Statistical joint evaluation of fracture results from distinct experimental programs: An application to annealed glass

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Abstract

In this work, the generalized local model (GLM), as a procedure to derive the Weibull primary failure cumulative distribution function (PFCDF) related to an adequate reference parameter is applied to predict failure of annealed glass specimens from two different experimental test types. The procedure allows a unique PFCDF, identified as a failure material characteristic, to be inferred irrespective from the test data being evaluated either for each single test type sample separately or for a unique sample pooled from both single samples as a whole, whereby the particular specimen features pertaining to the two samples concerning specimen shape and size are considered in the assessment. Nevertheless, higher reliability is ensured in the parameter estimation for the case of pooled evaluation as a result of the higher number of specimens being jointly evaluated. In this way, once a suitable generalized parameter is selected, failure of specimens or components may be reliably predicted independently of the specimen shape and size chosen for the tests. The quality of the generalized parameter selected can be also discerned using this methodology. The applicability of the approach proposed is demonstrated by a practical example comprising two different test types on glass plates.

Keywords: Probability; Fracture; Annealed Glass; Experimental Techniques.

Nomenclature

GLM	Generalized local model
GP	Generalized or reference parameter
PFCDF	Primary failure cumulative distribution function
EFCDF	Experimental failure cumulative distribution function
P_{fail}	Global probability of failure
P_{sur}	Global probability of survival
$P_{sur,\Delta S}$	Probability of survival for an elementary size ΔS
$P_{fail,\Delta S}$	Probability of failure for an elementary size ΔS
P_{int}	Global probability
λ	Location Weibull parameter
δ	Scale Weibull parameter
β	Shape Weibull parameter
S_{ref}	Reference size
S_{eq}	Equivalent size
RoR	Coaxial ring on ring test
4PB	Four point bending test

1. Introduction and motivation

The main objective of experimental programs in fracture mechanics consists in inferring the critical value of a reference generalized parameter (GP) identified with failure of the material, such as an equivalent stress, stress intensity factor, J-integral, etc., or any other one characterizing that critical condition. In any case, it is well known that the scatter of the critical reference parameter is omnipresent and must be evaluated statistically.

Aiming at achieving a more general overview of the parameter influence, experimental programs usually encompass distinct test batches or samples, each of them consisting in a low number of specimens of similar characteristics with certain parameter diversity (for instance, specimen shape and size or test type).

As a consequence of the relative specimen scarcity and sample diversity, the reliability of the statistical assessment of the failure phenomenon declines revealing a conflict of interest: from the point of view of achieving a reliable cumulative distribution function (cdf), a sample consisting in a large amount of similar tests is desirable but, on the other side, testing several sample classes, implying distinct shape, size and test type, would be ideal to confirm the universality and suitability of the generalized parameter though the latter implies the inherent inconvenience of their statistical assessment.

In previous works [1,2], the authors have introduced the so-called generalized local model (GLM) to derive the primary failure cumulative distribution function (PFCDF) for a generalized parameter based on experimental data even from specimens with different geometry and size. Accordingly, the primary cdf of the generalized parameter may be interpreted as a material property allowing failure of specimens or components to be predicted independently of the specimen shape and size and test type selected for the experimental program. Thus, a probabilistic failure prediction of components, when subjected to whatever, uniform or non-uniform, distribution of the generalized parameter can be accomplished.

In this paper, a methodology based on the GLM allows the primary failure cumulative distribution function of failure PFCDF of the material to be derived from a joint evaluation of the results obtained for the different samples as a whole. In this way, the joint PFCDF can be determined as a failure material characteristic pooling all the experimental results obtained from distinct test programs consisting in diversified samples concerning specimen shape, size and test type. In any case, the joint PFCDF allows the probability of failure for any of the samples to be predicted, irrespective of the particular features of the specimen involved whereas more confident parameter estimation is achieved as a result of the higher total number of results implied in the assessment.

The applicability of the approach proposed is corroborated by a practical example, which provides satisfactory results. The results of a large experimental program in which annealed glass specimens of different dimensions based on four-point bending and coaxial double ring tests (also known as ring on ring test) are tested and evaluated by applying the generalized local method. Annealed glass is selected as a suitable material for the experimental program since, first, several test types, as 3- and 4-point bending and ring on ring tests, are currently used to characterize glass failure strength, second, a large scatter is exhibited by the experimental data, and third, no satisfactory explanation is found till now in what concerns the lack of agreement among the results ensuing from the different test types currently used, what evidences to be an open issue.

Up to now, there is a certain lack in the regulation of structural use of glass [3]. The current methodology for structural glass design is basically represented by different approaches. Both the American code [4] and the Australian code [5] refer to glass as a panel not intended mainly for structural use while in Europe there is no consensus for a Eurocode for glass.

In European glass design concepts, it is mostly assumed that all cracks are oriented perpendicularly to the first principal stress. There are also global safety factors to diminish the risk of material failure, which are usually introduced by reducing the breaking strength in experiments to an allowable design stress. This concept, as opposed to the limit states one using probabilistic criteria based on a fracture approach, entails some important drawbacks [6] evidencing that the definition of criteria of probabilistic structural glass failure is still an open issue. In this paper, a contribution to overcome this situation is pursued.

The reliability level provided by two different fracture parameters and the corresponding failure criteria for glass under bending load is checked by comparing the failure results obtained experimentally owning to one test type with those predicted for this test type using the experimental results of the other test type, and vice-versa. Thereafter, the suitable failure criterion is selected and all results from both different experimental programs are fitted as a whole thus providing higher reliability for the PFCDF.

The main goal of this work consists in demonstrating the possibility of pooling results from different experimental programs to evaluate a reliable PFCDF and to check the quality of the reference parameter for failure prediction. It is also important to notice that the generalized parameter distribution does not need necessarily to be described analytically but can be determined by finite element computation under general loading conditions. As a result, it is suitable for components design rather than being restricted to the use for laboratory specimens.

2. Experimental results and selection of the generalized parameter

2.1. Material properties and experimental procedure

Annealed glass, characterized by brittle behaviour besides large scatter of the test results [7], is selected as the testing material for investigating the applicability of the statistical joint evaluation model. The unavoidable distribution of micro-cracks on the surface is the mainly reason of this scatter to be taken into account using the proposed probabilistic failure model.

The experimental program is subdivided in 2 batches, in the following denoted “samples”, comprising different shape of the specimens tested and the loading conditions, namely: a) four point bending test (4PB) and b) ring on ring test (RoR). A total of 30 specimens per sample are cut off from the same glass plate in order to reduce the variability of the material properties during the fabrication process. The main dimensions and characteristics for both samples are found in Fig.1. The Young modulus and the Poisson coefficient of the glass are E=72 GPa and μ=0.23 respectively. All test are performed according to the UNE-EN 1288-3:2000 and UNE-EN 1288-5:2000 standards [8,9].

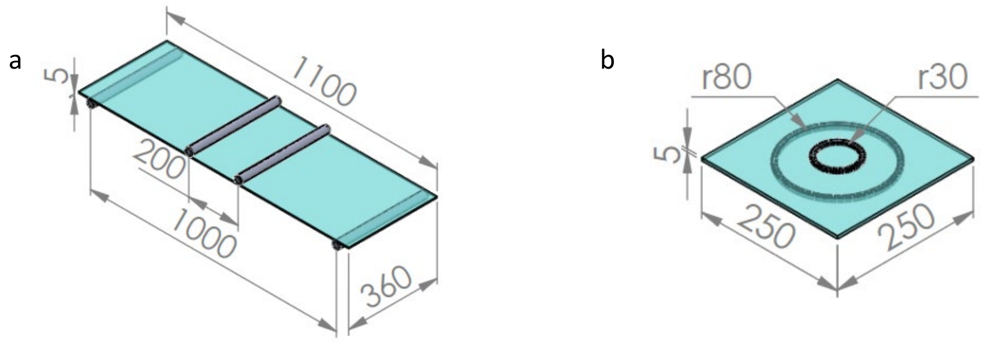

Fig 1. Geometry of the specimens used (in mm): a) 4-point bending tests; b) Ring on ring tests.

The following data measurements are performed during the experimental program carried out in a MTS Bionix uniaxial 100KN test machine. Load and displacement are measured directly from the test machine whereas, strains in the middle point and under load points are obtained, using strain gauges. Additionally, local displacements on the surface between the load rollers are recorded, by means of the digital image correlation equipment ARAMIS GOM 5M[10].

Former data are the starting point to proceed to fit and evaluate the experimental failure cumulative distribution function (EFCDF) and to check the usefulness of the iterative method applied here.

2.2. Results and simulation

Tables A1 and A2 in appendix A and Fig. 2 summarize the experimental results obtained in the laboratory for both samples. In order to define the critical stress and strain conditions at failure, the progression of each test is simulated by FEM using the commercial software ABAQUS 6.12 [11]. Hexahedral continuum shell elements with reduced integration (SC8R) are used in the numerical model, the load is applied to the rollers (4PB) or the load ring (RoR), respectively, as a constant ramp displacement. The validation of the FEM is achieved by comparing the reaction forces, the strains in the middle span and under the load points, and the local displacements between the rollers with those obtained experimentally. In all cases the differences found are less than 5%.

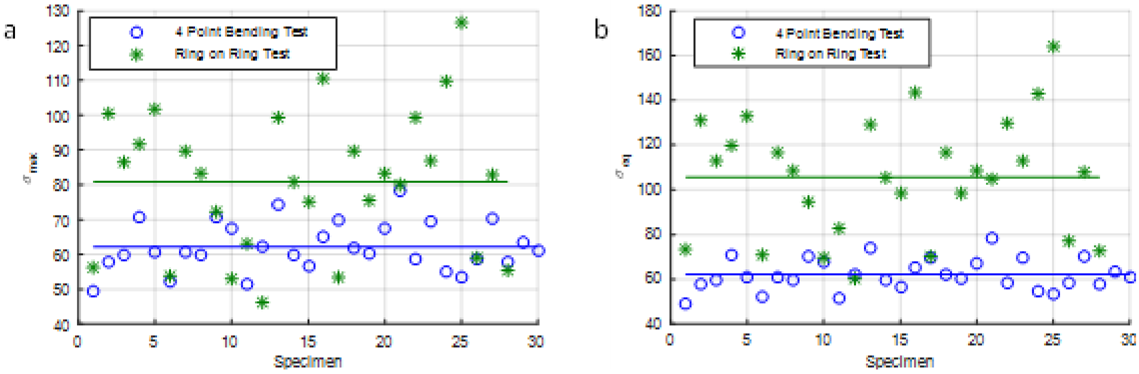

Fig 2. Experimental results using: a) Principal maximum stress criterion; b) Biaxial stress criterion.

2.3. Selection of the generalized parameter

As one of the most common generalized parameters used to evaluate the probability of failure for annealed glass, the principal maximum stress is firstly considered (See [12-14]):

$$GP = \sigma_{MAX} = \sigma_I \tag{1}$$

Taking into account the generalized parameter so defined, the local values for each element *j* in each test *i* (GP_{ij}) are obtained by applying the commercial finite element code ABAQUS 6.12. Thereafter, the generalized local model developed by the authors [1,2] is used to derive the primary failure cumulative distributions functions (PFCDF) for each sample, which are based on the three-parameter Weibull distribution:

$$P_{fail_i} = 1 - \exp\left[-\left(\frac{GP_{eq_i} - \lambda}{\delta_{ref}} \right) \right]^{\beta} \tag{2}$$

where β, λ and δ are the Weibull shape, location and scale parameters, respectively

Figure 3a) shows the PFCDF obtained for 4PB and RoR tests, besides the 5% and 95% reliability intervals calculated by means of the bootstrap method [15]. As can be seen, a clear difference appears between both PFCDFs, in contradiction with the GPLM, which states that a suitable selection of the generalized parameter leads to a unique PFCDF, this being shape and size independent and, as a consequence, a material property. Obviously, a perfect coincidence between both PFCDFs is only expected for infinite number of tests, whereas the results for a finite test number must be, at least, enveloped in the limits of the confidence intervals.

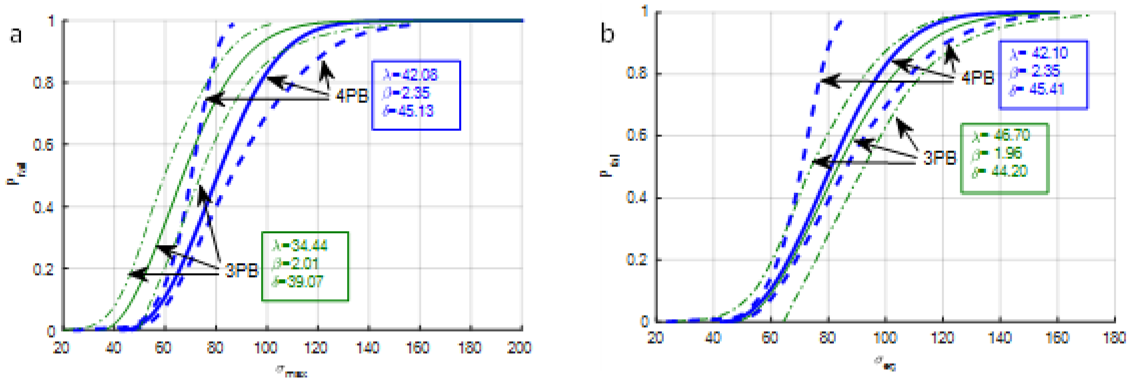

Fig 3. PFCDF (S_{ref}=10000 mm^2) and 5-95% confident intervals obtained for ring on ring and 4-point bending results based on different criteria: a) Principal maximum stress; b) Equivalent biaxial stress.

As evidenced by Figs. 4a) and 4b), the experimental cdfs for both test types differ substantially from those derived from the results obtained for the other test type. This fact evidences that the generalized parameter is not sound selected so that transferability of test results among different specimen shapes, sizes or test types is not ensured when the principal maximum stress is proposed as the generalized parameter. Therefore, the latter would be inadequate to predict failure of real components if failure characterization of glass is based on currently glass standard tests. In order to reinforce arguments leading to rejection of the principal maximum stress as the reference parameter, the quality and transferability of the PFCDFs obtained are additionally checked in two alternative ways by using: a) the PFCDF derived from 4PB results to predict failure in case of RoR tests (see Fig. 4a)) the PFCDF obtained from RoR results to predict failure in case of 4PB tests (see Fig. 4b)).

A possible reason for the poor correspondence between the failure cdf for both samples could be due to the biaxiality nature of the stresses with increasingly relevance in the case of ring on ring tests compared to four point bending tests. With the aim of taking into account such a biaxiality stress effect, an equivalent stress based on the principle of independent action (PIA) [16,17] is alternatively proposed as the generalized parameter:

$$GP = \sigma_{eq} = \left(\sigma_I^k + \sigma_{II}^k + \sigma_{III}^k\right)^{1/k} \tag{3}$$

According to our experience in the analysis of glass components using the multiaxial principal independent action, a value of K=3 could be recommended for Eq. (3). As in the previous generalized parameter case, the primary curves are now calculated for each test type. Figure 3b) represents the PFCDFs along with the 5% - 95% reliability intervals for each of them. In both cases, the confidence intervals envelop both curves proving good agreement between the two PFCDFs, so that the incomplete agreement could be assigned to the limited number of tests at disposal. In order to ensure the reliability of these PFCDFs and transferability of the results between both experimental samples, the PFCDF obtained from 4PB tests is used to derive the prediction of failure for RoR and vice-versa (see Figs. 4c) and 4d)) evidencing that experimental results and failure predictions fall much closer each other when the failure criterion each other.

From the point of view of reliability, a unique experimental program comprising sixty tests would provide a more reliable PFCDF than two different experimental programs each comprising thirty tests. Nevertheless, the second option offers the possibility of checking and comparing the suitability of candidate failure criteria. Both advantages are met in the generalized local model, which allows a single PFCDF to be evaluated with the reliability furnished by the sixty results by pooling both single samples comprising each thirty tests without renouncing to scrutinize the suitability of the failure criteria. The same reasoning can be applied to the joint evaluation when more than two samples have to be handled

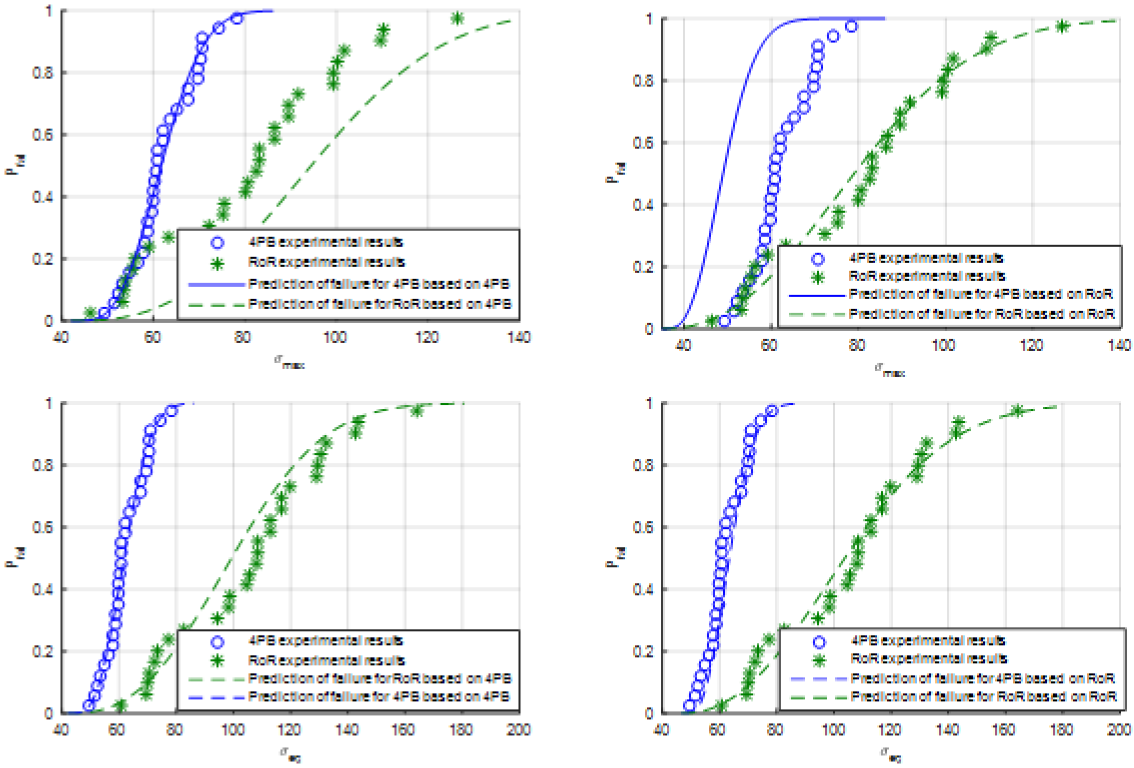

Fig 4. Prediction of failure: a) based on the maximum stress criterion and the PFCDF obtained from 4PB results; b) based on the maximum stress criterion and the PFCDF obtained from RoR results; c) based on the equivalent stress criterion and the PFCDF obtained from 4PB results; d) based on the equivalent stress criterion and the PFCDF obtained from RoR results.

3. Joint assessment of the single samples

In the following, the methodology applied for statistical joint evaluation of failure results of annealed glass from distinct experimental programs, implying different specimen types, is exemplified. The flow chart shown in Fig. 5 summarizes the iterative procedure comprising the steps detailed below.

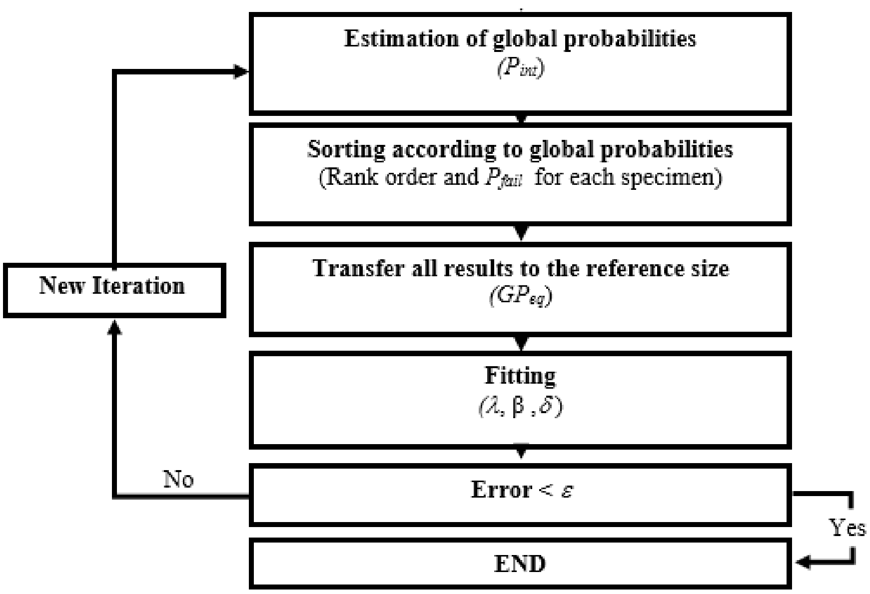

Figure 5. Flaw chart summarizing the joint assessment.

3.1. Estimation of global probabilities

Usually, the first step in a statistical evaluation consists in sorting all results in ascending order in accordance to the probability of failure. This entails the first difficulty to overcome in the joint assessment of experimental samples comprising heterogeneous specimen geometries and sizes, because the critical value of the generalized parameter reached at failure of each test is related to the particular size and geometry related to the specimen, so that the initial data ranking is not obvious. For example, it is not clear if the case of a RoR test failure for a high value of the GP referred to a small area represents higher or lower probability of failure than the failure of a 4-PB occurring for a low value of the GP referred to a big area. For this reason, the global probability of failure related to each test must be initially estimated from the following equation:

$$P_{Int\,j} = 1 - \prod (1 - P_{fail,\Delta S_{ij}}) = 1 - \prod \left(\exp\left[\frac{\Delta S_{ij}}{S_{ref}} \left(\frac{GP_{ij} - \lambda}{\delta_{ref}} \right)^{\beta} \right] \right) \quad (4)$$

This global probability of failure is a combination of the probability of failure implying all elements in the model, which depends on the element size (ΔS_{ij}), the local value of the generalized parameter (GP_{ij}) and the three Weibull parameters (λ, δ_{ref} and β). In the first iteration, the values of the δ_{ref} and β Weibull parameters can be assumed as the mean values corresponding to each sample in the previous section, whereas λ can be assumed as the minimum value resulting from the individual fitting. It is important to ensure that the initial values elude saturated values of the global probabilities, i.e. 0 or 1, for any test result, because in such cases sorting becomes unfeasible. The relation between the maximum GP values and the global probabilities of failure for each test is shown in Fig. 6 for both generalized parameter, i.e. maximum stress criterion (Fig. 6a) and biaxial equivalent stress criterion (Fig. 6b).

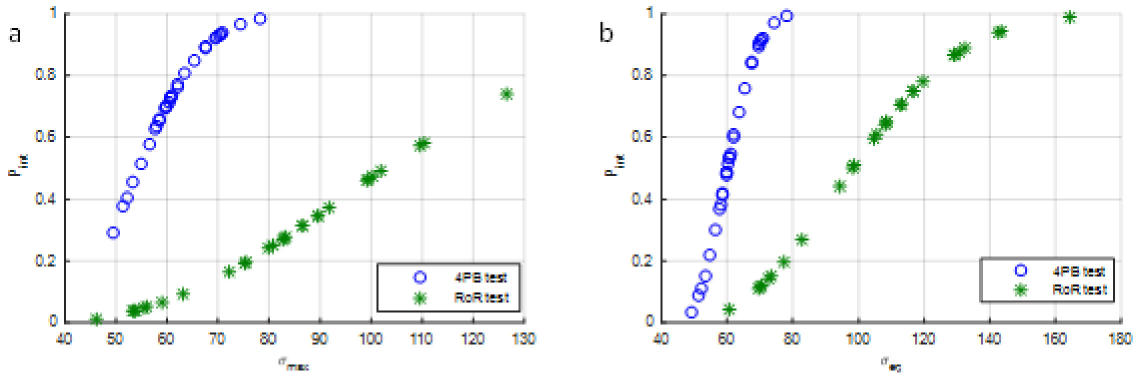

Fig. 6. Maximum values of GP and global probability of failure obtained from each test: a) for the maximum stress criterion and b) for the biaxial equivalent stress criterion.

3.2. Sorting according to global probabilities

Thereafter, it is possible to rank all results in increasing order according to their global probability of failure by assigning a new probability of failure ($P_{fail,i}$) to them using a plotting position rule [18]:

$$P_{fail,i} = \frac{i - 0.3}{n + 0.4} \quad (5)$$

It is worth mentioning, that the probability of failure assigned to each test changes its value from one iteration to the next, because their rank order is associated with the Weibull parameters, which change at each iteration.

3.3. Transfer all results to the reference size

At this point, each probability of failure is still associated with a non-homogeneous distribution of the GP all over the specimen. So, the next step is to relate these distribution to a unique reference size (S_{ref}) in such a way that if this size is uniformly subjected to an equivalent value of the generalized parameter, GP_{eq}, the resulting probability of failure equals that of the real specimen, i.e. $P_{fail,i}$. To do this, the critical values of the generalized parameter must be converted into GP_{eq} :

$$GP_{eq_i} = \delta_{ref}\left[-\log\left(1-P_{fail_i}\right)\right]^{1/\beta} + \lambda \qquad (6)$$

Figure 7 illustrates the conversion of all critical GP values from their respective sizes to the common reference size (S_{ref}) by applying Eq. (5).

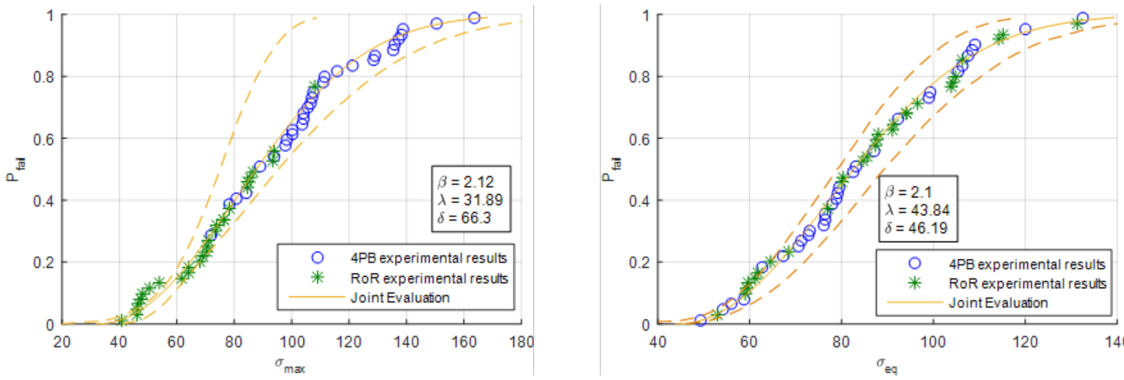

Fig. 7. Transferring all results to the common reference size (in this example S_{ref} = 10.000mm^2): a) for the maximum stress criterion and b) for the biaxial equivalent stress criterion.

The suitability of the generalized parameters studied is also reflected in Fig. 7. In case of an adequate selection of the generalized parameter, the results for each sample are randomly but more or less homogenously distributed along the cdf, i.e. in the probability range 0 to 1, as it happens for the biaxial equivalent stress criterion. On the contrary, if the selected generalized parameter is inadequate, the scale effect cannot be properly handled neither the different load conditions among samples. In this case, the results appear heterogeneously arranged tending to be pooled at lower or higher probabilities. This evidences once more that the biaxial equivalent stress, as a failure criterion, is superior to the maximum stress criterion.

3.4: Fitting the PFCDF

Once an equivalent GP_{eqi} and a probability of failure $P_{fail,i}$ are assigned to any specimen, the three Weibull parameters of the PFCDF are estimated by fitting the following equation [19]:

$$P_{fail_i} = 1 - \exp\left[-\left(\frac{GP_{eq_i} - \lambda}{\delta_{ref}}\right)\right]^{\beta} \qquad (7)$$

3.5. Evaluation of the convergence

During the iterative process, the convergence rate of the Weibull parameters is steadily inspected after each iteration with the aim of stopping the iterative process when the summation of the absolute values of the variations of the Weibull parameters falls below a certain prescribed threshold value ε according to the expression:

$$|\lambda_i - \lambda_{i-1}| + |\beta_i - \beta_{i-1}| + |\delta_i - \delta_{i-1}| < \varepsilon \quad (8)$$

In this case, the Weibull parameters obtained at the last iteration are assumed to represent their definitive estimation. Otherwise, the iterative process is reinitiated in order to improve the estimation of the global probability by modifying the rank order assigned taking into account the Weibull parameters ensuing from the previous iteration.

3.6. Results

The PFCDFs obtained as a result of the iterative process applied to both failure criteria are shown in Fig. 8. The dashed curves correspond to the 5% and 95% confidence intervals estimated by the bootstrap method. The confidence intervals are narrower for the biaxial criterion than for the maximum stress criterion. The main objective of the generalized local methodology consists in obtaining a unique PFCDF, as a material property, from joint evaluation of different experimental samples thus ensuring higher reliability than proceeding to a separate evaluation of the samples. The comparison of the confidence intervals resulting from join evaluation (Fig. 8b) with those obtained from separate evaluation of both RoR and 4PB samples (Fig. 3b) demonstrates definitely that this goal is fulfilled.

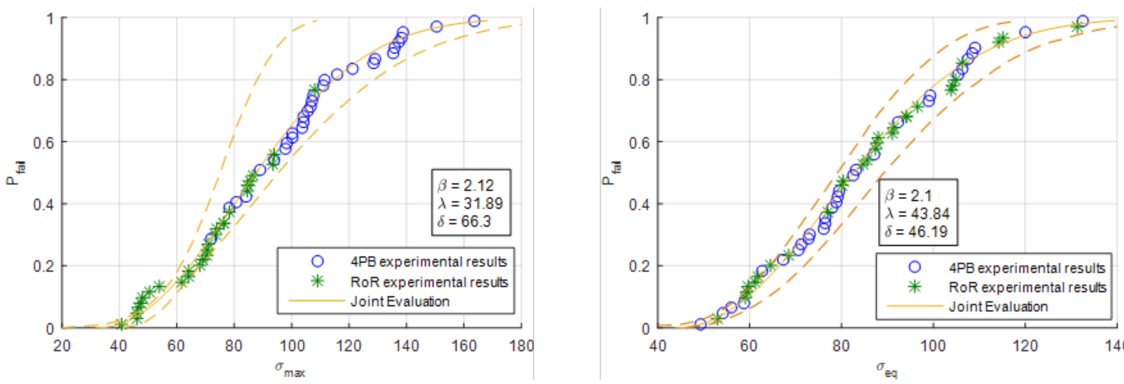

Fig 8. PFCDF (Sref =10000 mm2) obtained for a) maximum stress criterion and b) biaxial equivalent stress criterion.

The PFCDFs calculated through the iterative process are used to derive the failure cumulative distribution function predicted for 4-PB and RoR tests. A comparison between the real experimental failures and the predicted ones provides information about the quality of the solutions found, as shown in Fig 9.

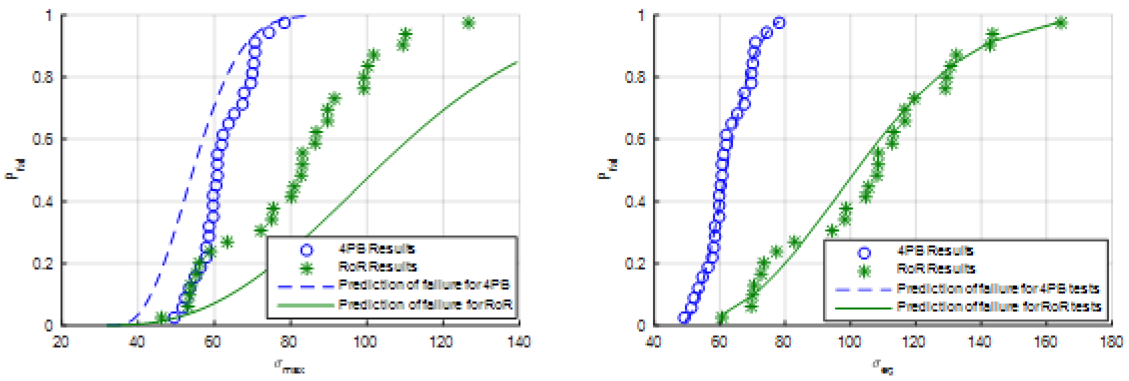

Fig 9. Comparison between the experimental results prediction of failure estimated from the joint PFCDF: a) for maximum stress criterion and b) for equivalent stress criterion.

4. ProJoint Software

In order to facilitate understanding, implementation and dissemination of the methodology proposed in this paper, a specific standalone application to Matlab software, denoted ProJoint, has been developed (see Fig. 10). On the one hand, the input values of this application are the values of the generalized parameter at any finite element for each experimental test and the size of the elements in the mesh. On the other hand, the output values are the PFCDFs obtained for any selected sample, the PFCDF calculated through the joint evaluation and the comparison of the experimental results with the PFCDF predicted for failure.

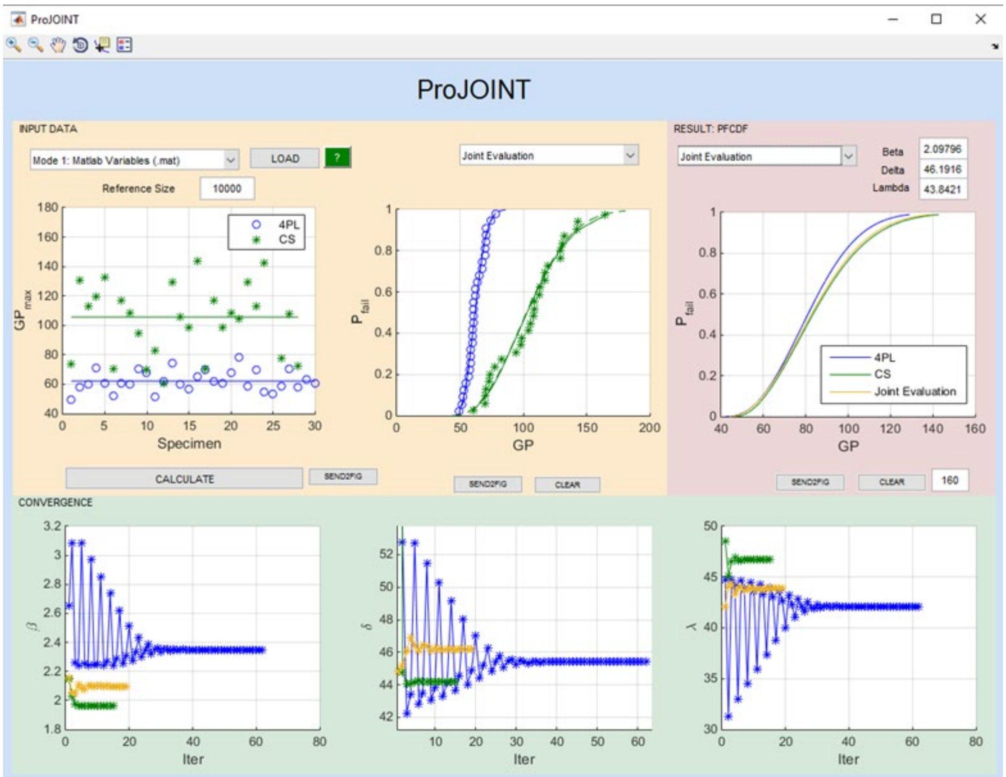

Fig 10. Main Window of ProJoint Matlab application.

ProJoint Matlab application, as a brief user tutorial, and the two examples carried out in this paper are found on the website: downloads.iemesgroup.com.

5. Discussion

In the light of the results, it can be concluded that the primary failure cumulative distribution function, PFCDF, is a material property, since the probability of failure and the value of the generalized parameter at failure for a given reference size are related to each other. The PFCDF is unique and can be deduced independently of the experimental test type used. On the contrary, the experimental failure cumulative distribution function, EFCDF, is the result of the application of the PFCDF to the particular test type chosen, thus being shape, size and test dependent.

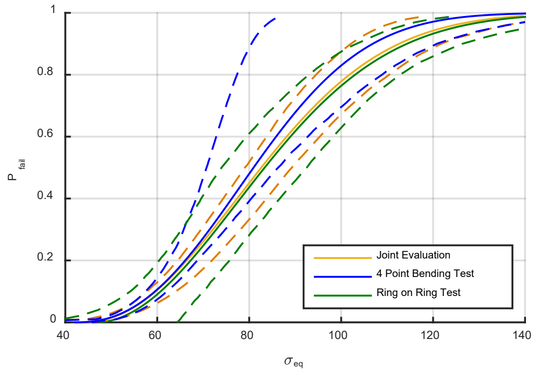

Fig 11. Comparison between the PFCDF derived from each of both test types and that resulting from the joint evaluation by pooling data from both tests.

Figure 11 shows a comparison between the PFCDF derived from each of both test types and that resulting from the joint evaluation pooling data from both tests. As can be seen, the joint cdf obtained does not represent the average of the two other single ones since its confidence interval shrinks, compared to that for the single distributions, as a result of the joint sample evaluation implying a total of sixty specimens as opposed to the thirty specimens comprised in each of the single tests.

As can be seen, the confidence interval for the joint cdf obtained does not represent the average of those for the two other single ones since the former confidence interval ensues from the joint sample comprising a total of sixty as opposed to the thirty specimens being comprised in the single tests.

6. Conclusions

The principal conclusions arising from this research are the following:

- The quality of alternative failure criteria for annealed glass is discerned by calculating the primary failure cumulative distribution function, PFCDF, in the frame of the generalized local methodology, GLM. Mutual transferability between the failure cdfs based on the criterion of maximum tensile stress criterion is rejected whereas an alternative failure criterion based on the Principle of Independent Actions (PIA) furnish promising and satisfactory results.

- A unique primary failure cumulative distribution function, PFCDF, is obtained when the proposed general local model, GLM, is applied to the experimental results irrespective of the test type being used proving the PFCDF to be a material property.

- The joint evaluation of data outcoming from the different test subsamples, as a unique sample, ensures higher reliability in the estimation of the parameter values.

- The methodology proposed proves to be applicable without loss of generality independently of the possible complexity of the failure criterion selected, specimen shape and geometry or test type being considered in the experimental program. The particular distribution of the generalized parameter, as determined by finite element calculations, plays no remarkable role.

- The methodology proposed is applied to a practical case consisting in failure assessment of annealed glass when failure data from two different test types are used. The satisfactory outcome obtained proves the suitability of the approach in spite of the high scatter inherent to the failure results for such material.

Acknowledgements

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Appendix A. Experimental results

Table A1. Experimental maximum GP values for four point bending tests.

Specimen Number	$\sigma_{\max}$ [MPa]	σ_{eq} [MPa]
1	49,33	49,33
2	58,04	58,04
3	59,85	59,85
4	70,76	70,76
5	60,68	60,68
6	52,23	52,23
7	60,80	60,80
8	59,83	59,83
9	70,54	70,54
10	67,63	67,63
11	51,51	51,51
12	62,14	62,14
13	74,32	74,32
14	59,73	59,73
15	56,52	56,52
16	65,29	65,29
17	69,84	69,84
18	61,97	61,97
19	60,33	60,33
20	67,48	67,48
21	78,26	78,26
22	58,60	58,60
23	69,58	69,58
24	54,92	54,92
25	53,40	53,40
26	58,62	58,62
27	70,29	70,29
28	57,78	57,78
29	63,54	63,54
30	60,93	60,93

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Table A2. Experimental maximum GP values for ring on ring test.

Specimen Number	σ_{max} [MPa]	σ_{eq} [MPa]
1	56,15	73,54
2	100,32	130,64
3	86,44	112,85
4	91,67	119,64
5	101,86	132,61
6	53,93	70,64
7	89,55	116,82
8	83,20	108,69
9	72,18	94,42
10	53,20	69,68
11	63,18	82,71
12	46,21	60,55
13	99,16	129,15
14	80,77	105,55
15	75,15	98,24
16	110,35	143,50
17	53,57	70,16
18	89,55	116,82
19	75,49	98,73
20	83,05	108,48
21	80,02	104,58
22	99,35	129,51
23	86,65	113,11
24	109,58	142,52
25	126,57	164,35
26	59,11	77,40
27	82,67	107,99
28	55,41	72,57

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