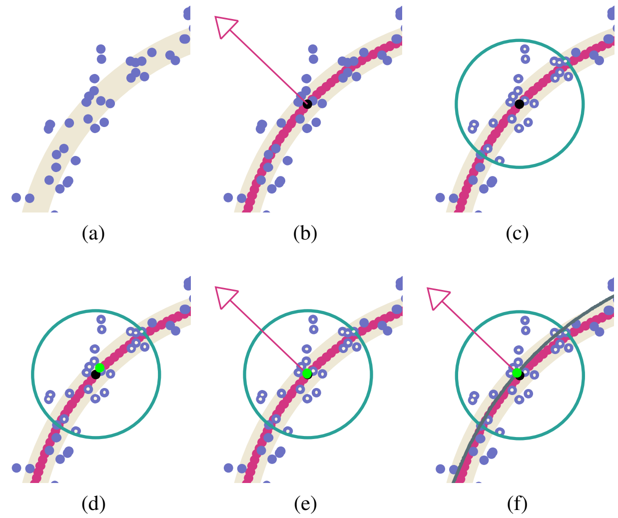


Fig. 3. Local fitting for model reconstruction. (a) Model and noisy profile. (b) Sampled points along the model perimeter and normal to the surface. (c) Computation of points closer to a point in the model using a kd-tree, given a radius distance. (d) Locally fitted point using the centroid. (e) Locally fitted point using a line. (f) Locally fitted point using a parabola.

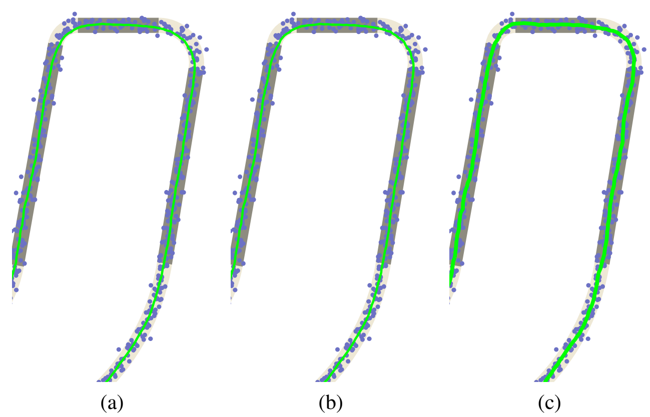


Fig. 4. Comparison of the three considered methods for local fitting. (a) Centroid. (b) Linear. (c) Quadratic.

they tend to smooth the shape, removing the sharpness. The quadratic approach, on the other hand, provides better results in curved areas. However, it is more sensitive to noise than the centroid and the linear method. The three methods are highly dependent on the considered radius distance. This value must be selected based on the amount of noise in the profile, which depends on the quality of the 3D reconstruction sensor used.

3) Reconstruction based on iterative piecewise linear approximation: This procedure is inspired in edge linking [8]. The proposed iterative piecewise linear approximation begins by fitting all the points in the profile by only one linear segment. Whenever, the largest error, $\epsilon_{\max}$, between the approximation line and the points in the profile is greater than a preselected threshold, ϵ, the linear segment is split into two new linear segments, from the location where $\epsilon_{\max}$ occurs. The procedure proceeds iteratively until $\epsilon_{\max}$ does not exceed the threshold ϵ.

This reconstruction procedure is very robust with noisy data. Moreover, the resolution of the reconstruction can be controlled depending on the value of the preselected threshold ϵ. This value is a trade-off between noise filtering and resolution. Figure 5 shows the resulting reconstruction of the noisy profile for different values of ϵ. As can be seen, as ϵ is decreased the reconstruction is more accurate, which requires more linear segments for the approximation.

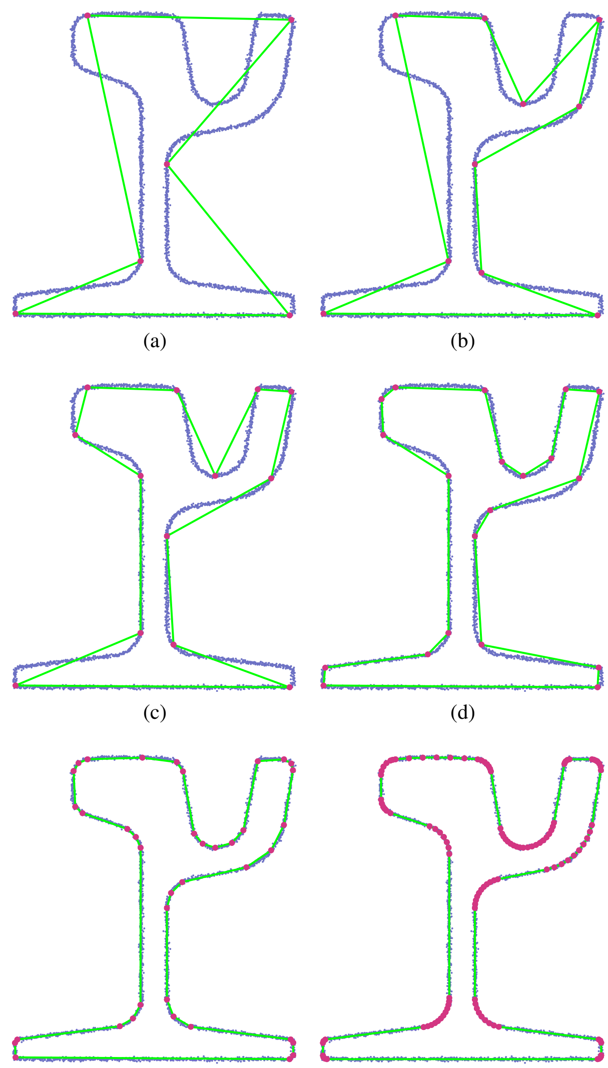
B. Complete 3D reconstruction

The described procedures calculate the reconstruction of one profile. While the long steel product is moved under the laser lines, these procedures are repeated, providing the reconstruction of the whole surface of the object. The result can be seen in Figure 6.

C. Measurement

Measurements provide information about the manufactured product. Quality control then establishes the quality of the product by comparing these measurements with the required specifications. The particular measurements depend on the product being inspected and the standard used. For example, the European Standard EN-13674-1-2011 defines the measurements required for rails. In addition, it indicates the maximum tolerances for each dimension: width, height, and others. Similar standards have been proposed in other parts of the world, such as the AREMA for America [9] or GOST for the Russian Federation. Because each particular standard describes specific dimensions and requires a slightly different procedure, this work cannot describe all of them in detail. However, the description of the measurement procedure for only some quality metrics can be easily adapted for any other required dimension. Therefore, the calculation of some of the most representative dimensions is described next.

1) Calculation of the area: One of the advantages of having the model reconstructed using geometric primitives is that measurements can be performed based on reconstructed geometric primitives rather than using points. This is not only easier, it is required in some cases, as points are not a



(a) (b) (c) (d) (e) (f)

Fig. 5. Reconstruction based on iterative piecewise linear approximation. (a) 50 mm distance threshold. (b) 20 mm distance threshold. (c) 10 mm distance threshold. (d) 5 mm distance threshold. (e) 1 mm distance threshold. (f) .1 mm distance threshold.

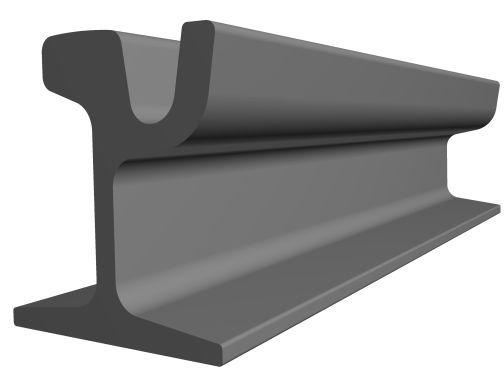
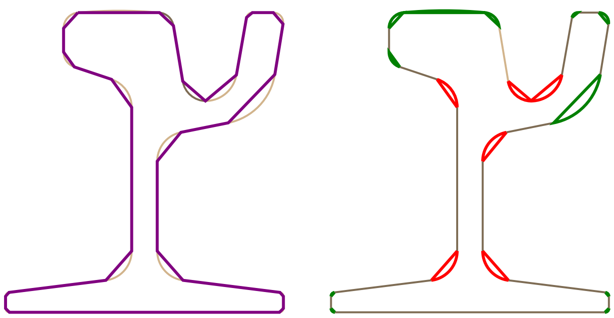


Fig. 6. 3D reconstruction of the grooved rail.



(a) (b)

Fig. 7. Calculation of the area. (a) Polynomial. (b) Outside (green) and inside circular arcs (red).

valid description to calculate some dimensions. One of these dimensions is the area, which can also be used to calculate the volume. In this work, an accurate method to calculate the area based on the reconstructed model is proposed.

There is no method to calculate the area of an irregular form. However, there is a very efficient method to calculate the area of a polygon [10]. A polygon can be decomposed into triangles. Thus, the area of the polygon is equal to the sum of the areas of all the triangles. The calculation of the area of an irregular polygon, A_P, can be efficiently expressed as (1), where $P_i = (x_i, y_i)$ represents the coordinates or the vertex i, N is the number of vertices of the polygon, and $P_{N+1} = P_1$.

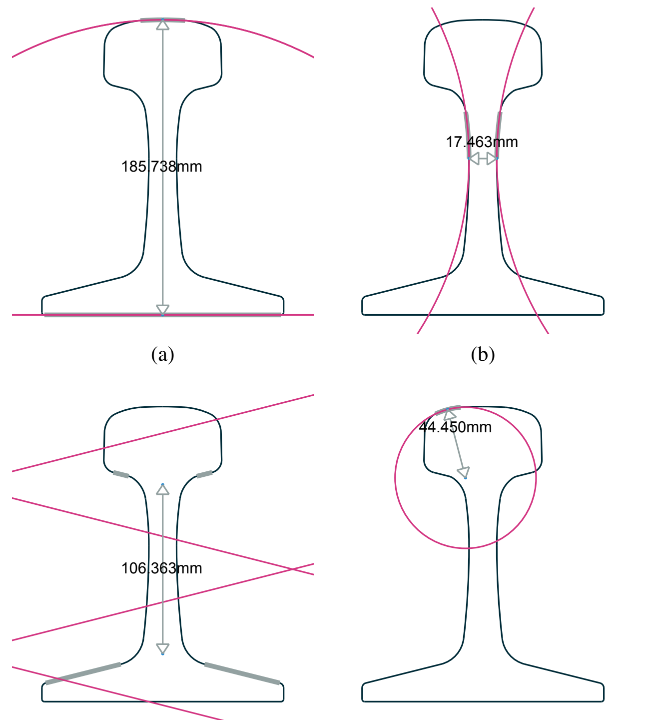
$$A_P = \frac{\sum_{i=1}^{N} x_i y_{i+1} - \sum_{i=1}^{N} x_{i+1} y_i}{2} \quad (1)$$

The reconstructed model with geometric primitives can be transformed into a polygon by connecting the boundary points describing each primitive. The result can be seen in Figure 7a. The area of this polygon can be calculated using the method described above. However, this not the area of the reconstructed model. As can be seen in Figure 7a, some circular segments contained in the model are outside the polygon, and some are inside the polygon. Therefore, in order to calculate the area of the model, the area of these circular segments needs to be added or subtracted to the calculated area of the polygon as necessary.

The calculation of the area of the circular segments, A_{CS}, in the reconstructed model can be calculated using (1), where R is the radius, and θ is the central angle in radians.

$$A_{CS} = \frac{R^2}{2} (\theta - \sin\theta) \quad (2)$$

Determining whether the area of a circular segment needs to be added or subtracted can be determined testing if the point in the center of the arc is outside or inside the polygon. If this point is outside the polygon, the area of the circular arc needs to be added and subtracted otherwise. Figure 7a shows the circular arcs that need to be added and subtracted.



(a) (b) (c) (d)

Fig. 8. Calculation of different dimensions in a reconstructed rail. (a) Height. (b) Web thickness. (c) Flange height. (d) Arc curvature.

Therefore, the area of the reconstructed model, A, can be calculated using (3).

$$A = A_P - \sum^{Inside} A_{CS} + \sum^{Outside} A_{CS} \quad (3)$$

The proposed method to calculate the area is analytic; therefore it is not an approximation. This method can only be applied because the geometric primitives of the model are reconstructed. Following this approach, any other measurement required can be performed accurately.

2) Calculation of the height: The height of a rail is calculated as the distance from the top arc in the head to the foot of the rail (bottom section). When the geometric primitives of the rail have been reconstructed, this dimension can be calculated as the distance from the intersection points of the top arc and the base segment with the vertical axis. Figure 8a shows an illustration for the 136 RE rail defined by AREMA.

3) Calculation of the web thickness: The web thickness is the width of the rail in the web section (middle of the rail). It is calculated as the distance from the left and right arcs in the web section. Some rail models are described using linear segments in this section. Thus, the calculation must be adapted accordingly. Using the corresponding reconstructed geometric primitives, this dimension can be calculated as the distance from the intersection points of the left and right arcs (or left and right linear segments) with the horizontal axis. Figure 8b shows an illustration.

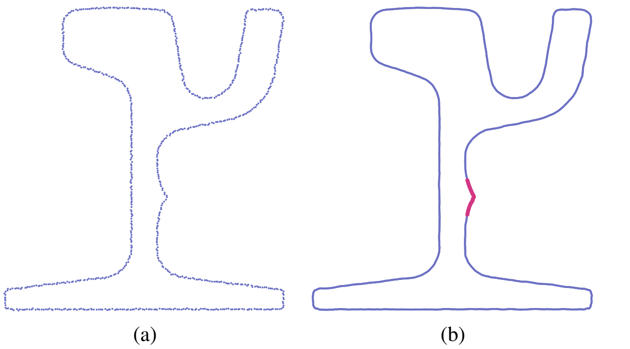


Fig. 9. Detection of deviation from the model. (a) Long product profile. (b) Reconstructed using local fitting based on a quadratic function and detection of deviations.

4) Calculation of the flange height: The flange height of a rail is a dimension used to measure the distance from two virtual points calculated from the intersections of the projections of linear segments in the bottom of the head and top of the foot. Using a similar procedure as in previous dimensions, the flange height is calculated using the corresponding reconstructed primitives. Figure 8c shows an illustration.

5) Calculation of the arc curvature: The arc curvature in the head is used to control the quality of the head where the contact with the wheels of the train is produced. The shape of this section of the rail is very important for a smooth and quiet vehicle running, also with minimum wheels and rail wearing. The arc curvature of the head can be calculated using the radius of the reconstructed arcs in the head. Figure 8b shows an illustration.

Deviations of the calculated dimensions from the specified standards indicate a failure in the manufacturing of the rail. The detection of these deviations, not only avoids unsafe operation, it also enables early detection and correction. Other dimensions for long steel products can be calculated and compared with the standard tolerances using similar straightforward approaches.

6) Calculation of deviations from the model: Deviations from the model are also important to be detected, as they represent surface defects on the manufactured long product. Deviations are generally small cracks that cannot be appreciated when the entire geometric primitives are reconstructed. In this case, it is better to perform reconstruction using local fitting, as it removes noise while providing a resolution that enables the detection of small deviations. Figure 9 presents and example of product with a deviation in the web section. The profile is reconstructed using local fitting based on a quadratic function. Deviations are then detected based on the distance from the reconstructed profile to the geometrical model of the inspected product. As can be seen, the deviation from the model is accurately located.

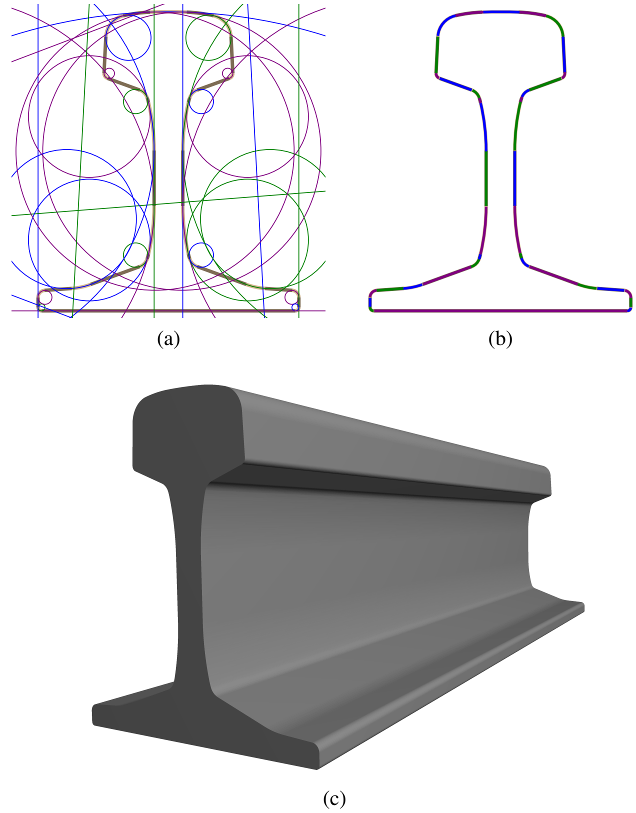


Fig. 10. Reconstruction of the rail 60E1. (a) Fitting to lines and circles. (b) Reconstruction of the model using circular arcs and line segments. (c) 3D reconstruction.

III. Results and discussion

A. Reconstruction of rails and beams

In order to test the reconstruction, two different long steel products are considered: a rail defined in the UNE EN13674-1 standard with identification code 60E1, and a wide flange H beam defined in the Euronorm 53-62 with identification code HEB140. The rail is used in railways and the beam is used in construction. Figure 10 shows the steps performed to obtain the reconstruction of the rail 60E1. The data in the profile is assigned to each geometric primitive in the model and fitting is performed based on the primitive type: either circle or line. Then, circular arcs and line segments are calculated. The model of the rail is complex, which requires many primitives to describe the shape. Combining the reconstruction of consecutive profiles, the complete 3D reconstruction is obtained, as can be seen in the figure.

The reconstruction results for the beam HEB140 are shown in Figure 11. In this case, the model is less complex than the rail. Thus, it is less computationally demanding. The set of reconstructions of consecutive profiles can be used to calculate the complete 3D reconstruction of the beam, as can be seen in the figure.

The two considered long products are quite different. Nevertheless, the proposed reconstruction procedure based on geometric primitives can perform reconstruction in the same

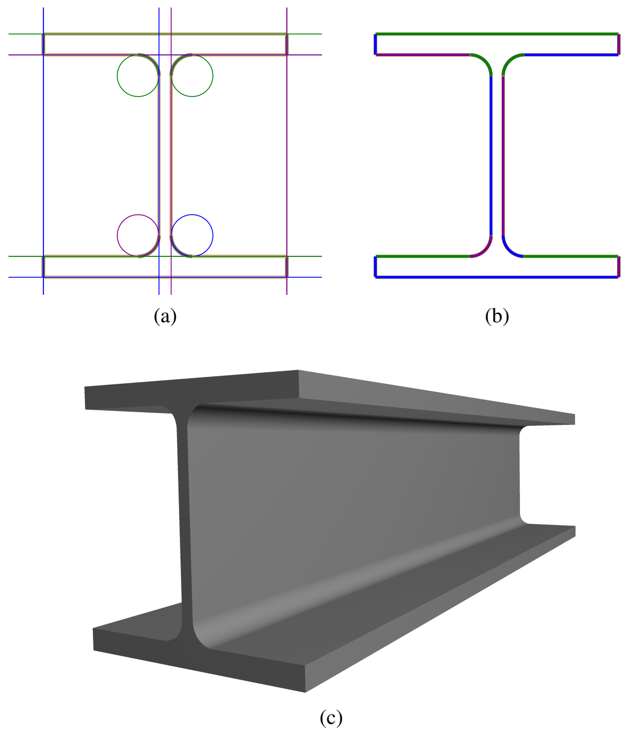


Fig. 11. Reconstruction of the beam HEB140. (a) Fitting to lines and circles. (b) Reconstruction of the model using circular arcs and line segments. (c) 3D reconstruction.

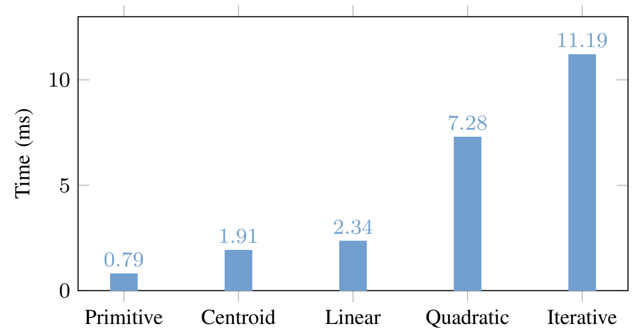


Fig. 12. Time required for the reconstruction of the inspected product.

way, no changes are required. Therefore, the proposed method is flexible enough to be applied to any long steel product easily. This flexibility not only enables efficient reconstruction of current long steel products, but it also makes the algorithm future-proof, as new products can be directly integrated in the inspection pipeline without any modifications in the quality control systems.

B. Time required for reconstruction

The proposed reconstruction procedure is designed to be applied for each acquired profile. Depending on the movement speed of the inspected long product and the desired resolution, the number of profiles acquired per second can be very high. Therefore, the reconstruction procedure requires to be fast. This way, measurements can be performed in real-time with the acquisition.

Figure 12 shows a comparison of the computational time required for model reconstruction using the four proposed procedures: reconstruction based on geometric primitives (Primitive), reconstruction based on local fitting using the centroid (Centroid), linear (Linear) and quadratic (Quadratic) functions, reconstruction based on iterative piecewise linear approximation (Iterative). The reconstruction if performed on a profile of rail 60E1 with 1200 points. All reported running times are for a implementation running on an Intel Core i7 4770 running at 3.4 GHz with 16 GB of RAM.

As can be seen in Figure 12, reconstruction based on geometric primitives is much faster than the other methods. One of the reasons is that the reconstruction procedure is based on an R-tree that is created statically before the reconstruction, as it only depends on the model of the inspected product. Thus, much computation is performed before reconstruction, which saves computational time. Moreover, this reconstruction procedure is designed using a lazy evaluation pattern. Based on this design pattern, the approximation of points in the profile to the corresponding geometric primitive is delayed until the primitive is required for measurement. The value indicated in Figure 12 includes the time required to approximate all geometric primitives. However, in a real applications, not all primitives need to be reconstructed, which greatly reduces the required time to less than 0.5 ms. This reconstruction procedure is very fast, which provides the opportunity for the application in mainstream computer systems at more than 1000 profiles per second.

The most computationally demanding reconstruction procedure is the iterative piecewise linear approximation. The threshold used in the comparison is 0.1 mm. However, this value does not affect the computational time significantly. The part of the reconstruction that is the limiting factor on performance is the required sorting of points before the iterative procedure begins. Assuming points are already sorted, the computational time required for reconstruction using this procedure would be reduced to 0.28 ms. However, this step is required, as no guarantees about the order of the points can be generally assumed.

C. Measurement

Figure 13 shows a comparison of the results for the calculation of the area when different reconstruction procedures are used. In this case, a profile of the grooved rail is corrupted with Gaussian noise of different intensities. The real area of the rail is 70.40 cm^2. When the area is calculated using the reconstruction based on the geometric primitives, the resulting are is not affected by the noise, which is effectively removed when points in the profile are approximated to the corresponding primitives. Moreover, using this reconstruction procedure provides the opportunity to apply an analytic approach for the calculation of the area, which produces very accurate results. In the case of the reconstruction based on local fitting, quadratic in this case, and the iterative approach, the calcu-

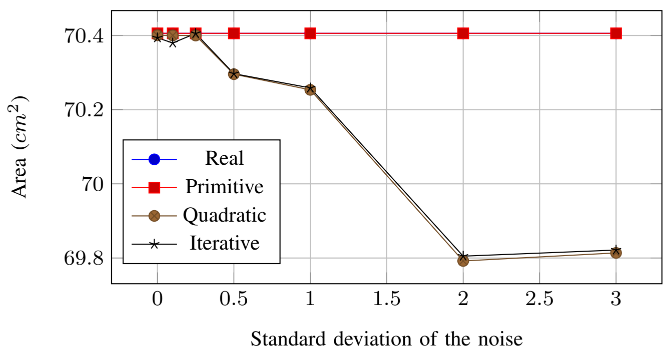


Fig. 13. Calculated area for a synthetic profile base on the grooved rail for different intensities of noise.

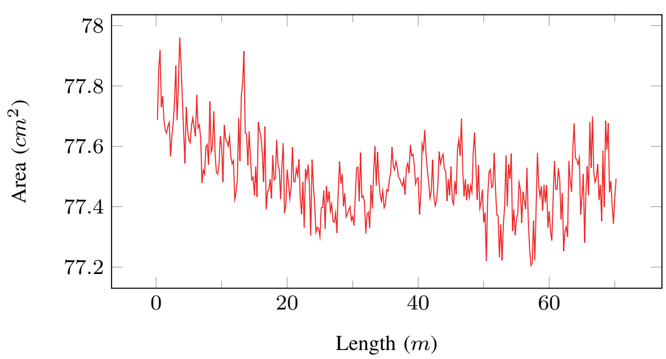


Fig. 14. Calculated area for the rail 60E1.

lation of the area needs to be carried out by approximating the reconstructed model to a polygon. As can be seen, the accuracy of the calculation of the area is greatly reduced using approach. Therefore, for the calculation of the area, the reconstruction based on geometric primitives should be the preferred approach.

Figure 14 shows the calculation of the area using the reconstruction based on geometric primitives applied to a real rail (type UIC 60). It is difficult to evaluate the accuracy of the reconstruction and measurement, as they depend on the quality of the 3D sensor used. However, the pattern that appears in the calculation of the area is consistent with other dimensions of the rail measured by different sensors. This periodic pattern in the area is caused by eccentricity in the rolls used in the manufacturing of the rail. The eccentricity also affects other dimensions such as the height, which can be measured using different methods. Thus, the results obtained with the proposed reconstruction and measurement procedures are consistent with other sensors, which indicates the results are reliable.

IV. Conclusions

In order to assure quality in the steel industry, long products must meet specific quality requirements. Many parameters need to be controlled, including the dimensions, which are of utmost importance. Therefore, 3D sensors are used to acquire geometric information about the shape of the long steel products. 3D sensors provide profiles with point coordinates around a cross-section of the long product.

In order to calculate geometric dimensions from the profiles acquired from 3D sensors, several tasks need to be performed, including calibration, registration, reconstruction and measurement. This work works proposed real-time methods to perform reconstruction and measurement. Three reconstruction methods based on geometric primitives, local fitting and iterative piecewise linear approximation are proposed. In addition, measurement procedures for some representative dimensions are proposed using the reconstructed models.

Results indicate that the reconstruction based on geometric primitives is much faster and accurate than the other considered methods. It can reconstruct the geometric model of the long product in less than 1 ms, enabling real-time operation. Moreover, the results are extremely accurate and consistent with other sensors. Tests with corrupted data also corroborate the robustness under the presence of heavy noise conditions. Reconstruction based on local fitting, using the centroid, or linear and quadratic functions, are good reconstruction procedures when small deviations from the model, such as crack, need to be detected. Reconstruction based on iterative piecewise linear approximation is a useful method when the geometric model of the inspected product is not available.

The proposed procedures are valid for any long product. Therefore, the proposed methods are likely to find potential applications in a number of different areas, where rigorous quality control and product inspection are starting to become more popular.

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