THE FOUNDATIONS OF DYNAMIC INPUT-OUTPUT REVISITED:
¿DOES DYNAMIC INPUT-OUTPUT BELONG TO GROWTH THEORY?

Mariano Blanc Díaz[1]

Carmen Ramos Carvajal[2]

Miembros del grupo de investigación en Análisis Input-Output.

Departamento de Economía Aplicada. Facultad de Ciencias Económicas y Empresariales

[1]mblanc@ovd.gdfsa.com

[2] cramos@correo.uniovi.es

THE FOUNDATIONS OF DYNAMIC INPUT-OUTPUT REVISITED:
¿DOES DYNAMIC INPUT-OUTPUT BELONG TO GROWTH THEORY?

TABLE OF CONTENTS

INTRODUCTION

Classical input-output comes in two flavors: static and dynamic.

The dynamic version was developed as an extension of the static one, to cope with time. But not with any influence of time: specifically with the effect of capital accumulation. Thus dynamic input-output models are considered among the early members of the family of growth models.

From mathematical considerations, we conclude in this paper that their long time accepted interpretation can hardly be sustained. Under certain assumptions, they do represent an economic reality but not growth.

Some directions for future empirical research are extracted from the analysis.

In the rest of the paper, we shall use capital letter **S**, as synonymous of "Leontief static model" and letter **D** as a substitute for "dynamic input-output model".

MATHEMATICAL BACKGROUND

Static formulae.

The characteristic balance equation of **S** is

$$X = A X + Y \qquad\qquad\qquad E.1.1.$$

where X is a vector of output, Y is a vector of final demands and A is a square matrix of interindustry coefficients.

The solution of E.1.1. is:

$$X = [I - A]^{-1} Y \qquad\qquad \text{E.1.2.}$$

Dynamic formulae

The characteristic balance equation of **D** is:

$$X(t) = A\, X(t) + Y(t) + B\, X'(t) \qquad\qquad \text{E.2.1.}$$

where B is a square matrix of capital coefficients. It represents the willingness of the economy to invest (1).

E.2.1 can also be written:

$$X'(t) = B^{-1} [I - A]\, X(t) - B^{-1} Y(t) \qquad\qquad \text{E.2.2.}$$

Naming $\boxed{M = B^{-1} [I - A]}$ and $\boxed{N = -B^{-1}}$, E.2.2. becomes:

$$X'(t) = M\, X(t) + N\, Y(t) \qquad\qquad \text{E.2.3.}$$

Its solution is:

$$X(t) = e^{Mt} X(0) + \int_{o}^{t} e^{M(t-\tau)}\, N\, Y(\tau)\, d\tau \qquad\qquad \text{E.2.4.}$$

(1) The rigorous technical definition is certainly different. It measures the involvement of each sector in the capital accumulation of the rest. For simplicity and better understanding we associate involvement with willingness.

ON THE CONSISTENCE OF S AND D

The process of understanding under which conditions **S** may be considered a particular solution of **D**, allows to surface some not so evident characteristics of **D**'s behavior.

There is more than one way to see the solutions of **S** and **D** coincide.

Let us review different approaches:

1. **Approach 1. Case 1.**

   Assuming that B = 0, in E.2.1.

   This is generally considered the obvious approach (2).

   This situation would correspond to an economy where production fluctuates but the willingness to invest is zero. Consequently investment itself is also zero.

   In fact, a situation like this can be depicted by **S** but not by **D**. This is clear when we consider **D** under its form E.2.2 . If B is singular, B^{-1} does not exist and **D** is not operational. We do not have then two working models which merge, but a model which works and another which does not.

   Therefore, this approach tries to eliminate the distance between the two models by annihilating one of the two terms of comparison. This is not acceptable as means to establish a parenthood relationship between the two models.

(2) See for instance this viewpoint in:

"Linear programming and economic analysis" Robert Dorfman, Paul A. Samuelson and Robert M. Solow . Mc Graw Hill 1958

2. **Approach 2. Case 2.**

An alternative way is to assume that $B \neq 0$ but $X'(t) = 0$ in E.2.1 . This situation would correspond to an economy with willingness to invest but where production needs do not grow. Therefore investment is again zero.

Under those assumptions, **D** truly satisfies the equation:

$$X(t) = [I - A]^{-1} Y(t)$$

and **S** and **D** describe equally the same reality from two different view points. Both models are operational and independent.

It is also true that the approach does not generate any information of interest about the dynamics of the economy: E.3 only works if $X(t) = C$ since otherwise $X'(t)$ would be $\neq 0$, which is against the assumptions; it also necessarily implies $Y(t) = K$.

We end up concluding that E.3 is in reality:

$$C = [I - A]^{-1} K$$

an expression which adds nothing to what we know from E.1.2.

3. **Approach 3. Different cases.**

Approaches 1 and 2 have something in common: they both try to approximate the balance equations of **S** (E.1.1) and **D** (E.2.1) by removing the term $BX'(t)$ from E.2.1.

But it is not necessary to make BX'(t)=0 to have **S** and **D** confirm each other.

To prove it, let us solve E.2.4 for different shapes of Y(t).

Case 3

To allow an easier comparison with the previous approach, let us solve first the case Y(t) = K.

This situation would represent an economy with willingness to invest, where production is allowed to fluctuate while final demand is constant. One must not take for granted that constant final demands lead to constant productions. As we shall see, under Leontief's dynamic formulae, production can grow in an explosive way even when final demands remain unchanged over time.

Under this assumption, E.2.4. becomes (demonstration in Appendix A):

$$X(t) = [I - A]^{-1} K + e^{Mt} [\ X(0) - [I - A]^{-1} K\ ] \qquad \text{E.4.}$$

If the second term of the right hand side of E.4 can be made zero, **S** and **D** do provide an identical solution:

This can happen in two ways:

3.1 When $X(0) = [\ I - A\ ]^{-1} K$ $\qquad\qquad$ E.5

that is when the economy is already working at time zero at the regime corresponding to the long term steady state.

This case is a redefinition of E.3. All assumptions coincide:

- $B \neq 0$
- $Y(t) = K$
- $X'(t) = M\,e^{Mt}[\,0\,] = 0$, therefore $X(t) = C$.

Nevertheless approach 3 has obviously enriched, with respect to approach 2, our information about the dynamic equilibrium. We know now that the validity of E.3. , which rests on the constancy of productions, implies two conditions: not only constant final demands but also very specific requirements about the initial situation of the economy.

3.2 **Additionally**, approach 3 tells us that even if E.5. does not hold and therefore $X'(t) \neq 0$, E.4. can still reproduce the solution of the static model **when** (and only when) **the system is stable**: the second term will fade away as time passes and become zero for practical purposes after a while.

Case 4.

Appendix B solves again **D** for steadily growing final demands :

$Y(t) = Kt$.

In this case, $B \neq 0$, final demands are not constant and $X'(t)$ is never zero. No restrictions are imposed on initial conditions.

Nevertheless, the conclusion is again the same: if the system is stable, the solution of **D** converges to that of **S** and the models provide consistent results (see Appendix B).

On the contrary, if the system is unstable, the static and dynamic input-output models will provide solutions that diverge continuously over time, even when identical final demands are applied.

A minor difference in initial conditions from the required state, leads also without real justification to contradictory solutions of the two models.

As mentioned before, stability requests that all eigenvalues of matrix $M = B^{-1}[I - A]$ have negative real parts.

ON THE STABILITY OF LEONTIEF'S DYNAMIC MODEL

Appendix C proves for the two sector case that if coefficients of matrix B are prevented from being negative, **D** is unstable. It can only exhibit the following types of behaviors, corresponding to a system with only this variety of singular points: unstable focus, unstable nodes or saddle points

Those behaviors would be:

- o Explosive diversion from the steady state with oscillations for unstable focus.

- o Explosive diversion, in either direction, without oscillations for unstable nodes.

- o "Strange" non oscillatory trajectories for saddle points, which start coming closer to the steady state, to depart always in the opposite direction in an explosive divergence before ever reaching it (3).

The situation described above occurs always under Leontief's hypothesis since the very definition of capital coefficients request from them to be positive or zero (although we point in this paper, in case of zero coefficients, they must not be situated in positions which make B singular).

If the coefficients of B are negative, it is still possible to have the same type of instabilities or under additional restrictions (see again Appendix C for the details), one can find the only cases of stability and consistency of the two models.

In Leontief's interpretation negative coefficients may have no meaning.

But alternative interpretations are possible: when production decays, X'(t) is negative and a negative B would turn into positive the component of output BX'(t), thus meaning a compensation of the slow down taking place.

In case of growing production, X'(t) is positive and B<0 makes BX'(t) negative, thus representing a correction effect to the expansion occurring.

Negative coefficients cannot represent growth but yes perhaps some kind of short term countercyclical policy.

(3) The saddle points include one case of stability. Out of the infinite numbers of possible trajectories, all of them are unstable except one, which happens when the system moves along one of the two separatrice lines.

This requests such stringent conditions on the proportions of the coefficients and their maintenance over time that the probability of such things occurring is not small or very small but absolutely negligible.
The situation, if occurring, would be more a consequence of a successful lottery draw than a case representative of the behavior of the system.

One specific case deserves attention: <u>when B is the negative identity matrix</u>.

Case 5. Assumptions: B = -I , Y(t) = K and X(t) is allowed to fluctuate.

The balance equation of **D**, 2.2.1, becomes:

$$X'(t) = [A - I] X(t) + Y(t) \qquad\qquad \text{E.6}$$

or $\qquad\qquad X'(t) = A X(t) + Y(t) - X(t) \qquad\qquad$ E.7

In the right hand side of E.7 :

- The term $AX(t) + Y(t)$ represents total demand of the economy.
- The term $X(t)$ represents total output of the economy.

Therefore E.7 is the representation of an economy that introduces short term charges in its production levels ($X'(t)$ is change of output over time) following the information received about the excess or default of demand over supply occurred in the previous period.

It is the general case of the static model whose equation E.1.1. can be rewritten:

$$0 = A X + Y - X \qquad\qquad \text{E.8.}$$

The static model assumes perfect equilibrium of supply and demand instant by instant. **D** with $B = -I$, covers that possibility but also more general situations of temporary unbalances.

The application of the stability criteria developed in Appendix C applied to the case $B = -I$, confirm fully that it is one of the situations of stability and therefore identical solution of **S** and **D.**

CONCLUSIONS

1. The largely accepted notion that **S** is equivalent to **D** with B = 0 is incorrect.

2. If the coefficients of B are not negative, **D** is unstable:

   It provides unrealistic descriptions of the world: the economy could grow indefinitely under unchanged final demands and minor differences in initial conditions lead to explosive growth whether oscillatory or not.

   In none of these cases **S** and **D** would provide coherent solutions.

   Thus **S** and **D** can be considered inconsistent models if   B > 0

3. **D** with negative B coefficients is the only approach which produces behaviors consistent with the static model, whatever the initial conditions and the final demands applied. Therefore B coefficients might be reinterpreted as an expression of short term countercyclical policy and not as long term growth agents.

4. **D** is the general version of **S**, when B is the negative identity matrix.

5. From the previous observations, it must be concluded that matrix B cannot be interpreted as a capital coefficient matrix.

6-DIRECTIONS FOR FUTURE EMPIRICAL RESEARCH

1. The equivalence of the static model and the dynamic model with B = - I, allows to tackle all the research typical of the static model by numerical computation of equation E.7 instead if going through the inversion of matrix [I − A].

   Numerical computation of E.7 allows the introduction of a number of non-linearities. In particular it is possible to substitute constant interindustry coefficients by ones variable with the level of occupation, reproducing some law of diminishing returns or any other behavior determined by parameter estimation techniques.

2. Equation E.2.4 reminds us that the output of one period does not depend only of the final demand of the period but also of the initial load of backlogs.

   In the real world the economy exhibits at any time some inertia. The influence of the original momentum vanishes if the period selected is very long since, after a while, the weight of the new flowing final demand plays the major role in the determination of production levels.

   Nevertheless, if the period is one year, the influence of the initial conditions certainly distorts the output calculated by the model if only final demand of the period is taken into account by the model.

   In empirical research some attention should be paid to the topic.

   Again, the use of numerical computation may make easier to introduce the influence of the initial loads.

3. The conventional dynamic model represented by E.2.1. assumes naively that the output of a period corresponds to the final demand of the same period, whether the period is long or short.

Since production consumes time, it is unrealistic to maintain such simultaneity, if the period considered is very short.

It is necessary to introduce production lags so that the research can trace the propagation over time of effects – demands and corresponding outputs – through the structure of the economy.

It is possible to introduce easily such a treatment, starting from equation E.7. but we leave for a further paper this presentation.

The inclusion of production delays would complete the timing picture: while the output of a year is influenced by situations inherited from the previous year, part of the consequences of the final demand of one year will be filtered through the industrial system in later periods.

Much of the difficulties attributed to an inappropriate selection of the level of aggregation or to practical problems in the collection of statistical data, could probably be transferred to an insufficient understanding and treatment of these timing factors, completely ignored in the static analysis.

Solution of Leontief's dynamic model for constant final demands: $Y(t) = K$

E.2.4 becomes

$$X(t) = e^{Mt} X(0) + e^{Mt} \int_{o}^{t} e^{-M\tau} N K d\tau \qquad \text{A.1}$$

$$\text{but } \int_{o}^{t} e^{-M\tau} N K d\tau = \left[-M^{-1} e^{-Mt} N K \right]_{o}^{t} \qquad \text{A.2}$$

We prove, before any further progress, that $M^{-1} N = -[I - A]^{-1}$ A.3

Proof of A.3.

$$M^{-1} N = [B^{-1} (I - A)]^{-1} \cdot [- B^{-1}] = [I - A]^{-1} B [- B^{-1}] = -[I - A]^{-1}$$

Therefore, the expression inside the parenthesis in A.2 is:

$$- M^{-1} e^{-Mt} N K = - M^{-1} e^{-Mt} [- M] \cdot [- M^{-1}] N K$$

$$= - M^{-1} [- M] e^{-Mt} [- M^{-1} N K]$$

$$= e^{-Mt} [I - A]^{-1} K$$

Therefore the solution of A.2 is:

$$\int_{o}^{t} e^{-M\tau} N K d\tau = e^{-Mt} \cdot [I - A]^{-1} K - e^{-M0} [I - A]^{-1} K =$$

$$= e^{-Mt} [I - A]^{-1} K - [I - A]^{-1} K \qquad\qquad A.4$$

If we replace in A.1 the expression A.2 by A.4 , we obtain:

$$X(t) = e^{Mt} X(0) + e^{Mt} \cdot \left[e^{-Mt}[I - A]^{-1} K - [I - A]^{-1} K \right]$$

$$= e^{Mt} \left[X(0) - [I - A]^{-1} K \right] + \left[I - A \right]^{-1} \cdot K$$

APPENDIX B

Solution of D when final demand grows steadily: Y(t) = Kt

Equation E.2 becomes: $X(t) = e^{Mt} X(0) + e^{Mt} \int_{o}^{t} e^{-M\tau} N K \tau \, d\tau$

We solve this equation in three steps:

1. Integration by parts of:

    $$\int_{o}^{t} e^{-M\tau} N K \tau \, d\tau$$

    to obtain

    $[- M^{-1} e^{-Mt} t - M^{-2} e^{-Mt} - M^{-2}] \cdot N \cdot K$

2. Multiplication by e^{Mt} to obtain:

    $- M^{-1} N Kt - M^{-2} N K - e^{Mt} M^{-2} N K$

3. Addition of $e^{Mt} X(0)$ and reordering:

    $[I - A]^{-1} Kt - M^{-1}[I - A]^{-1} K + e^{Mt} [X(0) - M^{-1}[I - A]^{-1} K]$ B.1

    (remember from Appendix A: $- M^{-1} N = [I - A]^{-1}$)

    If B = - I , then

    $X(t) = [I - A]^{-1} K t - [I - A]^{-2} K + e^{Mt} [X(0) - [I - A]^{-2} K]$ B.2

    In B.1 and B.2:

1. The first term coincides with the solution of the static model.

2. the second term represents a steady state error. It is constant. When the first term grows this component looses its weight.

3. The third term fades away to become zero if the system is stable.

Therefore the solution of **D** converges to that of **S** when t $\longrightarrow \infty$ again if the system is stable**.**

Determination of the stability conditions of D for the two sector case.

Determination of the sign of the eigenvalues of matrix $M = B^{-1}[I - A]$

We define:

$$*\quad B = \begin{bmatrix} b_{11} & b_{12} \\ & \\ b_{21} & b_{22} \end{bmatrix} \quad \text{therefore } B^{-1} = \begin{bmatrix} \dfrac{b_{22}}{|B|} & \dfrac{-b_{12}}{|B|} \\ \dfrac{-b_{21}}{|B|} & \dfrac{b_{11}}{|B|} \end{bmatrix}$$

where $|B| = b_{11}\, b_{22} - b_{12}\, b_{21}$

$$*\quad [I - A] = \begin{bmatrix} (1-a_{11}) & -a_{12} \\ & \\ -a_{21} & (1-a_{22}) \end{bmatrix}$$

therefore

$$M = \begin{bmatrix} m_{11} & m_{12} \\ & \\ m_{21} & m_{22} \end{bmatrix} = B^{-1}[I - A] =$$

$$= \begin{bmatrix} \dfrac{b_{22}(1-a_{11}) + b_{12}\, a_{21}}{|B|} & \dfrac{-b_{22}a_{12} - b_{12}(1-a_{22})}{|B|} \\ & \\ \dfrac{-b_{21}(1-a_{11}) - a_{21}b_{11}}{|B|} & \dfrac{b_{21}a_{12} + b_{11}(1-a_{22})}{|B|} \end{bmatrix}$$

We calculate the eigenvalues λ of M defined by:

$$| \lambda I - M | = 0 \qquad \text{that is:}$$

$$\begin{bmatrix} \lambda - m_{11} & - m_{12} \\ \\ - m_{21} & \lambda - m_{22} \end{bmatrix} \Rightarrow \text{and therefore:}$$

$$\lambda_1 = \frac{(m_{11} + m_{22}) + \sqrt{\Delta}}{2} \qquad \text{and}$$

$$\lambda_2 = \frac{(m_{11} + m_{22}) - \sqrt{\Delta}}{2}$$

where $\Delta = (m_{11} + m_{22})^{2} - 4 \, |M|$

where $|M| = m_{11} \, m_{22} - m_{12} \, m_{21}$

We do not develop the following demonstrations which are mechanical:

$$m_{11} + m_{22} = \frac{b_{22}(1-a_{11}) + b_{12} \, a_{21} + b_{21} \, a_{12} + b_{11}(1 - a_{22})}{b_{11} \, b_{22} - b_{12} \, b_{21}}$$

and

$$|M| = \frac{(1-a_{11})(1-a_{22}) - a_{12} \, a_{21}}{b_{11} \, b_{22} - b_{12} \, b_{21}}$$

We have to assume that the system complies with the two Hawkin-Simons conditions necessary for the existence of the underlying static model.

- $(1 - a_{11}) (1 - a_{22}) - a_{12} a_{11} > 0$

- $(1 - a_{11}) > 0$ and $(1 - a_{22}) > 0$

We analyze the following possible cases:

- (A) All coefficients b_{ij} are positive.
- (B) All coefficients b_{ij} are positive or zero.
- (C) All coefficients b_{ij} are negative.

(A) All b_{ij} positive leads to two possibilities:

A_1) When $b_{12} b_{21} > b_{11} b_{22}$

- $m_{11} + m_{22} < 0$ since its numerator is positive and the denominator is negative.

- But $|M| < 0$ for the same reason

- Therefore Δ is the addition of two positive quantities and $\sqrt{\Delta}$ is not complex. Both eigenvalues are real.

  Since $\sqrt{\Delta}$ is larger then $| m_{11} + m_{22} |$,

  λ_1 is real and positive.

  λ_2 is real and negative.

  Therefore the system exhibits a saddle point.

A_2) When $b_{12} b_{21} < b_{11} b_{22}$,

- $m_{11} + m_{22} > 0$, since both numerator and denominator are positive.

- $|M| > 0$ for the same reason

- Therefore Δ is the addition of one positive quantity $(m_{11} + m_{22})^2$

  and a negative quantity: $- 4 |M|$

Two cases are possible:

A.2.1. When $(m_{11} + m_{22})^2 > 4\,|M|$, Δ is positive

and $\sqrt{\Delta}$ is not complex. Both eigenvalues are real.

but since $\sqrt{\Delta} < |\, m_{11} + m_{22} \,|$,

both λ_1 and λ_2 are positive.

The system has an unstable node.

A.2.2. When $(\, m_{11} + m_{12} \,)^2 < 4\,|M|$, $\Delta < 0$, $\sqrt{\Delta}$ is complex,

and both eigenvalues are complex with real parts $[\, (\, m_{11} + m_{22} \,) \,]$
positive.

The model shows an unstable focus.

(B) Coefficients b_{ij} positive or zero.

B$_1$) At least all the coefficients of one of the diagonals must be non zero. Otherwise B would be singular and **D** would not exist.

B$_2$) When one or two coefficients of the main diagonal are zero and the rest positive:

- $m_{11} + m_{22} < 0$ since numerator is positive and denominator negative

- but $|M| < 0$ for the same reason.

We fall in case A1.

B$_3$) When one or two coefficients of the second diagonal are zero and the rest positive.

o $m_{11} + m_{12} > 0$ since numerator and denominator are positive

o   but $|M| > 0$ for the same reason

We fall in case A2.

(C) All coefficients b_{ij} are negative

Two alternatives:

C_1) When $b_{11} \; b_{22} \; < \; b_{12} \; b_{21}$, then

o $m_{11} + m_{22} > 0$ since both the denominator and the numerator are negative

o   but $|M| < 0$ since the numerator is positive and the denominator negative

o   therefore $\Delta = (m_{11} + m_{22})^2 - 4 \, |M|$ is the addition of two positive quantities

o $\sqrt{\Delta}$ is then real and both eigenvalues are real.

o   Since $\sqrt{\Delta} > | m_{11} + m_{22} |$,

  o λ_1 is real positive

  o λ_2 is real negative

Therefore we have a saddle point.

C_2) When $b_{11} \; b_{22} \; > b_{12} \; b_{21}$

o $m_{11} + m_{22} < 0$, since its numerator is negative and its denominator is positive

o   but $|M| > 0$ since both numerator and denominator are positive

Two cases are possible:

C.2.1. $(m_{11} + m_{22})^2 > 4 |M|$ and therefore $\sqrt{\Delta}$ is real.

Both eigenvalues are real and

since $\sqrt{\Delta} > |m_{11} + m_{22}|$,

λ_1 is real positive

and λ_2 is real negative.

We again have a saddle point as in A1.

C.2.2. $(m_{11} + m_{22})^2 < 4 |M|$ and therefore $\sqrt{\Delta}$ is complex

The two eigenvalues are complex with negative real parts.

Therefore the system shows **a stable focus.**

C.2.3. $(m_{11} + m_{22})^2 = 4 |M|$ and therefore $\sqrt{\Delta} = 0$.

The two eigenvalues are identical real and negative.

The system shows **a stable node.**

The specific cases C.2.2. and C.2.3. of B with negative coefficients are the only ones which prevent inconsistency of the static and the dynamic model, by representing a stable system where dynamic stationary behavior and static provide consistent representations of the same reality.

(4)   For the analysis included on the nature of the singular points we suggest contrast with "Modern Control Engineering" – Katsuhiko Ogata – Prentice Hall, 1970, Pages 583 – 584

REFERENCES

1. Blanc M. (1976) "Leontief Model: a dynamic, stochastic and non-linear version". Working paper. Alfred P. Sloan School of Industrial Management. Massachusetts Institute of Technology.

2. Blanc M. (1979) "Dynamic Input-Output: an alternative approach". Paper submitted to the "Seventh International Conference on "Changes in the structure of the world economy"". Innsbruck 9-13 April. Session 5. id 79 2117.

3. Dervis K., de Melo, J. & Robinson, S. (1982) "General equilibrium models for Development Policy". (Cambridge U.K., Cambridge University Press)

4. Dixon P., Parmenter B., Powell W. (1992) "Notes and problems in applied general equilibrium economics". North Holland. Amsterdam.

5. Dorfman R., Samuelson P.A. and Solow R.M. "Linear programming and economic analysis". McGraw Hill 1958.

6. Fossati A. (1996). "Economic modeling under the applied general equilibrium approach" (ed.) Avebury, Ashgate Publising Limited, England.

7. Hawkins, D. / Simon, H.A. "Note: Some Conditions of Macroeconomic Stability" in Econometrica vol. 17. Nr. 3-4 July – October 1949.

8. Kurz H.D. & Salvadori, N. (2000) "The dynamic Leontief model and the theory of endogenous growth", Economic Systems Research, 12, pp. 255-265

9. Leontief W., (1967), "An open dynamic system for long range projection of economic growth" In P.N. Mathur and R. Bharadwaj (eds) Economic Analisys in Input-Output Framework (P.N. Mathur-Poona. India)

10. Leontief W. (1970) "The Dynamic Inverse". In: Carter A.P., Brody et al. (eds), Contributions to Input-Output Analysis. North Holland, Amsterdam.

11. Leontief, W. (1989) Foreword, Economic Systems Research 1, pp. 3-4.

12. McKenzie, L. "Matrices with Dominant Diagonals and Economic Theory" in Arrow/Karlin/Suppes. Mathematical Method in the Social Sciences. Stanford 1960.

13. Ogata K. "Modern Control Engineering". Prentice Hall, 1970

14. Miller, R.E., Blair PD (1985) Input-Output analysis, foundations and extensions. Prentice Hall, Inc. Englewood Cliffs, New Jersey.

15. Sawyer J.A. (1992) "Forecasting with input-output matrices: are coefficients stationary?", Economic Systems Research, 8 pp. 33-62

16. Schwartz J.T. (1961) "Lectures on the mathematical Method in Analytical Economics" New York 1961

17. Ten Raa, T. & Mohnen, P. (1994) "Neoclassical input-output analysis", Regional Science and Urban Economics, 24-pp.135-158.