PARAMETER AFFECTING ANTI-LOCK BRAKE SYSTEM IN MATLAB/SIMULINK

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ABSTRACT

Anti-lock braking systems have been a major advance in the development of vehicle safety in recent years. Their mission is to prevent the wheels from locking during a braking situation keeping the slip ratio of the wheels under an optimum value. The ABS regulate the pressure inside the brake circuit to reach this task. Another great advance in recent years has been the development of new simulation techniques using software that allow evaluating the performance of certain systems without being implemented in a real vehicle, which saves time and money. In this work, two new control techniques are developed for an ABS system. These control techniques are a PID controller and a fuzzy logic controller. With the help of Matlab/Simulink software, a half-car model has been built and the two control strategies mentioned above have been implemented. The performance of these two control strategies is evaluated by performing several simulations. The effectiveness of the strategies has been evaluated in terms of the reduction in stopping time and distance of the vehicle.

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1. INTRODUCTION

1.1 ABS OVERVIEW

ABS systems (Antilock Braking System) were created thinking about the brake of the wheels of the aircrafts to avoid their wheels were blocked when braking during the manoeuvre of landing. At first, this idea arose in the 1930s by Bosch, but was not easily implemented due to the technology of the time. Later, in the 1970s, due to the advancement of digital electronics, this system began to be commercialized.

ABS systems are brake system regulation devices that prevent the wheels from locking when braking in extreme braking conditions: wet or slippery road, a reaction of the driver to an unexpected obstacle, etc. During these situations the blocking of some of the wheels may occur, resulting in the directional loss of the vehicle. The ABS system is able to detect a wheel lock situation before it occurs and increase or decrease the braking pressure to avoid it. The ability to transmit forces between the vehicle and the road depends fundamentally on the characteristics of the tyres and the condition of the road. If the braking torque applied to one of the wheels is too high, an angular big deceleration of the wheel occurs, causing it to lock. Once the wheel has locked it is necessary to reduce the braking torque so that the frictional force between the tyre and the road make the wheel start spinning again.

Locking the wheels of a vehicle limits the force available to brake the vehicle as the coefficient of friction between tyre and road decreases. This situation will cause both braking distance and braking time to be greater when the wheels lock during a braking situation. Only in the case of a road with snow the blocking of the wheels will reduce the braking distance due to the accumulation of snow in front of the wheels that opposes the movement of the vehicle. Even so, during this situation, ABS systems provide stability and directional control to the vehicle.

When the vehicle is braking during turning, both the longitudinal and transverse behaviour of the tyre must be taken into account. Figure 1.1 shows how the grip ellipse of the tyre is reduced when the wheel is locked, limiting the forces available to keep the vehicle stable. When the wheel locks, the tyre ability to request transverse forces is very small, so it will be easier the directional loss of the vehicle happens.
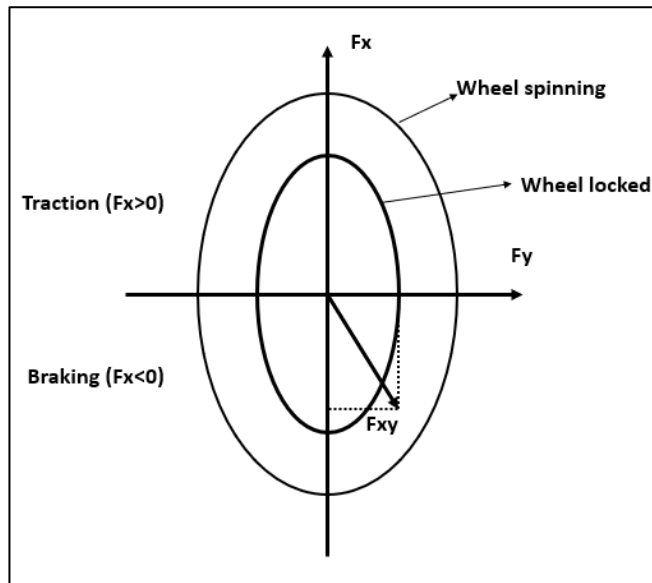
Figure 1.1- Grip ellipse during turning and braking.

1.1.1 ABS control systems

ABS systems let the braking torque be controlled by the driver until the wheel slip starts to increase excessively. When this situation occurs, the system interrupts communication between the master cylinder and the brakes so that a hydraulic modulator modifies the pressure sent to the wheel brake system. If, on one of the wheels, the wheel speed sensor shows excessive deceleration with the danger of wheel lock, the modulator releases pressure from the circuit. Figure 1.2 shows a schematic of the operation of an ABS.
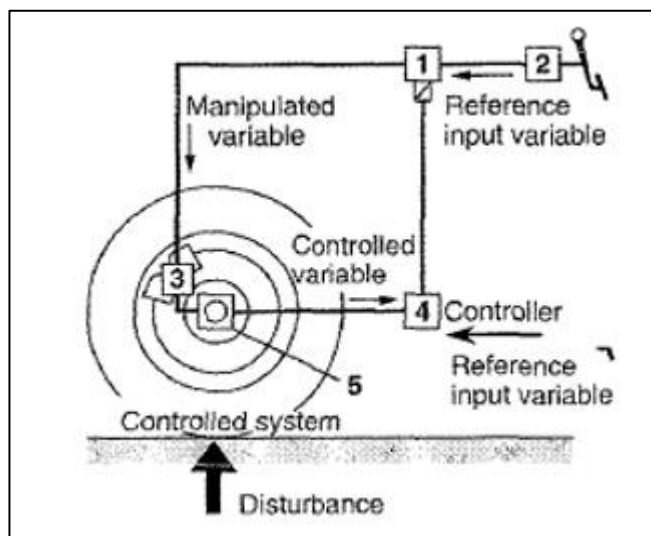
Figure 1.2- ABS control subsystems.

Once the danger of wheel locking has disappeared, the brake pressure is increased again until a situation of imminent wheel locking returns. Therefore, the actuation of an ABS system is a cyclic action. Figure 1.2 shows the evolution of the speed and angular acceleration of the wheels and the pressure of the brake circuit during the operation of the ABS.
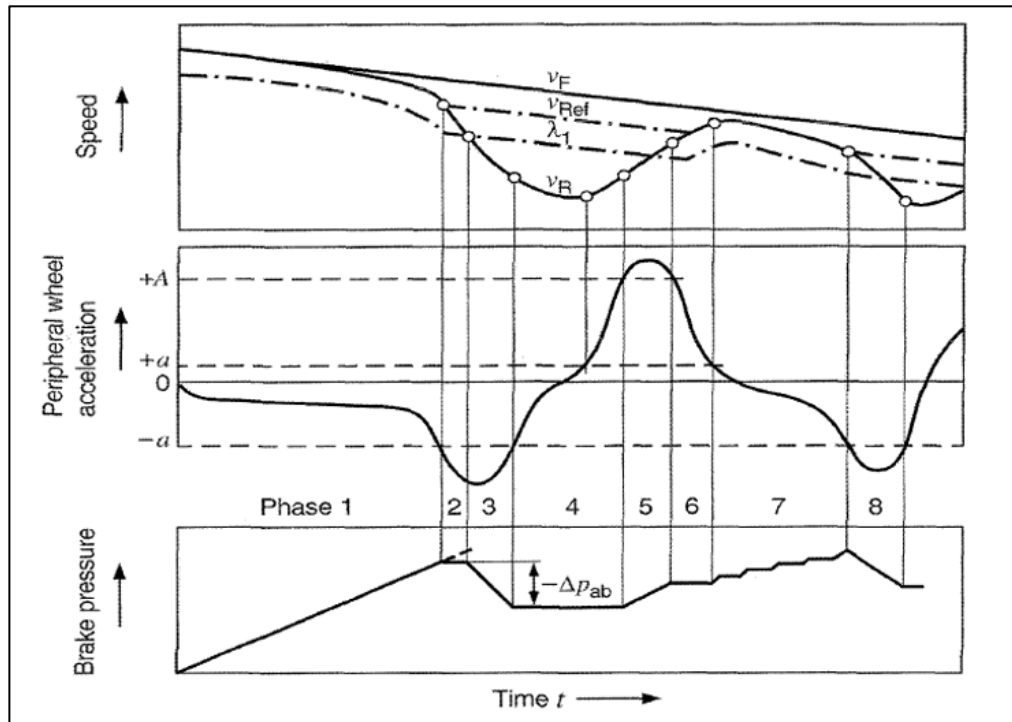
Figure 1.3- Variables of an ABS during braking.

The most important elements within the control system of an ABS are the following:

- Controlled systems: The complete vehicle, the wheels and the adhesion between the tyre and the road.
- Disturbances: Road conditions, braking conditions, vehicle load and tyres.
- Controllers: Wheel speed sensors and ABS control unit.
- Controlled variables: Wheel speed, tyre peripheral acceleration and slip ratio.
- Variable modified: braking pressure.

1.1.2 ABS components

An ABS, in addition to the elements already existing in a conventional brake system, incorporates 3 more elements:

- Wheel speed sensors

- Electronic control unit

- Pressure modulator

An ABS uses sensors to monitor the angular speed of the wheels. The control unit uses wheel speed variations to calculate wheel deceleration and slip ratio. This data will be the calculation basis for exerting the correct pressure on the brake system so as not to block the wheels.

Each of the wheel sensors consists of a magnet and a ton wheel. A magnetic sensor detects the variation of the electromagnetic field and converts it into pulses. The control unit calculates the pulses per second and calculates the speed of the wheel.
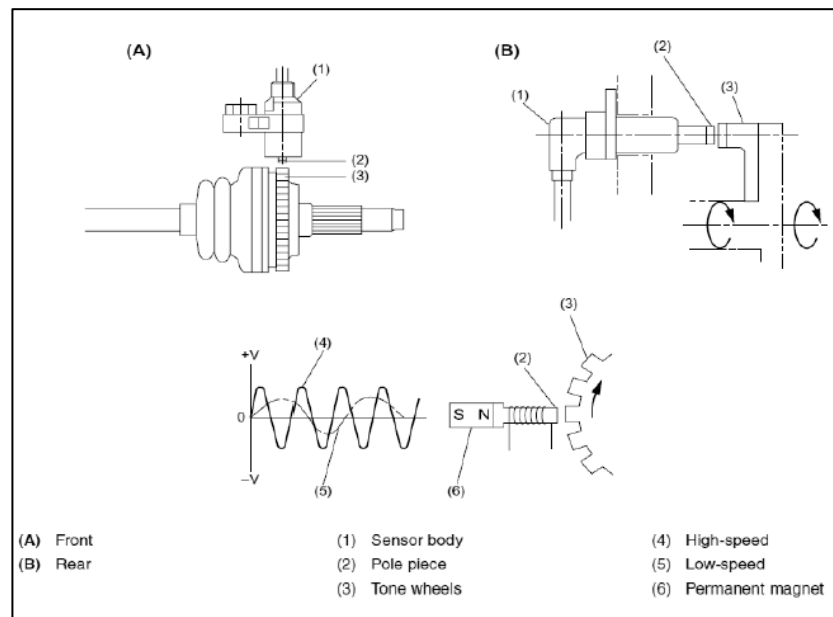
Figure 1.4- ABS wheel sensor (Chris828, 2009)

The electronic control unit (ECU) amplifies and filters the signals received by the sensors. This is a microcomputer that is capable of performing numerous calculations per second. It is in charge of calculating the slip ratio of the wheels. Based on the calculations

obtained, it sends pulses to the pressure modulator to indicate when it is necessary to increase, decrease or hold the pressure within the brake system. In addition, it is also able to completely shut down the system if necessary or detect fails in some parts.

The hydraulic pressure modulator is an electro-hydraulic device that has inside a solenoid valve capable of modifying the brake circuit pressure. This device receives electrical signals from the ECU. Figure 1.5 shows a real hydraulic modulator.
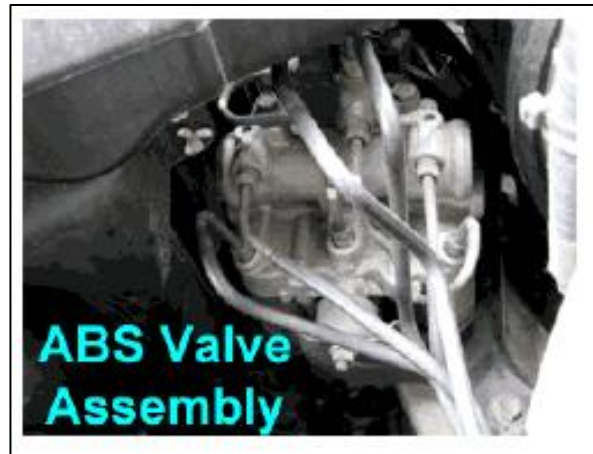
Figure 1.5- Hydraulic modulator.(Chris828, 2009)

1.2   AIMS AND OBJECTIVES

This project will explain the creation of an ABS control system. Within the Matlab/Simulink software a mathematical model of a vehicle will be created with the real data corresponding to the prototype for the Formula Student competition manufactured by Staffordshire University. The vehicle model and the control system will be made in the Matlab/Simulink environment, which is a widely known and widely used program in control engineering tasks. There are some works on similar applications in which this software is used as (Version, 2004) or (Zhang et al., 2012).

The main objectives of the project will be the following:

- Understand how an ABS system works and what components are necessary to implement it on a vehicle.
- Do research about technical papers that perform simulations using different mathematical models and different control techniques.

- Creation of a model in Matlab/Simulink that simulates the behaviour during braking.
- Creation of a control strategy to simulate the behaviour of an ABS.
- Perform various simulations varying some parameters and the control strategy and evaluate which one offers the best performance.

2. VEHICLE DYNAMICS

Some concepts necessary to understand the operation of ABS systems are explained below. These concepts are mainly related to the study of vehicle dynamics and it has been necessary a learning process to later make a vehicle model suitable to achieve the objectives of this work.

2.1 CAR MODELS

There are numerous mathematical models that can represent the dynamics of a vehicle in different situations. The main difference between all of them is the degree of precision and the number of simplifications they contain. Depending on the task for which you want to use the model, some models are more appropriate than others.

2.1.1 Bicycle model

The bicycle model is a model of two degrees of freedom. It is the simplest model in the study of vehicle dynamics. This model is suitable for the lateral behaviour of the vehicle as it represents lateral displacements and yaw movements. There is also a variant of the model with 3 degrees of freedom which includes the movements in the longitudinal direction of the vehicle. This model may be suitable for the study of the vehicle trajectory. Figure 2.1 shows the graphic representation of this model.
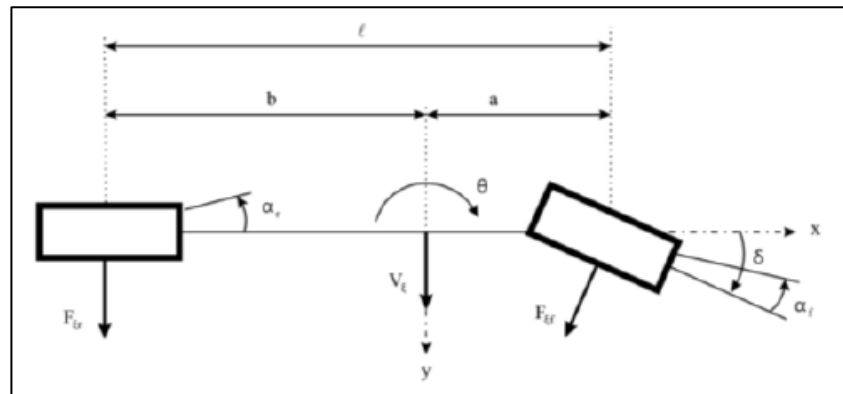
Figure 2.1- 2-DoF Bicycle model.

2.1.2 Quarter-car model

The quarter-car model is a model widely used above all for the analysis of suspensions and braking systems. This model assumes that the suspension systems and wheels of the vehicle can be considered equal and their effects multiplied by four. These models neglect the effects of vehicle pitch and roll.

In some quarter-car models, the effects of suspension are neglected and therefore only two degrees of freedom will be taken into account: wheel rotation and longitudinal displacement of the system. This type of model is widely used in the analysis of ABS systems.

Quarter-car models for suspension analysis neglect the degree of freedom associated with the longitudinal displacement of the system and add the vertical displacement of the wheel. Figure 2.2 shows a diagram of a quarter of a vehicle.
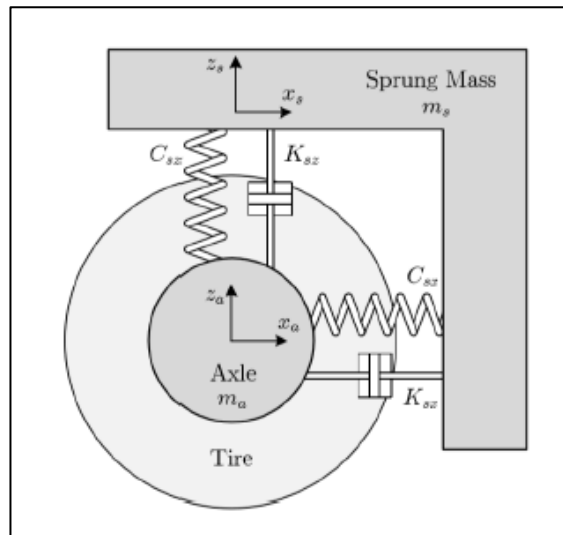
Figure 2.2- Quarter-car vehicle model.

2.1.3 Half-car model

Half-car models are also widely used for the analysis of vehicle suspensions. The only difference with the quarter car models is that they consider the pitching effect. In addition, they take into account the differences between the front and rear axles. When considering the pitching effect, the vertical load on the two axles will not be constant. For this reason, they are a good model for the analysis of braking systems as they take into account the load transfers due to longitudinal acceleration that occur between axles when performing a braking manoeuvre.
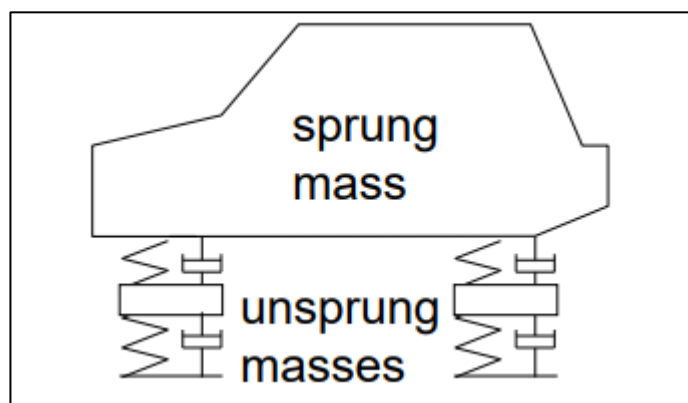
Figure 2.3- Half-car model

2.1.4 Full vehicle models

The complete vehicle models are used above all for a detailed analysis of the suspension of the vehicle. These models have 7 degrees of freedom: 3 degrees of freedom for the vertical movement of pitch and roll, and 4 degrees of freedom for the vertical movement of the wheels. This model does not imply lateral degrees of freedom.
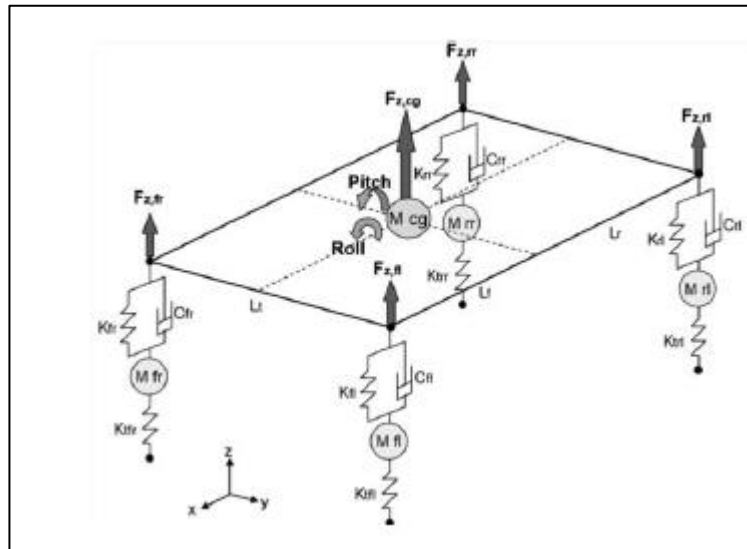
Figure 2.4- Full-car model.

2.2 BRAKING DYNAMICS

2.2.1 Slip ratio

The slip ratio is a parameter that measures the slip of each of the wheels of the vehicle. It is of great importance in the field of vehicle dynamics as it allows the relationship between wheel deformation and longitudinal force to be understood. It is a fundamental parameter within wheel anti-lock systems.

When a vehicle is accelerating the angular speed of the wheels it does not correspond to a speed applied in a pure rolling case and therefore the product of angular speed by the radius of the wheel will not be equal to the longitudinal speed of the wheel axle. This difference in speed expressed as a percentage shall be known as the longitudinal slip

coefficient. This coefficient plays an important role in the generation of longitudinal forces from the tyres and the evaluation of the tyre contact footprint.

According to the SAE (Society of Automotive Engineers), the longitudinal slip coefficient of a wheel is defined with the following equation:

$$\lambda = (\frac{\omega \times R}{V_x} - 1)$$

Where:

- λ is the slip ratio
- ω is the angular wheel velocity
- V_x is the longitudinal velocity of the vehicle
- R is the wheel radius

Figure 2.5 shows how the slip ratio affect the friction coefficient and how the road conditions modify the available maximum friction coefficient.
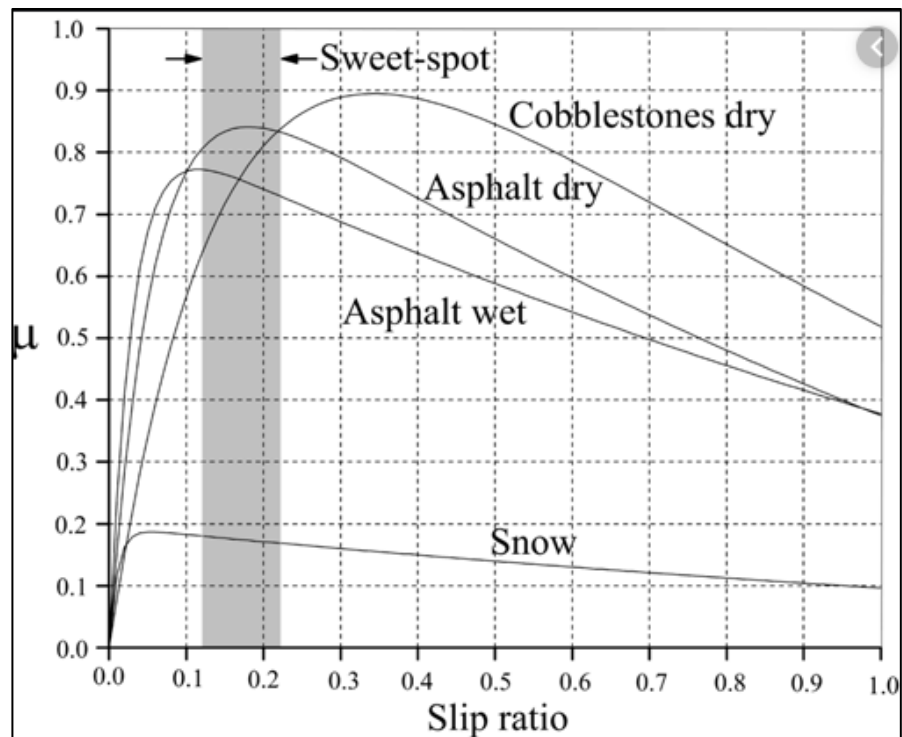
Figure 2.5- Mu-Slip curves for diferent roads.

3. CONTROL STRATEGIES

3.1 MODEL PREDICTIVE CONTROL (MPC)

Model predictive controllers (MPC) are a widely used solution in the world of control activities. In (Bhandari, Patil and Singh, 2012) a MPC applied to the ABS system of a vehicle is proposed. The following advantages of this type of controller can be highlighted:

- Formulation in the time domain, flexible, open and quite intuitive.
- Good performance in linear and non-linear systems.
- The control law responds to optimal criteria.
- Possibility of incorporating restrictions in the design.

Although it offers numerous advantages, this type of control also has drawbacks:

- It is necessary to have knowledge of the model on which you are going to work very precisely.
- It has a slow response capacity, which can cause problems in certain applications.

The problem of computational cost is currently not such a serious problem, as electronics is becoming faster and faster, and if a good design of the controller is made, it should not cause problems. Even so, the system needs a quick response, since it works in real time and quick actions will be necessary.

3.2 SLIDING MODE CONTROL

Sliding mode controllers have the ability to be an efficient tool for the complex, high-order control of non-linear dynamic plants operating under uncertain conditions, a common problem for many modern technology processes. This explains the high level of research and publication activity in the area and the interest of engineers in sliding mode controllers over the past two decades.

3.3 FUZZY LOGIC

Fuzzy logic represents a type of control based not so much on physical equations, but more on experimental results and the application of rules based on logic. Fuzzy logic is intended to serve as a solution for designing complex system controllers with greater simplicity than if they were designed with traditional techniques. To do so, it acts with logical rules, in a similar way to human thought. In this field of knowledge, there are no totally true or totally false assertions, there are only degrees of belonging to defined sets. It is observed that it is an eminently experimental procedure, because in order to design a controller, it is necessary to see the response of the system to the different inputs. Once we have the data, we proceed to create some rules for the system to work as we want.

The main advantages of this type of control are:

- Easy to understand, because it uses logical rules based on human language.
- Quick calculation. Logical conditions are easily implemented and take little time to be calculated. In addition, the output of the controller is done by adding areas and calculating the centres of gravity, operations that do not require much calculation time because the figures obtained are usually triangles or trapezoids.
- It allows mono-variable control as multivariable.
- It is based on experience. Other types of controllers present more restrictions when working with experimental data, fuzzy logic is based directly on them.
- It is compatible with other control systems, so we can use it as a complementary control.

However, it also has some disadvantages:

- If it is badly modelled, its structure is very prone to failure.
- If complex controllers are built it can be difficult to implement.

Looking at all the advantages, the main one is the speed of calculation and the simplicity of the control. Therefore, for a real time system and is used in some jobs for this type of applications like (Xiao, Hongqin and Jianzhen, 2016a).

3.4 PROPORTIONAL INTEGRAL DERIVATIVE CONTROL (PID)

Its operation is mainly based on evaluating the error between a previously calculated reference value and the actual value of the signal or signals to be monitored. Depending on this error, the controller output will be directed to the plant in order to reduce the error between the reference and the current value of the variable to be controlled.

This type of controllers combines their ease of implementation with their smooth operation in almost any type of application. These two characteristics make them a very valid solution for this job.

PID controllers have three modules, proportional control (P), integral control (I) and derivative control (D), each of these 3 controls has a specific function in stabilizing the system:

- Proportional control: This is the main control and its function is to create a response in the output proportional to the error.
- Integral Control: This part of the controler in charge of eliminating the residual error given by control P, reducing it to 0.
- Derivative Control: It is used to eliminate over oscillations generating a correction in the control signal proportional to the error.

PID controllers are very common for the design ABS systems in vehicles. In (Badie Sharkawy, 2010) this type of controllers are used.

4. PROPOSED MODEL

The proposed ABS system model consists of four parts. The first one is a mathematical model representing the vehicle braking system. This model has as input the force exerted by the driver on the brake pedal and the output is the braking torque applied to each of the wheels.

The second part of the model is the part that represents the longitudinal dynamics of the vehicle. It is a half vehicle model in which load transfer between the front and rear axles are taken into account and differences between wheels of the same axle are disregarded.

From this model, the angular speeds of the wheels and the braking distance of the vehicle are also obtained.

The third part is the tyre model. The use of Look-up has been chosen based on experimental data of the tyre for this task because of the simplicity and the good results obtained from its use. This technique is also used in (Pradeep Rohilla *et al.*, 2016).

The last part is in charge of the control system. For this part, it has been decided to implement two different control strategies in order to subsequently compare the performance of each of them. The first strategy implemented has been a Proportional Integral Derivative Controller (PID) like the one which is implemented in (Pradeep Rohilla *et al.*, 2016). The second strategy implemented has been a fuzzy logic controller like in (Xiao, Hongqin and Jianzhen, 2016b).

The following describes the process of creating both the vehicle model and the two control strategies within the Matlab/Simulink software.

4.1 BRAKELINES SYSTEM

The first system created has been the one that represents the dynamics that takes place within the braking system from the moment the driver applies force to the brake pedal until the wheels receive the corresponding braking torque. The type of brakes mounted on the actual vehicle to be simulated is disc brakes with fixed brake callipers as shown in Figure 4.1.
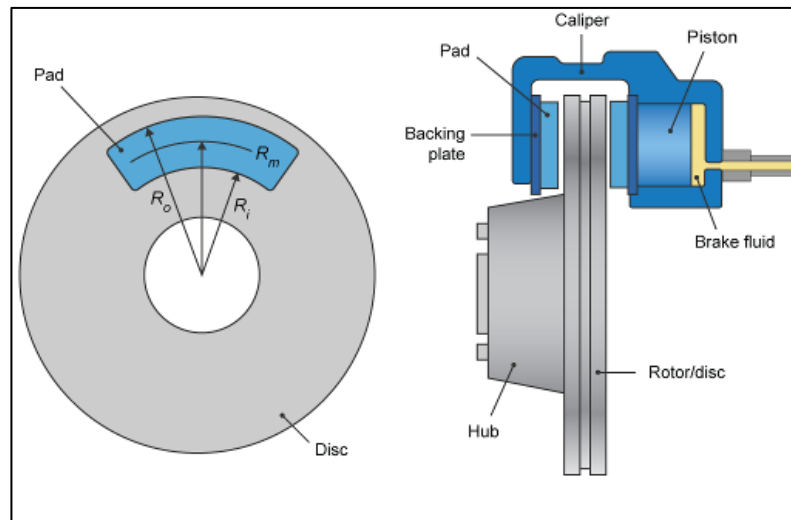
Figure 4.1- Brake disc and calliper.

The equations used to obtain the braking torque on the wheel are the following:

$$P = \frac{F_d \cdot R}{A_m}$$
(4.1)

$$T_b = 2 \cdot \mu_b \cdot P \cdot A_c \cdot r_{eff} \cdot n$$
(4.2)

Where:

- T_b is the braking torque

- μ_b is the friction coefficient of the pads

- P is the pressure in the line

- r_{eff} is the effective radius of the brake disc

- n is the number of pistons of the clamp on one side

- F_d is the driver force on the brake pedal

- R is the brake pedal ratio

- A_m is the master cylinder area

- A_c is the clamp piston area

Within the Matlab/Simulink software, a block has been created in which the input is the position of the brake pedal. This input is created as a step block as figure 4.2 shows. This input ranges from 0 to 1, where 1 is the value at which the driver exerts the maximum force on the pedal and 0 when no force is exerted. Within the created block the equations 4.1 and 4.2 have been implemented by gain blocks and the final result is shown in figure 4.3 and 4.4.
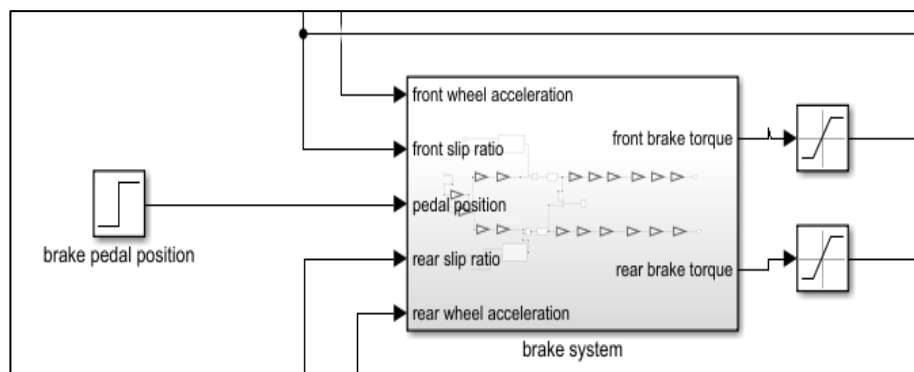
Figure 4.2- Brake lines block.

The brake lines block has two different channels inside. One for the front axle and another one for the rear axle. This is because this configuration is applied on the real prototype. A saturation block has been added in order to keep the braking torque calculated always positive. Otherwise, it would be a traction torque.
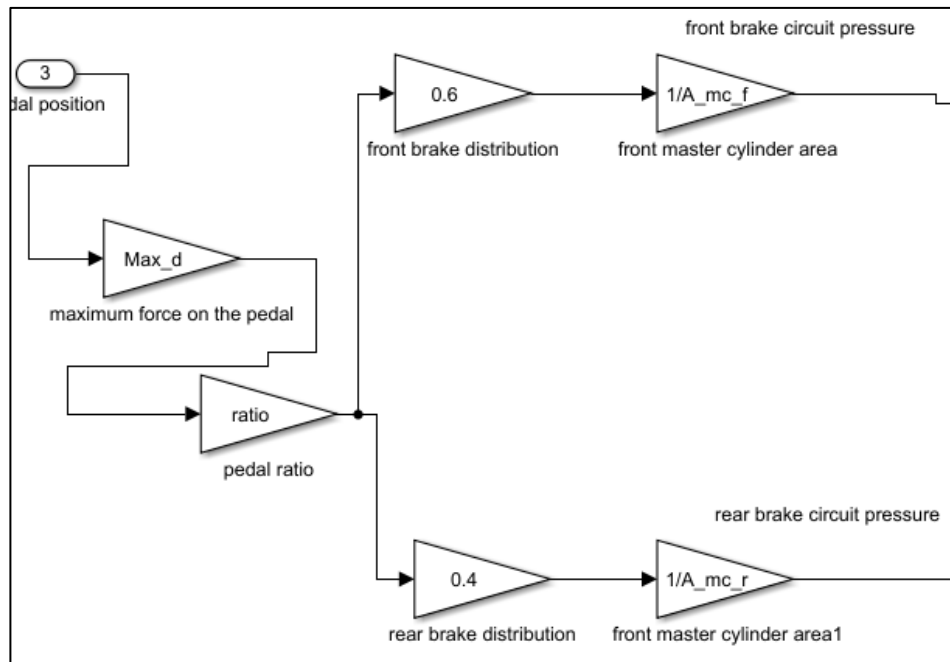
Figure 4.3- Equation 4.1 in Simulink.

Two gain blocks have been added to set pressure distribution between the front and rear axle. A pressure distribution of 60% to the front axle and 40% to the rear axle has been set.
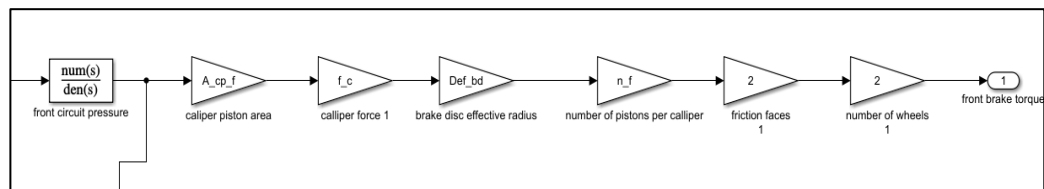
Figure 4.4- Equation 4.2 for the front axle in Simulink.

Within the brake lines there is a delay time in which the pressure travels from the master cylinder to the brake calliper cylinder. This delay can be represented by a first-order filter in the following form:

$$G = \frac{1}{\tau s + 1}$$

(4.3)

Where:

- τ is the retard in seconds

It has been supposed that the retard is 0.15 seconds as (Pedro and Ferro, 2014).

4.2 VEHICLE MODEL

The implemented vehicle model has been a half-car model like the one implemented in (Soliman, Kaldas and Mahmoud, 2009). In contrast to the quarter-vehicle model, this model allows to take into account the effects of load transfer between axles that occur during braking events. It is also necessary to implement a model that represents the forces on each of the wheels of the vehicle. As a half-car model is implemented, the forces on one of the wheels of each axle have been calculated and multiplied by 2 to simplify the model, since the differences between wheels of the same axle have been neglected. Figure 4.5 shows a scheme of the half vehicle model while figure 4.6 shows the forces acting on one of the wheels of the vehicle.
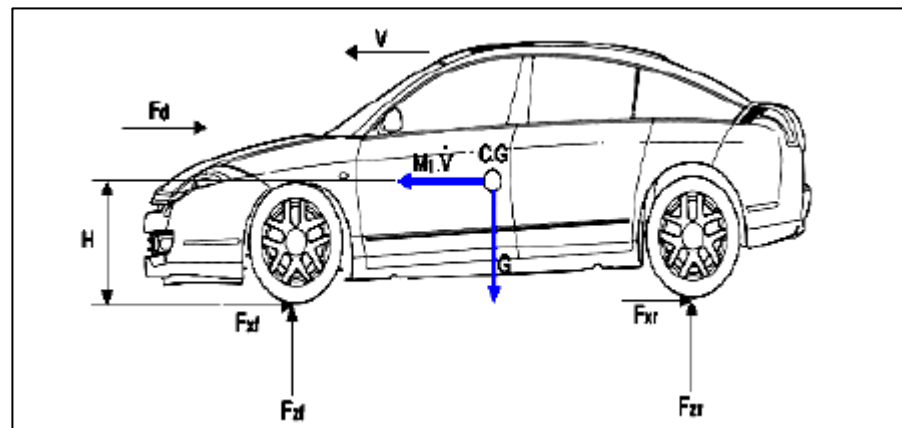
Figure 4.5- Half-car model
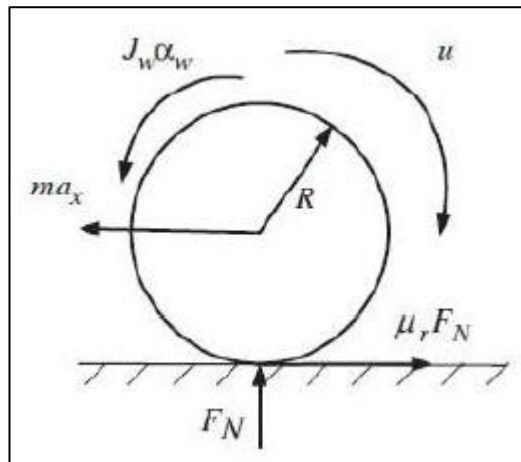
Figure 4.6- Wheel forces

The equations obtained from figures 4.5 and 4.6 are the following:

$$F_{N,f,r} = F_{Z,f,r} \pm \frac{m \cdot a_x \cdot H}{B} \qquad (4.4)$$

$$m \cdot a_x = \mu_r \cdot F_N \qquad (4.5)$$

$$J_W \cdot \omega_w = -T_b \cdot R \cdot F_N \cdot \mu_r \qquad (4.6)$$

Where:

- $F_{N,f,r}$ is the dynamic vertical load on the front or rear axle

- $F_{Z,f,r}$ is the static vertical load on the front or rear axle

- m is the mass of the vehicle

- a_x is the longitudinal acceleration of the vehicle

- H is the height of the centre of gravity of the vehicle

- B is the distance between axles

- μ_r is the friction coefficient between the tyre and the surface

- J_W is the moment of inertia of the wheel

- α_w is the angular acceleration of the wheel

- R is the wheel radius

Within Simulink a specific block to calculate de dynamic braking force has been created and shown in figure 4.7. The input of the block is the longitudinal acceleration of the vehicle while the output is the absolute vertical load on the selected axle.
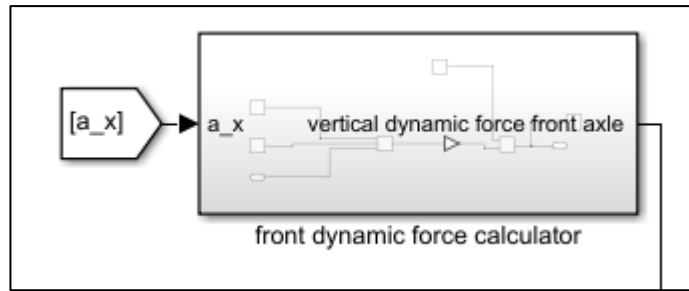
Figure 4.7- Vertical load calculator block.

Inside this block the equation 4.4 has been implemented as shown in figure 4.8.
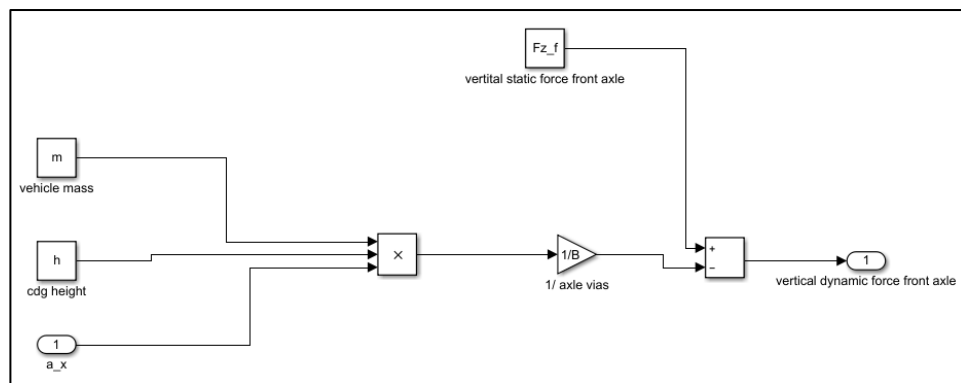
Figure 4.8- Equation 4.4 in Simulink for the front axle.

Using integrator blocks in Simulink is very useful to obtain certain variables from equations. In figure 4.9 it can be seen how the stopping time and the brake distance of the vehicle can be easily obtained from equation 4.5.
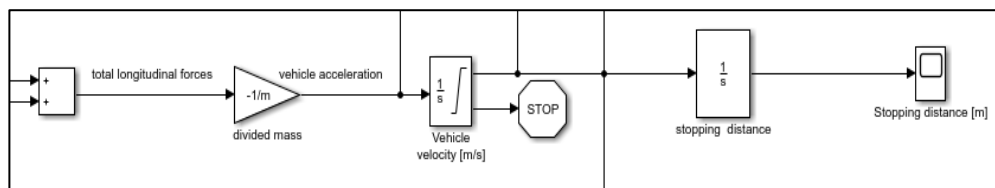
Figure 4.9- Use of integrator blocks.

4.3 TYRE MODEL

For the implementation of the previous models it is necessary to know the interaction between the tyre and the surface. In particular, it is necessary to know the friction coefficient that exists between the two materials. This friction coefficient is variable and depends fundamentally on the state of the surface and on the slip ratio of the wheel at each moment. There are numerous models capable of calculating the interaction of the tyre with the asphalt at any moment, but they are very complicated with numerous equations and parameters.

There is a method based on a Look-up tables to obtain the friction coefficient value of the tyre with the asphalt. Look-up tables are dynamic tables that assign an output value for each input value to the table. As the values of the Slip-Mu curves of the tyres used in the model are known, it is the easiest option to implement and there is no need to use a complex tyre model. The tyre data have been obtained from(*Milliken Research Associates, Inc. -- FSAE Tire Test Consortium*).

Two types of data have been loaded into the Simulink block. One type of data corresponds to an asphalt and echo road while another type corresponds to a wet road. In figure 4.10 it can be seen the Slip-mu friction curve obtained from the dry asphalt tyre data. The output of the block is the friction coefficient while the input of the block is the slip ratio of the wheel.
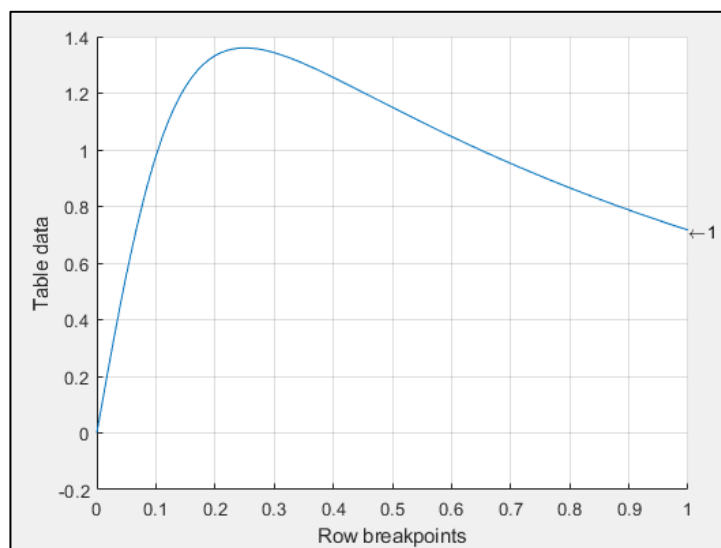
Figure 4.10- Slip-Mu curve for dry asphalt.

4.4 CONTROL STRATEGIES

4.4.1 PID controller

The first control strategy to be implemented is a PID controller. The general structure of this type of controller is shown in figure 4.11.
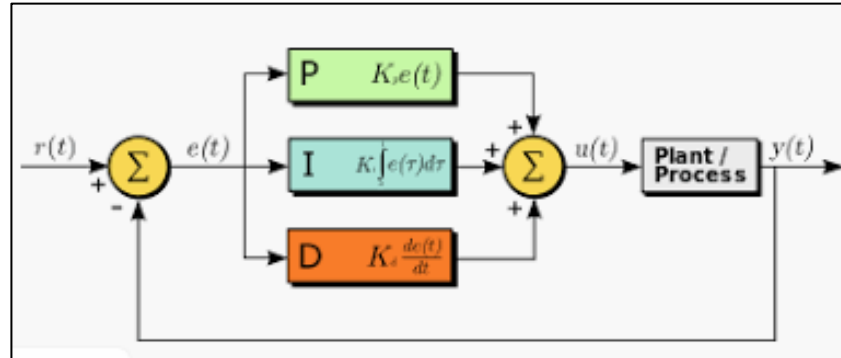
Figure 4.11- PID structure

Two different controllers have been created for each axis. The operation of each controller is to feed back the control variable, in this case the slip ratio of the wheels, and compare it to a desired constant value. The reference value has been set to 0.25 as it corresponds to the region of the curve in which the tyre has the highest friction coefficient. The controller compares the current slip ratio with the reference value and calculates the absolute error. This error is processed by the PID controller, which calculates the value of the control action. In this case the control action corresponds to the pressure that needs to be released or increased in the braking circuit to reduce or increase the braking torque of the wheel. The general equation for PID controller is:

$$u = K_P \cdot e + K_i \int e \cdot dt + K_d \frac{d}{dt} e \qquad (4.7)$$

Where:

- u is the control action
- e is the error
- K_P, K_i, K_d are the proportional, integral and derivative constants

Within the Matlab/Simulink software there is a toolbox that allows to include the equation 4.7 within a single block. This block will receive the error as input and will return the control action based on the constants defined within the block. Within the block in

charge of generating the control action there will also be included a 'switch' block whose function will be to disable the controller when the slip ratio value is below 0.25 since at that moment the pressure in the braking system will have to be regulated by the driver of the vehicle. A gain block has been added for convenience to enable and disable ABS control easily by changing the value from 1 (enabled) to 0 (disabled). The complete scheme can be seen in figure 4.12.
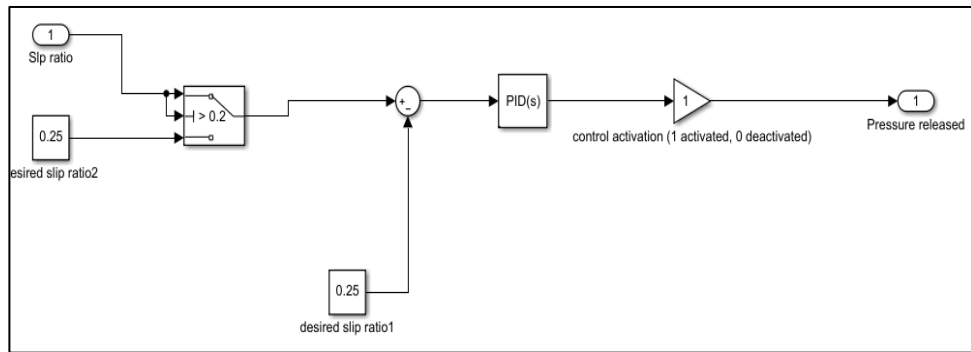
Figure 4.12- PID controller block in Simulink

To choose the constants used within the PID block, a tool provided by Simulink has been used to adjust these parameters according to the desired plant response to a single step input. The response of the front axle controller is shown in figure 4.14. By adjusting the desired robustness and speed level for the controller, the program automatically calculates the three constant values. In figure 4.13 it can be seen the chosen response to a unitary step for the front axle ABS system.
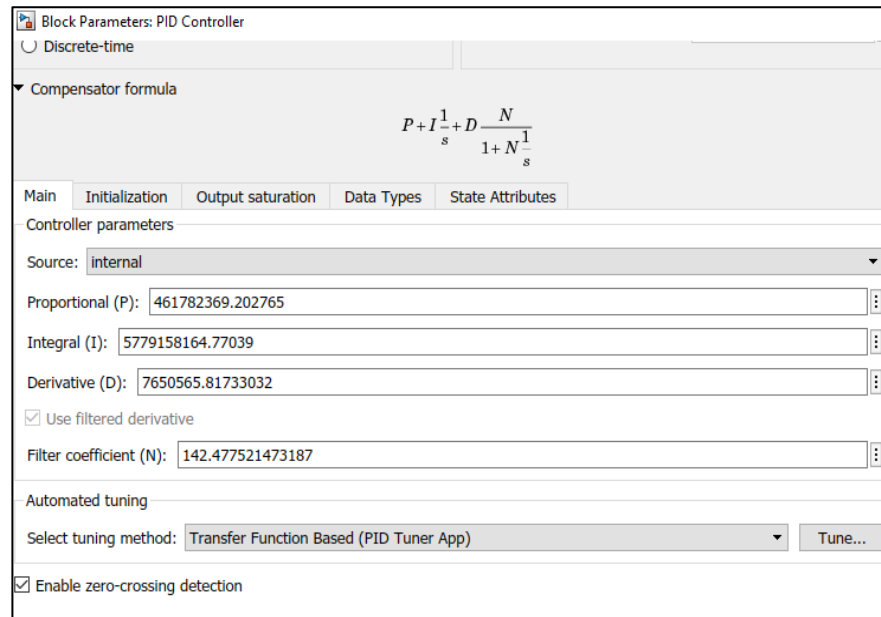
Figure 4.13- PID block in Simulink.
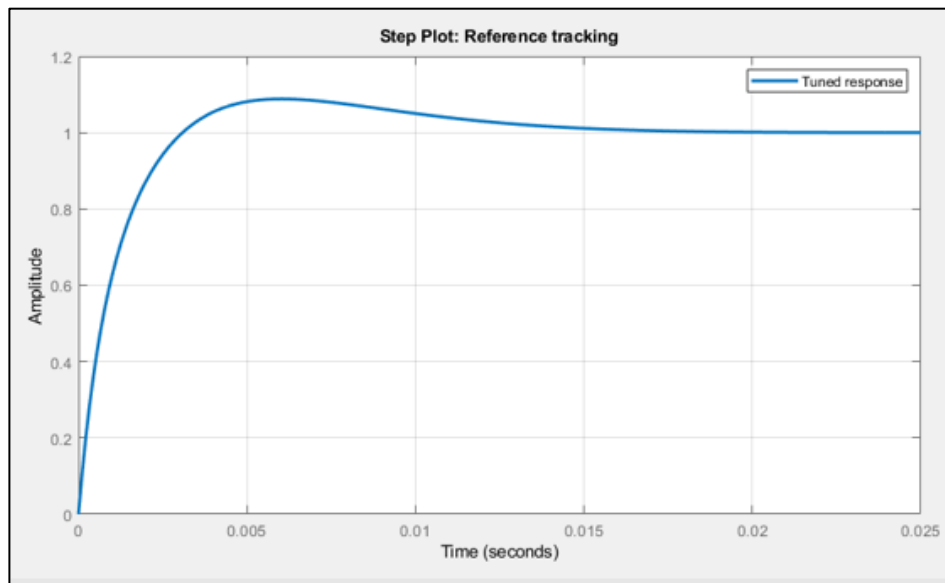
Figure 4.14- Step response of the front axle controller.

Los valores escogidos para las constantes del eje delantero y trasero se recogen en la tabla 4.1.

	Front axle	Rear axle
Proportional constant	461782369	535535670
Integral constant	5779158164	4993770285
Derivative constant	7650565	7209843

Table 4.1- Constant values of the PID controllers.

4.4.2   Fuzzy logic controller

The second controller implemented was a fuzzy logic controller. To do this, the base of the previous model has been used, although some modifications have been necessary, which will be explained below. Unlike a PID controller, a fuzzy logic controller can evaluate more than one input variable. The input variables chosen were the slip ratio error of the wheel and the angular acceleration of the wheel. The reason why the angular acceleration of the wheel has been chosen is that this variable is able to detect the previous moments a wheel lock when it registers very high values. It can also indicate that an increase in pressure is necessary when it registers positive values and when the wheel is starting to accelerate. The output variable is again the positive and negative increase in pressure necessary to adjust the braking torque and to prevent the wheel from locking.

For the implementation of the controller it is necessary to create some membership functions. The functions used have been be Gauss type functions since a smoother and progressive response is obtained from them. For this it will be necessary to delimit all the variables within a range and divide the range into subsets.

With the 'Fuzzy Logic Designer' tool in Matlab the task described above can be easily make in a very simple way. First, both input and output variables were created. The general schematic of the controller within the Fuzzy Logic Designer is shown in figure 4.15.

The first input variable corresponds to the slip ratio error. This variable has been dimensioned for a range from -1 to 1. The subdivision of the variable range into subsets is shown in table 4.2. The resulting membership functions can be seen in figure 4.16.

Figure 4.15- Fuzzy logic controller structure.

Figure 4.16- Membership functions of the slip ratio error.

Slip error ranges	Name

[-1 -0.3]	Negative Big (NB)
[-0.3 -0.01]	Negative Small (NS)
[-0.01 0.01]	Zero (Z)
[0.01 0.3]	Positive Small (PS)
[0.3 1]	Positive Big (PB)

Table 4.2- Slip ratio error ranges.

The second input variable is the angular acceleration of the wheel. This variable has been dimensioned between -1000 and 1000 rad/s^2. The division of the subsets is shown in table 4.3, while the membership functions are shown in figure 4.17.
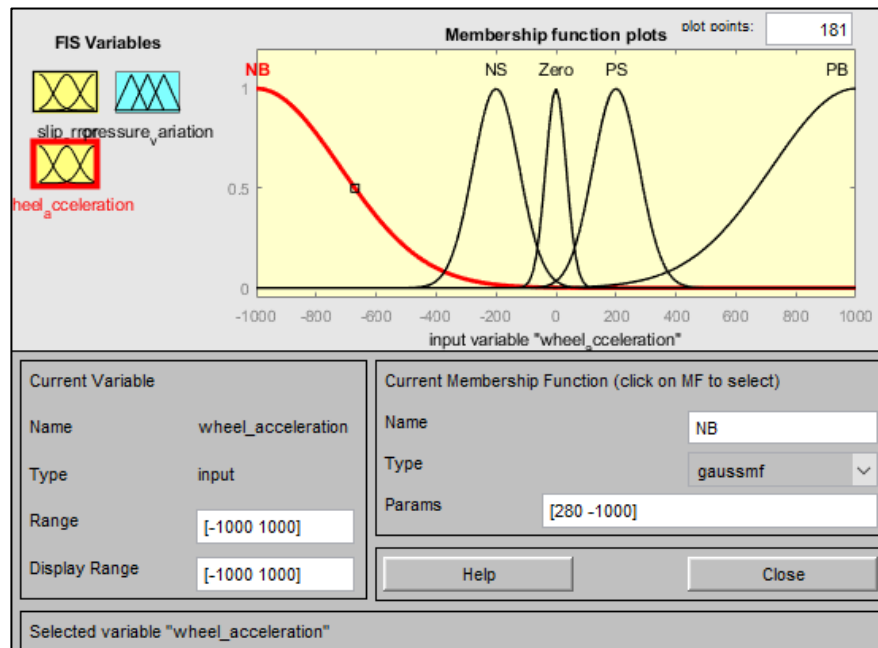
Figure 4.17- Membership functions for the wheel acceleration variable.

Wheel acceleration ranges (rad/s^2)	Name
[-1000 -300]	Negative Big (NB)
[-300 -50]	Negative Small (NS)
[-50 50]	Zero (Z)
[50 300]	Positive Small (PS)
[300 1000]	Positive Big (PB)

Table 4.3- Ranges for the wheel acceleration variable.

Finally, the output variable chosen was the pressure difference necessary in the circuit to keep the braking torque at optimum values. This variable has been divided into the subsets indicated in table 4.4 and its membership functions are shown in figure 4.18.
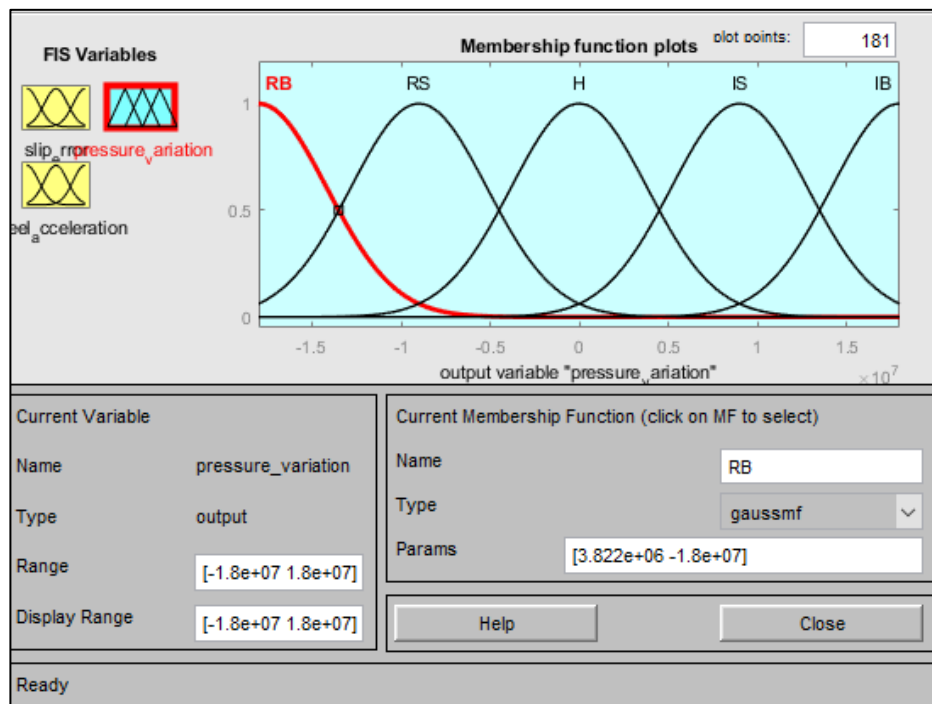
Figure 4.18- Membership functions for the output variable.

Pressure variation (N/m)	Name
[-1.8e+07 -0.9e+07]	Release pressure big (RB)
[-0.9e+07 -1e+03]	Release pressure small (RS)
[-1e+03 1e+03]	Hold pressure (H)
[1e+03 0.9e+07]	Increase pressure small (IS)
[0.9e+07 1.8e+07]	Increase pressure big (PB)

Table 4.4- Ranges for the pressure variation variable.

Once all the variables and their ranges have been defined, it will be necessary to create the logical rules that will determine the value of the output variable. For each value of the two input variables at a certain moment the controller will assign a pressure variation in the circuit according to the rules described in table 4.5.

PRESSURE VARIATION		WHEEL ACCELERATION				
		NB	NS	Z	PS	PB
SLIP RATIO ERROR	NB	H	H	IS	IS	IB
	NS	H	H	IS	IS	IB
	Z	H	H	IS	IS	IS
	PS	RS	RS	RS	IS	IS
	PB	RB	RB	RS	IS	IS

Table 4.5- Fuzzy logic rules for the output variable.

5. RESULTS AND DISCUSSION

In order to evaluate the performance of the different models implemented, a series some simulations have been carried out. Some parameters have been modified during the simulations in order to carry out a sensitive analysis. For the two proposed controllers, three different simulations were carried out. The simulated event in all the models was a straight-line braking event circulating at a certain initial velocity. Two braking simulations were performed at the initial speeds of 80 and 100 km/h on a dry surface and a third event at an initial speed of 80 km/h on a wet surface. These same manoeuvres have also been carried out without the intervention of any controller in order to be able to compare the effect of using the ABS system.

The variables monitored in each simulation were braking distance, braking time and slip ratio of both axles. Also, a comparison between the angular velocity of the wheel and the longitudinal velocity of the vehicle was made in each case. The variation of the pressure inside each circuit has been measured.

5.1 RESULTS WITHOUT ABS

5.1.1 80 km/h dry asphalt

Figure 5.1 shows the evolution of the distance covered by the vehicle during the braking manoeuvre on dry asphalt at an initial speed of 80 km/h.
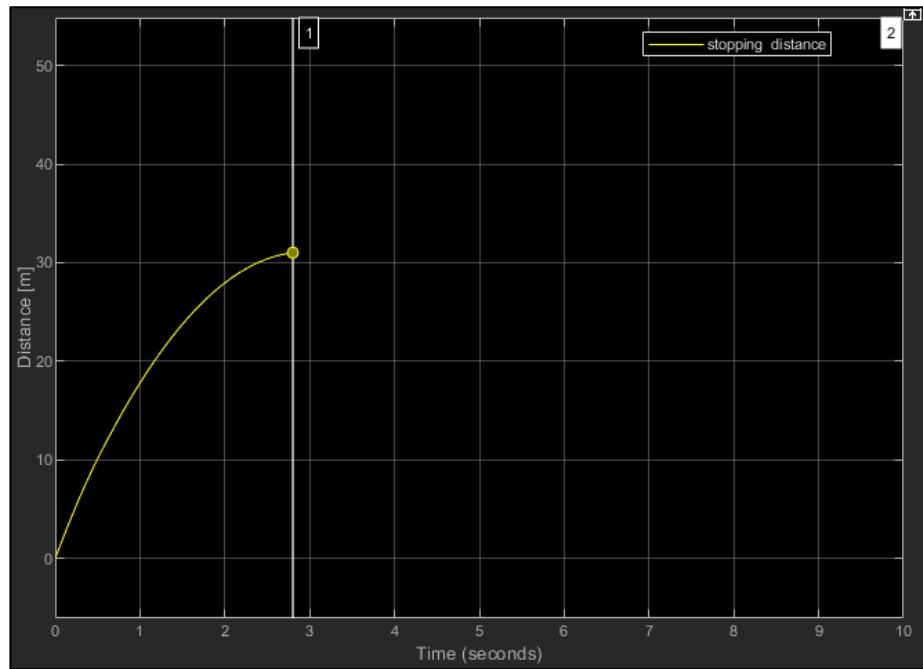
Figure 5.1- Stopping distance without ABS at 80 km/h on dry asphalt.

The vehicle brakes progressively until it stops in a time of 2.8 seconds in which it travelled 31 meters.

The slip ratio values for both the front and rear axles recorded during the manoeuvre are shown in figures 5.1 and 5.2 below.
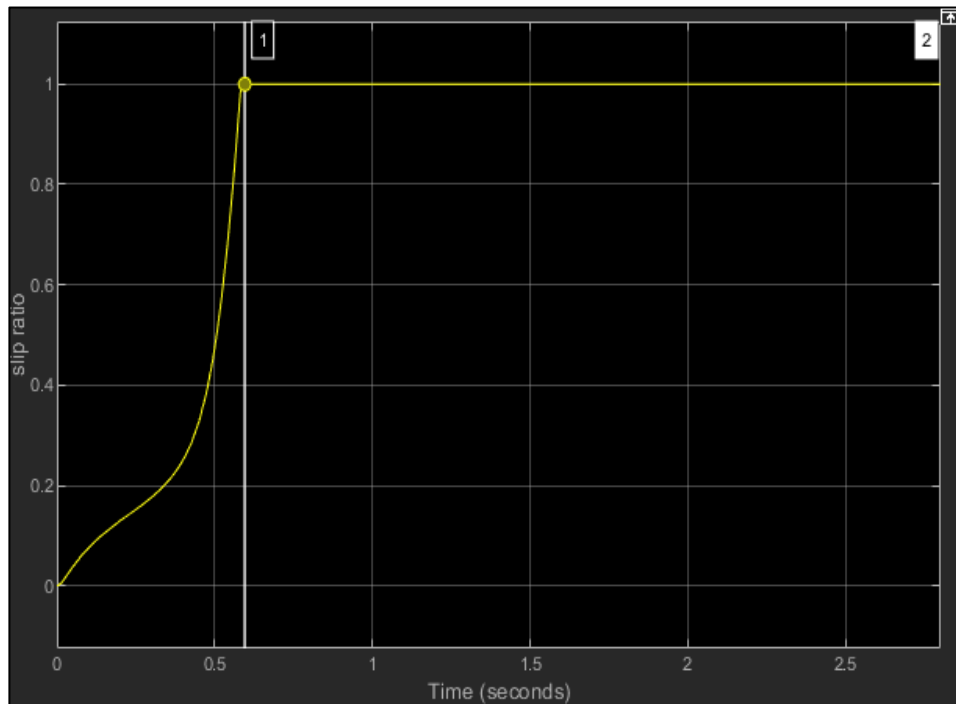
Figure 5.2- Front slip ratio without ABS at 80 km/h on dry asphalt.
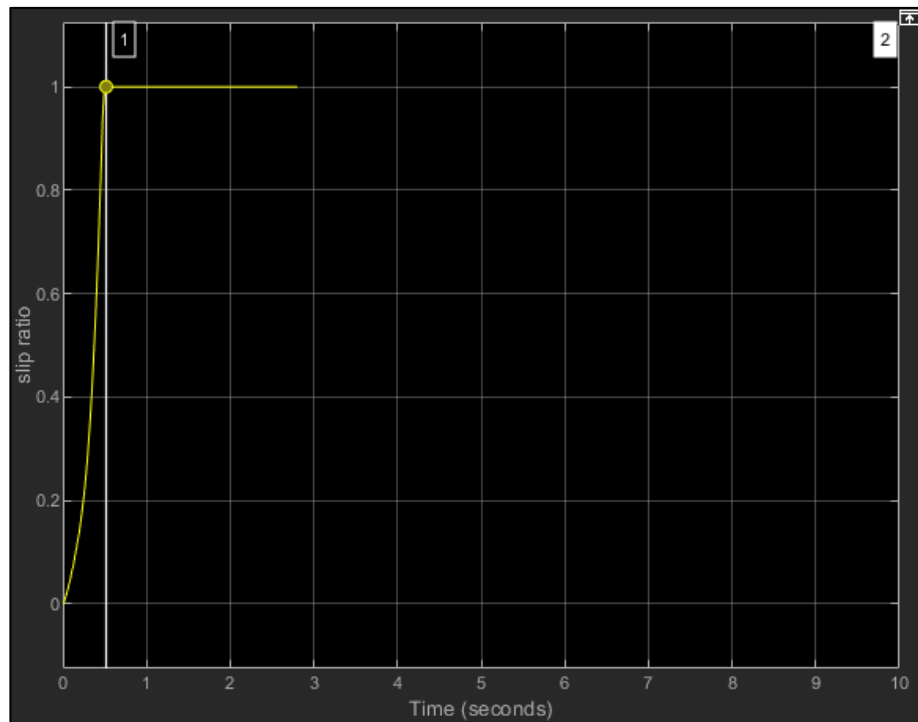
Figure 5.3- Rear slip ratio without ABS at 80 km/h on dry asphalt.

In the previous figures it can be seen that a value of 1 for the slip ratio is quickly reached and therefore the wheel locks when reaching that value. The wheels lock in a matter of tenths of a second because too much pressure is being introduced into the brake circuit and therefore excessive braking torque is applied. The front axle wheels lock after 0.595 seconds while the rear axle wheels lock after 0.517 seconds.

Figure 5.4 shows a comparison between the vehicle speed and the angular speed of the vehicle front wheels. It is a very interesting graph because it shows how the vehicle continues moving while the speed of the wheels is null and therefore, they are blocked. This situation increases the braking distance as well as cause directional loss of the vehicle. The vehicle circulates approximately 2.2 seconds with the wheels locked.

Figure 5.4- Comparison between vehicle and wheel velocities without ABS at 80 km/h on dry asphalt.

5.1.2 80 km/h wet asphalt

Figure 5.5 shows the evolution of the distance covered by the vehicle during the braking manoeuvre on dry asphalt at an initial speed of 80 km/h.
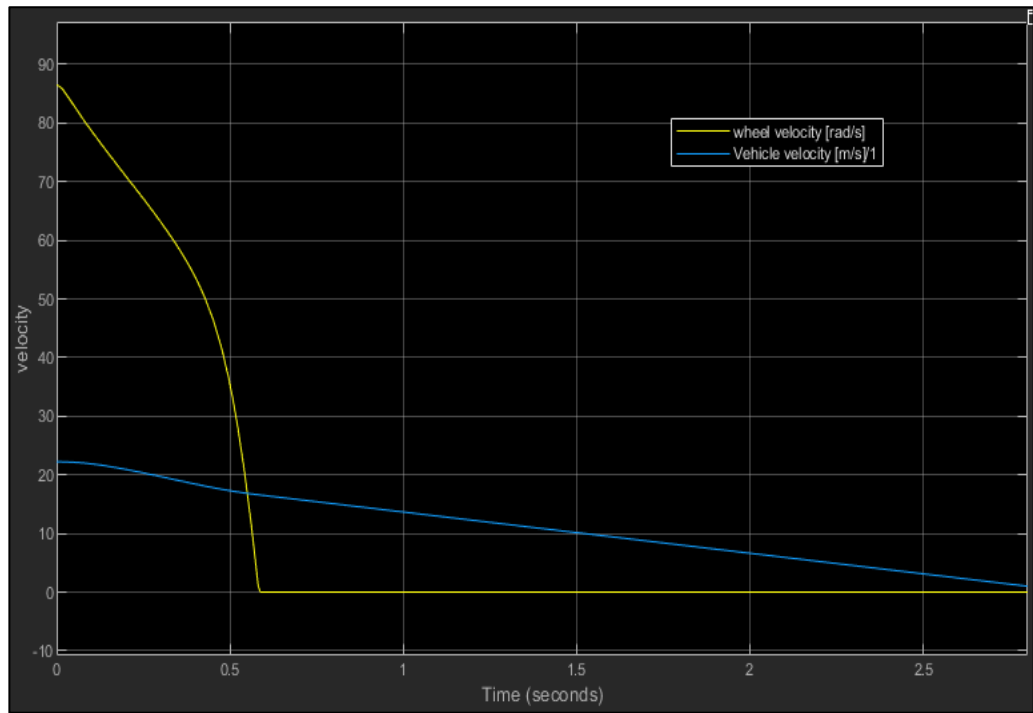

Figure 5.5- Stopping distance without ABS at 80 km/h on wet asphalt.

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The vehicle brakes progressively until it stops in a time of 6.23 seconds in which it travelled a distance of 71.4 meters. It can be seen that the difference with the distance and braking time recorded during the same event but with dry asphalt is quite large. The braking distance has been increased by 39.6 m and the braking time by 3.43 seconds. From this first graph it can already be concluded that the state of the road is a fundamental parameter during the braking event.

Figure 5.6 shows how the vehicle circulates most of the time with the wheels locked. When the asphalt is wet, the wheels lock even earlier than when the asphalt is dry.
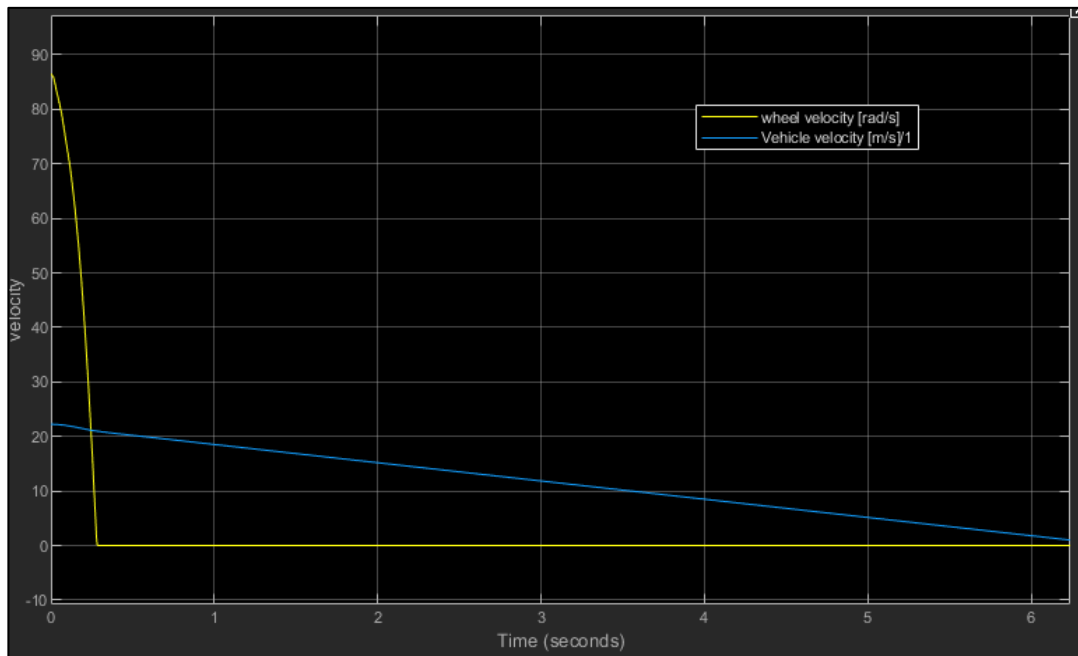
Figure 5.6- Comparison between vehicle and wheel velocities without ABS at 80 km/h on wet asphalt.

5.1.3   100 km/h dry asphalt

Figure 5.7 shows the evolution of the distance covered by the vehicle during the braking manoeuvre on dry asphalt at an initial speed of 100 km/h.
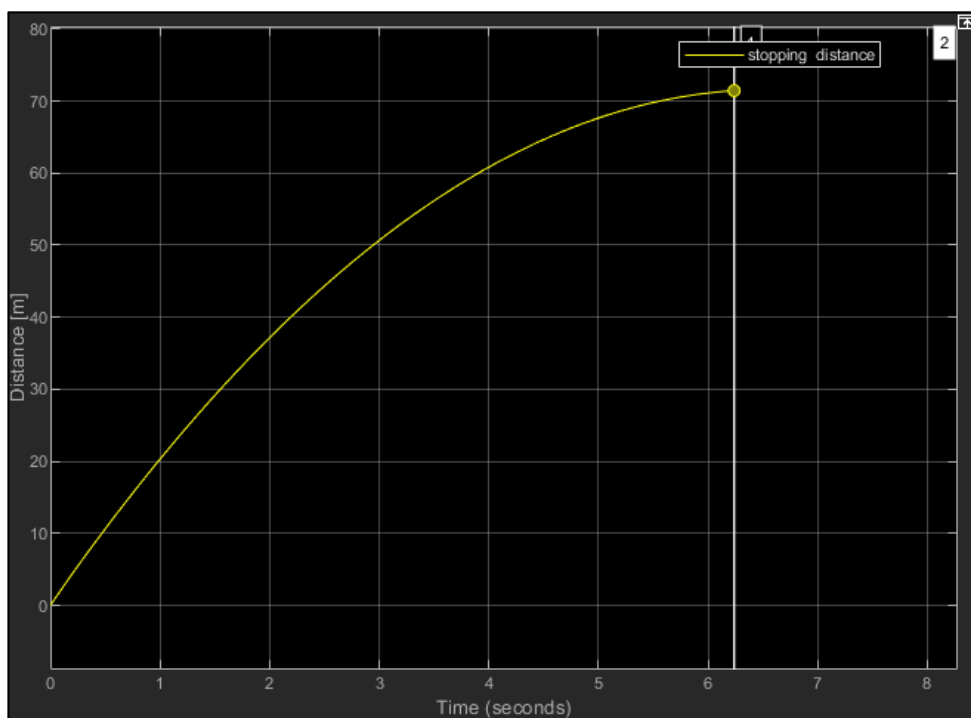
Figure 5.7- Stopping distance without ABS at 100 km/h on dry asphalt.

It can be noticed how the vehicle takes 3.55 seconds to stop at 48.7 meters. The time and braking distance are greater than in the simulation with an initial speed of 80 km/h, so it can be deduced that the initial speed is a parameter that influences the time of a sudden braking event.

In the 5.8 and 5.9 figures it can be seen the evolution of the slip ratios of the front and rear axle wheels. It takes 0.67 seconds for the front axle wheels to lock and therefore reach the maximum slip ratio. The rear axle wheels take 0.556 seconds. These times are slightly higher than those obtained during the event at 80 km/h although the difference is very small.
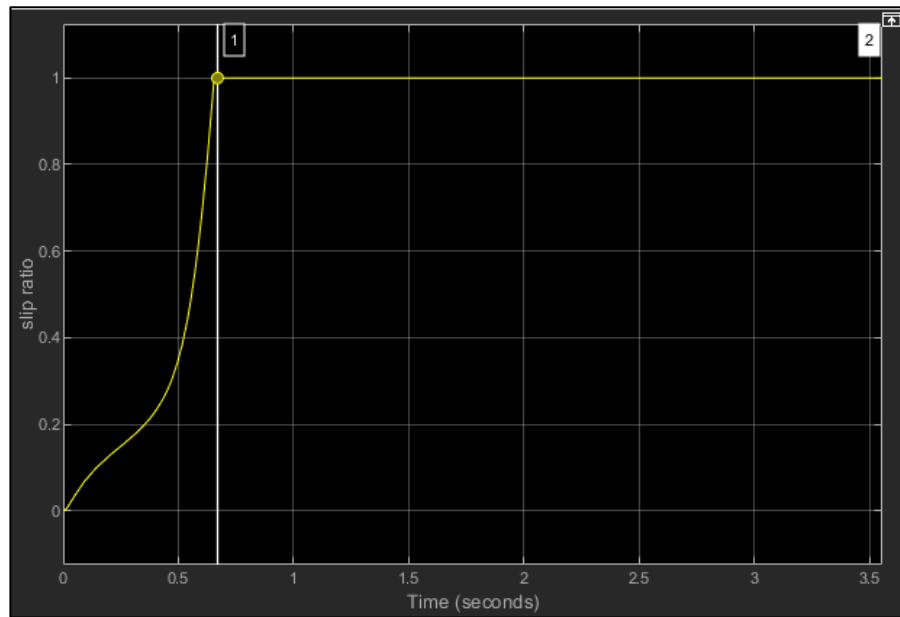
Figure 5.8- Front slip ratio without ABS at 80 km/h on wet asphalt.
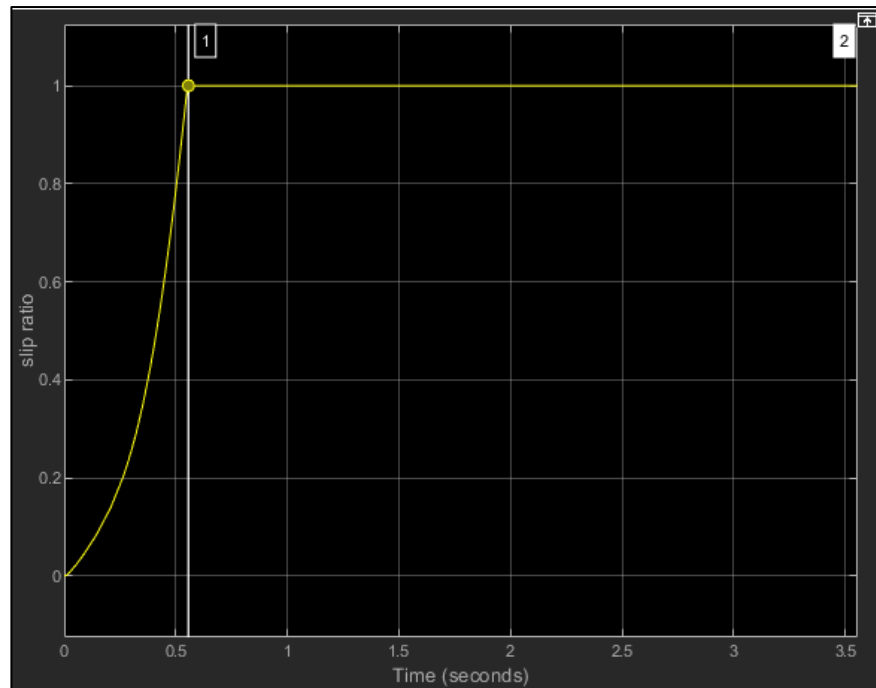
Figure 5.9- Rear slip ratio without ABS at 80 km/h on wet asphalt.

As in the previous cases, figure 5.10 shows the angular speed of the front wheels and the longitudinal speed of the vehicle. Again, as there is no ABS actuation, the vehicle runs most of the time with the wheels locked.
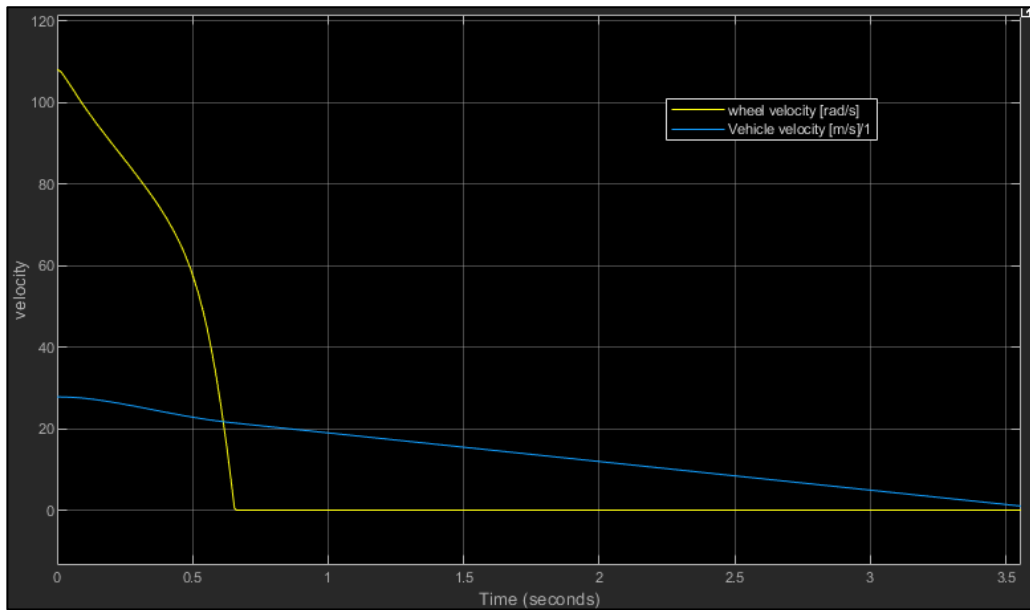
Figure 5.10- Comparison between vehicle and wheel velocities without ABS at 100 km/h on dry asphalt.

5.2 RESULTS WITH ABS (PID CONTROLLER)

The results shown below correspond to the response of the vehicle when the ABS system is actuating. In this case, the control strategy used is a PID controller.

5.2.1 80 km/h dry asphalt

Figure 5.11 shows the evolution of the distance travelled by the vehicle during the braking manoeuvre on dry asphalt at an initial speed of 80 km/h in which the ABS system operates.
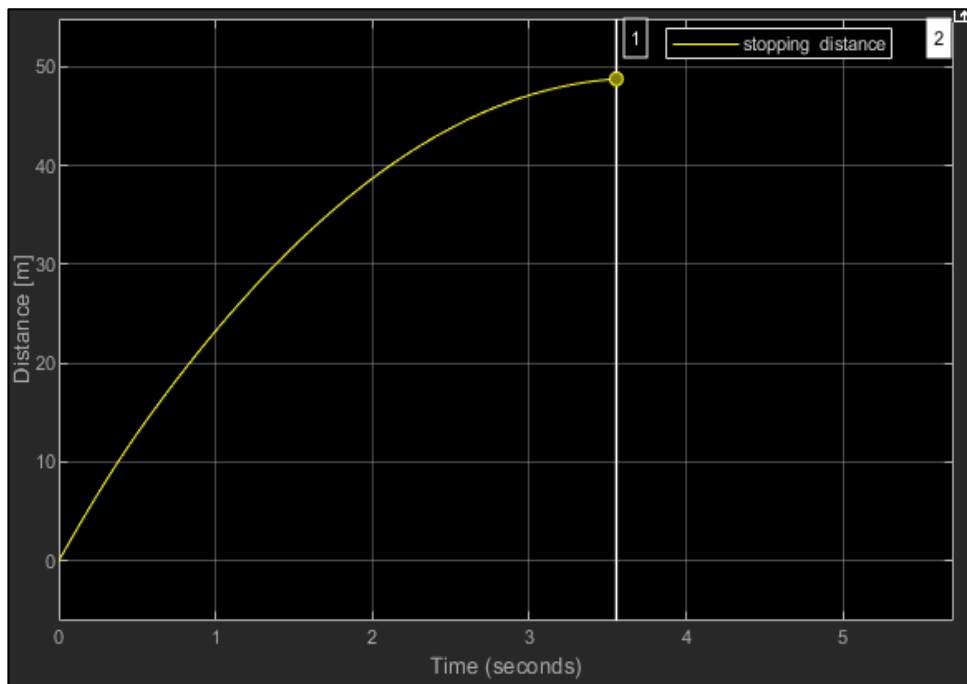
Figure 5.11- Stopping distance with PID ABS at 80 km/h on dry asphalt.

The graph shows that it took the vehicle 1.7 seconds to stop at which it travelled 20.93 meters. Both the braking time and the braking distance are much shorter than without the ABS system. This is due to the fact that the wheels do not lock at any time during the manoeuvre, as can be seen in figure 5.12.
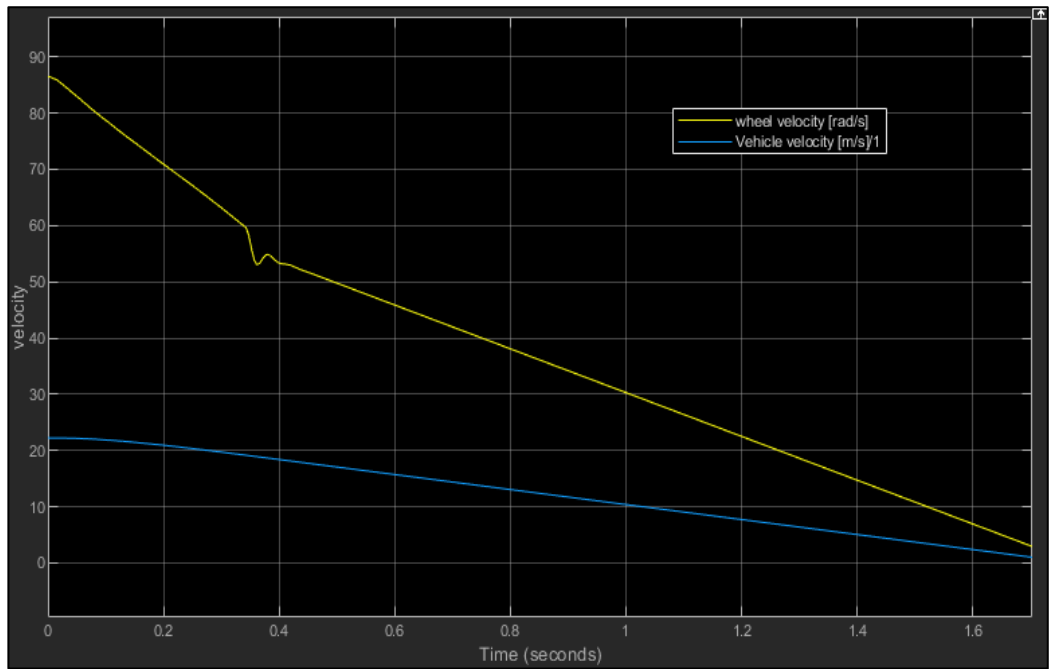
Figure 5.12- Comparison between vehicle and wheel velocities with PID ABS at 80 km/h on dry asphalt.

Figure 5.13 shows the slip ratio value of the two axes. It shows how the controller is able to maintain the slip ratio value around 0.25, which is the desired slip ratio value and when the tyre has a higher coefficient of friction with the asphalt. It is also observed that the rear axle reaches before the front axle the stationary state.

Figure 5.13- Slip ratios with PID ABS at 80 km/h on dry asphalt.

Figure 5.14 shows the evolution of the pressure within the front and rear axle circuits. The braking distribution is set at 60% for the front axle and 40% for the rear axle so the front axle will register the higher of the two. The graph shows how there are pressure variations generated by the controller to modify the final braking torque and reduce it if necessary, so that the wheels do not lock.
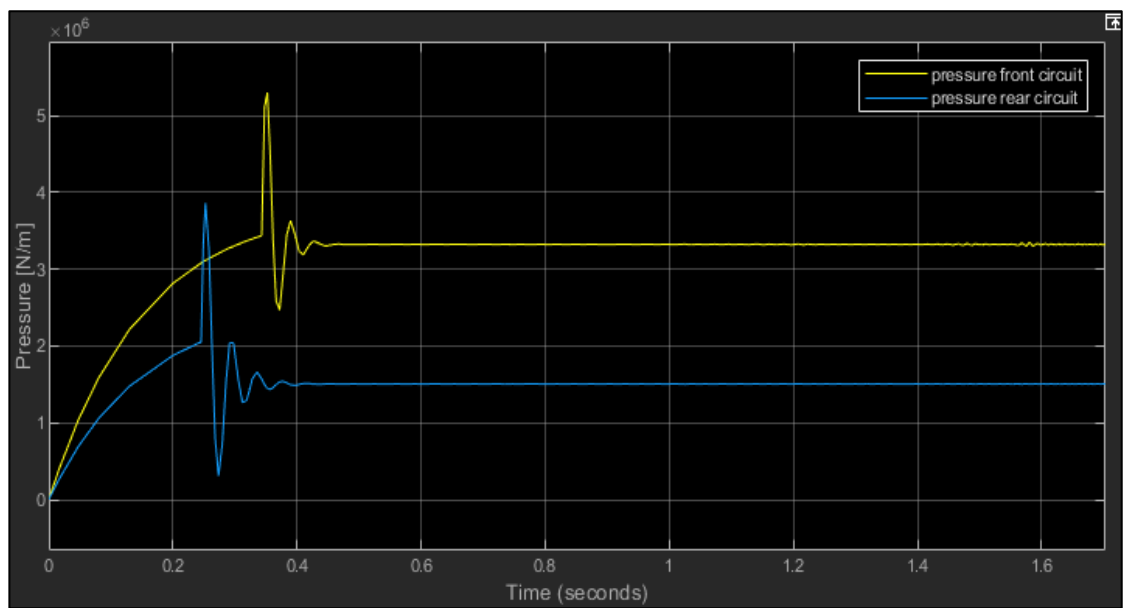
Figure 5.14- Pressure variation with PID ABS at 80km/h on dry asphalt.

5.2.2 80 km/h wet asphalt

Figure 5.15 shows the evolution of the distance travelled by the vehicle during the braking event on wet asphalt at an initial speed of 80 km/h in which the ABS system acts.
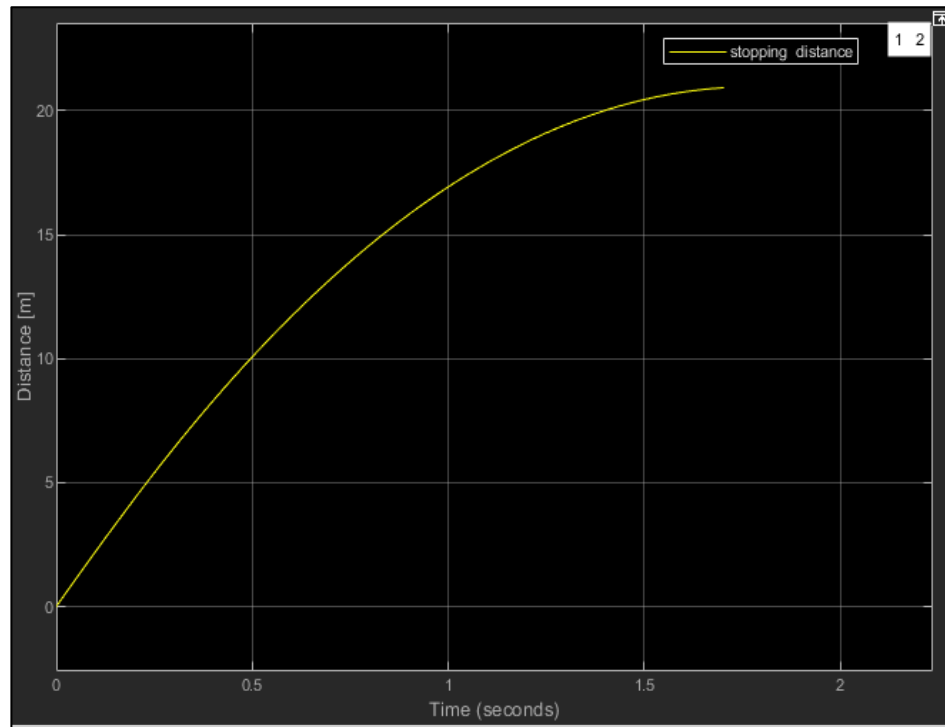
Figure 5.15- Stopping distance with PID ABS at 80 km/h on wet asphalt.

The graph shows that the vehicle took 3.47 seconds to stop at which it travelled 41 meters. The braking time and distance are longer than on dry asphalt, although with the ABS system it is possible to reduce these values substantially.
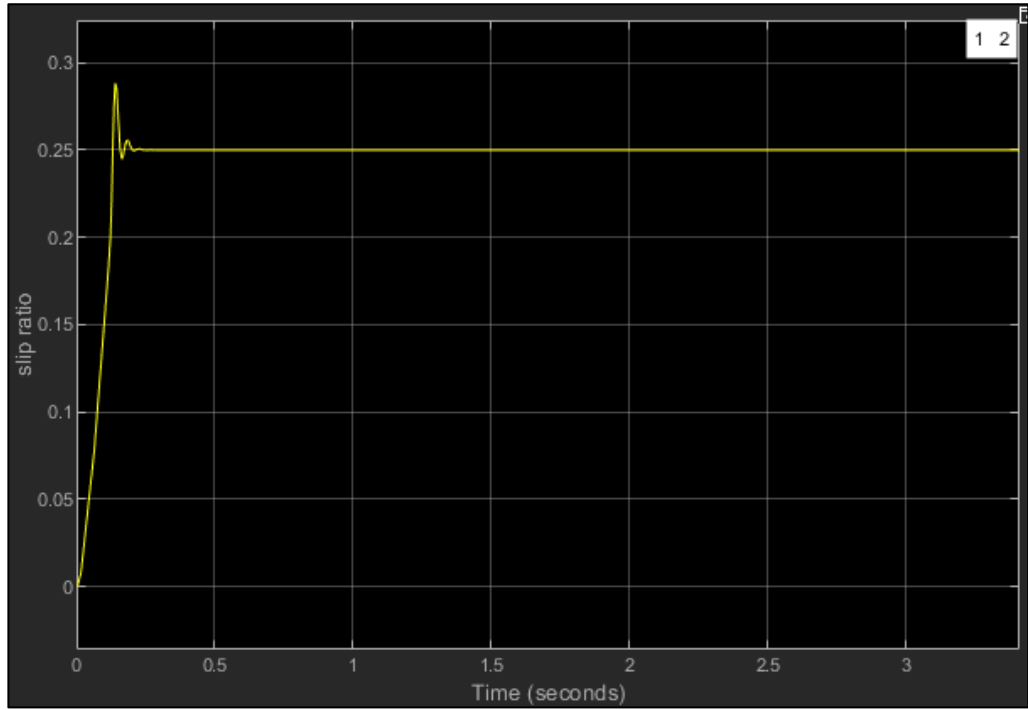
Figure 5.16- Front Slip ratio with PID ABS at 80 km/h on wet asphalt.
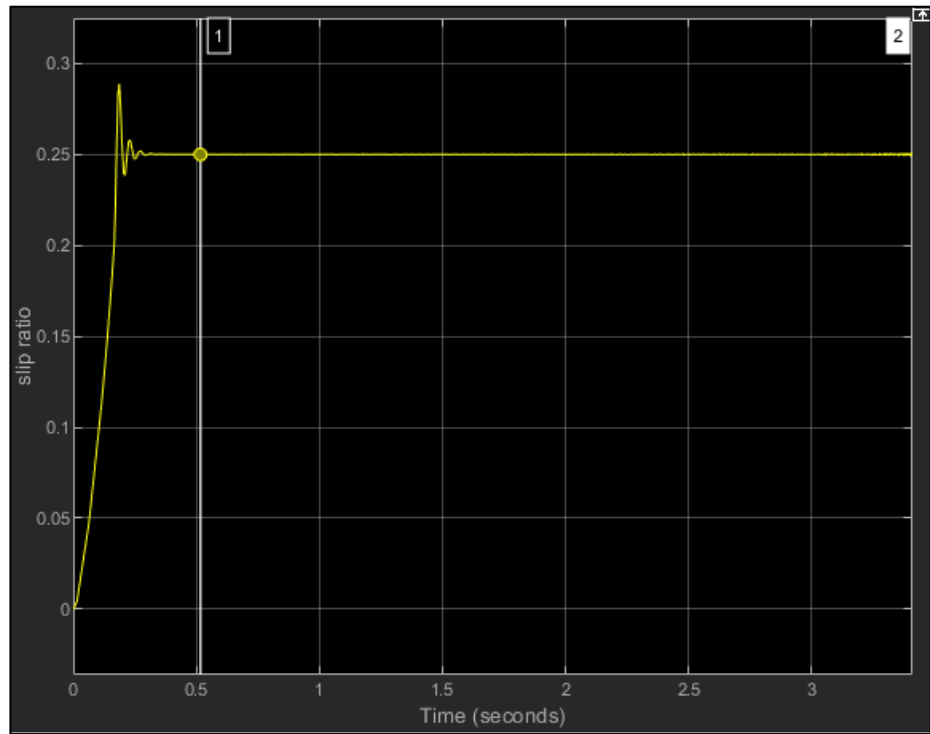
Figure 5.17- Rear Slip ratio with PID ABS at 80 km/h on wet asphalt.
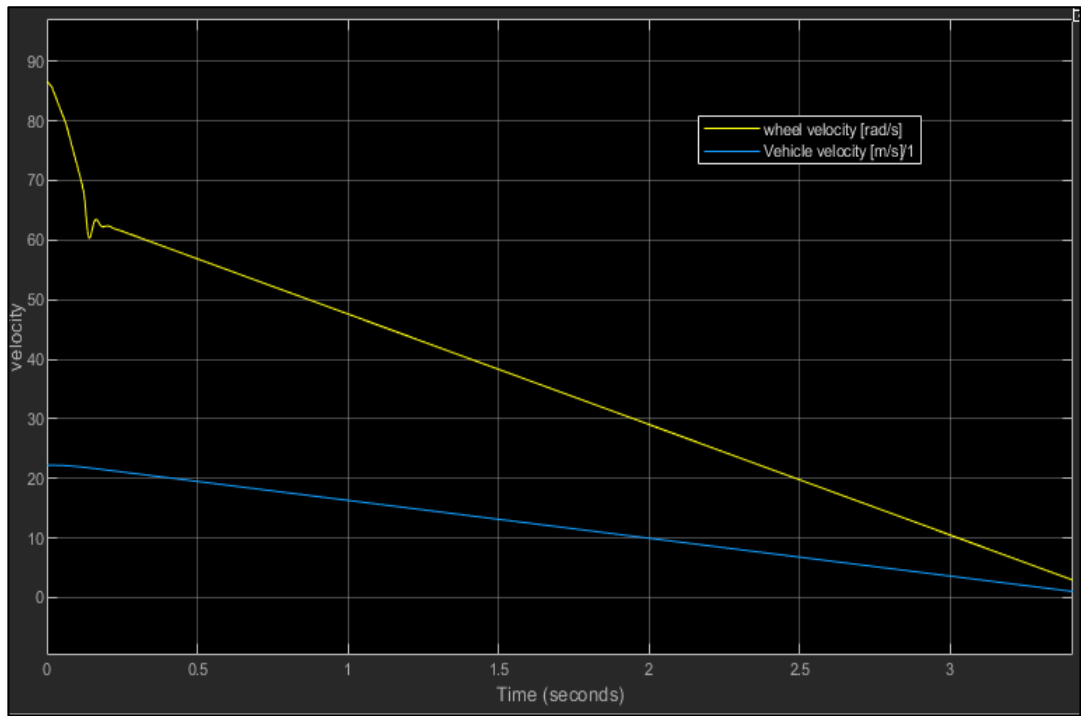
Figure 5.18- Comparison between vehicle and wheel velocities with PID ABS at 80 km/h on wet asphalt.
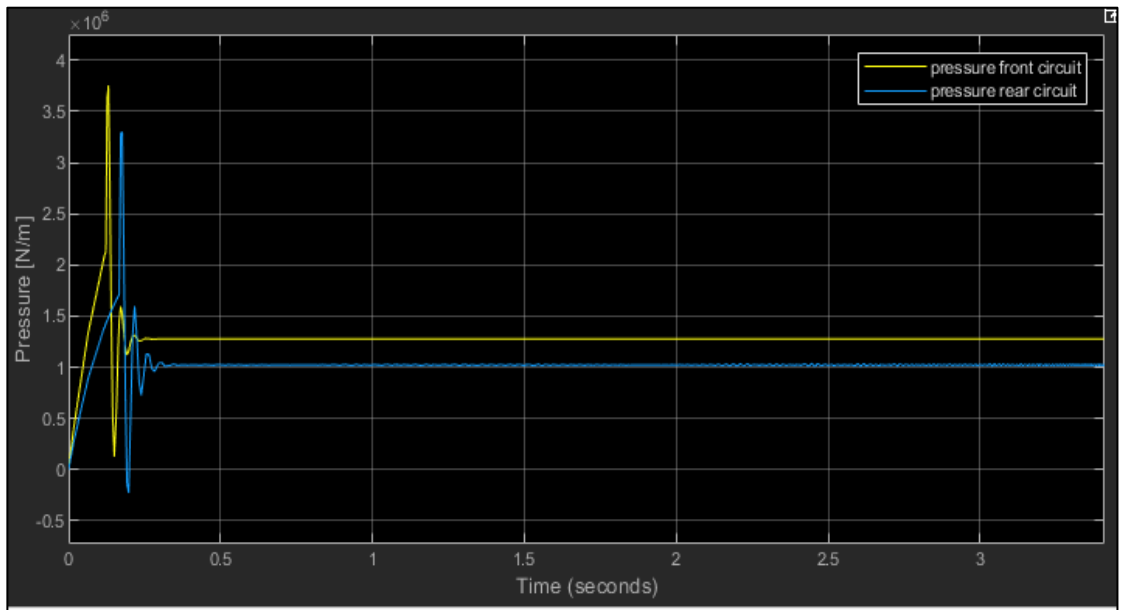
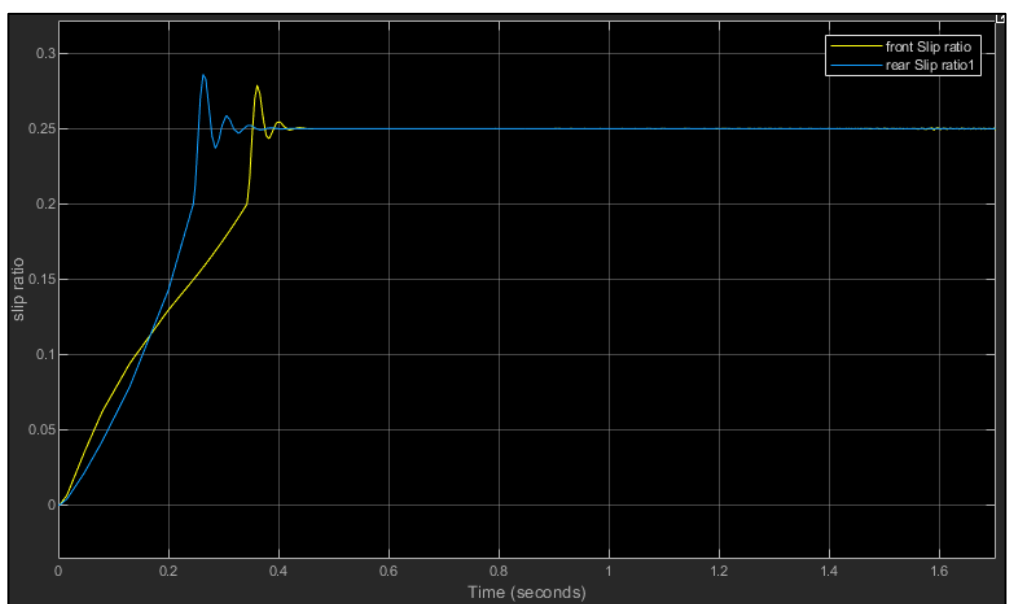*Figure 5.19- Pressure variation with PID ABS at 80km/h on wet asphalt.*

Figure 5.19 shows the evolution of the pressure within the front and rear axle circuits. The graph shows how there are pressure variations generated by the controller. The

number of oscillations is greater in this case because the wet road makes it more difficult to reach the optimum pressure value.

5.2.3 100 km/h dry asphalt

Figure 5.20 shows the evolution of the distance travelled by the vehicle during the braking event on dry asphalt at an initial speed of 100 km/h in which the ABS system operates.
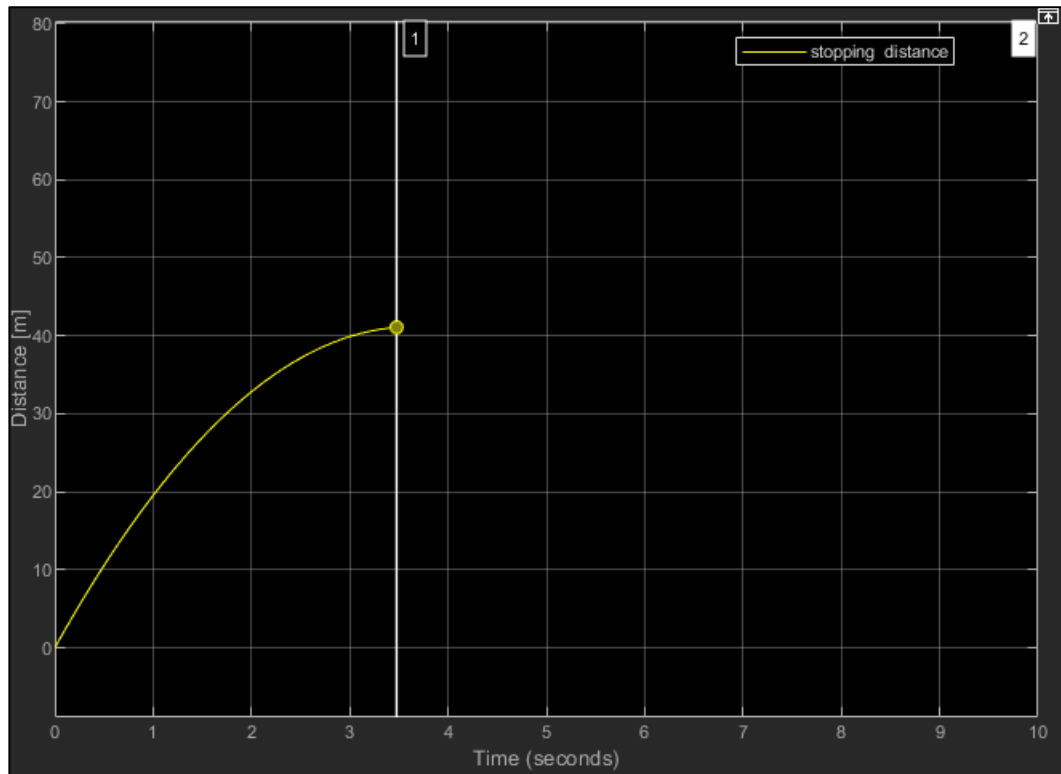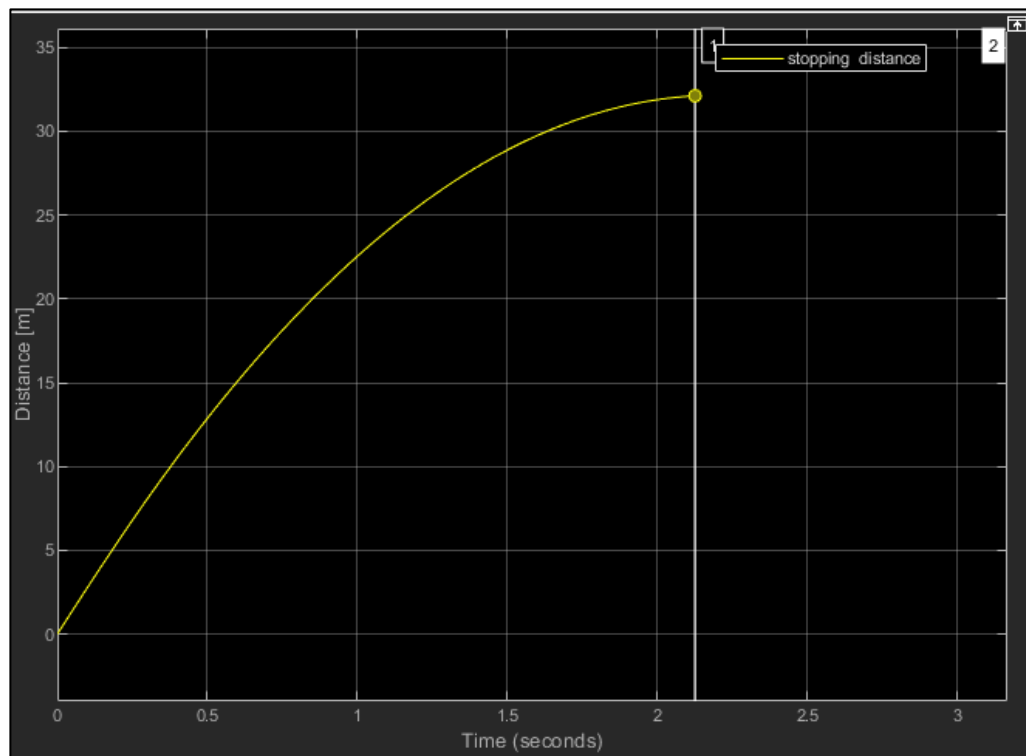

Figure 5.20- Stopping distance with PID ABS at 100 km/h on dry asphalt.

The graph shows that the vehicle took 2.12 seconds to stop at the 32.11 meters. The braking time and distance are longer than with dry asphalt, although with the action of the ABS system these values are substantially reduced.
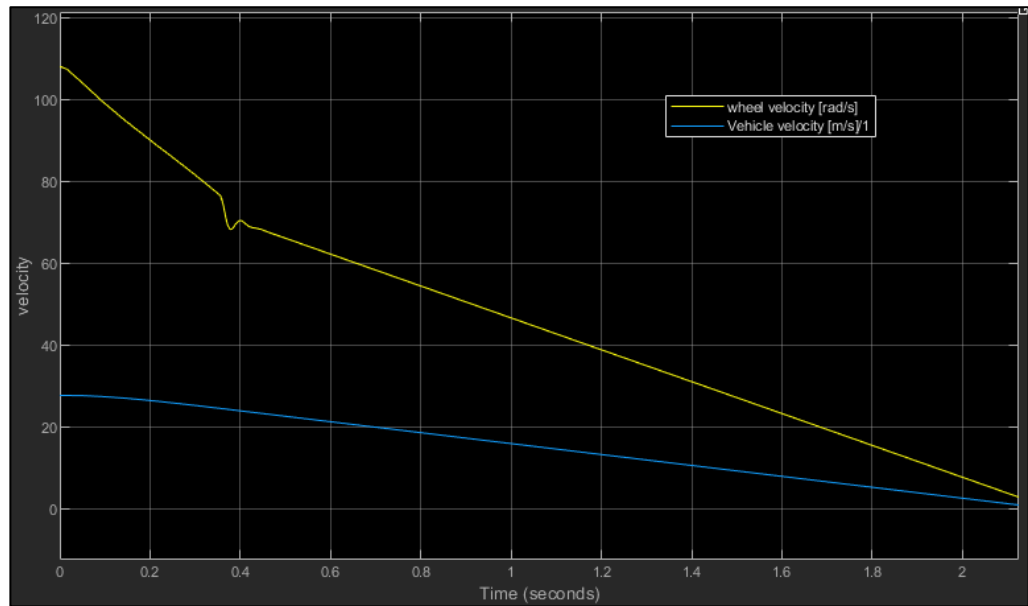
Figure 5.21- Comparison between vehicle and wheel velocities with PID ABS at 100 km/h on dry asphalt.
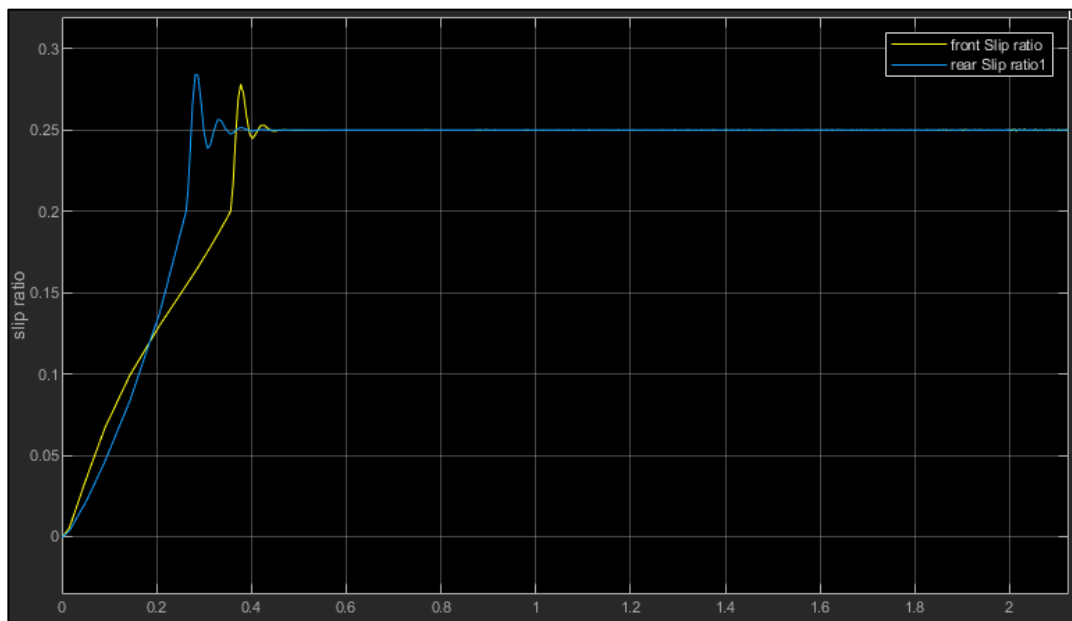
Figure 5.22- Slip ratios with Fuzzy Logic ABS at 80 km/h on dry asphalt.

Figure 5.22 shows the slip ratios values of the two axes. It shows how the controller is able to maintain the slip ratio value around 0.25, which is the desired slip ratio value and when the tyre has a higher coefficient of friction with the asphalt. It is also observed that the rear axle reaches before the stationary regime. The settling times are slightly higher than in the manoeuvre at 80 km/h.

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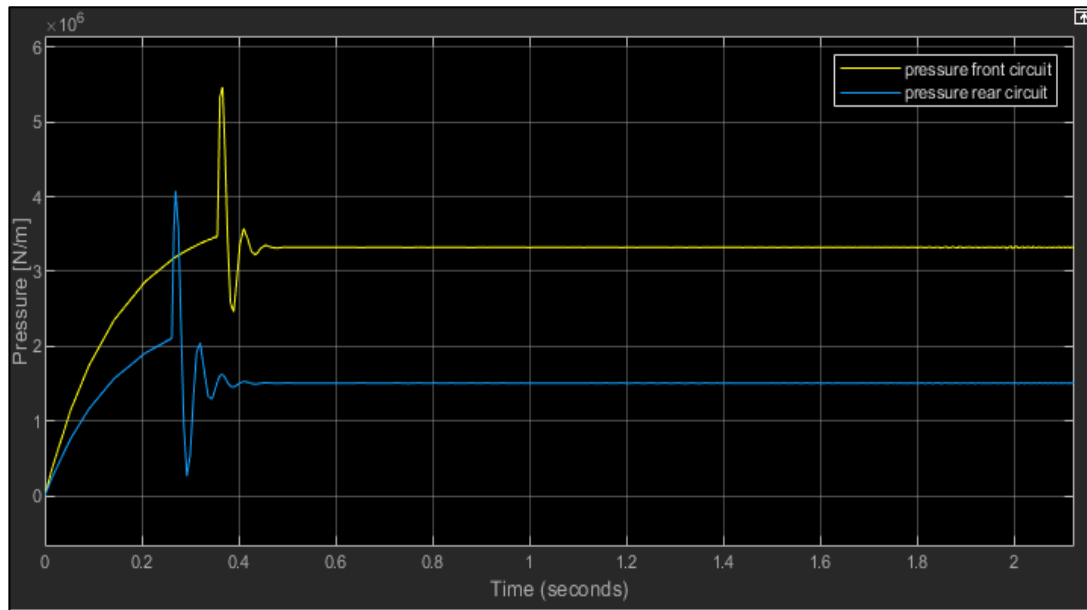
Figure 5.23- Pressure variation with PID ABS at 100km/h on dry asphalt.

Figure 5.23 shows again the evolution of the pressure in the two circuits. Once again it is observed how the system modifies the pressure until reaching an optimum constant value. The graph is very similar to the one recorded at a speed of 80 km/h.

5.3 RESULTS WITH ABS (FUZZY LOGIC CONTROLLER)

The results shown below correspond to the response of the vehicle when the ABS system is actuating. In this case the control strategy used the fuzzy logic controller.

5.3.1 80 km/h dry asphalt

Figure 5.24 shows the evolution of the distance travelled by the vehicle during the braking manoeuvre on dry asphalt at an initial speed of 80 km/h in which the ABS system operates.

Figure 5.24- Stopping distance with Fuzzy Logic ABS at 80 km/h on dry asphalt.

The graph shows that the vehicle took 1.68 seconds to stop at which it travelled 20.42 meters.

Figure 5.25 shows the evolution of the slip ratios of the wheels of the two axles. It can be seen that the controller keeps the average slip ratio value around the desired value although it produces small variations. This is due to the action of the controller on the pressure of the system, since it is constantly changing to obtain the desired braking torque.

Figure 5.25- Slip ratios with Fuzzy Logic ABS at 80 km/h on wet asphalt.

Figure 5.26 shows the speeds of the vehicle and the front wheels. The wheels never block. Certain vibrations can be observed in the wheel speed. This is due to the action of the controller on the braking torque. These vibrations are typical in real ABS systems.
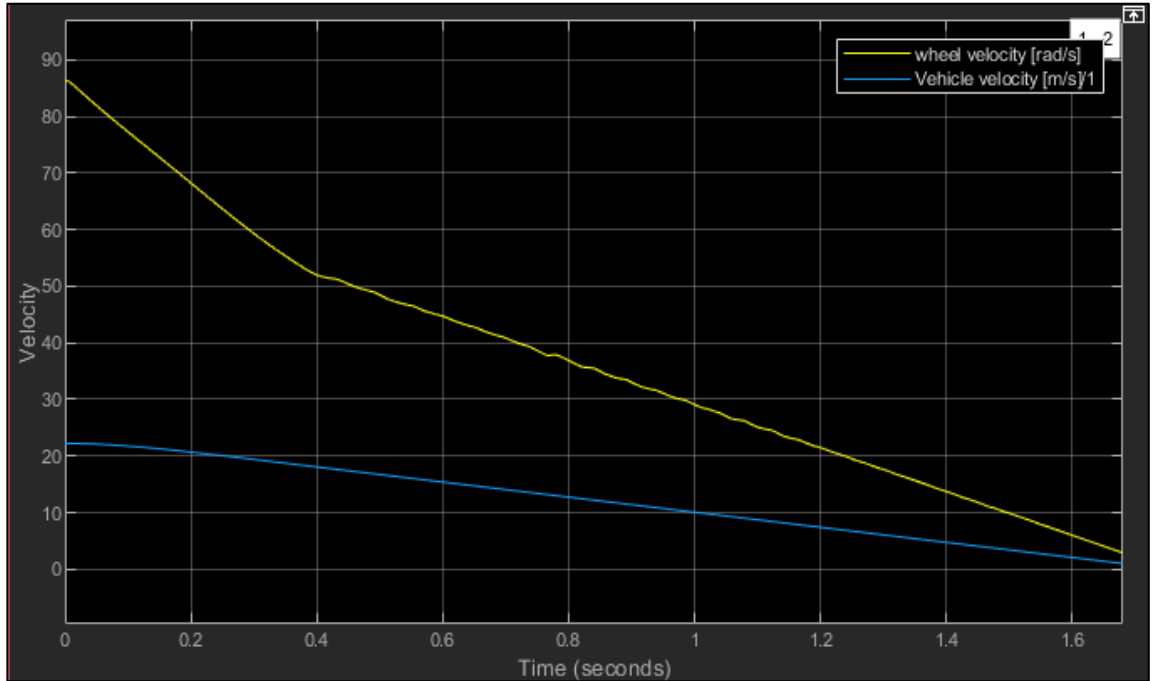
Figure 5.26- Comparison between vehicle and wheel velocities with Fuzzy Logic ABS at 80 km/h on dry asphalt.

Figure 5.27 also shows vibrations in the pressure of the front and rear circuits. The control releases or increases the pressure to obtain the optimum value.
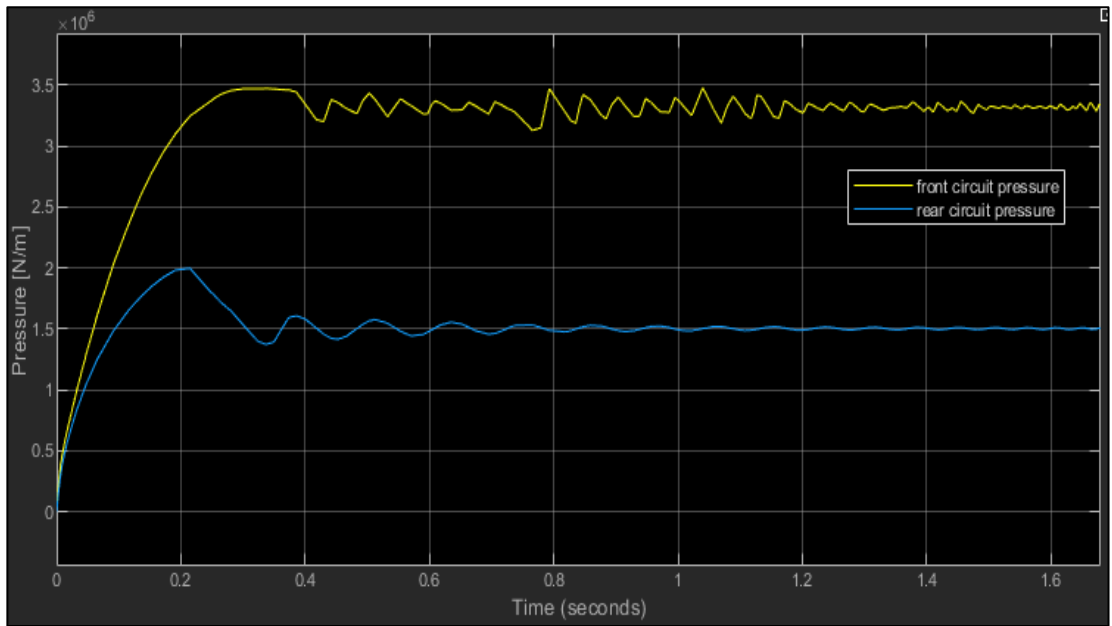
Figure 5.27- Pressure variation with Fuzzy Logic ABS at 80km/h on dry asphalt.

5.3.2 80 km/h wet asphalt

Figure 5.28 shows the evolution of the distance travelled by the vehicle during the braking manoeuvre on wet asphalt at an initial speed of 80 km/h in which the ABS system operates.
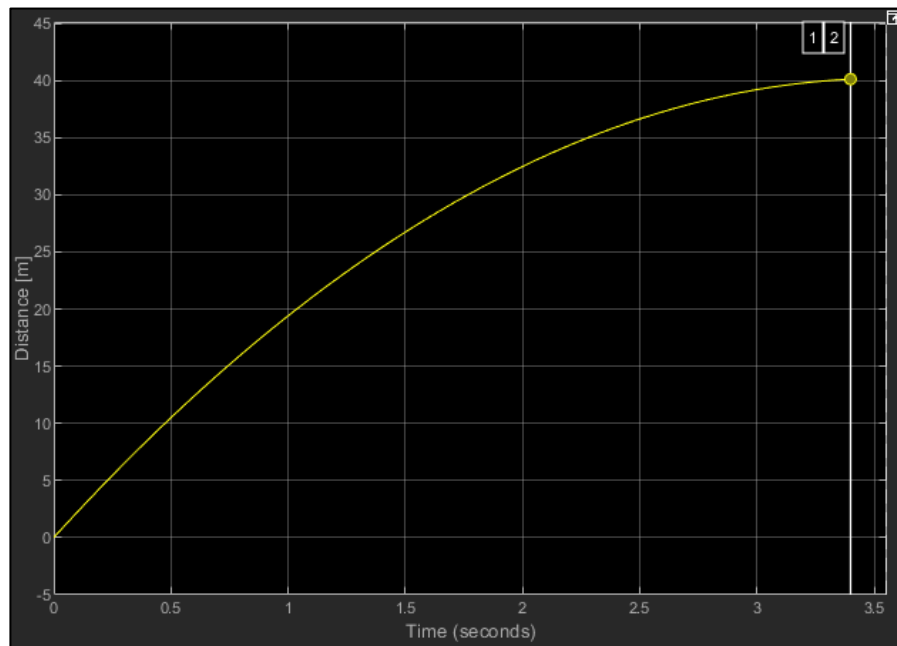

Figure 5.28- Stopping distance with Fuzzy Logic ABS at 80 km/h on wet asphalt.

The graph shows that it took 3.4 seconds for the vehicle to stop in 40.12 meters. These values are much higher than in dry asphalt conditions. Even so, the ABS system has managed to reduce these values substantially with respect to the values obtained from the same event but without the intervention of any control system.
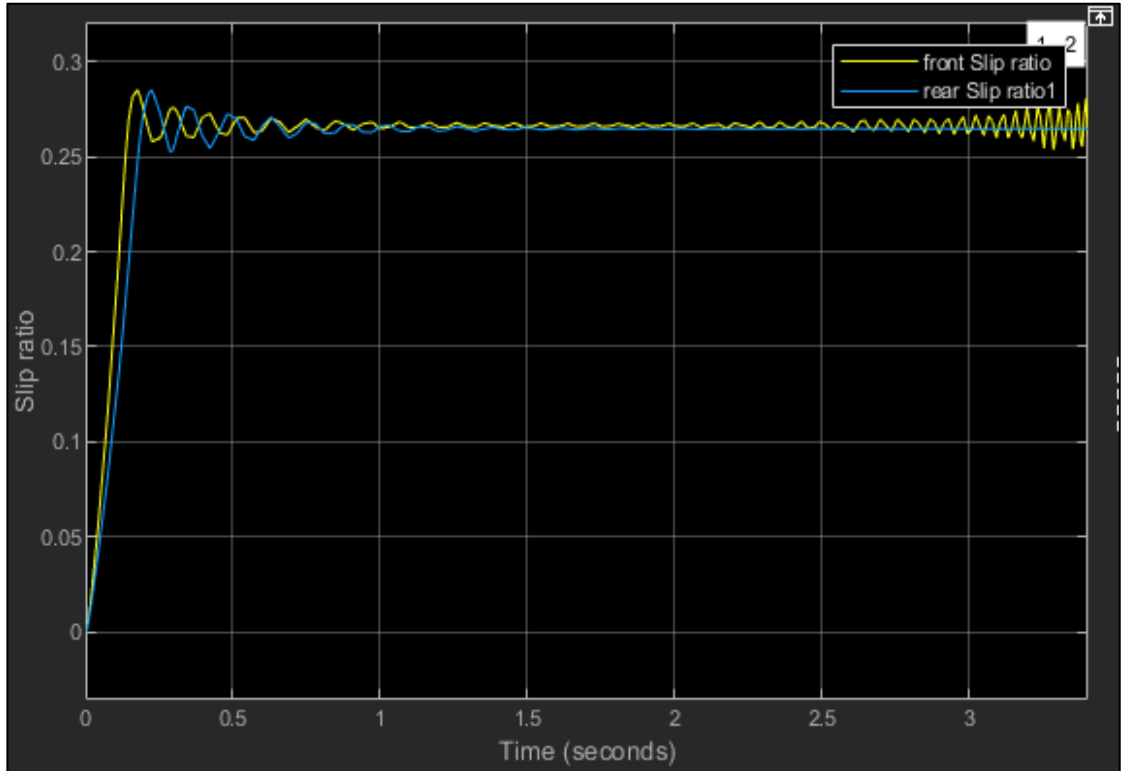
Figure 5.29- Slip ratios with Fuzzy Logic ABS at 80 km/h on wet asphalt.

In figure 5.29 it can be seen the evolution of the slip ratios of the wheels of the two axles. It can be seen that the controller keeps the average slip ratio value around the desired value although it produces small variations. This is due to the action of the controller on the pressure of the system, since it is constantly changing to obtain the desired braking torque.

Figure 5.30- Comparison between vehicle and wheel velocities with Fuzzy Logic ABS at 80 km/h on wet asphalt.

Figure 5.30 shows the speeds of the vehicle and the front wheels. The wheels are never blocked. Certain vibrations can be observed in the wheel speed. This is due to the action of the controller on the braking torque.
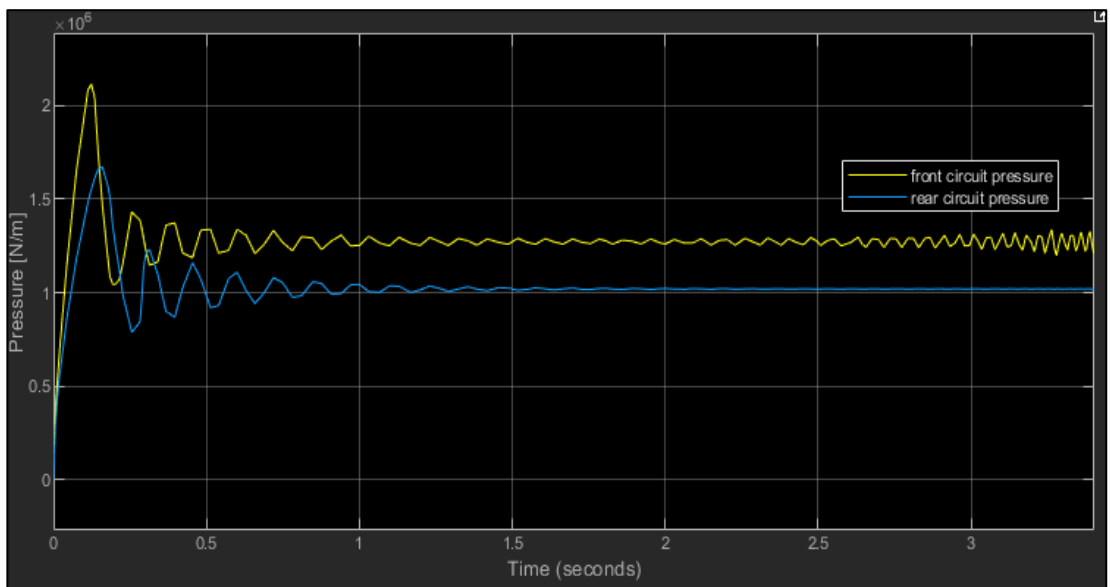
Figure 5.31- Pressure variation with Fuzzy Logic ABS at 80km/h on wet asphalt.

Figure 5.31 also shows vibrations in the pressure of the front and rear circuits. The control releases or increases the pressure to obtain the optimum value.

5.3.3 100 km/h dry asphalt

Figure X shows the evolution of the distance travelled by the vehicle during the braking manoeuvre on dry asphalt at an initial speed of 100 km/h in which the ABS system operates.
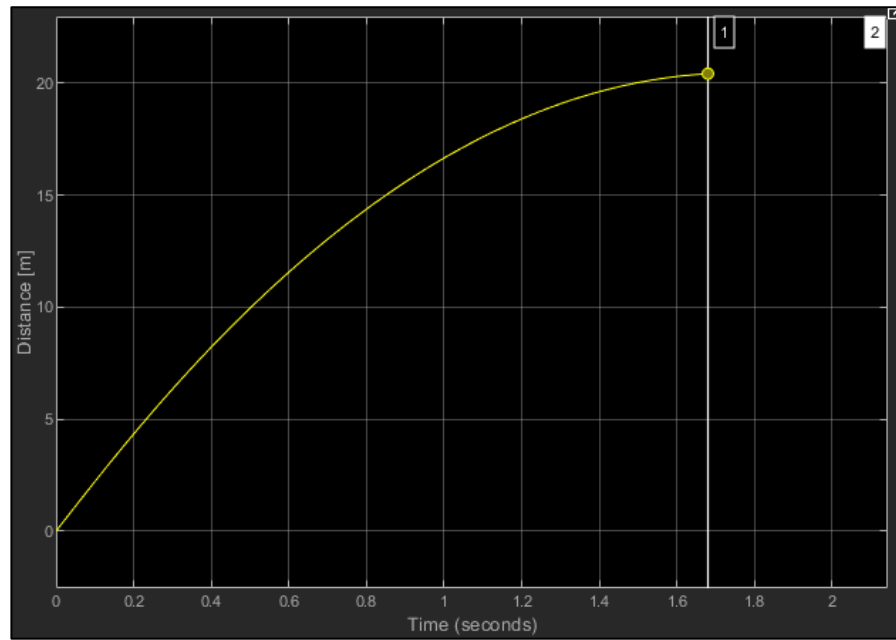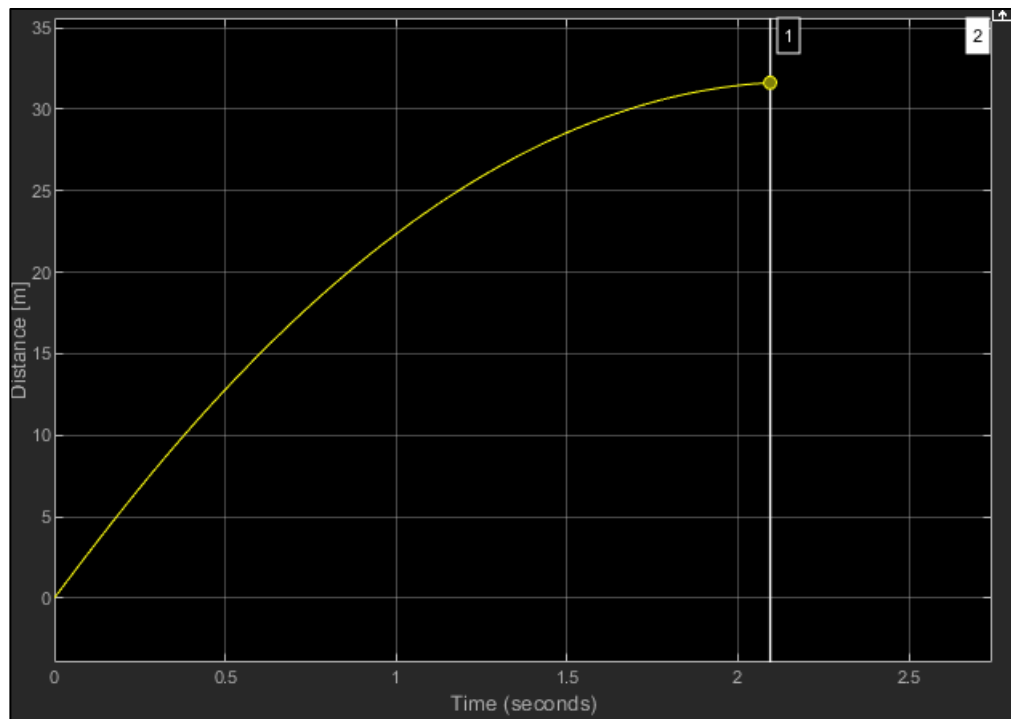

Figure 5.32- Stopping distance with Fuzzy Logic ABS at 100 km/h on dry asphalt.

The graph shows that the vehicle took 2.05 seconds to stop at which it travelled 31.6 meters. These values are slightly higher than during the manoeuvre at 80 km/h.
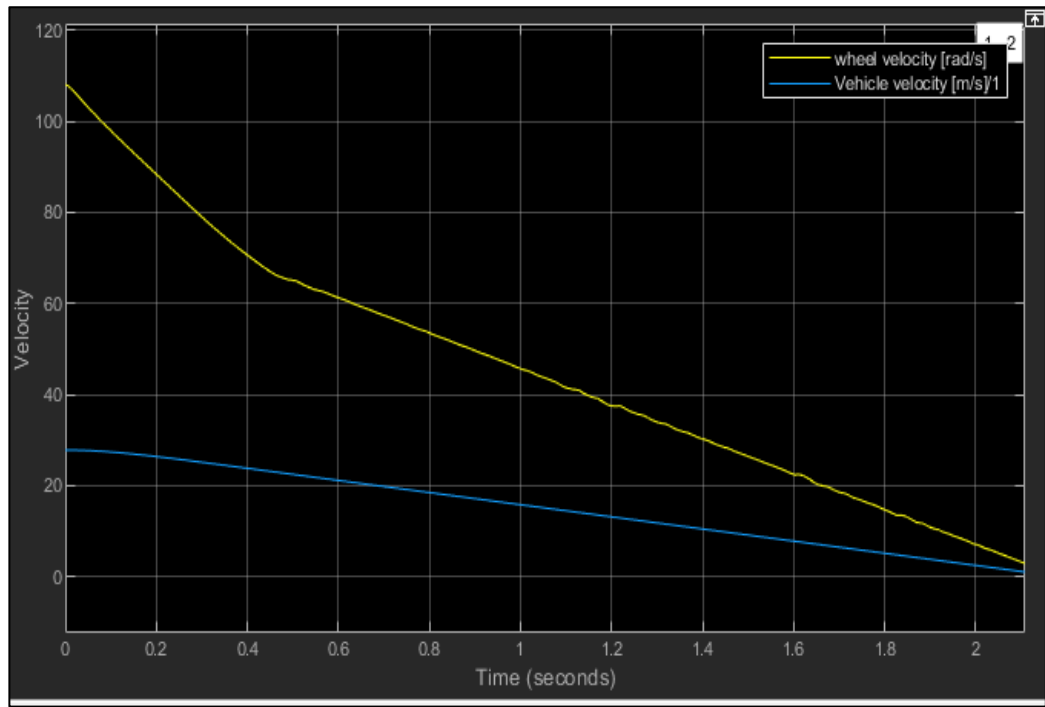
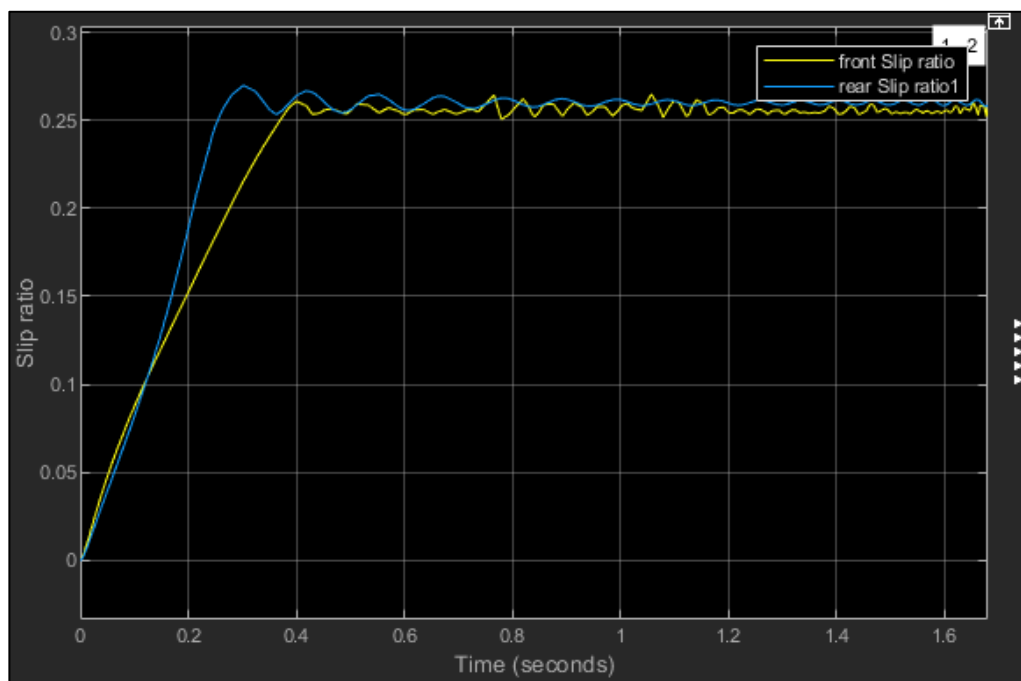*Figure 5.33- Comparison between vehicle and wheel velocities with Fuzzy Logic ABS at 100 km/h on dry asphalt.*

Figure 5.33 shows the speeds of the vehicle and the front wheels. The wheels are never blocked. Certain vibrations can be observed in the wheel speed. This is due to the action of the controller on the braking torque. These vibrations are typical of real ABS systems. These vibrations are small and should not affect the directional behaviour of the vehicle.
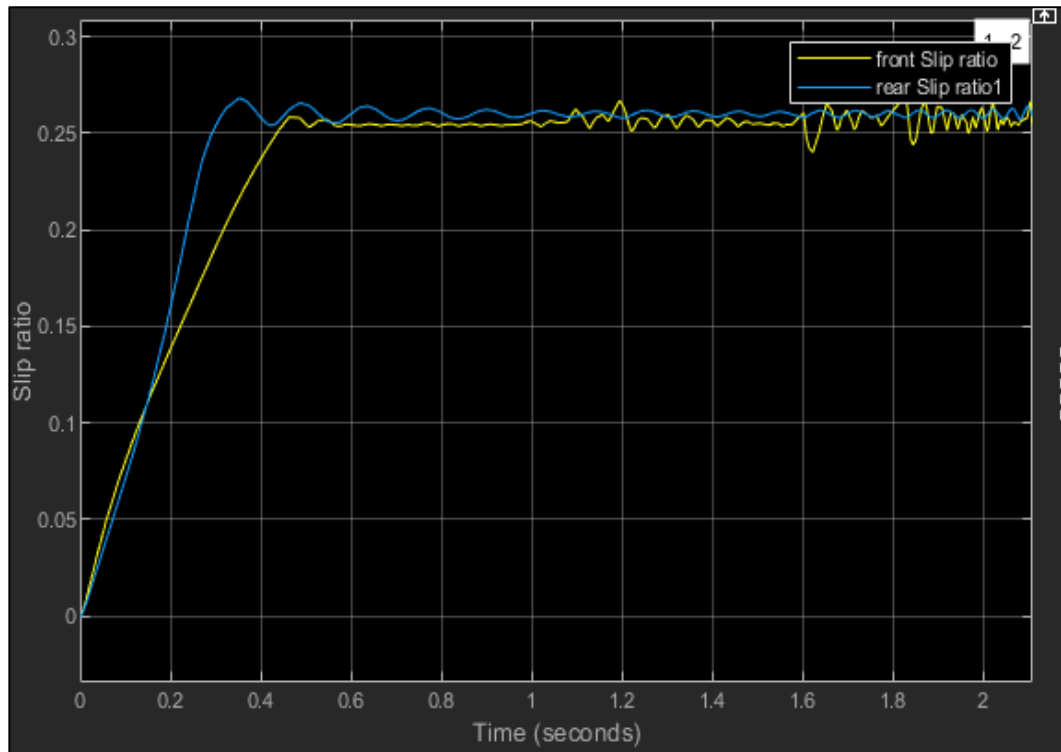
Figure 5.34- Slip ratios with Fuzzy Logic ABS at 100km/h on dry asphalt.

In figure 5.34 it can be seen the evolution of the slip ratios of the wheels of the two axles. It can be seen that the controller keeps the average slip ratio value around the desired value although it produces small variations. This is due to the action of the controller on the pressure of the system, since it is constantly changing to obtain the desired braking torque. The front slip ratio is more unstable than the rear slip ratio. This may be due to the fact that a greater action of the controller on this axis is necessary since it is the one that has the greatest braking capacity.
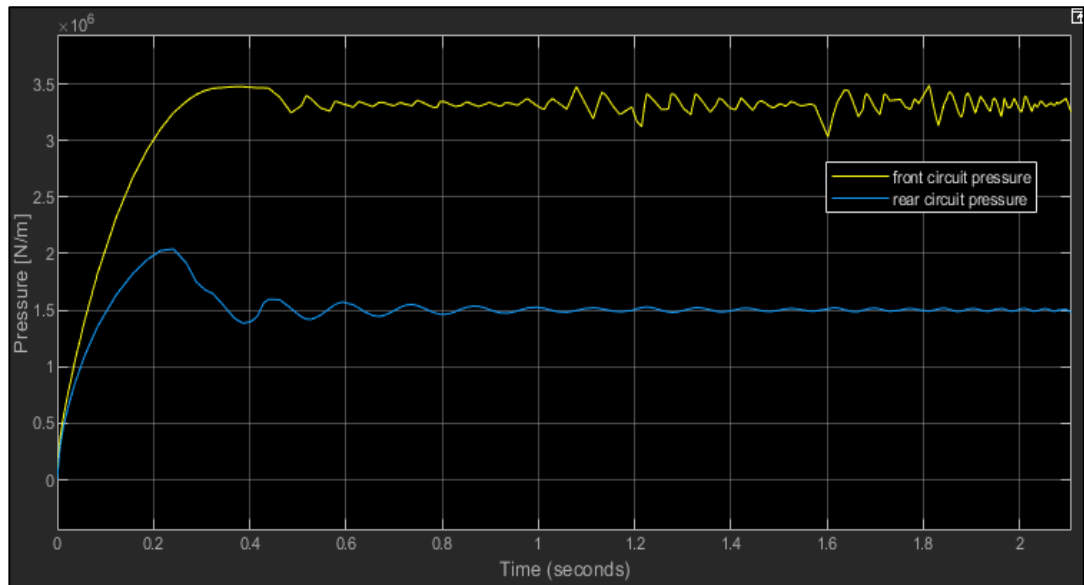
Figure 5.35- Pressure variation with Fuzzy Logic ABS at 100km/h on dry asphalt.

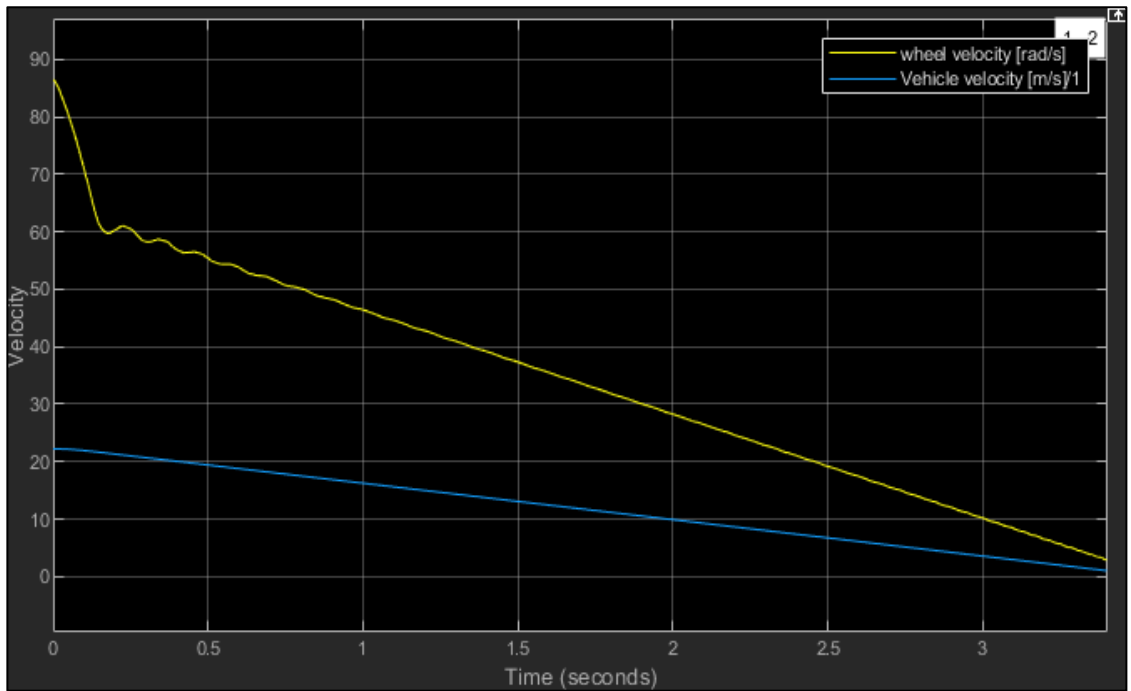Figure 5.35 also shows vibrations in the pressure of the front and rear circuits. The control releases or increases the pressure to obtain the optimum value.

6. CONCLUSIONS AND FUTURE WORK

Before making the final conclusions of the work, some summary tables have been constructed with the most important data collected during the 3 previously simulated manoeuvres in order to be able to make a comparison. These tables show the distance and braking time. These variables have been chosen because they have been considered the most important in terms of analysis of braking performance in vehicles.

	STOPPING DISTANCE [m]	STOPPING TIME [s]
WITHOUT ABS	31	2.8
WITH ABS (PID)	20.42	1.68
WITH ABS (FUZZY)	20.93	1.7

Table 6.1- Stopping distance and time of the 80 km/h event on dry road.

	STOPPING DISTANCE [m]	STOPPING TIME [s]
WITHOUT ABS	71.4	6.23
WITH ABS (PID)	41	3.47
WITH ABS (FUZZY)	40.12	3.4

Table 6.2- Stopping distance and time of the 80 km/h event on wet road.

	STOPPING DISTANCE [m]	STOPPING TIME [s]
WITHOUT ABS	48.7	3.55
WITH ABS (PID)	32.11	2.12
WITH ABS (FUZZY)	31.6	2.05

Table 6.3- Stopping distance and time of the 100 km/h event on dry road.

After analysing the data collected during the simulations, the following conclusions were drawn:

- ABS systems are a fundamental element in today's vehicles as they substantially reduce the number of fatal accidents.

- Through the use of appropriate software models can be implemented to simulate the dynamics of a vehicle which can save cost and time.

- There are numerous control strategies applied to ABS systems, each with different characteristics.

- It has been possible to create a half vehicle model within the Matlab/Simulink software and incorporate two control strategies so that they act as an ABS system would do.

- The state of the road is the parameter that has the greatest effect on distance and braking time. The speed at which the vehicle circulates is another parameter to take into account although less than the state of the road.

- Both controllers have managed to reduce both braking time and braking distance under different simulation conditions. Even so, the fuzzy logic controller has shown a slightly better performance as it has achieved lower values.

- The fuzzy logic controller actuation is more effective although it can cause vibrations during its actuation. As the objective of an ABS system is safety and not comfort it has been concluded that the best solution is the implementation of a fuzzy logic controller for use within an ABS system.

Possible future works to improve the present work could be the following:

- Implementation of the control strategies in a real vehicle ECU and compare the results obtained with the simulation results.
- Incorporate a more complex tyre model and evaluate the improvements.
- Refine the vehicle model trying to implement a model with more degrees of freedom.
- Incorporate a yaw control of the vehicle that acts in parallel with the ABS to improve the lateral dynamics.
- Perform simulations of braking during turning to check the operation of the controllers when the vehicle is subjected to lateral forces.

7. REFERENCES

1. Abro, M. A. *et al.* (2018) 'Design and Analysis of Anti-Lock Braking System Design and Analysis of Anti-Lock Braking System', (March).

2. Aly, A. A. *et al.* (2011) 'An Antilock-Braking Systems (ABS) Control: A Technical Review', *Intelligent Control and Automation*, 02(03), pp. 186–195. doi: 10.4236/ica.2011.23023.

3. Anwar, S. (2006) 'Anti-lock braking control of a hybrid brake-by-wire system', *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 220(8), pp. 1101–1117. doi: 10.1243/09544070D22704.

4. 'Article__Neural Networks for Intelligent Vehicles.pdf' (no date).

5. Badie Sharkawy, A. (2010) 'Genetic fuzzy self-tuning PID controllers for antilock braking systems', *Engineering Applications of Artificial Intelligence*. Elsevier, 23(7), pp. 1041–1052. doi: 10.1016/j.engappai.2010.06.011.

6. Bera, T. K., Bhattacharya, K. and Samantaray, A. K. (2011) 'Evaluation of antilock braking system with an integrated model of full vehicle system dynamics', *Simulation Modelling Practice and Theory*, 19(10), pp. 2131–2150. doi: 10.1016/j.simpat.2011.07.002.

7. Bhandari, R., Patil, S. and Singh, R. K. (2012) 'Surface prediction and control algorithms for anti-lock brake system', *Transportation Research Part C: Emerging Technologies*. Elsevier Ltd, 21(1), pp. 181–195. doi: 10.1016/j.trc.2011.09.004.

8. Chris828, (Web-Alias) (2009) 'Antilock Braking System', (October 2014). Available at: http://commons.wikimedia.org/wiki/File:Antilock_Braking_System.svg.

9. Colombo, D. (2007) 'Brake-by-Wire system development Introduction on by-Wire Technology Focus on CRF Brake-by-Wire Project System architecture Main components Control development and Validation (Brake-by-Wire Application) Application development process Software-in-the-Loo', *Development*.

10. Dalimus, Z. (2014) 'Braking System Modeling and Brake Temperature Response to Repeated Cycle', *Journal of Mechatronics, Electrical Power, and Vehicular Technology*, 5(2), p. 123. doi: 10.14203/j.mev.2014.v5.123-128.

11. van Duifhuizen, B. A. P. *et al.* (2014) 'Modeling, design and control of an individual wheel actuated brake system', 2014(2014).

12. John, S. and Pedro, J. O. (2013) 'Neural network-based adaptive feedback linearization control of antilock braking system', *International Journal of Artificial Intelligence*, 10(13 S), pp. 21–40.

13. Khanse, K. R. (2015) 'Development and Validation of a Tool for In-Plane Antilock Braking System (ABS) Simulations-master'.

14. *Milliken Research Associates, Inc. -- FSAE Tire Test Consortium* (no date). Available at: http://www.millikenresearch.com/fsaettc.html (Accessed: 21 June 2019).

15. Minh, V. T. *et al.* (2016) 'Development of Anti-lock Braking System (ABS) for Vehicles Braking', *Open Engineering*, 6(1), pp. 554–559. doi: 10.1515/eng-2016-0078.

16. Narwade, S. and Titare, P. (2017) 'PERFORMANCE EVALUATION OF AN ANTI-LOCK BRAKING', (10), pp. 52–54.

17. Pedro, J. and Ferro, C. (2014) 'Design and Simulation of an ABS Control Scheme for a Formula Student Prototype-master', (May), p. 107.

18. Pradeep Rohilla *et al.* (2016) 'Design and Analysis of Controller for Antilock Braking System in Matlab/Simulation', *International Journal of Engineering Research and*, V5(04), pp. 583–589. doi: 10.17577/ijertv5is041074.

19. Soliman, A. M. A., Kaldas, M. M. S. and Mahmoud, K. R. M. (2009) 'Active suspension and anti-lock braking systems for passenger cars', *SAE Technical Papers*, (July). doi: 10.4271/2009-01-0357.

20. Sundaram, G. and Sathyam, S. (2017) 'Modelling and Simulation of a Vehicle Powertrain and Anti-Lock Braking System', (November).

21. Version, D. (2004) *SIMULINK model of a quarter-vehicle with an Anti-lock Braking System Eindhoven University of Technology Mechatronic Design SIMULlNK Modei of a Quarter- ' Vehicle with an Anti-lock Braking System*.

22. Xiao, L., Hongqin, L. and Jianzhen, W. (2016a) 'Modeling and Simulation of Anti-lock Braking System based on Fuzzy Control', *Iarjset*, 3(10), pp. 110–113. doi: 10.17148/iarjset.2016.31021.

23. Xiao, L., Hongqin, L. and Jianzhen, W. (2016b) 'Modeling and Simulation of Anti-lock Braking System based on Fuzzy Control', *Iarjset*, 3(10), pp. 110–113. doi: 10.17148/iarjset.2016.31021.

24. Zhang, X. Q. *et al.* (2012) 'Research on ABS of multi-axle truck based on ADAMS/Car and Matlab/Simulink', *Procedia Engineering*, 37, pp. 120–124. doi:

10.1016/j.proeng.2012.04.213.

MATLAB SCRIPT FILE WITH CONSTANT VALUES

```matlab
%%constant values
g=9.81; % gravity (m/s2)

%%vehicle parameters
h=0.35; %heigth center of gravity
B= 1.75; %wheelbase
m=350;  %mass of the vehicle with driver (kg)
Fz_f=m*g*0.43;  %static front axle load
Fz_r=m*g*0.57; %static rear axle load
Fn=m*g;%total vertical load

%wheel constants
R=0.257;  %wheel radius (m)
Jw= 1.13; %moment inertia wheel (kgm2)

%initial values
Vx0=100/3.6; %longitudinal velocity (m/s)
w0=Vx0/R;  %initial angular velocity of the wheel (rad/seg)

%brake system constants
Max_d=250; %maximum driver force on the brake pedal
ratio=5; %brake pedal ratio
f_c=0.45; %friction coefficient of brake pads
Def_bd=0.0936; %effective diameter of the brake disk (same for four
wheels)

%front system
D_mc_f=0.0158; %front axle master cylinder diameter (m)
A_mc_f=(D_mc_f^2)*pi/4; %front axle master cylinder area (m2)
n_f=2; %number of pistons per front calliper
D_cp_f=0.03175; %front calliper piston diameter (m)
A_cp_f=(D_cp_f^2)*pi/4; %area of the front calliper piston (m2)

%rear system
D_mc_r=0.0158; %front axle master cylinder diameter (m)
A_mc_r=(D_mc_f^2)*pi/4; %front axle master cylinder area (m2)
n_r=2; %number of pistons per front calliper
D_cp_r=0.03175; %front calliper piston diameter (m)
A_cp_r=(D_cp_f^2)*pi/4; %area of the front calliper piston (m2)




% Import the data from Excel for lookup table
data = xlsread('slip-mu','Hoja1');
% Slip values for lookup table
Slip = data(1:end,1);
% Friction coefficient values for lookup table
Mu = data(1:end,2);
% wet Friction coefficient values for lookup table
Mu_wet = data(1:end,3);
```

COMPLETE SIMULINK MODEL
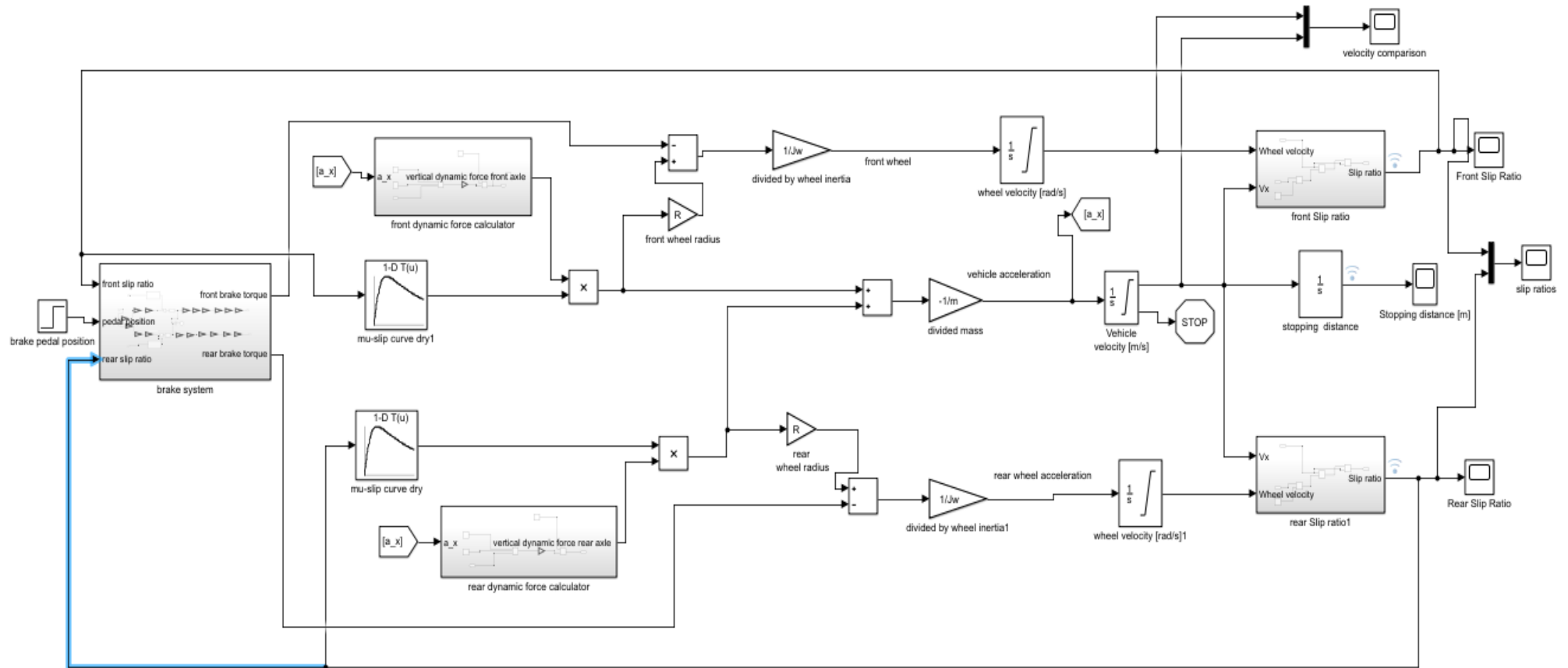
APENDIX C

Tyre data

Slip	Mu	Mu_wet
0.00	0.00	0.00
0.01	0.12	0.06
0.02	0.23	0.11
0.03	0.35	0.16
0.04	0.45	0.22
0.05	0.56	0.27
0.06	0.66	0.31
0.07	0.75	0.36
0.08	0.83	0.40
0.09	0.91	0.43
0.10	0.98	0.47
0.11	1.04	0.50
0.12	1.10	0.52
0.13	1.14	0.54
0.14	1.19	0.57
0.15	1.22	0.58
0.16	1.25	0.60
0.17	1.28	0.61
0.18	1.30	0.62
0.19	1.32	0.63
0.20	1.33	0.63
0.21	1.34	0.64
0.22	1.35	0.64
0.23	1.36	0.65
0.24	1.36	0.65
0.25	1.36	0.65
0.26	1.36	0.65
0.27	1.36	0.65
0.28	1.35	0.64
0.29	1.35	0.64
0.30	1.34	0.64
0.31	1.34	0.64
0.32	1.33	0.63
0.33	1.32	0.63
0.34	1.31	0.63
0.35	1.30	0.62
0.36	1.30	0.62
0.37	1.29	0.61
0.38	1.28	0.61
0.39	1.27	0.60

0.40	1.26	0.60
0.41	1.25	0.59
0.42	1.23	0.59
0.43	1.22	0.58
0.44	1.21	0.58
0.45	1.20	0.57
0.46	1.19	0.57
0.47	1.18	0.56
0.48	1.17	0.56
0.49	1.16	0.55
0.50	1.15	0.55
0.51	1.14	0.54
0.52	1.13	0.54
0.53	1.12	0.53
0.54	1.11	0.53
0.55	1.10	0.52
0.56	1.09	0.52
0.57	1.08	0.51
0.58	1.07	0.51
0.59	1.06	0.50
0.60	1.05	0.50
0.61	1.04	0.49
0.62	1.03	0.49
0.63	1.02	0.48
0.64	1.01	0.48
0.65	1.00	0.48
0.66	0.99	0.47
0.67	0.98	0.47
0.68	0.97	0.46
0.69	0.96	0.46
0.70	0.95	0.45
0.71	0.94	0.45
0.72	0.93	0.44
0.73	0.93	0.44
0.74	0.92	0.44
0.75	0.91	0.43
0.76	0.90	0.43
0.77	0.89	0.42
0.78	0.88	0.42
0.79	0.87	0.42
0.80	0.87	0.41
0.81	0.86	0.41
0.82	0.85	0.40
0.83	0.84	0.40
0.84	0.83	0.40
0.85	0.83	0.39

0.86	0.82	0.39
0.87	0.81	0.39
0.88	0.80	0.38
0.89	0.79	0.38
0.90	0.79	0.37
0.91	0.78	0.37
0.92	0.77	0.37
0.93	0.77	0.36
0.94	0.76	0.36
0.95	0.75	0.36
0.96	0.74	0.35
0.97	0.74	0.35
0.98	0.73	0.35
0.99	0.72	0.34
1.00	0.72	0.34