

## Research Article

# One-Step FE Model Updating Using Local Correspondence and Mode Shape Orthogonality

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In this paper, it is described how the matrix mixing model updating technique can be combined with the local correspondence (LC) mode shape expansion algorithm, to give a new finite element (FE) model updating method. The matrix mixing method uses that the inverse mass and stiffness matrices can be expressed as a linear combination of outer products of FE mode shape vectors, where the low-frequency part of these sums are substituted with expanded test modes. The approach is meant to update FE models in one-step and is exact, except for the following two approximations: the mode shape smoothing and the mass scaling of the expanded experimental mode shapes. A simulation study illustrates the errors from the two approximations and shows the ability of the technique to improve the modal assurance criterion (MAC) values so that they get very close to unity. Finally, the performance of the proposed updating method is assessed by means of an application example in which the FE model is updated based on the test modes of a real structure.

## 1. Introduction

Finite element models are numerical idealizations of real structures used in structural design or to predict the structural response under operational conditions. The accuracy of these models is highly dependent on how precise the localized and distributed imperfections are accounted for in the model. For example, joints and boundary conditions tend to be inaccurately modelled by standard components embedded in the FE software that may be different from the ones in the actual structure. Many other examples of inaccurate modelling can be found, and, in general, we accept that the discrepancies between the model and the modelled structure are not significant.

We do expect, however, that the model can simulate the structural behaviour of the modelled structure in every respect that matters. When the structure is built, we can

improve the model by adjusting the model parameters to match a subset of measured responses of the structure. This can be done by using the dynamic response of the structure [1]. A review of the existing updating techniques prior to 1993 is found in [2]. Nowadays, a commonly used updating technique is a sensitivity-based method explained in [3], in which the FE model spatial matrices are both parametrically and iteratively updated to match the experimental modal parameters estimated from vibration tests.

In this paper, a novel approach is proposed to update the FE model based on the test modal properties coming from an operational modal analysis (OMA) test [4]. Contrary to sensitivity-based updating techniques, the main advantage of this approach relies on the fact that the spatial matrices of the FE model are updated in one step, hence the name “one-step approach” was framed. The idea behind this approach is basically to replace the modal properties estimated with the FE

model by their experimental counterparts so that a good correlation between the updated FE and the test modal parameters is obtained.

In this proposed approach, the method of matrix mixing [5, 6] is used in combination with the local correspondence (LC) expansion technique [7]. The idea is to expand experimental mode shapes in a limited number of degrees of freedom (DOFs) to an entire structure using modes shapes from a FE model. Since the expansion procedure is a crucial task carried out by the one-step approach, it is also herein extensively and detailed described. In order to update the FE model in one step, the technique takes advantage of the matrix mixing method in which the inverse mass and stiffness matrices are written as a sum of outer products of mode shapes. This updating procedure is also discussed in [8, 9]. By making use of such method, the FE model modes in these sums can be replaced by the corresponding expanded test modes, yielding the updated inverse mass and stiffness matrices.

Since the inverse stiffness matrix is of interest in its own right in structural health monitoring (SHM) as shown in [10, 11], the accurate updating of  $\mathbf{K}^{-1}$  will also be of interest in SHM. The one-step algorithm presented in this paper is a perturbation technique since it relies on the LC principle [7]. The LC principle is a first-order perturbation technique based on the sensitivity equations created by Fox and Kapoor [12]. This means that one-step requires normal model updating to be done before the algorithm can be applied. The idea is that, after the application of the one-step updating approach, the updated FE model completely recreates all natural frequencies and mode shapes found from test measurements. Since no explicit mass scaling is done in the technique, errors in the modal mass are not adjusted by the technique.

Thus, the one-step technique ensures that the modal properties of the updated FE model match the ones estimated from measurements. The technique is meant to be applied only after classical FE model updating takes place, i.e., after the FE model is updated by making use of the analyst's engineering knowledge and/or of a classic sensitivity-based FE updating technique. This is to assure that the FE model being updated by the one-step approach is physically equivalent to the tested structure and that the discrepancies between model and real structure are caused by perturbations distributed over the FE model. Once these assumptions are fulfilled, the one-step approach can be applied to remove the existing discrepancies and bring the FE model closer to the experimental results.

It is worth also highlighting that, similarly to any sensitive-based FE model updating technique, the one-step FE updating approach herein proposed is suitable for cases where the structural system being updated can be modelled by a linear FE model with orthogonal mode shape vectors. This implies that the one-step technique cannot be used to update FE models of nonlinear structural systems with nonorthogonal mode shape vectors. This is the case of a fairly amount of constructed systems whose structural dynamic behaviour is dominated by nonlinearities.

The paper can be basically divided into three different parts. In the first part, the derivation and description of the one-step approach is presented. In order to illustrate the efficiency and

accuracy of the technique from a practical point of view, four application examples are presented in the second part. Finally, some remarks regarding the results obtained from the application examples are presented in the last part of the paper.

## 2. The One-Step Updating Approach

In the one-step updating approach, it is considered that the discrepancies between the modelled and the real structure are small. If this condition is satisfied, the LC will enable the mixing technique to directly update the mass and stiffness matrices of the FE model. This can be done by using the expanded test modal vectors, without introducing errors in the matrices. An iterative scaling technique ensures that the test mode shapes are correctly mass scaled relative to the new updated mass matrix. Once the test modal vectors are expanded and mass scaled, they are used to replace the modal properties estimated with the FE in order to improve the correlation between the FE and the test modal parameters. The derivation and the detailed description of the one-step approach are presented in the following subsections.

*2.1. Basic Equations.* An undamped multiple degree of freedom (MDOF) system in structural dynamics is described by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{0}, \quad (1)$$

where  $\mathbf{M}$  and  $\mathbf{K}$  are the  $N \times N$  mass and stiffness matrices, respectively.  $\mathbf{y}$  is an  $N$ -dimensional column vector containing the deformations in all DOFs of the system.  $\mathbf{y}$  is a function of time  $t$ , so we have  $\mathbf{y} = \mathbf{y}(t)$  for every instance in time. The solution of this linear system of equations is thoroughly explained in [4].

The solution  $\mathbf{y}$  is a linear combination of  $N$  mode shapes multiplied by harmonics at the corresponding eigenfrequencies. We order the mode shapes as the columns of the  $N \times N$  mode shape matrix  $\mathbf{B}$ , and the squares of the angular frequencies as the nonzero elements of the diagonal matrix  $[\omega_n^2]$ , where  $n \in 1, 2, \dots, N$ . The ordering of the frequencies on the diagonal corresponds to the ordering of the columns in the mode shape matrix. With this solution, we obtain the following orthogonality relations:

$$\mathbf{B}^T \mathbf{M} \mathbf{B} = \mathbf{I}, \quad (2)$$

$$\mathbf{B}^T \mathbf{K} \mathbf{B} = [\omega_n^2], \quad (3)$$

when the mode shapes are mass normalized [4].

*2.2. One-Step Updating Equations.* Pre- and postmultiplying equations (2) and (3), respectively, by  $\mathbf{B}^{-T}$  and  $\mathbf{B}^{-1}$ , and inverting the resulting equations yields

$$\mathbf{M}^{-1} = \mathbf{B} \mathbf{B}^T, \quad (4)$$

$$\mathbf{K}^{-1} = \mathbf{B} [\omega_n^{-2}] \mathbf{B}^T. \quad (5)$$

Equation (4) can be written as a sum of outer products:

$$\mathbf{M}^{-1} = \sum_{i=1}^N \mathbf{b}_i \mathbf{b}_i^T, \quad (6)$$

where  $\mathbf{b}_i$  is the  $i$ -th column in the mode shape matrix  $\mathbf{B}$ . If we split this sum into a sum containing contributions from the first  $m$  mode shapes, and a sum containing the remaining contributions, we have

$$\mathbf{M}^{-1} = \sum_{i=1}^m \mathbf{b}_i \mathbf{b}_i^T + \sum_{i=m+1}^N \mathbf{b}_i \mathbf{b}_i^T, \quad (7)$$

where the second sum has  $N - m$  elements.

Letting  $\mathbf{B}_1$  be the  $N \times m$  matrix with the first  $m$  mode shapes as columns and  $\mathbf{B}_2$  be the  $N \times (N - m)$  matrix with the remaining  $N - m$  mode shapes, we can write equation (7) as

$$\mathbf{M}^{-1} = \mathbf{B}_1 \mathbf{B}_1^T + \mathbf{B}_2 \mathbf{B}_2^T. \quad (8)$$

Using the same techniques, the stiffness equation (5) can be rewritten as

$$\mathbf{K}^{-1} = \mathbf{B}_1 [\omega_{m_1}^{-2}] \mathbf{B}_1^T + \mathbf{B}_2 [\omega_{m_2}^{-2}] \mathbf{B}_2^T, \quad (9)$$

where  $[\omega_{m_1}^{-2}]$  is an  $m \times m$  diagonal matrix containing the first  $m$  inverses of the squared angular frequencies and  $[\omega_{m_2}^{-2}]$  is an  $(N - m) \times (N - m)$  diagonal matrix containing the remaining  $N - m$  inverses of the squared angular frequencies. Equations (8) and (9) are central in the one-step updating approach.

**2.3. Mode Shape Expansion.** Using the notation from [4], we know from structural modification theory [13] that if we have a system with mode shape matrix  $\mathbf{B}$ , any perturbed system with mode shape matrix  $\mathbf{A}$  can be written as

$$\mathbf{A} = \mathbf{B}\mathbf{T}, \quad (10)$$

where  $\mathbf{T}$  contains the weights when writing the modes of  $\mathbf{A}$  as a linear combination of modes in  $\mathbf{B}$ . This is always possible for any modification.

Provided that  $\mathbf{A}$  has only a few columns and that the modes in  $\mathbf{A}$  are well described by the first  $m$  modes of  $\mathbf{B}$ , we can let  $\mathbf{B}$  be an  $N \times m$  matrix and use a smaller  $\mathbf{T}$  matrix of size  $m \times N$ .

If we measure the responses of the structure at a set of DOFs, we can represent the mode shape matrix  $\mathbf{B}$  as

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_a \\ \mathbf{B}_d \end{bmatrix}, \quad (11)$$

with  $\mathbf{B}_a$  and  $\mathbf{B}_d$  containing the modal coordinates of the FE mode shapes at the measured and unmeasured DOFs, respectively. We, hereinafter, designate them as active and deleted DOFs and denote them with subscripts a and d, respectively.

We can write an experimentally identified mode shape column vector  $\mathbf{a}$  as

$$\mathbf{a} \approx \mathbf{B}_a \mathbf{t}, \quad (12)$$

where the column vector  $\mathbf{t}$  is the linear combination of modes from  $\mathbf{B}_a$  that gives the identified mode. This is known from the system equivalent reduction-expansion process (SEREP) [14]. Having identified the mode shape  $\mathbf{a}$ , we solve equation (12) to get

$$\hat{\mathbf{t}} = \mathbf{B}_a^\dagger \mathbf{a}, \quad (13)$$

where  $\mathbf{B}^\dagger$  is the Moore–Penrose pseudoinverse [15] of matrix  $\mathbf{B}$ .

Inserting  $\hat{\mathbf{t}}$  into (12) gives the least squares smoothed identified mode shape:

$$\hat{\mathbf{a}} = \mathbf{B}_a \hat{\mathbf{t}} = \mathbf{B}_a \mathbf{B}_a^\dagger \mathbf{a}. \quad (14)$$

Extending equation (14) to all identified modes  $\mathbf{A}_a$  gives

$$\hat{\mathbf{A}}_a = \mathbf{B}_a \hat{\mathbf{T}}, \quad (15)$$

where  $\hat{\mathbf{T}} = \mathbf{B}_a^\dagger \mathbf{A}_a$ . Equation (15) that assumes that  $\mathbf{A}$  can be written as a linear combination of the first  $m$  modes of the full mode shape matrix is a reduced version of equation (10).

Equation (15) can now be used for expanding the measured mode shapes in  $\mathbf{A}_a$ . This is accomplished by appending the unmeasured or deleted DOFs to the linear combinations and holding  $\hat{\mathbf{T}}$  constant. This gives

$$\hat{\mathbf{A}} = \begin{Bmatrix} \hat{\mathbf{A}}_a \\ \hat{\mathbf{A}}_d \end{Bmatrix} = \begin{Bmatrix} \mathbf{B}_a \\ \mathbf{B}_d \end{Bmatrix} \hat{\mathbf{T}} = \mathbf{B}\hat{\mathbf{T}}. \quad (16)$$

Inserting  $\hat{\mathbf{T}}$  into (16) gives

$$\hat{\mathbf{A}} = \mathbf{B}\mathbf{B}_a^\dagger \mathbf{A}_a, \quad (17)$$

where  $\mathbf{B}\mathbf{B}_a^\dagger$  is the global mapping transformation matrix obtained by means of SEREP expansion [14]. The matrix  $\hat{\mathbf{A}}$  consists of the expanded test modes. The assumption in equation (16) is that the estimate of  $\hat{\mathbf{T}}$  based on the active subset is close to  $\mathbf{T}$  for the full set of DOFs. Solving equation (12) requires  $\mathbf{B}_a$  to have fewer mode shapes than DOFs to make this an overdetermined system. In case of few active DOFs, only a small set of modes can be used. A better way to solve equation (12) is to use the LC approach [7], which gives an alternative way to compute  $\hat{\mathbf{t}}$ .

**2.4. LC Expansion.** Based on the sensitivity equations [12], the LC principle [7] expands an experimental mode shape by selecting an optimal subset of the FE mode shapes in  $\mathbf{B}_a$  as expansion basis. Only FE mode shapes with frequencies close to those of the test mode shapes are considered to make sure that the number of modes is lower than the number of active DOFs. The idea behind this strategy is to obtain an overdetermined set of equations in (12) by selecting a small subset of FE modes that locally correspond to an experimentally estimated mode shape.

The best subset, or cluster, is chosen based on a resampling technique known as leave-one-out cross validation (LOOCV) [16]. The LOOCV technique leaves out one of the mode shape parameters, selects a subspace  $\mathbf{B}_{a,s}$ , of  $\mathbf{B}_a$ ,

and measures how well the expansion predicts the missing parameter. In this way, the LC principle chooses the optimal cluster based on optimality of predicting unknown responses. Each of the measured responses takes on the role of being unknown test data, one at a time. Using the LOOCV technique enables LC to do a high-level mode shape smoothing since the noise in the subspace of the optimal cluster is completely removed. This has been shown to be a significant advantage over SEREP.

It is possible to use the LC expansion in a simpler way, by only selecting a subspace of  $\mathbf{B}_a$  whose frequencies are located around that of the test mode  $\mathbf{a}$ . This subspace, containing a fixed number of modes, should then be used to compute the least squares fit and remove the elements of  $\mathbf{t}$  associated with the smallest contributions to  $\mathbf{a}$ . This is a kind of “noise floor” alternative to the traditional LC principle. The LOOCV technique will give its own optimality, and this is the principle used here.

In the simulation and test cases in this paper, there is no significant change, in terms of results, by choosing a fixed subspace of 5 modes locally around all test modes and solving equation (12) in a least-squares sense using the LC LOOCV technique. An illustration of using a 5-mode subspace in the LC expansion is shown in Figure 1. The subspace of modes used in the LC expansion of each experimental mode consists of a FE modal vector with the same natural frequency of the test modal vector being expanded, of one FE modal vector above this, and by three FE modal vectors below, giving a total of five modes. Once the subspace is defined, the LOOCV technique is then used to identify the modes that actually contribute to the expansion of the test mode shape. The empty circles indicate the modal vectors that do not contribute to expansion and, therefore, are not included in the (actual) subspaces used to expand the test mode shapes. The filled circles, on the contrary, show the modes used in the actual expansion. For test modes 1–3 fewer than 5 modes are shown since this gives a better visual illustration of the narrow-band nature of LC. The modes that are not shown are effectively unused in the expansion.

**2.5. One-Step Principle.** Referring to equation (8), the idea of one-step updating is to let the mode shapes corresponding to the lower  $m$  eigenfrequencies of a dynamical system be contained in  $\mathbf{B}_1$  and the remaining  $N - m$  mode shapes stored in  $\mathbf{B}_2$ . By modal testing, experimental modes shapes can be found. Using equation (16) or LC, the test modes can be expanded to the full set of FE DOFs. Since the modal properties of the physical structure are estimated from responses measured in a vibration test, the test mode shapes can be considered as the exact mode shapes contaminated with disturbances from, amongst other sources, measurement noise, identification errors, and errors in the expansion process. Despite these disturbances, the test mode shapes are expected to be more accurate than the FE model mode shapes contained in  $\mathbf{B}_1$ , whose accuracy is fairly affected by the modelling errors. Thus, we can use  $\hat{\mathbf{A}}$  as a substitute for  $\mathbf{B}_1$  and get an  $\mathbf{M}^{-1}$  that better corresponds to the physical structure for the lower frequencies.

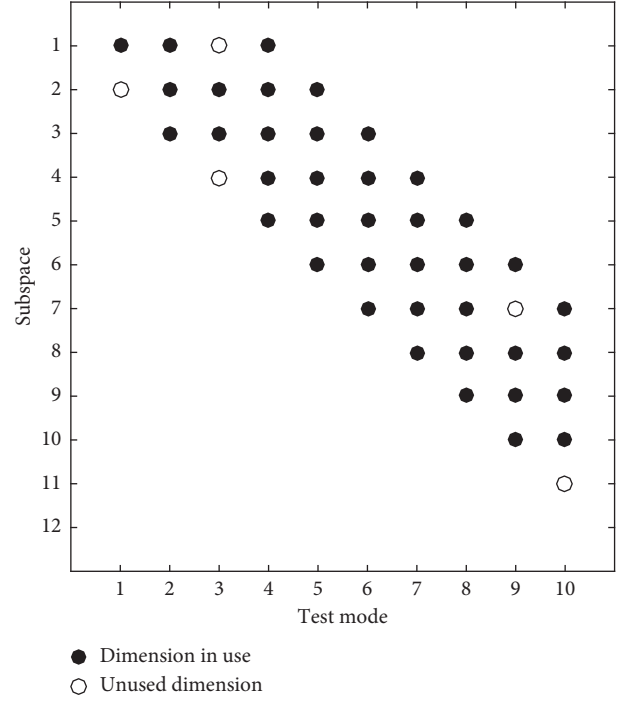


FIGURE 1: Plot of the structure of the  $12 \times 10$  matrix  $\mathbf{T}$  from equation (15). Only a limited number of FE mode shapes around the frequency of the test mode shape are used in LC. This means that the  $\mathbf{T}$  matrix will have nonzero entries in a narrow band only, illustrated by black circles. If the mode actually contributes to the expansion in this experiment, the circle is filled. Thus in this experiment, we use a 5-dimensional subspace for expansion and use anywhere from one to all five modes in the expansion, corresponding to columns of  $\mathbf{T}$ .

The same substitution can be made in equation (9), where the experimentally identified frequencies are also used.

The substitution is done by firstly using (8) to find  $\mathbf{B}_2\mathbf{B}_2^T$ :

$$\mathbf{B}_2\mathbf{B}_2^T = \mathbf{M}^{-1} - \mathbf{B}_1\mathbf{B}_1^T, \quad (18)$$

and then determining an updated mass matrix,  $\mathbf{M}_{\text{upd}}$ , by inserting  $\hat{\mathbf{A}}$  and equation (18) into (8)

$$\mathbf{M}_{\text{upd}}^{-1} = \hat{\mathbf{A}}\hat{\mathbf{A}}^T + \mathbf{B}_2\mathbf{B}_2^T. \quad (19)$$

Finally, the updated mass matrix is found by computing the inverse in equation (19). An analogous procedure is followed to determine the updated stiffness matrix,  $\mathbf{K}_{\text{upd}}$ , yielding

$$\mathbf{M}_{\text{upd}} = \left( \hat{\mathbf{A}}\hat{\mathbf{A}}^T - \mathbf{B}_1\mathbf{B}_1^T + \mathbf{M}^{-1} \right)^{-1}, \quad (20)$$

$$\mathbf{K}_{\text{upd}} = \left( \hat{\mathbf{A}}[\omega_{\text{exp}}^{-2}]\hat{\mathbf{A}}^T - \mathbf{B}_1[\omega_{m_2}^{-2}]\mathbf{B}_1^T + \mathbf{K}^{-1} \right)^{-1}, \quad (21)$$

where the diagonal matrix  $[\omega_{\text{exp}}^{-2}]$  contains the experimentally determined angular frequencies of the first  $m$  modes.

After updating, the mode shapes  $\hat{\mathbf{A}}$  are not mass scaled. This is done using the updated mass matrix  $\mathbf{M}_{\text{upd}}$ . When  $\hat{\mathbf{A}}$  has been mass scaled using the updated mass matrix from

equation (20), the scaled  $\hat{\mathbf{A}}$  can be used in equation (19) instead. Iterating this way will give correctly scaled test mode shapes in  $\mathbf{M}_{\text{upd}}$ . Only one loop is required in this fixed point iteration [17]. When correct mass scaling has been done, the frequency content of the new model, at the lower modes, can be found using the orthogonality relation in (3).

The one-step updating principle works best when the discrepancies between FE model and physical structural system are small. The reason for this is that the LC principle relies on locality coming from the sensitivity equations [12]. For small changes, only local modes contribute to the expansion of FE mode shape changes onto all the FE model mode shapes.

### 3. Application Examples

In the following sections, we explain the one-step principle by simulation and by experiment. Firstly, we show the simplest possible simulation based on a FE model. Then, we show a second simulation case in which a FE model is updated from the noise-contaminated FE mode shapes expanded with the LC expansion technique. Finally, we illustrate the technique by means of a complete experimental test case on a small specimen, where we update the FE model based on experimental modal parameters estimated from vibration measurements.

**3.1. FE Model.** For doing simulations, we have built an FE model of a T structure using FEMtools [18]. Figure 2 shows an image of the FE model from FEMtools. The model has 126 normal beam3d construction steel elements, and a spring element connecting the structure to the base. The spring has 3 rotational and 3 translational DOFs and connects the T structure to the base at the origin:  $(x, y, z) = (0, 0, 0)$ . These 127 elements are defined by 127 nodes on the structure and 1 boundary node. Each of the beam3d elements in the model has a total of 6 DOFs, i.e., three translational and three rotational DOFs. This gives a total of  $6 \times 127 = 762$  DOFs. Thus, the mass and stiffness matrices are of size  $762 \times 762$ . Half of the DOFs are rotational and half are translational.

The FEMtools model was imported into MATLAB [19] using SDTools [20], and the corresponding eigenvalue problem is solved to get the frequencies  $[\omega_n^2]$  and mode shapes  $\mathbf{B}$ . A perturbed version of the model was obtained by increasing the mass of some of the elements of the model and solving it again to get the new frequencies  $[\omega_{n,\text{exp}}^2]$  and the mode shapes  $\mathbf{A}$ . As in [4], we also use  $\mathbf{A}$  to indicate experimental mode shapes and  $\mathbf{B}$  to indicate model mode shapes. FE model updating using the one-step principle is done using equations (20) and (21), and the updated mass matrix  $\mathbf{M}_{\text{upd}}$  is adopted to mass scale the mode shape vectors in  $\mathbf{A}$ .

**3.2. Simulation without Noise.** An idealized noise-free simulation to illustrate updating using the one-step principle can be performed by using any FE model as the reference model and a perturbed version of the model as the “real” physical object. This setup is the same as having a FE

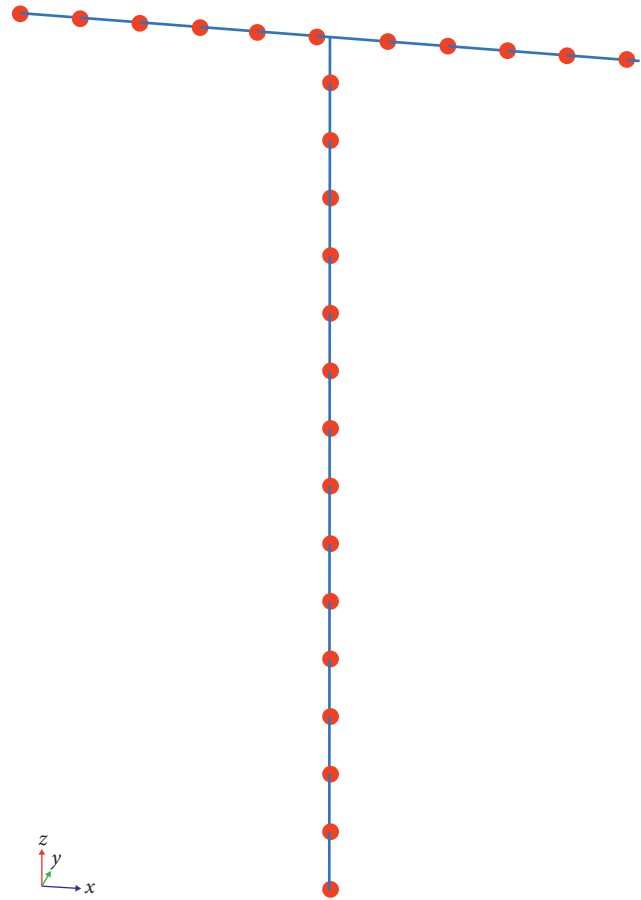


FIGURE 2: FE model of the T structure. To illustrate element size, every fifth node is marked with a red dot. The model contains 127 nodes and 1 boundary node, 126 beam elements, and one spring element. The spring has 3 rotational and 3 translational DOFs and connects the T structure to the base at the origin:  $(x, y, z) = (0, 0, 0)$ .

model which is always slightly different from the modelled structural system. In the terminology of equation (8), we start out by considering a model mass matrix  $\mathbf{M}$  constructed using outer products of all mode shapes in  $\mathbf{B}_1$  and  $\mathbf{B}_2$  combined. The mass matrix is perturbed to give  $\mathbf{M}_{\text{pert}}$ , and the stiffness matrix is unchanged.

Figure 3 shows the change in the mass matrix,  $\Delta\mathbf{M}$ , for a perturbation of the FE model corresponding to doubling the mass of one branch of the T structure. To make an intelligible plot of all  $762 \times 762$  entries in the mass matrix, the matrix was divided into submatrices of size  $15 \times 15$ . This division provides  $51 \times 51$  submatrices. Then, the maximum absolute value was chosen to describe each submatrix. Thus, the  $762 \times 762$  mass matrix is represented by the  $51 \times 51$  matrix shown in Figure 3. Note that the perturbation added to the mass matrix cannot be considered small and, therefore, the FE model does not correspond very well to the structure.

Figure 4 shows the MAC [21] value of  $\mathbf{B}_1$  and  $\mathbf{A}$ , where  $\mathbf{A}$  contains mode shapes of the perturbed system.

The one-step operation consists of substituting the mode shapes  $\mathbf{B}_1$  in  $\mathbf{M}$  by the mode shapes  $\mathbf{A}$  from the perturbed model. To do this, the perturbed model is solved and the first modes are extracted. This operation is synthesized by

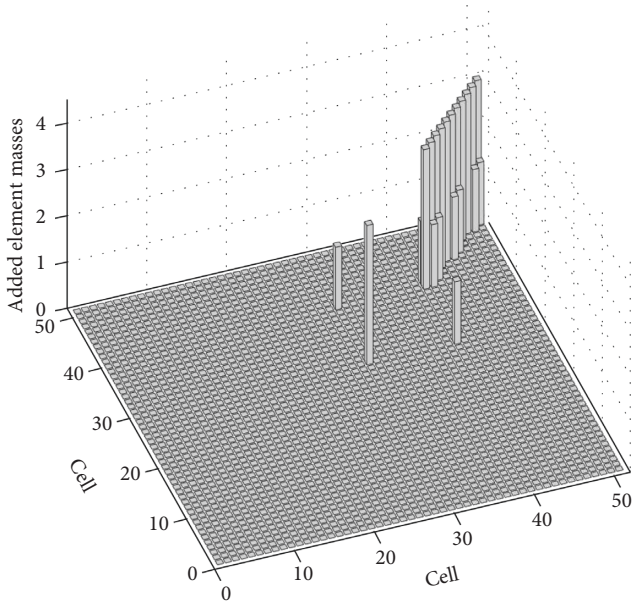


FIGURE 3:  $\Delta M$  for increasing the mass of one of the top branches of the T structure. The figure shows  $51 \times 51$  cells where the maximum absolute value in the cell is shown. Each cell is a  $15 \times 15$  submatrix of the  $762 \times 762$  matrix  $\Delta M$ . Every second element on the arm had its mass increased by 300% or three times the mass of a single element.

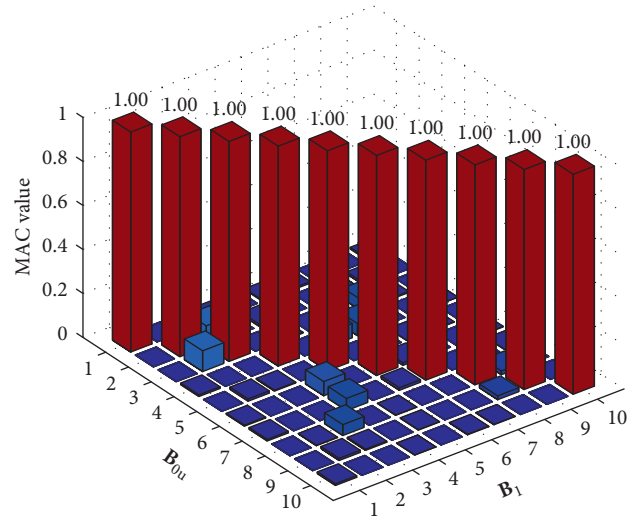


FIGURE 5: MAC value between the modal vectors from the FE model updated with the one-step updating approach and the modal vectors from the perturbed FE model. The perturbation added to the original FE model corresponds to an increase of mass on every second element on an arm of the T structure by 300%.

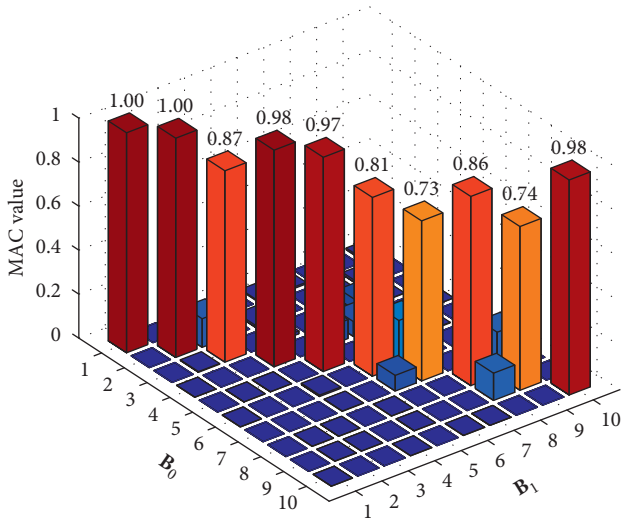


FIGURE 4: MAC value between the original and perturbed FE models before one-step updating. The perturbation added to the original FE model corresponds to an increase of mass on every second element on an arm of the T structure by 300%.

equation (20). The MAC values between the first modes of the updated and perturbed models are shown in Figure 5.

This procedure previously described constitutes one of the main ideas of the one-step approach, i.e., updating the mass and stiffness matrices so that the MAC values between the mode shapes of the updated and tested systems get close to 1. The example illustrates that, in the idealized case, this can be done perfectly. Note that since no update is done to high-frequency mode shapes, this part of the system will not be improved after updating.

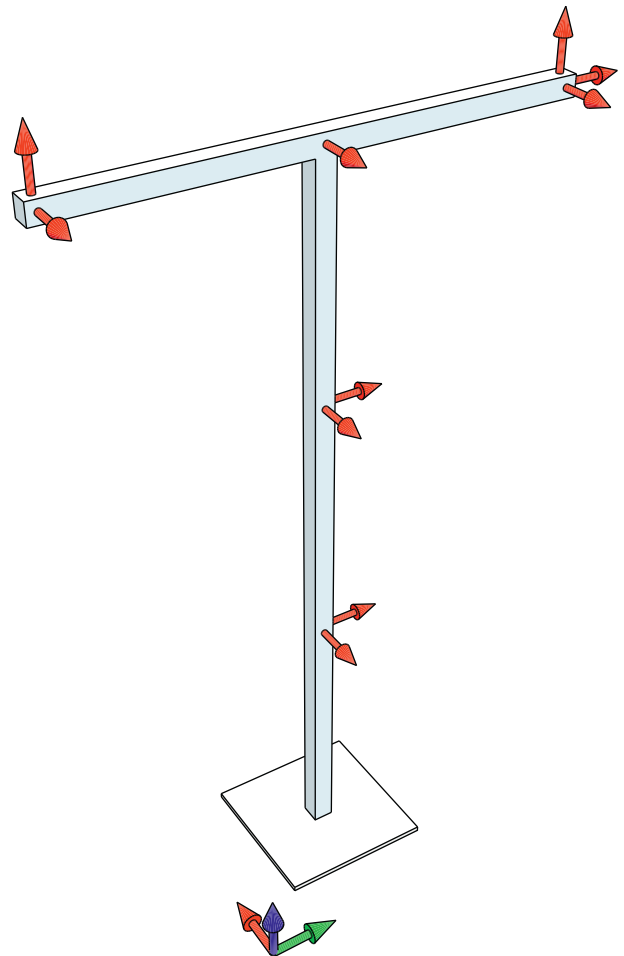


FIGURE 6: Measurement locations and directions indicated by red arrows on the T structure. Sensor groups are separated by 0.5 m. The green-red-blue arrows at the bottom of the figure denote the reference coordinate system of the T structure.

3.3. *Simulation with Noise and LC.* In this example, we extend the previous example to a complete one-step update of a structure. The example is also based on the FE model used in the previous example.

The test mode shapes in  $\mathbf{A}$  are generated as in the previous example by adding mass to one arm of the T structure and solving the eigenvalue problem to obtain the mode shapes and eigenfrequencies for the model with additional mass. In order to mimic modal parameter estimates obtained from measurements, white Gaussian noise was added to the mode shape coordinates with a standard deviation of 1%.

Since the perturbed model is known only in a relatively small subset of the 762 DOFs, in this particular case, we have used the 10 DOFs of the FE model that correspond best to the locations and directions shown in Figure 6. The 10 chosen DOFs have directions that coincide with the coordinate axes in the FE model. The chosen subset of the mode shapes  $\mathbf{A}$  are the active DOFs. This gives the test mode shapes  $\mathbf{A}_a$ . The MAC values between the test mode shapes  $\mathbf{A}_a$  from the perturbed system and the active DOFs of the model mode shapes  $\mathbf{B}_a$  from the unperturbed system are shown in Figure 7. The simulated data consisting of mass and stiffness matrices, eigenfrequencies, model mode shapes, and mode shapes from the 10 measured DOFs of the perturbed system constitute a complete setup required for one-step updating.

The FE model updating with the one-step approach is carried out in two phases. Firstly, the mode shape vectors in  $\mathbf{A}_a$  are expanded to the same size as the FE mode shape vectors of  $762 \times 1$ . This is done using LC, essentially by following the procedure synthesized by equation (17). This gives the smoothed and expanded test modes  $\hat{\mathbf{A}}$ . The MAC value between the expanded mode shapes and the original unperturbed model is shown in Figure 8.

Finally, the expanded mode shapes are inserted into equations (20) and (21) to update the mass and stiffness matrices of the unperturbed FE model. The MAC value between the mode shapes of the updated system and the perturbed system is shown in Figure 9.

This illustrates that the LC expansion has been able to express almost all perturbation of the system in terms of components of the local model modes. Thus, when updating the model with the LC expanded modes, model and test modes are very well correlated. The small drop in the MAC value visible for the 8-th mode is due to a perturbation in this mode that cannot be explained by expanding into the local modes, and therefore, LC misses a little of the perturbation. For perturbations that are significantly smaller than those related to doubling the mass of the arm of the T structure, the update can be expected to be much better. This is explained by the fact that the LC expansion technique relies on the sensitivity equations [12] that are a first-order approximation of the mode shape change under small perturbations.

The procedure previously described can be summarized in the following steps:

- (1) Extract  $\mathbf{M}$ ,  $\mathbf{K}$ ,  $\mathbf{B}$ , and  $\omega$  from the FE model

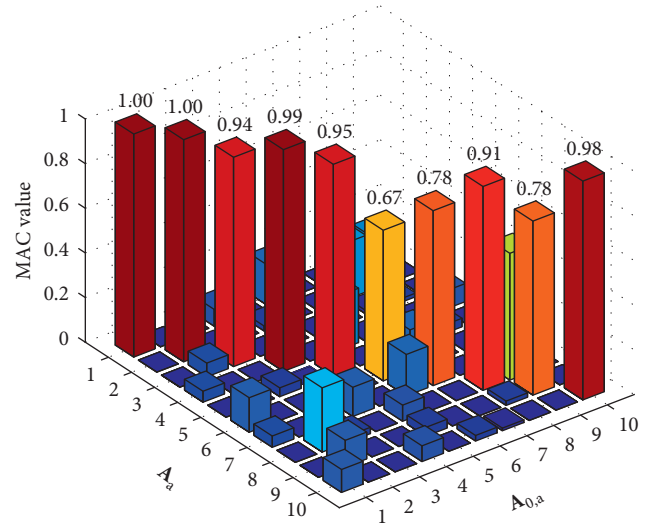


FIGURE 7: MAC between the measured mode shapes  $\mathbf{A}_a$  and unperturbed model  $\mathbf{B}_{0,a}$ .

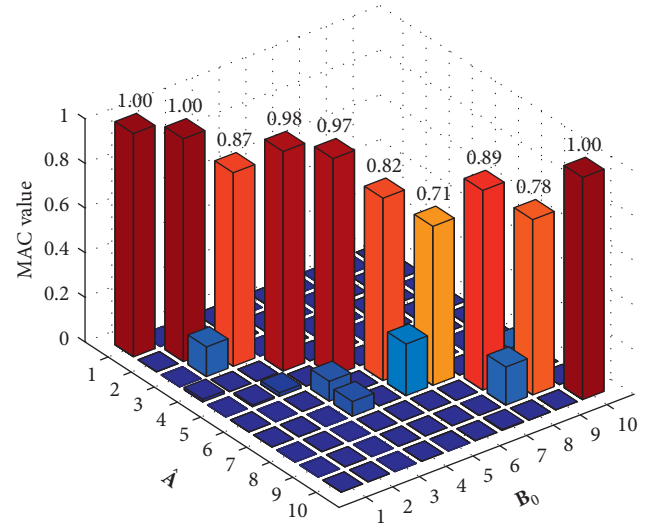


FIGURE 8: MAC between the LC expanded measured mode shapes  $\mathbf{A}$  and incomplete model  $\mathbf{B}_{0,a}$ .

- (2) Chose the subset  $\mathbf{B}_1$  and calculate  $\mathbf{M}^{-1}$  and  $\mathbf{K}^{-1}$
- (3) Determine  $\mathbf{A}_a$  and  $\omega_{\text{exp}}$  based on test data
- (4) Find LC subspace size and do LC expansion
- (5) Update  $\mathbf{M}$  and  $\mathbf{K}$  using equations (20) and (21)
- (6) Mass scale test mode shapes  $\mathbf{A}$  using the updated mass matrix in equation (20) and redo the one-step update using equations (20) and (21)

The number of modes used for LC expansion is chosen as described in [7]. For a dataset with 10 active DOFs, 4–6 modes are usually a good choice.

3.4. *Updating Test Case T Structure.* For illustrating the one-step technique for a real test case, we have used the T structure in Figure 10. It is a welded tubular steel structure constituted

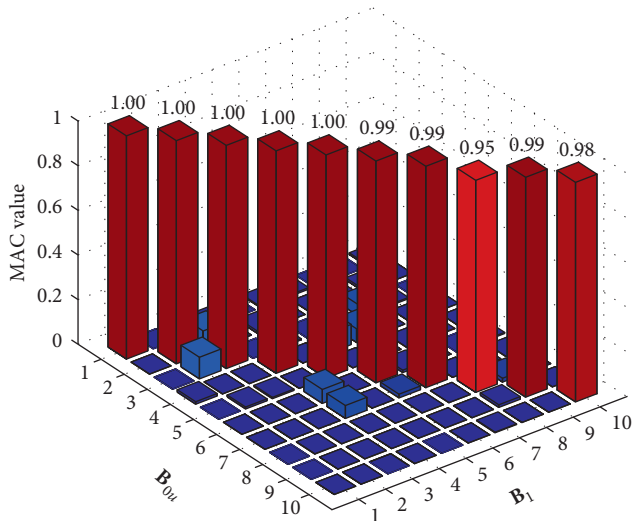


FIGURE 9: MAC values between the mode shapes from the updated FE model and the mode shapes from the perturbed FE model. The updating was done using 10 LC expanded modes. The LC expansion used 12 model modes from the unperturbed system.

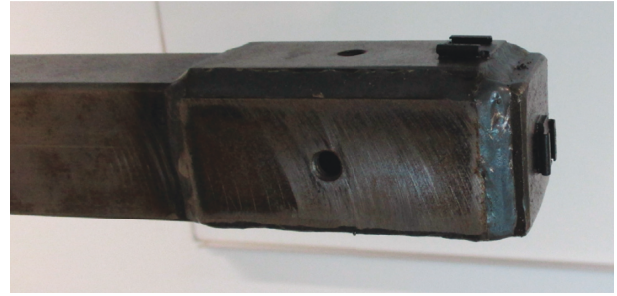


FIGURE 11: Close-up of the right arm of the T structure from Figure 10.



FIGURE 12: Close-up of the center of the T structure from Figure 10.

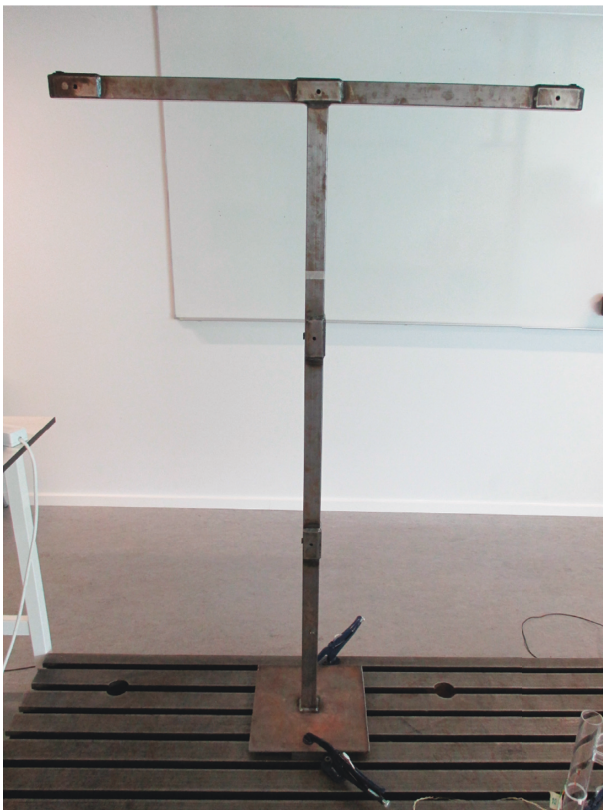


FIGURE 10: T structure used in the test experiment. The structure is a welded tubular steel structure of 2mm thick square pipes. Small masses are welded on in the areas where the sensors are placed. Sensor placement is shown in Figure 6. Close-ups of the additional masses at the sensor positions are shown in Figures 11 and 12.

of 2mm thick square pipes, with small additional masses welded on at the areas where the sensors are placed. Figures 11 and 12 show how the masses are welded on. The structure

closely resembles the FE model used in the previous simulation cases, except for the small additional masses that are not included in the model. Sensor placement and directions are shown in Figure 6. They are identical to the positions and directions chosen in the FE simulation case. For clamping, a 10 mm thick steel plate is welded to the bottom of the vertical pipe. The lower 8 modes are used to update the FE model of the T structure in this experiment.

Compared to the ideal FE model, the perturbation considered in Section 3.3 consisted of an increase of mass on one branch of the FE model of the T structure. In this section, however, the perturbation of the structure, with regard to the ideal FE model, is the small imperfections on the real T structure that are not captured by the FE model.

An output-only vibration test was conducted to estimate the experimental vibration modes of the structure. The vibration responses of the T structure were collected using three National Instruments (NI) 9234 DAQ modules with four channels each, placed in a NI cDAQ-9178 8 module carrier, with a total of 10 channels. Each channel consisted of a Brüel and Kjær 4508 Piezoelectric CCLD 100mV/g accelerometer mounted in Brüel and Kjær UA-1407 mounting clips. The NI-DAQmx driver from NI was used. The sampling frequency was set to 1652Hz. The structure was excited by air pressure coming from three different directions, to excite the structure along  $x$ ,  $y$ , and  $z$  directions. As expected, it was verified that the output data followed a Gaussian distribution. The modal identification was carried out using operational modal analysis (OMA) as described in [4]. A LabView [22] program was developed to automate the complete cycle of data collection, system identification, and saving to disk. The identification algorithms were written in MATLAB [19], and the LabView interface to MATLAB was used to make use of the algorithms.



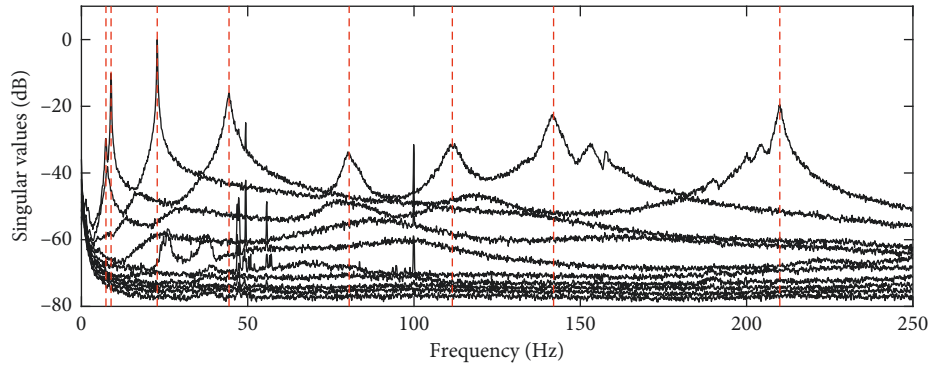


FIGURE 13: Plot of the singular values of the spectral matrix estimated from the vibration responses of the T structure. The natural frequencies of the identified modes are shown as the vertical dashed red lines.

System identification based on the Time Domain Poly Reference (TDPR) [4] algorithm was carried out to extract the natural frequencies and modal vectors from the test data. Bands containing approximately three natural frequencies were selected by filtering, and the TDPR algorithm with 10 poles was run on this block of filtered data. The physical natural frequencies of the T structure were selected, and the corresponding identified parameters were saved to the disk. For each block of data, 9 modes were saved, each consisting of a natural frequency, a damping ratio, and a 10 DOF mode shape vector. One of the 9 mode shapes was disregarded after identification as it was considered nonphysical by inspecting the mode shape animation, by comparing to the FE model modal vectors and by assessing the noise level of the identified parameters of the mode. The 8 identified physical modes physical are shown in Figure 13 as vertical dashed red lines.

The blocks of vibration data used for identification were 2 minutes long, where the length of each block was chosen according to the formula found in [4]:

$$T_{\text{recording}} > \frac{10}{\zeta f_{\min}} \approx \frac{10}{0.01 f_{\min}} \approx \frac{1000}{8} \text{ s} \approx 2 \text{ min}, \quad (22)$$

where  $f_{\min}$  is the lowest natural frequency of the structure and  $\zeta$  its corresponding damping ratio. The final identified modes were obtained as an average of 192 identifications. Typical standard deviations on the final parameters of the mode shapes were below 1/10000 for parameters of unit-scaled mode shapes.

One-step updating works best for FE models that are already in good correspondence with the modelled structure. Therefore, the FE model of the T structure was manual and iteratively updated using FEMtools [18] prior to the application of the one-step approach. This manual updating was done by placing a spring at the bottom of the model of the T structure, and optimizing stiffness in  $x$ - and  $y$ -axis directions. The Young's modulus of the steel beams was also used in the parametric optimization with FEMtools. The spring stiffness parameters and Young's modulus of the beams were updated based on the natural frequencies and mode shape vectors experimentally identified from the test data.

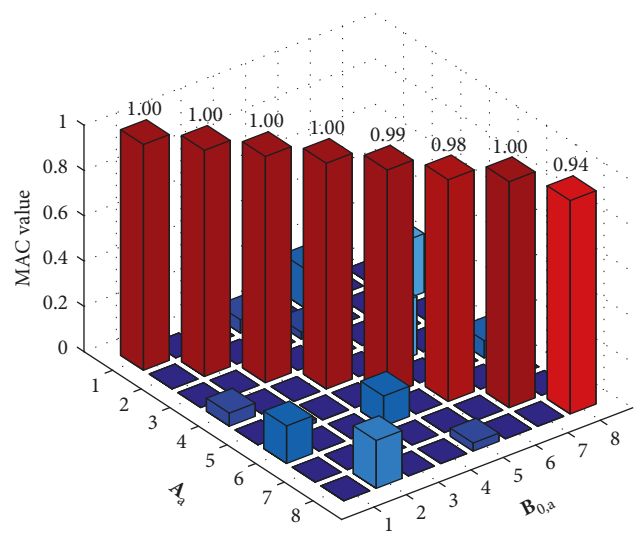


FIGURE 14: MAC between the measured mode shapes  $A$  and model  $B_{0,a}$ .

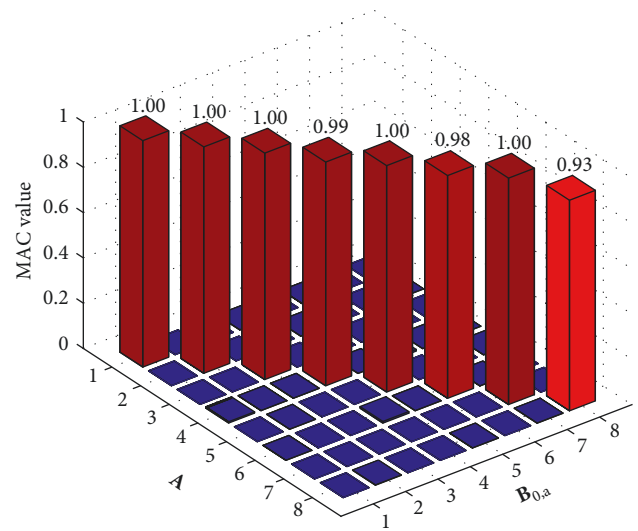


FIGURE 15: MAC between the LC expanded measured mode shapes  $A$  and model  $B_{0,a}$ .

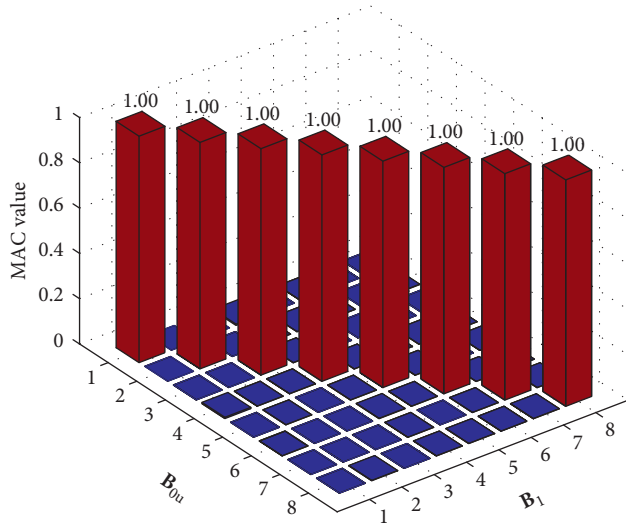


FIGURE 16: MAC values between the mode shape vectors of the FE model updated with the one-step principle and the test mode shapes. The FE model was updated with 8 modes using LC expansion for the actual test case.

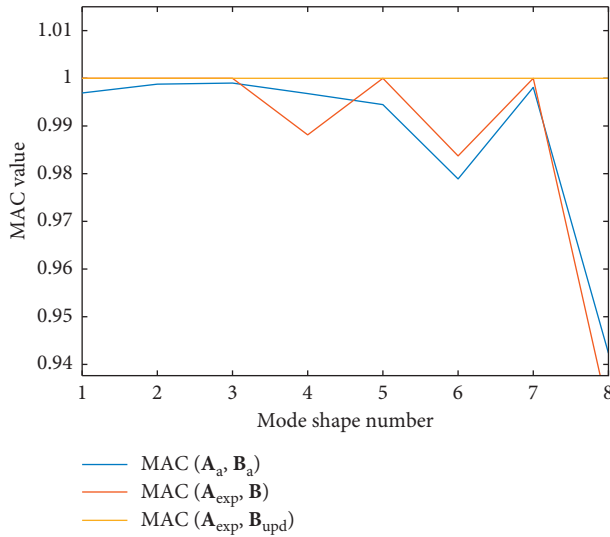


FIGURE 17: Variation of the MAC values between test and FE model mode shapes over the vibration modes: MAC values between the test and FE mode shapes (at the active DOFs only) before updating with one-step approach (blue line), MAC values between the LC expanded test mode shapes and the FE mode shapes before updating with the one-step approach (red line), and MAC values between the LC expanded test mode shapes and the FE mode shapes after updating with the one-step approach (yellow line).

The MAC values between the first 8 mode shapes estimated from test data and the corresponding modes from FE model updated with FEMtools are shown in Figure 14. These values were computed between modal vectors containing 10 modal components that correspond to the active DOFs. Doing LC expansion removes noise on the off-diagonal terms of the mass and stiffness matrices. The resulting MAC values between the model and the expanded test modes are shown in Figure 15. The MAC values

TABLE 1: Results obtained, in terms of natural frequencies, after one-step updating.

Mode	$f_{\text{exp}}$ (Hz)	$f_{\text{model}}$ (Hz)	$f_{\text{upd}}$ (Hz)
1	7.38	7.22	7.38
2	8.91	8.91	8.91
3	22.79	19.03	22.79
4	44.38	46.15	44.38
5	80.50	80.14	80.50
6	111.52	113.88	111.52
7	141.96	153.68	141.96
8	209.98	212.94	209.98
9	—	274.01	274.01
10	—	274.06	274.06
11	—	515.52	515.52
12	—	526.40	526.40
13	—	560.75	560.75
14	—	589.77	589.77
15	—	705.80	705.80

First column has experimentally determined frequencies. Second column contains frequencies from the original linear system  $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0$ . The third column contains frequencies from the updated linear system  $\mathbf{M}_{\text{upd}}\ddot{\mathbf{x}} + \mathbf{K}_{\text{upd}}\mathbf{x} = 0$ . After updating, the frequencies in the upper part of column three coincide with the frequencies in the first experimental column.

between the mode shapes computed from the FE model updated with the one-step approach and the test mode shapes are shown in Figure 16. To clearly see the result of this procedure, a plot of the diagonal MAC values of Figures 14–16 are shown in Figure 17. Analysing the latter, we see that if the LC technique successfully expands the mode, the updated system is in complete accordance with its perturbed counterpart.

The results obtained, in terms of natural frequencies, after one-step updating are presented in Table 1. The natural frequencies in the third column illustrates that, for the unscaled test mode shapes in  $\mathbf{A}$ , the frequencies  $\mathbf{A}_{\text{unsc}}^T \mathbf{K}_{0,u} \mathbf{A}_{\text{unsc}} = [\omega_{u,r}^2]$  should not be used. The frequencies in the updated model, however, are in perfect accordance with their experimental counterparts, as verified in column 1. It is worth highlighting that the frequencies below the second horizontal line in Table 1, i.e., from mode 9 onwards, have not been subjected to model updating and, thus, remained the same after updating.

## 4. Discussion

The one-step technique uses a limited number of modes and is therefore better suited to update FE models with distributed rather than with localized modelling errors. This implies that, in order to remove or minimize the localized imperfections, many experimental modes will be required. The reason for this is that higher frequency mode shapes carry more localized information than lower modes.

We have seen in Section 3.3 that when the perturbation is very large, the one-step technique will not handle all the imperfections in the FE model. This is illustrated by mode 8 in Figure 9 that has an MAC below one. The reason for this is that the LC expansion assumes small distributed

perturbations. When this assumption is not fulfilled, the one-step updating based on LC expansion can fail.

When updating a FE model using the one-step technique, the modes used for updating are mass scaled with respect to the original mass matrix. To scale the mode shapes using the updated mass matrix, the fixed point iteration explained in Section 2.5 is used. Since this procedure introduces only small changes in the mass matrix, the scaling of the mode shapes is only changed by a tiny amount, so almost no physical information in the system is lost by the fixed-point iteration.

The updating equations (20) and (21) require inversion of potentially very large matrices. Effective algorithms to do this inversion can be implemented by exploring the fact that the matrices  $\mathbf{M}$  and  $\mathbf{K}$  are sparse, symmetrical, and positive semidefinite. Alternatively, one could consider the following strategy: (1) reduce the FE spatial matrices using SEREP; (2) update the reduced compliance matrices with the test modal parameters; (3) invert the updated compliance matrices; and (4) finally, expand the resulting matrices with SEREP to obtain the full updated matrices. This strategy provides much better updating performance, both in terms of computational time and memory requirements.

Structural-change localization using the one-step technique could be developed by using a FE model to simulate the expected changes in the mass and stiffness matrices and by comparing with the changes suggested by the one-step updating procedure.

## 5. Conclusion

We have presented a method for updating finite element mass and stiffness matrices directly from experimentally identified mode shapes using FE models with local correspondence principle. The method uses the smoothed and expanded mode shapes to improve the correlation between FE model and test results. It is shown that, after mass scaling using the new updated mass matrix, the correct frequencies are found from the stiffness matrix. The method proved to work both with simulations and real vibration tests. One-step requires a fix-point mass scaling iteration and works well for non-localized changes that can be considered small distributed perturbations. It is also observed that, thanks to the LC approach, the one-step procedure introduces no errors in the higher frequency band of the model. Moreover, it is verified that, if the perturbations are small and globally distributed along the structure, the one-step technique is able to completely update the vibration modes with lower natural frequencies while keeping the modes with higher frequencies unaltered.

## Data Availability

The experimental data, i.e., the vibration responses of the T structure, used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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