GENERALIZED PROBABILISTIC MODEL ALLOWING FOR VARIOUS FATIGUE DAMAGE VARIABLES

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ABSTRACT

This paper proposes a generalization of the Castillo and Fernández-Canteli probabilistic fatigue model and shows how most fatigue models can be obtained as particular cases. Models that include mean-stress effects and multiaxial loading conditions are considered as examples of this general framework. Several fatigue damage parameters such as the Smith-Watson-Topper, the Walker-like strain, energy-based parameter in uniaxial and multiaxial loading conditions, and multiaxial critical plane parameters are proposed as reference parameters for the probabilistic model. It is shown that the Castillo & Fernández-Canteli probabilistic approach can be successfully extended to these advanced fatigue models.

KEYWORDS: Fatigue, Probabilistic fatigue model, Fatigue damage parameter, Multiaxial fatigue, Mean-stress effects.

1. INTRODUCTION

Probabilistic fatigue models are very important as they are able to incorporate different sources of uncertainty arising in the prediction procedures such as material properties and microstructures (e.g. large inclusions or defects) or geometrical features of a component. Most of the current fatigue damage models have essentially a deterministic basis. However, their application for design purposes requires subsequently additional statistical arguments in order to establish appropriate safety margins, not always based on rigorous criteria. In addition, in order to carry out reliability analyses the fatigue damage must be established in an appropriate probabilistic form. As a consequence, failure prediction, engineering design and risk analysis in fatigue are not possible without the support of probabilistic fatigue models.

Several authors have proposed probabilistic approaches to predict the fatigue life of materials and structural details using local models. Castillo and Fernández-Canteli proposed a probabilistic model based on Weibull or Gumbel distributions for constant and variable stress levels [1,2,3,4,5]. Using the same physical and statistical assumptions, Castillo and Fernández-Canteli extended their probabilistic model to the strain damage parameter [5]. These models were used to estimate fatigue life predictions for many applications, ranging from riveted joints of old steel bridges [6], structural details made of pressure vessel steel [7,8,9], and structural details made of old metallic materials [10,11,12]. Zhu et al. [13] proposed a probabilistic low-cycle fatigue life prediction based on the energy-based damage parameter using Bayes' theorem. Other probabilistic damage approach was developed by Amraoui et al. [14] based on a combination of Chaboche deterministic method and probabilistic model proposed by Castillo and Fernández-Canteli [5]. A probabilistic high-cycle multiaxial fatigue approach using the weakest link concept was introduced by Koutiri et al. [15] to model the competition between the different microstructural heterogeneities and porosity coexisting fatigue damage mechanisms. A probabilistic formulation of the multiaxial fatigue virtual strain energy damage model was proposed by Calvo et al. [16]. Other applications of the Castillo and Fernández-Canteli model [5] were proposed to reinterpret the Miner's linear damage summation [17,18].

Castillo and Fernández-Canteli probabilistic model [5] was originally developed for uniaxial stress/strain loading conditions using stress and strain amplitudes (or ranges) derived from Weibull or Gumbel distributions. Some of the papers summarized above extended it to other fatigue parameters. The current paper aims to provide a systematic formalism for the original model to include various fatigue damage parameters. Fatigue models that include mean-stress effects and accounting for multiaxial loadings are considered as examples.

The mean-stress effect can be accounted for by using various damage variables, such as the one proposed by Smith-Watson-Topper (SWT) [19]. Correia et al. [7,10] proposed a probabilistic SWT model to describe the mean stress effects. Although not formally, the SWT damage parameter can be classified as an energy-based parameter. Certainly, Koller et al. [20] also provided an extension of the probabilistic Weibull regression model to describe the stress level effects on S-N fields, but despite the rigorous analytical derivation of the model and the highly satisfactory fitting of the experimental S-N fields for different stress levels, this general approach implies a significant number of parameters the estimation of which requires a complex identification procedure and demanding test planning. This is why the availability of a probabilistic fatigue model with a simpler structure though referred to a higher-level reference parameter is attractive for it to be used in the design of structural components. A number of other energetic parameters are proposed in the literature, some of which are included in this paper to demonstrate the capability of the generalization concept based on the basic probabilistic model [5].

There is a large number of multiaxial damage parameters proposed in the literature covering low-cycle fatigue, high-cycle fatigue, proportional and non-proportional loading conditions [21]. However, most of the work is based on deterministic models [21]. Many of the damage parameters are correlated with the number of cycles to failure, by means of a power law. However, this type of representation can be also properly approximated by a hyperbolic function as given by the proposed probabilistic model. Therefore, this paper shows a simple way to integrate these multiaxial models into a probabilistic framework.

The software ProFatigue [22], initially developed for considering stress ranges or strain ranges as reference variables, can be applied straightforwardly to the fatigue data fitting using generalized fatigue parameters including energetic or multiaxial ones.

This paper is structured as follows: Section 2 details the proposed generalization model; Section 3 includes examples of uniaxial models with mean-stress effects; Section 4 describes models for multiaxial loading conditions whereas Section 5 shows various examples. Last section presents some concluding remarks.

2. GENERALIZATION OF THE PROBABILISTIC MODEL

In the case of one-parameter deterministic damage representation, the life to initiate a macro-crack, which is generally termed the fatigue failure criterion, can be written as [16,21]:

$$\psi = q(\qquad (1)$$

where represents a fatigue damage parameter and is a decreasing function of total life in terms of reversals to failure or cycles . For example, Figure 1 illustrates the generic power law function used in the field, which can be written as:

$$(2)$$

where is a fatigue (endurance) limit, κ and α are material constants [16,21]. This deterministic power-law model will be compared with the proposed probabilistic model for various fatigue damage parameters. Considerable amount of effort has been expended in defining a suitable empirical damage parameter ψ; few examples include stress or strain-based formulations, plastic work, strain energy density or some function of these [21,23].

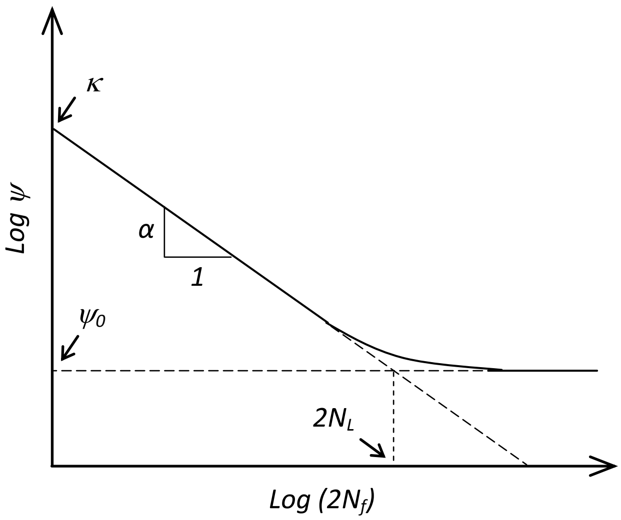

Figure 1: Schematic representation of the deterministic power-law fatigue failure criterion, showing a damage threshold.

Using the above argument, a probabilistic fatigue failure criterion can be defined by three variables: probability (p) of fatigue failure of a component when subjected to reversals, the number of cycles to failure (N_f), and a fatigue damage parameter (ψ). In addition, material constants such as a threshold value of (i.e. number of cycles for which the damage is theoretically infinite, denoted) and a fatigue damage threshold (i.e. limit for infinite number of cycles to failure, denoted) are needed to better define the correlation between the empirical damage parameter and the life to failure.

Employing the same rigorous statistical and physical assumptions of Castillo and Fernández-Canteli [5], a probabilistic Weibull regression model can be written in a compact form as:

(3)

where v is a regression parameter and the function f has a logarithmic form:

— (4)

for 2, , . These two constants can be determined from the experimental data using a constrained least-squares method. After this step, the set of values (where and T is the number of experiments) is fitted with a three-parameter Weibull distribution (Figure 2(a)):).

For the determination of parameters of this distribution (λ is the parameter defining the position of the corresponding zero-percentile hyperbola, corresponds to the scale factor, and is the Weibull shape parameter of the cumulative distribution function), the methods of maximum likelihood or probability weighted moments can be used [5]. It is noted here that the selection of Weibull distribution for variable is not random but it is based on statistical conditions that such a model should satisfy as detailed in [5]. These conditions are: (a) weakest link principle, (b) stability, (c) limit behavior, (d) limited range, and (e) compatibility. Among them, the latter condition requires that the cumulative distribution function of the lifetime given the value of the damage parameter to be compatible with the cumulative distribution function of the damage parameter given the lifetime. This condition translates into a functional equation which has only one acceptable solution, the 3-parameter Weibull distribution.

In the end, the generalized damage probabilistic field can be written as:

(5)

where p is the probability of failure, (A=lo) and (B=log) are normalizing values and λ, and are the non-dimensional Weibull model parameters.

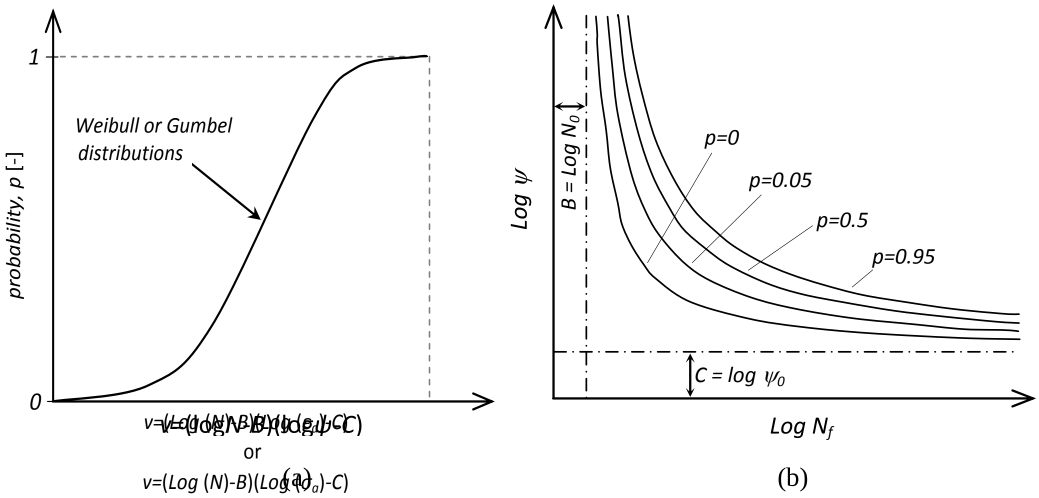

Figure 2: Schematic representation of (a) the probability of regression parameter *v*, (b) generalized probabilistic ψ-N field.

The main advantages of the generalised model are: (a) it provides an analytical probabilistic definition of the whole damage-life field as quantile curves, both in the low-cycle and high-cycle fatigue regions; (b) deals directly with the total damage, without the need of separating its elastic and plastic components as happens with the classical strain based fatigue damage parameters; (c) gives explicitly the probabilistic damage-life (p–ψ–N) field; (d) accounts for run-outs and specimens of different sizes.

Figure 2(b) illustrates the p–ψ–N Weibull field, which is characterized by percentile curves showing hyperbolic shape with two asymptotes: the horizontal one, having a clear physical meaning, represents the fatigue limit; the vertical one, denoted threshold value of lifetime, has a more controversial meaning as a limiting number of cycles. It is noted here and it will be shown in the examples below, that these model parameters B and C are not required to be positive; therefore these percentile curves can intersect the two axes.

Equation (5) is used below for various damage parameters and the results are compared with traditional deterministic models which in most of the cases are expressed using Equation (2). In the original contributions of Castillo and Fernández-Canteli [5], the stress and the strain amplitudes (σ and) are used as fatigue damage parameters and both the Weibull or Gumbel distributions satisfy the statistical and physical requirements. The Gumbel field is a limiting case of the Weibull field, when or even for values of higher than, say, 6; therefore the Gumbel distribution was not considered here. By applying the same model to describe the probabilistic stress and strain fields, the authors already implicitly assumed the possibility of generalization of the model, despite no combined/complex damage parameters were used.

3. DAMAGE PARAMETERS FOR UNIAXIAL LOADING CONDITIONS

A short overview of the stress and strain notations is summarized here for a complete presentation. As illustrated in Figure 3, for a cyclically varying stress, and are the maximum and minimum stresses, respectively, is the stress amplitude, is the mean stress, is the stress range, is the stress ratio and:

; (6)

Similar definitions are applied to strain: is the strain amplitude, is the strain range, is the mean strain, and is the strain ratio.

The stress and strain amplitudes) have been proposed as basic fatigue damage parameters to handle respectively with high- and low-cycle fatigue behaviours and those parameters were also selected by Castillo and Fernández-Canteli for the formulation of their probabilistic model [5]. Despite being basis choices to tackle fatigue damage, quickly one may find various limitations that can be overcome with composed fatigue damage parameters which some will be briefed in the following sections.

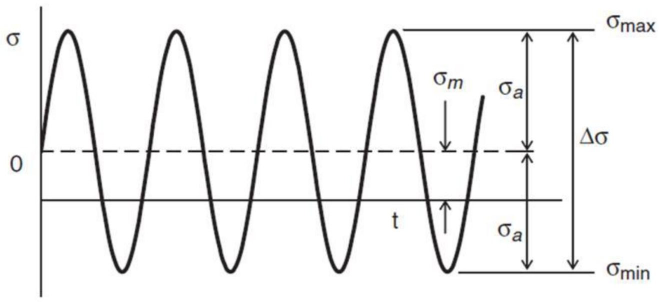

Figure 3: Constant amplitude cyclic stressing and definitions of stress variables [24].

3.1. Smith-Watson-Topper damage parameter

To account for mean-stress effects, Smith *et al.* [19] proposed the following damage parameter:

(7)

which can be related to the fatigue life by means of the deterministic Morrow-Basquin equation [25,26]:

(8)

The same SWT parameter can be used with the proposed probabilistic formulation. In order to compare the deterministic model (8) with the proposed probabilistic model given by equation (5) for

(9)

an example is included in Section 5.

3.2. Walker-like strain damage parameter for uniaxial loading conditions

Another strain-life fatigue model with mean-stress effect is the Walker model which can be written as [23,27]:

(10)

where, , , and are material parameters and is called the Walker fitting constant. For a given set of experimental results, the constants and are obtained first by the stress-life data fitting as the first term of equation (10) corresponds to the elastic strain. The second term of equation (10) corresponds to the plastic strain amplitude, therefore a linear-least-squares fit is performed in order to determine constants and . The main advantage of this model is that parameter introduces the sensitivity of the material to mean stress, giving this approach a versatility that is not possessed by the other common mean stress methods [28]. SWT-life model is a particular case for 5.

A probabilistic Walker model was proposed by Apetre *et al.* [24], using equation (5) as a framework. The model assumes that the damage variable is:

(11)

Using this variable with equation (5), the following percentile curves are obtained:

(12)

A comparison of classical model (eq. (10)) and current model (eq. (12)) explains the difference between them. Both are based on five parameters and the left sides of these equations have the same expression. Nevertheless, the classical model separates elastic and plastic terms and their corresponding *log-log* variations are assumed linear, whereas the current model deals with total strain and the corresponding *log-log* variation is non-linear. Because these two equations are different, γ and $\hat{\gamma}$ are similar yet different (in other words, for a given data set, and $\hat{\gamma}$ cannot be compared). An example is included in Section 5 to illustrate this distinction.

3.3. Energy-based parameters for uniaxial fatigue loading conditions

Energy-based criteria can be classified into two categories depending upon which of the following hypotheses is used: the total absorbed energy to fracture is (1) constant and independent of the number of cycles to failure or (2) dependent on the number of cycles to failure, the latter being the most plausible hypothesis. Different energy-based parameters have been proposed within the framework of the second hypothesis; among them, the plastic strain energy range, ΔW^P [29] and the total strain energy range per reversal, [30] are mentioned here.

In order to include mean-stress effects, Golos and Ellyin [31,32] defined an alternative version of the total strain energy range, , resulting from the superposition of the plastic strain energy range, and the elastic strain energy range associated with the tensile stress, :

(13)

where is the plastic strain range, is the stress range, is the cyclic strain-hardening exponent, and E is the elastic modulus.

Inspired by Morrow's relation, a deterministic fatigue failure criterion can be written as:

(14)

where and 0, are material constants. Section 5 includes an example that compares the deterministic Equation (13) with the proposed probabilistic model Equation (5) for:

(15)

4. FATIGUE DAMAGE PARAMETERS FOR MULTIAXIAL LOADING CONDITIONS

In this chapter a few representative damage parameters are included to illustrate a generalization of the Weibull field to describe multiaxial fatigue.

4.1. Energy-based damage parameters

4.1.1. *Proportional and biaxial non-proportional loading*

For multiaxial fatigue, Ellyin [21] proposed a model based on a modification of energy density associated to one cycle of the strain history , which is a summation of plastic strain energy , divided by a multiaxial constraint ratio and the positive elastic strain energy , which allows the inclusion of mean stress effects:

(16)

where σ_{ij} and are the stress and plastic strain tensors, and ε_i^e are the principal stresses and the principal elastic strains, is the period of one cycle and is the Heaviside function (H for 0 and $H(x) = 1$ for 0). The multiaxial constraint ratio $\bar{\rho}$ is defined as:

(17)

where $\bar{\nu}$ is an effective Poisson's ratio calculated from:

(18)

and where ε_a and are the principal in-plane axial and transversal strain parallel to the free surface, and is the radial strain (perpendicular to the free surface), given by:

(19)

The multiaxial constraint ratio demonstrates the importance of the orientation of the free surface with respect to the imposed principal strains and for the following particular cases is defined as:

(20)

A deterministic fatigue failure criterion can be written as:

(21)

where κ, α and C are material parameters to be determined from appropriate tests and $2N_f$ is the number of reversals to failure. To allow the generalization of the probabilistic Weibull model for multiaxial fatigue, the damage parameter $\psi = \Delta W^t$ should be computed using Equations (16) –(20).

4.1.2. *Non-proportional loading*

The application of the energy-based parameter proposed by Ellyin to non-proportional loading requires the following modification of the multiaxial constraint factor, $\bar{\rho}$ [21]:

(22)

This means that the $\bar{\rho}$ parameter is evaluated using a ratio $\varepsilon/$ taken at the instant when the shear strain in the direction 45° to the surface reaches its maximum value.

Section 5 includes examples that compare the deterministic equation (21) for proportional and non-proportional loading with the proposed probabilistic model, Equation (5), with damage parameter defined as:

(23)

where the two energy densities associated to one cycle of the channels of the strain history are given by Equation (16).

4.2. Critical plane-based damage parameters

Another group of damage parameters for multiaxial fatigue is represented by the critical-plane parameters, among which two are presented here. Fatemi and Socie [33,34] and Fatemi and Kurath [35] proposed a shear-strain based multiaxial fatigue criterion that uses the following fatigue parameter (FP):

(24)

where $\Delta\gamma/2$ is the shear strain amplitude, $\sigma_{n,max}$ is the maximum normal stress on the critical plane, σ is the yield stress of the material and K is a material constant. Equation (24) defines the critical plane as the plane associated with the maximum shear strain amplitude. However Jiang *et al.* [36] defined the critical plane as the material plane where the fatigue parameter (FP) expressed by Equation (24) reaches a maximum. Jiang *et al.* [36] demonstrated the suitability of a power fatigue failure criterion to predict multiaxial proportional and non-proportional multiaxial loading paths:

(25)

where the model parameters $\alpha_p, \beta_p, \varphi$ are determined by fitting experimental data. Section 5 compares this power-law deterministic model with the proposed probabilistic, model Equation (5), where the damage parameter is defined by:

(26)

5. APPLICATIONS

In this section several sets of experimental data are used to demonstrate the accuracy of the proposed probabilistic framework to correlate distinct types of fatigue parameters with lifetime, covering uniaxial and multiaxial fatigue loading [37, 41-43]. The five models summarized above and used here are: SWT Equation (9), Walker-like Equation (11), energy-based for uniaxial loading Equation (15), energy-based for multiaxial loading Equation (23) and critical plane-based Equation (26).

The parameters of the probabilistic $p-\psi-N$ fields (B, C, β, λ, δ) are estimated using the procedures proposed by Castillo and Fernández-Canteli [5], namely the constrained least square method (B, C) and the maximum likelihood method (β, λ, δ). For each damage parameter, the probabilistic field is presented using the percentile curves corresponding to probability of failures of 1%, 5%, 50%, 95% and 99%.

In addition to these percentile curves, the deterministic fitting curves described above are included [37]. Although shown on the same plots, the deterministic and the probabilistic models cannot be directly compared as they represent different mathematical expressions. Nevertheless, both models are included in the same plots for a pictorial comparison. As both models are based on fitting-data algorithms, these curves can be improved by using additional experimental data, for example in the very-low and very high cycle fatigue regions.

The fatigue experimental data used in this section were collected from the several references [10,21,23,28,29,36,37,39,40]. All experimental data were obtained at room temperature in laboratory air. The material from Eiffel bridge, ASTM A516 Gr. 70 and S460 steels were tested under constant amplitude loading, whereas the 2024-T3 aluminium was tested under variable and constant amplitude loading.

5.1. Smith-Watson-Topper damage parameter

Figure 4 illustrates the probabilistic fields correlating the parameter with the number of cycles to failure, for puddle iron from the Eiffel bridge [10,40]. Experimental data was obtained from smooth specimens tested under strain-controlled conditions ($R_{\varepsilon}=-1$). The figure also includes the deterministic fitting according to Equation (8). Both models correlate well with the data in the range of the experimental data, however they diverge for very low-cycle fatigue regimes, the probabilistic model producing more consistent results.

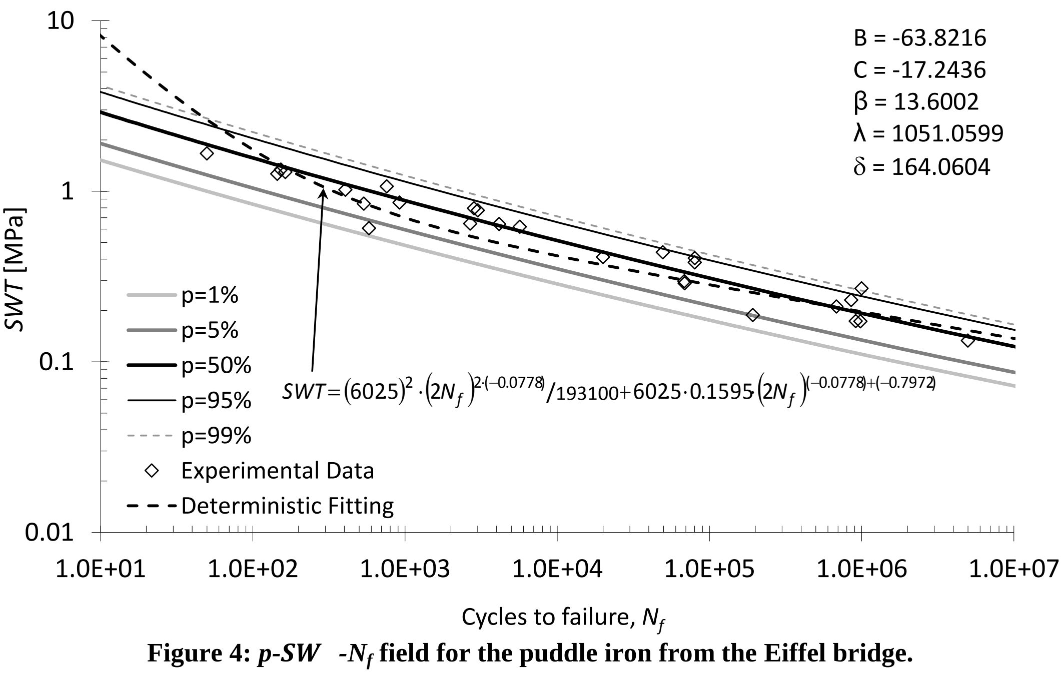

Figure 4: *p-SW* -N_f field for the puddle iron from the Eiffel bridge.

5.2. Walker-like strain damage parameter

The probabilistic fields using Walker-like strain damage parameter Equation (11) was applied for the fatigue experimental results of the 2024-T3 aluminium alloy that were collected in reference [37] and used in various references [23,28,38,39]. Figure 5 presents the probabilistic fields of the Walker-like strain amplitude with the fatigue life, , and shows a very good correlation with the experimental data. The value for the Walker-like strain damage parameter $\hat{\gamma}$ is equal to 0.60 for the range of data between 77 and 2.7×10^{6} cycles to failure. This data set illustrates the importance of the model as the $\hat{\gamma}$ value is not 0.5 as in the Smith-Watson-Topper model [19].

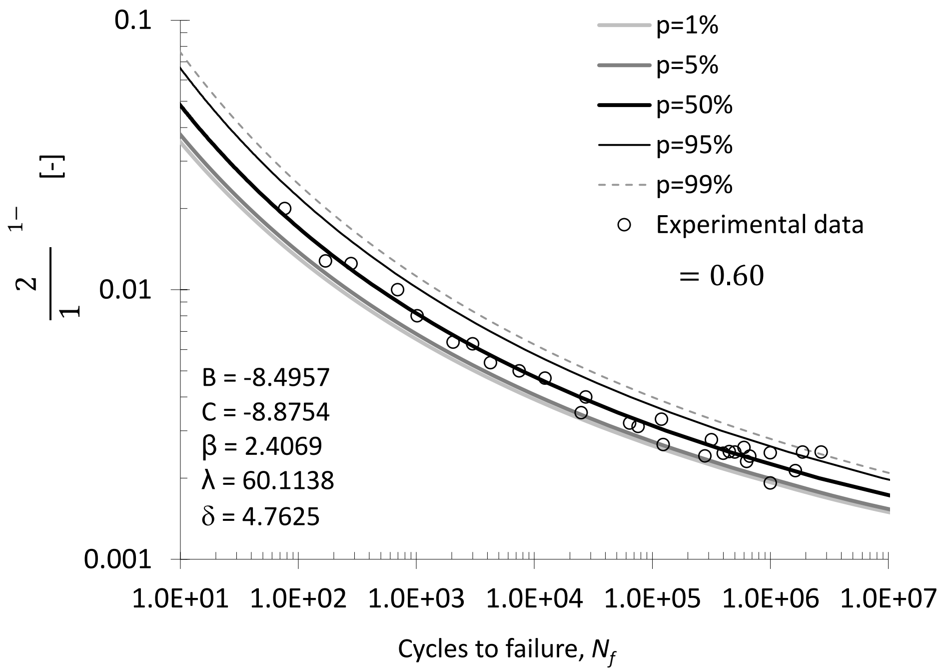

Figure 5: p-ψ-N_f field for 2024-T3 aluminium, where

5.3. Energy-based parameter for uniaxial loading conditions

Figure 6 shows the probabilistic field correlating the energy-based fatigue damage parameter , Equation (15), with the number of cycles to failure, using the experimental data obtained for the Eiffel bridge material [40]. The original deterministic relation proposed by Ellyin [21] underestimates the fatigue lives in the medium region and overestimates fatigue lives for the low and high-cycle fatigue regimes. The 50% percentile curve of the proposed model fits in between the two deterministic lines for very low-cycle fatigue. For very high-cycle fatigue, the 50% percentile curve falls below the two deterministic fitted lines, suggesting a lower fatigue limit. However, the probabilistic model is able to correlate the fatigue limit region if adequate data, including run-outs is available for this region. Only one data point falls outside this band, proving the accuracy of the probabilistic model.

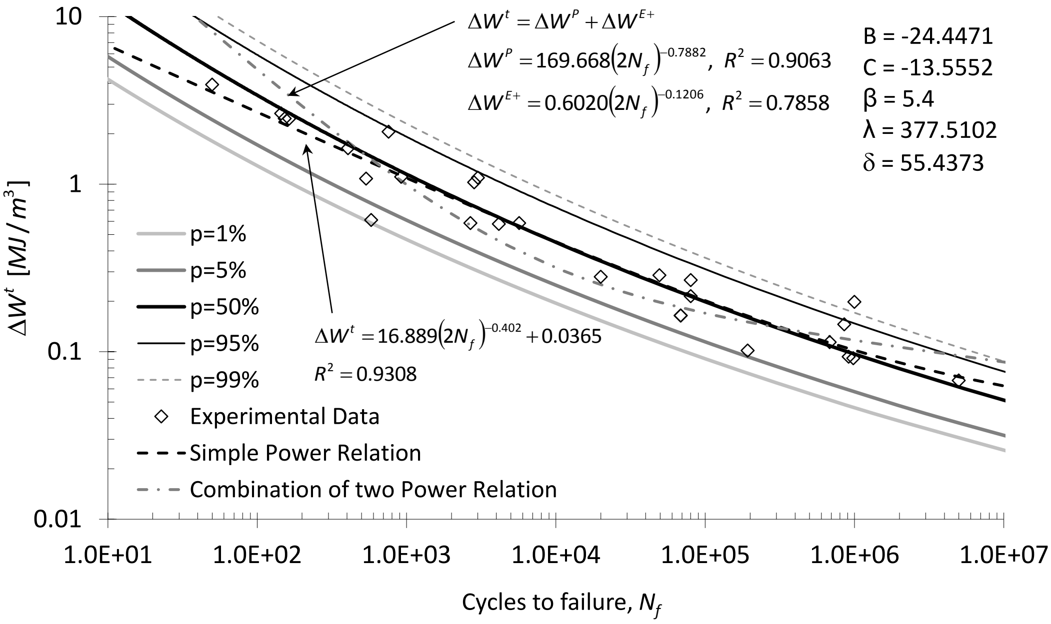

Figure 6: p-Δ -N_f field for the puddle iron from the Eiffel Bridge.

5.4. Energy-based fatigue damage parameters for multiaxial loading conditions

Figures 7 and 8 represent the probabilistic fields correlating the energy-based fatigue damage parameter, Equation (23), and the number of cycles to failure, using the multiaxial experimental data available from the ASTM A516 Gr. 70 [21]. Figure 7 includes only proportional data whereas Figure 8 includes both proportional and non-proportional data. In both cases, the comparison between the 50% percentile curve and the deterministic line of Equation (21) shows good agreement below 1×10^6 cycles. However, a deviation is observed above this fatigue life, with the 50% percentile curve falling below the deterministic line which seems to have a fatigue limit plateau.

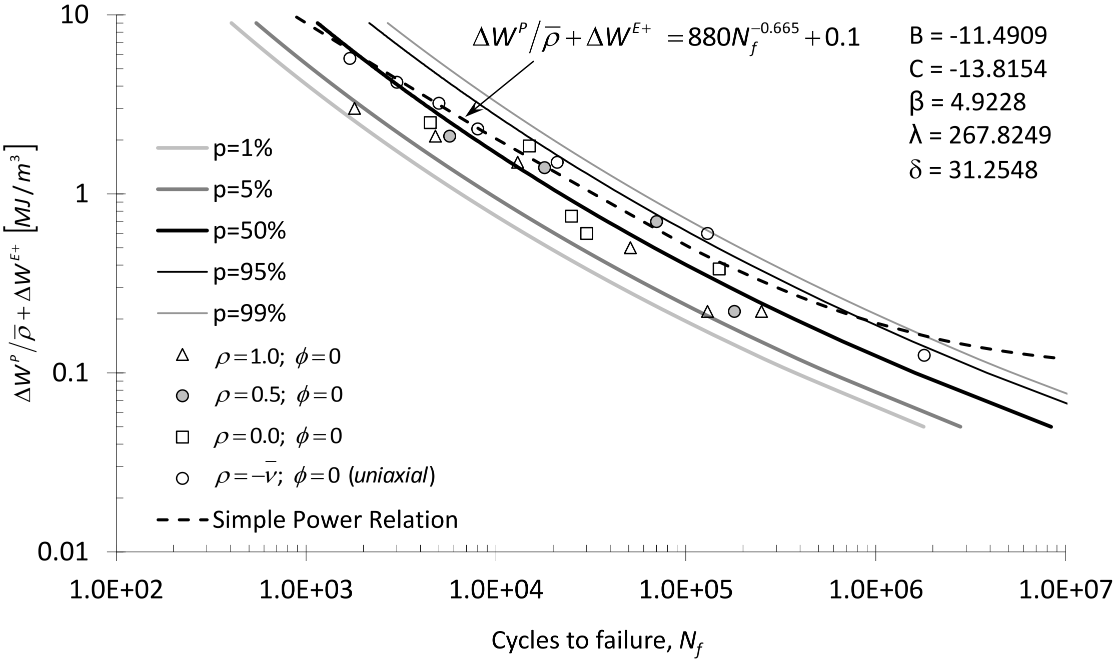

Figure 7. ***p*-(** **-N_f field proposed for the A516 Gr. 70.**

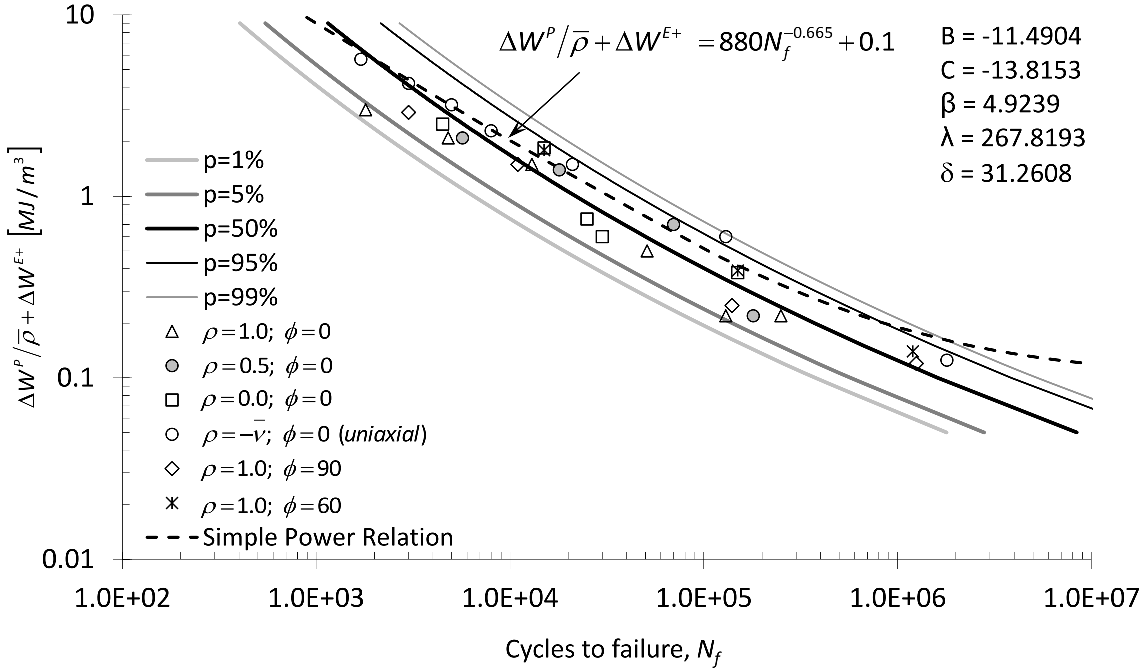

Figure 8. ***p*-(** **-N_f field proposed for the A516 Gr. 70 including non-proportional loading data.**

5.5. Critical plane based fatigue damage parameter for multiaxial loading conditions

Using data from tubular specimens made of S460 steel tested under pure tension-compression and pure torsion and trial-and-error procedure, Jiang *et al.* [36] identified the model parameters (K 98, a) and obtained the following power law:

(27)

where

(28)

Jiang *et al.* [36] demonstrated the suitability of Equation (27) to predict multiaxial proportional and non-proportional multiaxial loading paths. Figure 9 shows the probabilistic field correlating the critical plane fatigue parameter, Equation (26), proposed by Fatemi *et al*. [34,35] and the number of cycles to failure obtained for the S460N structural steel grade. This figure shows a good agreement of the model with the experimental data. The experimental data has a very narrow scatter band with one exception point corresponding to the lowest fatigue life. Censoring this lowest lifetime data point, the probabilistic field of Figure 10 shows an even better fit of the model with the data. In both cases, the 50% percentile line and the deterministic power relation show a good agreement.

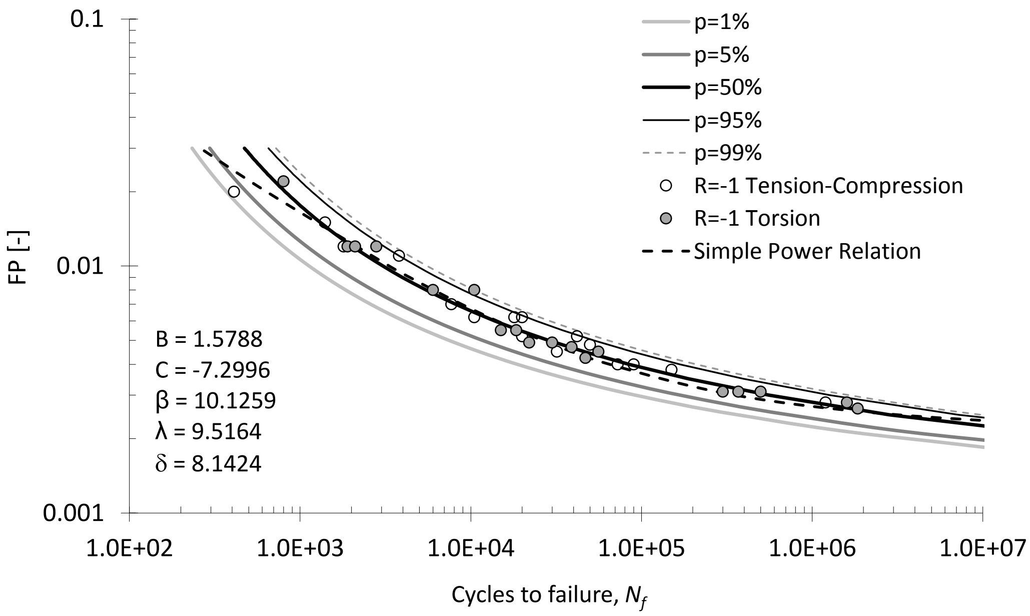

Figure 9. ***p-FP-N**$_f$* **field proposed for the S460N.**

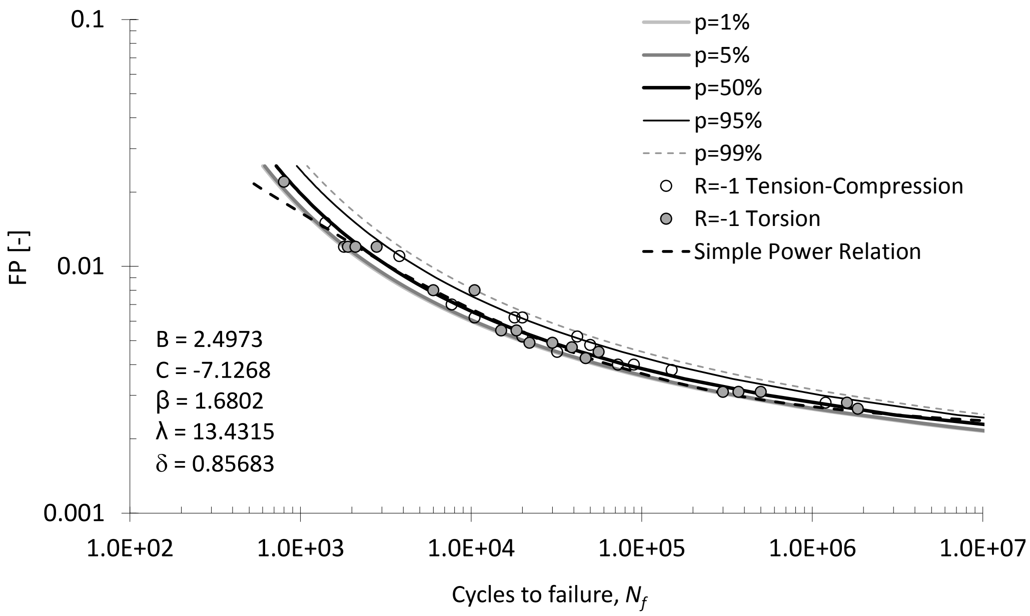

Figure 10. ***p-FP-N_f*** **field proposed for the S460N with one experimental data point censored.**

6. CONCLUSIONS

A generalization of the Castillo and Fernández-Canteli probabilistic model is proposed in this paper by considering a generic fatigue damage parameter ψ and obtaining a family of Weibull percentile curves, p–ψ–N_f. This proposal opens new perspectives for the application of the probabilistic model to a number of very general problems for estimating fatigue life of structural components. In particular, the proposed probabilistic model can be used as a suitable alternative to replace existing deterministic approaches to fatigue relating a damage parameter with the number of cycles. The model is applied to uniaxial fatigue damage parameter with mean-stress effects, and to uniaxial and multiaxial fatigue energy-based and critical plane based parameters. The model gives excellent results for various alloys (steel, puddle iron and aluminum) and can deal with the experimental scatter.

With this proposed generalized approach a straightforward probabilistic procedure is made available for fatigue problems were probabilistic approaches still have little penetration, which is the case of multiaxial approaches, which have been more concerned with new damage parameters development than tackle fatigue scatter in an appropriate way. Also, the model generalization to account for a diversity of fatigue damage parameters can also be extended to deal with size (scale) effects and non-uniform damage parameters fields, which is a topic to be addressed in forthcoming publications.

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REFERENCES

[1] Lawless JF. Statistical models and methods for lifetime data. John Wiley & Sons, 2011.

[2] Pascual FG, Meeker WQ. Estimating fatigue curves with the random fatigue limit model. Technometrics, 41, 277-290, 1999.

[3] Paolino DS, Chiandussi G, Rossetto M. A unified statistical model for the S-N fatigue curves: Probabilistic definition. Fat. and Fract. of Eng. Mat. and Structures, 36 (3), 187-201, 2013.

[4] Rémy F, Gilles C, Patrick P. Probabilistic modelling of S-N curves. Int. J. of Fatigue, 68, 217-223, 2104.

[5] Castillo E, Fernández-Canteli A. A Unified Statistical Methodology for Modeling Fatigue Damage. Springer, 2009.

[6] De Jesus AMP, Pinto H, Fernández-Canteli A, Castillo E, Correia JAFOC. Fatigue Assessment of a Riveted Shear Splice Based on a Steel Probabilistic Model. International Journal of Fatigue, Issue 2, Vol. 32, 2010, 453-462.

[7] Correia JAFO, De Jesus AMP, Fernández-Canteli A. Local unified model for fatigue crack initiation and propagation: application to a notched geometry. Engineering Structures, Vol. 52, 2013, 394-407.

[8] Correia JAFO, De Jesus AMP, Fernandez-Canteli A, Brighenti R, Moreira PMGP, Calçada RAB. A procedure to obtain the probabilistic Kitagawa-Takahashi diagram. UPB Scientific Bulletin, Series D: Mechanical Engineering, Vol. 78, Iss. 1, pags. 3-12, 2016.

[9] Correia JAFO, De Jesus AMP, Moreira PMGP, Calçada RAB, Fernández-Canteli A. Fatigue Crack Propagation Rates Prediction Using Probabilistic Strain-Based Approaches, Chapter 11, pp. 245-273, in: Fracture Mechanics – Properties, Patterns and Behaviours, Lucas Alves (Ed.), ISBN 978-953-51-2709-3, Print ISBN 978-953-51-2708-6, 330 pages, Publisher: InTech, 2016.

[10] Correia JAFO, De Jesus AMP, Fernández-Canteli A. A procedure to derive probabilistic fatigue crack propagation data. International Journal Structural Integrity, Issue 2, Vol. 3, 2012, 158-183.

[11] Sampayo LMCMV, Monteiro PMF, Correia JAFO, Xavier JMC, De Jesus AMP, Fernandez-Canteli A, Calçada RAB. Probabilistic S-N field assessment for a notched plate made of puddle iron from the Eiffel bridge with an elliptical hole. Procedia Engineering, 114 (2015) 691–698.

[12] Correia JAFO, De Jesus AMP, Fernández-Canteli A, Calçada RAB. Modelling probabilistic fatigue crack propagation rates for a mild structural steel. Frattura ed Integrita Strutturale, Vol. 31, 2015, 80-96.

[13] Zhu Shun-Peng, Huang Hong-Zhong, Ontiveros V, He Li-Ping, Modarres M. Probabilistic Low Cycle Fatigue Life Prediction Using an Energy-Based Damage Parameter and Accounting for Model Uncertainty. International Journal of Damage Mechanics, Nov-2012, Vol. 12, no. 8, pp. 1128-1153.

[14] Amraoui A, Gharad A, Bensalah MO. Probabilistic model of damage from a deterministic model and estimated lifetimes (case SAE 1137 carbon steel), Applied Mathematical Sciences, Vol. 7, 2013, no. 90, pp. 4465-4474.

[15] Koutiri I, Bellett D, Morel F, Pessard E. A probabilistic model for the high cycle fatigue behaviour of cast aluminium alloys subject to complex loads. International Journal of Fatigue, 2013, 47, pp. 137-147.

[16] Calvo S, Canales M, Gómez C, Valdés JR, Núñez JL. Probabilistic formulation of the multiaxial fatigue damage of Liu. International Journal of Fatigue 33 (2011) 460–465.

[17] Fernández-Canteli A, Blasón S, Correia JAFO, De Jesus AMP. A probabilistic interpretation of the Miner number for fatigue life prediction. Frattura ed Integrita Strutturale, Vol. 30, 2014, 327-339.

[18] Balsón S, Correia JAFO, De Jesus AMP, Fernandez-Canteli A, Calçada RAB. A probabilistic analysis of Miner's law for different loading conditions. Structural Engineering and Mechanics, An International Journal, Vol. 60, No. 1, 71-90, 2016.

[19] Smith KN, Watson P, Topper TH. A Stress-Strain Function for the Fatigue of Metals. Journal of Materials 1970; 5(4): 767-78.

[20] Koller R, Ruiz-Ripoll ML, García A, Fernández Canteli A, Castillo E. Experimental validation of a statistical model for the Wöhler field corresponding to any stress level and amplitude. Int. J. of Fatigue, 31, 231-241, 2009.

[21] Ellyin F. Fatigue damage, crack growth and life prediction. Chapman & Hall, 1997.

[22] Fernández Canteli A, Przybilla C, Nogal M, López Aenlle M, Castillo E. ProFatigue: A software program for probabilistic assessment of experimental fatigue data sets. Procedia Engineering, 74, 236-241, 2014.

[23] Dowling NE. Mechanical Behavior of Materials. (4th Edition), 2012, Prentice Hall.

[24] Apetre N, Arcari A, Dowling N, Iyyer N, Phan N. Probabilistic Model of Mean Stress Effects in Strain-Life Fatigue. Procedia Engineering 114 (2015) 538 – 545.

[25] Morrow JD. Cyclic plastic strain energy and fatigue of metals. Int Frict Damp Cyclic Plast ASTM STP 1965; 378: 45–87.

[26] Basquin OH. The exponential law of endurance tests. Proc Am Soc Test Mater 1910;10:625–30.

[27] Walker K. The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7075-T6 aluminum. Effects of Environment and Complex Load History on Fatigue Life. ASTM STP 462, Am. Soc. for Testing and Materials, Philadelphia, PA, pp. 1-14, 1970.

[28] Dowling NE. Mean stress effects in strain-life fatigue. Fatigue Fract Engng Mater Struct 32 (2009) 1004-1019.

[29] Ellyin F, Kujawski D. Plastic strain energy in fatigue failure. Journal Pressure Vessel Technology, Trans. ASME 1984; 106, 342-7.

[30] Ellyin F, Kujawski D. An energy-based fatigue failure criterion. Microstructure and Mechanical Behaviour of Materials, Vol. II (eds. Gu H, He J), EMAS, West Midlands, UK; 541-600.

[31] Golos K, Ellyin F. Generalization of cumulative damage multilevel cyclic loading. Theor. Appl. Fract. Mech. 1987; 7, 169-76.

[32] Golos K, Ellyin F. A total strain energy density theory live damage. J. Pressure Vessel Technol, Trans. ASME 1988; 110, 36-41.

[33] Socie DF, Marquis GB. Multiaxial Fatigue. Society of Automotive Engineers, Inc, 2000.

[34] Fatemi A, Socie DF. A Critical plane approach to multiaxial fatigue damage including out of phase loading. Fatigue Fract Eng Mater Struct 1988; 11: 149–165.

[35] Fatemi A, Kurath P. Multiaxial fatigue life predictions under the influence of mean stresses. ASME J Eng Mater Technol 1988; 110: 380–388.

[36] Jiang Y, Hertel O, Vormwald M. An experimental evaluation of three critical plane multiaxial fatigue criteria. International Journal of Structural Integrity, 2009; No. 29: 15–183.

[37] Baumel A, Seeger T. Materials Data for Cyclic Loading, Supplement 1, (1990) Elsevier Science Pubs., Amsterdam.

[38] Dowling NE. Mean stress effects in stress-life and strain-life fatigue. Fatigue 2004: Second SAE Brazil International Conference on Fatigue, São Paulo, Brazil, June 2004. Also SAE Paper No. 2004-01-2227, SAE International, Warrendale, PA, 2004.

[39] Dowling NE, Calhoun CA, Arcari A. Mean stress effects in stress-life fatigue and the Walker equation. Fatigue Fract. Engng. Mater. Struct. 32 (2009) 163-179.

[40] Correia JAFO. An Integral Probabilistic Approach for Fatigue Lifetime Prediction of Mechanical and Structural Components. Ph.D. Thesis, Faculty of Engineering, University of Porto, Porto, Portugal.

[41] Wang R, Li D, Dianyin Hu D, Meng F, Liu H, Ma Q. A combined critical distance and highly-stressed-volume model to evaluate the statistical size effect of the stress concentrator on low cycle fatigue of TA19 plate. International Journal of Fatigue, Volume 95, February 2017, Pages 8–17.

[42] Hu D, Ma Q, Shang S, Gao Y, Wang R. Creep-fatigue behavior of turbine disc of superalloy GH720Li at 650 °C and probabilistic creep-fatigue modelling. Materials Science and Engineering: A, Volume 670, 18 July 2016, Pages 17–25.

[43] Hu D, Wang R, Tao Z. Probabilistic design for turbine disk at high temperature. Aircraft Engineering and Aerospace Technology, 2011, 83, 199-207.