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Proposal of a fatigue crack propagation model taking into account crack closure effects using a modified CCS crack growth model

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Abstract

This paper proposes a modification of the fatigue crack growth model proposed by Castillo-Canteli-Siegele (CCS) to take into account crack opening and closure effects as well as the influence of the stress *R*-ratio. The theoretical model to obtain the effective stress intensity range, ΔK_{eff}, which takes into account the effects of the mean stress and the crack closure and opening effects proposed by *Correia et al*. was taken in consideration in this new proposal. This last model is based on the same initial assumptions of the analytical models proposed by *Hudak et al.* and *Ellyin*. This modified CCS crack propagation model is a new version of an explicit fatigue crack propagation model, supported by mathematical and physical assumptions.

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Keywords: Fatigue; Crack Growth; Effective Stress Intensity; Crack Closure; Gumbel Distribution.

1. Introduction

A reliable design of structural details taking into account fatigue damage requires a thorough knowledge of the materials behavior being used. The Fracture Mechanics based fatigue approaches are very common to assess the fatigue behavior/strength of structural details, however adequate/accurate fatigue crack propagation laws are required. Many fatigue crack propagation laws have been proposed in the literature, since the pioneer proposal by

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Paris (Paris et al. (1963)). Reference Beden et al. (2009) provides a compilation of fatigue crack propagation models that have been proposed in the literature. Recently, based on a normalization of the crack growth rate curve and its identification as a cumulative distribution function, particularly of the Gumbel family, a new fatigue crack propagation law was proposed by *Castillo-Canteli-Siegele* (Castillo et al. (2014)) allowing *S-N* propagation curves to be obtained for design. This model resulted from a dimensional analysis and was based on non-dimensional parameters, which allowed dimensionless constants. This is an original model in the way it results from the application of a statistical cumulative distribution function to represent the S-shaped fatigue crack propagation law. Under certain conditions, the model requires a unique integration of the differential equation to provide an analytical expression of a reference crack growth curve from which any other curve for any given initial crack size and stress range can be obtained without making use of any similarity or self-similarity assumptions. An extension of the CCS original model is feasible to take into account crack opening and closure effects (Elber (1970), Elber (1971)) as well as the influence of the *R* factor by considering an effective stress intensity factor range ΔK_{eff}, as proposed recently by *Correia et al.* (Correia et al. (2016)). In this way, the crack growth rates (*da/dN*) versus the effective stress intensity factor range (ΔK_{eff}) may be derived.

2. Review about crack closure on fatigue crack growth laws

The crack closure and opening effects have been intensively studied in the scientific community interested by fatigue. These effects are taken into account in the crack growth laws of the materials which are typically based on the stress intensity factor range, ΔK. The crack propagation laws are obtained using experimental results from fatigue crack growth tests of the materials.

The *Paris* law was the first crack growth model proposed in literature, which give a good description of the fatigue crack propagation in regime II (see Figure 1), correlating the fatigue crack growth rate with the stress intensity factor range (Paris et al. (1963)):

$$\frac{da}{dN} = C_p \cdot \Delta K^{m_p} \tag{1}$$

where C_p and m_p are material constants. *Walker* (Dowling (1998)) proposed an alternative relation to take into account the stress ratio effects in *Paris* law:

$$\frac{da}{dN} = C_w \left[\frac{\Delta K}{(1-R_\sigma)^{1-\gamma}}\right]^{m_w} = C_w[\overline{\Delta K}]^{m_w} \tag{2}$$

where C_w, m_w and γ are constants. Another relation (Pereira et al. (2012), Alves et al. (2015)) consisted on a modification of the *Paris* law to account for crack propagation regime I and stress ratio effects:

$$\frac{\mathrm{da}}{\mathrm{dN}} = C_w(\overline{\Delta \mathrm{K}} - \Delta \mathrm{K}_{\mathrm{th}})^{m_w} \tag{3}$$

Hartman and *Schijve* (Hartman et al. (1970)) proposed a law to cover the three crack propagation regimes, wherein the results of this relation are a sigmoidal shaped curve with vertical asymptotes at $K_{max}=K_c$ and $\Delta K=\Delta K_{th}$, resulting in the typical S-shaped relation:

$$\frac{da}{dN} = \frac{C \cdot (\Delta K - \Delta K_{th})^m}{(1-R)K_c - \Delta K} \tag{4}$$

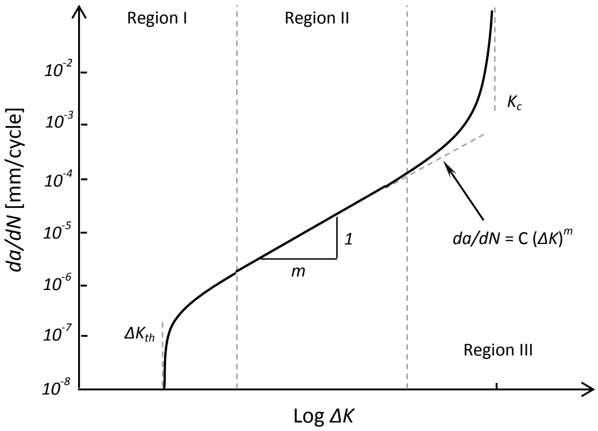

Figure 1. Fatigue crack propagation regimes.

A recent model for fatigue crack propagation as proposed by *Castillo-Canteli-Siegele* (CCS) (Castillo et al. (2014), Blasón et al. (2015)) is a new solution for the fatigue crack propagation based on the assumption that the crack growth follows a cumulative distribution function – the Gumbel distribution. The inconvenience of dimensional parameters in existing crack propagation models, some of them previously described, is overcome in this new proposal by means of an appropriate dimensional analysis, carried out over the influent variables leading to adimensional normalized parameters. The identification of the crack growth rate curve, as a cumulative distribution function in which ΔK^{*+} is identified as the normalizing variable defined in the interval [0,1] leads to the consideration of log (*da/dN*) as the random variable. The proposed model represents an explicit fatigue cracks growth relation that was supported by mathematical and physical assumptions. This model is given by the following expressions:

$$\Delta K^{*+} = \frac{log\ \Delta K^{*} - log\ \Delta K^{*}_{th}}{log\ \Delta K^{*}_{up} - log\ \Delta K^{*}_{th}}$$

$$\Delta K^{*+} = F\left(log\frac{da^{*}}{dN^{*}}\right) = exp\left[-exp\left(\frac{\alpha - log\ \frac{da^{*}}{dN^{*}}}{\gamma}\right)\right] \tag{5}$$

This model depends on four parameters, α, γ, ΔK^{*}_{th} and ΔK^{*}_{up} which may be computed by the least-squares technique (Castillo et al. (2014), Blasón et al. (2015)). The normalized variables of the model suggested by the authors are given by the following relations:

$$a^{*} = \frac{a}{W}$$

$$N^{*} = \frac{N}{N_0}$$

$$\Delta K^{*} = \frac{K_{max} - K_{min}}{K_c} \tag{6}$$

$$\Delta K^{*}_{th} = \frac{\Delta K_{th}}{K_c}$$

$$\Delta K^{*}_{up} = \frac{\Delta K_{up}}{K_c}$$

where, a is the crack length, W is a characteristic length (e.g. specimen width), N is the number of cycles, N_0 is a reference number of cycles, K_{max} and K_{min} are the maximum and minimum stress intensity factors, respectively, ΔK_{th} is the threshold stress intensity factor range, ΔK_{up} is the limit stress intensity factor range, and finally, K_c is the material characteristic fracture toughness. The equation of the model can be written of the following form:

$$\frac{da^*(N^*)}{dN^*} = exp\left[F^{-1}\left(\frac{log\ K - log\ \Delta K^*_{th}}{log\ \Delta K^*_{up} - log\ \Delta K^*_{th}}\right)\right] \tag{7}$$

According to the approach of *Elber* (Elber (1970), Elber (1971)), the fatigue crack growth rate, da/dN, is a function of the effective stress intensity factor range, ΔK_{eff}, according to the following expression:

$$\frac{da}{dN} = f(\Delta K_{eff}) \tag{8}$$

Elber suggested (Elber (1970), Elber (1971)) that crack closure and opening effects could be characterized in terms of the effective stress intensity factor range which is normalized by the applied stress intensity factor, resulting the U ratio, with the following form:

$$U = \frac{\Delta K_{eff}}{\Delta K} \tag{9}$$

with,

$$\Delta K_{eff} = K_{max} - K_{op} \tag{10}$$

$$\Delta K = K_{max} - K_{min} \tag{11}$$

where, ΔK_{eff} is the effective stress intensity factor range, ΔK is the applied stress intensity factor range, K_{max} is the maximum stress intensity factor, K_{min} being the minimum stress intensity factor, and K_{op} is the crack opening stress intensity factor.

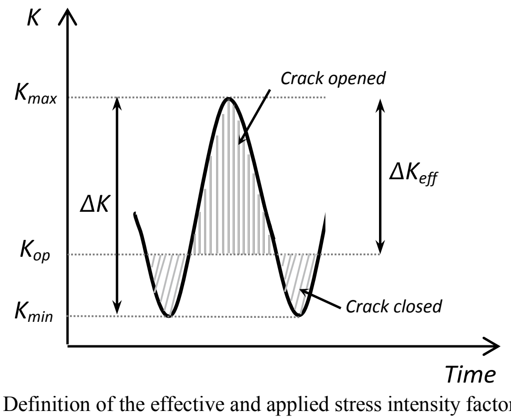

Figure 2. Definition of the effective and applied stress intensity factor ranges.

The studies developed by *Elber* were for the 2024-T3 aluminum alloy, using stress R-ratios, R, between -0.1 and 0.7. In this study was observed that U depends on R, as follows:

$$U = 0.5 + 0.4R \tag{12}$$

Schijve (Schijve (2004)) proposed an improved expression of Equation (12), which it represents a more realistic behavior, accounting for the crack closure and opening effects for -1⩽R⩽1,

$$U = 0.55 + 0.33R + 0.12R^2 \tag{13}$$

Another proposal was presented by ASTM (Geerlofs (2004), Ellyin (1997)):

$$\begin{aligned} &\Delta K = K_{max} \;\; for\ R < 0 \\ &\Delta K = K_{max} - K_{min} \;\; for\ R \geq 0 \\ &U = 0.576 + 0.015R + 0.409R^2 \end{aligned} \tag{14}$$

Newman (Newman (1984)) proposed a general crack opening stress approach to correlate fatigue crack growth rate data for other materials and thicknesses, under constant amplitude loading, once the proper constraint factor has been determined. This approach is based on plasticity of the materials.

Elastoplastic analysis based on analytical (Newman (1984), Vormwald et al. (1991), Vormwald (2015), Savaidis et al. (1995)) or numerical (McClung et al. (1991), Nakagaki (1979)) approaches can be used to estimate the crack closure and opening effects.

Other approaches were proposed by *Hudak et al.* (Hudak et al. (1988)), *Ellyin* (Ellyin (1997)) and more recently by *Correia et al*. (Correia et al. (2016)) supported by experimental results from fatigue crack propagation tests and theoretical assumptions. The expression proposed by *Hudak et al.* (Hudak et al. (1988)) has the following form:

$$\begin{aligned} &U = \gamma\left(1 - \frac{K_o}{K_{max}}\right), \qquad K_{max} \leq K_L \\ &U = 1, \qquad K_{max} \geq K_L \end{aligned} \tag{15}$$

where K_o is a constant related to the pure Mode I fatigue crack growth threshold. *Ellyin* (Ellyin (1997)) proposed a modified version of proposal by *Hudak et al.* (Hudak et al. (1988)) to define the effective stress intensity range, ΔK_{eff}, taking into account the stress ratio, R, and the threshold value of stress intensity factor range, ΔK_{th}, with constant amplitude loading:

$$\begin{aligned} &\Delta K_{eff,0} = (\Delta K^2 - \Delta K_{th}^2)^{1/2} \approx \Delta K\left[1 - \frac{1}{2}\left(\frac{\Delta K_{th}}{\Delta K}\right)^2\right] > \Delta K - \Delta K_{th} \quad for\ R \approx 0 \\ &\Delta K_{eff} = \frac{\Delta K_{eff,0}}{\left[1 - \left(\sigma_m/\sigma_f'\right)\right]} = \frac{\Delta K_{eff,0}}{\left[1 - \left((1+R)\,\sigma_{max}/2\sigma_f'\right)\right]} \quad for\ R \neq 0 \end{aligned} \tag{16}$$

where, σ_m is the mean stress, σ_{max} is the maximum stress, and σ_f' is the fatigue strength coefficient. *Correia et al*. (Correia et al. (2016)) proposed recently a new law to obtain the U parameter. This model has the same initial assumptions of the analytical models proposed by *Hudak et al.* (Hudak et al. (1988)) and *Ellyin* (Ellyin (1997)):

$$\begin{aligned} &U = \left(1 - \frac{\Delta K_{th,0}}{K_{max}}\right)(1-R)^{\gamma-1} \;\; for\ K_{max} \leq K_L \\ &U = 1 \;\; for\ K_{max} \geq K_L \end{aligned} \tag{17}$$

where, K_L is the limiting K_{max}.

3. Update of the CCS crack growth model to account for crack closure effects

In this paper, an update of the CCS crack propagation model is proposed in order to extend its applicability, taking into account the stress ratio effects and consequent crack opening and closure effects of the material. This proposal to modify the CCS crack growth model consists in updating the applied stress intensity factor range, ΔK, by the effective stress intensity factor range, ΔK_{eff}, in the original law. Thus, the modified proposal is given by the following expression:

$$\Delta K_{eff}^{*+} = \frac{log \Delta K_{eff}^{*} - log\ \Delta K_{th}^{*}}{log\ \Delta K_{up}^{*} - log\ \Delta K_{th}^{*}}$$

$$\Delta K_{eff}^{*+} = F\left(log \frac{da^*}{dN^*}\right) = exp\left[-exp\left(\frac{\alpha - log\ \frac{da^*}{dN^*}}{\gamma}\right)\right] \tag{18}$$

which depends on the same four parameters α, γ, ΔK_{th} and ΔK_{up} of the original model proposed by Castillo-Canteli-Siegele (Castillo et al. (2014), Blasón et al. (2015)), but with changes in the new normalized variables suggested by the authors in this paper, as given by the following relations:

$$\Delta K_{eff}^{*} = \frac{\Delta K \cdot U}{K_c}$$

$$\Delta K_{th}^{*} = \frac{\Delta K_{th,0} \cdot \left(1 - R_{eff}\right)^{\gamma m}}{K_c} \tag{19}$$

$$R_{eff} = \frac{K_{op}}{K_{max}} = \frac{K_{op}}{K_{min}} \cdot R$$

where, a, W, N, N_0, ΔK_{th}, ΔK_{up} and K_c have the same characteristics of the original CCS crack growth model, $\Delta K_{th,0}$ is the pure Mode I stress intensity factor range threshold for null stress ratio, γ is the material constant, R is the stress R-ratio, R_{eff} is the effective stress R-ratio that accounts for higher minimum stress intensity factor range due to crack closure, K_{max} is the maximum stress intensity factor, K_{min} is the minimum stress intensity factor, K_{op} is the crack opening stress intensity factor, and finally, U is the $\Delta K_{eff}/\ \Delta K$ ratio. The U parameter can be obtained with experimental fatigue tests using local measuring techniques according to ASTM E 647 (Geerlofs et al. (2004)) or with analytical and numerical approaches present in literature. The second Equation (19) for the threshold value of the stress intensity range, ΔK_{th}, function of the stress R-ratio, R, is based on the original relation proposed by *Klesnil* and *Lukáš* (Klesnil et al. (1992)):

$$\Delta K_{th} = \Delta K_{th,0} \cdot (1 - R)^{\gamma} \tag{20}$$

The updated CSS model can then be written in the following explicit form for the fatigue crack propagation rates:

$$log \frac{da^*}{dN^*} = F^{-1}\left(\Delta K_{eff}^{*+}\right) = F^{-1}\left(\frac{log\ \Delta K_{eff}^{*} - log\ \Delta K_{th}^{*}}{log\ \Delta K_{up}^{*} - log\ \Delta K_{th}^{*}}\right)$$

$$\frac{da^*}{dN^*} = exp\left[F^{-1}\left(\Delta K_{eff}^{*+}\right)\right] = exp\left[F^{-1}\left(\frac{log\ \Delta K_{eff}^{*} - log\ \Delta K_{th}^{*}}{log\ \Delta K_{up}^{*} - log\ \Delta K_{th}^{*}}\right)\right] \tag{21}$$

where, *F()* is a cumulative distribution function, which was selected as the Gumbel function.

4. Conclusions

The main conclusions of this paper can be summarized as follows:

- The proposal model is based on CCS crack growth model. This model provides an analytical expression of the crack propagation rate curve by fitting experimental data using a least squares technique, to be used in the design of structural elements;
- An identification of new variables to be introduced in the fatigue crack model was carried out. The new variables take into account the crack closure and opening effects using as well the influence of stress R-ratio;
- Proposing the amendment of the normalized variable ΔK^{*+}, which takes values in the range [0,1], allows us to use "S-shaped" cumulative distribution functions to reproduce the relation between ΔK^{*+}_{eff} and *da/dN*. This permits incorporation of a wide range of models coming from the statistical field to solve the crack growth problem;
- Using this proposal it is possible the derivation of the *a-N* and *S-N* curves for different initial crack size values and load, or remote stress range, taking into account the crack closure effects and influence of the stress ratio of the materials.
- Future works will seek the experimental validation of the proposed fatigue crack propagation model.

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