ESTIMATING FACTOR'S CONTRIBUTIONS TO TEMPORAL STRUCTURAL CHANGES

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1. INTRODUCTION

The analysis of economic structure of a country (or region) can be extended and improved if information about it is available at different dates. So, it would be possible to make comparisons over time and measure the observed changes. The structural change analysis lies basically in this process. A relevant matter in this kind of studies is to identify the sources of the change and quantify their influence over it. In other words, to decompose on several factors the observed change in a variable over time.

On the input-output framework, a variable Z usually may be written as the product of several ones, that is:

$$Z = x_1 x_2 \ldots x_n.$$

The traditionally used methods to determinate the respective contributions of each x_i to the whole change in Z, as the well-known Structural Decomposition Analysis (SDA), present a set of disadvantages. Specifically, the main weakness is that contributions are arbitrarily obtained (Dietzenbacher & Los, 1998).

In this paper an alternative general method to calculate contributions of a set of exogenous variables to the change in an endogenous one between two dates. In the next section, basing on the work of Harrison et al. (2000) and writing the temporal paths of the variables as a function of unknown parameters, a general technique to break down temporal changes in a variable into changes in its determinants. In section 3 the analysis will be reconsidered as an estimation problem whit limited data, which will be solved using additional information about temporal behaviour of variables. In this stage, in search of achieving a solution without the disease of arbitrarily calculated contributions, some of the econometric tools obtained from mathematical information theory and presented in Golan et al. (1996) will be applied. Finally, in section 4 the main conclusion will be resumed.

2. A GENERAL METHOD OF DECOMPOSITION

This method is proposed to attribute changes in a variable between two points in time to the sources of this change. Let's assume Z is a variable that can be expressed as a function of a set of n independent variables x_i, $Z = F(x_1,\ldots,x_n)$, and observations of Z at the initial (Z_0) and final (Z_1) date are available. So, the observed change is $\Delta Z = Z_1 - Z_0$ and the objective is to quantify the influence of every x_i over ΔZ. In this paper we consider only the case when the function $F(x_1,\ldots,x_n)$ is a product (a very usual state in input-output framework), so $Z = x_1\ldots x_n$.

To determinate the contribution of every x_i to the whole variation in Z we use an extension of the technique explained in Harrison et al. (2000). In this work a method to calculate contributions is proposed basing on an auxiliary variable t that reflects the (proportion of) time passed from initial point (t will take values between 0 at the initial point and 1 at the final point). This variable is a temporal support to x_i. So we yield the values of x_i as a function of t, and so the value of Z for a time *t* can be expressed as $Z(t) = x_1(t)\ldots x_n(t)$.

Since $F(x_1,\ldots,x_n)$ is a product, a diferenciable function, we can write any infinitesimal change in Z as:

$$dZ = \frac{\partial Z}{\partial x_1} dx_1 + \ldots + \frac{\partial Z}{\partial x_n} dx_n \tag{1}$$

As we have seen x_i is a function of time, $x_i = H_i(t)$, and if every $H_i(t)$ is diferenciable we yield:

$$dZ = \frac{\partial Z}{\partial x_1}\frac{dx_1}{dt} dt + \ldots + \frac{\partial Z}{\partial x_n}\frac{dx_n}{dt} dt \tag{2}$$

As (2) shows the variation of Z in infinitesimal terms, the sum of all these changes along the interval since t = 0 to the point when t = 1 will reflect the whole variation in Z, this is, ΔZ:

$$\Delta Z = \int_{t=0}^{t=1} \frac{dZ}{dt} dt = \int_{t=0}^{t=1} \sum_{i=1}^{n} \frac{\partial Z}{\partial x_i}\frac{dx_i}{dt} dt \tag{3}$$

Basing on (3) it is possible to calculate contributions of every x_i to the whole change. Focusing on x_i's contribution, this one will be:

$$\text{Cont.}x_i = \int_{t=0}^{t=1} \frac{\partial Z}{\partial x_i}\frac{dx_i}{dt}dt = \int_{t=0}^{t=1}\prod_{j\neq i}^{n} x_j \frac{dx_i}{dt}dt \quad (4)$$

In expression (4) the relevance of temporal path $H_i(t)$ can be seen when we want to obtain the contribution of x_i, because this path determinates the value of $\frac{dx_i}{dt}$. Therefore, the election of the functional form of temporal paths will have a big influence over every x_i's contribution.

Having account the description done about the characteristics of variable t, let's propose the following path[1] for x_i:

$$x_i(t) = x_{i0} + \Delta x_i t^{\theta_i};\ \forall\,\theta_i > 0 \quad (5)$$

So, if t is in the initial point $x_i(t) = x_{i0}$ and if t is in the final point $x_i(t) = x_{i0} + \Delta x_i = x_{i1}$, being $\Delta x_i = x_{i1} - x_{i0}$, in other words, the temporal change of x_i. We will suppose θ_i takes only positive values, since with this assumption x_i always will fluctuate between x_{i0} and x_{i1}. For simplicity, also we'll suppose that $\Delta x_i \geq 0;\ \forall i = 1,2,...,n$. Taking expression (5) and inserting in (4), we obtain:

$$\text{Cont.}x_i = \int_{t=0}^{t=1}\prod_{j\neq i}^{n} x_j \frac{dx_i}{dt}dt = \left[\prod_{j<i}^{i-1} x_{j_0}\right]\Delta x_i\left[\prod_{j>i}^{n} x_{j_0}\right] + \frac{\theta_i}{\sum_{j=2}^{n}\theta_j} x_{1_0}\left[\prod_{j=2}^{n}\Delta x_j\right] + \frac{\theta_i}{\sum_{j>2}^{n}\theta_j}\Delta x_1 x_{2_0}\left[\prod_{j>2}^{n}\Delta x_j\right] + \dots$$

$$\dots + \frac{\theta_i}{\sum_{j=1}^{n-1}\theta_j}\left[\prod_{j=1}^{n-1}\Delta x_j\right]x_{n_0} + \frac{\theta_i}{\sum_{j=1}^{n}\theta_j}\left[\prod_{j=1}^{n}\Delta x_j\right] \quad (6)$$

The first sum's term shows the minimum contribution that can reach x_i; it will be the change in x_i weighted by the rest of exogenous variables at initial values. Remaining terms show a set of interaction effects among variables' variations weighted by initial

[1] In the Harrison's work, only the line path is considered. Here we extend this analysis considering that the rates of growth of exogenous variables may change along the path.

values of the other variables, sharing these effects among variables is done depending on the relative value of every θ_i. It must be noticed that there are several interaction terms; specifically are $\binom{n}{n-2}$ between x_i with every of the n-1 remaining variables, $\binom{n}{n-3}$ interaction terms between x_i with sets of n-2 variables, etc. The last term of (6) shows the joint contribution of every all the changes on the variables (as a product) and the part corresponding to x_i is obtained by the quotient value $\frac{\theta_i}{\sum_{j=1}^{n}\theta_j}$ (the previous interaction terms are shared on an equivalent way).

So, the relevance of the θ_i values it's clear when we want to calculate the contributions to the whole change: as comparatively bigger is θ_i than the rest of coefficients, greater is the allocated to x_i of the set of interactions and greater will be its contribution to whole change in Z. To illustrate this idea, let's suppose θ_i is very close to zero, so in (6) we obtain:

$$
\begin{aligned}
\text{Cont.}x_i = \lim_{\theta_i \to 0} \Bigg[& \left[\prod_{j<i}^{i-1} x_{j_0}\right] \Delta x_i \left[\prod_{j>i}^{n} x_{j_0}\right] + \frac{\theta_i}{\sum_{j=2}^{n}\theta_j} x_{1_0} \left[\prod_{j=2}^{n} \Delta x_j\right] + \frac{\theta_i}{\sum_{j>2}^{n}\theta_j} \Delta x_1 x_{2_0} \left[\prod_{j>2}^{n} \Delta x_j\right] + \ldots \\
& \ldots + \frac{\theta_i}{\sum_{j=1}^{n-1}\theta_j} \left[\prod_{j=1}^{n-1} \Delta x_j\right] x_{n_0} + \frac{\theta_i}{\sum_{j=1}^{n}\theta_j} \left[\prod_{j=1}^{n} \Delta x_j\right] \Bigg] = \left[\prod_{j<i}^{i-1} x_{j0}\right] \Delta x_i \left[\prod_{j>i}^{n} x_{j0}\right] = \\
& = x_{10} x_{20} \ldots x_{(i-1)_0} \Delta x_i x_{(i+1)_0} \ldots x_{n0}
\end{aligned}
$$

In this case we have the x_i's minimum contribution, and its increase is weighted by the remaining variables at the initial values. Notice that is one of the n! possible solutions that SDA can obtain, specifically, one of the called "polar decompositions". The opposed solution will be achieved if we suppose that θ_i takes an infinitely big value; on this stage the contribution of x_i to the change in Z would be expressed as:

$$\text{Cont.}x_i = \lim_{\theta_i \to \infty} \left[\left[\prod_{j<i}^{i-1} x_{j_0} \right] \Delta x_i \left[\prod_{j>i}^{n} x_{j_0} \right] + \frac{\theta_i}{\sum_{j=2}^{n} \theta_j} x_{1_0} \left[\prod_{j=2}^{n} \Delta x_j \right] + \frac{\theta_i}{\sum_{j>2}^{n} \theta_j} \Delta x_1 x_{2_0} \left[\prod_{j>2}^{n} \Delta x_j \right] + \ldots \right.$$

$$\left. \ldots + \frac{\theta_i}{\sum_{j=1}^{n-1} \theta_j} \left[\prod_{j=1}^{n-1} \Delta x_j \right] x_{n_0} + \frac{\theta_i}{\sum_{j=1}^{n} \theta_j} \left[\prod_{j=1}^{n} \Delta x_j \right] \right] =$$

$$= \left[\prod_{j<i}^{i-1} x_{j0} \right] \Delta x_i \left[\prod_{j>i}^{n} x_{j0} \right] + x_{1_0} \left[\prod_{j=2}^{n} \Delta x_j \right] + \Delta x_1 x_{2_0} \left[\prod_{j>2}^{n} \Delta x_j \right] + \ldots + \left[\prod_{j=1}^{n-1} \Delta x_j \right] x_{n_0} + \left[\prod_{j=1}^{n} \Delta x_j \right] =$$

$$= x_{11} x_{21} \ldots x_{(i-1)_1} \Delta x_i x_{(i+1)_1} \ldots x_{n1}$$

Now the x_i's contribution is as big as it can be, because its increase is weighted by the final values of the remaining variables (the other "polar decomposition" in SDA). Between both extreme solutions there is an infinitely wide range of feasible solutions depending on the relative sizes of each θ_i. Among all this variety, it can be proved that the rest of decompositions obtained in SDA are included. So we can conclude that SDA's solutions are just a set of particular cases of this more general decomposition method.

3. ESTIMATING FACTOR'S CONTRIBUTIONS WITH ADTITIONAL INFORMATION.

In the previous section we have seen the way to calculate the contributions of exogenous variables to temporal changes between two times. These contributions depend on the temporal path of the variables, more explicitly, on the values of θ_i coefficients. If the only available information is initial and final values, we can hypothesize infinite temporal paths between these two points. So, the problem of not uniqueness in the solutions arises.

It must be clear that we have reduced the decomposition problem to give values to unknown parameters (coefficients θ_i). If additional information about values of variables at middle points (between initial and final point) were available, it would be possible to estimate these parameters. However, it wouldn't be a classical econometric problem, because the limited data wouldn't allow applying an inferencial treatment based on limit theorems. The econometric techniques based on mathematical information theory are an alternative approach to this problem, and allow obtaining robust estimations without additional assumptions (Golan et al., 1996).

Let's suppose some additional information about the variables is available and we'll consider two different stages.

a) One intermediate observation of Z.

Suppose we know the value of Z at a point t between 0 and 1. So the observed value of Z in this point will be:

$$Z(t) = x_1(t)x_2(t)...x_n(t) \tag{7}$$

Inserting expression (5) in (7) we obtain:

$$Z(t) = \left(x_{10} + \Delta x_1(t)\right)\left(x_{20} + \Delta x_2(t)\right)...\left(x_{k0} + \Delta x_n(t)\right) \tag{8}$$

and calculating the product, we yield:

$$Z(t) = x_{10}x_{20}...x_{n0} + \Delta x_1(t)x_{20}...x_{n0} + x_{10}\Delta x_2(t)...x_{n0} + ... + x_{10}x_{20}...\Delta x_n(t) + J(t) \tag{9}$$

Where J(t) is the called "joint effect" using traditional terminology. The expression of J(t) will be:

$$J(t) = \Delta x_1(t)\Delta x_2(t)...x_{n0} + ... + \Delta x_1(t)x_{20}...\Delta x_n(t) + ... + \Delta x_1(t)\Delta x_2(t)...\Delta x_n(t) \tag{10}$$

and includes the effect of every interaction terms of the changes in exogenous variables. It's clear that we can write ΔZ(t) as $Z(t) - Z_0 = Z(t) - x_{10}x_{20}...x_{n0}$, transforming (10) on the following expression:

$$\Delta Z(t) = \Delta x_1(t)x_{20}...x_{n0} + x_{10}\Delta x_2(t)...x_{n0} + ... + x_{10}x_{20}...\Delta x_n(t) + J(t) \tag{11}$$

If we consider what have been explained in section 2, every term $\Delta x_i(t)$ in (11) can be written as:

$$\Delta x_i(t) = \Delta x_i t^{\theta_i} \tag{12}$$

and for this reason expression (11) can be changed to[2]:

$$\begin{aligned}\Delta Z(t) = {} & \Delta x_1 t^{\theta_1} x_{20} \ldots x_{n0} + x_{10} \Delta x_2 t^{\theta_2} \ldots x_{n0} + \ldots + x_{10} x_{20} \ldots \Delta x_n t^{\theta_n} + \\ & \Delta x_1 t^{\theta_1} \Delta x_2 t^{\theta_2} \ldots x_{n0} + \ldots + \Delta x_1 t^{\theta_1} x_{20} \ldots \Delta x_n t^{\theta_n} + \ldots + \Delta x_1 t^{\theta_1} \Delta x_2 t^{\theta_2} \ldots \Delta x_n t^{\theta_n}\end{aligned} \tag{13a}$$

Just multiplying the powers we obtain:

$$\begin{aligned}\Delta Z(t) = {} & \Delta x_1 t^{\theta_1} x_{20} \ldots x_{n0} + x_{10} \Delta x_2 t^{\theta_2} \ldots x_{n0} + \ldots + x_{10} x_{20} \ldots \Delta x_n t^{\theta_n} + \\ & \Delta x_1 \Delta x_2 t^{\theta_1+\theta_2} \ldots x_{n0} + \ldots + \Delta x_1 x_{20} \ldots \Delta x_n t^{\theta_1+\theta_n} + \ldots + \Delta x_1 \Delta x_2 \ldots \Delta x_n t^{\sum_{i=1}^{n} \theta_i}\end{aligned} \tag{13b}$$

Now, we're going to call β_i each t^{θ_i} and so:

$$\begin{aligned}\Delta Z(t) = {} & \Delta x_1 x_{20} \ldots x_{n0} \beta_1 + x_{10} \Delta x_2 \ldots x_{n0} \beta_2 + \ldots + x_{10} x_{20} \ldots \Delta x_n \beta_n + \\ & \Delta x_1 \Delta x_2 \ldots x_{n0} \beta_{12} + \ldots + \Delta x_1 x_{20} \ldots \Delta x_n \beta_{1n} + \ldots + \Delta x_1 \Delta x_2 \ldots \Delta x_n \beta_{12\ldots n}\end{aligned} \tag{13c}$$

where $\beta_{ij} = \beta_i \beta_j$, ...; $\beta_{12\ldots n} = \prod_{i=i}^{n} \beta_i$. Hence, expression (13c) may be written in a more compact form as:

$$Z(t) = \mathbf{x}'\boldsymbol{\beta} \tag{13d}$$

where $\mathbf{x}$ is a $(k \times 1)$ vector with the variations of exogenous variables between initial and final times weighted by remaining variables at initial values and with all the interaction terms. On the other side, $\boldsymbol{\beta}$ is the $(k \times 1)$ vector of β_i parameters to be estimated. Once

[2] The number of terms in this expression is $\left(n + \binom{n}{2} + \ldots + \binom{n}{n-1} + 1\right)$ because we're considering n exogenous variables. For reasons of simplicity, this number will be renamed as k.

the estimation is realized, it is immediate to obtain as the θ_i values as the x_i's contributions.

To complete this estimation process we start from a discrete probability distribution for every β_i. This distribution is defined over the parametric space [0,1] by a set of $M \geq 2$ discrete values equally distanced $\mathbf{b} = [b_1, b_2, ..., b_M]$ with corresponding probabilities $\mathbf{p_i} = [p_{i1}, p_{i2}, ..., p_{iM}]$. So, vector $\boldsymbol{\beta}$ can be written as:

$$\boldsymbol{\beta} = \mathbf{Bp} \tag{14a}$$

$$\text{where } \mathbf{Bp} = \begin{bmatrix} \mathbf{b}' & \mathbf{0} & . & \mathbf{0} \\ \mathbf{0} & \mathbf{b}' & . & \mathbf{0} \\ . & . & . & . \\ \mathbf{0} & \mathbf{0} & . & \mathbf{b}' \end{bmatrix} \begin{bmatrix} \mathbf{p_1} \\ \mathbf{p_2} \\ \dots \\ \mathbf{p_n} \end{bmatrix} \tag{14b}$$

Therefore, the value of each β_i will be:

$$\beta_i = \mathbf{b}'\mathbf{p_i} = \sum_{m=1}^{M} b_m p_{im} \tag{14c}$$

and now (13d) can be expressed as:

$$Z(t) = \mathbf{x}'\boldsymbol{\beta} = \mathbf{x}'\mathbf{Bp} \tag{15}$$

So, the unknown parameters' estimation problem has been changed to a new estimation problem, where the unknown is a set of k probability distributions. To solve it and recover $\mathbf{p}$, we'll base on Shannon's entropy measure. Therefore, the subsequent constrained maximization problem can be stated

$$\max_{\mathbf{p}} H(\mathbf{p}) = -\sum_{i=1}^{k} \sum_{m=1}^{M} p_{im} \ln(p_{im}) \tag{16a}$$

subject to

$$\Delta Z(t) = \mathbf{x'Bp} \tag{16b}$$

$$0 \le p_{im} \le 1 \quad \forall i = 1,...,k; \; m = 1,...,M \tag{16c}$$

$$\sum_{m=1}^{M} p_{im} = 1 \quad \forall i = 1,...,k \tag{16d}$$

$$\beta_{i...j} = \prod_{i=i}^{j} \beta_i \tag{16e}$$

We're following the Maximum Entropy (ME) approach to recover an unknown probability distribution, and thus, to calculate the θ_i coefficient values. This allows us to obtain non-arbitrary contributions of the explicative variables, because this approach among all the feasible solutions selects the one that "*could have been generated in the greatest number of ways consistent with what we know (the data)*" (Golan et al., 1996).

b) Several intermediate observations of Z.

In the previous stage only one additional observation of Z between initial and final point was available. We can extend this analysis to the case when there are several observations of Z. Let's suppose we observe the Z value at point t and also at another point t^*. Now, it wouldn't be possible to use the same ME approach applied before because this would mean to include a new constraint on the preceding programme:

$$\Delta Z(t^*) = \sum_{i=1}^{k} \sum_{m=1}^{M} x_i b_m p_{im} \tag{17}$$

and it implies an inconsistency, for the reason that we're saying that $\beta_i = (t)^{\theta_i} = (t^*)^{\theta_i}$, which it's not true unless $t = t^*$. The solution to this problem resides in modifying the estimation technique, using now the Cross Entropy (CE) approach.

Starting from a situation where we know the value of Z in two points, t and t^*. Since it's no possible to apply the same ME technique just including a new constrain, we propose to use a two-stage estimation method: in the first stage the ME technique can be applied just employing information at point t. In the second one, once this prior estimation has been realized, this estimation would be employed as starting point to apply the CE method and, so, to obtain final estimation of θ_i values.

Let's suppose that using the ME approach and available information at point t we've obtained a prior estimation, therefore we have a matrix $\hat{\mathbf{p}}$ that:

$$\hat{\mathbf{p}} = \begin{bmatrix} \hat{\mathbf{p}}_1 \\ \hat{\mathbf{p}}_2 \\ \\ \hat{\mathbf{p}}_k \end{bmatrix} \tag{18}$$

being $\hat{\mathbf{p}}_i = [\hat{p}_{i1}, \hat{p}_{i2}, ..., \hat{p}_{iM}]$ a vector with estimations of probability distribution corresponding to β_i values, where $\hat{\beta}_i = \sum_{m=1}^{M} b_m \hat{p}_{im}$ allows obtaining the estimation of every $\hat{\theta}_i$. It's possible to make use of $\hat{\mathbf{p}}$ as prior information to modify this estimation when further information (data at point t^*) is available. So, we have:

$$\begin{aligned} \Delta Z(t^*) = {} & \Delta x_1 x_{20} ... x_{n0} \beta_i^* + x_{10} \Delta x_2 ... x_{n0} \beta_2^* + ... + x_{10} x_{20} ... \Delta x_n \beta_n^* + \\ & \Delta x_1 \Delta x_2 ... x_{n0} \beta^*{}_{12} + ... + \Delta x_1 x_{20} ... \Delta x_n \beta^*{}_{1n} + ... + \Delta x_1 \Delta x_2 ... \Delta x_n \beta^*{}_{12...n} \end{aligned} \tag{19}$$

where $\beta_i^* = (t^*)^{\theta_i}$. To estimate the β_i^* values a constrained minimization programme will be solved, where the objective function is the Kullback-Liebler distance. The prior distribution will be $\hat{\mathbf{p}}$, and the goal is to minimize the distance between $\hat{\mathbf{p}}$ and $\mathbf{q}$, being:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_2 \\ \\ \mathbf{q}_k \end{bmatrix}, \text{ where } \mathbf{q}_i = [q_{i1}, q_{i2}, ..., q_{iM}]$$

and the equally $\beta_i^* = \mathbf{b}'\mathbf{q}_i = \sum_{m=1}^{M} b_m q_{im}$ is satisfied.

Therefore, the minimization problem will be:

$$\min_q \; I(\mathbf{q}, \hat{\mathbf{p}}) = \sum_{i=i}^{k} \sum_{m=1}^{M} q_{im} \ln\left(\frac{q_{im}}{\hat{p}_{im}}\right) \tag{20a}$$

subject to

$$\Delta Z(t^*) = \sum_{i=1}^{k}\sum_{m=1}^{M} x_i b_m q_{im} \quad (20b)$$

$$0 \leq q_{im} \leq 1 \quad \forall i = 1,...,k; \; m = 1,...,M \quad (20c)$$

$$\sum_{m=1}^{M} q_{im} = 1 \quad \forall i = 1,...,k \quad (20d)$$

$$\beta^*{}_{i...j} = \prod_{i=i}^{j} \beta_i^* \quad (20e)$$

Solving this problem we obtain the estimations of θ_i coefficients ($\hat{\theta}_i$) basing on estimations of β_i values ($\hat{\beta}_i^*$). These estimation satisfy the expression:

$$\hat{\beta}_i^* = \sum_{m=1}^{M} b_m \hat{q}_{im} \quad (21)$$

Following this way, we may obtain non-arbitrary contributions of every x_i to the whole change in Z. Information at point t is useful to get a prior estimation of these contributions (using the ME approach). These estimations are the starting point to realize the second stage of the process, when we obtain the **q** "closest" distribution to **p** that at the same time is consistent with data in t^* point. Obviously, if new data were available (for instance, additional observation of Z in a new point t^{**}), now the problem would be solved just repeating the process one more time.

4. CONCLUSIONS

In this paper the problem of calculating the factor's contributions to a temporal change in a variable is analysed, focusing on the case when the functional form is a product (a very usual state in input-output framework). The traditionally used method in this field is the Structural Decomposition Analysis (SDA), a technique that presents a set of disadvantages very often ignored. The most important weakness is that contributions are arbitrarily obtained (Dietzenbacher & Los, 1998).

To solve this problem, a new general decomposition method is presented as an extension of Harrison's work (2000). This method calculates the factor's contribution depending on it temporal path between initial and final dates. As the changes analysed are structural changes, it's not rare that additional intermediate information about the variable's values is available.

Rearranging the problem as an estimation problem of unknown parameters with limited data, it's possible to obtain non-arbitrary contributions using that additional information. Basing on the econometric techniques derived from mathematical information theory, and depending on the amount of available information, we may use the ME approach or the CE approach.

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