STATISTICAL PREFERENCE AS A COMPARISON METHOD OF TWO IMPRECISE FITNESS VALUES

Ignacio Montes[1] **Davide Martinetti**[1] **Susana Díaz**[1] **Susana Montes**[1]

[1] Dpto. Estadística e I.O. y D.M., Universidad de Oviedo,
{uo182551,martinettidavide.uo,diazsusana,montes}@uniovi.es

Abstract

The fitness value of a knowledge base (KB) can be unknown and only some imprecise information about it can be obtained. In some cases this information is given by means of an interval where we know the fitness is contained. Thus, the comparison of two randomly distributed intervals is necessary in this context in order to be able to determine the preferences among individuals.

This contribution is a first approach to the use of statistical preference as a tool to compare this kind of intervals. We consider the probabilistic relation associated to the stochastic comparison of every pair of intervals and we study the cycle-transitivity of this relation. The defuzzification of this probabilitic relation, that is, the statistical preference relation, is studied and some properties are obtained. Our studies are particulary detailed for the case of the uniform distribution.

Keywords: vague data, fitness value, probabilistic relation, transitivity, statistical preference.

1 INTRODUCTION

Genetic algorithms are a powerful tool, capable of solving complex optimization tasks such as learning or tuning fuzzy rule bases and their corresponding fuzzy partitions (see [1]). When a genetic algorithm is used to generate or adapt a fuzzy system, the technique is referred to as a Genetic Fuzzy System (GFS) [2]. The use of GFSs has widely accepted, given that these algorithms are robust and can search efficiently large solution spaces (see [18]).

Although in this context the linguistic granules or information are represented by fuzzy sets, the input data and the output results are usually crisp [7]. However, some recent papers (see [12, 13, 14, 15]) have dealt with fuzzy-valued data to learn and evaluate GFS. In that approach the function that quantifies the optimality of a solution in the genetic algorithm, that is, the fitness function, is fuzzy-valued. In particular, in [15], the authors have considered that the fitness values are partly unknown, since there only exists interval valued information about them. In this context some kind of order between two fitness values is necessary for determining if one individual precedes the other. Since the information about the fitness values is imprecise and given by means of intervals, a procedure for comparing two intervals is required. Initially, this procedure was based on estimating and comparing two probabilities [15]. In this work we will consider a more general and flexible way to compare two intervals which is based on a probabilistic relation [5, 6, 9]. When the defuzzification is required, we will consider the statistical preference [6, 10], which is obtained as a cut of this relation.

In this contribution we study how these concepts are applied to compare two intervals, which represent an imprecise information about the fitness valued of two KBs. In particular, we will not assume knowledge of the joint distribution for the two fitness values and then the uniform distribution is used. Not only the independent, but also the countermonotone and comonotone cases (see [4]) are considered for defining the joint distribution. The assumption of a uniform distribution is not an artificial requirement and it can be considered in many situations as a consequence of lack of information (see, for instance, [15, 17]). When this distribution is considered, we will obtain the specific expression of the associated probabilistic and fuzzy relations. Since transitivity is a very simple, but also very important property when comparing elements, we will study whether and how the transitivity propagates between these two types of relations. Moreover, we will generalize the concept of statistical preference, as a defuzzification of the probabilistic relation and we will study the general connection between this way to compare random elements and the classical stochastic dominance. Finally we study in detail the case of the uniform distribution.

The work is organized as follows: Section 2 collects some definitions and notions involved in following sections. In Section 3 we include the results obtained about the transitivity of the fuzzy and the probabilistic relations associated to two imprecise fitness values. In Section 4 we study the generalization of the concept of statistical preference, its connections with stochastic dominance and a particular analysis for the uniform distribution. The last section contains some final remarks.

2 BASIC CONCEPTS

Let us consider two fitness values θ_1 and θ_2 of two Fuzzy Systems in a regression problem. For instance, θ_1 and θ_2 are the mean squared errors of two Fuzzy Rule Based Systems (FRBS) on the same training set. In many situations, these values θ_1 and θ_2 are unknown, but we have some imprecise information about them. Thus, we cannot know the value of θ_1 and θ_2, but we know two intervals where each of them is contained. These intervals can be obtained by means of a fuzzy generalization of the mean squared errors (for a more detailed explanation, see Sections 4 and 5 in [15]) and they will be denoted by $FMSE_1$ and $FMSE_2$, respectively.

If these two intervals are disjoint, then we have not any problem for determining the preferred interval and therefore the decision is trivial. The problem arises when the intersection is non-empty. In that case, a prior knowledge about the probability distribution of the fitness $P(\theta_1, \theta_2)$ can be considered. In that situation, a decision rule considered in [15] was to decide that $FMSE_1 \succ FMSE_2$ if and only if

$$P((\theta_1, \theta_2) : \theta_1 < \theta_2) > P((\theta_1, \theta_2) : \theta_1 \geq \theta_2).$$

However, in that approach we consider a crisp order between the intervals, but if we are in a fuzzy context, with imprecise data, some kind of gradual comparison could be more appropriate as the starting point of the comparison. Thus, we will use multivalued (also called fuzzy) relations to compare the intervals, that is, binary relations whose image is the real interval $[0, 1]$. These relations express the stochastic dependence between the alternatives with any value in that interval $[0, 1]$. The closer the value to 0, the weaker the stochastic dependence between the alternatives. In [9] a fuzzy relation was considered and it can be defined, by using the usual notation in this context, by $R(FMSE_1, FMSE_2) = 1$ if $P((\theta_1, \theta_2) : \theta_1 > \theta_2) \geq P((\theta_1, \theta_2) : \theta_1 < \theta_2)$ and $R(FMSE_1, FMSE_2) = 1 + P((\theta_1, \theta_2) : \theta_1 > \theta_2) - P((\theta_1, \theta_2) : \theta_1 < \theta_2)$ otherwise.

Let us remark that R is reflexive (for any $FMSE_i$ we have that $R(FMSE_i, FMSE_i) = 1$) and strongly complete ($R(FMSE_1, FMSE_2) = 1$ or $R(FMSE_2, FMSE_1) = 1$), so that, any pair of fitnesses can be compared.

A different way to obtain a gradual comparison between fitness values is by means of the probabilistic relations. Let us recall that given a set of alternatives A, a probabilistic relation in A is a mapping $Q : A \times A \to [0, 1]$ such that $Q(a, b) + Q(b, a) = 1$ for every pair of alternatives a and b in A. These relations are sometimes called reciprocal or ipsodual relations. The interpretation of a probabilistic relation is quite different from the interpretation of a fuzzy relation. If Q is a probabilistic relation, $Q(a, b) = 1$ expresses that alternative a is totally preferred to b. But the value 0 does not mean absence of connection. For a probabilistic relation Q, $Q(a, b) = 0$ is identified with a clear preference of b over a. It is equivalent to $R(b, a) = 1$ and $R(a, b) = 0$. It also holds that $Q(a, b) = \frac{1}{2}$, reflects indifference between both alternatives.

De Schuymer et al. introduced in [6] the probabilistic relation generated by a collection of dice. The collection of dice is called a discrete dice model for the probabilistic relation defined. The definition generalized to a set of random variables can be seen in [5]. In our context it can be expressed as follows:

$$Q(FMSE_1, FMSE_2) = P((\theta_1, \theta_2) : \theta_1 > \theta_2) + \tfrac{1}{2} P((\theta_1, \theta_2) : \theta_1 = \theta_2).$$

From this probabilistic relation, we can obtain a way to compare two fitness values (the general definition was given in [5]). Thus, we say that $FMSE_1$ is statistically preferred to $FMSE_2$ if $Q(FMSE_1, FMSE_2) > \frac{1}{2}$. We will denote it $FMSE_1 >_{\mathrm{SP}} FMSE_2$. $FMSE_1$ and $FMSE_2$ are statistically indifferent if $Q(FMSE_1, FMSE_2) = \frac{1}{2}$. We will denote $FMSE_1 \geq_{\mathrm{SP}} FMSE_2$ if $Q(FMSE_1, FMSE_2) \geq \frac{1}{2}$.

This way to compare two random elements can be seen as an alternative to the classical stochastic dominance [8]. Let us recall that stochastic dominance does not take into account the possible relationship between the random elements being compared, but statistical preference depends on the stochastic dependence between the random elements. We recall that for every joint distribution function F, there is a copula C that connects the marginal probability distribution functions (F_1 and F_2): $F(x,y) = C(F_1(x), F_1(y))$. Therefore statistical preference depends on the copula that connects the random elements. We recall that a copula (see for example [11]) is an operation $C : [0,1]^2 \to [0,1]$ satisfying

- $C(x, 0) = C(0, x) = 0$ for all $x \in [0, 1]$,
- $C(x, 1) = C(1, x) = x$ for all $x \in [0, 1]$,
- the property of moderate growth: $C(x_1, y_1) + C(x_2, y_2) \geq C(x_1, y_2) + C(x_2, y_1)$ for every $(x_1, x_2, y_1, y_2) \in [0, 1]^4$ such that $x_1 \leq x_2$ and $y_1 \leq y_2$.

If C is the minimum operator, the two elements are said to be comonotonic and if $C(x, y) = \max(x + y - 1, 0)$ they are countermonotonic. The case C is the product is the usual case of independence.

In [9] it was proven that there is a one-to-one correspondence between these fuzzy relations (R) and these probabilistic relations (Q), but we should recall they are considering two different ways to measure the relationship degree. Thus, although an important property to provide coherence to relations is the transitivity, the definition has to be totally different for probabilistic and fuzzy relations.

For probabilistic relations, De Baets et al. [3] developed a general framework that includes as particular cases several types of transitivity for probabilistic relations. In [6] the same authors provide an example based on a game with dices where no one “classical” transitivity is satisfied, but a particular type of cycle-transitivity holds. Analogously to FG-transitivity developed by Switalski [16], cycle-transitivity is based on an upper and a lower bound function. But the value bounded is not the usual $Q(a, c)$, but the sum $Q(a, b) + Q(b, c) + Q(c, a)$.

Thus, in [3] they defined that a probabilistic relation Q is cycle-transitive with respect to the upper bound function U if for all a, b, c in the set of alternatives,

$$\alpha_{abc} + \beta_{abc} + \gamma_{abc} - 1 \leq U(\alpha_{abc}, \beta_{abc}, \gamma_{abc}),$$

where

$$\begin{aligned}\alpha_{abc} &= \min(Q(a, b), Q(b, c), Q(c, a)),\\ \beta_{abc} &= \mathrm{median}(Q(a, b), Q(b, c), Q(c, a)),\\ \gamma_{abc} &= \max(Q(a, b), Q(b, c), Q(c, a)).\end{aligned}$$

and U is an upper bound function, that is, it fulfils

1. $U(0, 0, 1) \geq 0$,
2. $U(0, 1, 1) \geq 1$,
3. $U(\alpha, \beta, \gamma) + U(1 - \gamma, 1 - \beta, 1 - \alpha) \geq 1$.

The most important upper bounds are:

- $U_{\mathbf{M}}(\alpha, \beta, \gamma) = \beta$,
- $U_{\mathbf{P}}(\alpha, \beta, \gamma) = \alpha + \beta - \alpha\beta$,
- $U_{\mathbf{L}}(\alpha, \beta, \gamma) = \min(\alpha + \beta, 1)$.

The notation is derived from the associated t-norms (see [3]). We briefly recall that a t-norm is a binary $[0, 1]^2 \to [0, 1]$ operator such that it is commutative, associative, non-decreasing in each argument and has 1 as neutral element. The three most common t-norms are the minimum defined as: $T_{\mathbf{M}}(x, y) = \min(x, y), \quad \forall\, x, y \in [0, 1],$, the product t-norm defined as: $T_{\mathbf{P}}(x, y) = x \cdot y, \quad \forall\, x, y \in [0, 1]$, and the Łukasiewicz t-norm: $T_{\mathbf{L}}(x, y) = \max(x + y - 1, 0), \quad \forall\, x, y \in [0, 1]$.

This topic is essential in the most usual definition of transitivity for a fuzzy relation: a fuzzy relation R is transitive with respect to the t-norm T (T-transitive, for short) if it holds that

$$T(R(a, b), R(b, c)) \leq R(a, c)$$

for any three alternatives.

3 TRANSITIVITY

In this section we will obtain the type of cycle-transitivity and the type of T-transitivity fulfilled by Q and R, respectively, if we assume $P(\theta_1, \theta_2)$ is obtained by copulation of two uniform distributions (see, for instance, [15, 17]). We will consider three different approaches to this problem: independence, comonotony and countermonotony. Moreover, by coherence with the studies developed in [4, 5], we will consider in this section that the two random elements are uniformly distributed in two intervals with the same width.

Thus, let $FMSE_1 = [a_1, a_1+\lambda]$ and $FMSE_2 = [a_2, a_2+\lambda]$ be two intervals where we know the fitnesses θ_1 and θ_2 of two KBs are content and let us consider a uniform distribution on each of them. As we commented in Section 2, the joint distribution is defined by means of a copula. In this work we will consider the minimum, product and Łukasiewicz copulas (which is the same of the minimum, product and Łukasiewicz t-norms) in order to define this joint distribution.

3.1 THE INDEPENDENT CASE

The case when the copula is the product was already studied in [5], where they obtained the expression of the probabilistic relation associated to this kind of random variables. Thus, in our context,

$$Q^P(FMSE_1, FMSE_2) = \begin{cases} 0 & \text{if } a_2 > a_1 + \lambda \\ \frac{(\lambda + a_2 - a_1)^2}{2\lambda^2} & \text{if } a_1 < a_2 < a_1 + \lambda \end{cases}$$

It also was proven in [5] that Q^P is cycle-transitive with respect to the upper bound function

$$U^P(\alpha,\beta,\gamma) = \begin{cases} \beta+\gamma-1+\frac{1}{2}(T_L(\sqrt{2(1-\beta)},\sqrt{2(1-\gamma)}))^2, & \text{if } \beta \geq 1/2, \\ \alpha+\beta-\frac{1}{2}(T_L(\sqrt{2\alpha},\sqrt{2\beta}))^2, & \text{if } \beta < 1/2. \end{cases}$$

In this case, by using [9], the expression of the associated fuzzy relation is:

$$R^P(FMSE_1, FMSE_2) = \begin{cases} 0 & \text{if } a_2 > a_1+\lambda \\ 2(1-\frac{(\lambda+a_1-a_2)^2}{2\lambda^2}) & \text{if } a_2 < a_1 < a_2+\lambda \\ 1 & \text{otherwise} \end{cases}$$

3.2 THE COMONOTONIC CASE

In the case the joint distribution function is obtained as the minimum between both marginal distribution functions, the expression of the probabilistic relation is crisp. In such situation, we can assure that it is always cycle-transitive with respect to any upper bound function, since it is transitive in the classical sense.

Proposition 3.1 *Let $FMSE_1 = [a_1, a_1+\lambda]$ and $FMSE_2 = [a_2, a_2+\lambda]$ be two intervals where we know the fitnesses θ_1 and θ_2 of two KBs are content and let us consider a uniform distribution on each of them. If the joint distribution is obtained by means of the minimum copula, then the associated probabilistic relation Q^M is crisp with*

$$Q^M(FMSE_1, FMSE_2) = \begin{cases} 0 & \text{if } a_1 \leq a_2, \\ 1 & \text{if } a_1 > a_2. \end{cases}$$

Moreover Q^M is transitive.

In this case, the associated fuzzy relation R^M is also crisp (see [9]) and it is transitive.

3.3 THE COUNTERMONOTONIC CASE

Let us now consider the case the joint distribution is obtained by means of the Łukasiewicz copula.

Proposition 3.2 *Let $FMSE_1 = [a_1, a_1+\lambda]$ and $FMSE_2 = [a_2, a_2+\lambda]$ be two intervals where we know the fitnesses θ_1 and θ_2 of two KBs are content and let us consider a uniform distribution on each of them. If the joint distribution is obtained by means of the Łukasiewicz copula, then the associated probabilistic relation Q^L is*

$$Q^L(FMSE_1, FMSE_2) = \begin{cases} \frac{a_1-a_2+\lambda}{2\lambda} & \text{if } a_2 < a_1+\lambda, \\ 0 & \text{if } a_1+\lambda \leq a_2. \end{cases}$$

Moreover, Q^L is cycle-transitive with respect to the upper bound function

$$U^L(\alpha,\beta,\gamma) = \max(\beta, 1/2).$$

In [9] it was proven that the cycle-transitivity with respect to U^L is equivalent to the T_L-transitivity of the associated fuzzy relation R^L. Thus,

Corollary 3.3 *Let $FMSE_1 = [a_1, a_1+\lambda]$ and $FMSE_2 = [a_2, a_2+\lambda]$ be two intervals where we know the fitnesses θ_1 and θ_2 of two KBs are content and let us consider a uniform distribution on each of them. If the joint distribution is obtained by means of the Łukasiewicz copula, then the associated fuzzy relation R^L is T_L-transitive.*

4 STATISTICAL PREFERENCE

As we showed in Section 2, the classical statistical preference [5] is defined as the 0.5-cut of the probabilistic relation Q. This defuzzification is very strict and we could consider appropriate any other cut as the minimum level of relationship. Thus, we could generalize this definition as follows.

Definition 4.1 *Let X and Y be two random variables and let Q be its associated probabilistic relation. X is said to be α-statistical preferred or indifferent to Y, $X \geq_{\alpha SP} Y$, if it holds that $(X,Y) \in Q_\alpha$, where $Q_\alpha = \{(X,Y) : Q(X,Y) \geq \alpha\}$.*

It is immediate that α-statistical preference implies α'-statistical preference if $\alpha \geq \alpha'$, but the converse is not true.

The relationship of this concept with the stochastic dominance (see [8]) is developed in the following result.

Proposition 4.2 *If $\alpha \leq 1/2$, then*

- *the first degree stochastic dominance implies α-statistical preference but the converse is not true;*
- *there is not any implication between the second degree stochastic dominance and the α-statistical preference.*

In the case $\alpha > 1/2$, even the first implication is not fulfilled and any relationship can be established.

Although we have considered here a more general definition, the most usual case (see, for instance, [5, 6, 10]) is the case of the 1/2-cut, which is simply called statistical preference. In that case, the meaning of this definition can be clarify by the following proposition.

Proposition 4.3 *Let X and Y be two random variables. In general, it holds that*

$$X \geq_{SP} Y \Longrightarrow P(X < Y) \leq \frac{1}{2}.$$

For continuous variables, the converse implication is also fulfilled.

Let us now present two results which can be use as a tool to manage statistical preference under translations and dilations. Both of them are natural properties for this relation.

Proposition 4.4 *Let X and Y be two random variables. It holds that*

- $X \geq_{SP} Y \Longleftrightarrow X - Y \geq_{SP} \underline{0}$.
- $X + Y \geq_{SP} X \Longleftrightarrow Y \geq_{SP} \underline{0}$.
- $\lambda X \geq_{SP} \underline{\mu} \Longleftrightarrow \begin{cases} X \geq_{SP} \underline{\mu/\lambda} & \text{if } \lambda > 0 \\ \underline{\mu/\lambda} \geq_{SP} X & \text{if } \lambda < 0 \\ \mu \leq 0 & \text{if } \lambda = 0 \end{cases}$

As a consequence of this result, we have

Corollary 4.5 *Let X be a random variable and let α be a constant. It holds that*

- $X \geq_{SP} X + \alpha \Longleftrightarrow \alpha \leq 0$.
- $\lambda X \geq_{SP} \alpha X \Longleftrightarrow \begin{cases} \underline{0} \geq_{SP} X & \text{if } \alpha > 1 \\ X \geq_{SP} \underline{0} & \text{if } \alpha < 1 \end{cases}$

In the following of the section, we will apply this definition in our context. Thus, we will consider two imprecise fitness values and we will obtain the requirement to be able to assure one of them is statistical preferred to the other. We will consider again a uniform distribution, that is, no any prior information about the distribution over the observed interval, but in this case we will not restrict our study to intervals with the same width, but we will consider the most general case.

Thus, $FMSE_1 = [a, b]$ and $FMSE_2 = [c, d]$ will denote now two intervals where we know the fitnesses θ_1 and θ_2 of two KBs are content. Let us assume a uniform distribution on each of them. We will consider again three possible ways to obtain the joint distribution: minimum, product and Łukasiewicz copulas. In these three cases we will obtain the condition over a, b, c and d to assure the statistical preference of the interval $FMSE_1$ to the interval $FMSE_2$. To do that, the expression of Q^P, Q^M and Q^L obtained in the previous section will be an essential part of the proof.

4.1 THE INDEPENDENT CASE

Proposition 4.6 *Let $FMSE_1 = [a, b]$ and $FMSE_2 = [c, d]$ be two uniformly distributed intervals which represent the information we have about the fitnesses θ_1 and θ_2 of two KBs. If the joint distribution is obtained by means of the product copula, then $FMSE_1 \geq_{SP} FMSE_2$ if, and only if,*

$$\begin{cases} (b-c)^2 \geq (b-a)(d-c) & \text{if } a \leq c < b \leq d, \\ a+b \geq c+d & \text{if } a \leq c < d < b \\ a+b \geq c+d & \text{if } c < a < b \leq d, \\ (d-c)(b-a) \geq (d-a)^2 & \text{if } c < a < d \leq b. \end{cases}$$

4.2 THE COMONOTONIC CASE

Proposition 4.7 *Let $FMSE_1 = [a, b]$ and $FMSE_2 = [c, d]$ be two uniformly distributed intervals which represent the information we have about the fitnesses θ_1 and θ_2 of two KBs. If the joint distribution is obtained by means of the minimum copula, then*

- *If $a \leq c < b \leq d$, then $FMSE_1 \not\geq_{SP} FMSE_2$.*
- *If $a \leq c < d < b$, $FMSE_1 \geq_{SP} FMSE_2$ if, and only if, $b(d-b+c) \geq a(c+d-a)$.*
- *If $c < a < d \leq b$, then $FMSE_1 \geq_{SP} FMSE_2$.*
- *If $c < a < b \leq d$, $FMSE_2 \geq_{SP} FMSE_1$ if, and only if, $3ad + (a-b)^2 + 2c^2 + bc \geq 2cd + 3ac + bd$.*

4.3 THE COUNTERMONOTONIC CASE

Proposition 4.8
Let $FMSE_1 = [a, b]$ and $FMSE_2 = [c, d]$ be two uniformly distributed intervals which represent the information we have about the fitnesses θ_1 and θ_2 of two KBs. If the joint distribution is obtained by means of the Łukasiewicz copula, then $FMSE_1 \geq_{SP} FMSE_2$ if, and only if, $c(c-b-a) \geq d(d-a-b)$.

5 CONCLUDING REMARKS

In this work we have developed a complete study of a method to compare two random intervals, which can be applied in particular to compare the imprecise information about two fitness values of two KBs. This comparison is fuzzy and it is established by means of a probabilistic relation. When the appropriate cut is considered, we obtain the statistical preference as a defuzzification of this relation. In all our studies the particular expressions and results for the usual case of uniform distributions are obtained. Thus, for the probabilistic and fuzzy relations associated to any set of interval ($FMSE_i$) we have characterized the transitivity fulfilled for these relations. For statistical preference, the requirements to be ordered two intervals are totally detailed.

As a future work, we would like to extend our studies in two different directions. The first one and simpler is an extension to other families of distributions. The second one would be to develop some parallel method to compare other kind of imprecise information, which could be determined by any fuzzy set instead of an interval.

Acknowledgements

The research reported on in this communication has been partially supported by Project FEDER-MEC-MTM2007-61193.

References

[1] O. Cordón, F. Herrera, F. Gomide, F. Hoffmann, L. Magdalena. Ten years of genetic-fuzzy systems: a current framework and new trends. In: *Proceedings of Joint 9th IFSA World Congress and 20th NAFIPS International Conference*, Vancouver - Canada, 2001, pp. 1241–1246.

[2] O. Cordón, F. Herrera, F. Hoffmann, L. Magdalena. Genetic Fuzzy Systems: Evolutionary Tuning and Learning of Fuzzy Knowledge Bases. World Scientific Publishing, Sigapore, 2001.

[3] B. De Baets, H. De Meyer, B. De Schuymer, S. Jenei. Cyclic evaluation of transitivity of reciprocal relations. *Social Choice and Welfare* **26** (2006) 217–238.

[4] H. De Meyer, B. De Baets, B. De Schuymer. On the transitivity of the comonotonic and countermonotonic comparison of random variables. *Journal of Multivariate Analysis* **98** (2007) 177–193.

[5] B. De Schuymer, H. De Meyer, B. De Baets. Cycle-transitive comparison of independent random variables. *Journal of Multivariate Analysis* **96** (2005) 352–373.

[6] B. De Schuymer, H. De Meyer, B. De Baets, S. Jenei. On the cycle-transitivity of the dice model. *Theory and Decision* **54** (2003) 261–285.

[7] F. Herrera. Genetic fuzzy systems: taxonomy and current research trends and prospects. *Evolutionary Intelligence* **1** (2008) 27-46.

[8] H. Levy. *Stochastic dominance.* Kluwer Academic Publishers. 1998.

[9] D. Martinetti, I. Montes, S. Díaz. Transitivity of the fuzzy and the probabilistic relations associated to a set of random variables. In: *Proc. EUROFUSE 2009 Workshop: Preference modelling and decision analysis*, Pamplona, Spain, 2009.

[10] I. Montes, D. Martinetti, S. Díaz. On the statistical preference as a pairwise comparison for random variables. In: *Proc. EUROFUSE 2009 Workshop: Preference modelling and decision analysis*, Pamplona, Spain, 2009.

[11] R. Nelsen. *An Introduction to Copulas.* Lecture Notes in Statistics. Vol. 139. Springer, New York.

[12] L. Sánchez, I. Couso. Advocating the use of imprecisely observed data in genetic fuzzy systems. *IEEE Transactions on Fuzzy Systems* **15** (2007) 551-562.

[13] L. Sánchez, I. Couso, J. Casillas. A multiobjective genetic fuzzy system with imprecise probability fitness for vague data. In: *Proc. 2006 IEEE Internat. Conf. on Evolutionary Fuzzy Systems*, Ambleside, UK, 2006, pp.131-137.

[14] L. Sánchez, I. Couso, J. Casillas. Modeling vague data with genetic fuzzy systems under a combination of crisp and imprecise criteria. In: *Proc. 2007 IEEE Sympos. on Computational Intelligence in Multicriteria Decision Making*, Honolulu, USA, 2007, pp.30-37.

[15] L. Sánchez, I. Couso, J. Casillas. Genetic learning of fuzzy rules based on low quality data. *Fuzzy Sets and Systems* **160** (2009) 2524–2552.

[16] Z. Switalski. General transitivity conditions for fuzzy reciprocal preference matrices. *Fuzzy Sets and Systems* **137** (2003) 85–100.

[17] J. Teich. Pareto-front exploration with uncertain objectives. In: *Proc. First Internat. Conf. on Evolutionary Multi-Criterion Optimization*, 2001, pp. 314-328.

[18] Y. Yuan, H. Zhuang. A genetic algorithm for generating fuzzy classification rules. *Fuzzy Sets and Systems* **84** (1996) 1–19.