Metaheuristic Strategies for Scheduling Problems with Uncertainty

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1 Introduction

Scheduling problems have formed an important body of research during the last decades. A *scheduling problem* consists in scheduling a set of jobs $\{J_1, \ldots, J_n\}$ on a set of physical resources or machines $\{M_1, \ldots, M_m\}$. Each job J_i is composed of m tasks or operations $\{\theta_{i1}, \ldots, \theta_{im}\}$ with processing time p_{ij}. At the same time, we usually have constraints that establish that two task belonging to the same job cannot overlap their execution in time and that each task requires the uninterrupted and exclusive use of one of the machines for its whole processing time. Depending on the additional constraints we define, we may obtain different families of problems. The most popular in the literature are the job shop (JSP), the open shop (OSP) and the flow shop (FSP) but there exists also variants of them as the flexible job shop (FJSP) among others. Commonly, the objective function to optimise is the earliest time in which all jobs can be finished or *makespan*. However many other objectives may be optimised, being the most popular the *tardiness*, the *idleness* and the *total flow time*.

In classical scheduling problems all input data are assumed to be well defined and all constraints are assumed to be hard, which is not so common in real-life applications. To reduce the gap between theory and practice, this thesis focuses on solving scheduling problems considering that uncertainty and vagueness. For instance, we shall consider uncertain task durations as well as flexible due-date constraints.

2 Related Work

The complexity of scheduling problems means that practical approaches to solving them usually involve heuristic strategies. Metaheuristics have been extensively used and evolved during the last decades and in consequence there exists a wide range of them [12]. Among many others, we highlight a tabu search in [8] for the JSP, a local search in [7] for the flexible FJSP, and a hybridized ACO algorithm in [3] for the OSP.

In the last years, *fuzzy sets* have emerged as a very interesting tool to model uncertainty and have been used in different manners. This approach appears to be more flexible as it needs less information from a human expert or past data for modelling a problem and it involves a lesser computational cost. Some attempts have been made to extend metaheuristics designed for the deterministic problem to the case where uncertain durations are modelled via fuzzy intervals, for instance a genetic algorithm in [11] and a local search strategy in [10] for the fuzzy JSP; or a PSO in [9] for the fuzzy OSP. When managing uncertainty, robustness and stability are important concepts to take in account. A solution is robust if it performs relatively well under a wide range of possible problem realizations [1]. However in the literature we find many discrepancies between different authors when applying this concept to scheduling problems. For instance it can be measured as the variation of the objective function depending on the real scenario, but also as the variation of the starting time of each single operation with respect to the predicted value or the best possible performance in the worst scenario.

3 Objectives

The main goal for this thesis is to efficiently solve different scheduling problems taking in account the uncertainty pervading real world problems. To achieve this goal, we may establish the following more detailed objectives:

- **Uncertainty model:** Having adopted fuzzy sets as a framework for dealing with uncertainty and vagueness, we intend to propose and develop analytic measures for optimality in this fuzzy framework, as well as studying the search spaces and related concepts such as dominance, criticality, etc.
- **Robustness and Stability:** When dealing with uncertainty, robustness and stability are important concepts to take in account. Unfortunately, there is no consensus in the definitions of these concepts and associated measures. Therefore the objective is to acquire knowledge of the different proposals in the literature, analyse them and adopt definitions for using during the thesis. Here we intend to fix metrics for these concepts that allow us to improve solutions accordingly.
- **Design of meta-heuristics:** The objective is to design different algorithms for solving fuzzy scheduling problems based on the adopted uncertainty model. This is not trivial, as the search spaces are now different and the properties of the deterministic problems may not hold in the fuzzy case. In addition, we plan to design multi-objective algorithms for optimising at the same time classical objective functions and the robustness of the solutions.

4 Methodology

In this section we describe the methodology to achieve each of the previous objectives. In addition, we briefly expose the preliminary results obtained in the early stages of the thesis.

4.1 Uncertainty model

The starting point for this objective is to make a review of the literature in order to know different techniques to deal with uncertainty and vagueness. The main source of uncertainty we shall consider is in task durations. The crudest representation for uncertain processing times would be a human-originated confidence interval. If some values appear to be more plausible than others, a natural extension is a fuzzy interval or a fuzzy number. For the thesis we adopt the use of *triangular fuzzy numbers* or *TFN* denoted as $N = (n^1, n^2, n^3)$, where $[n^1, n^3]$ defines an interval of possible values and n^2 the most plausible value in it. The number of authors using this model is increasing, which suggests that it is a good representation model. A first issue to be addressed in this framework is the arithmetic of TFNs and, in particular, the two operations needed in scheduling: addition and maximum. Extending the addition on real numbers to TFNs is straightforward, but this is not the case for the maximum. Indeed, the set of TFNs is not closed under the maximum and its computation can be computationally expensive. The main approaches in the literature are based on approximating the maximum by a TFN [4] [6]; we need to analyse the soundness and advantages and dissadvantages of each approach. Another issue is to define a total ordering in the set of TFNs, in order to optimise fuzzy objective values. In principle, we shall use the expected value to this end, taking into account its relationship with various ranking methods used by other authors.

A source of vagueness may be due-dates, as they can be flexible. A common approach for modelling flexible due-dates is to use a fuzzy set with linear decreasing membership function representing a satisfaction level for the ending date of a job. If task durations are modelled as TFNs, the completion time of a job is also a TFN, so we measure the satisfaction level using the *agreement index* [11].

To study the different search spaces of fuzzy scheduling problems where durations are modelled as TFNs, we have started to analyse and adapt schedule generation schemes, in special the well-known Giffler & Thompson algorithm (G&T), to those problems. So far we have seen that a minor change in the way we adapt the G&T has a great impact on the associated search space and we have identified different search spaces. We are currently studying their properties, with special emphasys on their dominance (i.e., if they contain at least one optimal solution).

4.2 Robustness and Stability

Regarding robustness and stability, the first step is to make a review of the different definitions and measures that are present in the literature and study the relation between those concepts and the adopted uncertainty model. However, so far we have found many discrepancies in the way these concepts are defined and measured in the scheduling literature. Our starting point has been the semantics for fuzzy schedules from [5], by which solutions should be understood as a-priori solutions, also called predictive schedules, providing a fuzzy interval of possible values for the starting time of each task. We have used it as a basis to naturally adapt the definition of ϵ-robustness from [2] and associated metrics to the fuzzy framework. For evaluating the robustness of a predictive schedule we perform it on different scenarios using a MonteCarlo simulation and take the largest obtained ϵ value as robustness measure. In addition, when evaluating the performance of an algorithm we also run it without considering the uncertainty, taking as real duration for each task the most probable one (the modal value). Until now we have seen that solutions obtained considering the uncertainty are more robust than if we do not take it in account. Nevertheless, we are still studying different measures of robustness and in the near future we intend to start studying stability concepts.

4.3 Design of Metaheuristics

The plan is to start studying the most relevant algorithms published for deterministic scheduling problems and try to adapt and develop different extensions of them to the fuzzy framework. Based on the ideas and results obtained with these first approaches and the experience of our research group in designing metaheuristics for scheduling problems, we intend to develop new strategies to efficiently solve fuzzy scheduling problems. Our preliminary experience tell us that extend metaheuristics to the fuzzy framework is actually a challenge. Previously to the beginning of the thesis we had already developed a couple of metaheuristics for solving the fuzzy OSP and presented in IWINAC and Fuzz-IEEE conferences. During the firsts steps of the thesis we have developed different multi-objective approaches based on our previous experience. The first approach optimises the well known objective functions makespan and tardiness which in our case are both TFNs. To deal with both objectives, we have designed a swarm based algorithm with lexicographical goal programming which have obtained satisfactory results. Following a similar methodology, we have adapted our algorithm to deal with flexible due-dates instead of deterministic ones, considering the agreement index as objective function. The obtained results are again positive when comparing with other algorithms. The first approach has been presented in 2012 at the RCRA workshop and an extension of it, including a robustness analysis, is currently under review in a JCR indexed journal. The second approach has been sent to a journal and is currently under revision. Recently we have also implemented different search algorithms, including a Scatter Search, for the fuzzy OSP. Unfortunately, results are not competitive enough and it is open to future work how they can be improved.

We have also implemented different search algorithms for the fuzzy job shop problem. We have developed a tabu search algorithm based on the neighbourhood structures for the fuzzy framework proposed in [10], and then combined it with a genetic algorithm, obtaining very competitive results with the state-of-the-art both in quality and computational cost. This work has been already presented at the IWINAC 2013 conference.

Currently we are working on the flexible job shop problem with uncertainty, which is a topic of interest for many researchers. We have designed a neighbourhood for the fuzzy problem and developed a tabu search which we combine again with a genetic algorithm. The results clearly outperform the current state of the art for the fuzzy problems, and are also competitive in the most common crisp benchmarks. We are also working on a co-evolutionary algorithm with specific local search for each populations. Here the preliminary results are promising but we still testing and developing the algorithm.

For evaluating the results we shall obtain during the development of the thesis, we use the most extended benchmarks in the literature. For the fuzzy job shop, there are several problem instances of common use, but this is not the case for other fuzzy scheduling problems, with no specific benchmarks available for researchers. When this is the case, following a common approach in the literature, we generate a set of problems from the best-known benchmarks for the deterministic problem by transforming the original crisp processing times into TFNs such that their modal value corresponds to the original duration. It is desirable that the generated TFNs holds that $n^3 - n^2 \geq n^2 - n^1$ because in this case we can use the best known lower bound for the crisp problem as a lower bound for the fuzzy instance. This allows to better assess the quality of the obtained solutions.

For the future, once we adopt a measure for robustness, we intend to develop multi-objective algorithms using that measure as an objective function. Here we plan to adopt lexicographical goal programming with different hierarchies between the objective functions and dominance-based algorithms, and compare the obtained results so we can find the best way to optimise a classical objective function having robust solutions. In addition, based on the fact that multi-core processors are widely extended, parallel computing is one of our main targets.

5 Relevance

Fuzzy scheduling has been a topic of interest for many researchers during the last years. However it is still a relatively new line of research and there are many properties that have not been yet well-studied nor defined. For instance, even if many authors are adopting fuzzy numbers to deal with uncertainty, they are using different approaches and arithmetic. Also, it is well-known the importance of the different search spaces when facing complex problems, however in the case of fuzzy scheduling they have not been deeply studied yet, as well as schedule generator schemes.

Robustness and stability have not been well-defined yet for the scheduling problems as different authors propose different definitions. As a consequence, most researchers in fuzzy scheduling focus on optimise classical objectives, as makespan, but not on optimising the robustness or the stability of the obtained schedules, which are critical aspects when dealing with uncertainty. This supports the relevance of looking for an adequate measure of robustness and stability and designing multi-objective algorithms optimising not only a classical objective function, but also its robustness.

Meta-heuristics are important for solving fuzzy scheduling problems as they are NP-hard problems. Good meta-heuristics can yield good solutions for our problems in a short period of time, however the introduction of uncertainty in the problem makes it harder and a deep study is needed to develop new strategies. It is also remarkable that multi-core processors are widely extended nowadays, however few authors design meta-heuristics that exploit that computing capacity, therefore we consider relevant the design of parallel algorithms for solving our problems using that technology.

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