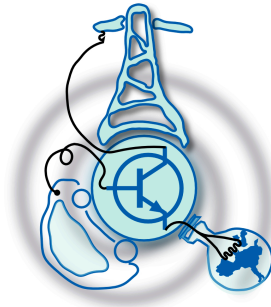# Development and implementation of a 2x25kV AC bivoltage traction system simulator for high speed railways

by

Mariia Plakhova

Submitted to the Department of Electrical Engineering, Electronics,
Computers and Systems
in partial fulfillment of the requirements for the degree of

Erasmus Mundus Master Course in Sustainable Transportation and
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Abstract

This thesis describes the development and implementation of a 2x25kV AC bivoltage traction system simulator for a high speed railways using the Forward/Backward Sweep load flow method. Its objective is to design the tool that allows to simulate the power flow for a high speed railways during the movement of the trains. The thesis has been built as following: the first part provides an overview of the load flow calculation techniques and computer-based simulators for the high speed railways. The second part represents the description of a 2x25kV bivoltage traction system, while the third part provides the mathematical model (for both, static and dynamic systems) of the network. The forth part explains the solving procedure and describes two techniques (derivative-based and Backward/Forward Sweep methods) that have been used to calculate system. The fifth part describes the testing procedure and validation of results. Finally, the last part gives conclusions and proposes steps that can be done in order to improve developed simulator.

Thesis Supervisor: Pablo Arboleya Arboleya
Title: Associate Professor

Acknowledgments

I would like to express my appreciation to my advisor, Prof. Pablo Arboleya, for his support and direction for this work. I would also like to thank Bassam Mohamed for sharing his experience and his patience while working with me. And to Minoru Tsuru for his valued support during the development of this thesis.

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Chapter 1

Introduction

1.1 Motivation

Transportation system is a significant part of the modern world. Since mankind created a wheel, its further developments and tools were aimed to make life easier, faster and simpler. Railway transport system has had a long history and large progress to become one of the most popular public transport nowadays. The demand of railway transportation system increases every year for passengers travels and cargo due to its numerous advantages, such as: economy, comfort, better control on travel time and schedule, less risk factors (heavy traffic or road accidents). Moreover, there is no need for the train to carry the source of energy during the journey time, unlike the other modes of transportation. All these combined with the fact that over the past years there has been a growing trend towards the use of clean energy sources, motivates to improve the current existing railway systems by developing number of tools, simulators and software products. With development of high performance computers and its accessibility for the commercial purposes it became possible to enhance and upgrade any railway or to plan and design new one with less time and cost consumed. Currently, simulation and computer modelling are used not only to project new system, but also for its maintenance and control. Taking into account the growing complexity of traction drives and power supplies that are used in railway system nowadays, more precise computer-based models and algorithms are required.

With improvement of these tools, more efficient and accurate analyse and forecast of the electric railway system performance is possible, which will allow to design and upgrade the railway system effectively.

1.2 Aim of Research and Structure of Thesis

The aim of research was to develop a 2x25kV AC bivoltage traction system simulator for high speed trains. Most of the current existing software products model the train and the power system separately. This thesis is focused on the power flow and power system simulation, therefore, the complex train model has not been built and trains are represented as non-linear power sources. The objective of this thesis is to develop the mathematical model of the bivoltage power system for the traction purpose and afterward solve it, using two different approaches: a commercial algorithm "fsolve" of MATLAB software and an alternative algorithm, based on Backward/Forward Sweep (BFS) technique, which has been specially adopted for this thesis. The first part of the thesis represents the current existing load flow algorithms and railway simulators, describes AC traction power systems and gives an overview of load flow solution methods and computer-based models. In the second chapter, a 2x25kV AC bivoltage traction system together with its pros and cons have been has been explained. The third part provides the mathematical model of the given system, both static and dynamic. It also includes the brief explanation of the graph theory and gives a description of the line splitting procedure. The topology of the system changes dynamically with the movements of the trains. However, it is very important to be able to obtain the system during each instance of the simulation and the line splitting procedure helps to track all changes in the position of the trains. The next chapter describes the computer simulation (with the aid of MATLAB software) of the system, based on the usage of two methods (MATLAB function "fsolve" and BFS technique). This chapter is mostly focused on the suggested BFS algorithm due to its compact matrix formulation and good performance. The fifth chapter of the thesis is dedicated to the testing procedure and validation of the results. The last part of the

thesis gives the conclusions and proposes a future work that could potentially improve the overall performance of the system and make the simulation more accurate.

1.3 General Review and History

Humanity has known about the railway system starting from 6th century BC [26]. However, long time has passed till the railway system took a familiar form. The inventing of the steam engine and later creation of the first steam locomotive was just the beginning of the new era for the transport system. Further progress in railway and locomotive technologies entailed the development of the first electrified railway, that was introduced by Siemens in 1879 [38]. Thru technical revolution, the power supply system and the traction motor become essential parts of the modern electrified transport system. The early traction systems used DC motors supplied from a low-voltage DC (1.5 kV from the beginning of XX century and 3 kV from 1930s) mostly because of its simplicity and ease to control. There are two main directions in railway systems: low-voltage DC transmission networks for drives with DC traction motors, and low-frequency (16.7 Hz in Central Europe and 25 Hz in the United States) high-voltage AC transmission networks, which with the establishment of high-voltage electrification allowed to increase the operation frequency up to the industrial values (50 and 60 Hz respectively) [17]. Nowadays, the standard range of voltages is defined by standards EN 50163 [42] and IEC 60850 [43]. Though, the most common railway systems are following:

- DC high voltage systems: 3kV with the distance between substations 15-30 km.

- DC medium voltage systems: 1.5kV with the distance between substations 15-30 km.

- DC low voltage systems: 0.6 - 1.4kV with the distance between substations 1-6 km.

- AC single phase systems: 15kV/16.7Hz, 25kV/50Hz.

- AC three-phase systems: 25kV/50Hz.

For most of the tram and metro systems 750V DC is a common value. However, in few cases railway systems operate with non-standard values of voltage. For instance, metro-transit system of Milan and London use 630V DC [11].

Figure 1-1 shows the basic structure of both, DC and AC, traction power systems.
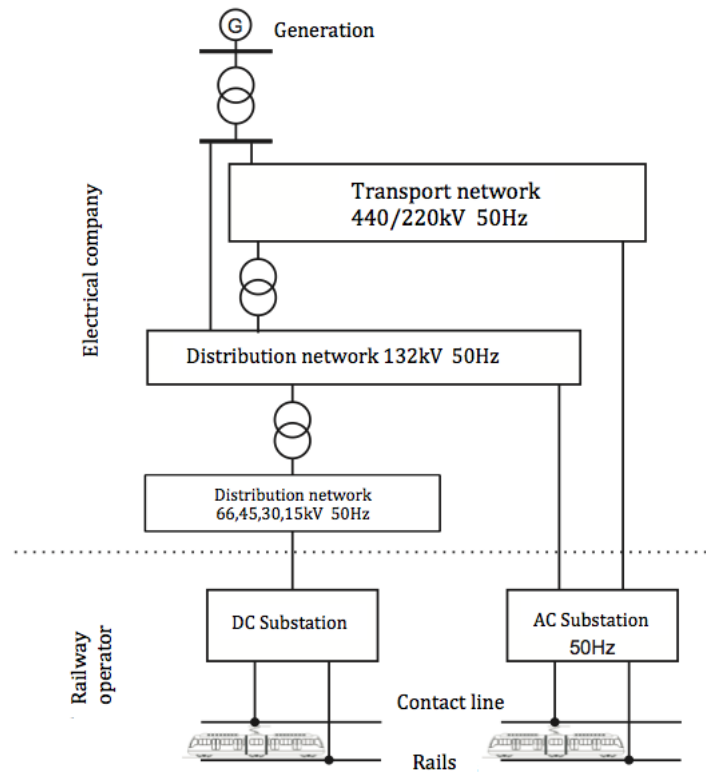
Figure 1-1: Basic Structure of DC and AC Power Systems [3]

There has been large transformation of the railway traction system during the last 20 years due to significant progress in power electronics and microprocessor fields. However, DC power supply is still the most common, and economic for the public transportation, whereas AC at industrial frequency is more useful and efficient for long-distance lines. Currently, there are no alternatives to the fast travel on the land. Therefore, it is indeed interesting and important to work on improvement of AC traction systems.

1.4 Review of Existing Load Flow Algorithms and Railway Simulators

The development of traction technology helped AC electrical railways powered by 25kV 50Hz power supplies to become one of the important mode of urban transportation system. New railways are faced with the range of technical considerations [18]:

- Operational requests (depending on the operation field: urban metro, high-speed passenger or heavy-haul freight).

- Physical route characteristics (gradients, slope, curves, and bridge and tunnel clearances).

- Closeness of power generating utility or railway-owned grids.

- Traction technology (power electronics, traction motors and regenerative capability).

The aim of these considerations is not only successful design and implementation of railways by itself, but also to facilitate further control and management of the load flow. Solving the load flow problems in such systems is difficult task due to numerous possible variety of trains, routes, power consumption, different schedules [23, 24, 47, 27]. It is also complicated depending on the type of feeding network. However, the research is still going and new methods and algorithms have been developed recently.

1.4.1 AC Traction Power System

An electrified railway line is similar to a typical power transmission and distribution system. However, trains (loads) move and change operation modes constantly, thus varying the power demand over a wide range in such way, that a train may become a power source during the regenerative braking [1]. Power demand can also be affected

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[1]Regenerative braking is a method of braking in which energy is extracted from the parts braked to be stored and reused

by the number of other factors, such as train speed, track layout, service scheduling, traffic demand, drivers' behaviour etc [25].

Several countries in Europe have been using AC power-supply systems (Great Britain, Spain, Portugal, Italy etc.). The development of the commercial high-speed railways in 1980s has spread the use of AC power-supply systems, involving larger power flows compare to DC systems. The most commonly adopted AC traction system (1x25 kV or 2x25kV, that can be called dual [12] or bivoltage at 50Hz) was designed for the lines with high power requirements.

The railway system is usually divided into a number of electrically isolated sectors. Each sector is fed from the single phase or three-phase network through traction substations, that are modified to guarantee the operation in case of failure. The direct connection of the feed transformer is common in AC railways. Booster transformers (BTs) and autotransformers (ATs), set into a feeding section are widely used in AC systems to improve transmission efficiency and system regulation, decrease rail-to-earth voltage and prevent electromagnetic interference to the adjacent telecommunication circuits [25].
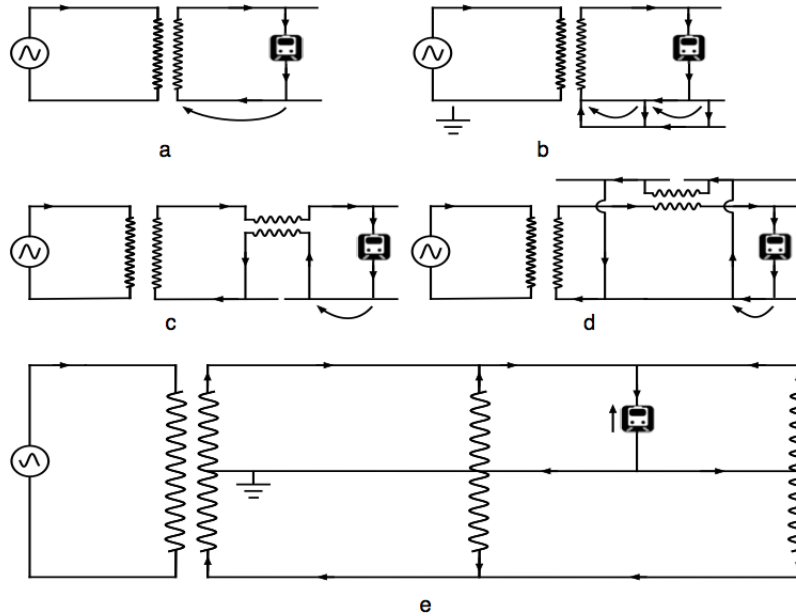
Figure 1-2: AC railway feeding systems: (a) Direct feed; (b) Direct feed with return conductor; (c) Booster transformer feed; (d) Booster transformer with return conductor; (e) Autotransformer feed [3]

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Simple/direct feeding. This the least cost option of feeding power. It achieves by the direct connection of the traction feed transformer to the catenary and rails of each substation (Figure 1-2a). However, it has following cons: high feeding impedance with large losses, high rail-to-earth voltage (safety issues) and the earth currents as the side-product that can cause interference in the telecommunication circuits, that are located nearby. The purpose of the addition of a return conductor is to reduce the leakage current by forcing the current to flow rather in a conductor, than in a rail, reducing the impedance traction current return path (Figure 1-2b). There are several conductors in the line and they are usually located at regular intervals (5 or 6 km) [18]. A return conductor also has a screening effect [2] that helps to reduce inductive interference in parallel telecommunication cables by about 30%.

Booster transformers (BTs). First BT, rated at about 150 kVA has been used in Japan in 1964 and it could improve the characteristics of the feeding circuit. BTs are usually located along the catenary at distance 3-4 km. The primary and the secondary windings are connected across a gap of the contact wire and across the insulated rail section respectively (Figure 1-2c). The purpose of a return conductor is the same as in case of direct feeding. It is preferable to incorporate a conductor in parallel with the rails for the return current (see Figure 1-2d).

Autotransformer (AT) power feeding. The first AT was presented in Philadelphia in early of XX century [18], after it was installed in Japan in 1972, and starting from 1981 more and more countries included ATs in their railway systems, developing new standards for the AC electrification system.

AT feeding is shown on Figure 1-2e. It combines the advantage of higher-voltage power transmission with the benefit of using standard 25 kV/50 kV equipment. The AT winding is connected between the catenary and an auxiliary feeder, with the rails tied to an intermediate point. The principle of AT operation is following: the train draws current from the two ATs, located nearby, the supply current from each

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[2]Screening is the damping of electric fields caused by the presence of mobile charge careers

of AT depends on the location of the train. Rail currents flow through the AT windings as illustrated in order to maintain Ampere-turn balance in cores. The AT system operates by balancing voltage [18]. This is its main advantage over BT feeding systems. As for another benefits of the AT system, it is easier to maintain, because this topology allows to separate a lot of substations.

1.4.2   Load Flow Solution Methods and Algorithms

Modelling and simulation of electrified traction systems are widely used as design instruments in order to analyse train movement regarding existing limits of the system. Numerous railway system models have been developed recently aiming to study power flow processes. In order to solve this problem all system models are usually separated into two models: electro-mechanic simulator/model and electric simulator/model as it is illustrated on the Figure 1-3. [11, 19].

Train Data
Track Data → *Electro − mechanical Simulator*

Power Profile
Train Dynamics

Power System Data → *Electrical Simulator*
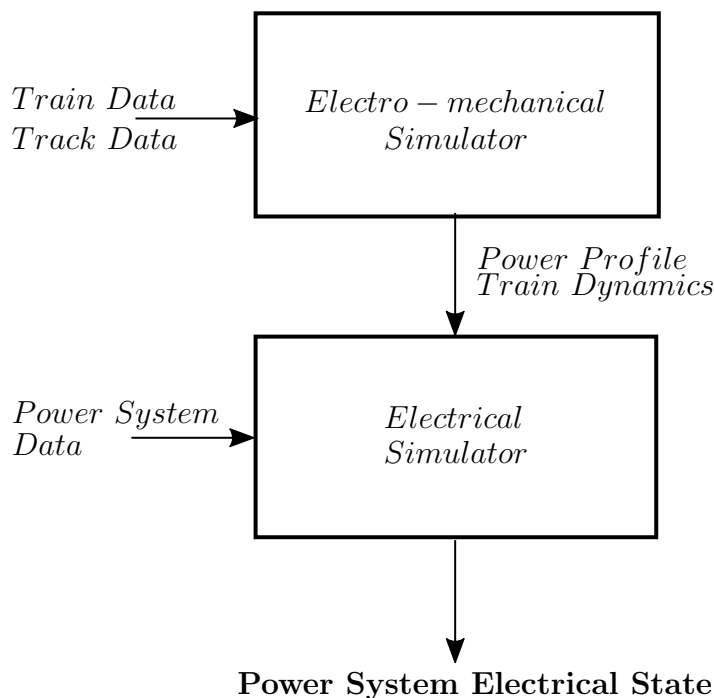
Power System Electrical State

Figure 1-3: Power Flow Modelling Technique

Electro-mechanical simulator obtains the physical dynamics of train movement, also providing as an output the timetable of the line for a giving traffic scenario

and energy consumption. Moreover, it can draw the position-time curve for each train, presents on the track during the simulation time. In order electro-mechanical simulator to work, the input data of train and track should be set up, such as: the trains departure time on two ways, specific trains sequence, train stop time period on each station, train shift time on the two ways. The data from electro-mechanical tool (power profile and position of each train for each period of time) is an input for the electric simulator, that works toward the evaluation of the energy state of the traction system. Among the information, provided by the electro-mechanical simulator, the data for electrical infrastructure's definition is required. It includes rated voltage of the system, number and position on the line of electrical substations (ESSs), number and size of the groups in ESS, number and section of conductors of the traction line, etc. Combining the consumption data for each train, the traffic scenario and the electrical topology of the supplying power system, the complete simulator is capable of performing power flow calculation for each time step. The tool uses some numerical techniques in order to solve the power flow problem, such as: Gauss-Seidel methods, Newton-Raphson methods, decoupled methods, multiple-conductor techniques and forward/backward sweep method. The brief review of each approach will be given further.

Gauss-Seidel (GS) technique has the advantages of clarity, good performance and there is no need to store previous values [36]. It is a procedure for solving n equations of the linear system of equations Ax = b one at the time in sequence, and uses previously obtained results as soon as they are available. There are two key aspects of the GS method, that should be noted. Firstly, the estimations are serial, so each component of new iterations depends on all previous results. Secondly, the new iteration x^k depends on the order in which the equations are calculated. If the order will change, the components of new iterations will also change. In terms of matrices, the GS method can be expressed as:

$$x^k = (D - L)^{-1}(Ux^{(k-1)} + b), \tag{1.1}$$

where the matrices D, -L, and -U are the diagonal, strictly lower triangular and strictly upper triangular parts of A respectively.

Strictly diagonally dominant, or symmetric positive definite matrices A can be solved by GS method. However, this condition may not be satisfied by real power networks due to combinations of very high and low series impedances, large capacitances, caused by cable circuits, long extra-high-voltage transmission lines, series and shunt compensation.

Applying GS technique to the traction system, the value that will be updated with each iteration, is usually voltage. This method has been implemented in probabilistic load flow technique (PLF) [47, 25, 13, 16]. The first mention of this technique has been done in [4]. As it is very common in this field, the first research was done for the DC system, followed by simulation for the AC systems [2]. Solving the power load flow problem with the help of the PLF gives as a result a full range of all possible values of bus voltages, power flows and losses, with their respective probabilities, with respect to load uncertainties and control-action effects [47]. Among with GS this study has also adopted Monte-Carlo simulation technique in order to set probability power distribution function of each train [25].

Newton-Raphson (NR) method is sometimes known as **Newton's iteration** is an algorithm for solving a set of nonlinear equations in an equal number of unknowns [36]. It is a root-finding technique that applies the Taylor series expansion of a function f(x) in proximity of a suspected root. Applying this method to the power flow question in railway systems, the real and reactive power flow equations are approximated by collecting the first two terms and neglecting the other high-order components, though some interaction between them is still presented. The compact form of NR can be expressed as:

$$
\begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} A_1 & A_3 \\ A_2 & A_4 \end{bmatrix} \times \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}
\tag{1.2}
$$

The power flow solution by NR was firstly described in [45, 9], and later imple-

mented in [47, 1, 33]. Compare to GS technique, that operates slowly due to updating each bus voltage separately, from the first to the last, NR quadratic convergence proceeds faster [36]. However, with the current existing railway traction systems it is still not enough.

Decoupled power flow or Fast Decoupled power flow approach. Any electrified traction system that operates in steady state has a strong interconnection between active and reactive powers and bus voltage angles and magnitudes respectively [36]. The coupling between these P-θ and Q-V components is not strong, thus for power flow calculation it has been decided to solve these tasks separately ("decoupling" them):

$$[\Delta\theta] = [A_1]^{-1}[\Delta P] \tag{1.3}$$

$$[\Delta|V|] = [A_4]^{-1}[\Delta Q] \tag{1.4}$$

This method is simple, fast and more reliable than NR or GS. It is used to solve DC traction systems [41] and also for AC railway systems [37, 28].

Multiple-conductor technique is based on representation of the whole power system as the combination of small separated in frequency domain network components. There are different types of conductors, presented in feeding circuits, such as: trolleys, feeders, rails, soils and protective wires. Although there are some of assumptions, that should be considering during application of this technique [23]:

- The supply source is represented as an ideal sinusoidal voltage source in series with an impedance.

- Bus is a set of conductor nodes with a particular distance.

- The propagation effects along the conductors are neglected.

- The sum of all conductor currents including the leakage current is zero.

- There is no mutual coupling between conductors.

This approach has been used to calculate power flow for railway vehicles in Japan [30] and in United Kingdom [20].

Backward/Forward sweep (BFS) method. There are a lot of load flow calculation that are based on this technique [21, 6, 7, 29]. It consists two steps, backward sweep and forward sweep, which are repeated until the calculation is done. Current or power flow summation with possible voltage updates are represents by the backward sweep, while a voltage drop calculation with possible current or power flow updates are introduced by the forward sweep. BFS approach can be implemented as following:

1. Initialization of all bus voltages.

2. Backward sweep: calculation of the currents based on the assigned voltages.

3. Forward sweep: updating voltage values, based on calculated current and for the next iteration using them as the initial values.

4. Repeat steps 2 and 3 until the convergence is performed.

[31] describes the implementation and prove the effectiveness of this method for Tehran-Golshahr suburban electrical railway.

1.4.3 Computer-based Railway Simulations

Design, management and maintenance of the railroad systems is difficult task that requires high quality and efficient computer-based simulators. Development of a complete simulator means to create a tool that is able to model numerous aspects: train movement, signalling, interlocking, power supply, regenerative braking etc., keeping the results according to existing limits and needed accuracy within realistic conditions. Thus, it can provide significant data about traction system behaviour and help to evaluate different possible railroad upgrade strategies. Due to constant improvement of railroads in locomotives, infrastructure and operating procedures, it is quite important to verify and understand all changes immediately in order not to increase further expenses.

A good computer model should be prepared to perform next steps:

1. Precisely recreate the real railway performance according to the existing trains, substations, their location and schedule.

2. Analyse possible issues, including timetables stability, infrastructure requirements for a given schedule, changes, that can be caused by trains' movement.

Such modelling and simulations are useful for railway design, because once developed and calibrated, models can be used to compare the advantages, effects, and costs of numerous update packages. The effective traction system simulator is capable of determination more options and alternatives, guiding to more complete and efficient problem solutions.

There are couple of commercial railway system simulators that are successfully used today. Briefly review of them will be given below.

OpenTrack [32] developed by Swiss Federal Institute of Technology's Institute for Transportation Planning and Systems, is able to perform the behaviour of all railway components (infrastructure network, rolling stock and schedules) and all operations between them. Besides, it also provides information about power and energy consumption of train services, simulates railway power supply systems (using Open-PowerNet) and analyses the effects of system failures. OpenTrack is the most popular railway simulator, that used by several railway and metro companies, railway administrations, such as Swiss Federal Railways, Finnish Railway Administration (RHK), NSB (Oslo), RFI - Rete Ferroviaria Italiana (Rome, Italy), Transport of London, Taiwan High Speed Rail Corporation etc.; and railway supply industries, such as: Siemens, ABB, Alstom and many others.

TracFeed/Simtrac [44, 46] has the same abilities as OpenTrack, providing the real time simulation of electric traction power system models of trains, train movements and electric components. It uses SIMPOW®platform by ABB Power System, Sweden. However, the simulator still requires some modifications due to its problems with modelling heavily loaded systems, caused by difficulties from electrical point of view. It has been used by Norwegian railway administration and Swedish railway administration.

Sitras Sidytrac [39] developed by Siemens, has abilities to make a detailed design of AC or DC railway systems with electrical measurements of substations and contact line systems, providing planning of the whole system together with estimation of power, needed for further improvement. As an output it is possible to get the graphical control of railway and electrical networks, vehicles, schedules' information, train data (currents, voltages, total power, traction poser, losses in braking resistance, efficiencies, position, velocity, acceleration etc.) and electrical network results (currents in network branches, supply voltages, total power, active power, energy balance, losses etc).

μPas/TDS - Train Drive Simulator [34] developed by PROLEITEC AG, is the network calculation package that provides the practical simulation of selected lines and obtains the effects of train drives and their influence on lines and high-voltage network. It is also possible to use the train-drive simulation separately, i.e. without consideration of the electrical grid.

Fabel [10] is a software developed by ENOTRAC that simulates electrical transport systems and can be useful for design new lines and extensions, supporting of existing railroads, evaluation of vehicle propulsion equipment and design of protection system. It combines the train movement modelling with the electric network estimations, describing train behaviour and providing the power requirements. It has been used by several operators and companies in UK, India and Switzerland.

Chapter 2

A 2x25kV AC Bivoltage Traction System

For the past years, there has been a significant increase of implementation and usage of high speed railways. It leads to higher demands of power, thus higher feeding voltages are expected. For this reason, nowadays, new lines are often electrified in AC. As for existing AC lines, there is a growing tendency to convert them into bivoltage systems (e.g. 2x25kV, 50 Hz), which will allow to get higher feeding voltage. In such type of systems power is transferred using high voltage, which then is reduced by autotransformers to the suitable for the trains or distribution network level. This configuration has been described in [12, 31, 35, 48].

However, such countries as Germany and Sweden use 15kV, 16.7 Hz electric systems for their railways. This type of traction network is similar to the bivoltage systems, but has difference in terms of feeding: it carries through their own single-phase transmission system that can be connected to the dedicated stations or to frequency converter stations. The size of the converter station defines its type. Stations that supply energy to substations are called central, while stations that supply energy directly to the traction systems are called decentralised. Also, there are several feeding schemes, such as simple or direct feeding, feeding through the booster transformer or AT with or without an external return conductor. Each scheme has its pros and cons, which were discussed in previous chapter.

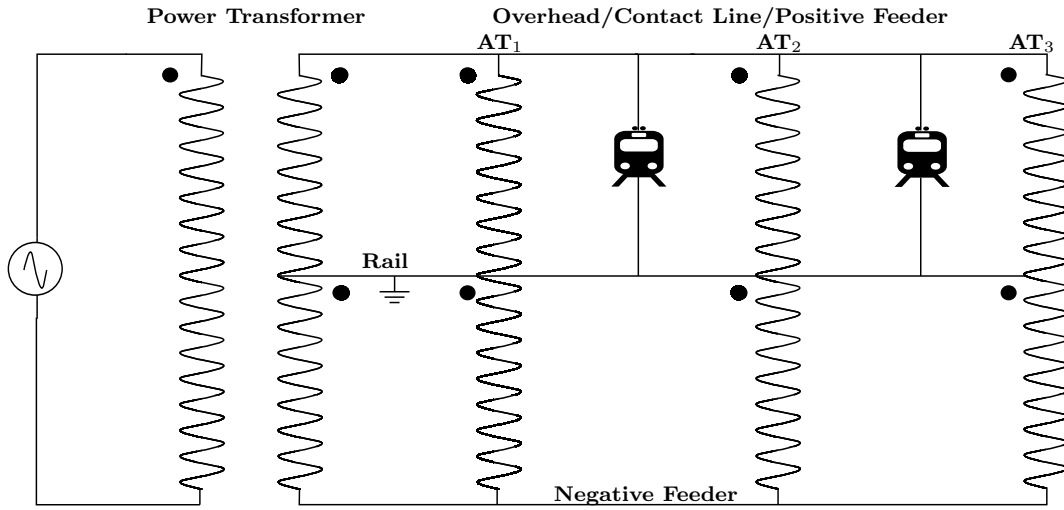The 2x25kV 50Hz bivoltage traction system is illustrated on Figure 2-1.

Figure 2-1: AC bivoltage traction system

As it is showed on Figure 2-1 the secondary windings of the power transformers are placed at ESSs and have a central tap connected to the rails (ground), while the poles are connected to the contact/overhead line (or positive feeder) and the negative feeder respectively. In general, the positive voltage is defined by standards and negative one can be chosen freely. However, from the design and maintenance point of view, it is common to set up the same voltage for the both, positive and negative feeder. ATs that are used in this configuration usually have a unity turn ratio, and this is a specific characteristic for such schemes. ATs are connected to contact line, rail and negative feeder at certain intervals, and break the electrical section fed by one substation into different compartments. These compartments are called cells and in general the length of each cell can be from 10 km up to 50 km.

The Figure 2-2 shows an example of the current distribution in bivoltage configuration. There are two cells in the scheme, represented in the Figure 2-2: the first one is between the substation and the first AT and the second one is between the first and the second ATs. The most important part to study is placed in the second cell, where the train is located. The train is consuming a current $I = 400A$, which consists of two parts: the first part is current aI comes from the first AT, while the second

part *(1 - a)I* comes from the second AT. The value of current, supplying by each AT, depends on the factor *a*, which defines the location of the train. In this example *a = 0.75* has been assumed, thus the values of current, supplying by the first and the second ATs, are *300A* and *100A* respectively. The return current flows through the rails to the closest ATs, then it splits into two, so the current flowing through the rails of the first cell is zero. Also, the current flowing from the second cell to the power transformer through the positive and negative feeders is *200A*, half of the train current. The empty cell (without train) has the feeding and return system of 50kV, so it is possible to increase the transport capacity of the line together with the distance between the nearby stations. There are five different loops in this configuration: two in the first cell (without train) with the current *I/2* and three in the second cell (with train), and two of them have *(1 - a)I* current and one loop has *aI* current. Thus, the current distribution of this scheme can be represented by the current flow in these five loops.
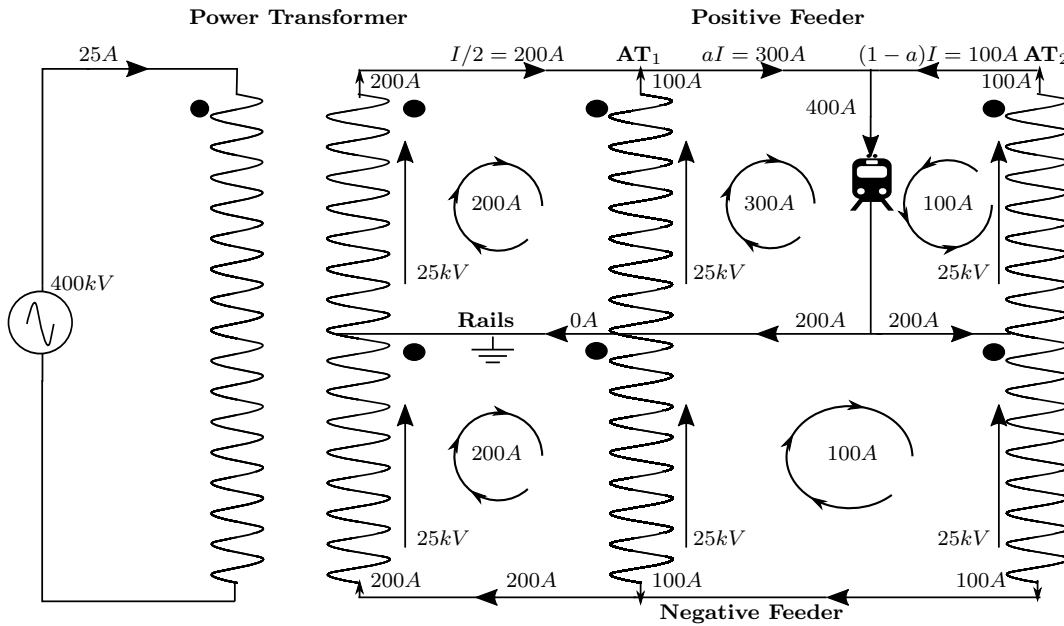
Figure 2-2: Current distribution in an AC bivoltage traction system

There are a lot benefits of such configuration, and the reduction of the current through the rails is one of them. It makes the rails potentials and the losses also to be

reduced. Among the other advantages, the bivoltage design for high speed railways makes the maintenance procedures simpler and decreases the level of interference with the telecommunication systems.

It is also possible to implement bivoltage configuration in DC traction systems, using DC/DC controlled converters instead of the ATs.

Chapter 3

Mathematical Model

3.1 Static Mathematical Model

Any electrical section fed by one substation (one power transformer) can be broken into different cells by the ATs. In order to avoid the problem of unbalanced network and do not complicate estimations, cells are usually assumed to be independent. It allows to make calculation of voltage, current and load flow easier.

Figure 3-1 represents a bivoltage AC system with one cell and one train in it. The network can be divided into two parts: the high voltage feeding network, which is connected to the AC traction network through the primary side of the power transformer, and the AC traction network by itself, that includes the secondary side of the power transformer, ATs and trains. The source voltage, primary side current and impedance are represented by V_{src}, I_p and Z_p respectively, while all values, that describe the AC traction network, can be separated into different groups, such as node voltages (explanation and enumeration of the nodes will be given in the following parts of this chapter), line currents (current trough the positive feeder I_{PF}, current trough the rails I_{GND} and current through the negative feeder I_{NF}) and line impedances (Z_{PF}, Z_{GND} and Z_{NF}), also taking into account ATs currents and voltages ($I_{AT1}, I_{AT2}, V_{AT1}, V_{AT2}$) together with the characteristics of the train (V_t and I_t).

The mathematical model of the static system[1] is represented by a system of

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[1] by static means the system that does not consist trains
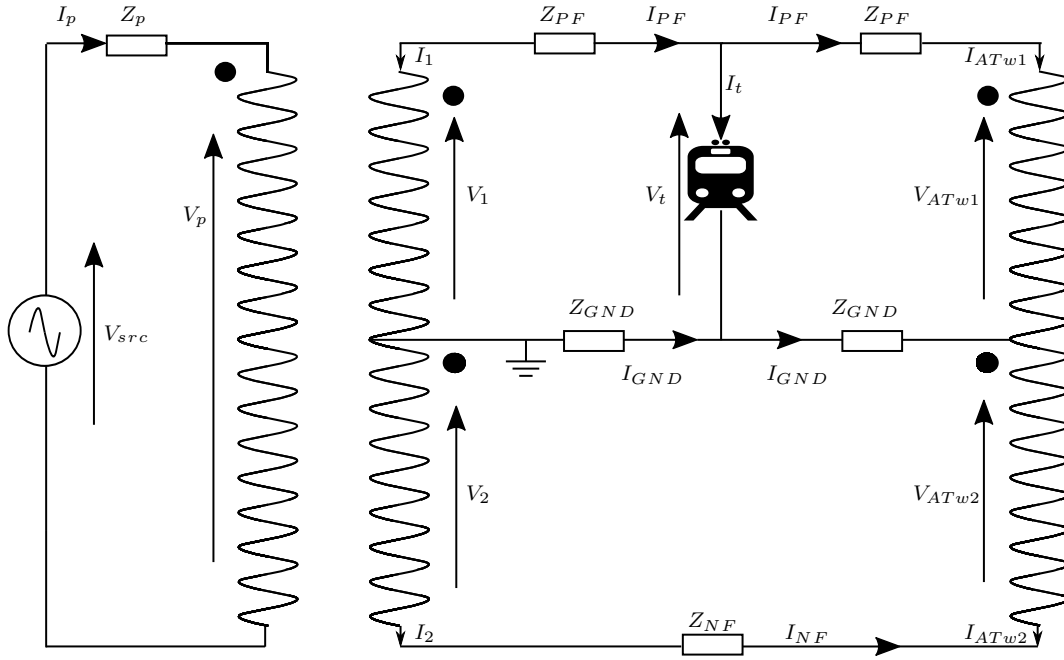
Figure 3-1: AC bivoltage system with one cell and one train

equations that describes the network. This set of equations includes Kirchhoff Voltage
and Current Laws (KVL and KCL) for all branches and nodes, power transformer
and ATs' equations. The general form of the equations, which describe the power
transformer and ATs', is given below.

Two of the power transformer equations describe the relation between the primary
side and each group of the windings on the secondary side:

$$V_p = \frac{N_p}{N_{p1}} V_1 \tag{3.1}$$

$$V_p = \frac{N_p}{N_{p2}} V_2 \tag{3.2}$$

where N_p is the number of turns of the power transformer primary winding,
while N_{p1} and N_{p2} are the number of turns of the power transformer secondary wind-
ing first and second groups respectively (here and later the winding that connects the
positive feeder and the rails will be considered as the first winding group, and the
winding that connects the rails and the negative feeder will be the second winding
group. This will also work for the ATs windings). From the Figure 3-1 V_p, V_1 and

V_2 are the voltage of the primary side of the power transformer and voltages of the first and second winding groups of the secondary side of the power transformer.

Power equation of the power transformer will be also included into the set of equations, which describe the network:

$$I_p V_p = I_1 V_1 + I_2 V_2 \qquad (3.3)$$

I_1 and I_2 are the currents through the power transformer secondary windings.

Each AT will add two equations to the system (one that describes the voltage relation between first and second winding groups, and the power equation):

$$V_{ATw1} = \frac{N_2}{N_1} V_{ATw2} \qquad (3.4)$$

$$I_{ATw1} V_{ATw1} + I_{ATw2} V_{ATw2} = 0 \qquad (3.5)$$

In order to obtain the network, i.e. obtain all currents and voltages in the system, the next data is required:

- Cell Data that consists all information about all cells: number of cells in the network, length of each cell, turn ratio of ATs and resistance and impedance values for the contact line, rails and the negative feeder.

- Train Data that includes all information about trains: number of trains, number of cells, in which each train is located, together with distance and active and reactive power values. Active and reactive power of the train can be obtained, knowing the power consumption of the train and its power factor.

- Value of the source voltage.

- Number of turns for the primary and secondary side of the power transformer.

Any developed and properly working simulator is a tool that can be successfully used by any railway company. In order to use this tool the company should provide necessary information, required to solve the system.

3.2 Breaf Introduction to the Graph Theory

Graph theory describes properties of networks using graphs, which are symbolic representations of systems and connections within these systems. Graph consists vertices or nodes (substations or trains in case of railways), and lines or edges, which join nodes. Any graph in graph theory can have the next properties:

- Graph G will be an oriented/directed graph (or digraph), when its edges are identified with ordered pairs of nodes. Otherwise graph G will be undirected/non-oriented graph.

- Graph G is a simple graph, when there is no self-loops or multi-edges, i.e. there is no edges starting and ending in the same node.

- Graph G is called dense, if the number of edges is twice higher than the order of the graph[2]. Otherwise graph is called sparse.

Figure 3-2 shows a simple graph G, that includes two finite sets N and E (nodes and edges accordingly). Vertices are represented as 1, 2..n, while edges are represented as $e_1, e_2..e_k$. Each edge is defined by a pair of nodes (i, j), where i is a source node/tail, and j is a destination node/head.

Thus, simple graph G (Figure 3-2) consists:

- Nodes N = {1, 2, 3, 4, 5}

- Edges E = $\{e_1, e_2, e_3, e_4, e_5, e_6\}$, where :

  $e_1 = (1, 2)$

  $e_2 = (3, 1)$

  $e_3 = (3, 5)$

  $e_4 = (4, 2)$

  $e_5 = (4, 3)$

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[2]The order of the graph is defined by the number of nodes
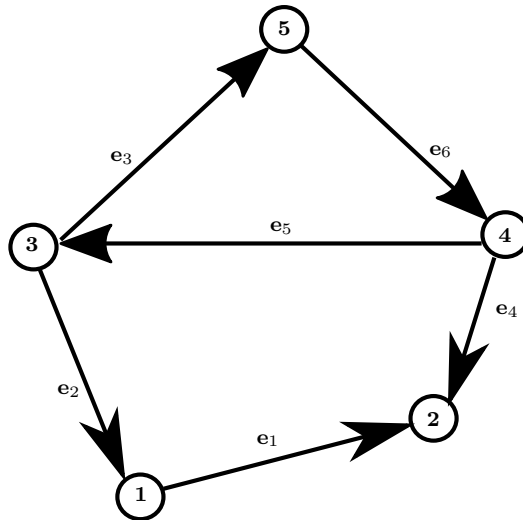
Figure 3-2: Simple graph G

$$e_6 = (5, 4)$$

The direction of the edges will define a positive reference for currents in a given system.

It is possible to divide graph G into subgraphs, in which nodes and edges are subsets of a present simple graph G. Figure 3-3 shows one of the subgraphs from a graph G: a subgraph H consists nodes $N_h = \{3, 4, 5\}$ and edges $E_h = \{e_3, e_5, e_6\}$.
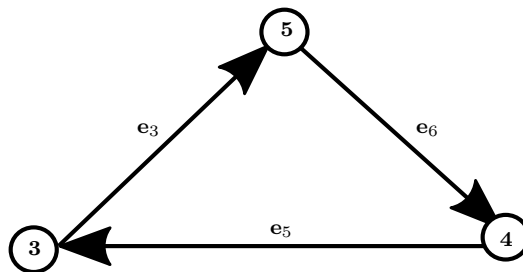
Figure 3-3: Subgraph H of a simple graph G

Numeration of the nodes and edges in any graph can be done in any order, depending on the designer of the system. Nevertheless, enumeration of the network elements (nodes/lines) plays a significant role in the description of the railway systems, especially when it comes to the solving of the system with the large amount of trains. Irrelevant numeration can cause difficulties during the solving procedure.

A pair of nodes are adjacent if there is an edge, that connects them (e.g. node 1 and node 2 in a graph G are connected by an edge e_1).The number of edges that connects a node with another vertices verifies the degree of a node (d(i)).Node with a degree one is called a leaf or pendant node. Depending on the direction of the edge, whether it goes in or go out from the node, a vertices can have the following types of degree: in-degree (number of incoming edges) and out-degree (number of outgoing edges).

For a graph G with n nodes the matrix Λ with dimensions *(n, n)* can be built.This matrix Λ relates the connection between nodes.

$$\Lambda_{i,j} = \begin{cases} \text{number of edges between } i \text{ and } j & i \neq j \\ \text{number of self-loops in } i & i = j \end{cases}$$

However, in most of studied cases in order to simplify the solution procedure, all graphs are usually simple digraphs. Thus:

$$\Lambda_{i,j} = \begin{cases} 1 \; \exists \text{ adjacency between } i \text{ and } j \wedge i > j \\ 0 \text{ other cases} \end{cases}$$

Adjacency matrix Λ can be represented as following:

$$\Lambda_{i,j} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.6}$$

Usually, adjacency matrices are defined as symmetric. However, to get the greater efficiency obtaining the sparse matrices, it is better to define them as an upper triangular, thus avoiding the redundant information storage.

Furthermore, the graph G with n nodes and k edges can be fully defined by the k, n dimension incidence matrix Γ :

$$\Gamma_{k,n} = \begin{cases} 1 \text{ the tail/source of the edge } k \text{ is the node } n \\ -1 \text{ the head/destination of the edge } k \text{ is the node } n \\ 0 \text{ other cases} \end{cases}$$

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$$\Gamma_{k,n} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \tag{3.7}$$

Applying graph theory to solve railway systems trains and substations will be consider as nodes, and the lines, connecting them, as edges. The network by itself can be considered as a fixed network, i.e. the number of trains and topology can be fixed, even if the number of trains changes. Thus, the number of trains during the simulation will be consider constant, even if some of trains will be out of stage.

3.3 Static System Formulation based on Graph Theory Approach

This thesis was developed, based on the graph theory. First approach of the concept, applying to the traction systems, has been described in [15]. According to the graph theory the substations and the trains are considered as nodes of the graph and the lines, which connect them, as the edges.

As it was mentioned above, using the graph theory approach for the railways it is important to choose a proper way to numerate network elements, so the further solving of equations that describe the network will not be complicated. Before start the numeration procedure, it is critically to point out that all changes will occur in the AC traction system (right side of the power transformer on the Figure 3-1) and not in the high voltage feeding network. Due to this fact the whole high voltage system can be described just by one equation of KVL, which will remain constant, and the numeration procedure will be applied to the elements of the AC traction system.

The first node is a central tap in the secondary side of the power transformer, so the nodes that belongs to the ground line will be numerated first, then the nodes of

the positive feeder and the nodes of the negative feeder will be numerated the last. However, in case of line numeration, the procedure will start from the negative feeder, following by the ground lines and only then the lines of the positive feeder. This way of numeration has been chosen to simplify the further verification of the results for the dynamic system (trains are connected only to the rails and positive feeder, since the values of the negative feeder are not going to be affected by the allocation of the trains). The graphic representation of the numeration procedure is shown on the Figure 3-4.
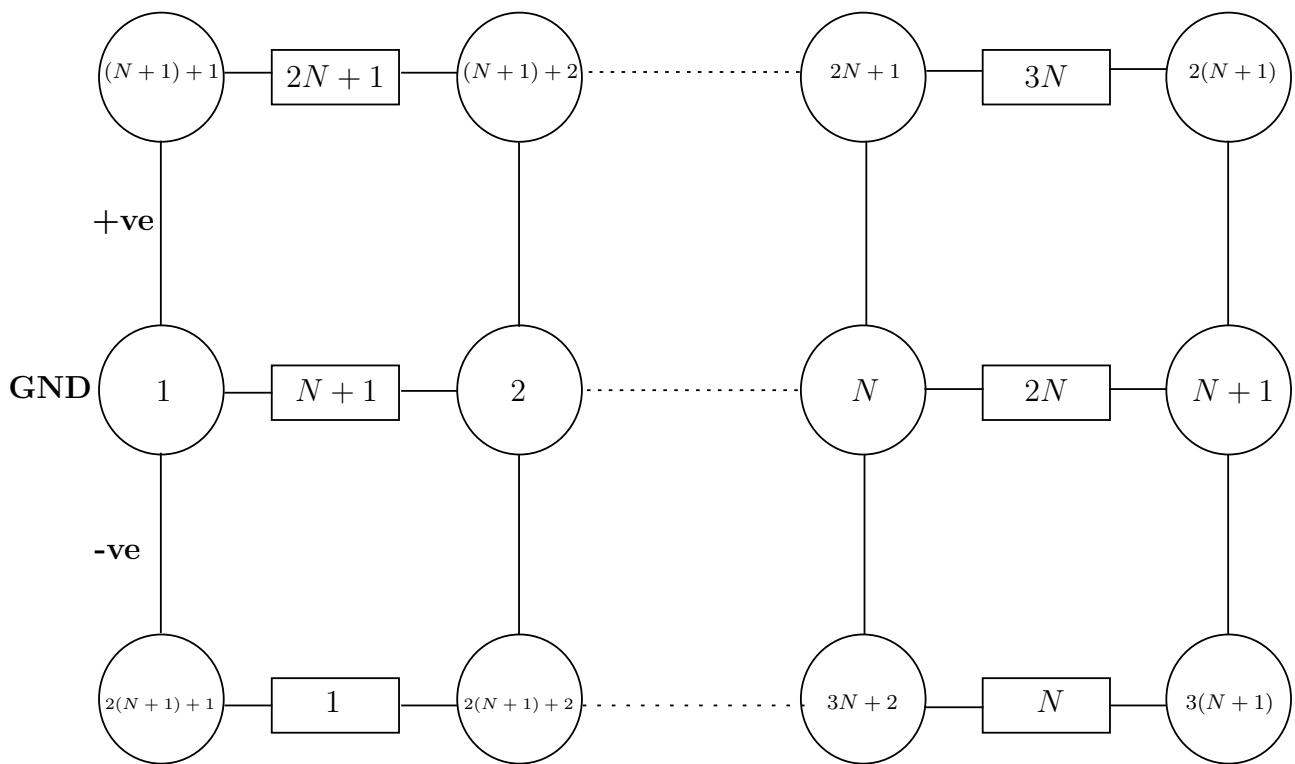
Figure 3-4: The graphic representation of the numeration procedure

N in this figure represents the number of cells. The index of the node depends on the number of the cells and the last node of the static system has $3(N + 1)$ index.

Figure 3-5 illustrates the static AC network with two cells. Node and line numeration have been done following the procedure, described previously.

In case of the static system three node incidence matrices are presented: the first one Γ_{line} represents the real connections between ATs and between the secondary side

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Figure 3-5: Static AC network

of the power transformer and ATs, while the second and the third ones (Γ_{w1} and Γ_{w2}) define the connections within winding groups of the ATs and the secondary side of the power transformer. The n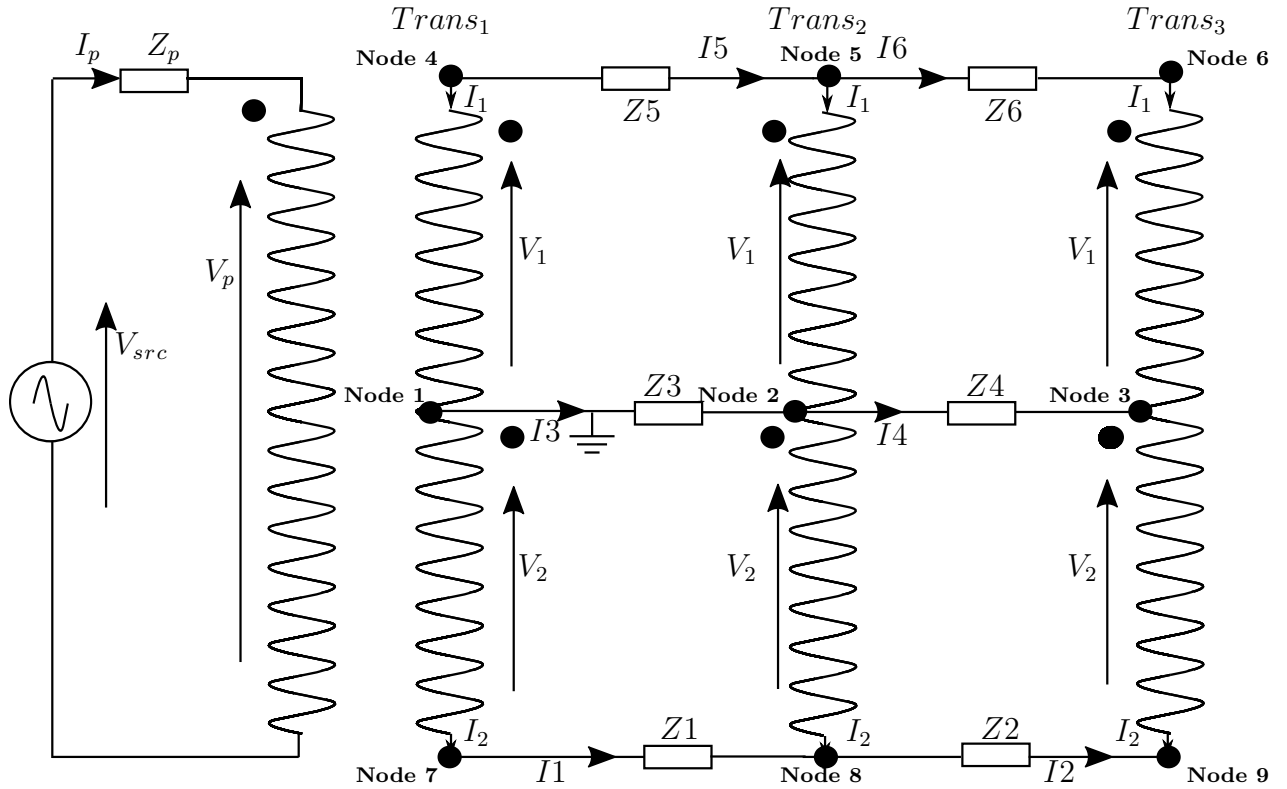umber of columns in these matrices is equal to the number of nodes in the network, while the number of rowss is different: for line incidence matrix Γ_{line} it is equal to the number of lines, and for the transformer incidence matrices Γ_{w1} and Γ_{w2} it is equal to the number of transformers (it includes ATs and the windings of the secondary side of the power transformer).

Each line is defined by two nodes, the first one is the source node ("from" node) and the second one is the destination node ("to" node). These names refer to the direction of the currents. Thus, following the procedure of building incidence matrix, the next result for the configuration, shown on the Figure 3-5, is achieved.

$$\Gamma_{line} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \tag{3.8}$$

$$\Gamma_{w1} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{3.9}$$

$$\Gamma_{w2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{3.10}$$

Using defined node incidence matrices all the KVL's and KCL's can be written in a general form.

$$V_{src} = I_p * Z_p + V_p \tag{3.11}$$

$$\Gamma_{line} * V_N = Z * I_{line} \tag{3.12}$$

$$(\Gamma_{line})^T * I_{line} + (\Gamma_{w1})^T * I1_{trans} + (\Gamma_{w2})^T * I2_{trans} \tag{3.13}$$

Equations (3.10) and (3.11) represent the KVL for the feeding system and for the lines of the AC traction system, while equation (3.12) interprets the KCL in the traction network.

The vector V_N includes the voltages in the traction network nodes (nodes 1 to 9) and the vector I_{line} contains line currents (lines 1 to 6), whereas vectors $I1_{trans}$ and $I2_{trans}$ hold the values of the transformers currents for the first and the second winding groups respectively. Z is the impedance vector that includes the impedances of the lines (first impedance of the lines of the negative feeder, then rails and afterwards the contact line). The impedances of the lines that belong to negative feeder are static and indicate the real topology of the network. They are not going to be changed with

the introduction of the trains into the system.

For the final description of the static system, equations of the power transformer and ATs should be included (equations (3.1) - (3.5)).

3.4   Line Splitting Procedure

Any railway system can be described by the set of equations. Its electrical analysis, however, is usually complicated by the presence of trains, which are moving loads that continuously change their positions, speed and as a result their power consumption. Thus a set of equations that describes the system will also vary within simulation steps, making the solving of equations a difficult task. Having constant number of trains will not make problem easy to resolve due to the fact that trains will still move and a set of equations will still change. However, it is not a problem for the small system (system with small amount of trains and substations), but with the increasing size of the system it becomes significant to manage this task and to get the proper results. Usually, there is a particular routine to specify the number of trains and their positions in the system for each time interval and there are different ways to achieve it, developed by different authors [40, 15].

A special algorithm to introduce trains into the system has been developed for this thesis. It is called a line splitting procedure and based on the splitting of the lines of the positive feeder and the rails depending on the location of the trains. The algorithm takes as input the train data, which includes total amount of trains, their location (which cell[3] and at what distance), values of an active and reactive powers, and gives as an output so-called train and line informations, and also counts the amount of new dynamic nodes that was created by trains. Train information includes train index, the source node and the destination node, train distance (counted from the beginning of the line of the cell, in which the train is located), and the values of an active and reactive powers. Line information consists the data about new lines, which are created due to the presence of the trains, such as: the start node of the

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[3]Cell that consists train or trains will be called active

line/source and the end node of the line/destination, its length and the values of the line resistance and impedance.

First, the procedure will be done for the active cells of the positive feeder. Exactly the same strategy should be followed for the active cells of the rails, and afterwards all information can be combined into one matrix, which will represent line or train data.

The most significant information out of all train data for this algorithm is the information about the position of the train, i.e. at which distance train is located. This distance is usually given in per unit system, thus it cannot be higher then 1. For each cell each train (trains) places between the source and destination nodes of this particular positive feeder line of the cell accordingly to its distance. For this procedure the source node places at the distance 0 and the destination node places at the distance 1. Then the difference between the distances (dX) within the same cell is obtained sequentially (see the Figure 3-6). Whether the new node in the line will be created or not totally depends on this value.

Source Node   Train 1   Train 2   Train n   Destination Node

Distance X 0 —— X_1 —— X_2 – – – – X_n —— 1

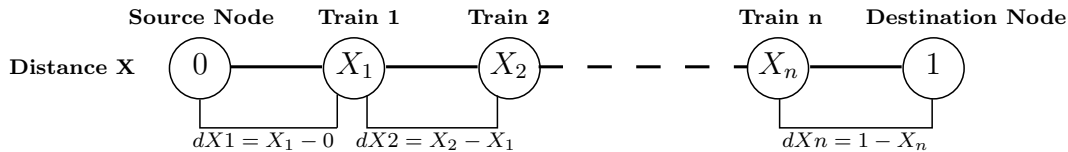$dX1 = X_1 - 0$ $dX2 = X_2 - X_1$ $dXn = 1 - X_n$

Figure 3-6: Evaluation of the difference of the distances

It is also important to set up the minimum required distance between the trains or between the train and the node. This distance, which is going to be called tolerance, defines the grouping procedure for the trains. All values of dX will be divided into two vectors a and b, where vector a contains all values that are less or equal to the tolerance, and vector b includes all values that are greater than the tolerance. For all terms of the vector b a new dynamic node will be represented. Trains will create a group, if the difference of the distances between them is less then the tolerance. For those terms of vector a that represent dX between node and train, two distinct scenarios are possible:

- Train or group of trains will be located at the source node, if the first term of the

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vector a is equal or less then the tolerance.

- Train or group of trains will be located at the destination node, if the last term of the vector a is equal or less then the tolerance.

Following the same procedure for the positive feeder and the rails, each train will give two new dynamic nodes, located at the same distances at the contact line and the rails respectively. The procedure of numeration of the new nodes and lines will be described below. Above that, with the introduction trains to the system, the impedance of the dynamic lines will be depend of the train distance and should be update at each simulation step.

3.5   Final Formulation of Dymanic Model

The representation of the electrical trains and investigation of their interaction with the traction system is very important for modelling, operation and analysis of the railway systems. There are several ways, developed by researchers, to describe the behaviour of trains within the electrical networks [14, 5]. However, in order to separate the train model from the network model and make it possible to simulate the train behaviour and impacts that it has to the electrical network (voltage drops in the catenaries, voltage increase in the catenary during braking, reversible power flows in substations that have bidirectional devices etc.), the following assumptions should be made:

- No overcurrent protection for the trains.

- No squeeze control for the trains.

- No catenary voltage constraints.

In this case, the impact of the network over the train is not taken into account at all. Depending on the train mode, i.e. traction or braking, this effect will be or will not be considered. As the result, the list of assumptions can be changed.

In order to analyse the effect of network traffic over the schedule variation, the train model and the network model should be connected and the list of assumptions should be modified. New suppositions will include the trains torque limitation constraints, the trains overcorrect protection, the trains squeeze control and the catenary voltage constraints.

Train by itself can be represented from different sides: from mechanical point of view, that considers forces, that are involved into train's movement, track conditions (slopes, curves etc.), from electrical point of view, that includes information about electrical equipment (electrical machines and train power electronics), and from control point of view, that includes constant tracking of the position of the train at each time interval, given by the schedule, and correction of this position by accelerating or decelerating. Thus, the total model of the train should consider all three sub-models (mechanical, electrical and control) in order to get results as closely to the real life as possible. However, in this work trains are simply represented as a non-linear power sources, because the thesis is focused on the simulation of power flow and power system.

As it was mentioned in previous sections, the graph theory strategy has been used in this thesis. It allows to consider the network as a system with a fixed number of trains and an established topology, even if the real number of the trains in the network changes together with the positions of the trains. The set of equations that describes a static system has already been built. However, with an introduction of the trains into the network, this equations require some changes.

Figure 3-7 illustrates the dynamic traction system. It is similar to the Figure 3-5, but there is a train located in the first cell.

The indexes of the static nodes have not been changed, however there are two new nodes (node 10 and node 11), which represent the train. As it can be noticed from the Figure 3-7, the new node of the contact line is numerated first, and afterwards, the new node of the rails. Their indexes of the dynamic nodes have the numbers that follows the index of the static node, which was enumerated last. Last static node is node 9, thus the first dynamic node (one that belongs to the overhead line) has index
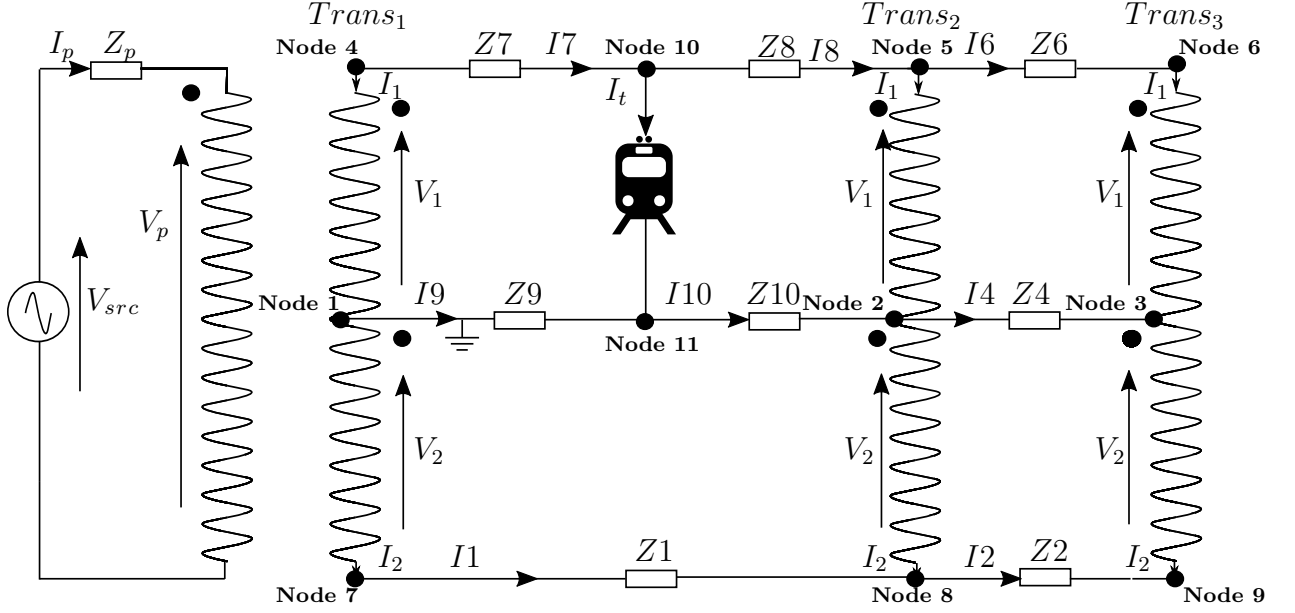
Figure 3-7: Dynamic AC network

10, and the new dynamic node of the rails has index 11. If more trains are present in the system, the new dynamic nodes in all cells of the contact line will be numerated first, and only then the new dynamic nodes of the rails will get their indexes in the same sequence. The static lines of the positive feeder and the rails of the first cell have been split by one train into four new lines (two for the positive feeder and two for the rails). First, the dynamic lines of the contact line are numerated, and then the lines of the rails. The procedure of their numeration is similar to the numeration of the dynamic nodes: the index of the first dynamic line has the number, which is sequential to the index of the last static line. For the configuration, represented on the Figure 3-7, the static lines 3 and 5 (see Figure 3-5) have been split into dynamic lines 7 and 8, 9 and 10 respectively.

In case of the dynamic system four node incidence matrices are presented: the first one Γ_{line} defines represents the real connection between ATs, between the secondary side of the power transformer and ATs, and between trains and ATs and the secondary side of the power transformer, the second and the third matrices (Γ_{w1} and Γ_{w2}) define the connections within winding groups of the ATs and the secondary side of the power transformer, while the fourth matrix Γ_{train} defines the connections of the trains. The

number of columns in these matrices are equal to the number of nodes in the network, but the number of rows is different: for the incidence matrix Γ_{line} it is equal to the total number of lines[4], for the transformer matrices Γ_{w1} and Γ_{w2} are equal to the number of transformers (it includes ATs and the secondary windings of the power transformer), and for the train incidence matrix Γ_{train} the number of columns is equal to the number of the trains in the network.

The transformer incidence matrices Γ_{w1} and Γ_{w2} are going to be the same, as for the static model. They are represented by equations (3.9) and (3.10). The line incidence matrix Γ_{line} changes dynamically with the position of the trains, and its final version is described by an equation (3.14). The new train matrix Γ_{train} is represented by an equation (3.15).

$$\Gamma_{line} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.14}$$

$$\Gamma_{train} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{3.15}$$

A general form of the KVL's and KCL's now will look the next way:

$$V_{src} = I_p * Z_p + V_p \tag{3.16}$$

$$\Gamma_{line} * V_N = Z * I_{line} \tag{3.17}$$

$$(\Gamma_{line})^T * I_{line} + (\Gamma_{w1})^T * I1_{trans} + (\Gamma_{w2})^T * I2_{trans} + (\Gamma_{train})^T * I_{train} \tag{3.18}$$

---

[4]by total means the sum of the static and dynamic lines

Equations (3.16) and (3.17) represent the KVL for the feeding system and for the lines of the AC traction system and in general form look exactly as equations (3.10) and (3.11), while equation (3.18) interprets the KCL in the traction network.

The vector V_N includes the voltages in the traction network nodes (nodes 1 to 9) and the vector I_{line} contains line currents (lines 1 to 6), whereas vectors $I1_{trans}$ and $I2_{trans}$ hold the values of the transformers currents for the first and the second winding groups respectively, and finally, vector I_{train} contains trains' currents. Z is the impedance vector that includes the impedances of the lines (first impedance of the lines of the negative feeder, then rails and afterwards the contact line). The impedances of the lines that belong to negative feeder are static and indicate the real topology of the network, while the impedance of the dynamic lines depends on the position of the train, thus this value will vary at every simulation step.

For the final description of the dynamic system, equations of the power transformer and ATs should be included (equations (3.1) - (3.5)) together with equation of the train model (3.19):

$$S_{ti} = (V_{n_i}^{gnd} - V_{n_i}^{+ve})I_{train} \tag{3.19}$$

where S_{ti} is the apparent power of the train i, $V_{n_i}^{gnd}$ and $V_{n_i}^{+ve}$ are the voltages of the nodes that represent train i on the rails and the positive feeder respectively.

The other important thing to mention is that all equations, described in this chapter are complex ones, because all variables are the complex magnitudes.

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Chapter 4

Solving Procedure

The task of the script, which has been developed for this thesis, is to calculate the node voltages and line currents for all nodes and all branches. The problem has been solved by applying two techniques: a derivative method (with a help of the function "fsolve" of MATLAB software) and by the implementation of BFS method.

4.1 Derivative-based Technique

There are several derivative-based techniques [8], but in this thesis the Newton-Raphson method has been used. This strategy has been implemented many times by different researches [45, 9, 47, 1, 15, 33]. An algorithm requires to provide the same number of equations and unknowns.

Based on the chapter 3 and defined incidence matrices Γ, the set of equations, which must be solved, is following:

1. KVL of the feeding system (primary side of the power transformer):

$$V_{src} - I_p * Z_p - V_p = 0 \tag{4.1}$$

2. The relation between the primary side of the power transformer and the first

winding group of the secondary side of the power transformer:

$$V_p - N_{p1}V_1 = 0 \qquad (4.2)$$

3. The relation between the currents of the primary and secondary sides of the power transformer, which can be derived from the equations (3.1) - (3.3):

$$\frac{1}{N_{p1}}I_1 + \frac{1}{N_{p2}}I_2 = 0 \qquad (4.3)$$

4. Relation between the voltages of the secondary side of the power transformer and ATs' winding groups:

$$V2 - N * V1 = 0 \qquad (4.4)$$

where V1 and V2 are the single column vectors that include voltages of the first and second winding groups of the power transformer and the ATs, while N a single column vector that includes turn ratio of the ATs and power transformer.

5. ATs' current equation, which can be expressed from the ATs' equations (3.4) and (3.5):

$$I1_{trans} + N_{trans} * I2_{trans} = 0 \qquad (4.5)$$

where N_{trans} is a single column vector that includes turn ratio of the ATs.

6. KVL for all lines of the AC traction system:

$$\Gamma_{line} * V_N - Z * I_{line} = 0 \qquad (4.6)$$

7. KCL for all nodes of the AC traction system:

$$(\Gamma_{line})^T * I_{line} + (\Gamma_{w1})^T * I1_{trans} + (\Gamma_{w2})^T * I2_{trans} + (\Gamma_{train})^T * I_{train} = 0 \quad (4.7)$$

It can be noticed that first three equations ((4.1) and (4.3)) are related to the power transformer, while next two equations ((4.4) and (4.5)) describe AT.

An initial approximation to the voltage and current solutions is assigned. Thus, all nodes of the ground line have an initial voltage 0kV, while all nodes of the positive and negative feeders have an initial voltages +25kV and -25kV respectively. Therefore, voltage across the primary side of the power transformer is initially equal to the source voltage.

Function "fsolve" of MATLAB software solves a problem, defined by $F(X) = 0$ for X, where $F(X)$ is a function that returns a vector value, and X is a vector or a matrix of unknowns. Solver syntax for the developed scrip looks like this [X,fval,exitflag,output] = fsolve(@myfun,X0,Opt), where @myfun is the set of equation, written in the form, required by the solver, X0 is the initial values for the solution, and Opt is a list of the optimisation options.

4.2   Backward/Forward Sweep Technique

It is common to use BFS to solve the radial traction systems. An application of this strategy has been described in [21, 6, 7, 31, 29]. In general, BFS follows the next steps:

1. Assign voltage values.

2. Backward sweep: calculation of the currents based on the assigned voltages.

3. Forward sweep: updating voltage values, based on calculated current and for the next iteration using them as the initial values.

4. Repeat steps 2 and 3 until the convergence is performed.

In order to implement BFS in the developed script, two additional matrices have been built. Square matrix A has the number of rows that is equal to the number of equations, which describes the system (see equations (4.1) - (4.7)), and the number of columns that is equal to the number of unknowns (all nodes voltages and line currents, currents through the primary and secondary sides of the power transformer, currents though the ATs, and voltage across the primary side of the power transformer).

Each unknown has its own index that corresponds to the number of the column of the matrix A. Square matrix A comprises those known values (source voltage, impedances, incidence matrices Γ and turn ratios), which in the equations (4.1) - (4.7) have an interaction with the unknowns. Each of these known terms places to the row that corresponds to the number of equation, from which it is taken, and to the column that corresponds to the unknown, which is either multiplied or divided by this value. A single column vector B has the same number of rows that matrix A. Vector B is filled with known values, which in the equations (4.1) - (4.7) do not have any additional unknown terms. These known values places to the row that corresponds to the number of equation, from which they are taken. The detailed explanation of how these two matrixes A and B have been built will be given further together with its' constructions.

Equations (4.2)-(4.7) have been slightly updated to the form suitable to build matrixes A and B. Equation (4.1), KVL of the primary side of the power transformer, remains constant due to the fact that all changes occur in the AC traction system and not in the high voltage feeding network. Thus, the matrixes are build, based on following equations:

1. KVL of the feeding system (primary side of the power transformer):

$$V_{src} - I_p * Z_p - V_p = 0 \tag{4.8}$$

2. The relation between the primary side of the power transformer and the first winding group of the secondary side of the power transformer can be represented through a power transformer incidence matrix Γ_{w1}^{PT} :

$$V_p + \Gamma_{w1}^{PT} = 0 \tag{4.9}$$

where a power transformer incidence matrix Γ_{w1}^{PT} has been calculated as following:

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$$\Gamma_{w1}^{PT} = -N_{p1}V1^{PT} \tag{4.10}$$

where $V1^{PT}$ is a voltage of the first winding group of the power transformer, while N_{p1} is the number of turns of the power transformer secondary winding (first group). According to the numeration criteria, which has been described in detail in the previous chapter, $V1^{PT}$ is the first term of the column vector V1:

$$V1 = \Gamma_{w1}V_N \tag{4.11}$$

V_N is a column vector that includes the voltages in the nodes of the traction network. Only the first term of the vector V1 is taken into account for the power transformer incidence matrix Γ_{w1}^{PT}.

3. The relation between the currents of the primary and secondary sides of the power transformer, taking into account the first and the second winding groups of the secondary side of the power transformer:

$$\frac{1}{N_{p1}}I1^{PT} + \frac{1}{N_{p2}}I2^{PT} + I_p = 0 \tag{4.12}$$

where $I1^{PT}$ and $I2^{PT}$ are the first terms of the vectors $I1_{trans}$ and $I2_{trans}$, and correspond to the currents through the first and second winding groups of the secondary side of the power transformer.

4. In the same way as equation (4.11), the voltages of the second winding group of the secondary side of the power transformer and ATs can be obtained as:

$$V2 = \Gamma_{w2}V_N \tag{4.13}$$

It should be mentioned that for the following equation, which describes the relation between the voltages of winding groups of the secondary side of the ATs, all terms of the vectors V1 and V2 are used, except first ones, because they

are related to the power transformer. Thus, equation (4.4) can be represented through the AT voltage incidence matrixes:

$$\Gamma_V^{AT} = \Gamma_{w2} - N_{trans}\Gamma_{w1} \tag{4.14}$$

where N_{trans} is a single column vector that includes turn ratio of the ATs.

5. ATs' current equation follows the same logic as the ATs' voltage equation, taking into account all terms of the vectors $I1_{trans}$ and $I2_{trans}$, except the first ones, which correspond to the currents of the secondary side of the power transformer. Thus, equation (4.5) can be updated with help of AT current incidence matrixes:

$$\Gamma_{I1}^{AT} + \Gamma_{I2}^{AT} = 0 \tag{4.15}$$

where Γ_{I1}^{AT} is an identity matrix with number of rows and columns equal to the number of ATs, while Γ_{I2}^{AT} has the same structure as Γ_{I1}^{AT} with the main diagonal filled with values of N_{trans}. [1]

6. KVL for all lines of the AC traction system:

$$\Gamma_{line} * V_N - Z * I_{line} = 0 \tag{4.16}$$

where Z is the impedance vector that includes the impedances of the lines in order, which has been set up during the numeration procedure.

7. KCL for all nodes of the AC traction system has rewritten as following:

$$(\Gamma_{train})^T * I_{train} = -(\Gamma_{line})^T * I_{line} - (\Gamma_{w1})^T * I1_{trans} - (\Gamma_{w2})^T * I2_{trans} \tag{4.17}$$

As it was mentioned before, matrix A is filled with those known terms that in the equations (4.8), (4.9), (4.12), (4.14)-(4.17) either multiplied or divided by any unknown terms. Meantime, matrix B is filled with those known terms (from the

---

[1]This comes from the relation between the first and second group of winding of the ATs.

same equations (4.8), (4.9), (4.12), (4.14)-(4.17)) that do not have any additional unknown terms.

The constructions of matrixes A and B are represented in the expressions (4.18) and (4.19) respectively.

$$
A = \begin{pmatrix}
1 & Z_p & & & & & \\
1 & & & & \Gamma_{w1}^{PT} & & \\
& 1 & \dfrac{1}{N_{p1}} & \dfrac{1}{N_{p2}} & & & \\
& & & & \Gamma_V^{AT} & & \\
& \Gamma_{I1}^{AT} & \Gamma_{I2}^{AT} & & & & \\
& & & & \Gamma_{line} & -Z & \\
& (-\Gamma_{w1})^T & (-\Gamma_{w2})^T & & & (-\Gamma_{line})^T &
\end{pmatrix} \qquad (4.18)
$$

$$
B = \begin{bmatrix}
V_{src} \\
\\
\\
\\
\\
(\Gamma_{train})^T
\end{bmatrix} \qquad (4.19)
$$

Summarizing all discussed above, suggested implementation of the BFS method for solving the power flow for the AC traction systems, allows to write all equations

in a compact matrix form. This helps to reduce simulation time and, eventually, will allow to increase the size of the systems that need to be obtained.

In the developed script the BFS follows the next steps:

1. Assign the voltage for all nodes of the AC network, i.e. assign values to the vector V_N : 0kV for all nodes of the ground line, +25kV for all nodes of the positive feeder and -25kV for all nodes of the negative feeder.

2. Calculate train voltage:

$$V_{train} = \Gamma_{train} \times V_N \tag{4.20}$$

3. Calculate train current:

$$I_{train} = (S_{train}/V_{train})^* \tag{4.21}$$

where S represents the apparent power of the train.

4. Update a single column vector B with the obtained values of trains' currents, because after the step 3 train currents are considered as known values. In the vector B trains' currents will be placed in a row that corresponds to the KCL:

$$B(Index\_KCL) = \Gamma_{train}^T * I_{train} \tag{4.22}$$

5. Solve a system of a linear equations

$$A * X = B \tag{4.23}$$

with respect to a single column vector of unknowns X:

$$X = A^{-1} * B \tag{4.24}$$

According to the chosen order for the numeration of unknowns, a single column

vector of unknowns X has been build as following:

$$X = \begin{bmatrix} V_p \\ I_p \\ I1^{PT} \\ I1^{AT} \\ I2^{PT} \\ I2^{AT} \\ V_N \\ I_{line} \end{bmatrix} \qquad (4.25)$$

6. Update a node voltage vector V_N with the new calculated values of the node voltages from the vector X.

7. Repeat steps 2 - 7 until the convergence is performed.

Figure 4-1 illustrates the flowchart of the load flow calculations done by BFS.

A backward sweep estimates train currents, while forward sweep updates the train voltages.

The next chapter will cover BFS steps, mentioned above, in detail, solving as an example small test case (only 1 train and 2 cells in the system).
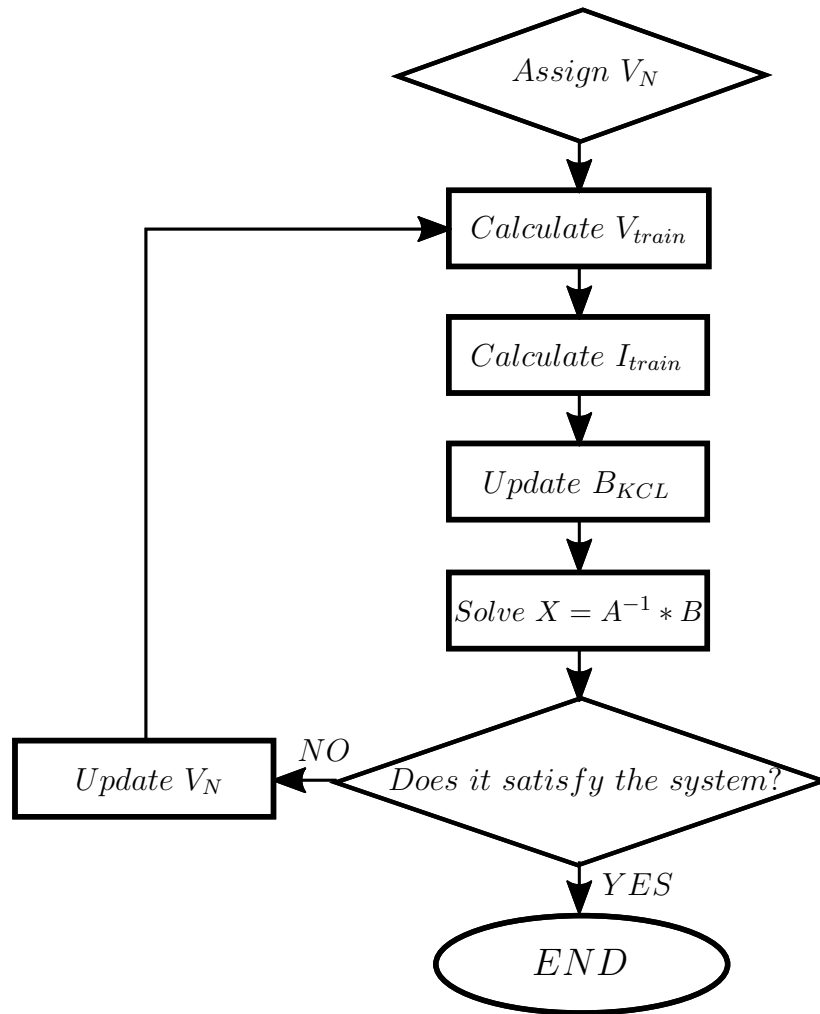
Figure 4-1: Flowchart of the BFS to solve the load flows in the AC traction system

Chapter 5

Test and Validation of the Results

The simulator that has been developed for this thesis, based on the application of the two different techniques to solve the power flow (a derivative method with a help of the function "fsolve" of MATLAB software and a backward/forward sweep method), which were described in the Chapter 4.

The initial conditions are the same for both methods:

- There are no losses on the primary side of the power transformer, thus the voltage across the primary side of the power transformer is equal to the source voltage.

- The voltage of the positive feeder, ground rail and the negative feeder are equal to 25kV, 0kV and -25kV respectively.

- All branch currents are equal to zero.

The simulator was developed using MATLAB. All tests have been done in a laptop with a processor Intel Core i5 @ 2.5GHz.

To analyse the performance of the simulator, various tests with the different network parameters have been done. The source voltage is 400kV in all cases for all tests.

First test has been run for the small system (2 cells and 1 train), represented on the Figure 5-1.
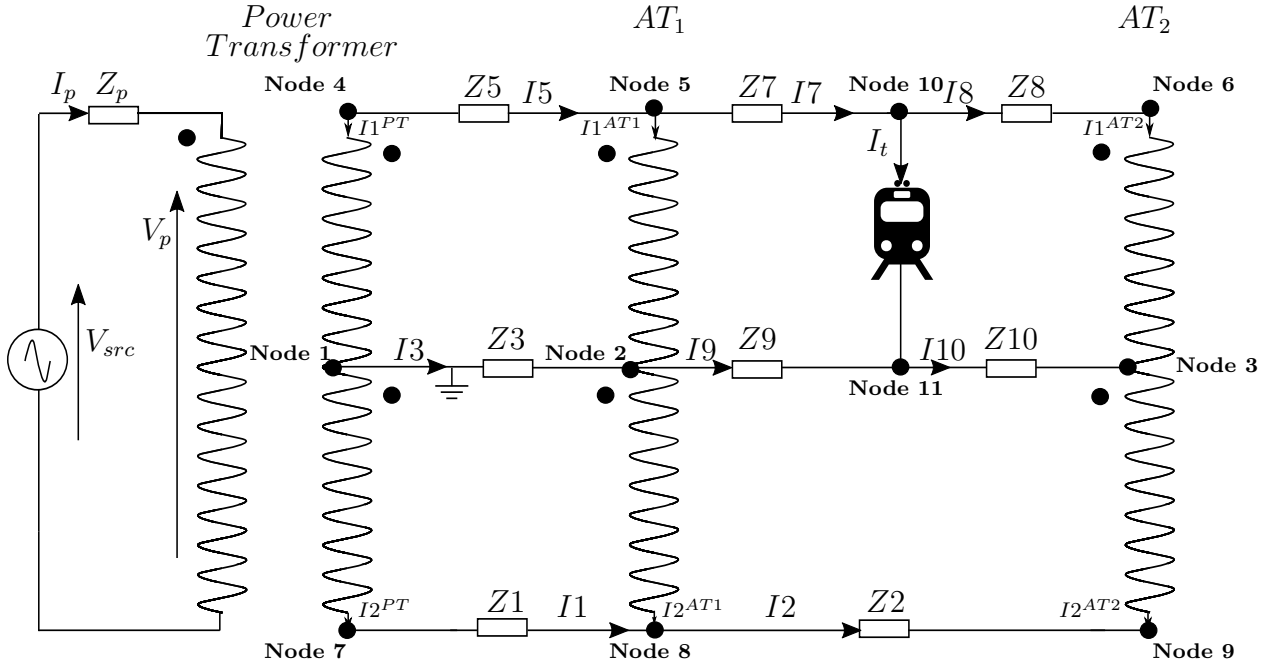
Figure 5-1: Test 1 network topology

Each cell from the Figure 5-1 has a length of 40 km. Resistance and the inductance of each line of the cell are assumed to be to be 93.63 mΩ/km and 0.92 mH [22]. The turn ratio of all ATs is 1 and the turn ratio of the power transformer is 16. The train is located in the cell number 2 at the distance 0.1576 (p.u.) from the beginning of the line and its active and reactive power of the train are 7.6617 MW and 2.5 MW respectively.

The network, illustrated on the Figure 5-1 will be used as an example to explain the BFS approach. The represented network has been solved in the following way:

1. For the first iteration voltage for all nodes of the AC network has been assigned, i.e. next values of the vector V_N : 0kV for all nodes of the ground line, +25kV for all nodes of the positive feeder and -25kV for all nodes of the negative feeder.

2. Train voltage, obtained from the equation (4.20), is equal to 25kV.

3. Since power is given and train voltage is calculated, it is possible to find train current, using equation (4.21) I_{train} = 304.71 - 100i

4. Next step is to update a single column vector B with the obtained values of trains'

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currents, because after the step 3 train currents are considered as known values. In the vector B trains' currents will be placed in a row that corresponds to the KCL:

$$
B =
\begin{bmatrix}
400 * 10^3 + 0i \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
304.71 - 100i \\
-304.71 + 100i
\end{bmatrix}
\tag{5.1}
$$

5. Thus, all unknowns (voltage across the primary side of the power transformer,

currents through the primary and secondary sides of the power transformer, currents though the ATs, all nodes voltages and line currents) will get their first values:

$$X = \begin{bmatrix} 399.64 * 10^3 - 0.49 * 10^3 i \\ 0.02 * 10^3 - 0.01 * 10^3 i \\ -0.28 * 10^3 + 0.09 * 10^3 i \\ -0.02 * 10^3 + 0.01 * 10^3 i \\ -0 + 0i \\ -0.02 * 10^3 + 0.01 * 10^3 i \\ 0.02 * 10^3 - 0.01 * 10^3 i \\ 0 - 0i \\ 0.05 * 10^3 + 0.07 * 10^3 i \\ 0.05 * 10^3 + 0.07 * 10^3 i \\ 24.98 * 10^3 - 0.03 * 10^3 i \\ 24.92 * 10^3 - 0.1 * 10^3 i \\ 24.92 * 10^3 - 0.11 * 10^3 i \\ -24.98 * 10^3 + 0.03 * 10^3 i \\ -24.82 * 10^3 + 0.24 * 10^3 i \\ -24.82 * 10^3 + 0.25 * 10^3 i \\ 24.92 * 10^3 - 0.11 * 10^3 i \\ 0.05 * 10^3 + 0.07 * 10^3 i \\ -0.02 * 10^3 + 0.01 * 10^3 i \\ -0 + 0i \\ 0.28 * 10^3 - 0.09 * 10^3 i \\ 0.3 * 10^3 - 0.1 * 10^3 i \\ -0 + 0i \\ -0.26 * 10^3 + 0.09 * 10^3 i \\ -0.3 * 10^3 + 0.1 * 10^3 i \\ 0 - 0i \end{bmatrix} \qquad (5.2)$$

6. Therefore, a node voltage vector V_N can be updated with the new calculated

values of the node voltages from the vector X:

$$
V_N =
\begin{bmatrix}
0.05 * 10^3 + 0.07 * 10^3 i \\
0.05 * 10^3 + 0.07 * 10^3 i \\
24.98 * 10^3 - 0.03 * 10^3 i \\
24.92 * 10^3 - 0.1 * 10^3 i \\
24.92 * 10^3 - 0.11 * 10^3 i \\
-24.98 * 10^3 + 0.03 * 10^3 i \\
-24.82 * 10^3 + 0.24 * 10^3 i \\
-24.82 * 10^3 + 0.25 * 10^3 i \\
24.92 * 10^3 - 0.11 * 10^3 i \\
0.05 * 10^3 + 0.07 * 10^3 i
\end{bmatrix}
\tag{5.3}
$$

7. Steps 2 - 7 will be repeated until the convergence will be performed.

It took 6 iterations to obtain the system, taking for each of them only 0.1288 ms. The calculated values are given in the Table 5.1.

As it can be noticed from the time results of Test 1, BFS requires relatively small time to obtain network. The total calculation time is less than 1 ms.

In Test 2 the simulated system has been increased. Figure 5-2 illustrates the network topology for Test 2. In this particular case the network consists 3 cells, each of them has a length of 40 km. Resistance and the inductance of each line of the cell are assumed to be to be 93.63 mΩ/km and 0.92 mH [22]. The turn ratio of all ATs is 1 and the turn ratio of the power transformer is 16. There are 6 trains in the system, which locations (in which cell and at what distance) and power are generated randomly. The active power can vary from -5 MW up to 8 MW, while reactive power remains constant 2.5 MW. Table 5.2 summarises all data that has been generated for the trains. The first column shows the index of the train, the second column represents the number of the cells, where trains are located, while the third column shows trains' distance. The forth and fifth columns provide an information about active and reactive power respectively.

Table 5.1: Test 1. Results

Unknown	Calculated Value
V_p	$399.64*10^3 - 0.49*10^3 i$
I_p	19.0987 - 6.4181i
$I1^{PT}$	-285.21 + 95.84i
$I1^{AT1}$	-19.85 + 6.67i
$I1^{AT2}$	-0.52 + 0.18i
$I2^{PT}$	-20.372 + 6.846i
$I2^{AT1}$	19.8506 - 6.6708i
$I2^{AT2}$	0.5214 - 0.1752i
V_{Node1}	0 + 0i
V_{Node2}	$0.051*10^3 + 0.068*10^3 i$
V_{Node3}	$0.052*10^3 + 0.07*10^3 i$
V_{Node4}	$24.997*10^3 - 0.031*10^3 i$
V_{Node5}	$24.923*10^3 - 0.104*10^3 i$
V_{Node6}	$24.921*10^3 - 0.106*10^3 i$
V_{Node7}	$-24.977*10^3 + 0.031*10^3 i$
V_{Node8}	$-24.822*10^3 + 0.241*10^3 i$
V_{Node9}	$-24.818*10^3 + 0.246*10^3 i$
V_{Node10}	$24.921*10^3 - 0.106*10^3 i$
V_{Node11}	$0.052*10^3 + 0.07*10^3 i$
I1	-20.37 + 6.85i
I2	-0.52 + 0.18i
I5	285.21 - 95.84i
I7	305.06 - 102.51i
I8	-0.52 + 0.18i
I3	-264.84 + 89i
I9	-304.54 + 102.34i
I10	1.04 - 0.35i

Figure A-1 and Figure A-2, given in the Appendix, represent the results of the case, described by Figure 5-2 and Table 5.2. Figure A-1 shows the currents in the system, obtained with help of the derivative method, while Figure A-2 illustrates the currents in the system, calculated by applying the BFS technique.

As it can be noticed from the Figure A-1 and Figure A-2, the results, obtained by the both methods are the same. Moreover, the error in static nodes' voltage calculations for the both algorithms is very small and equal 2.458×10^{-9}. Table 5.3 illustrates the comparison of the both techniques, derivative-based and BFS-based in terms of time and iterations spending to solve the cases.
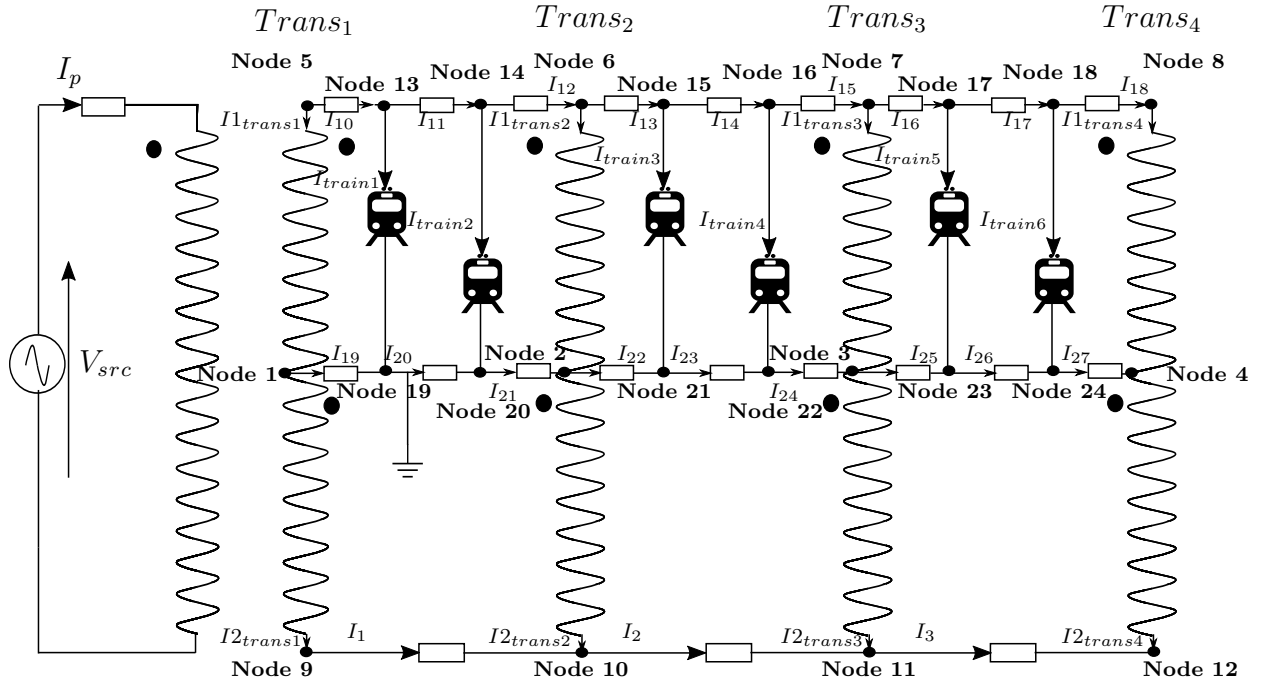
Figure 5-2: Test 2 network topology

Table 5.2: Train Data

Train Index	Cell Number	Length (p.u.)	Active Power (MW)	Reactive Power (MW)
1	3	0.7655	4.2217	2.5
2	1	0.7952	4.8109	2.5
3	3	0.1869	- 1.4117	2.5
4	1	0.4898	3.8361	2.5
5	2	0.4456	3.5163	2.5
6	2	0.6463	-2.8860	2.5

It can be noticed from the Table 5.3 the BFS method is much faster and invests almost twice less iterations than the derivative-based one.

Next step to test the simulator is to increase the number of cells and the number of trains in the system and run the test for more different cases. The following tested system has 10 cells, each of them has a length of 40 km. Resistance and inductance of each line of the cell are assumed to be 93.63 mΩ/km and 0.92 mH [22]. The turn ratio of all ATs is 1 and the turn ratio of the power transformer is 16. There are 10 trains in the system, which locations (in which cell and at what distance) and power are

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Table 5.3: Comparison of the two methods for the topology, represented in Figure 5-2

	Derivative-based method	Backward/ Forward Sweep method
Total time (ms)	332.7	2.23
Number of iterations	13	6
Time per iteration (ms)	25.6	0.3725

generated randomly. The power varies from -5 MW up to 8 MW. In total 10^3 cases with random train data have been generated to check the work of the simulator.

Figure 5-3 represents the voltage error in all static nodes between the derivative-based algorithm and the BFS-based one, considering 10^3 cases. It can be noticed, the maximum error is lower than 2.5×10^{-5}.
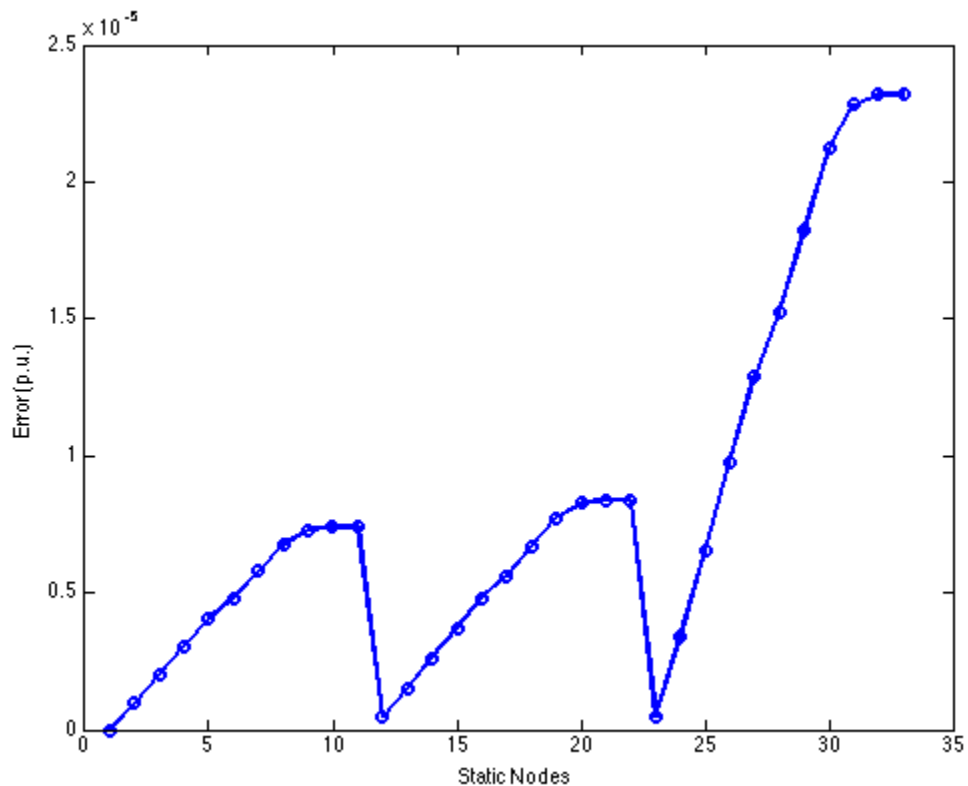

Figure 5-3: Static node voltage error of derivative-based method relative to BFS-based results

The Table 5.4 illustrates the comparison of the both techniques, derivative-based

and BFS-based in terms of time and iterations spending to solve the cases.

Table 5.4: Summary of the comparison between two methods

	Derivative-based method	Backward/ Forward Sweep method
Minimum total time (ms)	544.79	4.69
Mean total time (ms)	498.65	5.82
Maximum total time (ms)	1461.55	42.89
Minimum number of iterations	14	8
Mean number of iterations	11.78	8.35
Maximum number of iterations	18	55
Minimum time per iteration (ms)	389.14	0.587
Mean time per iteration (ms)	42.34	0.696
Maximum time per iteration (ms)	81.19	0.779

In order to verify the results of the calculations, the values of static nodes' voltage, obtained by two different methods, are compared and the average error is computed. Figure 5-4 shows the cumulative time that is required for all cases to be solved. It can be observed that the BFS methods takes less time to solve each case.

Based on results, provided by Figures 5-4 and Table 5.4, it can be observed that the BFS method is much faster than the derivative-based one. The total mean time for solving one instant with the BFS is 5.82 ms, while the NR method requires almost 0.5 s. Besides, the mean number of iterations for the derivative-based method is 11.78, while the BFS technique needs only around 8.

The values of the voltage of the static nodes are almost the same, giving the error in 8.2×10^{-5}.

Thereby, BFS technique shows a good performance in solving AC traction networks due to its compact matrix form. Based on the obtained results, the application of the BFS algorithm for the further development of the simulator for a bivoltage AC traction system would be worthwhile.
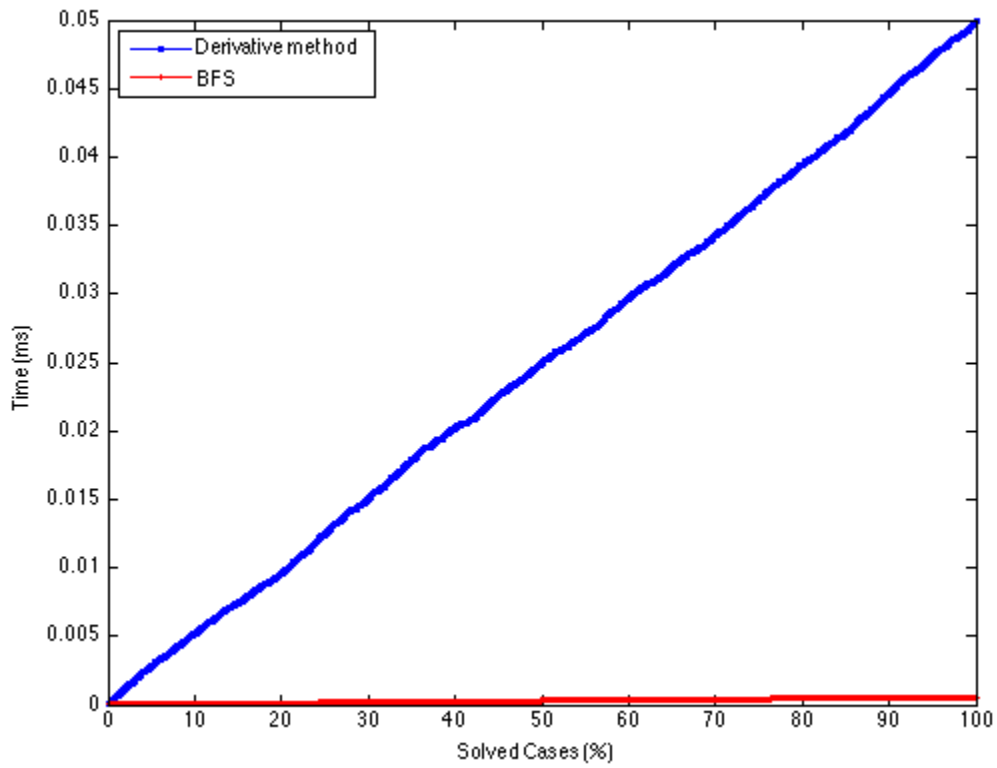
Figure 5-4: Derivative-based method versus BFS. Variation of the solved cases with time

Chapter 6

Conclusions

Railways have taken a significant part of the transportation system in the modern world. Economy, comfort, less risk factors and a lot of the other advantages are the reason for this. Together with the grown tendency towards the use of clean energy sources, it becomes important to update an existing railway systems with help of the developed tools, simulators and software products.

The objective of this work was to develop a simulator and to find an efficient algorithm to solve a power flow in a 2x25kV bivoltage traction system. The objective has been achieved through the implementation of the Newton-Raphson and Backward/Forward Sweep algorithms.

The main advantage of the developed simulator is the compact matrix formulation of the performed methods. A suggested BFS power flow algorithm is equivalent to the standard derivative-based one in terms of accuracy. However, it is faster and more robust. These benefits of the BFS method proves its efficiency to solve the power flow for the high speed railways.

Aside from further testing of the simulator and the algorithms, there are several modifications, which can be explored further. They contain:

- Extend the network modelling.

- Extend the train modelling.

- Explore the possibilities for improved contingency analysis.

- Implement for the industry use.

There are also some improvements that can be made. In particular, with the proper data form the railway company, it may be possible to obtain more accurate results that will be closer to the real life scenarios.

One of the most obvious ways of the future work based on this thesis is to convert the MATLAB program, used to build this simulator, to a C++ program, suitable for everyday use in industry. A developed script could potentially be a very useful tool for the railway planning and operation.
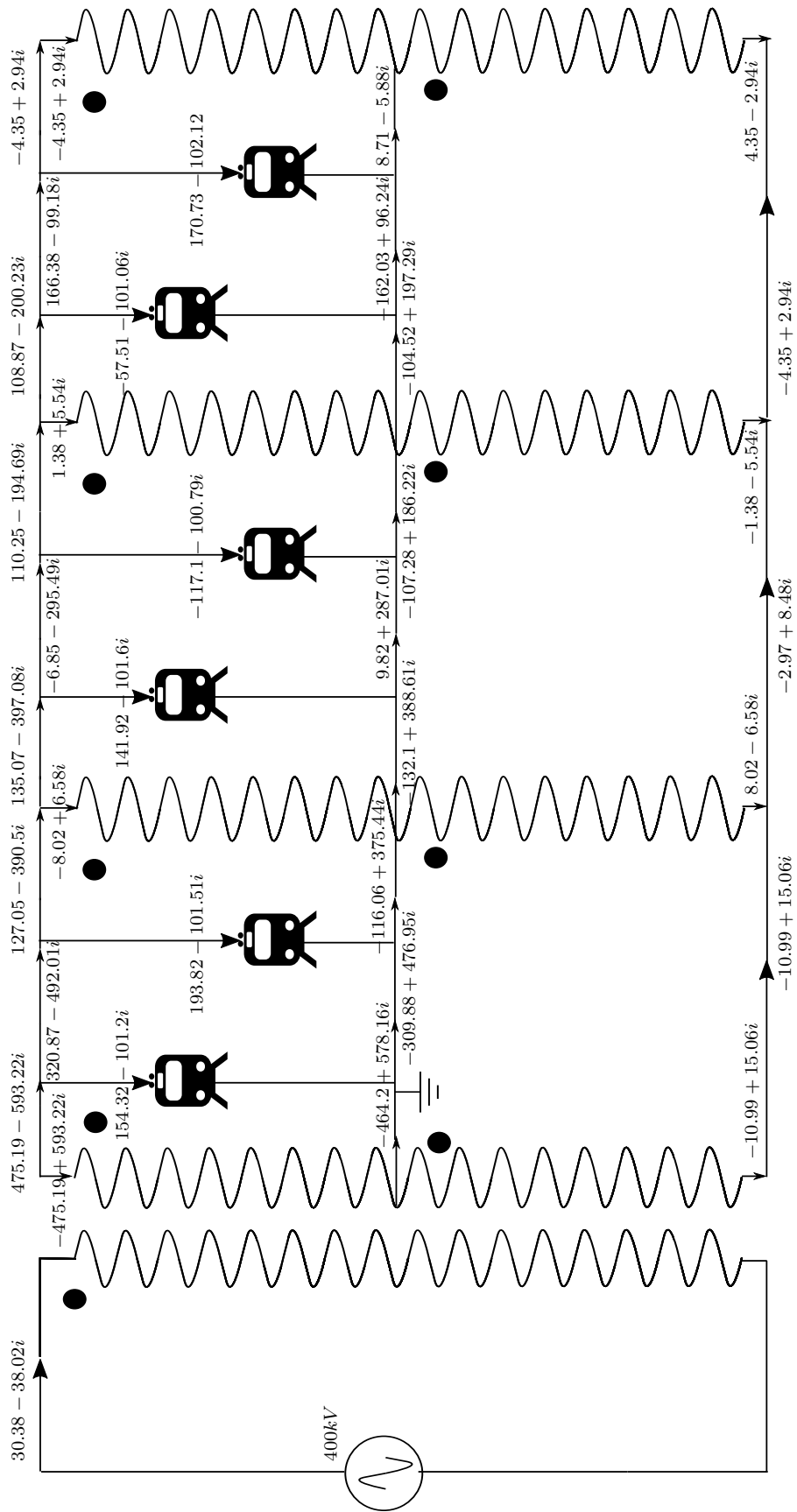
Appendix A

Figures

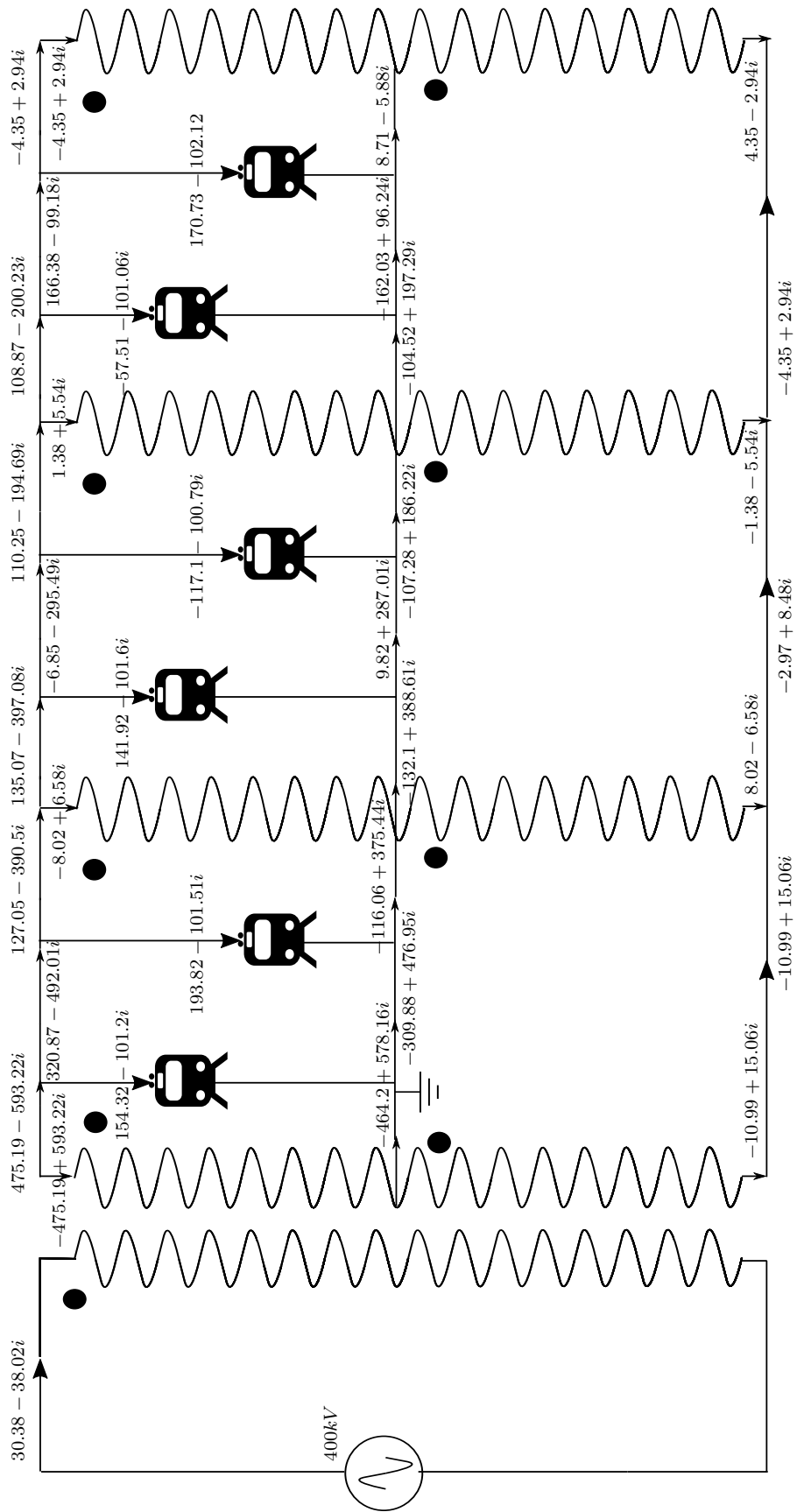Figure A-1: Current distribution in an AC bivoltage traction system, obtained by derivative method

Figure A-2: Current distribution in an AC bivoltage traction system, obtained by Backward/Forward Sweep method

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