Benchmarking Formula One auto racing circuits: A two stage DEA approach

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Abstract

Formula One (F1) World Championship has become one of the most successful sport tournaments over the last decade. Races take place in modern-day closed racing circuits, whose design plays a key role in racing results. This paper proposes a framework for the design efficiency assessment of the more representative racing circuits that hosted Grands Prix during the recent F1 seasons. The proposed approach considers two basic circuit features (namely, circuit length and number of turns) and combines car performance and race safety data. The methodology used is based on Data Envelopment Analysis (DEA). The number of inefficient circuits is small, five in the case of variable returns to scale and nine (out of 21) when constant returns to scale are assumed. Potential improvements in terms of speed, fuel consumption and safety targets are computed. For each inefficient circuit its reference set is identified. Also, a second-stage DEA fractional regression analysis is carried out to study the influence of the circuit type (race or street circuit), the track orientation (clockwise or anticlockwise) and the number of red-flagged races due to rainfall on the circuits' efficiency. The results indicate that all three variables are significant. The implications of the results for track designers and F1 organizers are also discussed.

Keywords: Formula One race circuits; design efficiency; Data Envelopment Analysis; SBM; fractional regression analysis

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1. Introduction

Formula One (F1) has gained significant global recognition in the last decades. F1 auto races are considered mega sporting events offering a large positive impact for host cities across different dimensions, such as branding for tourism destination, social-cultural impact and impact on the local economy (Cheng and Jarvis, 2010). The mass media have had a leading contribution to its increasing expansion worldwide (Henry et al., 2007). Thus, for example, in terms of audience, the F1 series captured 390 million viewers worldwide in 185 countries in the 2016 season (FOM, 2017).

The F1 season comprises a series of outdoors city-to-city races, known as Grands Prix, that take place annually from to March to November. The results of each race provide a rankingboth for the drivers and the car constructors. The racing position of every F1 driver/constructor depends on a number of important factors such as quality of the cars (including engine and aerodynamic performance, brakes, etc.), characteristics of the race track, talent of racing drivers, effectiveness of race strategies and others (weather conditions, tyre performance, etc.).

In particular, the F1 circuit layout is considered to be one of the most important components of the race competition. Closed-circuit racing tracks are licensed by the FIA (FédérationInternationale de l’Automobile) according to the style of track and the classes of cars to be raced on them. Currently, the FIA’s rules on circuit design are harsh, looking at the improvement in the caroperation safety as much as the car performance.

The success of the F1 World Championship as the highest auto racing tournament class derives from the performance of the drivers, constructors and the F1 race circuits. The F1 world has attracted a significant amount of attention from academics in recent years due to its highly competitive environment, partially induced by the openness to new urban environments (Lefebvre andRoult, 2011). These studies analyse various F1 issues, such as, F1 car designs and failure analysis (Jenkins and Floyd, 2001;

PerantoniandLimebeer, 2014; Savage, 2010), car drivers' injuries (MinoyamaandTsuchida, 2004), practices of intellectual capital (SolitanderandSolitander, 2010), estimation of the effect of aging on productivity (Castellucci et al., 2011), public relations strategies (Pfahland Bates, 2008), brand profiles (Rosenberger andDonahay 2008), stakeholder theory (Xue and Mason, 2011), social-cultural impacts (Cheng and Jarvis, 2010; Fairley et al., 2011; Jago et al., 2003; Liu and Gratton, 2010), etc. Although, the references cited above encompass many F1 features, contributions regarding F1 circuits are still scarce. Two exceptions are studies by Casanova et al. (2001), who propose two methods for the reconstruction of Barcelona (Spain) and Suzuka (Japan) F1 circuits based on race car speed and lateral acceleration, and Lefebvre andRoult (2011), who analyse F1 circuits' expansion in the last decades.

More research has been carried out as regards performance analysis in F1: Kladroba (2000) focuses on ordinal multicriteria methods and illustrates the case of the 1998 F1 drivers' world championship; the contribution of Gomes Junior and Soares de Mello (2007) and Chaves et al. (2010) assess2007 F1 world drivers' championship usingthe ELECTRE II multicriteria decision making method;Sitarz (2013) proposes a system of points for rankings in sports, presenting as an example the 2011 F1 championship;Phillips (2014) compares F1 driver performances during 1950-2013 using a statistical model, providing lap-time data predictions; Anderson (2014) applies three statistical models, two based on paired-comparison and one based on the rank-ordered logit function, to rank F1 driver performance in the 2012 season;Soares de Mello et al. (2015) present an adaptationof the Condorcet method in a weakly rational decision maker environment to establish the ranking of the F1 constructors in the 2013 championship; more recently, Bell et al. (2016) propose across-classified multilevel model to evaluate the F1 Driver and Constructor performance across the period 1950-2014.

There have, however, been fewer studies using a deterministic frontier analysis approach, such as Data Envelopment Analysis (DEA), to assess F1 performance, despite its wide use in many other sports,such as football (e.g. Espitia-EscuerandGarcía-Cebrián, 2010; Villa and Lozano 2016), basketball (e.g. Chen et al., 2017; Moreno and Lozano, 2014), baseball (e.g. Lewis et al., 2009), cricket (Amin and Sharma. 2014), Olympic games (e.g. Lozano et al., 2002; Wu et al., 2010), sport federations (de Carlos et al., 2017), tennis (e.g. Klaasenand Magnus, 2009; Ruiz et al., 2013), etc. To the best of our knowledge, there are only two F1 DEA studies in the literature: Gomes Junior and Soares de Mello (2007) that assesses F1 world drivers' championship using DEA (F1 season 2006), and Gutiérrez and Lozano (2014), that analyses F1 teams' performance (using Principal Component Analysis-based variable selection and DEA) over a number of F1 seasons up to 2011.

The proposed approach is related to those DEA applications in which different products are benchmarked. Thus, DEA has been used to compare printers (Doyle and Green 1991), car models (e.g. Hwang et al., 2013; Papahristodoulou, 1997), industrial robots (e.g. Braglia and Petroni, 1999), computer programs (Herrero and Salmerón, 2005), facility layouts (e.g. Ertay et al., 2006), etc. In our case, the entities to be benchmarked are F1 circuits. The attributes considered are basically three: speed, safety and environmental impact. Speed is considered because F1 events are, mainly and above all, races and speed is the key element in a race. Safety, for both the drivers and the public, is clearly also a main concern. Finally, although the duration and frequency of F1 events are limited, that is no reason for ignoring the fact that some F1 races may generate more pollution than others. The main environmental impact considered has been CO_2 emissions, which can be considered to be proportional to fuel consumption (Demir et al., 2014).

The idea behind the proposed approach for F1 circuits benchmarking is to estimate, in a non-parametric way, the relationship (i.e. the so-called production function) between key circuit design features and main circuit performance measures. This allows not only assessing the efficiency of the circuits overall, along with the speed, environmental and safety dimensions, but also, by regressing the efficiency scores obtained with some explanatory variables, the effects of these exogenous factors can be tested and estimated. In particular, it has been found that race tracks are more efficient than street racing circuits, thatanticlockwise-oriented circuits are less efficient than clockwise-oriented circuits and that circuits in rainy weather regions are less efficient than circuits in dry weather regions.

As regards the methodology used, first of all, the proposed DEA approach allows the identification of the efficient circuits, i.e. those with a superior design in terms of speed, fuel efficiency and safety. For those circuits that are found to be inefficient, their shortfalls in each of these dimensions have been assessed. The reference set (i.e. peer group) of each inefficient circuit, i.e. those efficient circuits that may act as benchmarks for improvement, are also reported. Also, fractional regression models are used for the second stage of the DEA model. The regression model analysishas been carried out to describe the association between the DEA efficiency scores and relevant characteristics of F1 circuits, i.e. the type of circuit (race or street), the track direction(clockwise or anticlockwise) and the number of red-flagged races due to rain (in the last five races). The selection of this specific methodology of the two stage regression model is based on the fractional nature of the DEA efficiency scores and overcomes several highly restrictive assumptions of linear and censored regression models.

Summarising, the efficiency of a racing circuit is a key issue in the F1 competition since the overall success of the F1 series depends on the performance of its races. This paper analyses the relative efficiency of the race circuits that host the F1 competition, both in terms of cars' performance (i.e. speed and fuel efficiency) and circuit safety. The remainder of the paper is organized as follows. Section 2 describes the proposed approach. Section 3 presents the results. The proposed approach and implications of the study are further discussed in Section 4. Finally, in Section 5 the main conclusions are drawn and further research outlined.

2. Proposed approach

2.1. First stage: DEA model

DEA is a non-parametric mathematical tool for assessing the relative efficiency of a number of comparable entities. The entities to be benchmarked can represent factories, countries, departments, industry sectors, football teams, tennis players, etc. and are usually designated as Decision Making Units (DMUs). All DMUs consume inputs in order to produce outputs. From the set of observations,the Production Possibility Set (also called the technology) is inferred using some basic assumptions such as envelopment, free disposability, convexity or scalability (e.g. Cooper et al., 2006). Considering or not the latter assumption leads to the two most common technologies, which are labelled, respectively, Constant Returns to Scale (CRS) (e.g. Charnes et al., 1978) and Variable Returns to Scale (VRS) (e.g. Banker et al., 1984) technologies.

Given the technology, DEA models aim to project the DMUs onto the efficient frontier, which corresponds to the best practice. Thus, DMUs for which no potential improvement is feasible are deemed relatively efficient and therefore belong to the efficient frontier. In contrast, those DMUs for which a reduction in input consumption and/or an increase in output production are considered feasible are assessed as inefficient and an efficiency score is computed. The efficiency score measures the distance to the frontier and depends on the estimated amount of potential input reductions and output increases. There are different DEA models depending on the technology, the projection direction and the metric used to compute the relative efficiency (Cooper et al.,2004).

A crucial step in DEA modelling is the selection of the input and output variables as everything that follows depends on that selection. As Cook et al. (2014) indicate, the selection of the inputs and outputs is not always discussed, given the importance it has, and also the selection of the proper inputs and outputs of a DEA study generally depend on the aims of the study and on the nature of the DMUs being analysed.There are a number of variables that can, in principle, be considered for an F1 DEA application. Thus, one can say that the elevation change along the circuit or the longest straight may have an effect. The run-off area available may also be a variable of interest, as it can affect the drivers' safety (Perantoni and Limebeer, 2014). Other factors that can affect the cars' performance are the altitude of the location and the weather conditions (temperature, or rain, for example) (Judde et al., 2013; Wloch and Bentley, 2004). It is difficult, however, to include the weather conditions because they are not constant for a given circuit, i.e. the temperature or the occurrence of rain is different from one season to the next, and it can be argued that these uncontrollable variables are not strictly speaking circuit design features. Regarding F1 design features, the FIA provides an updated list of requirements for the F1 circuit drawing (FIA, 2018) even providing an AUTOCAD template; in this regard, Casanova et al. (2001) develop the reconstruction of the Barcelona and Suzuka race circuits from racing car characteristics and Alnaser et al. (2007) highlight the success of the Bahrain International F1 circuit from its architectural characteristics.

After careful evaluation, the result was five main variables: two of them are key circuit design features (circuit length and number of turns) and the other three represent basic dimensions that we consider important for benchmarking F1 circuits (i.e. speed, safety and environmental impact). More specifically, as shown in Figure 1, fastest lap time (in seconds) (http: //www. fia.com) and fuel use per lap (in kilograms) (http: //f1-facts.com\results) are considered as inputs. These variables correspond to speed and fuel consumption, the latter beinga surrogate for greenhouse gases emissions and their related environmental impact. On the output side, two types of variables can be distinguished: non-discretionary outputs related to track characteristics (namely, number of turns and lap length, in kilometres) (http://fia.com), and an undesirable output related to circuit safety (namely, number of car withdrawals per 100 laps due toaccidents and collisions) (http://en.espnf1.com). The inclusion of the number of accidents as a variable in the DEA model proposed in this paper is inspired by the permanent concerns of Formula One Management Ltd. (FOM), FIA and other F1 stakeholders regarding the issue of safety.

**********************Insert Figure 1 around here**********************

The reason for considering lap length and number of turns stems from the fact that these two circuit design features affect the race (Castelluci et al., 2011; Papachristos, 2014). The fastest lap is an input which measures the extent to which the design is aimed at speed. Of course, the duration of a lap depends on the circuit length (which is why that variable is included as an input).

Note that in order to make the DMUs homogeneous and comparable all the variables are measured per lap (100 laps in the case of accidents-caused withdrawals) and that the two non-discretionary variables considered (which represent the main physical/geometric attributes of the circuit) are of the internal type (Camanho et al., 2009) and, hence, follow the Banker and Morey (1986) approach. Alternative ways of modelling this type of internal non-discretionary variable, especially for the CRS case, are discussed in Camanho et al. (2009). In the end, the inputs and outputs selected imply that a circuit is inefficient if there exists some other circuit (with the same length or longer and with the same or a larger number of turns) that involve less time, less fuel consumption and fewer accidents.

As will be commented on in Section 3, one of the variables considered (namely, the number of accidents/100 laps) refers to a certain timespan (1998-2014). Since we are very interested in considering the safety dimension of circuit design, in order to include the number of accidents that have occurred we have to consider several years because, fortunately, accidents do not occur too frequently. However, in this period some circuits have hosted more races than others. That is why the (undesirable) output is not the absolute number of accidents but it is normalized by considering how many accidents occur in every 100 laps. Thus, circuits that have held more races can be compared with circuits that have held fewer and also the benchmarking is fair because in this way this output variable refers to an intrinsic feature of the circuit design (the lap, or in the case of this variable, 100 laps).

The specific DEA model used is a weighted Slacks-Based Measureof efficiency (SBM), (Tone, 2001) DEA model, which is a common approach when some outputs are undesirable (e.g. Lozano and Gutiérrez 2011). There are different ways of modelling undesirable outputs in DEA (e.g. Färe and Grosskopf, 2003; Scheel, 2001;Seiford and Zhu, 2002). One of them is to consider the undesirable outputas weakly disposable, i.e. efficient DMUs can only reduce the undesirable output if they also reduce the desirable outputs. In particular, in the proposed approach, the weak disposability of the undesirable output is modelled using the approach in Kuosmanen (2005). Alternatively, the approach in Färe and Grosskopf (2003), which does not use separate abatement factors for the different DMUs, can be used.

In addition to the chosen SBM approach, let us recall that there are severalother types of DEA models that can handle undesirable outputs, such as the Directional Distance Function (DDF) model (e.g. FäreandGrosskopf, 2003; Lozano et al., 2013) and the Slacks-Based Inefficiency (SBI) model (e.g. Fukuyama and Weber, 2010; Lozano, 2016). Each of them has its pros and cons. A disadvantage of DDF models is that input and desirable output slacks may remain. This does not happen in SBM or SBI. DDF has the advantage of using a directional vector, which allows computing the distance to the frontier in several directions. Note that we are referring to the conventional case thatassumes that the directional vector is exogenously given and not to those DDF approaches that endogenously compute the directional vector (see Wang et al., 2017), neither to the so-called reversed DDF approach (Pastor et al., 2016)nor to the non-radial DDF model (e.g. Ferreira and Marques 2016).Although SBI also uses a direction vector, its role is more of a normalizing nature rather than defining the projection direction (Pastor andAparicio, 2010). What can be used in SBM as a surrogate of the direction vector are the weights used in the objective function. These weights are assumed to be normalized, i.e. their sum is unity.

In order to formulate the mathematical model, let

Data

$I = \{1,2\}$	set of inputs
$i \in I$	index on inputs
w_i	relative weight of improving input $i \in I$
0	index of DMU being projected
$K = \{1,2\}$	set of non-discretionary outputs
$k \in K$	index on non-discretionary outputs
$B = \{1\}$	set of undesirable outputs
$b \in B$	index on undesirable outputs
$\hat{w}_b$	relative weight of improving undesirable output $b \in B$
$J = \{1,2,...,n\}$	set of DMUs
$j \in J$	index on DMUs
x_{ij}	amount of input $i \in I$ consumed by DMU $j \in J$
y_{kj}	amount of non-discretionary output $k \in K$ produced by DMU $j \in J$
z_{bj}	amount of undesirable output $b \in B$ produced by DMU $j \in J$

<u>Variables</u>

λ_j , μ_j	multiplier variable used to compute the target inputs and outputs of DMU 0
s_i	improvement (i.e. slack) of input i
$\hat{s}_b$	improvement (i.e. slack) of undesirable output b

The proposed VRS SBM DEA model is thus,

$$\theta = Min \quad 1 - \sum_{i \in I} w_i \frac{s_i}{x_{i0}} - \sum_{b \in B} \hat{w}_b \frac{\hat{s}_b}{z_{b0}} \tag{1a}$$

s.t.

$$\sum_j \left(\lambda_j + \mu_j\right) \cdot x_{ij} = x_{i0} - s_i \qquad \forall i \tag{1b}$$

$$\sum_j \lambda_j \, y_{kj} \geq y_{k0} \qquad \forall k \tag{1c}$$

$$\sum_j \lambda_j \, z_{bj} = z_{b0} - \hat{s}_b \qquad \forall b \tag{1d}$$

$$\sum_j \left(\lambda_j + \mu_j\right) = 1 \tag{1e}$$

$$\lambda_j , \mu_j \geq 0 \quad \forall j \qquad s_i \geq 0 \quad \forall i \qquad \hat{s}_b \geq 0 \quad \forall b \tag{1f}$$

This linear programming model computes a target operating point within the production possibility corresponding to assuming VRS, non-discretionary desirable outputs and weak disposability of the undesirable outputs.The objective function provides a weighted SBM efficiency score. This type of non-radial, non-oriented DEA model has the indication property (i.e. the efficiency score is unity if and only if the DMU is efficient), apart from other properties directly inherited from using SBM efficiency, namelyunits invariance, monotonicity and reference-set dependence(see Tone, 2001).

Note that the above DEA model tries to simultaneously decrease the inputs (e.g. the fastest lap time) and the undesirable output, i.e. to increase speed and safety at the same time. This may seem contradictory and actually it is not possible if the DMU is efficient. However, the optimization model sees if there is an operating point in which that occurs and tries to maximize the weighted improvements in both the inputs and the undesirable output.

A subtler issue is the interpretation of reducing the undesirable outputs when, as in our case, the desirable outputs are non-discretionary. That a certain variable is non-discretionary means that its value cannot be changed. But that refers to a specific DMU, the one that is being assessed. However, when projecting that DMU,DEA considers the whole production possibility set, which includes all virtual operating points whose input-output mixesare theoretically possible. It is when searching among those feasible operating points that it makes sense to allow for a reduction of an undesirable output compatible with possible increases of the desirable outputs, even if those outputs are non-discretionary. That is, of course, provided that the technology (i.e. the production possibility set) considers such operating point feasible. In other words, when computing the target,the proposed DEA model does not consider a fixed operating point (whose non-discretionary outputs could not be changed) but it has freedom (within the production possibility set) to choose any operating point with its corresponding input and output variables (including the non-discretionary).

The computed DEA projection provides targets which imply potential improvements whose interpretation is the following. Given the length and number of turns of the circuit, and based on the observed dataset, it would be feasible to reduce (in the amount given by the respective slacks) the observed fastest lap time, the observed fuel consumption and the observed number of accident-caused withdrawals. In other words, for that length and number of turns, the design of the circuit is not as speedy, nor as fuel-efficient nor as safe as it could be according to the production possibility set inferred from the given observations.

For each inefficient circuit, looking at the optimal values of the λ_j variables, its corresponding reference set can be identified. This information is useful to identify, for each inefficient DMU, which circuits to consider as benchmarks.

Although the above models assume VRS, without much effort (just deleting the convexity constraint $\sum_j \left(\lambda_j + \mu_j\right) = 1$) a CRS efficiency score can be obtained. The CRS analysis, which corresponds to ignoring the possible scale size effects in the design of the circuits, always produces lower efficiency scores and a lower number of efficient DMUs.

Another interesting possibility is to estimate the maximum possible improvement along each of the three improvable dimensions. That can be done using a specific weight vector that assigns a weight of 1.0 to that dimension and 0.0 to the rest. Thus, considering a weight vector $w_1 = 1, w_2 = \hat{w}_1 = 0$ a speed efficiency score efficiency score θ_0^{speed} can be obtained. Similarly, using $w_2 = 1, w_1 = \hat{w}_1 = 0$ and $w_1 = w_2 = 0, \hat{w}_1 = 1$ a fuel efficiency score θ_0^{fuel} and a safety efficiency score θ_0^{safety}, respectively, can be computed.

Finally, in order to check for the presence of outliers in the data the method in Anh Tran et al. (2010) was applied and it was found that all the efficient DMUs had small λ-count and λ-sum indexes. When outliers were present a robust frontier method (e.g. Ferreira et al. 2018, Ferreira and Marques 2018) can be applied.

2.2. Second stage: Regression models

In order to study the impact of factors that can influence efficiency, a second stage analysis is performed regressing the efficiency scores on some contextual variables.In the scientific literature about second stage DEA efficiency analysis, several regression models have been considered. Standard linear models based on ordinary the least squares estimation procedure offer best linear unbiased estimates upon statistical distributional assumptions and biased estimates when the assumptions do not hold. In general, linear regression models are not suitable for second stage efficiency analysis because the estimates may lie outside the closed unit interval. Truncated and censored regression models based on the maximum likelihood estimates have been used to take into account the bounded nature of the efficiency score as response variable; however, those models are actually mis-specified when modelling efficiency DEA scores and are not exempted from distributional assumptions (Hoff, 2007; Simar and Wilson, 2008; McDonald, 2009; Ramalho et al., 2010). A semi-parametric bootstrapped regression model was proposed by Simar and Wilson(2008) to make inferential statements.In this study, the frontier is considered as an observed best-practice concept, hence DEA efficiency scores are treated as observed measures of technical efficiency (McDonald, 2009;Ramalho et al., 2010). Several statistical regression models are considered. A linear conditional mean model could be used to describe the DEA efficiency scores (*EFF*):

$$E(EFF_i | \mathbf{x}_i) = \mathbf{x}_i\boldsymbol{\beta} \text{ , } i = 1, 2, ..., n$$

$\boldsymbol{x}_i$ denotes the k-dimensional vector of the variables of the *i-th* DMU observation and $\boldsymbol{\beta}$a k-dimensional vector of unknown parameters. However, the DEA efficiency is not generated from a truncated process but rather is the outcome of a fractional process (McDonald, 2009).

$$E(EFF|_i \mathbf{x}_i) = H(\mathbf{x}_i\boldsymbol{\beta}) \text{ , } i = 1, 2, ..., n$$

where$H(\cdot) \in [0, 1]$ is a nonlinear function, and may adopt a logit (2.a), probit (2.b), log-log (2.c) or complementary log-log (2.d) specification.

$$H_{logit}(\mathbf{x}_i\boldsymbol{\beta}) = \frac{exp(\mathbf{x_i}\boldsymbol{\beta})}{(1 + exp(\mathbf{x_i}\boldsymbol{\beta}))} \tag{2.a}$$

$$H_{probit}(\mathbf{x}_i\boldsymbol{\beta}) = \Phi(\mathbf{x}_i\boldsymbol{\beta}) \tag{2.b}$$

$$H_{loglog}(\mathbf{x}_i\boldsymbol{\beta}) = exp(-exp^{(-\mathbf{x}_i\boldsymbol{\beta})}) \tag{2.c}$$

$$H_{cloglog}\left(\mathbf{x}_i\boldsymbol{\beta}\right)=1-exp(-exp^{(\mathbf{x}_i\boldsymbol{\beta})}) \tag{2.d}$$

One-part standard fractional regression models are statistically suitable for conducting this type of fractional regression analysis based on the following fundamentals: a) no underlying assumption is required about the conditional distribution of DEA efficiency scores or heteroskedasticity patterns; b) the specification of the model can adopt the asymmetric character of the efficiency scores; c) the estimates can be computed by quasi-maximum likelihood. In addition, fractional models proportion better performance results than other regression models when efficiency DEA scores are concentrated at unity (Papke and Wooldridge, 1996; Ramalho et al., 2010).

In order to examine the link specification for the conditional mean of efficiency scores the Ramsey RESET test (Ramsey Regression Equation Specification Error Test) is tested and uses the null hypothesis $E(EFF|\mathbf{x}_i)=H(\mathbf{x}_i\boldsymbol{\beta})$ and the alternative hypothesis $E(EFF|\mathbf{x}_i,\mathbf{z})=H(\mathbf{x}_i\boldsymbol{\beta}+\mathbf{z}\boldsymbol{\gamma})$, where $\boldsymbol{\gamma}\neq\mathbf{0}$ and $\mathbf{z}=\left[\left(\mathbf{x}\hat{\boldsymbol{\beta}}\right)^2,\left(\mathbf{x}\hat{\boldsymbol{\beta}}\right)^3,...,\left(\mathbf{x}\hat{\boldsymbol{\beta}}\right)^{J+1}\right]$ allow using a higher-order polynomial regression specification (for further details see Ramsey, 1969).

3. Assessment of F1 racing circuits' efficiency

This section discusses the dataset used and the results obtained from the DEA analysis.

3.1 Dataset

The circuits considered in the analysis, their main features and corresponding descriptive statistics are listed in Table 1. The dataset comprises 21 circuits, selected on the basis of the availability of as many DMUsas possible through the 17 F1 seasons 1998-2014. New regulations related to engines, cars, penalties and testing have taken place since then, producing a gap in the times series. Table 1 also shows the main data sources. Whenever possible, the collected data were double-checked against other F1-related sources.

The database includes all the circuits that hosted 2014 F1 races (except for Sochi Autodrom, Russia, that joined the F1 World Championship in 2014) plus three other circuits, namely Korea International Circuit (South Korea), Istanbul Park (Turkey) and Buddh International Circuit (India), that have regularly hosted F1 World Championship in the latest seasons. The track design of each of the circuits considered is considerably different (FIA, 2017). Thus, for example, Silverstone (U.K.) is a generally fast circuit with some slow corners and several fast wide turns, while Shanghai International circuit (China) features medium-speed corners and a straight that is flat out for almost 1.2 kilometres.

**********************Insert Table 1 around here**********************

The number of seasons in which a Grand Prix was held in a circuit and the number of car withdrawals/100 laps exhibit significant variability among the circuits. In this regard, the withdrawals data used consist of the number of recorded car withdrawals from the race per 100 laps, excluding technical problems (e.g., gearbox, engines, broken wing). Table 2 lists the number of registered car withdrawals due to accidents and collisions occurring in each F1 season from 1998 through to 2014. From 1998 to 2009, the number of these F1 circuits' car withdrawals represents over 60% of the total for the 1998-2014 period, while the accumulated number during the 2010-2014 period represents around 40%.

**********************Insert Table 2 around here**********************

Note that although the dataset considered includes up to the 2014 season, this does not mean, that, for example, the fastest lap input refers to the fastest lap that year. Actually, it refers to the fastest lap recorded in the history of the circuit up to 2014. Analogously, as indicated above, the number of accidents refers to the period 1998-2004. Fuel use figures are estimations and refer to the 2014 season. Although there may be some inconsistency, in that the variables do not refer to the same time interval that should not be a problem, provided that the same intervals are used for all DMUs. In particular, given the lumpy and infrequent character of accidents it has been deemed preferable to accumulate them for as long a period as possible, which is equivalent to averaging them.

In order to obtain an idea about the data distribution,Figures2 and 3 show the boxplots of the five input and output variables considered and their corresponding scatterplots, respectively. In addition, Figure 3 shows the correlation coefficient between each input and output variable. Note that the variables where there is more variability are the fastest lap and the number of turns. There is positive correlation between fastest lap and fuel use/lap, i.e. as a lap takes longer then more fuel is consumed. There are also positive correlations between fastest lap and lap length and number of turns, indicating that if the circuit length is of great length or has many turns then a lap takes longer. Fuel use/lap is positive correlated with lap length and, to a small extent, with the number of turns. The number of turns and the circuit length are only slightly positively correlated. The correlation of the number of accidents/100 laps with the fastest lap and fuel use is negative but very small. One would expect the number of accidents/lap (or per 100 laps) to depend on the average speed but the two inputs (fastest lap and fuel use/lap) are related to the average speed but mediated by the circuit length. Thus, for example, the fastest lap corresponds to the lap length divided by the average speed. Similarly, the fuel consumption grows with the average speed but also with the lap length. The correlation of the number of accidents/100 laps with the number of turns is slightly negative and with the lap length is positive, although small.

**********************Insert Figure 2 around here**********************

**********************Insert Figure 3 around here**********************

3.2. Efficiency scores

The proposed SBM DEA model has been solved for both VRS and CRS using uniform weights $w_1 = w_2 = \hat{w}_1 = 1/3$. Table 3 shows the corresponding efficiency scores as well as the potential improvements (of the discretionary variables) corresponding to the target operating points computed by the VRS DEA model. Only five circuits are found to be technically inefficient by the VRS analysis. The CRS efficiency scores are lower andassess more circuits (up to nine) as inefficient. The potential improvements estimated are rather modest (i.e. even the inefficient circuits are not far from efficiency) except in the case of circuit C19, for which significant improvements are deemed possible.

**********************Insert Table 3 around here**********************

Table 4 shows, for the five technically inefficient circuits, their reference set and corresponding optimal values of λ_j and μ_j variables. The reference set represents the subset of efficient DMUs from which the target efficient operating point is computed (using the optimal values of the λ_j and μ_j variables as coefficients of the corresponding convex linear combination). Note that the μ_j variables are always zero and that each inefficient circuit has a different reference set. In some cases, as in the cases of C3, C15 and C19 there is a main benchmark (whose corresponding λ_j is close to unity) and thus represents the specific efficient circuit that should be taken as the basic reference. In the cases of C5 and C21, however, there is no a single main benchmark but several. Note that among the different efficient circuits the ones with a higher peer count (i.e. that intervene in the reference set of more inefficient DMUs) are C10, C16 and C20. Note also that there are several efficient circuits (namely C1, C4, C8, C13, C14, C17 and C18) that do not belong to any of the reference sets of the inefficient circuits.

**********************Insert Table 4 around here**********************

Figure 4 shows the variable-specific efficiency scores along the speed, fuel consumption and safety dimensions. The average value for each dimension is also shown so that the circuits with efficiency scores above or below the average can be identified. Note that the VRS efficient DMUs are efficient in all these three specific dimensions. Note also that $Aver.\theta^{speed} = 0.992 > Aver.\theta^{fuel} = 0.974 > Aver.\theta^{safety} = 0.939$. This means that safety is the dimension with the largest efficiency improvement potential, followed by fuel consumption. In contrast, the efficiency as regards speed is very high. The only circuit that seems to have some improvement potential in that dimension is C19 (Korea International circuit).

**********************Insert Figure 4 around here**********************

3.2. Second-stage analysis

In a second stage, a regression analysis has been carried out to understand why F1 circuits differ in their efficiency scores and to investigate the effects of contextual variables. The variables considered in the models include twodesign-related variables, namely the type of circuit (i.e. street or race circuit) andthe track orientation (clockwise or anticlockwise),as well asa proxy climate variable, represented by the number of red-flagged racesdue tounsafe track conditions caused by rainfallin the past five races. As shown in Table 1, the dataset includes four street circuits (namely, Albert Park, Monte Carlo, Gilles Villeneuve and Marina Bay) while the rest are race circuits. Similarly, from a track orientation perspective, a majority of the circuits has a clockwise orientation, with the exception of Marina Bay, José Carlos Pace, Yas Marina and Korea International. The hazard of unsafe track conditions caused by the rainfall is not a frequent event, except forSepang, Albert Park, Circuit of Catalunya,Suzuka and Korea International.

The corresponding fractional models were estimated using *frm*(Ramalho, 2017), an R package.The quasi-maximum likelihood estimation results of the fractional models for the different functional specifications, besides the Ordinary Least Squares (OLS) linear regression model estimation, are presented in Table 5.

The estimation results of the linear model and the fractional models differ significantly. In the OLS linear regression, the number of red-flagged races due to unsafe track conditions caused by rainfall is the only variable considered statistically significant and the percentage of efficiency variability explained by the linear model is 45.1% indicating a poor model fit. In addition, the 55% of the estimated efficiency scores in OLS linear model do not belong to (0,1].

However, all four fractional regression models identify as statistically significant the three variables considered, with no disagreement between the sign of theireffects. The coefficient of the type of circuit is highly statistically significant, and negative, in all fractional models.This means the efficiency scores of street circuits and race circuits are different, i.e. that street circuits have better efficiency results than race circuits. As regards the red-flagged races due to the rain variable, the results indicate that it can significantly affect the track conditions and reduce the efficiency of F1 circuits, i.e. rain likelihood also has a negative effect on circuit efficiency. Clockwise orientation, on the other hand, affects the circuit efficiency score positively.Moreover, the fractional regressions models describe a better association between the observed model and the estimated model than the OLS linear model.

The RESET test, using second order (J=1) and the Lagrange multiplier version, reveals that as fractional models specifications do not differ greatly, each of these could be chosen to perform second stage efficiency scores (p-value$_{\text{Logit}}$=0.273; p-value$_{\text{Probit}}$=0.202; p-value$_{\text{log-log}}$=0.279; p-value$_{\text{clog-log}}$=0. 106).

**********************Insert Table 5 around here**********************

4. Discussion

From the selection of the input and output variables it can be seen that the proposed DEA approach corresponds to considering each circuit as an entity that consumes time and fuel to produce a lap that has a certain length, certain number of turns and a certain probability of accident. The latter is considered an undesirable output so that the smaller the better. As regards the other two outputs, they have been considered non-discretionary because doing otherwise would imply that, *ceteris paribus*, a circuit would be more efficient if it is longer and has more turns, something which we do not mean. The proposed model considers that, *ceteris paribus*, a circuit is more efficient if it is faster, safer and less polluting. Also, the non-discretionary character of the length and number of turns outputs implies that we are not considering the possibility of remodelling the circuit, i.e. we are benchmarking the current circuit designs.

An interesting question, posed by one of the reviewers, is that some of the variables used for benchmarking the circuits (such as speed or fuel consumption) are more dependent on drivers and constructors than on the circuits themselves. Thus, while some of the variables (i.e. number of turns and circuit length) completely fall under the responsibility of the circuits, others (such as the fastest lap, the fuel consumption or the number of accidents) also depend on the drivers and constructors. However, provided that the drivers and constructors are the same in all the races, the differences in fuel consumption, fastest laps and accidents between the different circuits can be attributed to the circuits themselves. Therefore, considering those variables for circuit benchmarking is a reasonable assumption that, admittedly, ignores that actually the drivers vary over time and sometimes even within a season or that the cars suffer modifications and improvements between seasons and even between races.

Since the different stakeholders have differing (and sometimes even conflicting) aims, a different DEA model may result depending on the perspective adopted. The perspective adopted in this paper is that of the public in general and auto racing fans in particular. It is assumed that they are interested in three main aspects: speed, safety and environment; hence the three discretionary variables that the DEA model tries to improve. However, that the model focuses on one stakeholder does not mean that other stakeholders may not also be interested in some of these aspects. The best example is safety, which is a concern probably shared by all stakeholders. Note also that the weighted nature of the proposed SBM model allows taking into account some preference structure among the three aims considered. In the paper we have reported the results for the equal weights case as well as for giving all the weight to each of the three aspects separately. Of course, the corresponding results only vary in the case of inefficient circuits, as for the efficient circuits the results are the same for any weighting.

About the results obtained, we have, on the one hand, the identification of the circuits that are inefficient, the assigning of an overall efficiency score, the quantification of their margin for improvement in each dimension and the reference DMUs they can use as benchmarks. On the other hand, from the second stage, it has been found that race tracks are more efficient than street racing circuits, that a clockwise orientation increases the efficiency of the circuits and that weather conditions (particularly, rain) negatively affect the circuit efficiency. These findings can be useful for track designers and F1 organizers. Thus, hosting races in street racing circuits should be avoided as they lead to more fuel consumption, more accidents and a lower average speed than an equivalent race track (with the same length and number of turns). Also, when designing the circuit it is preferable, *ceteris paribus*, to adopt a clockwise orientation as this leads to faster laps, less fuel consumption and increased safety. And, finally, when choosing a location for a race (or when scheduling the different Grands Prix of a season) it should be borne in mind that bad weather affects efficiency negatively byincreasing accidents and fuel consumption and reducing speed. Two of the above recommendations/effects are reasonable and to a certain extent unsurprising. The fact that they have been confirmed empirically is nevertheless interesting and supports the validity of the proposed approach. As regards the influencing effect of the orientation of the circuit, this finding is new and calls for further study to find the reasons behind it.

5. Conclusions

In this paper a benchmarking model for F1 circuits has been proposed. It involves carrying out an efficiency assessment of the circuits'designs along three key dimensions: speed, fuel consumption and safety. Efficient circuits can be identified and for the inefficient ones specific targets for improvement as well as a reference set are computed. In addition, separate speed, fuel and safety efficiency scores have been determined. CRS efficiency scores have been also computed. The results show that, in the VRS case, all but five circuits are technicallyefficient. In the CRS case four additional circuits are deemed inefficient.When considering each dimension of improvement separately then significant potential improvements have been estimated for the inefficient circuits. The inefficiencies, i.e. the margins for improvement, are highest for the safety dimension and lowest for the speed dimension. These are general remarks about the obtained results but specific figures for each inefficient circuit are provided by the proposed approach, allowing a case by case analysis of the results of each circuit.Moreover, since the proposed approach can rank the circuits based on their efficiency score and since, now and in the future, it can happen that there are more circuits than the actual number of races that can take place in a given season, the circuits' efficiency scores might be used, together with other factors, to select the circuits to be included in the F1 championship. The proposed DEA model can also be used to estimate the changes in fastest lap, fuel consumption and number of accidents to be expected as a result of a circuit redesign (e.g. removing or adding a turn).

The DEA efficiency scores have beenregressed, using a fractional regression model, to measure the influence of the circuit type, track orientation and rainfall likelihood on the circuit efficiency. The results of this second-stage DEA regression indicate, with a reasonable goodness of fit, that all three variables appear to significantly affect efficiency across F1 circuits, with the corresponding implications for track designers and F1 organizers.

There are, however, limitations of the study that one should be aware of. Some circuits could not be included in the analysis due to missing data on input or output variables. Also, even though the number of withdrawals due to accidents and collisions used as a measure of safety excluded the mechanical failure causes, not all the accidents and collisions may be ascribed to circuit safety issues. Thus, some accidents may have been due to driver error and not the fault of the circuit design and maintenance. This means that the computed safety efficiency may have been underestimated.

Finally, as topics for further research, we can mention the possibility of selectinga different set of variables, reflecting the perspective of some other stakeholder. Also, a network DEA approach that considered two stages in series, namely a design stage and a racing stage, each one with its corresponding inputs, outputs and intermediate measures, can also be conceived.

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List of figures and table captions

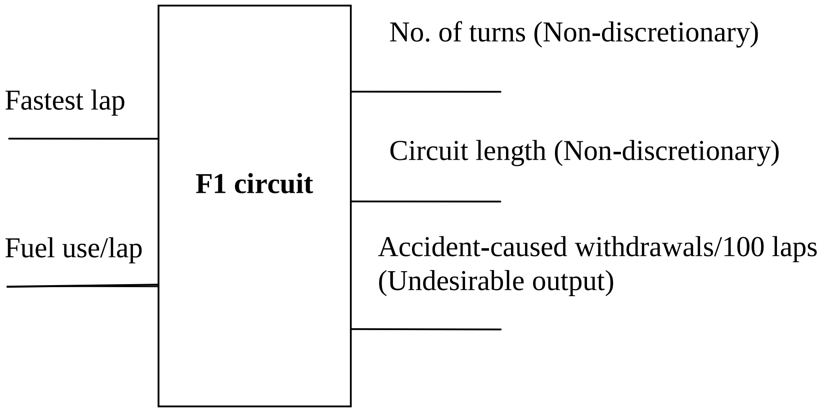


Figure 1. DEA inputs and outputs considered

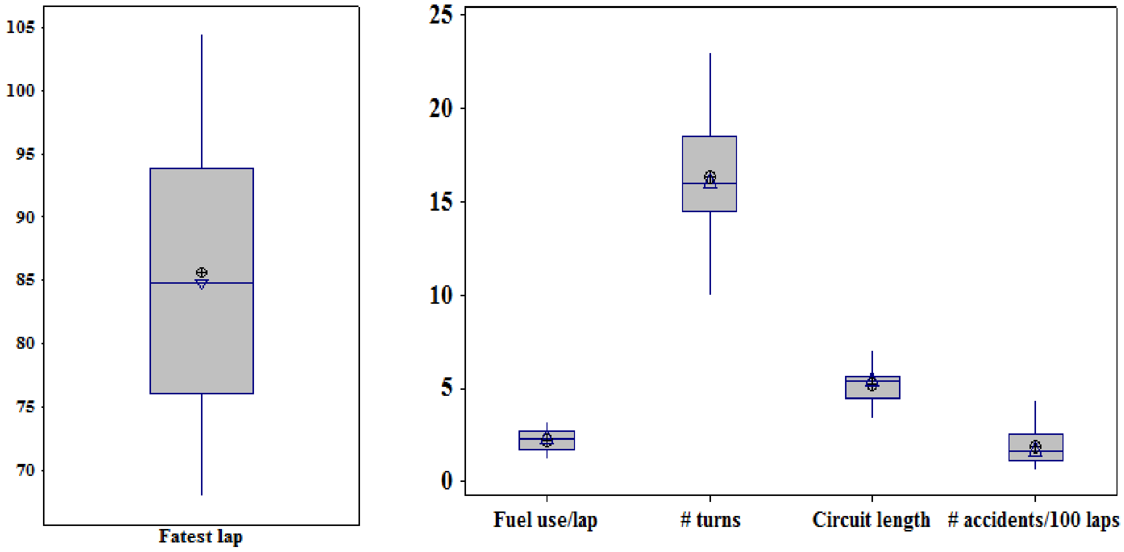
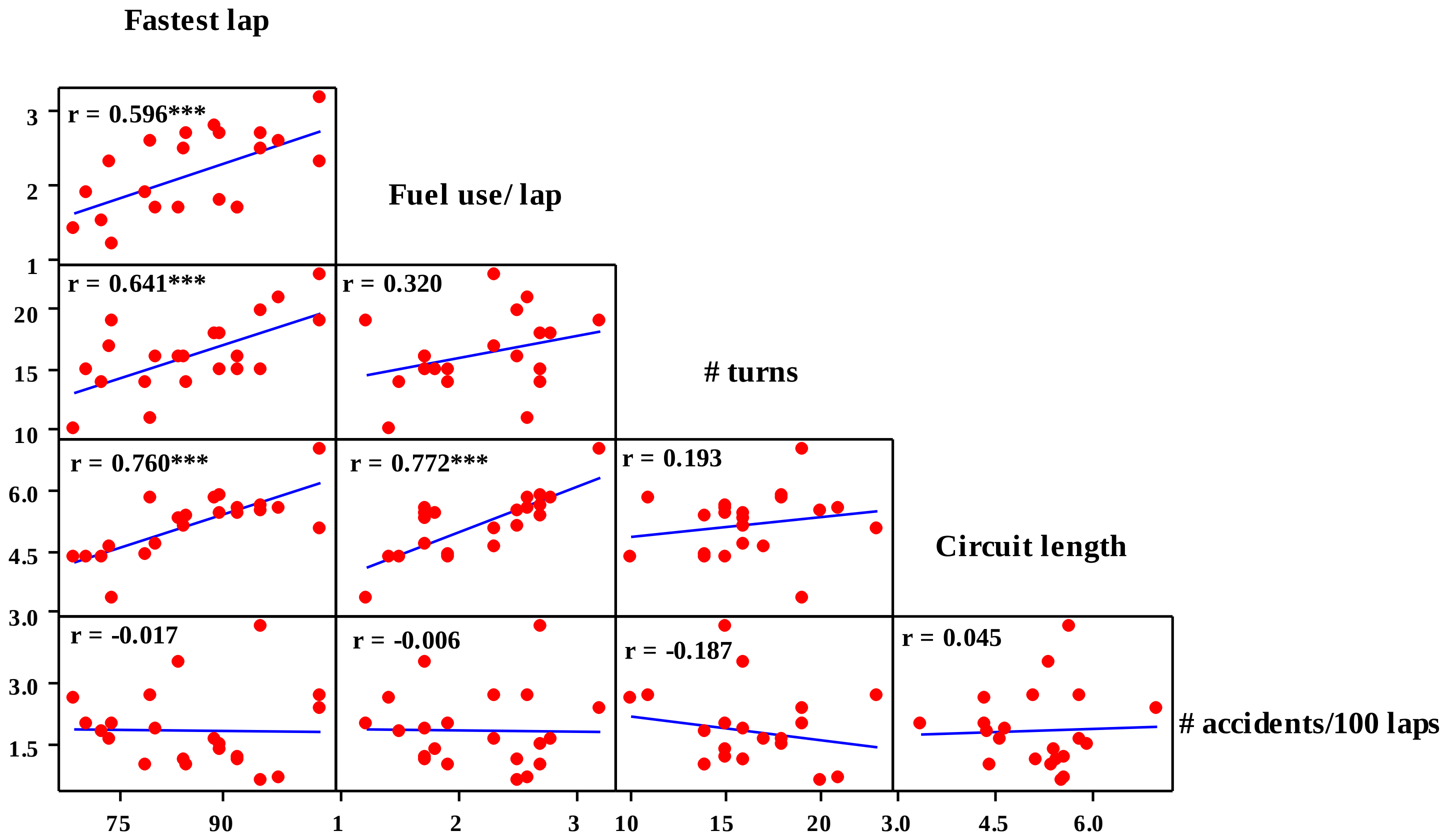


Figure 2. Boxplots of inputs and output variables



Note: "r" indicates the Pearson correlation coefficient. "***" indicates statistical significance at the 1% levels.

Figure 3. Scatterplots of input and output variables

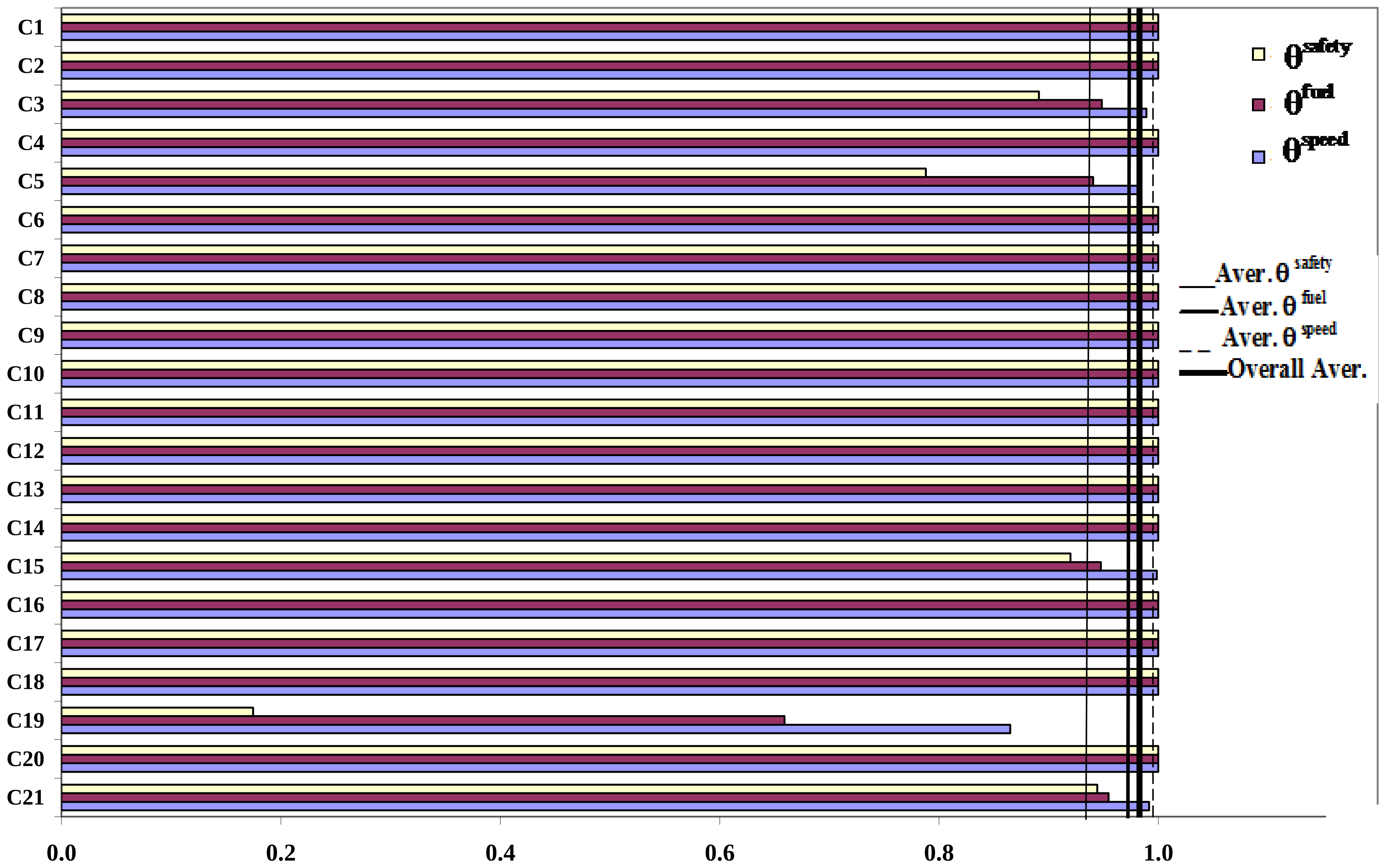


Figure 4. Variable-specific directional distance function along safety, fuel and speed dimensions

Table 1. Dataset of auto racing circuits in Formula One motor racing series (*N*=21)

DMU	Country	Circuit name	Circuit type	Track direction	No.of red-flagged F1 races due to rain (in the last five races)	No. of times hosting a Grand Prix[a]	No. of turns	Lap length[b]	Fastest lap[c]	Fuel use per lap[d]	# accidents-caused withdrawals per 100 laps [e]
C1	Australia	Albert Park	Street	Clockwise	1	19	16	5.303	83.529	1.7	3.5
C2	Malaysia	Sepang	Race	Clockwise	1	16	15	5.543	92.282	1.7	1.2
C3	Bahrain	Sakhir	Race	Clockwise	0	10	15	5.412	89.527	1.8	1.4
C4	China	Shanghai	Race	Clockwise	0	11	16	5.451	92.238	1.7	1.1
C5	Spain	Circuit of Catalunya	Race	Clockwise	1	24	16	4.655	79.954	1.7	1.9
C6	Monaco	Monte Carlo	Street	Clockwise	0	61	19	3.340	73.532	1.2	2.0
C7	Canada	Gilles Villeneuve	Street	Clockwise	0	35	14	4.361	72.275	1.5	1.8
C8	Austria	Red Bull Ring	Race	Clockwise	0	26	10	4.326	67.908	1.4	2.6
C9	Great Britain	Silverstone	Race	Clockwise	0	48	18	5.891	89.615	2.7	1.5
C10	Germany	Hockenheimring	Race	Clockwise	0	33	17	4.574	73.306	2.3	1.6
C11	Hungary	Hungaroring	Race	Clockwise	0	29	14	4.381	78.436	1.9	1.0
C12	Belgium	Spa-Francorchamps	Race	Clockwise	0	47	19	7.004	104.503	3.2	2.4
C13	Italy	Monza	Race	Clockwise	0	64	11	5.793	79.525	2.6	2.7
C14	Singapore	Marina Bay	Street	Anticlockwise	0	7	23	5.065	104.381	2.3	2.7
C15	Japan	Suzuka	Race	Clockwise	1	26	18	5.807	88.954	2.8	1.6
C16	USA	De las Americas	Race	Anticlockwise	-	3	20	5.513	95.657	2.5	0.6
C17	Brazil	José Carlos Pace	Race	Anticlockwise	0	32	15	4.309	69.822	1.9	2.0
C18	Abu Dhabi	Yas Marina	Race	Anticlockwise	0	6	21	5.554	98.434	2.6	0.7
C19	South Korea	Korea International	Race	Anticlockwise	0	4	15	5.621	95.585	2.7	4.4
C20	Turkey	Istanbul Park	Race	Anticlockwise	-	7	14	5.338	84.771	2.7	1.0
C21	India	Buddh	Race	Clockwise	-	3	16	5.125	84.178	2.5	1.1
					Mean	24.3	16.3	5.160	85.638	2.16	1.8
					Median	24	16	5.338	84.771	2.30	1.6
					St.Dev.	18.7	3.1	0.780	10.886	0.55	0.9
					IQR[f]	27.0	4.0	1.110	17.95	0.95	1.4

Sources: http: //www. fia.com; http://en.espnf1.com/; http://f1-facts.com\results (last accessed December, 2017).

Notes: [a]from 1950 till 2014; [b] in kilometres; [c] in seconds; [d] in kilograms; [e] covers the period 1998-2014 and excludes mechanical failure causes; [f] Interquartile range

Table 2. Number of car withdrawals from the race due to accident/collision causes during the period 1998-2014

Circuit	Circuit name	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001	2000	1999	1998
C1	Albert Park	1	0	0	1	3	1	8	2	3	2	0	0	8	2	0	3	1
C2	Sepang	1	0	0	1	0	0	1	1	1	2	0	0	0	0	3	1	
C3	Sakhir	3	0	0		1	0	2	2	0	0	0						
C4	Shanghai	0	0	0	0	3	1	1	0	1	0	1						
C5	Circuit of Catalunya	0	0	0	1	0	4	5	1	0	0	0	4	0	2	2	0	2
C6	Monte Carlo	1	0	4	3	3	2	2	0	0	1	5	0	3	1	0	1	1
C7	Gilles Villeneuve	2	0	0	3	0		3	1	2	0	1	1	0	3	1	2	2
C8	Red Bull Ring	0											0	3	0	5	0	5
C9	Silverstone	3	0	1	0	1	2	0	1	3	0		0	0	2	0	0	0
C10	Nürburgring	1		0		0	0	1	0	0	0	0	5	0	3	4	2	0
C11	Hungaroring	4	0	0	0	0	0	0	1	2	2	0	1	0	0	0	0	0
C12	Spa-Francorchamps	0	1	5	0	3	4	0	0		3	5		0	0	1	0	5
C13	Monza	0	1	0	4	0	1	1	1	0	0	1	0	2	1	5	3	0
C14	Marina Bay	0	0	2	2	5	1	0										
C15	Suzuka	2	2	1	0	4	0	1	0	0	1	2	0	0	1	0		2
C16	De las Americas	0	1	0														
C17	José Carlos Pace	0	1	0	0	0	4	1	2	1	2	1	0	1	1	3	2	0
C18	Yas Marina	0	0	3	0	0	0											
C19	Korea International	0	0	2	0	10												
C20	Istanbul Park				0	1	0	2	0	1	0							
C21	Buddh		1	0	1													

Source: http://en.espnf1.com/

Table 3. Efficiency scores and potential improvements

Circuit	VRS				CRS
	Efficiency score	Potential reductions			Efficiency score
		Fastest lap	Fuel use/km	# accidents/100 laps	
C1	1.000	-	-	-	1.000
C2	1.000	-	-	-	1.000
C3	0.963	0.0	0.0	0.1	0.938
C4	1.000	-	-	-	1.000
C5	0.929	0.000	0.0	0.4	0.920
C6	1.000	-	-	-	1.000
C7	1.000	-	-	-	1.000
C8	1.000	-	-	-	1.000
C9	1.000	-	-	-	1.000
C10	1.000	-	-	-	1.000
C11	1.000	-	-	-	0.833
C12	1.000	-	-	-	0.954
C13	1.000	-	-	-	1.000
C14	1.000	-	-	-	0.820
C15	0.958	0.000	0.1	0.1	0.957
C16	1.000	-	-	-	1.000
C17	1.000	-	-	-	0.921
C18	1.000	-	-	-	1.000
C19	0.640	2.651	0.9	3.1	0.631
C20	1.000	-	-	-	1.000
C21	0.981	0.000	0.0	0.1	0.909

Table 4. Reference set and optimal values of ofλ_j and μ_j variables

Circuit	**Reference Set (λ_j, μ_j)**
C3	C2 (0.802, 0.0) C7 (0.120, 0.0) C9 (0.050, 0.0) C20 (0.028, 0.0)
C5	C4 (0.225, 0.0) C6 (0.112, 0.0) C7 (0.459, 0.0) C10 (0.077, 0.0) C16 (0.127, 0.0)
C15	C9 (0.909, 0.0) C10 (0.052, 0.0) C16 (0.035, 0.0) C20 (0.004, 0.0)
C19	C2 (0.947, 0.0) C12 (0.053, 0.0)
C21	C10 (0.215, 0.0) C11 (0.092, 0.0) C16 (0.226, 0.0) C20 (0.467, 0.0)

Table 5. Regression models estimates

	Ordinary Least Squares	Fractional regression model [c]			
		logit	probit	loglog	cloglog
Model intercept	0.960*** (0.046)	22.576*** (0.779)	6.407*** (0.387)	22.520*** (0.776)	3.284*** (0.245)
Circuit type[a]	-0.036 (0.039)	-18.263*** (0.468)	-4.366*** (0.247)	-18.152*** (0.456)	-2.020*** (0.148)
Track orientation[b]	0.084 (0.039)	2.334*** (0.477)	1.087*** (0.324)	2.181** (0.437)	0.690** (0.351)
Number of red-flagged races due to rain	-0.093** (0.036)	-3.622*** (0.134)	-1.514** (0.512)	-3.487*** (1.091)	-0.927** (0.398)
% of fitted values out the range [0,1]	55.55%	-	-	-	-
R^2	0.451	0.963	0.947	0.964	0.900
Sum of Squared Residuals	0.067	0.005	0.009	0.004	0.020

Notes: Dependent variable: WSBM efficiency scores. Sample: 18 F1 circuit cases.
"*", "**" and "***" indicate statistical significance at the 10%, 5% and 1% level, respectively.
Corresponding robust standard error is reported within parentheses.
[a]Dummy variable coded one for circuits with race tracks and zero for street racing circuits.
[b]Dummy variable coded one for clockwise-oriented circuits and zero for anticlockwise-oriented circuits.
[c]Standard one-part fractional regression model. Quasi-maximum likelihood estimation method.