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Stress Estimation in Structures Using Operational Modal Analysis

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ABSTRACT: In the fatigue design and life prediction of structures, the stresses at each point are one of the main sources of uncertainty. This is due to the difficulty of estimating the stiffness, mass and damping properties with accuracy, as well as the use of simplified load models. In this work, a methodology to improve the fatigue design reliability using operational modal analysis to estimate the stress time-histories is presented. The modal parameters of the structure, estimated by operational modal analysis and the responses, usually acceleration, corresponding to several degrees of freedom, are needed. The methodology was applied to a steel S275 cantilever beam. The stresses estimated with the proposed method are compared with those obtained from the experimental strains recorded using strain gages attached to some points of the beam.

1 INTRODUCTION

In fatigue design of structures and mechanical components, the main sources of uncertainty are the fatigue material characterization and the real stress time histories present in the most stressed points.

The fatigue material characterization can be reasonably improved using an optimized test strategy and analysing the results with appropriated statistical fatigue models (Schijve 2001).

As regards the stress histories, the uncertainty is considerable high. On one hand, the dynamic properties (stiffness, mass and damping) of the structure can only be estimated in an approximate way. On the other hand, simplified load models are commonly used in fatigue design, which not represent, with the needed accuracy, the load characteristics (variable amplitude, random nature, frequency bandwidth, sequence effect, etc.)

In recent years, operational modal analysis (OMA) has become a powerful tool that can be utilized in many civil (Cantieni 2005, Cunha et al. 2005) and mechanical applications (Møller et al. 2001). OMA makes use of the natural or operating loads to excite the structure, which can be considered an important advantage in large structures (Cantieni 2005, Cunha et al. 2005), where the use of artificial excitation devices may be expensive or impractical. Another advantage is that the modal tests can be performed with the structure in operation subject to natural or operational loads. Thus, the operational responses of the structure are measured and used to perform an OMA identification (Brincker et al. 2003).

In this work, a methodology to estimate stresses in any arbitrary point of the structure, which combines a numerical model and operational modal analysis, is presented. Initially, a finite element model (FEM) of the structure is assembled. Then, the numerical model is updated using the modal parameters estimated by operational modal analysis. Finally, the stresses are estimated using the updated numerical model and the experimental displacements measured at several discrete points of the structure.

2 STRESS ESTIMATION METHOD

The methodology proposed in this paper is applicable to beam structures. The estimation process is detailed below.

2.1 Theory

For an Euler-Bernoulli beam, see Fig. 1, the bending moment and the curvature are related by the equation:

$$EI_z \frac{d^2 y}{d^2 x} = M_z \quad (1)$$

where E is the Young's modulus, I_z is the second moment of the cross section about z axis, y is the vertical displacement and M_z is the bending moment.

The stresses can be calculated by means of the Navier's Law equation:

$$\sigma(x) = \frac{M_z}{I_z} h \quad (2)$$

where h is the beam thickness.

If Eqs. (1) and (2) are combined, a relation between the stress and the curvature is obtained by:

$$\sigma(x) = -E h \frac{d^2 y}{d^2 x} \quad (3)$$

If a finite element model is used, the displacement in any arbitrary point of the element (see Fig. 2) is obtained as (Clough and Penzien 1993):

$$y(x) = N_e(x) u_e \quad (4)$$

where $N_e(x)$ and u_e are vectors containing the element displacement functions and the nodal displacements, respectively.

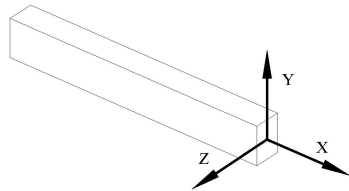


Figure 1: Example Of Euler-Bernoulli beam.

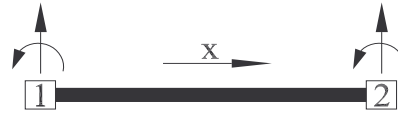


Figure 2. Beam element, two nodes.

If Eq. (4) is combined with Eq. (3), the following expression is inferred:

$$\sigma(x,t) = -E N_e''(x) u_e(t) h \quad (5)$$

If mode-superposition is used (Clough and Penzien 1993), the vector $u_e(t)$, corresponding to each element, can be expressed in terms of mode shapes, Φ_e , and the modal coordinates, $q(t)$, as follows:

$$u_e(t) = \Phi_e q(t) \quad (6)$$

Finally, if Eq. 6 is substituted in Eq. 5, the stresses in any point of the beam element can be calculated by means of the expression:

$$\sigma(x,t) = -E N_e''(x) \Phi_e q(t) h \quad (7)$$

2.2 Finite Element Model

The first stage of the proposed method is to assemble a numerical model. In this work, a cantilever beam is used in the tests, which is modelled by finite linear elements with two DOF's per node.

2.3 Modal Parameters

A modal identification is performed to obtain the natural frequencies, f_{exp} , and mode shapes, Φ_{exp} , of the structure. The identification may be performed by e.g. Stochastic Subspace Identification (SSI) (Van Overschee 1996) or Frequency Domain Decomposition (FDD) (Brincker et al. 2001). The FDD is used in this paper as implemented in the ARTeMIS Extractor software. The FDD is based on calculation of Spectral Density Matrices of the measured data series by discrete Fourier transformation. For each frequency line the Spectral Density Matrix is decomposed into auto spectral functions corresponding to a single degree of freedom system (SDOF).

2.4 Scaling Factors

An important disadvantage of OMA is that not all modal parameters can be estimated (Brincker and Andersen 2003, Aenlle et al. 2007)]. Since the forces are unknown, the mode shapes can not be mass normalized and only the un-scaled mode shapes can be obtained for each mode. A new approach (Brincker and Andersen 2003, Aenlle et al. 2007) has been published based on modifying the dynamic behaviour of the structure adding masses in the points of the structure where the mode shapes are known and performing repeated modal tests on both the original and the perturbed structure. From the modal parameters of both structures, the scaling factors can be estimated by:

$$\alpha = \sqrt{\frac{\omega_0^2 - \omega_1^2}{\omega_1^2 \cdot \Psi^T \Delta M \Psi}} \quad (8)$$

where ω_0 y ω_1 are the natural frequencies of the unmodified and modified structure, respectively, Ψ are the un-scaled modes shapes and ΔM is the mass change matrix.

2.5 Model Up-dating and Modal Expansion of the Mode Shapes

In this stage, a finite element model is assembled and updated using the experimental modal parameters estimated with OMA (Friswell and Mottershead 1995).

After updating, a transformation matrix T is obtained by:

$$\Phi_{\text{exp}}^m = \Phi_{FE}^m T \quad (9)$$

where subscripts 'exp' and 'FE' indicates experimental and numerical mode shapes, respectively, and superscript 'm' indicates measured DOF's. In expression (9), the experimental mode shapes are assumed to be a linear combination of the numerical mode shapes (Friswell and Mottershead 1995).

Applying the pseudo inverse ($^+$) to Eq. (9), matrix T is obtained as:

$$T = \Phi_{FE}^{m+} \Phi_{\text{exp}}^m \quad (10)$$

Then, the experimental mode shapes are expanded (Friswell and Mottershead 1995) to the unmeasured degrees of freedom by the expression:

$$\Phi_{\text{exp}}^{um} = \Phi_{FE}^{um} T \quad (11)$$

where superscript 'um' indicates unmeasured DOF's.

It must be emphasized that only the mode shapes need to be updated in this stage. The information corresponding to the natural frequencies and damping is included in the modal coordinates.

2.6 Modal Coordinates

The experimental displacements modal coordinates, $q_{\text{exp}}(t)$, are calculated from the measured acceleration, $\ddot{u}_{\text{exp}}(t)$. The acceleration modal coordinates, $\ddot{q}_{\text{exp}}(t)$, can be obtained by means of the expression:

$$\ddot{q}_{\text{exp}}(t) = \Phi_{\text{exp}}^{-1} \ddot{u}_{\text{exp}}(t) \quad (12)$$

where $\ddot{u}_{\text{exp}}(t)$ is the measured acceleration and Φ_{exp} is the experimental mode shape matrix. Then, the displacement coordinates, $q(t)$, are estimated by a double integration of $\ddot{q}(t)$. In this work a double integration in the frequency domain is used as follows:

$$q_{\text{exp}}(\omega) = -\ddot{q}_{\text{exp}}(\omega) / \omega^2 \quad (13)$$

The corresponding modal coordinates in the time domain are obtained by inverse Fourier Transform.

2.7 Avoiding the use of Scaling Factors

The proposed method to estimate stresses requires that the mode shapes be mass normalised so that the scaling factors have to be estimated. However, the method can also be applied using un-scaled mode shapes. The scaled and the un-scaled mode shape are related by the equation:

$$\Phi = \alpha \Psi \quad (14)$$

where Φ is the scaled or mass normalised mode shape, Ψ is the un-scaled mode shape and α is the scaling factor. If Eq. (14) is substituted in Eq. (6), results:

$$u_e(t) = \alpha \Psi_e q(t) \quad (15)$$

or, alternatively:

$$u_e(t) = \Psi_e q^*(t) \quad (16)$$

where

$$q^*(t) = \alpha q(t) \quad (17)$$

is denoted here as scaled modal coordinate, which is estimated by means of the expression:

$$\ddot{q}_{\text{exp}}^*(t) = \Psi_{\text{exp}}^{-1} \ddot{u}_{\text{exp}}(t) \quad (18)$$

Finally, combining Eqs. (7) and (16), the stresses can be determined, using un-scaled mode shapes, by the equation:

$$\sigma(x, t) = -E N_e''(x) \Psi_e q^*(t) h \quad (19)$$

3 EXPERIMENTAL RESULTS

A steel cantilever beam is used to perform the tests (see Fig. 3). The beam is 1.875 m long, showing a 100x40x4 mm tube rectangular section. A numerical model of the beam was assembled using 16 linear elements with two nodes, eight of each correspond with the position of the accelerometers.

3.1 Operational modal analysis

The beam is excited moving a small saw blade forward and backward along the beam trying to apply a stationary broad banded load. The responses are measured using 8 accelerometers 4508B Brüel & Kjær, located as shown in Fig. 3 and recorded with a data acquisition card (National Instruments PCI4472) controlled by LabView. The tests are carried out at a sampling frequency of 1500 Hz during a period of approximately 4 minutes.

The modal parameters were identified by Enhanced Frequency Domain Decomposition (EFDD). The normalized singular values are represented in Fig. 4. The first five natural frequencies together with the scaling factors estimated by the mass change method [9, 10] are shown in Table 1. The scaling factors correspond to mode shapes normalised making the largest element in each vector equal to unity.

Table 1: Scaling factors and Natural frequencies

MODE	1	2	3	4	5
Scaling Factor	0.49	0.51	0.48	0.48	0.47
f_0 (Hz)	11.4	72.62	201.5	386.7	611.9

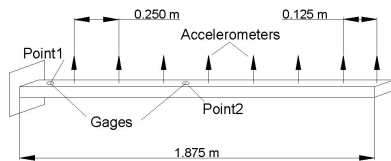


Figure 3. Accelerometers and gage configuration in testing.

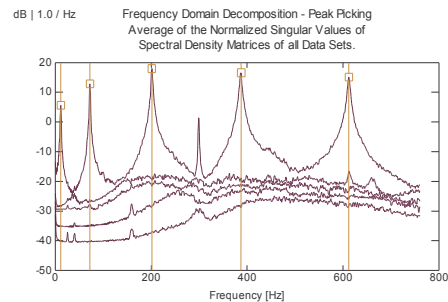


Figure 4. Modes identified for the beam structure by the EFDD technique.

3.2 Free Vibration Tests.

The method proposed in this paper was validated subjecting the cantilever beam to free vibration. The stress in points 1 & 2 (see Fig. 3) were measured using two 350 Ω strain gages, disposed in quarter bridge configuration and acquired with a gage conditioner and amplifier VISHAY 2100, and LabView.

The stress history corresponding to point 1, measured with a strain gage, is shown in Fig. 5a. As it can be seen, the stress is a damped sine, where the contribution of the first mode is predominant. Thus, only the first mode contributes significantly to the stress at this point. Fig. 5b presents a detail of Fig. 5a. The acceleration recorded on the top of the cantilever beam is presented in Fig. 6. The first two acceleration modal coordinates in the frequency domain, estimated using Eq. (12), are shown in Fig. 7.

To avoid the problems related to integration at low frequencies (only the frequencies in the range $0.02f_s < f < 0.1f_s$, where f_s is the sampling frequency, are treated accurately), a high pass band FIR filter at 8 Hz is applied to the acceleration modal coordinates.

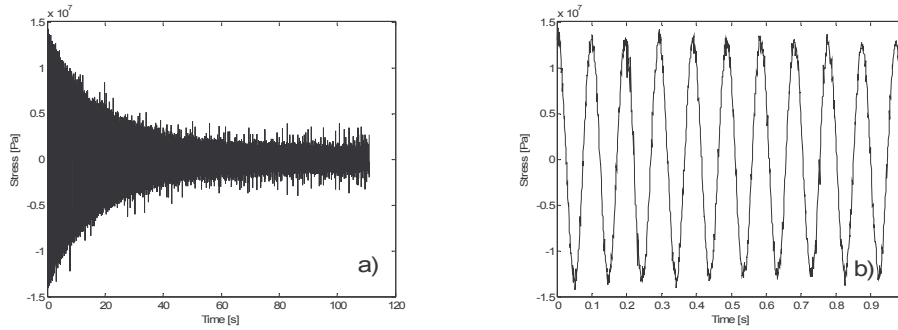


Figure 5: Stress measured at point 1.

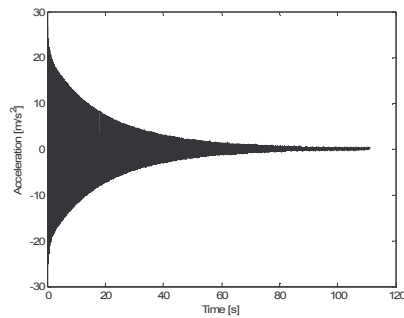


Figure 6. Acceleration of the top cantilever beam

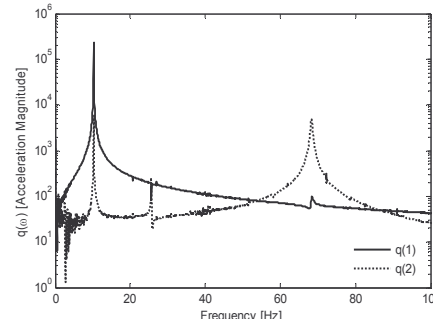


Figure 7. Acceleration modal coordinates. (Frequency Domain)

In a first stage, the numerical model was not updated. The estimated stresses in point 1 are shown in Fig. 8, where it can be observed that the proposed method overestimates considerably the stress at this point. The MAC between the experimental and numerical model is presented in Table 3a.

Then, the model was updated locating a rotational spring at the clamped of the beam. The MAC between the experimental and the updated numerical model is presented in Table 3b, from which is inferred that a good correlation was achieved

The stress estimated using the updated model is shown in Fig. 9. It can be concluded that the methodology used in this paper predicts quite well the stress in the beam.

Table 3: Modal Assurance Criterion (MAC)

	MEF (a)					Updated MEF (b)				
EXP	0,9986	0,1202	0,093	0,094	0,0794	0,9991	0,0826	0,0943	0,0849	0,0742
	0,0952	0,9921	0,14	0,0698	0,0967	0,1223	0,9976	0,0794	0,0842	0,0867
	0,1165	0,0687	0,9702	0,1731	0,0593	0,1055	0,1172	0,9971	0,0858	0,0788
	0,1053	0,1203	0,052	0,9527	0,211	0,1089	0,0956	0,1107	0,998	0,1019
	0,1063	0,0887	0,1273	0,0331	0,9207	0,1042	0,1016	0,0925	0,0888	0,9951

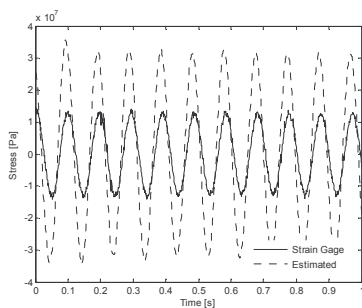


Figure 8. Comparison between estimated and measured stresses point 1.

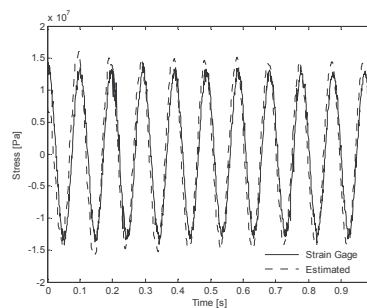


Figure 9. Comparison between estimated and measured stresses after model up-dating, point 1.

3.3 Impact tests

In this case, several hits were applied to the beam in random positions, using an impact hammer. The strain measured at point 1, using a strain gage, is presented in Fig. 10a, whereas the spectral density is shown in Fig. 10b.

The experimental and predicted stress at point 1 is presented in Fig. 11. Again, the proposed method predicts quite well the stress in this point of the beam.

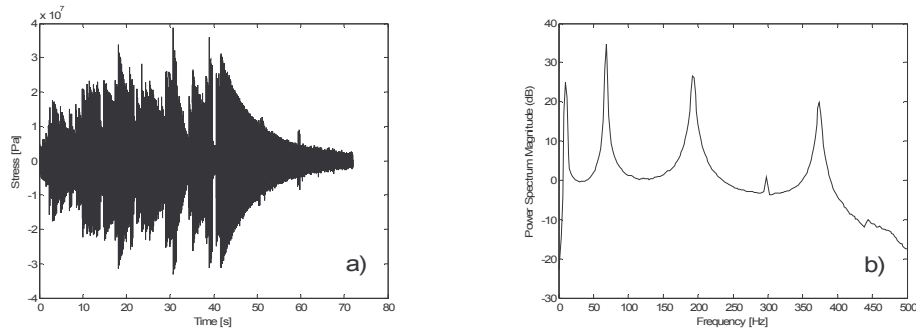


Figure 10. Stresses at point 1

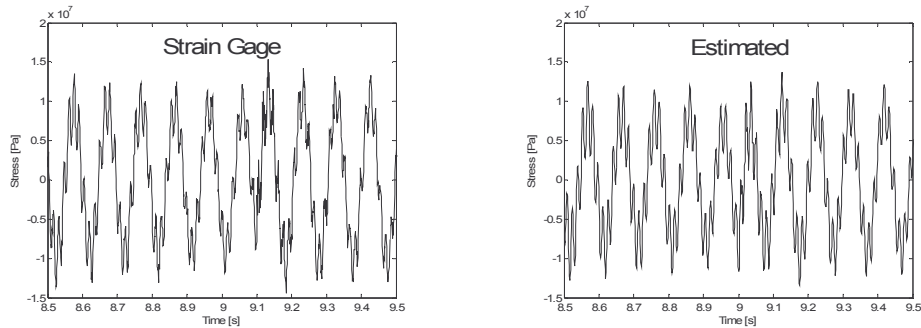


Figure 11: Stresses at point 1 for impact load

4 CONCLUSIONS

- A method to estimate stresses at any arbitrary point of a structure, which combines operational modal analysis and a finite element model, is proposed.
- The method is validated by several vibration tests carried out on a steel beam cantilever. Good results are obtained but the method requires performing a good updating of the numerical model.
- The method has many potential applications in fatigue design and remain life estimation of in-service structures.

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