Applying statistical methods with imprecise data to quality control in cheese manufacturing

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Abstract. Sensory analysis entails subjective valuations provided by qualified experts which in most of the cases are given by means of a real value. However personal valuations usually present an uncertainty in their meaning which is difficult to capture by using a unique value. In this work some statistical techniques to deal with such kind of information are presented. The methodology is illustrated through a case-study, where some tasters have been proposed to use trapezoidal fuzzy numbers to express their perceptions regarding the quality of the so-called Gamonedo blue cheese. In order to establish an agreement between the tasters a weighted summary measure of the information collected is described. This will lead to assign a weight to each expert depending on the influence they have when the weighted mean is computed. An example of the real-life application is also provided.

Keywords: Fuzzy sets; Subjective valuations; Quality control of cheese; Weighted central tendency measure; Statistical techniques

1 Introduction

Sensory analysis (or sensory evaluation) is useful to interprete some reactions of the people to those features of food which are perceived by the senses of sight, hear, smell, taste and touch [2]. The aim of this technique is to control the acceptability of certain product in the market, taking into account some requirements regarding hygiene, harmlessness and quality. Sometimes a product requires the mention of Protected Designation of Origin (PDO) to be commercialized, which increases the importance of making a sensorial analysis study [8,31].

Likert scales are widely used to measure attributes or attitudes which are associated with opinions, perceptions, valuations and so on [25]. A Likert scale-based questionnaire is based on a set of pre-fixed categories which are usually coded by means of integer numbers from a scale usually ranging from 1 to 5, or from 1 to 7. This kind of closed format questionnaires have well-known advantages: they are easy to conduct and the meaning of the answers does not need to be explained in general. Nevertheless, its usefulness presents some drawbacks as it has been pointed out, for instance, in [1,4,13,18,38].

Fuzzy set theory was firstly introduced by Zadeh [44] as an extension of the classical set theory. It is a wide field of study in which different tools have been developed to deal with several problems during the last 20 years. Concerning product quality in the food industry, several applications involving fuzzy sets and fuzzy logic have been studied in some works. Most of them are summarized in [32].

The application of fuzzy concepts to the sensory evaluation field is quite recent. On one hand, there are some works based on the employment of fuzzy logic for representing the semantics of human assessment in sensory analysis. For instance, Davidson *et al.* [6] proposed a linguistic format for the sensory assessment of food and processing methods for analyzing taste panel opinions by using fuzzy inference systems. On the other hand, there are other works showing how fuzzy sets can be used as a way of interpreting sensory data. In this line, Tan *et al.* [42] considered sensory scales described as fuzzy sets, sensory attributes as fuzzy random variables and sensory responses as sample membership grades. Then, the set of responses is formulated as a fuzzy histogram of response and neural networks are used to provide an effective tool for modeling sensory responses. Additionally, a sensory evaluation model that manages evaluation frameworks applying a fuzzy linguistic approach is developed in [28].

In the literature some authors have suggested to establish a correspondence between each Likert response category and a fuzzy value chosen from a class of flexible fuzzy sets previously stated by experts; see, for instance, [23,24]. This is also the case of the works [6,42] presented above, where it is suggested to identify each sensory response with a fuzzy subset chosen from a class of operational and flexible fuzzy sets which has been stated by 'experts' either individually or by consensus.

However, the employment of a value chosen from either a Likert scale or a previously fixed fuzzy scale to describe each sensorial perception presents some drawbacks, as the ones described below.

- Sometimes it is difficult for the taster to summarize his/her perception in a unique value. This summary leads to a loss of information that could be useful since the subjective perceptions include certain imprecision that cannot be capture by using a single value.
- The transition from one category to another one is abrupt in general and two different categories may not be perceived in the same way by two observers.
- Most of the statistical tools cannot be applied directly when Likert scales are employed and, when it is possible, the interpretability and reliability of the results obtained is considerably reduced (see, for instance, [41]).

To overcome these problems, it is proposed a natural way for describing each individual opinion/perception/valuation by means of a fuzzy set capturing the subjectiveness or imprecision involved in the answer and without taking into account a pre-fixed enumeration of answers, neither crisp nor fuzzy. That is, in this case there is not a pre-specified list of possible answers but the proposed questionnaire has a free-response format.

This work is focused on presenting different statistical techniques for sensory analysis based on fuzzy data. The methodology will be illustrated by considering the case of the quality analysis of the Gamonedo cheese which was made by some tasters of the ALCE CALIDAD company (http://www.alcecalidad.com/empresa) in order to preserve the Protected Designation of Origin (PDO). Thus, it should be remarked that the tools used to carry out the perceptual analysis are qualified people.

The Gamonedo cheese is a kind of blue cheese which is produced in Onís and Cangas de Onís, councils of Asturias, Spain (see, for instance, [14,37]). The cheese experiences a smoked process and later on it is let settle in natural caves or a dry place (see Figure 1).

Fig. 1. Production of the Gamonedo cheese

The difficulties in keeping the quality of the Gamonedo cheese - due to its sensitivity and complexity - support the necessity of developing a solid tasting system to determine its quality.

Most of the tasters of the ALCE CALIDAD company are expert people, although some of them are unexperience tasters. They were proposed to soften their subjective perceptions about the quality of the Gamonedo cheese by using a scale of free-response fuzzy sets. In addition, random fuzzy sets (RFSs for short) in Puri and Ralescu's sense [34] were introduced to formalize imprecise experimental data which can be described by means of fuzzy sets.

We will present some descriptive and inferential statistical approaches for fuzzy data which are applicable to the sensorial analysis of the Gamonedo cheese. On one hand, we will propose a robust measure of central tendency for fuzzy values which will be very useful in our experiment since there are some non-expert tasters in the tasting process. Later, we will focus the attention on the application of different hypothesis testing procedures for the mean and the variance of fuzzy sets.

The rest of the manuscript is organized as follows. Section 2 gathers some preliminaries about fuzzy sets. A procedure for computing weights associated with the opinions of the tasters regarding the quality of different features of the Gamonedo cheese is included in Section 3. This will lead to construct a weighted mean to summarize the opinions of the tasters through a robust summary measure. Hypothesis testing procedures for fuzzy data and their applicability to the sensory evaluation of the Gamonedo cheese are presented in Section 4. Finally, some remarks and open problems are shown in Section 5.

2 Preliminary concepts

Formally, the space of fuzzy numbers $\mathcal{F}_c(\mathbb{R})$ is the class of functions $U : \mathbb{R} \to [0, 1]$ such that $U_\alpha \in \mathcal{K}_c(\mathbb{R})$ for all $\alpha \in (0, 1]$, where $\mathcal{K}_c(\mathbb{R})$ is the family of intervals of $\mathbb{R}$. The α-levels of U are defined as $U_\alpha = \{x \in \mathbb{R} | U(x) \geq \alpha\}$ if $\alpha \in (0, 1]$, and U_0 is the closure of the support of U.

Two of the most usual shapes of fuzzy numbers considered in the literature are trapezoidal and triangular (see, for instance, [9,16]). A trapezoidal fuzzy number, from now on denoted by $\tau(a, b, c, d)$, fulfils that the interval $[a, d]$ is the 0-level whereas $[b, c]$ is the 1-level. Mathematically, the expression for the trapezoidal fuzzy number with vertices in $\{a, b, c, d\}$ is

$$\tau(a,b,c,d) = \begin{cases} (x-a)/(b-a) & \text{if } x \in [a,b) \\ 1 & \text{if} x \in [b,c] \\ (d-x)/(d-c) & \text{if } x \in (c,d] \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

A description of a trapezoidal fuzzy number and one of its α-cuts ($\alpha = 0.4$) is provided in Figure 2. A triangular fuzzy number is a particular case with $b = c$.

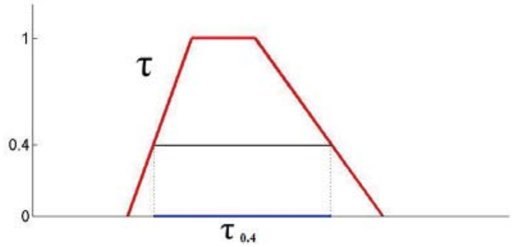

Fig. 2. Representation of a trapezoidal fuzzy set

The usual arithmetic between fuzzy numbers is based on Zadeh's extension principle and it agrees levelwise with the Minkowski addition and the product by scalars for compact intervals [29].

The space of fuzzy numbers is not linear due to the lack of symmetric element with respect to the Minkowski addition. It is very useful then to consider a distance between fuzzy numbers. The distance $D_\theta^\varphi(U,V)$ between the fuzzy numbers U and V, firstly introduced in [43], is defined by

$$\sqrt{\int_{(0,1]}\left(\left(\mathrm{mid}U_\alpha - \mathrm{mid}V_\alpha\right)^2 + \theta\left(\mathrm{spr}U_\alpha - \mathrm{spr}V_\alpha\right)^2\right) d\varphi(\alpha)}, \tag{2}$$

where $\theta > 0$ determines the relative weight of the distance between the spread of the α-cuts, i.e. $\mathrm{spr}Y_\alpha = (\sup U_\alpha - \inf U_\alpha)/2$, with respect to the distance between the corresponding mid-points $\mathrm{mid}U_\alpha = (\sup U_\alpha + \inf U_\alpha)/2$. In addition, φ is associated with a bounded density measure with support (0,1] which allows us to weigh the importance given to each α-level of the fuzzy numbers.

Let us consider a probability space $(\Omega, \mathcal{A}, P)$. Random fuzzy sets - RFSs, also called fuzzy random variables - were firstly introduced in [34]. $\mathcal{X} : \Omega \to \mathcal{F}_c(\mathbb{R})$ is a RFS if it is a Borel measurable mapping with respect to D_θ^φ [34,43]. One of the most important advantages of considering Borel measurable metric-space-valued mappings is that concepts such as induced distribution, independence, etc., can be stated as usual.

The *expected value* of $\mathcal{X}$ [3] is the unique fuzzy number $E(\mathcal{X})$ so that for each $\alpha \in [0,1]$, $(E(\mathcal{X}))_\alpha = [E(\inf \mathcal{X}_\alpha), E(\sup \mathcal{X}_\alpha)]$. To assure the existence of the last expectation, the integrability condition $\sup_{x\in\mathcal{X}_0} \|x\| \in L^1(\Omega, \mathcal{A}, P)$ must be satisfied. Besides, if $\sup_{x\in\mathcal{X}_0} \|x\| \in L^2(\Omega, \mathcal{A}, P)$ the *Fréchet-type variance* [12] - or simply *variance*, inspired on [22] - is defined as the real value

$$\sigma_{\mathcal{X}}^2 = E\big(D_\theta^\varphi(\mathcal{X}, E(\mathcal{X}))\big)^2. \tag{3}$$

2.1 Collecting the data

The case study addressed in this work was firstly suggested on a project in the European Centre for Soft Computing (Mieres, Spain), made in collaboration with the ALCE CALIDAD company. In that project, we were proposed to develop a method in order to improve the sensory analysis of the Gamonedo blue cheese produced in Asturias.

So far, the experts of the company provided an ordinal number ranging from 1 to 7 to describe their perceptions about different characteristics of the cheese. These characteristics included visual parameters (shape, rind and appearance), texture parameters (hardness and crumbliness), olfactory-gustatory parameters (smell intensity, smell quality, flavour intensity, flavour quality and aftertaste) and an overall impression of the cheese.

An example of the categorical opinion of an expert about the appearance of a specific cheese is shown in Figure 3. It should be remarked that the experts of the company apply different weights to each one of the features of the cheese, according to the experience accumulated on the previous tastings.

Appearance of the cheese

	M	D	R	AC	B	MB	EX
COEFICIENTE x 3.3	1	2	3	4	5	6	7
	3.3	6.6	9.9	13.2	16.5	19.8	23.1

Fig. 3. Categorical opinion of an expert about the appearance of a specific cheese

During the project, the tasters were proposed to use trapezoidal fuzzy numbers for describing their perceptions. This kind of fuzzy numbers is one of the most commonly used for fuzzy descriptions and their employment is easy to understand by the tasters [7,16,17]. It would be also possible to consider other shapes of fuzzy values.

Thus, the valuation of the different features of each cheese is made over a graduate scale ranging from 0% (lowest quality) to 100% (highest quality). The taster designs a trapezoidal fuzzy set, $A = Tra(\inf A_0, \inf A_1, \sup A_1, \sup A_0)$. The 0-level $[\inf A_0, \sup A_0]$ is the set of values that he/she considers to be compatible with his/her opinion at some extent (that is, the taster thinks that it is not possible that the quality is out of this set), and the 1-level $[\inf A_1, \sup A_1]$ is the set of values that he/she considers to be fully compatible with his/her opinion. These two levels are linearly interpolated to get the trapezoidal fuzzy set representing his/her personal valuation. An example of this representation is shown in Figure 4.

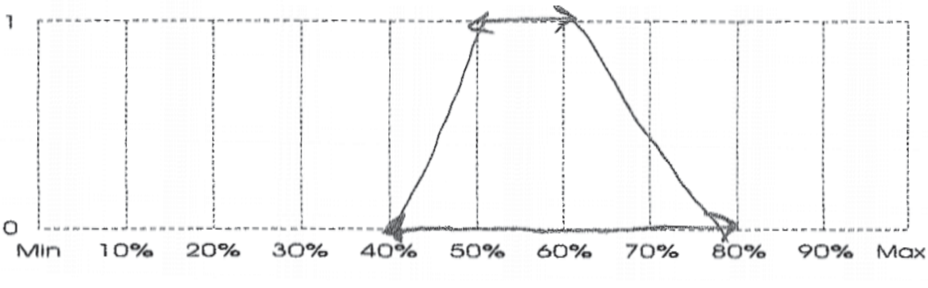

Fig. 4. Opinion of a taster given by means of a trapezoidal fuzzy set

The lower intervals of the trapezoidal fuzzy sets employed for describing the opinion of the tasters about certain feature of the cheese can be interpreted as a measure determining the consensus that exists between them, while the upper intervals represents their personal opinion.

Tastings in the period 2009-2011 have been collected in order to develop suitable statistical analysis to determine the quality of the Gamonedo cheese. We would like to remark that the database is not very large since the experts of the ALCE company developed only a few tastings during this period of time. In addition, to better check the suitability of our proposal it could be convenient to compare the results of the study with previous results. Although the previous (fuzzy) information does not exist until the moment, it is possible to compare in some sense the results obtained with the categorical information provided by previous tastings.

3 Construction of the summary measure of the opinions of the tasters

As it has been shown in Section 2.1, the consensus existing between the experts involved in the tasting procedure can be analyzed by considering the 0-levels of the trapezoids provided by those experts. For summarizing such observations, we suggest to define a robust central location measure. The sample mean, defined in terms of the natural arithmetic analogously to the classical concept, i.e. $\overline{X} = \sum_{i=1}^{n} X_i/n$, is an estimator of the population mean that is usually influenced by outliers or small departures from the model assumptions. The proposed summary measure will be useful for analyzing how good are the opinions of one taster with respect to the others.

Several approaches to robust estimation of the population mean have been proposed in the literature. For instance, the median, trimmed estimators, Winsorised estimators or M-estimators [19,20,27]. Combining robustness and fuzziness, some methods have been explored in particular problems [10,11,40]. The complexity of the space of fuzzy sets jointly with the lack of a sound total order in that space, makes some of the classical robust statistical methods - such the ones based on M-estimators - quite difficult to be tackled. In [40] a median for random fuzzy sets is defined. In addition, a robust population central location measure and a trimming approach have been proposed in [5] in order to reduce the impact of the outliers in the fuzzy data analysis.

Classical M-estimators have been shown to have a higher breakdown point and to be more efficient [20] than other alternatives. We propose to consider a robust estimator of the mean for the infima and suprema of the 0-levels of the trapezoids as follows. As a first attempt, the well-known Huber M-estimator [20] is employed, although other M-estimators could be taken into account as, for instance, the Tuckey, Hampel, Cauchy or Welsch M-estimators.

Let X^l be the real random variable "lowest value that the taster is willing to accept regarding the quality of a specific feature of the cheese". Suppose that there are n tasters and that the lowest values of their opinions are $\left\{X_1^l, \ldots, X_n^l\right\}$.

In the framework of this work the interest is focused on assigning lower weights to those opinions which are far from the mean value. A robust estimator of the mean is considered, based on the Huber M-estimator and taking into account the following Huber loss function:

$$\psi(x) = \begin{cases} x & if\ |x| \leq 1 \\ 1 & if\ |x| > 1 \end{cases} \tag{4}$$

The loss function defined in (4) is illustrated in Figure 5. It describes the penalty incurred by the mean estimation procedure.

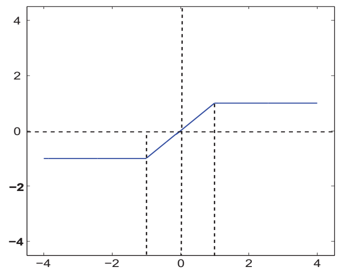

Fig. 5. Huber loss function

The robust estimator of the mean, denoted by T_n^l, is the one satisfying the equation

$$\sum_{i=1}^{n} \psi\left(\frac{X_i^l - T_n^l}{\widehat{\sigma}}\right) = 0, \tag{5}$$

where $\widehat{\sigma}$ is an estimator of the standard deviation of the population data. The classical estimator $\widehat{\sigma} = \sqrt{\sum_{i=1}^{n}(X_i^l - \overline{X^l})^2/(n-1)}$ is also generally affected by extreme observations. Thus, a robust measure of the variability of the data is also employed. The median absolute deviation (MAD) is based on the median and it is defined as

$$\text{MAD} = \text{median}\Big(\Big|X_i^l - \text{median}\big(\{X_1^l, \ldots, X_n^l\}\big)\Big|\Big). \tag{6}$$

The MAD was firstly promoted in [19]. It can be used as a consistent estimator of the standard deviation by taking

$$\widehat{\sigma} = K \cdot \text{MAD}, \tag{7}$$

where K is a constant scale factor which depends on the distribution. In general $K = 1.4826$ is the most employed number in practice; it is the required value to make the estimator consistent for the parameter of interest when Gaussian distributions are considered.

Equation (5) is equivalent to

$$\sum_{i=1}^{n} \left(\frac{\Psi\big(\frac{X_i^l - T_n^l}{K \cdot MAD}\big)}{\frac{X_i^l - T_n^l}{K \cdot MAD}}\right)(X_i^l - T_n^l) = 0, \tag{8}$$

where ψ is the function defined in (4), and so the Huber robust estimator for the mean is given by

$$T_n^l = \frac{\sum_{i=1}^n w(X_i^l) X_i^l}{\sum_{i=1}^n w(X_i^l)}, \tag{9}$$

where

$$w(X_i^l) = \begin{cases} \frac{\Psi\left(\frac{X_i^l - T_n^l}{K \cdot MAD}\right)}{\frac{X_i^l - T_n^l}{K \cdot MAD}} & \text{if } X_i^l \neq T_n^l \\ \Psi'(0) & \text{if } X_i^l = T_n^l \end{cases}$$

are the weights associated with each value of the sample. It is straightforward to show that $w(X_i^l) \in [0,1]$, for all $i \in \{1, \ldots, n\}$.

The weights can be gathered in a graphic - as the ones provided in Figures 11-14 - by taking into account the following points:

- If the weight is equal to 1 (which means that the infimum of the lower level of the trapezoid is "relatively close" to the robust estimate T_n^l) we will represent the weight with a completely shaded rectangle.
- If the weight is lower than 1, its representation will be a proportionally shaded rectangle (for instance, if the weight is .3 we will shade the 30% of the rectangle).
- The position of the rectangle will be either left or right depending on the position of the value of the sample with respect to the robust estimate T_n^l.

The proposed methodology could be analogously applied to the variable $X^r \equiv$"highest value that the taster is willing to accept regarding the quality of a specific feature of the cheese", leading to a robust summary measure of the highest values of the trapezoids drawn by all the tasters. In this case the Huber estimator is denoted by T_n^r and a set of weights $\{w_1^r, \ldots, w_n^r\}$ is obtained.

The aim is to compute a general weight on the basis of w_i^l and w_i^r to associate to the opinion of each taster. One possibility entails the consideration of a t-norm, which is a concept firstly introduced in [39], and defined as an operation in $[0,1]^2$ satisfying good properties. Two well-known t-norms have been selected to deal with the problem proposed here. The first one is the *product t-norm* which is the ordinary product of real numbers. Thus, if w_i^l and w_i^r are the weights for the taster i, the general weight for this taster will be $w_i^P = w_i^l \cdot w_i^r$. On the other hand, the *minimum t-norm* (also called Gödel t-norm) is considered which has the expression $w_i^M = \min\{w_i^l, w_i^r\}$.

The computed general weights are employed to calculate a weighted mean of the sample $\{X_1, \ldots, X_n\}$, where each X_i is the opinion of the taster i given by a trapezoidal fuzzy number.

Let $\{w_1, \ldots, w_n\}$ be the general Huber robust weights obtained by the procedure presented before. Without loss of generality, let X_i^l be the infimum of the 0-level of the trapezoid described by the taster i (as it was presented previously). Then, the weighted mean of the sample $\{X_i^l\}_{i=1}^n$ is

$$\frac{\sum_{i=1}^n w_i \cdot X_i^l}{\sum_{i=1}^n w_i}. \tag{10}$$

The same procedure could be applied to the other extremes of the trapezoids. Figure 6 shows a summary of the procedure followed in this section.

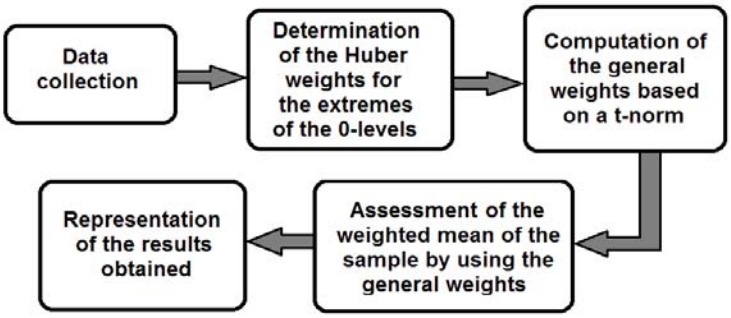

Fig. 6. Flow chart of the process

3.1 Application to the case study

In one of the tastings carried out in the ALCE company, a specific cheese of one of the Gamonedo's cheese factories has been analyzed by 7 tasters. Eleven features of the cheese have been considered: its shape, its rind, its appearance, the intensity and quality of its smell, its hardness, its friability, the intensity and quality of its taste, its aftertaste, and its overall impression. The tasters were proposed to express their opinions about the previous features by means of trapezoidal fuzzy values. We will focus our analysis in four of the features: the appearance, the quality of the smell, the taste and the overall impression. The data are gathered in Table 1 as vectors containing the four vertex of the trapezoids.

Expert	Appearance	Smell quality	Flavour quality	Overall impression
1	$(40, 46, 50, 57)$	$(34, 40, 50, 60)$	$(40, 46, 50, 57)$	$(30, 35, 45, 50)$
2	$(49, 52, 56, 61)$	$(50, 53, 56, 61)$	$(47, 50, 53, 60)$	$(60, 63, 67, 72)$
3	$(70, 75, 84, 90)$	$(65, 80, 87, 90)$	$(60, 68, 74, 80)$	$(70, 75, 85, 90)$
4	$(60, 70, 76, 80)$	$(65, 70, 74, 80)$	$(60, 70, 80, 84)$	$(65, 75, 85, 85)$
5	$(50, 50, 55, 65)$	$(50, 50, 65, 75)$	$(49, 50, 62, 75)$	$(50, 50, 63, 75)$
6	$(40, 44, 50, 57)$	$(40, 46, 55, 56)$	$(30, 40, 46, 50)$	$(40, 44, 50, 55)$
7	$(50, 55, 56, 56)$	$(47, 51, 54, 55)$	$(49, 54, 57, 61)$	$(49, 52, 54, 56)$

Table 1. Perceptions of the tasters concerning four features of the cheese

The previous trapezoidal fuzzy data are depicted in Figures 7, 8, 9 and 10.

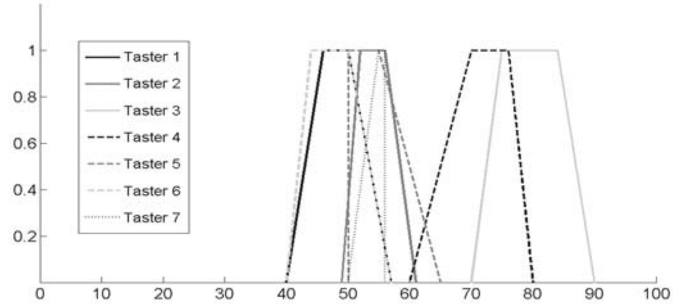

Fig. 7. Opinions about the appearance of the cheese

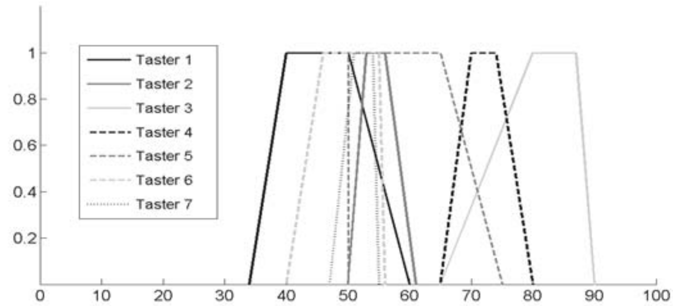

Fig. 8. Opinions about the smell quality of the cheese

The representation of the Huber weights associated to each taster (w_i^l, w_i^r, w_i^P and w_i^M) in the analysis of the four features of the cheese is provided in Figures 11, 12, 13 and 14. From them, we can conclude that to the tasters 3 and 4 are assigned, in general, smaller weights than to the other tasters, which means that their influence in the computation of the weighted mean is lower. This can be also noticed in Figures 7-10, where the trapezoids drawn by the tasters 3 and 4 describe significant higher perceptions about the quality than the other tasters. On the other hand, taster 1 presents a small influence in the case of his/her opinion about the smell quality and the overall impression (as it can also be observed in Figures 8 and 10), and the opinion of taster 6 about the flavour quality of the cheese also affect the computation of the weighted mean and for this reason its weight is smaller than the others.

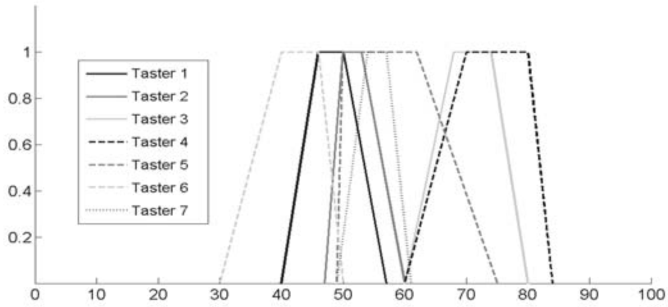

Fig. 9. Opinions about the flavour quality of the cheese

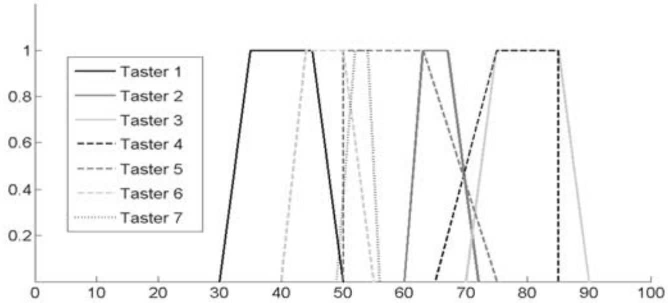

Fig. 10. Opinions about the overall impression of the cheese

Fig. 11. Representation of the Huber weights of the appearance opinions

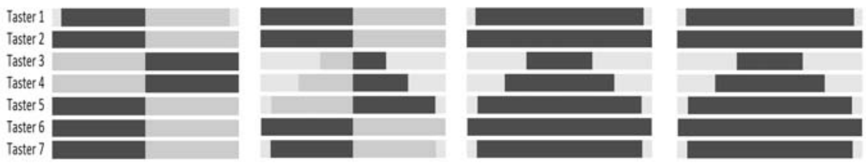

Fig. 12. Representation of the Huber weights of the smell quality opinions

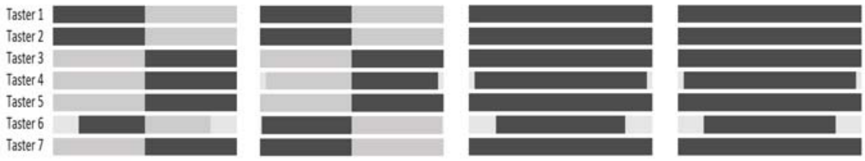

Fig. 13. Representation of the Huber weights of the flavour quality opinions

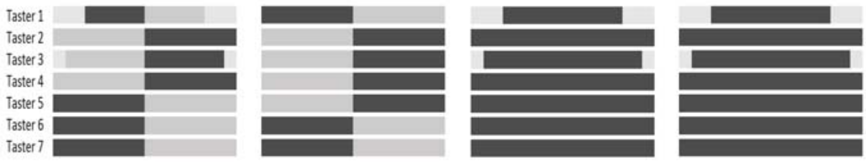

Fig. 14. Representation of the Huber weights of the overall opinions

Finally, the corresponding weighted means (taking into account the minimum t-norm) for the four features are gathered in Figure 15. It shows that the appearance and the quality of this specific cheese is not very high since the 0-levels of the trapezoids representing the weighted means for each case are between the values 47 and 63 approximately. The flavour quality and the overall impression are a little bit higher than the previous ones since the weights of tasters 3 and 4 (whose perceptions about the quality are better than the others in all the situations) are either 1 or a value closer to 1 in this two situations. In addition, the overall impression is not out of the interval [53,70] and the interval [57,65] is fully compatible with the opinions of the tasters.

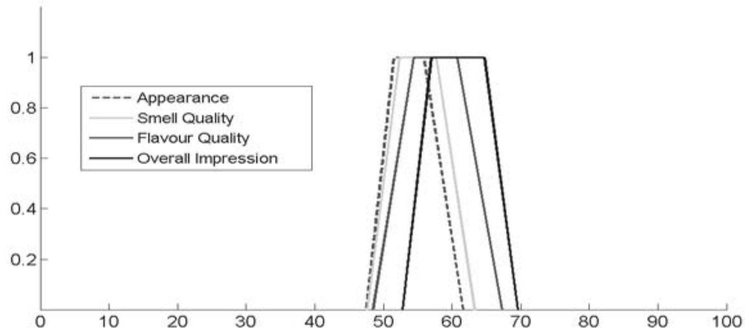

Fig. 15. Weighted means of the four features of the cheese

4 Hypothesis testing procedures applied to the quality control of the cheese

Some approaches have been developed in the literature for testing statistical hypotheses with fuzzy data. Concerning the fuzzy expectation, the one-sample test has been developed in [21,30,15], and the equality of fuzzy means of k populations (one-way ANOVA) for general RFSs has been presented in [16]. On the other hand, concerning the variance of RFSs, one-sample tests for the Fréchet variance have been analyzed in [26,35]. A procedure for testing the equality of variances (or homoscedasticity) of k populations based on ANOVA techniques has been introduced in [36].

Given a probability space $(\Omega, \mathcal{A}, P)$, let $\mathcal{X}_1, \ldots, \mathcal{X}_k$ be k independent RFSs associated with it and let $\{X_{ij}\}_{j=1}^{ni}$ be a simple random sample drawn from each RFS $\mathcal{X}_i$, where $\sum_{i=1}^{k} n_i = N$ is the total sample size. The following sample moments are defined:

- The *sample mean* associated with the $i-th$ variable is

$$\overline{\mathcal{X}_{i\cdot}} = \sum_{j=1}^{n_i} \mathcal{X}_{ij}/n_i.$$

- The *total sample mean* is given by

$$\overline{\mathcal{X}_{\cdot\cdot}} = \sum_{i=1}^{k}\sum_{j=1}^{n_i} \mathcal{X}_{ij}/N.$$

- The *sample variance* of the $i-th$ variable is defined as

$$\widehat{\sigma}^2_{\mathcal{X}_i} = \frac{\sum_{j=1}^{n_i}\left(D_\theta^\varphi\left(\mathcal{X}_{ij}, \overline{\mathcal{X}_{i\cdot}}\right)\right)^2}{n_i}.$$

- The *sample quasi-variance* of the $i-th$ variable is equal to

$$\widehat{S}^2_{\mathcal{X}_i} = n_i\widehat{\sigma}^2_{\mathcal{X}_i}/(n_i-1).$$

- The *total sample variance* is given by

$$\widehat{\sigma}^2 = \frac{\sum_{i=1}^{k} n_i\widehat{\sigma}^2_{\mathcal{X}_i}}{N}.$$

The one-sample and multi-sample tests for the mean and the variance of RFSs are recalled in the following lines.

One-sample tests

Given $A \in \mathcal{F}_c(\mathbb{R})$ and $\sigma_0 \in \mathbb{R}$, the following hypotheses are to be tested:

$$H_0^{\mathbf{1}} : E(\mathcal{X}) = A \ vs.\ H_1^{\mathbf{1}} : E(\mathcal{X}) \neq A, \tag{11}$$

$$H_0^{\mathbf{2}} : \sigma^2_{\mathcal{X}} = \sigma^2_0 \ vs. \ H_1^{\mathbf{2}} : \sigma^2_{\mathcal{X}} \neq \sigma^2_0. \tag{12}$$

To solve Test (11), the following statistic has been considered in [21]:

$$T_n^{\mathbf{1}} = \frac{n[D_\theta^\varphi(\overline{\mathcal{X}}_n, A)]^2}{\widehat{S}^2_{\mathcal{X}}}, \tag{13}$$

which converges in law to $\|\mathbf{Z}\|^2$ as n tends to ∞ - $\mathbf{Z}$ being a Gaussian variable in $\mathcal{L}^2(\{-1,1\} \times (0,1])$ with null expected value and the same covariance operator than $s_{\mathcal{X}}$, provided that $E\left(\|s_{\mathcal{X}}\|^2_{\theta,\varphi}\right) < \infty$ (where $\|\cdot\|_{\theta,\varphi}$ is a norm on the Hilbert space $\mathcal{L}^2(\{-1,1\} \times (0,1])$).

Lubiano *et al.* [26] have analyzed the problem of solving Test (12) for simple FRVs, i.e. those taking on a finite number of different values, in a particular class. These studies were extended in [35] to a wider class of non-necessarily simple FRVs leading to the following statistic:

$$T_n^{\mathbf{2}} = \frac{\sqrt{n}\left(\widehat{S}^2_{\mathcal{X}} - \sigma^2_0\right)}{\widehat{\sigma}^2_{\left(D_\theta^\varphi(\mathcal{X},\overline{\mathcal{X}_n})\right)^2}}. \tag{14}$$

where $\widehat{\sigma}^2_{\left(D_\theta^\varphi(\mathcal{X},\overline{\mathcal{X}_n})\right)^2} = \frac{1}{n}\sum_{i=1}^{n}\left(\left(D_\theta^\varphi\left(\mathcal{X}_i,\overline{\mathcal{X}_n}\right)\right)^2 - \frac{1}{n}\sum_{i=1}^{n}\left(D_\theta^\varphi\left(\mathcal{X}_i,\overline{\mathcal{X}_n}\right)\right)^2\right)^2$.

It has also been shown that $T_n^{\mathbf{2}}$ converges to a standard normal variable $\mathcal{N}(0,1)$ as n tends to ∞ whenever $E\left(\|s_{\mathcal{X}}\|^4_{\theta,\varphi}\right) < \infty$ (see [35]). The corresponding one-sided tests for the variance of an RFS can be analogously developed.

The difficulties in handling the limit distribution of the statistic $T_n^{\mathbf{1}}$ and the slow approximation to the sample distribution of $T_n^{\mathbf{2}}$ using the asymptotic approach support the necessity of developing bootstrap testing methods. Some bootstrap procedures are provided in [15,35] which have been shown to be useful for moderate sample sizes.

Multi-sample tests

Given k independent RFSs $\mathcal{X}_1, \ldots, \mathcal{X}_k$, the ANOVA and homoscedasticity tests for RFSs are stated as follows. The hypotheses to be tested are

$$H_0^{\mathbf{3}} : E(\mathcal{X}_1) = \ldots = E(\mathcal{X}_k) \ vs. \ H_1^{\mathbf{3}} : \exists\, i,j \in \{1,\ldots,k\} \text{ s.t. } E(\mathcal{X}_i) \neq E(\mathcal{X}_j), \tag{15}$$

$$H_0^{\mathbf{4}} : \sigma^2_{\mathcal{X}_1} = \ldots = \sigma^2_{\mathcal{X}_k} \ vs. \ H_1^{\mathbf{4}} : \exists\, i,j \in \{1,\ldots,k\} \text{ s.t } \sigma^2_{\mathcal{X}_i} \neq \sigma^2_{\mathcal{X}_j}. \tag{16}$$

In order to solve Test (15) for general RFSs the following test statistic has been proposed in [16]:

$$T^{\mathbf{3}}_{(n_1,\ldots,n_k)} = \sum_{i=1}^{k} n_i \left(D_\theta^\varphi\left(\overline{\mathcal{X}_{i\cdot}}.\overline{\mathcal{X}_{\cdot\cdot}}\right)\right)^2. \tag{17}$$

In [16] it is shown that $T^{\mathbf{3}}_{(n_1,\ldots,n_k)}$ converges in law to

$$\sum_{i=1}^{k}\left(\left\|Z_i-\sum_{l=1}^{k}\beta_{li}Z_l\right\|_{\theta}^{\varphi}\right)^2,$$

whenever $n_i \to \infty$ and $n_i/N \to p_i > 0$ as $N \to \infty$ for $i \in \{1,\ldots,k\}$. The definition of the Gaussian variables $Z_1,\ldots,Z_k$ and the coefficients β_{li} are also established in [16].

On the other hand, the homoscedasticity test for RFSs (16) has been introduced in [36] on the basis of the Levene's classical test for real variables. The test statistic considered in this case is

$$T^{\mathbf{4}}_{(n_1,\ldots,n_k)} = \frac{\displaystyle\sum_{i=1}^{k} n_i\left(\widehat{\sigma}^2_{\mathcal{X}_i}-\widehat{\sigma}^2\right)^2}{\displaystyle\sum_{i=1}^{k}\widehat{\sigma}^2_{(D_\theta^\varphi(\mathcal{X}_i,\overline{\mathcal{X}_i}\cdot))^2}}, \tag{18}$$

which converges in law to

$$\sum_{i=1}^{k}\left(\tilde{Z}_i-\sum_{l=1}^{k}\sqrt{p_ip_l}\tilde{Z}_l\right)^2 \Bigg/ \sum_{i=1}^{k}\sigma^2_{(D_\theta^\varphi(\mathcal{X}_i,E(\mathcal{X}_i)))^2},$$

whenever $E\left(\|s_{\mathcal{X}_i}\|^4_{\theta,\varphi}\right) < \infty$, and $n_i/N \to p_i \in (0,1)$ when $n_i \to \infty$ for all $i \in \{1,\ldots,k\}$, and $\tilde{Z}_1,\ldots,\tilde{Z}_k$ have the expressions specified in [36].

Again, bootstrap techniques have been applied and they have been shown to be useful for small and moderate sample sizes (see [16] and [36] for details).

4.1 Application to the case study

As a first study, to determine if the Gamonedo cheese deserves to keep its Protected Designation of Origin (PDO for short) it could be interesting to analyze if the quality conditions of the cheese are also kept. For this purpose, some hypothesis tests for the expected value and the variability of the opinions of three different experts (that we call Expert 1, Expert 2 and Expert 3) concerning their overall impression about the cheese are carried out. Due to the lack of previous fuzzy information to compare with, some summarizing information drawn from the categorical answers obtained in previous tastings are taken into account. Specifically, the categorical mean values are empirically associated with trapezoidal fuzzy sets. In this context, three trapezoidal RFSs are considered, namely $\mathcal{X}_i \equiv$'perception of Expert i about the overall impression of the cheese', for $i \in \{1,2,3\}$.

Suppose that we know from previous studies that the expected value of the overall impression of the cheese in past tastings was $A = Tra(60,65,70,75)$

and that the variance was approximately equal to 200. Then, in our study it could be interesting to check if the new conditions are at least as good as the previous ones. For that we will test if the corresponding expected values do not differ significantly from A and if the variabilities of those variables are reduced. Therefore, the aim is to analyze the following hypotheses:

(a) $H_0^{1,1} : E(\mathcal{X}_1) = A$, $H_0^{1,2} : E(\mathcal{X}_2) = A$, $H_0^{1,3} : E(\mathcal{X}_3) = A$,

(b) $H_0^{2,1} : \sigma^2_{\mathcal{X}_1} \leq 200$, $H_0^{2,2} : \sigma^2_{\mathcal{X}_2} \leq 200$, $H_0^{2,3} : \sigma^2_{\mathcal{X}_3} \leq 200$.

The different opinions of the three experts about the overall impression of the Gamonedo cheese are gathered in Table 2.

Firstly, the null hypotheses provided in **(a)** are analyzed. The sample means of the data gathered in Table 2 are

$$\overline{\mathcal{X}_1} = Tra(57.65, 63.18, 69.18, 73.48),\ \ \overline{\mathcal{X}_2} = Tra(47.34, 51.21, 59.86, 66.84) \ \text{ and}$$

$$\overline{\mathcal{X}_3} = Tra(57.24, 62.38, 67.95, 73.52).$$

The distance has been chosen to be D_{10}^{λ} (where λ denotes the Lebesgue measure in $(0, 1]$) since the difference between the mids is in the order of 10 times larger than the one corresponding to the spreads. Thus, if we apply the bootstrap approach introduced in [15], we obtain the p-values $p_{(1,1)} = .323$, $p_{(1,2)} = 0$ and $p_{(1,3)} = .156$. In light of these results, we can conclude that the hypotheses $H_0^{1,1}$ and $H_0^{1,3}$ are not rejected at the usual significance levels, whereas the null hypothesis $H_0^{1,2}$ is rejected at the usual levels. However, if we consider as trapezoid of reference $A = Tra(50, 50, 60, 70)$ for the Expert 2, we obtain a p-value of $p_2 = .299$, leading to a non-rejection decision of $H_0^{1,2}$ at the usual significance levels.

At a second stage, we are going to test the hypotheses $H_0^{2,1}$, $H_0^{2,2}$ and $H_0^{2,3}$ in **(b)**. The corresponding sample quasi-variances are $\widehat{S}^2_{\mathcal{X}_1} = 185.6401$, $\widehat{S}^2_{\mathcal{X}_2} = 170.8639$ and $\widehat{S}^2_{\mathcal{X}_3} = 142.964$. By applying the bootstrap procedure proposed in [35], the following p-values have been obtained: $p_{(2,1)} = .596$, $p_{(2,2)} = .699$ and $p_{(2,3)} = .953$. Therefore, the null hypotheses $H_0^{2,1}$, $H_0^{2,2}$ and $H_0^{2,3}$ are not rejected at the usual significance levels.

Other aspect to be considered in order to keep the PDO of the Gamonedo cheese is related to the agreement between several experts with respect the quality of the characteristics of the cheese. Thus, to analyze the coherence between the different tasters it is necessary to test the equality of variances and the equality of means of such opinions. We have taken into account again variables $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{X}_3$ defined as above as well as the opinions given by Experts 1, 2 and 3 which are provided in Table 2.

First, the equality of variances of the opinions of the three experts is analyzed. Then, the aim now is to test the following hypothesis:

(c) $H_0^3 : \sigma^2_{\mathcal{X}_1} = \sigma^2_{\mathcal{X}_2} = \sigma^2_{\mathcal{X}_3}$.

Table 2. Sample of the opinions of Expert 1, 2 and 3 concerning the overall impression of the Gamonedo cheese

Opinion number	Expert 1	Expert 2	Expert 3
1	$(65, 75, 85, 85)$	$(50, 50, 63, 75)$	$(60, 63, 67, 72)$
2	$(35, 37, 44, 50)$	$(39, 47, 52, 60)$	$(53, 58, 63, 68)$
3	$(66, 70, 75, 80)$	$(60, 70, 85, 90)$	$(43, 47, 54, 58)$
4	$(70, 74, 80, 84)$	$(50, 56, 64, 74)$	$(70, 76, 83, 86)$
5	$(65, 70, 75, 80)$	$(39, 45, 53, 57)$	$(54, 60, 65, 70)$
6	$(45, 50, 57, 65)$	$(55, 60, 70, 76)$	$(76, 80, 83, 86)$
7	$(60, 66, 70, 75)$	$(50, 50, 57, 67)$	$(65, 68, 73, 80)$
8	$(65, 65, 70, 76)$	$(65, 67, 80, 87)$	$(77, 80, 86, 90)$
9	$(60, 65, 75, 80)$	$(50, 50, 65, 75)$	$(76, 80, 85, 90)$
10	$(55, 60, 66, 70)$	$(50, 55, 64, 70)$	$(70, 76, 80, 85)$
11	$(60, 65, 70, 74)$	$(39, 46, 53, 56)$	$(50, 51, 55, 64)$
12	$(30, 46, 44, 54)$	$(19, 29, 41, 50)$	$(43, 47, 51, 58)$
13	$(60, 65, 75, 75)$	$(40, 47, 52, 56)$	$(50, 55, 60, 64)$
14	$(70, 75, 85, 85)$	$(54, 55, 65, 76)$	$(65, 67, 73, 80)$
15	$(44, 45, 50, 56)$	$(59, 65, 75, 85)$	$(65, 70, 75, 80)$
16	$(51, 56, 64, 70)$	$(50, 52, 57, 60)$	$(50, 55, 60, 65)$
17	$(40, 46, 54, 60)$	$(60, 60, 70, 80)$	$(65, 70, 75, 80)$
18	$(55, 60, 65, 70)$	$(50, 54, 61, 67)$	$(74, 80, 85, 90)$
19	$(80, 85, 90, 94)$	$(40, 46, 50, 50)$	$(46, 50, 55, 60)$
20	$(80, 84, 90, 90)$	$(44, 50, 56, 66)$	$(50, 57, 64, 70)$
21	$(65, 70, 76, 80)$	$(60, 64, 75, 85)$	$(65, 74, 80, 84)$
22	$(75, 80, 86, 90)$	$(54, 56, 64, 75)$	$(55, 58, 64, 70)$
23	$(65, 70, 73, 80)$	$(50, 50, 60, 66)$	$(65, 73, 80, 85)$
24	$(70, 80, 84, 84)$	$(44, 46, 55, 57)$	$(54, 57, 62, 70)$
25	$(55, 64, 70, 70)$	$(59, 63, 74, 80)$	$(73, 80, 85, 90)$
26	$(64, 73, 80, 84)$	$(49, 50, 54, 58)$	$(54, 60, 65, 70)$
27	$(50, 56, 64, 70)$	$(55, 60, 70, 75)$	$(50, 55, 60, 64)$
28	$(55, 55, 60, 70)$	$(44, 47, 53, 60)$	$(65, 74, 80, 84)$
29	$(60, 70, 75, 80)$	$(19, 20, 30, 41)$	$(40, 47, 53, 60)$
30	$(64, 71, 80, 80)$	$(40, 44, 50, 60)$	$(46, 50, 57, 64)$
31	$(50, 50, 55, 65)$	$(50, 50, 59, 66)$	$(55, 60, 65, 74)$
32	$(50, 54, 60, 65)$	$(50, 53, 60, 66)$	$(50, 57, 63, 70)$
33	$(65, 75, 80, 86)$	$(50, 52, 58, 61)$	$(40, 47, 53, 60)$
34	$(50, 55, 60, 66)$	$(60, 65, 72, 80)$	$(65, 70, 76, 80)$
35	$(40, 44, 50, 50)$	$(50, 50, 55, 60)$	$(55, 60, 65, 70)$
36	$(70, 76, 85, 85)$	$(30, 34, 43, 47)$	$(70, 74, 83, 90)$
37	$(44, 50, 53, 60)$	$(19, 25, 36, 46)$	$(60, 66, 74, 81)$
38	$(34, 40, 46, 46)$	$(53, 63, 74, 80)$	$(64, 70, 75, 80)$
39	$(40, 45, 51, 60)$		$(40, 44, 51, 56)$
40	$(84, 90, 95, 95)$		$(35, 40, 46, 50)$
41			$(35, 44, 50, 55)$
42			$(66, 70, 75, 85)$

The bootstrap procedure presented in [36] has been applied, leading to a p value of $p_{(3)} = .557$. Therefore, the null hypothesis H_0^3 is not rejected at the usual significance levels. Thus, we can conclude that there exists coherence in terms of variability between the three experts.

Since the homoscedasticity hypothesis is not rejected, the ANOVA test will be carried out to check the equality of means of the three RFSs $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{X}_3$ by using the test proposed in [16]. Then, taking into account once again the distance D_{10}^{λ}, the hypothesis to check is:

(d) $H_0^4 : E(\mathcal{X}_1) = E(\mathcal{X}_2) = E(\mathcal{X}_3)$.

By applying the bootstrap approach introduced in [16], it has been obtained a p value of $p_{(4)} = 0$. This could be due to the inclusion of Expert 2 to our study, since the sample mean corresponding to the opinions of that expert differs significantly from the ones corresponding to Experts 1 and 3. If we test the equality of means between Experts 1 and 3 by using the same procedure, we obtain a p-value of .868 which supports the agreement between those two experts.

5 Conclusions and open problems

The results provided in this work have shown that fuzzy sets and the different statistical methodologies to deal with this kind of data are a powerful tool to be employed in the field of quality control of several products. On one hand, fuzzy data are able to capture the imprecision inherent to personal valuations about features of different products. On the other hand, some classical statistical techniques could be easily adapted to the fuzzy sets framework.

As future work, further statistical studies involving fuzzy information could be carried out in the quality control field as, for instance, clustering, discriminant analysis and other classification methods, regression models, and so on. In addition, other robust procedures could be applied and compared in order to deal with anomalous or irregular data.

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