New type IIB backgrounds and aspects of their field theory duals

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Abstract: In this paper we study aspects of geometries in Type IIA and Type IIB String theory and elaborate on their field theory dual pairs. The backgrounds are associated with reductions to Type IIA of solutions with G_2 holonomy in eleven dimensions. We classify these backgrounds according to their G-structure, perform a non-Abelian T-duality on them and find new Type IIB configurations presenting *dynamical* SU(2)-structure. We study some aspects of the associated field theories defined by these new backgrounds. Various technical details are clearly spelled out.

Contents

1 Introduction and general idea of this paper

The Maldacena conjecture [1–3] substantially changed the panorama of theoretical physics. In the last fourteen years, the area has been dominated by ideas tightly associated with gauge-Strings dualities. In most of the examples, the idea is to use dualities to study interesting aspects of quantum field theories (QFTs) which cannot be approached by perturbative techniques. A massive amount of work deals with theories with minimal SUSY in different number of dimensions. This lead to the discovery of new string backgrounds [4–6] that encode phenomena as diverse and nontrivial as confinement, breaking of global symmetries, presence of domain-wall like objects, diverse correlation functions, etc. In all these

cases the QFT is strongly coupled but the duality relates these nontrivial phenomena to semi classical calculations on the string theory side. This line of work evolved in many directions, with many applications to different branches of Theoretical Physics.

One of these directions is the construction of duals to field theories in four dimensions realized on the worldvolume of $D_{p>3}$ branes, where $(p-3)$ directions have been compactified on a small manifold. The compactification is (usually) performed in a way that preserves the smallest amount of SUSY. These field theories, are higher-dimensional in disguise; the whole construction is in spirit, similar to the Kaluza-Klein idea. In this paper, we will deal with one such example. We will consider the case in which $D6$ branes wrap a calibrated three-cycle inside the deformed conifold. Extensions of this case to different number of dimensions, different number of preserved SUSY, etc; have been studied. In particular, if these configurations in Type IIA string theory are lifted to eleven dimensions, the configurations become purely geometric, leading to the associated seven-dimensional spaces possesing G_2 holonomy. This line of research [7]–[14], was quite fertile, specially on the mathematical side where it lead to the construction of new metrics with G_2 holonomy. However, it did not give as many physically interesting result as its Type IIB counterparts [4–6]. In this work we present a family of those 'old' G_2 metrics, reduce the system to Type IIA and study some of its physical implications, making sharper the reasons for which they failed to capture some of the phenomena their Type IIB counterpart were able to calculate.

In parallel with these 'physically motivated' discoveries, a powerful line of research was developed, aiming to a complete classification of different backgrounds by specifying their G-structure [15–18]. In particular, in these four dimensional and SUSY preserving examples, it is possible to encode *all* the information about the background (BPS equations, metric, fluxes, calibrated sub-manifolds, etc), in a set of forms defined on the space 'external' to the Minkowski coordinates. Furthermore, the SU(2) and SU(3) structures typical of these backgrounds, their associated pure spinors and forms encode in subtle ways quite common operations in QFT [19–25]. In this paper we complement the above mentioned study of the type IIA backgrounds associated with the wrapped D6 branes and their precise description in terms of G-structures.

We also perform an operation on the geometry called non-Abelian T-duality. For a sample of old and recent research on the topic, see [26]–[40]. We generate new Type IIB solutions that preserve four supercharges; hence it is dual to a minimally SUSY 4-d QFT . We describe the result of the non-Abelian T-duality in terms of the generated G-structure. We believe, ours is one of the first few examples of *dynamical* SU(2)-structure in Type IIB. We will use the word 'dynamical' to denote the fact that the quantities $k_\perp, k_\parallel$ defined in eq. (3.20), are point dependent, changing value through out the internal manifold. We will propose a relation between the 'dynamical' character of the SU(2)-structure and the phenomena of confinement in the dual QFT.

The structure of the paper is the following. In section 2 — that contains a fair amount of review but also various original pieces, we will sumarise the eleven-dimensional and Type IIA supergravity solutions that will act as the 'seed backgrounds' for our non-Abelian T-duality generating technique. Their G-structure will be carefully discussed. We will also present the explicit numerical solutions to the BPS equations and clarify their asymptotics.

In section 3, the action of non-Abelian T-duality on the Type IIA backgrounds, the new generated solutions in Type IIB and a discussion of their G-structure will be spelled-out in detail. Different dual field theory aspects of the original and of the generated solution will be described in section 4. Finally, we close the paper in section 5 with some global remarks and proposing topics to be investigated. An appendix that discusses the delicate numerical study, complements the presentation.

2 Presentation of the background

We will start with the pure metric configuration in eleven-dimensions found in [10, 11]. We consider the family called $\mathcal{D}_7$. The notation we will adopt is that of [11]. We will have two sets of left invariant forms of SU(2),

$$
\begin{aligned}
\sigma_1 &= \cos\psi_1 \, d\theta + \sin\psi_1 \, \sin\theta \, d\varphi \, , & \Sigma_1 &= \cos\psi_2 \, d\tilde\theta + \sin\psi_2 \, \sin\tilde\theta \, d\tilde\varphi \\
\sigma_2 &= -\sin\psi_1 \, d\theta + \cos\psi_1 \, \sin\theta \, d\varphi \, , & \Sigma_2 &= -\sin\psi_2 \, d\tilde\theta + \cos\psi_2 \, \sin\tilde\theta \, d\tilde\varphi & (2.1) \\
\sigma_3 &= d\psi_1 + \cos\theta \, d\varphi \, , & \Sigma_3 &= d\psi_2 + \cos\tilde\theta \, d\tilde\varphi
\end{aligned}
$$

which satisfy the SU(2) algebras

$$
d\sigma_1 = -\sigma_2 \wedge \sigma_3 \ + \text{cyclic perms.}, \quad d\Sigma_1 = -\Sigma_2 \wedge \Sigma_3 \ + \text{cyclic perms.} \tag{2.2}
$$

The eleven dimensional metric is of the form $ds_{11}^2 = dx_{1,3}^2 + ds_7^2$, with

$$
ds_7^2 = dr^2 + a^2 \left[(\Sigma_1 + g \, \sigma_1)^2 + (\Sigma_2 + g \, \sigma_2)^2 \right] + b^2 \, (\sigma_1^2 + \sigma_2^2) + c^2 (\Sigma_3 + g_3 \, \sigma_3)^2 + f^2 \sigma_3^2 \, , \tag{2.3}
$$

where a, b, c, f, g and g_3 are functions only of the radial variable r. The six functions are not all independent, the relations

$$
g(r) = \frac{-a(r)f(r)}{2b(r)c(r)}, \quad g_3(r) = -1 + 2g(r)^2. \tag{2.4}
$$

are necessary for the BPS system

$$
\begin{aligned}
\dot a &= -\frac{c}{2a} + \frac{a^5 f^2}{8b^4 c^3}, & \dot b &= -\frac{c}{2b} - \frac{a^2(a^2 - 3c^2)f^2}{8b^3 c^3}, \\
\dot c &= -1 + \frac{c^2}{2a^2} + \frac{c^2}{2b^2} - \frac{3a^2 f^2}{8b^4}, & \dot f &= -\frac{a^4 f^3}{4b^4 c^3},
\end{aligned} \tag{2.5}
$$

to satisfy the equations of motion. We have checked that these equations imply that the eleven dimensional metric satisfies $R_{\mu\nu} = 0$.

2.1 The Type IIA version

For our purposes, we need the Type IIA version of the configuration presented above and we need to pick a U(1) isometry to reduce on. The relevant U(1) isometry is generated by the Killing vector $\partial_{\psi_1} + \partial_{\psi_2}$. Having this in mind we rewrite the metric in a way which makes the isometry manifest,

$$
ds_{11}^2 = dx_{1,3}^2 + dr^2 + b^2 \left[(\sigma_1)^2 + (\sigma_2)^2 \right] + a^2 \left[(\Sigma_1 + g\sigma_1)^2 + (\Sigma_2 + g\sigma_2)^2 \right]
$$

$$+\frac{f^2c^2}{f^2+(1+g_3)^2c^2}\left(\sigma_3-\Sigma_3\right)^2$$

$$+\frac{1}{4}\left[f^2+(1+g_3)^2c^2\right]\left[\sigma_3+\Sigma_3+\frac{f^2-c^2(1-g_3^2)}{f^2+(1+g_3)^2c^2}\left(\sigma_3-\Sigma_3\right)\right]^2,\qquad(2.6)$$

Note that in this metric nothing depends on the combination $(\psi_2+\psi_1)$. Now Kaluza-Klein reduction simply amounts to dropping the last line in eq. (2.6) which has been written as a complete square for that purpose. In particular we can now read off the dilaton and the RR one-form gauge field,

$$e^\phi=2^{-3/2}\left[f^2+(1+g_3)^2c^2\right]^{3/4},\quad A_1=\frac{f^2-c^2(1-g_3^2)}{f^2+(1+g_3)^2c^2}\left(\sigma_3-\Sigma_3\right)+\cos\theta d\phi+\cos\tilde\theta d\tilde\phi.$$
$$(2.7)$$

The ten-dimensional metric in string frame is given by

$$ds^2_{IIA}=\frac{1}{2}\left\{dx^2_{1,3}+b^2\left[(\sigma_1)^2+(\sigma_2)^2\right]+a^2\left[(\Sigma_1+g\sigma_1)^2+(\Sigma_2+g\sigma_2)^2\right]\right.$$

$$\left.+\frac{f^2c^2}{f^2+(1+g_3)^2c^2}\left(\sigma_3-\Sigma_3\right)^2+dr^2\right\}\times\left[f^2+(1+g_3)^2c^2\right]^{1/2}\qquad(2.8)$$

Notice that the metric depends explicitly on $\psi=\psi_2-\psi_1$ and not on the coordinate on which we reduced, $\psi_+=\psi_2+\psi_1$. It is then advantageous to introduce a third set of one-forms:

$$\tilde\omega_1=\cos\psi\,d\tilde\theta+\sin\psi\,\sin\tilde\theta\,d\tilde\varphi,\quad\tilde\omega_2=-\sin\psi\,d\tilde\theta+\cos\psi\,\sin\tilde\theta\,d\tilde\varphi,\quad\tilde\omega_3=d\psi+\cos\tilde\theta d\tilde\varphi.\quad(2.9)$$

It should be pointed out here that the metric is written in terms of two-pairs of left-invariant forms of SU(2). In the following section, we will perform a non-Abelian T-duality on the SU(2) described by the coordinates (θ,φ,ψ).

Upon rescaling the Minkowski part of the space by a constant μ and reinstating the factors of α',g_s, the full metric, dilaton and RR field strength are,[1]

$$ds^2_{IIA,st}=\alpha'g_sNe^{2A}\left[\frac{\mu}{\alpha'g_sN}dx^2_{1,3}+dr^2+b^2(d\theta^2+\sin^2\theta d\varphi^2)+\right.$$

$$\left.+a^2(\tilde\omega^1+gd\theta)^2+a^2(\tilde\omega^2+g\sin\theta d\varphi)^2+h^2(\tilde\omega^3-\cos\theta d\varphi)^2\right]$$
$$(2.10)$$

$$h^2=\frac{c^2f^2}{f^2+c^2(1+g_3)^2},\quad e^{4/3\phi}=\frac{c^2f^2}{4(g_sN)^{2/3}h^2},\quad e^{4A}=\frac{c^2f^2}{4h^2}$$

$$\frac{F_2}{\sqrt{\alpha'g_sN}}=-(1+K)\sin\theta d\theta\wedge d\varphi+(K-1)\tilde\omega^1\wedge\tilde\omega^2-K'dr\wedge(\tilde\omega^3-\cos\theta d\varphi).$$

where

$$K(r)=\frac{f^2-c^2(1-g_3^2)}{f^2+c^2(1+g_3)^2}.$$

---

[1]One can send $ds^2_6\to A_1ds^2_6$, $F_2\to A_2F_2$, $e^{-4\phi/3}\to A_3e^{-4\phi/3}$ and still have a solution of IIA supergravity, preserving $\mathcal{N}=1$ SUSY provided $A_1^2A_3^3=A_2^4$. We choose $A_1=\alpha'g_sN$, $A_2=\sqrt{\alpha'}g_sN$ and $A_3=(g_sN)^{2/3}$, so that the dilaton is independent of α'. The parameter μ is just a scaling the $R^{1,3}$ coordinates.

Note that F_2 contains two components with no 'legs' on the radial coordinate r:

$$F_2\Big|_{r=r_0} = -Ng_s\sqrt{\alpha'}\Big[(K+1)\sin\theta d\theta \wedge d\varphi - (K-1)\sin\tilde{\theta}d\tilde{\theta} \wedge d\tilde{\varphi}\Big]. \qquad (2.11)$$

Thus, we only have flux quantisation on cycles for which the $K(r)$ parts mutually cancel. For example on $\Sigma_2 = [\tilde{\theta} = \theta,\ \tilde{\varphi} = \varphi]$, $\psi =$ constant, which is a SUSY cycle in the IR, we have

$$F_2\Big|_{\Sigma_2} = -2g_s N\sqrt{\alpha'}\sin\theta d\theta \wedge d\varphi. \qquad (2.12)$$

As we will see below, under the non-abelian T-duality, these two terms in F_2 will not be mapped to the same dual flux. We require that the flux on Σ_2 is quantised in the usual fashion

$$-\int F_2 = 2\kappa_{10}T_6 N. \qquad (2.13)$$

To achieve this we use,

$$T_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}, \qquad 2\kappa_{10} = 4(2\pi)^7 \alpha'^{\,4} g_s^2. \qquad (2.14)$$

So that we may associate the charge of the D6 branes N with an $SU(N)$ gauge group in the dual QFT.

2.2 G-structures: from G_2 to $SU(3)$

We derive the G-structures and SUSY conditions at each step going from M-theory to type-IIA. For clarity in presentation, in this section $g_s = \alpha' = N = 1$.

As is shown in [11], the M-theory background obeys the condition of G_2 holonomy. Hence, following [11], but in notation suggestive of dimensional reduction, we introduce a set of vielbeins for the 7d internal space as defined in eq. (2.6) — here we call $z = \psi_+$,

$$\begin{aligned}
\hat{e}^r &= dr, & \hat{e}^\theta &= b\sigma_1, & \hat{e}^\varphi &= b\sigma_2, & \hat{e}^z &= e^{2\phi/3}(dz + A_1) \\
\hat{e}^1 &= a(\Sigma_1 + g\sigma_1), & \hat{e}^2 &= a(\Sigma_2 + g\sigma_2), & \hat{e}^3 &= h(\Sigma_3 - \sigma_3).
\end{aligned} \qquad (2.15)$$

Here we have used the definitions introduced in previous sections (the reason for the cluttered by tildes definition will become clear shortly). The following three-form can be constructed from the projections on the SUSY spinor, needed to derive the BPS system [11],

$$\tilde{\Phi}_3 = \hat{e}^r \wedge (\hat{e}^{1\theta} + \hat{e}^{2\varphi} + e^{3z}) + (\hat{e}^{12} - \hat{e}^{\theta\varphi}) \wedge (\alpha\hat{e}^3 + \beta\hat{e}^z) + (\hat{e}^{1\varphi} - \hat{e}^{2\theta}) \wedge (\alpha\hat{e}^3 - \beta\hat{e}^z) \quad (2.16)$$

where

$$\alpha(r) = \frac{ag}{\sqrt{b^2 + a^2g^2}}, \quad \beta(r) = \frac{b}{\sqrt{b^2 + a^2g^2}}, \quad \alpha^2 + \beta^2 = 1. \qquad (2.17)$$

It is then simple to show that the three-form obeys

$$d\tilde{\Phi}_3 = 0, \quad d\star_7 \tilde{\Phi}_3 = 0, \qquad (2.18)$$

once the BPS equations (2.5) are imposed. We would now like to dimensionally reduce the G_2 SUSY conditions to find the corresponding conditions in type-IIA. Fortunately, this was done in full generality in [41] and in a rather similar scenario in [42]. The corresponding conditions are those of an SU(3)-structure. All one needs is to convert eq. (2.16) to Scherk-Schwarz gauge then follow the prescription of [41]. This is achieved through a rotation in both the $\hat{\tilde{e}}^\theta, \hat{\tilde{e}}^\varphi$ and $\hat{\tilde{e}}^1, \hat{\tilde{e}}^1$ planes such that:

$$
\begin{aligned}
\hat{e}^\theta &= \cos\psi\hat{\tilde{e}}^\theta - \sin\psi\hat{\tilde{e}}^\varphi = bd\theta \\
\hat{e}^\varphi &= \sin\psi\hat{\tilde{e}}^\theta + \cos\psi\hat{\tilde{e}}^\varphi = b\sin\theta d\varphi \\
\hat{e}^1 &= \cos\psi\hat{\tilde{e}}^1 - \sin\psi\hat{\tilde{e}}^2 = a(\omega^1 + gd\theta) \\
\hat{e}^2 &= \sin\psi\hat{\tilde{e}}^1 + \cos\psi\hat{\tilde{e}}^2 = a(\omega^2 + g\sin\theta d\varphi).
\end{aligned}
\tag{2.19}
$$

The corresponding three-form, Φ_3 is the same as eq. (2.16) with $\hat{\tilde{e}} \to \hat{e}$ and is obviously still both closed and co-closed. The vielbeins of the new 6-d internal space can be neatly expressed as

$$
\begin{aligned}
e^r &= e^{\phi/3}dr, & e^\theta &= e^{\phi/3}bd\theta, & e^\varphi &= e^{\phi/3}b\sin\theta d\varphi \\
e^1 &= e^{\phi/3}a(\tilde{\omega}^1 + gd\theta), & e^2 &= e^{\phi/3}a(\tilde{\omega}^2 + g\sin\theta d\varphi), & e^3 &= e^{\phi/3}h(\tilde{\omega}^3 - \cos\theta d\varphi),
\end{aligned}
\tag{2.20}
$$

while the 11-D vielbeins are of the form $\hat{e}^A = (e^a, \hat{e}^z)$. The SU(3) structure is then given in terms of the 3-form by:

$$
J_{ab} = \Phi_{abz}, \qquad (\Omega_{\text{hol}})_{abc} = \Phi_{abc} - i(\star_6\Phi)_{abc}.
\tag{2.21}
$$

which amounts in this case to

$$
\begin{aligned}
J &= -e^{3r} + (\alpha e^2 + \beta e^\varphi) \wedge e^\theta + e^1 \wedge (-\alpha e^\varphi + \beta e^2) \\
\Omega_{\text{hol}} &= (-e^3 + ie^r) \wedge ((\alpha e^2 + \beta e^\varphi) + ie^\theta) \wedge (e^1 + i(-\alpha e^\varphi + \beta e^2)).
\end{aligned}
\tag{2.22}
$$

These can be used to construct two pure-spinors,

$$
\Psi_+ = \frac{e^A}{8}e^{-iJ}, \qquad \Psi_- = \frac{e^A}{8}\Omega_{\text{hol}},
\tag{2.23}
$$

that can be shown to satisfy the pure spinors SUSY conditions

$$
\begin{aligned}
d(e^{2A-\phi}\Psi_+) &= 0 \\
d(e^{2A-\phi}\Psi_-) &= e^{2A-\phi}dA \wedge \bar{\Psi}_- + i\frac{e^{3A}}{8} \star_6 F_2,
\end{aligned}
\tag{2.24}
$$

which, collecting forms of equal size, gives

$$
\begin{aligned}
dJ &= 0 \\
d(e^{3A-\phi}) &= 0 \\
d(e^{2A-\phi}Re\Omega_{\text{hol}}) &= 0 \\
d(e^{4A-\phi}Im\Omega_{\text{hol}}) - e^{4A} \star_6 F_2 &= 0
\end{aligned}
\tag{2.25}
$$

these relations are all satisfied once eqs. (2.5) are taken into account. We will choose $3A = \phi$. Also, notice that $F_4 = 0$ for backgrounds of SU(3)-structure.

2.2.1 Potential and calibrations

It is useful to derive an expression for the seven form C_7 that acts as a potential for F_8, i.e. $F_8 = \star F_2 = dC_7$. One finds,

$$C_7 = e^{4A-\phi} vol_4 \wedge Im\Omega_{\text{hol}}. \tag{2.26}$$

The calibration form of space-time filling D branes is given by [17, 18],

$$\Psi_{\text{cal}} = -8e^{3A-\phi}(Im\Psi_-) = -e^{4A-\phi}Im\Omega_{\text{hol}}. \tag{2.27}$$

Clearly we have $vol_4 \wedge \Psi_{\text{cal}} + C_7 = 0$ so any space-time filling D6 brane wrapping a 3-cycle Σ^3 such that the calibration condition

$$e^{4A-\phi}\sqrt{detG_{\Sigma^3}} = e^{4A-\phi}Im\Omega_{\text{hol}}\Big|_{\Sigma^3} \tag{2.28}$$

is satisfied will be SUSY.[2] The same condition must be satisfied for any odd cycle and so the only non vanishing odd cycles are 3-cycles (if B_2 were turned on we could also have 5-cycles). A similar calculation shows that potential even SUSY cycles are Σ^2, Σ^6 such that (these are calibrated by $Im\Psi_+$),

$$\sqrt{detG_{\Sigma^2}} = J\Big|_{\Sigma^2}, \qquad \sqrt{detG_{\Sigma^2}} = -\frac{1}{6}J \wedge J \wedge J\Big|_{\Sigma^6}. \tag{2.29}$$

All the information above, only relies on the backgrounds in eqs. (2.3), (2.10) and their BPS equations (2.5). We will now describe some solutions to this system of first order, ordinary and non-linear equations.

2.3 Explicit solutions

Let us first describe a couple of known exact solutions. There is a simple solution to eqs.(2.5) given by,

$$a(r) = c(r) = -\frac{r}{3}, \qquad b(r) = f(r) = \frac{r}{2\sqrt{3}}. \tag{2.30}$$

This solution corresponds to a $R^{1,4} \times \mathcal{M}_7$ space with metric (2.3),

$$ds_{11}^2 = dx_{1,3}^2 + dr^2 + \frac{r^2}{9}\left[\left(\Sigma_1 - \frac{1}{2}\sigma_1\right)^2 + \left(\Sigma_2 - \frac{1}{2}\sigma_2\right)^2\right] + \frac{r^2}{12}(\sigma_1^2 + \sigma_2^2) + \frac{r^2}{9}\left(\Sigma_3 - \frac{1}{2}\sigma_3\right)^2 + \frac{r^2}{12}\sigma_3^2. \tag{2.31}$$

When reduced to ten dimensions the resulting IIA dilaton behaves as $e^{4\phi/3} \sim r^2$. This solution present a singularity at $r = 0$ and the need to lift this background to M-theory for large values of the radial coordinate, to avoid strong coupling in IIA. This solution is the 'unresolved' version of the one written in — for example — eqs. (3.16)–(3.17) of [43]. In that case, we will have

$$dr = \frac{d\rho}{\sqrt{1 - \frac{a^3}{\rho^3}}}, \quad b^2 = f^2 = \frac{\rho^2}{12}, \quad a^2 = c^2 = \frac{\rho^2}{9}\left(1 - \frac{a^3}{\rho^3}\right). \tag{2.32}$$

---

[2]We work in conventions where the DBI and WZ actions have a relative sign difference.

This solution avoids the singularity by ending the space at $\rho = a$. Still, the behavior of the dilaton is such that it the Type IIA description is strongly coupled for large values of the radial coordinate r. To avoid this last issue and to have a background fully contained in type IIA, we will describe new solutions that are both non-singular and with bounded dilaton. These new solutions, turn out to not be known in exact form, but semi-analytically, that is as series expansions for large and small values of r, complemented with a careful numerical interpolation. We will study them below.

2.4 Semi-analytical solutions

Since our goal is to work with trustable backgrounds in Type IIA we will be mostly interested in solutions with bounded dilaton and everywhere finite Ricci and Riemann invariants. The asymptotic large radius $r \to \infty$, form of these solutions is,

$$
\begin{aligned}
a(r) =& \frac{r}{\sqrt{6}} - \frac{\sqrt{3}q_1 R_1}{\sqrt{2}} + \frac{21\sqrt{3}R_1{}^2}{\sqrt{2}\,16r} + \frac{63\sqrt{3}q_1 R_1{}^3}{\sqrt{2}\,16r^2} + \frac{9\sqrt{3}\left(672q_1{}^2 + 221\right)R_1{}^4}{\sqrt{2}\,512r^3} + \\
&+ \frac{81\sqrt{3}q_1\left(224q_1{}^2 + 221\right)R_1{}^5}{\sqrt{2}\,512r^4} + \frac{\sqrt{3}\left(2048h_1 + 1377\left(768q_1{}^4 + 1632q_1{}^2 + 137\right)R_1{}^6\right)}{\sqrt{2}\,8192r^5} + \dots \\
b(r) =& \frac{r}{\sqrt{6}} - \frac{\sqrt{3}q_1 R_1}{\sqrt{2}} - \frac{3\sqrt{3}R_1^2}{4\sqrt{2}r} - \frac{9\sqrt{3}q_1 R_1^3}{4\sqrt{2}r^2} - \frac{9\sqrt{3}(37 + 96q_1^2)R1^4}{128\sqrt{2}r^3} - \frac{81\sqrt{3}q_1(37 + 32q_1^2)R_1^5}{128\sqrt{2}\,r^4} + \\
&+ \frac{\sqrt{3}(512h_1 - 81(133 + 1920q_1^2 + 960q_1^4)R_1^6)}{2048\sqrt{2}\,r^5} + \dots \\
c(r) =& -\frac{r}{3} + q_1 R_1 - \frac{9R_1^2}{8r} - \frac{27q_1 R_1^3}{8r^2} - \frac{9(17 + 36q_1^2)R_1^4}{32r^3} - \frac{81q_1(17 + 12q_1^2)R_1^5}{32r^4} + \frac{h_1}{r^5} + \dots \\
f(r) =& R_1 - \frac{27R_1{}^3}{8r^2} - \frac{81q_1 R_1{}^4}{4r^3} - \frac{243R_1{}^5\left(12q_1{}^2 + 1\right)}{32r^4} - \frac{729R_1{}^6\left(4q_1{}^3 + q_1\right)}{8r^5} + \dots
\end{aligned} \tag{2.33}
$$

where q_1, R_1 and h_1 are constants.

Close to $r \to 0$ one has

$$
\begin{aligned}
a(r) &= \frac{r}{2} - \frac{(q_0^2 + 2)r^3}{288R_0^2} - \frac{(-74 - 29q_0^2 + 31q_0^4)r^5}{69120R_0^4} + \cdots, \\
b(r) &= R_0 - \frac{(q_0^2 - 2)r^2}{16R_0} - \frac{(13 - 21q_0^2 + 11q_0^4)r^4}{1152R_0^3} + \cdots, \\
c(r) &= -\frac{r}{2} - \frac{(5q_0^2 - 8)r^3}{288R_0^2} - \frac{(232 - 353q_0^2 + 157q_0^4)r^5}{34560R_0^4} + \cdots, \\
f(r) &= q_0 R_0 + \frac{q_0^3 r^2}{16R_0} + \frac{q_0^3(-14 + 11q_0^2)r^4}{1152R_0^3} + \cdots, \\
g(r) &= \frac{q_0}{2} + \frac{q_0(q_0^2 - 1)}{24R_0^2}r^2 + \cdots, \\
g_3(r) &= \frac{q_0^2 - 2}{2} + \frac{q_0^2(q_0^2 - 1)}{12R_0^2}r^2 + \cdots.
\end{aligned} \tag{2.34}
$$

Note that $a(r)$ and $c(r)$ collapse in the IR and the other two functions do not. The constants q_0 and R_0 determine the IR behavior. Similarly, q_1, R_1 and h_1 are the UV parameters. Not for every set of q_0, R_0, q_1, R_1, h_1 there will exist a solution that interpolates between (2.33) and (2.34). For example, as seen in figure 1, if we numerically integrate forward from

the IR, not every value of R_0, q_0 leads to a stabilized dilaton. Similarly if we integrate back from the UV using eq.(2.33) as boundary conditions we do not necessarily get to an IR like that in eq.(2.34). Nevertheless, it is possible to show numerically that solutions interpolating between the behavior of eqs.(2.33) and (2.34) do exist. In figure 2 we present representatives of such solutions. To obtain these numerical solutions we shoot from the IR and minimize the mismatch between this forward integrated solution and the required UV behavior. This minimization procedure determines the UV parameters (see appendix A for more details). Also, we have defined some other functions in terms of the above, their expansions read, for $r \to \infty$

$$
\begin{aligned}
e^{4A} &= (g_s N)^{3/2} e^{4\phi/3} \\
e^{4A} &= \frac{R_1^2}{4} - \frac{9R_1^4}{8r^2} + O\left(\frac{1}{r}\right)^3 \\
h^2 &= \frac{r^2}{9} - \frac{2}{3}r\left(q_1 R_1\right) + \frac{1}{2}\left(2q_1^2 + 1\right)R_1^2 + O\left(\frac{1}{r}\right)^1 \\
K &= \frac{6561 R_1^4}{64 r^4} + O\left(\frac{1}{r}\right)^5 \\
g &= \frac{3R_1}{2r} + \frac{9q_1 R_1^2}{2r^2} + \frac{27\left(16q_1^2 - 1\right)R_1^3}{32r^3} + O\left(\frac{1}{r}\right)^4 \\
g_3 &= -1 + \frac{9R_1^2}{2r^2} + \frac{27q_1 R_1^3}{r^3} + \frac{81\left(24q_1^2 - 1\right)R_1^4}{16r^4} + O\left(\frac{1}{r}\right)^5
\end{aligned}
\tag{2.35}
$$

and for $r \to 0$, we have

$$
\begin{aligned}
e^{4A} &= \frac{1}{4}q_0^2 R_0^2 + \frac{3}{64}q_0^4 r^2 + \frac{q_0^4\left(37q_0^2 - 40\right)r^4}{3072 R_0^2} + O\left(r^6\right) \\
h^2 &= \frac{r^2}{4} + \frac{\left(q_0^2 - 16\right)r^4}{576 R_0^2} + O\left(r^6\right) \\
K &= 1 - \frac{r^2}{4R_0^2} + \frac{\left(40 - 7q_0^2\right)r^4}{576 R_0^4} + O\left(r^6\right) \\
g &= \frac{q_0}{2} + \frac{q_0\left(q_0^2 - 1\right)r^2}{24 R_0^2} + \frac{q_0\left(91q_0^4 - 179q_0^2 + 88\right)r^4}{13824 R_0^4} + O\left(r^6\right) \\
g_3 &= \frac{1}{2}\left(q_0^2 - 2\right) + \frac{q_0^2\left(q_0^2 - 1\right)r^2}{12 R_0^2} + \frac{q_0^2\left(115q_0^4 - 227q_0^2 + 112\right)r^4}{6912 R_0^4} + O\left(r^6\right)
\end{aligned}
\tag{2.36}
$$

The numerical solutions presented in figures 2 satisfy $R_0 q_0 = 2$. This corresponds to choosing the normalization of the dilaton such that $(g_s N)^{3/2} e^{4\phi_0/3} = 1$, where ϕ_0 is the value of the dilaton at $r = 0$. Also, since we want solutions with monotonically increasing dilaton, we require comparing eq.(2.35) with eq. (2.36), that $R_1^2 > q_0^2 R_0^2$.

2.4.1 Asymptotic behaviour

After reducing to ten dimensions the simple exact solution mentioned above leads to a background with metric easily obtained from eq. (2.31), dilaton $e^{\frac{4\phi}{3}} = \frac{r^2}{36(g_s N)^{2/3}}$ and
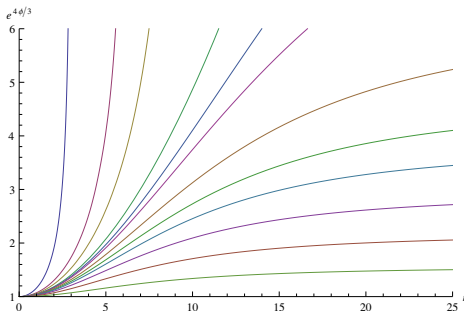
Figure 1. $e^{\frac{4\phi}{3}}$ for different values of q_0 and R_0. We keep $q_0 R_0 = 2$ fixed which amounts to fixing the normalization of the dilaton in the IR.

$F_2 = -\sqrt{a'}g_s N(\sin\theta d\theta \wedge d\varphi + \tilde\omega^1 \wedge \tilde\omega^2)$. Notice that the space is not asymptotically $T^{1,1}$ for the exact solutions. On the other hand, the numerical solutions with stabilized dilaton behave in the UV as,

$$ds^2_{IIA,st} = \alpha' g_s N \frac{R_1}{2}\left[\frac{\mu dx^2_{1,3}}{\alpha' g_s N} + dr^2 + r^2\left(\frac{1}{6}(d\theta^2 + \sin^2\theta d\varphi^2 + (\tilde\omega^1)^2 + (\tilde\omega^2)^2)\right.\right.$$
$$\left.\left. + \frac{1}{9}(\tilde\omega^3 - \cos\theta d\varphi)^2\right) + \dots\right] \quad (2.37)$$

with

$$(g_s N)^{2/3}e^{\frac{4\phi}{3}} = \frac{R_1^2}{4} - \frac{9R_1^4}{8r^2} - \frac{27q_1 R_1^5}{4r^3}\dots$$
$$F_2 = -\sqrt{\alpha'}g_s N\left[\left(1 - \frac{81R_1^2}{8r^2} + \dots\right)\sin\theta d\theta \wedge d\varphi + \left(1 + \frac{81R_1^2}{8r^2} + \dots\right)\tilde\omega^1 \wedge \tilde\omega^2\right.$$
$$\left. - \left(\frac{81R_1^2}{4r^3} + \dots\right)dr \wedge (\tilde\omega^3 - \cos\theta d\varphi)\right]$$
$$\sim -\sqrt{\alpha'}g_s N(\sin\theta d\theta \wedge d\varphi + \tilde\omega^1 \wedge \tilde\omega^2) \quad (2.38)$$

In the UV the five dimensional internal space is $T^{1,1}$. Thus, the space is asymptotically $R^{1,3} \times CY_6$ with a constant dilaton and constant F_2. This 'flat space' asymptotics is characteristic of duals to QFTs whose UV behavior is controlled by an irrelevant operator — this will come back when dealing with the QFT analysis. Somehow the field theory is taken out of the 'decoupling limit'. On the other hand, in the IR the metric, dilaton and RR form asymptote to,

$$ds^2_{IIA,str} = \frac{q_0 R_0 \alpha' g_s N}{2}\left[\mu dx^2_{1,3} + dr^2 + R_0^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) + \frac{r^2}{4}d\Omega_3\right] + \dots$$
$$d\Omega_3 = (\tilde\omega_1 + d\theta)^2 + (\tilde\omega_2 + \sin\theta d\varphi)^2 + (\tilde\omega_3 - \cos\theta d\varphi)^2, \quad (g_s N)^{2/3}e^{4\phi/3} = \frac{q_0^2 R_0^2}{4} + \frac{3q_0^4}{64}r^2 + \dots$$
$$F_2 = -2\sqrt{\alpha'}g_s N \sin\theta d\theta \wedge d\varphi + \dots$$
$$(2.39)$$
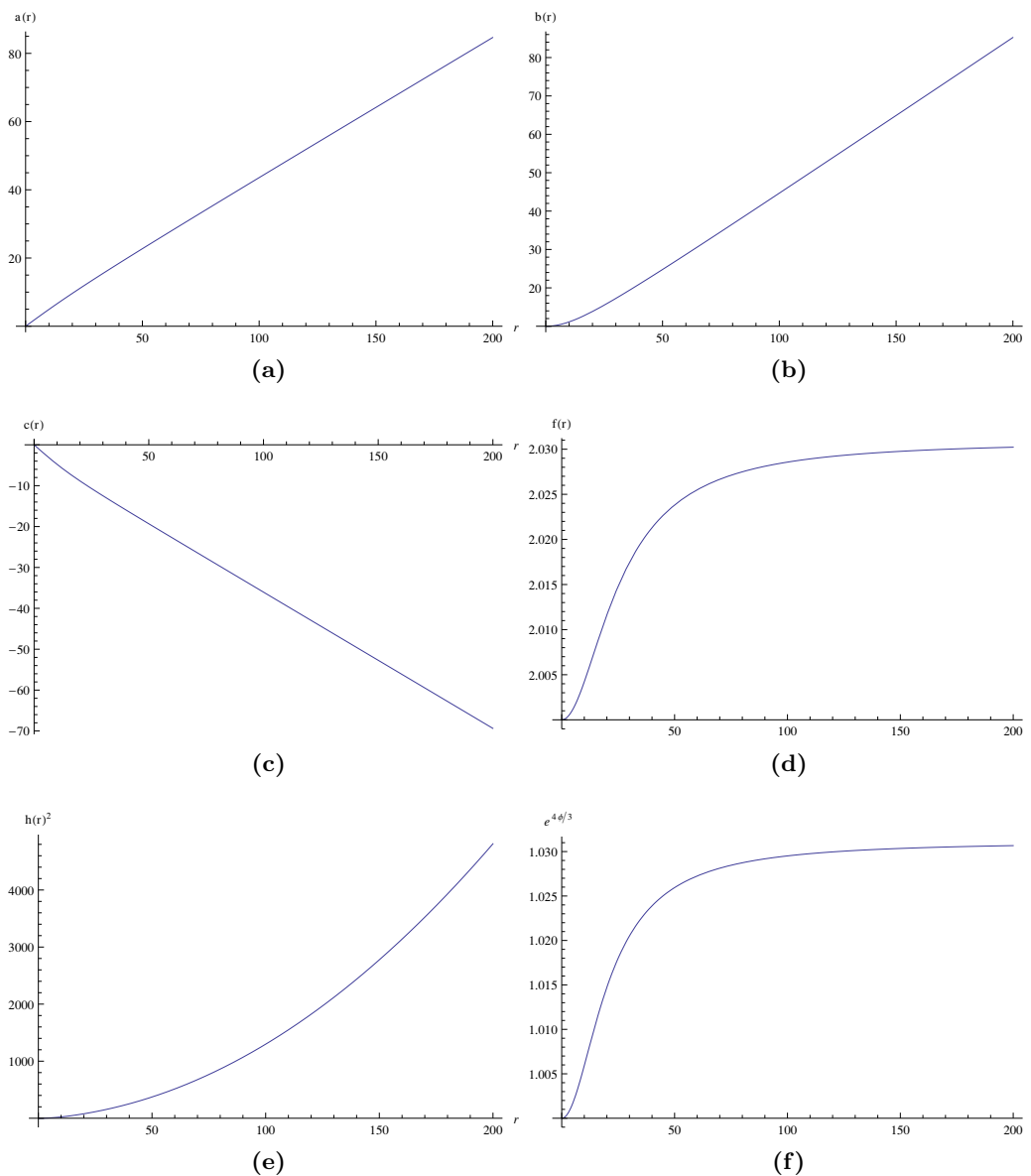
Figure 2. A numerical solution for $a(r), b(r), c(r)$ and $f(r)$ obtained by forward integration of the BPS equations with (2.34) as boundary conditions, $R_0 = 10$, and $q_0 = 1/5$. After the minimization procedure explained in the appendix A we find that for the UV parameters $q_1 = 1.31946$, $R_1 = -2.03087$, $h_1 = -1.9733$ this solution has the required UV behavior (2.33). We also plot $h(r)^2$ and $e^{4\phi/3}$ defined in (2.10).

The material discussed in this section is not all original; we have rewritten some of it to ease the analysis of the next section. However, we should point out that the semi-analytic solutions with stabilized dilaton and no singularities (though have been discussed in [10] and [11]) are found explicitly — with the explicit delicate numerics — in this paper. These solutions will play an important role in the next sections.

3 Non-Abelian T-duality

In this section, we will present completely original material. We will construct a *new* solution in Type IIB supergravity preserving four supercharges. This background will have SU(2)-dynamical structure. We believe this type of solution is new in the literature.

The technique we will use to construct this new background is non-Abelian T-duality, see [26]–[39] for a partial sample of papers. The reader unfamiliar with this technology should read section 2 in [33] for a clear explanation of the whole procedure.

We will straightforwardly present the new background in type IIB supergravity. Following the conventions of section 2 of the paper [33] and starting from the background in eq. (2.10) we perform a non-Abelian T-duality on the SU(2) isometry parametrised by (θ, φ, ψ) and gauge fix such that $\theta = \varphi = v_1 = 0$, so that the solution generated still depends on the angles $(\tilde{\theta}, \tilde{\varphi}, \psi)$ and on the new coordinates (v_2, v_3) — see the short discussion below eq. (2.8) about the explicit SU(2) invariances of the background. We remind the reader that $\tilde{\omega}^i$,

$$\tilde{\omega}_1 = \cos\psi\, d\tilde{\theta} + \sin\psi\, \sin\tilde{\theta}\, d\tilde{\varphi}, \quad \tilde{\omega}_2 = -\sin\psi\, d\tilde{\theta} + \cos\psi\, \sin\tilde{\theta}\, d\tilde{\varphi}, \quad \tilde{\omega}_3 = d\psi + \cos\tilde{\theta} d\tilde{\varphi}. \tag{3.1}$$

In the process of doing this non-Abelian T-duality, we generate an entirely new NS and RR sector and type-IIB metric. The T-dual metric is given by (we take $g_s = \alpha' = \mu = 1$ and we remind the reader that below eq. (2.25) we set $3A = \phi$),

$$ds^2_{IIB,st} = e^{2A}\left[dx^2_{1,3} + Ndr^2 + N\hat{a}^2(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\varphi}^2)\right] + \frac{1}{\det M}\left[2(v_3 dv_2 + v_3 dv_3)^2 + \tag{3.2}\right.$$

$$\left. + 4N^2 e^{4A}\hat{b}\left(\hat{b}^2(dv_3 + \hat{c}v_2\tilde{\omega}_2)^2 + h^2(\hat{c}^2 v_3^2\tilde{\omega}_1^2 + (dv_2 - \hat{c}v_3\tilde{\omega}_2)^2) + 2\hat{c}v_2 v_3\tilde{\omega}_1\tilde{\omega}_3 + v_2^2\tilde{\omega}_3)\right)\right]$$

where

$$\det M = 4e^{2A}N\left(2e^{4A}N^2\hat{b}^4 h^2 + \hat{b}^2 v_2^2 + h^2 v_3^2\right), \tag{3.3}$$

which also appears in the definition of the dual dilaton

$$e^{-2\Phi} = \det M e^{-2\phi}, \tag{3.4}$$

and we have introduced the following functions for convenience of presentation

$$\hat{a} = \frac{ab}{\sqrt{b^2 + a^2 g^2}}, \quad \hat{b} = \sqrt{b^2 + a^2 g^2}, \quad \hat{c} = \frac{a^2 g}{b^2 + a^2 g^2}. \tag{3.5}$$

The many and complicated forms that this background supports can be expressed in a relatively compact manner through a judicious choice of dual vielbein basis $\hat{e}^a$, namely

$$e^{x^\mu} = e^A dx^\mu, \qquad e^r = e^A\sqrt{N}dr, \qquad e^{1,2} = e^A\sqrt{N}\hat{a}\tilde\omega_{1,2}$$

$$e^{\hat{1}} = \frac{2\sqrt{N}e^A\hat{b}}{\det M}\Bigg[-\sqrt{2}v_2(v_2 dv_2 + v_3 dv_3) - 2\sqrt{2}e^{4A}N^2\hat{b}^2h^2(dv_2 - \hat{c}v_3\tilde\omega_2)$$
$$+ 2e^{2A}Nh^2v_3(v_3\hat{c}\tilde\omega_1 + v_2\tilde\omega_3)\Bigg]$$

$$e^{\hat{2}} = \frac{4e^{3A}N^{3/2}\hat{b}}{\det M}\Bigg[v_2\hat{b}^2(dv_3 + \hat{c}v_2\tilde\omega_2) + h^2(\hat{c}v_3^2\tilde\omega_2 - v_3 dv_2) \tag{3.6}$$
$$- 2\sqrt{2}e^{2A}Nh^2\hat{b}^2(\hat{c}v_3\tilde\omega_1 + v_2\tilde\omega_3)\Bigg]$$

$$e^{\hat{3}} = \frac{2e^A\sqrt{N}h}{\det M}\Bigg[-\sqrt{2}v_2(v_2 dv_2 + v_3 dv_3) - 2\sqrt{2}e^{4A}N^2\hat{b}^4(dv_3 + \hat{c}v_2\tilde\omega_2)$$
$$- 2e^{2A}N\hat{b}^2v_2(\hat{c}v_3\tilde\omega_1 + v_2\tilde\omega_3)\Bigg].$$

With respect to this basis the NS two-form is given by[3]

$$B_2 = \frac{1}{\hat{a}\hat{b}v_2}\Bigg[\hat{a}hv_2 e^{\hat{1}\hat{3}} - \hat{b}\hat{c}hv_3 e^{1\hat{3}} - \hat{b}^2\big(\hat{c}v_2 e^{1\hat{1}} + \sqrt{2}e^{2A}N\hat{a}he^{\hat{2}\hat{3}}\big)\Bigg] \tag{3.7}$$

The RR sector is given by,

$$F_1 = \frac{2e^{-A}\sqrt{N}(K+1)}{\hat{a}\hat{b}}\Bigg[\hat{b}\big(\hat{c}v_2 e^2 + \sqrt{2}e^{2A}N\hat{a}he^{\hat{3}}\big) - \hat{a}v_2 e^{\hat{2}}\Bigg] - 2e^{-A}\sqrt{N}K'v_3 e^r,$$

$$F_3 = \frac{2(K+1)e^{-A}\sqrt{N}}{\hat{a}\hat{b}^2}\Bigg[\hat{b}^2\hat{c}v_2 e^{1\hat{1}\hat{2}} + \hat{b}\hat{c}hv_3\big(e^{2\hat{1}\hat{3}} - e^{1\hat{2}\hat{3}}\big) - \sqrt{2}e^{2A}N\hat{b}^3\hat{c}\big(e^{1\hat{1}\hat{3}} + e^{2\hat{2}\hat{3}}\big) + \hat{a}hv_3 e^{\hat{1}\hat{2}\hat{3}}\Bigg] +$$
$$+ \frac{2e^{-A}\sqrt{N}\hat{b}K'}{h}\Bigg[\sqrt{2}e^{2A}N\hat{b}he^{r\hat{1}\hat{2}} + v_2 e^{r\hat{1}\hat{3}}\Bigg] + \frac{2e^{-A}\sqrt{N}U}{\hat{a}}\Bigg[\hat{b}v_2 e^{12\hat{1}} + hv_3 e^{12\hat{3}}\Bigg],$$

$$F_5 = \frac{2\sqrt{2}e^A N^{3/2}\hat{b}hU}{\hat{a}^2}\Bigg[e^{tx^1 x^2 x^3} - e^{12\hat{1}\hat{2}\hat{3}}\Bigg],$$

$$\tag{3.8}$$

where

$$U = \hat{c}(K+1) - (K-1), \tag{3.9}$$

has been defined for convenience. We also note that the potential such that $F_1 = dC_0$ is actually very simple, namely $C_0 = -2N(K+1)v_3$. We have checked using Mathematica that this background solves the Einstein, dilaton, Maxwell and Bianchi equations of Type IIB, once the eqs. (2.5) are imposed.

Notice, that like in the paper [50], our background's warp factors and dilaton depend on more than one coordinate — (r, v_2, v_3) — in our case.

---

[3]Note that the procedure of [33] actually gives the NS two from up to an exact $B_{2,eq.\ (3.7)} = B_{2,NATD} + \frac{1}{\sqrt{2}}d\psi \wedge dv_3$. The choice we make is merely more simple in vielbein basis.

3.1 Asymptotics

In the IR the new 3 manifold that is generated has induced metric

$$ds_3^2 = \frac{1}{2Nq_0R_0^3v_2^2}\left(2v_2^2dv_2^2 + 4v_2v_3dv_2dv_3 + (N^2q_0^2R_0^6 + 2v_3)dv_3^2\right) + \dots \qquad (3.10)$$

The form of this metric suggests that $v_2 = 0$ produces a singularity and indeed calculating the curvature invariants in the IR are all inversely proportional to some power of v_2. For instance

$$R = \frac{q_0^2(2N^2R_0^6 - 15v_2) + 4v_3^2}{2Nq_0R_0^3v_2^2} + \dots \qquad (3.11)$$

One may want to restrict the range of the coordinate $v_2 > 0$ to ensure our solution is non singular. This is a physical requisite on a coordinate, that the process of non-Abelian duality gives no information on. But, imposing that $v_2 > 0$ may lead to a space that is not consistent geometrically, namely the manifold would not be well defined (probably geodesically incomplete). It should be interesting to determine if there is any geometrical obstruction to such restriction. We will elaborate more on this point below.

The appearence of this possible-singular behavior at $v_2 = 0$ is due to the fact that we are T-dualising on a manifold (θ, φ, ψ) with a shrinking fiber ψ. See eq. (2.10) together with eq. (2.36). Since the non-Abelian T-duality (at least at the supergravity level as we are doing it) does not restrict the range of the coordinates, we may propose to restrict $v_2 > 0$. Recent developments on the sigma model side of the formalism [40] may illuminate these issues, but still more work on the topic is needed. It may be that the restriction $v_2 > 0$ is not feasible as discussed above and/or generates a manifold with a boundary. In that case, our solution would present a singularity at $v_2 = 0$. Physical observables would be trustable as long as they do not 'sit' on the point $v_2 = 0$.

In the UV the 3 manifold has induced metric

$$d^2s_3 = \frac{3}{NR_1r^2}\left(2dv_2^2 + 3dv_3^2 + 2v_2(d\psi + \cos\psi d\tilde\theta)^2\right) + \dots \qquad (3.12)$$

Although this is vanishing, in line with our expectations from dualising a manifold which blows up, all the curvature invariants remain finite. Related to this is the fact that, whilst the induced metric g_3 vanishes, the string volume $e^{-\Phi}\sqrt{\det g_3}$ is finite.

Finally, let us quote the asymptotics of the dilaton of Type IIB. For small values of r, we have

$$e^\Phi = \frac{q_0}{4Nv_2} - \frac{r^2\left(q_0\left(N^2q_0^2R_0^6 + 2\left(v_3^2 - (q_0^2 - 1)v_2^2\right)\right)\right)}{64\left(NR_0^2v_2^3\right)} + \dots. \qquad (3.13)$$

while the dual dilaton for $r \to \infty$ is,

$$e^\Phi = \frac{9}{\sqrt{2}N^2r^3} + \frac{81q_1R_1}{\sqrt{2}N^2r^4} + \frac{243\sqrt{2}q_1^2R_1^2}{N^2r^5} + \dots. \qquad (3.14)$$

3.2 G-structure

The seed type-IIA solution of section 2.1 exhibits confinement and supports an SU(3) structure as discussed in section 2.2. The results of [48] suggest that the T-dual solution should support a dynamical SU(2)-structure, defined by a point dependent rotation between the two 6-d internal killing spinors. This is indeed the case, we will present the structure here and refer the reader to appendix D of [48] for the details of the calculation[4] To express the structure succinctly it is useful to introduce a new set of vielbeins, which are a rotation of eq. (3.6),

$$
e^r = e^A \sqrt{N} dr \,, \quad \check{e}^\theta = e^A \sqrt{N} \hat{a} \, d\tilde{\theta} \,, \quad \beta e^\varphi + \alpha e^2 = e^A \sqrt{N} \hat{a} \sin\tilde{\theta} d\tilde{\varphi} \,,
$$
$$
e^{1'} = \frac{2e^A \sqrt{N} \hat{b}}{\det M} \Bigg[-2\sqrt{2} e^{4A} N^2 \hat{b}^2 h^2 \big(\cos\psi dv_2 - \hat{c} v_3 (\sin\psi \tilde{\omega}_1 + \cos\psi \tilde{\omega}_2) - v_2 \sin\psi \tilde{\omega}_3 \big)
$$
$$
- \sqrt{2} v_2 \cos\psi (v_2 dv_2 + v_3 dv_3) + 2e^{2A} N \Big(- \hat{b}^2 v_2 \sin\psi (dv_3 + \hat{c} v_2 \tilde{\omega}_)
$$
$$
+ h^2 v_3 \big(\sin\psi dv_2 + \hat{c} v_3 (\cos\psi \tilde{\omega}_1 - \sin\psi \tilde{\omega}_2) + v_2 \cos\psi \tilde{\omega}_3 \big) \Big) \Bigg]
$$
$$
\alpha e^\varphi - \beta e^{2'} = \frac{2e^A \sqrt{N} \hat{b}}{\det M} \Bigg[-2\sqrt{2} e^{4A} N^2 \hat{b}^2 h^2 \big(\cos\psi dv_2 - \hat{c} v_3 (\sin\psi \tilde{\omega}_1 + \cos\psi \tilde{\omega}_2) - v_2 \sin\psi \tilde{\omega}_3 \big)
$$
$$
- \sqrt{2} v_2 \cos\psi (v_2 dv_2 + v_3 dv_3) + 2e^{2A} N \Big(- \hat{b}^2 v_2 \sin\psi (dv_3 + \hat{c} v_2 \tilde{\omega}_)
$$
$$
+ h^2 v_3 \big(\sin\psi dv_2 + \hat{c} v_3 (\cos\psi \tilde{\omega}_1 - \sin\psi \tilde{\omega}_2) + v_2 \cos\psi \tilde{\omega}_3 \big) \Big) \Bigg]
$$
$$
e^{3'} = \frac{2e^A \sqrt{N} h}{\det M} \Bigg[-\sqrt{2} v_2 (v_2 dv_2 + v_3 dv_3) - 2\sqrt{2} e^{4A} N^2 \hat{b}^4 (dv_3 + \hat{c} v_2 \tilde{\omega}_2)
$$
$$
- 2e^{2A} N \hat{b}^2 v_2 (\hat{c} v_3 \tilde{\omega}_1 + v_2 \tilde{\omega}_3) \Bigg] \,.
$$

(3.15)

One then takes these vielbeins ordered as $(r\theta\varphi 1'2'3')$ and rotates to define another basis of vielbeins as

$$
\tilde{e} = R.e'. \tag{3.16}
$$

The matrix with which this rotation is performed is

$$
R = \frac{1}{\sqrt{\Delta}} \begin{pmatrix} \beta & 0 & 0 & \zeta_1 & -\zeta_2\beta & \zeta_3 \\ 0 & \sqrt{\Delta} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\Delta} & 0 & 0 & 0 \\ -\zeta_1 & 0 & 0 & \beta & \zeta_3 & \zeta_2\beta \\ \zeta_2\beta & 0 & 0 & -\zeta_3 & \beta & \zeta_1 \\ -\zeta_3 & 0 & 0 & -\zeta^2\beta & -\zeta^1 & \beta \end{pmatrix} \tag{3.17}
$$

where

$$
\Delta = \beta^2 + \zeta_1^2 + \zeta_2^2 \beta^2 + \zeta_3^2 \tag{3.18}
$$

---

[4]Actually it is the isometry defined by $(\tilde{\theta}, \tilde{\varphi}, \psi)$ that is dualised in appendix D of [48], but this calculation is completely analogous to our's. Our result is non-singular in the radial coordinate r.

and

$$\zeta_1 = -\frac{e^{-2A}v_2\cos\psi}{\sqrt{2}N\hat{b}h}, \quad \zeta_2 = -\frac{e^{-2A}v_2\sin\psi}{\sqrt{2}N\hat{b}h}, \quad \zeta_3 = -\frac{e^{-2A}v_3}{\sqrt{2}N\hat{b}^2}. \tag{3.19}$$

Let us now express the forms of the geometric structure, following the conventions of [44] we have

$$\begin{aligned}
k_\parallel &= \frac{\alpha}{\sqrt{1+\zeta.\zeta}} \qquad k_\perp = \sqrt{\frac{\beta^2+\zeta.\zeta}{1+\zeta.\zeta}} \\
z &= w - i\,v = \frac{1}{\sqrt{\beta^2+\zeta.\zeta}}\big(\sqrt{\Delta}\tilde{e}^r + \zeta_2\alpha\tilde{e}^\varphi - i(\sqrt{\Delta}\tilde{e}^3 + \zeta_2\alpha\tilde{e}^\theta)\big) \\
j &= \tilde{e}^{r3} + \tilde{e}^{\varphi\theta} + \tilde{e}^{21} - v \wedge w \\
\omega &= \frac{-i}{\sqrt{\beta^2+\zeta.\zeta}}\big(\sqrt{\Delta}(\tilde{e}^\varphi + i\tilde{e}^\theta) - \zeta_2\alpha(\tilde{e}^r + i\tilde{e}^3)\big) \wedge (\tilde{e}^2 + i\tilde{e}^1).
\end{aligned} \tag{3.20}$$

In terms of those forms, we can define two 6-d pure spinors as:

$$\begin{aligned}
\Phi_+ &= \frac{ie^A}{8}e^{-iv\wedge w}\big(k_\parallel e^{-ij} - ik_\perp\omega\big) \\
\Phi_- &= \frac{ie^A}{8}(v+iw)\wedge\big(k_\perp e^{-ij} + ik_\parallel\omega\big)
\end{aligned} \tag{3.21}$$

Notice that because $k_\parallel$ is point dependent we have a dynamical SU(2)-structure. To have a good idea of the dynamical character of the SU(2)-structure, we can expand the quantities $k_\parallel, k_\perp$. For the solution in eq. (2.32), we have for $\rho \to \infty$,

$$k_\perp = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}a^3}{16\rho^3} + \dots, \qquad k_\parallel = -\frac{1}{2} + \frac{3a^3}{16\rho^3} + \dots. \tag{3.22}$$

While for $\rho \to a$ we have,

$$k_\perp = 1 - \frac{\left(a^4 N^2\right)(\rho-a)^2}{1728 v_2^2} + \dots, \qquad k_\parallel = -\frac{\left(a^2 \mathrm{Nc}\right)(\rho-a)}{12\left(\sqrt{6}v_2\right)} + \dots. \tag{3.23}$$

On the other hand for the semi-analytic solutions we have,

$$k_\perp = 1 - \frac{9R_1^2}{8r^2} + \dots, \qquad k_\parallel = -\frac{3R_1}{2r} + \dots \tag{3.24}$$

for the large radius expansion and

$$k_\perp = 1 - \frac{r^4\left(q_0^4 R_0^2\right)}{256 v_2^2} + \dots, \qquad k_\parallel = -\frac{r^2\left(q_0^2 R_0\right)}{8\left(\sqrt{2}v_2\right)} + \dots \tag{3.25}$$

for the case of $r \to 0$. These expansions make clear the dynamical character of the structure. Also very descriptive is the quantity $\alpha(r)$ shown in figure 3.

It is interesting to notice that for the non-Abelian T-dual of the exact and singular solution in eq. (2.30), the SU(2)-structure is not dynamical. It is precisely the deformation of the space, displayed by the non-singular solution or the semi-analytical ones that makes
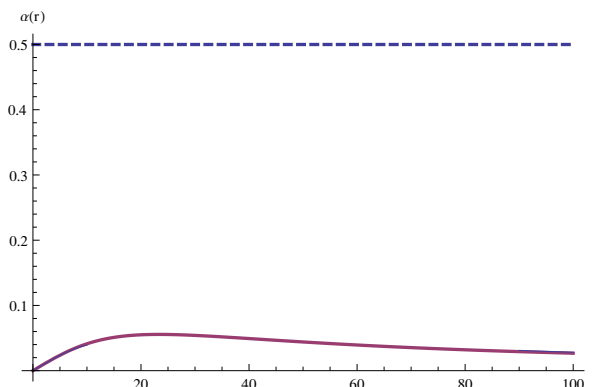
Figure 3. Solid line: $\alpha(r)$ for the numerical solution with $R_0 = 10$, $q_0 = 1/5$. Dashed line: $\alpha(r)$ for the exact solution of eq.(2.30), $\alpha_{\text{exact}}(r) = \frac{1}{2}$.

the structure dynamical. This may be related with the phenomena of 'confinement' and 'symmetry breaking' that occur in the dual field theory.

The calibration forms of SUSY cycles in the 6-d internal space are defined by

$$\Psi_n = -8e^{-\Phi}\text{Im}\left(\Phi_{\pm}\right) \wedge e^{-B_2}\bigg|_n \tag{3.26}$$

where on the left hand side it should be understood that we restrict to the part with n-legs and the even/odd calibrations are given by $\Psi_{\pm}$ respectively. In the bibliography, one can find *compactifications* with SU(2)-dynamical structure [45–47]. Here we have constructed a non-compact manifold with that characteristic.

3.3 SUSY sub-manifolds

Here we present a list of supersymmetric sub-manifolds, that while not exhaustive, gives at least some indication of the types of SUSY subspaces that this type IIB solution supports. Attention shall be restricted to manifolds with no legs in the r-direction.

In following sections, we will analyse different quantities derived from our background that can be put in correspondence with observables in the dual QFT. This analysis will *suggest* to impose certain conditions on the coordinates v_2, v_3. Indeed, we will define sub-manifolds-that will be referred to as '*cycles*', though technically they may present boundaries. The issues of the periodicities (or not) of the coordinates v_2, v_3, the presence of boundaries in our sub-manifolds, etc are difficult to sort out in the present system and with the present choice of coordinates.

Hence, the analysis in the sections below is to be read as a field theoretical-way to get some hint on the ranges and periodicities (if any) of these coordinates introduced by the dualisation procedure. A more dedicated analysis — perhaps in a more symmetric system [64, 65] — is in order, but beyond the scope of the present work.

One-cycles. These may be defined by imposing $v_2(v_3)$ with all other coordinates constant. The DBI action is given by

$$S^1_{DBI} = T_1 \int dv_3 \mathcal{L}^1_{DBI} = T_1 \int dv_3 e^{-\phi}\sqrt{2e^{4A}N^2(\hat{b}^4 + \hat{b}^2 h^2 v_2'^2) + (v_3 + v_2 v_2')^2} \tag{3.27}$$

and the behaviour of the integrand in the IR and and UV is

$$\mathcal{L}^1_{DBI} = \begin{cases} 4\sqrt{\dfrac{N(n^2 q_0^2 R_0^6 + 2(v_3 + v_2 v_2')^2)}{2q_0^3 R_0^3}} + \dots & \text{as } r \to 0 \\ \dfrac{1}{3}\sqrt{\dfrac{2}{3}}\sqrt{\dfrac{N^3(3 + 2v_2')}{R_1}} r^2 + \dots & \text{as } r \to \infty \end{cases} \tag{3.28}$$

A one-cycle is SUSY when $\mathcal{L}^1_{DBI} dv_3 = \Psi_1$ on that cycle. This may be used to fix $v_2(v_3)$. The calibration 1-form on $\{v_3 | v_2 = v_2(v_3)\}$ is given by

$$\Psi_1 = \frac{4e^{6A-\phi} N b \hat{b}^2}{4b^2 + \frac{a^4 f^2}{b^2 c^2}} dv_3 = \begin{cases} \dfrac{R_0^2 (Nq_0 R_0)^{3/2}}{\sqrt{2}} dv_3 + \dots & \text{as } r \to 0 \\ \dfrac{(NR_1)^{3/2}}{6\sqrt{2}} r^2 dv_3 + \dots & \text{as } r \to \infty \end{cases} \tag{3.29}$$

It is a simple matter to show that a 1-cycle which is SUSY in the UV is given by

$$v_2 = \frac{1}{4}\sqrt{\frac{3}{2}}\sqrt{R_1^4 - 16} v_3 + C \tag{3.30}$$

where C is any real constant and a real solution requires $|R_1| \geq 2$ which is consistent with the numerical solutions presented in section 2.4. Whilst there is a one cycle which is SUSY in the IR whenever

$$v_2^2 = \frac{1}{4}\left(Nq_0 R_0^3 \sqrt{2R_0^4 q_0^4 - 32} v_3 - 4v_3^2 + 4C^2\right) \tag{3.31}$$

where C is a different real constant. Notice that this simplifies to $v_2^2 + v_3^2 = C^2$ when $R_0 q_0 = 2$ and then the cycle defines a circle, a similar cycle was defined for a flavour D6 brane in [51].

Two-cycles. There are some cycles which preserve SUSY for large values of r. One of them is given by $(\tilde{\theta}, v_3)$ such that $\psi = 0$ and $v_2 = \frac{2\sqrt{6}}{R_1^4 - 16} v_3$.[5] For this cycle the DBI action is obtained by integrating

$$e^{-\Phi}\sqrt{g + B_2}\Big|_{\Sigma_2} = B\sqrt{v_3^2 + C} \tag{3.32}$$

where

$$B = e^{A-\phi}\frac{8 + R_1^4}{R_1^4 - 16}\sqrt{2N(\hat{a}^2 + \hat{b}^2 \hat{c}^2)}$$

$$C = \frac{2e^{4A} N^2 (R_1^4 - 16)\hat{b}^2 (24\hat{b}^2 \hat{c}^2 h^2 + \hat{a}^2((R_1^4 - 16)\hat{b}^2 + 24h^2))}{(r_1^4 + 8)^2 (\hat{a}^2 + \hat{b}^2 \hat{c}^2)} \tag{3.33}$$

One can integrate this to get the volume of the cycle to behave as

$$\int dv_3 e^{-\Phi}\sqrt{g + B_2}\Big|_{\Sigma_2} = \begin{cases} \dfrac{N^2 R_1^2 \Delta v_3}{3\sqrt{6}\sqrt{R_1^4 - 16}} r^3 + \dots, & r \to \infty \\ \left(\mathcal{F}(v_{3a}) - \mathcal{F}(v_{3b})\right) r + \dots, & r \to 0 \end{cases} \tag{3.34}$$

---

[5]Or equivalently $(\tilde{\varphi}, v_3)$ such that $\tilde{\theta} = \psi = \pi/2$, $v_2 = \frac{2\sqrt{6}}{R_1^4 - 16} v_3$.

where v_{3a}, v_{3b} are the two values determining the range of the coordinate v_3.

$$\mathcal{F}(v_3) = \frac{N\left(R_1^4+8\right)\left(v_3\sqrt{v_3^2+\alpha}+\alpha\log\left(\sqrt{v_3^2+\alpha}+v_3\right)\right)}{\sqrt{2}q_0R_0\left(R_1^4-16\right)}, \quad \alpha = \frac{N^2q_0^2R_0^6\left(R_1^4-16\right)^2}{2\left(R_1^4+8\right)^2}. \quad (3.35)$$

The behavior is similar for the exact solution, although that is not SUSY on this cycle. In all cases the cycle blows up in the UV and contracts to zero in the IR. Here again, we should notice that the assumed range for the coordinate v_3 might imply that the cycle has a boundary. We do not report about calibrated three-cycles or higher.

4 Comments on the Quantum Field Theory

In this section, we will study some aspects of the four dimensional QFTs dual to the background we presented in eq. (2.10). Comparisons with a suitable analysis for the solution after the non-abelian T-duality written in eqs. (3.2)–(3.8), will be made when possible.

We emphasize that the field theory dual to the Type IIA backgrounds is characteristically non-local or 'higher-dimensional'. This should not come as a surprise, as it was already observed in [52], full decoupling of the gravity modes is not achieved for the case of flat D6 branes. We will make this point via the study of some observables that will be sensitive to the high energy properties of the QFT. We will analyse Wilson loops, with emphasis on its UV behavior. We will then study the entanglement entropy and central charge. Both observables will present signs of non-locality. We will also discuss the behaviour of Wilson, 't Hooft loops, domain walls and gauge couplings, when studied as IR effects. These observables are well-behaved for the solutions presented in this work. In other words, the dual QFT to our background in eq. (2.10) or our new background in eqs. (3.2)–(3.8) — together with the solutions in section 2.3, behave as QFTs that at low energies show signs of the expected four dimensional behaviour, like confinement and symmetry breaking, but need to be defined with a UV-cut off, or need a UV-completion.

Various properties are 'inherited' (in a sense that will become clear) by the new Type IIB solution that we have constructed. We will finally calculate the Page charges of this new solution. We will propose a possible quiver suggested by these charges.

It will be clear by analising the backgrounds that the initial QFT, corresponding to the compactified D6 branes has global symetries given by $SU(2) \times SU(2)$, while the QFT dual to the Type IIB background will only have $SU(2)$. This reduction of global symmetries (isometries, for the dual backgrounds) is characteristic of non-Abelian T-duality.

4.1 Some useful sub-manifolds

It will be useful for the analysis below, to define some sub-manifolds of the metric in eq. (2.10). We can define then

$$\Sigma_3 = [\theta, \varphi, \psi], \quad \tilde{\Sigma}_3 = [\tilde{\theta}, \tilde{\varphi}, \psi], \quad \hat{\Sigma}_3 = [\theta = \tilde{\theta}, \varphi = \tilde{\varphi}, \psi]. \quad (4.1)$$

The volume element of each of these cycles is (we take $3A = \phi$),

$$\sqrt{\det g_{\Sigma_3}} = 16\pi^2(\alpha'g_sN)^{3/2}e^{\phi}h(b^2+a^2g^2), \quad \sqrt{\det g_{\tilde{\Sigma}_3}} = 16\pi^2(\alpha'g_sN)^{3/2}e^{\phi}ha^2,$$

$$\sqrt{\det g_{\hat{\Sigma}_3}} = 16\pi^2(\alpha' g_s N)^{3/2} e^\phi h\left(b^2 + a^2(g^2 + 1)\right). \tag{4.2}$$

We can see using the IR expansions that each of these cycles vanish at $r \to 0$ and diverge as $r \to \infty$ for the explicit solutions presented in section 2.1.

If we consider the three-cycles after the non-Abelian T-duality, we have the submanifold defined by the coordinates $(\tilde{\theta}, \tilde{\varphi}, v_2)$. This cycle is not calibrated and probably has a boundary in the coordinate v_2.

4.2 Wilson and 't Hooft loops

The type IIA background in eq. (2.10), ends in a smooth way, with finite values for the combinations $F^2 = g_{tt}g_{xx}$, $G^2 = g_{tt}g_{rr}$. This might suggest that the system confines as usual. But there are some subtleties. Indeed, when calculating the Wilson loop with the prescription of hanging a fundamental string from a brane very far away in the UV of the geometry, we are assuming that this string will end on the D-brane satisfying the boundary condition of ending 'perpendicularly' to the brane. This is discussed, for example in [53]. Following the formalism in [53], the boundary condition boils to defining $V_{\text{eff}} = \frac{F}{G}\sqrt{F^2 - F_0^2}$ and imposing that for large values of the radial coordinate V_{eff} diverges. In our present case, $F^2 = G^2 = (\alpha' g_s N)^2 e^{\frac{4}{3}\phi}$ (we choose $\mu = 1$). The value of

$$V_{\text{eff}} \sim \sqrt{e^{4\phi/3} - e^{4\phi_0/3}}\, \alpha' g_s N$$

is a finite constant for the semi-analytic solutions. This suggests, that the QFT needs to be UV-completed or be supplemented by a hard UV-cutoff which in turn suggests that the QFT is afflicted by the presence of an irrelevant operator. Conversely, one can consider the case in which the dilaton diverges at infinity, as described by eq. (2.32). In that case, the UV-boundary conditions are satisfied, but one will find that there is a minimal length-separation for the quark-antiquark pair. For r_* close to the boundary $L_{QQ}(r_*)$ is finite, instead of vanishing. This indicates the presence of a minimal length in the dual QFT. Hence, some form of non-locality. In summary, regardless the solution we choose, the high energy behaviour of the dual field theory seems to be not the expected one for a 4-dimensional QFT.

Once assumed a UV-cutoff, the Wilson loop can be calculated. The QCD string tension is finite (suggesting confinement) and given by,

$$\sigma = \frac{1}{2\pi\alpha'}\sqrt{g_{tt}g_{xx}}|_{IR} = \frac{1}{2\pi\alpha'}e^{2A(0)} = \frac{(q_0 R_0)^2}{4\pi\alpha'}\ .$$

The components of the metric that enter this particular Wilson loop calculation are g_{tt}, g_{xx}, g_{rr}. These components are not changed by the non-Abelian T-duality. We should then expect that the comments above should be valid also for the QFT dual to the background in eq. (3.2).

In contact with the discussion on the dynamical character of the SU(2)-structure, notice that this is a consequence of the deformation of the space associated with the confining behavior. Relations of this kind have been reported in [48].

4.2.1 't Hooft loops

In a very similar way as described above, we could wrap a D4 brane on any of the three-cycles in eq. (4.1) and extend the brane on $[t, x_1]$, to form a magnetic string-like object. We propose that this object computes the 't Hooft loop in the QFT. On the type IIA side, let us consider the different three-manifolds in eq. (4.1), we will have that the effective tension of the 't Hooft string-like object is

$$\frac{T_{eff,\Sigma_3}}{16\pi^2 T_{D4}(\alpha' g_s N)^{3/2}} = e^{5A-\phi}h(b^2 + a^2 g^2)|_{r=0}.$$

$$\frac{T_{eff,\tilde{\Sigma}_3}}{16\pi^2 T_{D4}(\alpha' g_s N)^{3/2}} = e^{5A-\phi}ha^2|_{r=0}.$$

$$\frac{T_{eff,\hat{\Sigma}_3}}{16\pi^2 T_{D4}(\alpha' g_s N)^{3/2}} = e^{5A-\phi}h\big(b^2 + a^2(g^2+1)\big)|_{r=0}.$$

Notice that all these present a vanishing tension — hence screening — of the monopole-antimonopole pair. Again, the behavior of this low energy observable is in line with the expected.

We can define a screened magnetic string in the Type IIB picture. To do so, we will use the two cycle described below eq. (3.32) and wrap a D3 brane on it, also extending the brane on the two directions (t, x_1). For the effective tension we will get,

$$\frac{T_{\text{eff}}}{T_{D3}} = e^{A-\Phi} \int d\theta d\varphi \sqrt{\det[g+B]_{\Sigma_2}}|_{r=0}. \tag{4.3}$$

We observe using the asymptotics associated with this cycle a tensionless magnetic string or conversely, a 'screened' force between a pair of monopoles, as expected. Let us move to study another IR-observable.

4.3 Domain walls

In our Type IIA geometry of eq. (2.10), there is a natural two-cycle defined by

$$\Sigma_2 = [\theta = \tilde{\theta}, \varphi = \tilde{\varphi}],$$

for some fixed value of the angle $\psi = \psi_0$, which is SUSY in the IR.

The objects of potential interest to represent Domain Walls, are D4 branes that wrap the two-cycle above and that extend on the Minkowski directions (t, x_1, x_2). If this object has finite tension, then it may act as a Domain Wall, separating different vacua. Let us study the object in more detail.

The induced metric (for constant radial coordinate and constant angle ψ) is,

$$ds^2_{ind,st} = e^{2A}\Big[\mu dx^2_{1,2} + \alpha' g_s N\Big(b^2 + a^2(g^2 + 1 + 2g\cos\psi)\Big)(d\theta^2 + \sin^2\theta d\varphi^2)\Big]. \tag{4.4}$$

So, the action of the object (choosing $\mu = 1$) is,

$$S = -T_{\text{eff}} \int d^3x, \quad T_{\text{eff}} = 16\pi^2 e^{5A-\phi}(\alpha' g_s N)T_{D4}\Big(b^2 + a^2(g^2 + 1 + 2g\cos\psi_0)\Big)\Big|_{r=0}. \tag{4.5}$$

We can use the IR expansions of eq. (2.34), to check that this object has a constant tension in the far IR of the geometry. If we follow the logic presented in [54] and add a gauge field (a_1, with curvature $f_2 = da_1$) on the Minkowski part of the world volume of the brane This will create a Wess-Zumino term of the form

$$S_{WZ} = T_{D4} \int C_1 \wedge f_2 \wedge f_2 = -T_{D4} \int d\theta d\varphi F_2 \int d^3 x f_2 \wedge a_1. \tag{4.6}$$

Using that on the particular cycle $F_2 = -2N \sin\theta d\theta \wedge d\varphi$, we have induced a Chern-Simons term. These domain walls, should separate vacua coming from the breaking of some global (discrete) symmetry, see [55].

After the non-Abelian T-duality, we can define Domain Walls by using the calibrated one-cycle defined around eq. (3.28) and extend a D3 brane on the (t, x_1, x_2) directions, also wrapping the one-cycle parametrised by v_3. We will have a simple induced metric

$$ds_{D3}^2 = e^{2A}(-dt^2 + dx_1^2 + dx_2^2) + d\Sigma_1^2. \tag{4.7}$$

The Action and effective tension of this object will be given by,

$$S_{D3} = -T_{D3} e^{3A-\Phi} \sqrt{\det g_{\Sigma_1}} \int dv_3 \int d^{2+1}x,$$
$$T_{\text{eff}} = T_{D3} \int dv_3 e^{3A-\Phi} \sqrt{\det g_{\Sigma_1}}|_{r=0}. \tag{4.8}$$

Notice that imposing that the Domain Wall has a finite tension implies a finite range of values (or periodicity) for the coordinate v_3. Here again, like when we restricted the range of v_2 to avoid singularities — see around eq. (3.11), we find that a 'physical' requirement implies conditions on the range of coordinates. These conditions are not imposed by non-Abelian T-duality when thought as a solution generating technique in supergravity. In other words, the periodicty of the corodinate v_3 is being *imposed* by the requirement that the domain-wall objects in the dual QFT have finite tension. This type of requirements may give hints about the Type IIB geometry we have generated.

We can also turn on a gauge field $\mathcal{A}$ with curvature $\mathcal{F}_2$ on the $R^{1,2}$ directions. The Wess-Zumino term will read

$$S_{WZ} = \left(T_{D3} \int_{v_3} F_1|_{r=0} \right) \int d^{2+1}x \mathcal{A}_1 \wedge \mathcal{F}_2 = \kappa \int d^{2+1}x \mathcal{A}_1 \wedge \mathcal{F}_2. \tag{4.9}$$

Using that the Ramond form $C_0 = 2N(K+1)v_3$ — see below eq. (3.9) — implies that the 'charge' of the Domain Wall (or the coefficient of the Chern-Simons term induced on it) is

$$\kappa = 2N(K(0) + 1) \oint dv_3. \quad K(0) = 1 \tag{4.10}$$

Let us move now to the definition of a gauge coupling.

4.4 A gauge coupling

We can define the gauge coupling of the QFT, by wrapping a D6 brane on any of the three-cycles in eq. (4.1). We turn on a gauge field on the brane (for the argument, it is enough to turn on just F_{tx_1}), and we also turn on a pure gauge C_3-field of the form

$$C_3 = \frac{k}{16\pi^2} \sin\tilde{\theta} d\tilde{\theta} \wedge d\tilde{\varphi} \wedge d\psi,$$

we will have, for the cycle $\tilde{\Sigma}_3$ in eq. (4.1)[6] that the induced metric and Born-Infeld-Wess-Zumino-action are (we use $3A = \phi$),

$$ds^2_{\tilde{\Sigma}_3} = e^{2\phi/3}\Big[\mu dx^2_{1,3} + \alpha' g_s N\Big(a^2(\omega_1^2 + \omega_2^2) + h^2\omega_3^2\Big)\Big],$$

$$S_{BIWZ} = -T_{D6}\int e^{-\phi}\sqrt{-\det[g_{ab} + 2\pi\alpha' F_{ab}]} + T_{D6}\int C_7 + C_3 \wedge F_2 \wedge F_2. \quad (4.11)$$

$$S_{BIWZ} \sim -T_{D6}(\alpha' g_s N_c)^{3/2} 16\pi^2 \int e^{\frac{4\phi}{3}}\mu^2 h a^2 (1 - \frac{1}{2\mu^2}e^{-4\phi/3}4\pi^2\alpha'^2 F_{\mu\nu}F^{\mu\nu})$$

$$+T_{D6}k\int F_2 \wedge F_2 + T_{D6}\int C_7.$$

where the last contraction $F_{\mu\nu}F^{\mu\nu}$ is in Minkowski space and we have expanded for small field strengths (equivalently for small values of α'). This leaves us with a gauge coupling of the form,

$$\frac{1}{g_{YM}^2 N_c} = (g_s N)^{1/2}\frac{a^2 h}{2\pi^4}. \quad (4.12)$$

with asymptotic behaviour as $r \to \infty$,

$$\frac{1}{g_{YM}^2 N_c} \sim \frac{(g_s N)^{1/2}}{2\pi^4}\left(\frac{r^3}{18} - \frac{1}{2}q_1 R_1\ r^2 + \frac{3}{16}(3R_1^2 + 8q_1^2 R_1^2)\ r - \frac{3}{16}(9q_1 R_1^3 + 8q_1^3 R_1^3) + \frac{801R_1^4}{256}\frac{1}{r} + \cdots\right) \quad (4.13)$$

and as $r \to 0$

$$\frac{1}{g_{YM}^2 N_c} \sim \frac{(g_s N)^{1/2}}{2\pi^4}\left(\frac{r^3}{8} + \frac{(-8 - q_0^2)}{768R_0^2}\ r^5 + \frac{(1792 - 208q_0^2 - 93q_0^4)}{(737280R_0^4)}\ r^7 + \cdots\right) \quad (4.14)$$

Notice that there is no effect of the rescaling by μ. This is expected, because this defines a a four-dimensional gauge coupling, that should be classically invariant under dilations.

We can run this calculation for the other three-cycles defined in eq. (4.1) and get analogous expressions. All these expressions present a divergent gauge coupling in the IR — in the solution of eq. (2.32) it diverges at $\rho = a$ — while vanishing in the far UV. This should not be taken as a sign that the QFT is weakly coupled in the far UV. Indeed, these QFTs contain also superpotential couplings that make the whole system strongly interacting. This is in agreement with the dual spacetimes being weakly curved and trustable in the far UV.

After the non-Abelian T-duality, we can define a gauge coupling in the type IIB dual by using D5 branes; extend them on $R^{1,3}$ and wrapping the calibrated two cycle defined below

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[6]We found that this cycle fails to be calibrated, in far UV, by a factor of $1/2$.

eq. (3.32). We should also turn on a gauge field on the $R^{1,3}$ directions and also consider the projection of the NS B_2 field on the two-cycle. We find that this gauge coupling reads,

$$\frac{1}{g^2} = 4\pi^2 \alpha'^2 T_{D5} e^{-\Phi} \int \sqrt{\det[g_{\Sigma_2} + B_2]} \tag{4.15}$$

Using the asymptotics associated with the cycle above, we see that this gauge coupling 'confines' in the IR and vanishes in the far UV. The Wess-Zumino term for this D5 brane should define the Θ-angle.

In summary, we see that these observables, behave in the far IR as expected for a confining four dimensional QFT. Nevertheless, the Wilson loop indicates the need for a UV-completion. Below, we will briefly discuss another observable showing the same need for UV-completion.

4.5 Central charge and entanglement entropy

A couple of quantities that characterise nicely the QFT dual to a geometry are the central charge and entanglement entropy of the QFT. These quantities have been studied in many different papers. Let us quote a couple of original references [56, 57].

We will follow the systematic treatment summarised in [58]. Consider a metric of the form,

$$ds_{st}^2 = \alpha\beta dr^2 + \alpha dx_{1,d}^2 + g_{ij} dy^i dy^j, \tag{4.16}$$

we can compute the following quantities in our generic background of eq. (2.10)

$$
\begin{aligned}
V_{\text{int}} &= \int d^{8-d}y \sqrt{\det[g_{ij}]} = (4\pi)^3 b^2 a^2 h (\alpha' g_s N)^{5/2} e^{5\phi/3}, \\
\alpha &= \mu e^{2A}, \quad \beta = \frac{\alpha' g_s N}{\mu}, \quad d = 3, \\
H &= e^{-4\phi} V_{\text{int}}^2 \alpha^d = (4\pi)^6 \mu^3 b^4 a^4 h^2 (\alpha' g_s N)^5 e^{16A - 4\phi}, \\
ds_5^2 &= \kappa[dx_{1,3}^2 + dr^2], \quad \kappa^3 = H
\end{aligned}
\tag{4.17}
$$

This implies that the central charge is given by,

$$c \sim 27 N^{3/2} \frac{H^{7/2}}{(H')^3}. \tag{4.18}$$

The UV and IR behavior of the central charge for the solution with stabilized dilaton is

$$r \to \infty$$

$$\log(c) \sim -8\log(1/r) + \frac{7}{2}\log\left(\frac{R_1^2}{11664}\right) - 3\log\left(\frac{5R_1^2}{5832}\right) - \log(2) - 24\frac{q_1 R_1}{r} + \frac{\left(\frac{297R_1^2}{40} - 36q_1^2 R_1^2\right)}{r^2} + \cdots$$

$$r \to 0$$

$$\log(c) \sim 6\log(r) + \frac{7}{2}\log\frac{q_0^2 R_0^6}{64} - 3\log\frac{3q_0^2 R_0^6}{32} - \log(2) + \frac{-4 + q_0^2}{24R_0^2}r^2 + \cdots \tag{4.19}$$
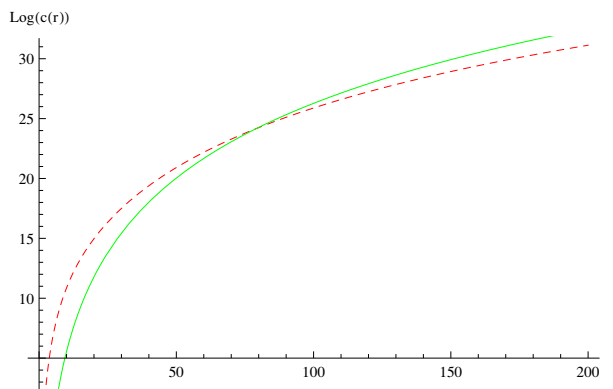
Figure 4. The central charge for a numerical solution with stabilized dilaton (red dashed curve) and for the exact solution with linear dilaton (green curve).

For comparison, we note that the central charge of the exact solution is, in the UV, $\log(c_{\text{exact}}) \sim \log(\frac{r^9}{2239488\sqrt{3}})$. In figure (4) we plot the central charge for a numerical solution with stabilized dilaton and for the exact solution with linear dilaton.

If we calculate the central charge after the non-abelian T-duality using the background of eq. (3.2), we follow [58] and write the relevant quantities are,

$$\alpha = e^{2A}\mu, \quad \beta = \frac{\alpha' g_s N_c}{\mu}, \quad V = \int d\theta d\varphi d\psi dv_1 dv_2 e^{-2\hat{\Phi}}\sqrt{g_{\text{int}}}. \tag{4.20}$$

and the we will have

$$H = V^2 \alpha^3, \quad c \sim \frac{H^{7/2}}{(H')^3}.$$

Following the algebra, one gets

$$c_{new} = \pi \mathcal{N} c_{old}.$$

Where $\mathcal{N}$ is an radius (energy) independent factor. Then, the central charges of the original and T-dual solutions differ by a constant with no much dynamical content. This can be traced to the invariance under NATD of the quantity $\sqrt{g_{initial}}e^{-\phi_{initial}}$, being equal, up to a Fadeev-Popov like factor to the same quantity in the dual background. This is explained in [33]. The Fadeev-Popov factor is associated with the scale independent number $\mathcal{N}$ above. This central charge and the entanglement entropy described below are two observables whose behavior is 'inherited' by the non-Abelian T-dualised background QFT pair.

4.5.1 Entanglement entropy

We now turn to the entanglement entropy. Consider a boundary region $\mathbb{R}^{d-1} \times \mathcal{I}_L$ where $\mathcal{I}_L$ is a line segment of length L. We calculate the entanglement entropy following [58] and obtain,

$$L(r_*) = 2\sqrt{H(r_*)N} \int_{r_*}^{\infty} \frac{dr}{\sqrt{H(r) - H(r_*)}}, \tag{4.21}$$

$$S_{\text{conn}} - S_{\text{disc}} \sim \int_{r_*}^{\infty} dr\sqrt{H}\left[\frac{\sqrt{H}}{\sqrt{H - H(r_*)}} - 1\right] - \int_{r_0}^{r_*} dr\sqrt{H}. \tag{4.22}$$

Evaluating (4.21) using the numerical solutions with stabilized dilaton found in section 2.3 we can show that $L(r_*)$ grows indefinetely and has not a maximum value. The non-existence of a maximum and hence the absence of double-valuedness for $L(\rho_*)$, suggests the absence of a first order phase transition in the entanglement entropy. This falls within the description of [60] for the entanglement entropy of non-local QFTs. Same behavior will present the background of eq. (3.2).

A tricky point that should not confuse the diligent reader is that if a UV cutoff is imposed on the geometry, numerically a double valuedness of $L(r_*)$ is obtained and correspondingly, a first order transition in the entanglement entropy will be observed. But a more detailed analysis will show that changing the position of the cutoff, moves also the position of the maximum of the separation $L(r_*)$ and the maximum of the phase transition. Hence, this is a cutoff effect and should perhaps be taken as non-physical. The resolution is that a cutoff in the radial direction is needed to solve some stability problems in the configurations that compute the Entanglement Entropy. At the same time a Volume-law for the divergent part of the Entanglement Entropy will take place. A more detailed analysis of these issues appears in [59].

4.6 Page charges

Finally, we will study some global quantities in the QFT that are defined using the background of eqs. (3.2)–(3.9). Following [61] we write some given currents at constant radial position,

$$
\begin{aligned}
\star \mathcal{J}_{D7}^{\text{Page}} &= dF_1, \\
\star \mathcal{J}_{D5}^{\text{Page}} &= d(F_3 - B_2 \wedge F_1) \\
\star \mathcal{J}_{D3}^{\text{Page}} &= d(F_5 - B_2 \wedge F_3 + \frac{1}{2} B_2 \wedge B_2 \wedge F_1).
\end{aligned}
\tag{4.23}
$$

In terms of these we can define three Page charges,

$$
Q_{D7}^{\text{page}} = \frac{1}{2\kappa_{10}^2 T_{D7}} \int_{V_2} \star \mathcal{J}_{D7}^{\text{Page}}, \quad Q_{D5}^{\text{page}} = \frac{1}{2\kappa_{10}^2 T_{D5}} \int_{V_4} \star \mathcal{J}_{D5}^{\text{Page}}, \quad Q_{D3}^{\text{page}} = \frac{1}{2\kappa_{10}^2 T_{D3}} \int_{V_6} \star \mathcal{J}_{D3}^{\text{Page}}.
\tag{4.24}
$$

where V_{9-p} is the transverse space of the corresponding Dp brane. Using Stokes theorem these may be expressed as integrals over three compact spaces. Notice that we demand that v_2 and v_3 are compact to have these charges well-defined. Let us propose the following cycles at constant radius (the coordinates not mentioned are kept at constant values),

$$
\Sigma_1 = (v_3), \quad \Sigma_3 = (\tilde{\theta}, \tilde{\varphi}, v_2 = v_3), \quad \Sigma_5 = (\tilde{\theta}, \tilde{\varphi}, v_2, v_3, \psi)
\tag{4.25}
$$

Then the Page charges may be expressed as in the paper [62, 63] by the following quantities,

$$
\begin{aligned}
Q_{D7} &= \frac{1}{4} \int F_1, \quad Q_{D5} = \frac{1}{16\pi^2} \int F_3 - B_2 \wedge F_1 \\
Q_{D3} &= \frac{1}{64\pi^4} \int F_5 - B_2 \wedge F_3 + \frac{1}{2} B_2 \wedge B_2 \wedge F_1.
\end{aligned}
\tag{4.26}
$$

We then get that the relevant quantities are,

$$
\begin{aligned}
F_1 &= -2N(K+1)dv_3 \\
F_3 - B_2 \wedge F_1 &= \sqrt{2}N(K-1)\sin\tilde{\theta}\big(v_2 dv_2 + v_3 dv_3\big) \wedge d\tilde{\theta} \wedge d\tilde{\varphi} \\
F_5 - B_2 \wedge F_3 + \tfrac{1}{2}B_2 \wedge B_2 \wedge F_1 &= 0.
\end{aligned} \tag{4.27}
$$

Performing explicitly the integrals, we get

$$
Q_{D7} = -N\hat{A}(K+1), \quad Q_{D5} = N(K-1)\hat{B}, \quad Q_{D3} = 0. \tag{4.28}
$$

Importantly, we have imposed that the range of the coordinates v_2, v_3 is finite. We have defined them as periodic with periodicity of the coordinate v_3 being $\hat{A}$ and that for v_2 being $\hat{B}$, according to,

$$
\hat{A} = \frac{1}{2}\int dv_3, \quad \hat{B} = \frac{1}{\sqrt{2}\pi}\int v_2 dv_2 \tag{4.29}
$$

The integrals are performed over the range of those variables v_2, v_3. If the manifolds in eq. (4.25) were strictly 'cycles' the Page charges above defined should all be quantised, see our comments above about the presnces of boundaries in these submanifolds. We will then impose a quantisation on a combination of Q_{D7} and Q_{D5}. Indeed, we can form the combination,

$$
Q_{\text{int}} = -(Q_{D7} + Q_{D5}) \tag{4.30}
$$

If we impose that the periods $\hat{A}, \hat{B}$ are equal and integer, we have defined a quantised quantity Q_{int}. This together with Q_{D3}, suggest a situation reminiscent of the Klebanov-Strassler QFT, with two gauge groups and one of the Page charges (that associated with D3 branes), vanishing.

This suggests that we are dealing with a two-nodes quiver, plus some bifundamental matter. It is certainly not the KS-field theory. We leave for future studies to describe the precise matter content and interactions of the bifundamental matter.

5 Conclusion and future directions

Let us start by briefly summarising what we have done in this paper. We started with backgrounds in M-theory, reduced them to Type IIA, wrote the conditions for these backgrounds to preserve minimal SUSY in four dimensions (this was material already present in the bibliography). The first piece of new material consisted in explicitly solving the differential equations with a careful numerical integration that used as boundary conditions the asymptotic solutions, obtained analytically by solving (asymptotically) the BPS system. This is why we called our solutions 'semi-analytical'. We then studied the transition between G_2 structure (in eleven dimensions) to SU(3) structure in Type IIA. We constructed explicit expressions for the potential and calibration forms.

Then, we performed a non-Abelian T-duality on this Type IIA background. We obtained a family of backgrounds in Type IIB with all Ramond and Neveu-Schwarz forms

turned on. This is a *new* family of solutions. We established its SU(2)-dynamical structure, pure spinors, calibration forms and found some calibrated cycles. Restrictions on the range of the T-dual coordinates were imposed, by requiring the smoothness of the generated space and the good behavior of field theoretical observables.

After that, we moved into the study of the correspondence between the family of Type IIA solutions and its dual QFT, also extending the study of various observables to the QFT's dual to the new family of IIB backgrounds. In this line, we made clear that the QFTs are non-local and in the need of a UV-completion (this is specially clear from the behaviour of the Wilson loop and central charges at high energies). On the other hand, observables relevant to the IR dynamics show the expected four-dimensional behaviour. Finally, based on global charges, we loosely proposed a possible two-nodes quiver describing the QFT dual to the new Type IIB background. Notice that in the logic we are advocating, the background is *defining the QFT* via its observables at strong coupling.

A couple of points emerged as specially interesting from the previous study. If we impose that some physical observables of the QFT dual to our new background behave as expected, this in turn imposes constraints on the new coordinates 'after the duality'. We also restricted the range of one of the dual coordinates v_2 in order to avoid singularities. This is not free of ambiguities, unlike the restriction imposed on v_3 to be periodic, such that the domain wall charge is quantised.

. These new coordinates originally play the role of Lagrange multipliers in the sigma model Action. Working at the genus-zero level in the sigma model gives no information on the periodicity (or not), of such new coordinates. It is quite nice to find some conditions imposing the good-behaviour of the dual QFT.

It is also quite interesting to have found an SU(2)-dynamical structure in Type IIB for a solution preserving four supercharges. It is our understanding that such backgrounds are not easy to come by. The technique presented here suggests a way of generating these and other backgrounds with similar features.

What could be nice to study (and at the same time feasible)? It seems natural to explore further the quiver structure of the QFT. It would be interesting to search in our backgrounds other well defined strong coupling effects and observables believed to appear in those QFTs. In this way, try to make sharp the dual QFT (matter content, superpotential, etc).

Even more interesting, but perhaps more difficult, would be to find a UV completion to our Type IIB dual QFT. Thinking about the lines of the papers [19–25] one may find a way to transform our system into one presenting AdS_5-like asymptotics.

Extending our results to examples in $2+1$ and $1+1$ dimensions seems a natural way to proceed. All these studies mentioned above will give important clues into the understanding of non-Abelian T-duality.

A   On numerics

Our goal is to numerically find some particular solutions of the equations

$$
\begin{aligned}
\dot{a} &= -\frac{c}{2a} + \frac{a^5 f^2}{8b^4 c^3}, & \dot{b} &= -\frac{c}{2b} - \frac{a^2(a^2 - 3c^2)f^2}{8b^3 c^3}, \\
\dot{c} &= -1 + \frac{c^2}{2a^2} + \frac{c^2}{2b^2} - \frac{3a^2 f^2}{8b^4}, & \dot{f} &= -\frac{a^4 f^3}{4b^4 c^3}.
\end{aligned}
\tag{A.1}
$$

In general this system will have four integration constants. We can find series solutions of these equations as $r \to 0$ and choose the the zeroth order term in each expansion to be the independent parameter. Thus, generally the IR expansions will have the form,

$$
a(r) \sim a_0 + a_1(a_0, b_0, c_0, f_0)r + a_2(a_0, b_0, c_0, f_0)r^2 + a_3(a_0, b_0, c_0, f_0)r^3 + \cdots \tag{A.2}
$$

and similar expressions for all the other functions. However, we are interested in solutions dual to a 4 dimensional field theory, thus we want the 3-cycle that the D6 brane wraps to shrink to zero as $r \to 0$. From the IIA metric (2.10) we see that this requirement fixes $a_0 = 0$, $c_0 = 0$ and we are left with only two independent parameters in the IR, b_0 and f_0 that we label R_0 and $q_0 R_0$ respectively. Similarly, in the UV generically we have 4 independent parameters but since we want solutions with a stabilized dilaton, we set the coefficient of the linear term in the dilaton expansion to zero and are left with three independent parameters R_1, q_1, h_1 in terms of which a UV solution to arbitrary order can be found.

To find numerical solutions we have the choice of starting in the IR and integrate forward or start in the UV and integrate backwards. We choose to solve the equations of motion starting from the IR, using the IR expansions as boundary conditions. Our motivations for doing so are two-fold. First, the parameter space in the IR is smaller, $\{R_0, q_0\}$, than the one in the UV, $\{R_1, q_1, h_1\}$, this facilitates the search of a solution with the required behavior. Second, the expansion of the equations of motion around $r = 0$ is less computationally-intensive than the one around $r \to \infty$ allowing us to use very high order expansions as boundary conditions. More precisely, in our code we use IR expansions of the functions $a(r)$, $b(r)$, $c(r)$, $f(r)$ up to order $\mathcal{O}(r^{27})$ as boundary conditions. By way of illustration, we present here the IR expansions up to order $\mathcal{O}(r^{13})$,

$$
a(r) = \frac{r}{2} - \frac{(2 + q_0^2)r^3}{(288R_0^2)} + \frac{(74 + 29q_0^2 - 31q_0^4)r^5}{(69120R_0^4)} + \frac{(-7274 + 546q_0^2 + 5043q_0^4 - 2473q_0^6)r^7}{(34836480R_0^6)}
$$

$$+ \frac{(-2767396 + 2066644q_0^2 + 1326639q_0^4 - 2267840q_0^6 + 761969q_0^8)r^9}{(60197437440R_0^8)} + \frac{P_{10}(q_0)r^{11}}{(158921234841600R_0^{10})}$$

$$- \frac{P_{12}(q_0)r^{13}}{(297500551623475200R_0^{12})}$$

$$P_{10}(q_0) = -1732820552 + 2661492292q_0^2 - 714674162q_0^4 - 1616450167q_0^6 + 1494468524q_0^8$$
$$- 388078387q_0^{10}$$

$$P_{12}(q_0) = 809180302184 - 1936619471316q_0^2 + 1686929485098q_0^4 + 13678188077q_0^6$$
$$- 1046636256642q_0^8 + 694139577405q_0^{10} - 148147907158q_0^{12} \tag{A.3}$$

$$b(r) = R_0 - \frac{(-2 + q_0^2)r^2}{(R_0 16)} - \frac{(13 - 21q_0^2 + 11q_0^4)r^4}{(1152R_0^3)} + \frac{(3268 - 8866q_0^2 + 9149q_0^4 - 3209q_0^6)r^6}{(1658880R_0^5)}$$

$$+ \frac{Pb_8(q_0)r^8}{(557383680R_0^7)} + \frac{Pb_{10}(q_0)r^{10}}{(1203948748800R_0^9)} + \frac{Pb_{12}(q_0)r^{12}}{(3814109636198400R_0^{11})}$$

$$Pb_{12}(q_0) = -96075595496 + 555977381336q_0^2 - 1393711678048q_0^4 + 1890154422552q_0^6$$
$$- 1451154850145q_0^8 + 596013842074q_0^{10} - 102144488257q_0^{12}$$

$$Pb_{10}(q_0) = 120346756 - 576435426q_0^2 + 1165086146q_0^4 - 1196194108q_0^6 + 617593365q_0^8 - 127804976q_0^{10}$$

$$Pb_8(q_0) = -235082 + 885868q_0^2 - 1355526q_0^4 + 938210q_0^6 - 244621q_0^8 \tag{A.4}$$

$$c(r) = -\frac{r}{2} + \frac{(8 - 5q_0^2)r^3}{(288R_0^2)} - \frac{(232 - 353q_0^2 + 157q_0^4)r^5}{(34560R_0^4)} + \frac{(31168 - 76440q_0^2 + 68637q_0^4 - 21286q_0^6)r^7}{(17418240R_0^6)}$$

$$+ \frac{Pc_{10}(q_0)r^{11}}{(39730308710400R_0^{10})} + \frac{Pc_8(q_0)r^9}{(15049359360R_0^8)} + \frac{Pc_{12}(q_0)r^{13}}{(74375137905868800R_0^{12})}$$

$$Pc_{10}(q_0) = 5716032512 - 24717750400q_0^2 + 44863517744q_0^4 - 41761366916q_0^6 + 19753037956q_0^8$$
$$- 3779455283q_0^{10}$$

$$Pc_8(q_0) = -7527424 + 25507072q_0^2 - 34570320q_0^4 + 21451291q_0^6 - 5080615q_0^8$$

$$Pc_{12}(q_0) = -3137711476736 + 16500424668672q_0^2 - 37556084710560q_0^4 + 46609546892530q_0^6$$
$$- 33023463748437q_0^8 + 12612429685326q_0^{10} - 2023272290207q_0^{12} \tag{A.5}$$

$$f(r) = q_0 R_0 + \frac{q_0^3 r^2}{R_0 16} + \frac{q_0^3(-14 + 11q_0^2)r^4}{(1152R_0^3)} + \frac{q_0^3(2152 - 3473q_0^2 + 1492q_0^4)r^6}{(829440R_0^5)}$$

$$+ \frac{Pf_8(q_0)r^{10}}{(1203948748800R_0^9)} + \frac{Pf_6(q_0)r^8}{(557383680R_0^7)} + \frac{Pf_{10}(q_0)r^{12}}{(238381852262400R_0^{11})}$$

$$Pf_8(q_0) = q_0^3(170283008 - 568700672q_0^2 + 744979116q_0^4 - 446064434q_0^6 + 102094739q_0^8)$$

$$Pf_6(q_0) = q_0^3(-329536 + 813288q_0^2 - 705252q_0^4 + 210349q_0^6)$$

$$Pf_{10}(q_0) = q_0^3(-8376443008 + 35383047296q_0^2 - 62151718900q_0^4 + 55981055275q_0^6 - 25662630839q_0^8$$
$$+ 4767879802q_0^{10}) \tag{A.6}$$

Using 40-digit **WorkingPrecision** in **NDSolve**, Mathematica 8, we generate, using the IR expansions as boundary conditions, solutions that extend in the UV. We observe that not for all values of $\{R_0, q_0\}$ we get solutions with stabilized dilaton. Thus, the behavior of the dilaton serves as a first indication of a potential solution with the required UV behavior. We use UV expansions up to order $\mathcal{O}(1/r^9)$ for all the functions. We show here, as an example, the UV expansion for $a(r)$.

$$a(r) = \frac{r}{\sqrt{6}} - \frac{\sqrt{3}q_1 R_1}{\sqrt{2}} + \frac{21\sqrt{3}R_1^2}{\sqrt{2}\,16r} + \frac{63\sqrt{3}q_1 R_1^3}{\sqrt{2}\,16r^2} + \frac{9\sqrt{3}\left(672q_1^2 + 221\right)R_1^4}{\sqrt{2}\,512r^3}$$

$$+ \frac{81\sqrt{3}q_1\left(224q_1^2 + 221\right)R_1^5}{\sqrt{2}\,512r^4} + \frac{\sqrt{3}\left(2048h_1 + 1377\left(768q_1^4 + 1632q_1^2 + 137\right)R_1^6\right)}{\sqrt{2}\,8192r^5} +$$

$$
+ \frac{3\sqrt{\frac{3}{2}}q_1 R_1(10240 h_1 + 81(11645 + 68000 q_1^2 + 22272 q_1^4)R_1^6)}{8192 r^6}
$$

$$
+ \frac{27\sqrt{\frac{3}{2}}R1^2(8192 h_1(27 + 560 q1^2) + 27(583399 + 17765952 q_1^2 + 68376576 q_1^4 + 20299776 q_1^6)R_1^6)}{3670016 r^7}
$$

$$
+ \frac{27\sqrt{\frac{3}{2}}q_1 R_1^3(8192 h_1(81 + 560 q_1^2) + 81(583399 + 7332032 q_1^2 + 20121600 q_1^4 + 5849088 q_1^6)R_1^6)}{524288 r^8}
$$

$$
\frac{9\sqrt{\frac{3}{2}}R_1^4}{8388608 \; r^9} Pa_9^{UV}(q_1, R_1, h_1) + \cdots \tag{A.7}
$$

where,

$$
\begin{aligned}
Pa_9^{UV}(q_1, R_1, h_1) = \big(& 4096 h_1(3941 + 93312 q1^2 + 322560 q_1^4) \\
& + 243(3297681 + 129163840 q_1^2 + 912975360 q_1^4 + 1851260928 q_1^6 + 528482304 q_1^8)R_1^6)\big)
\end{aligned} \tag{A.8}
$$

We then have to analyze if this candidate solution obtained by forward integration has indeed a UV where the functions are given by eq. (2.33) or not. To this end, we define a mismatch function,

$$
m = \sum_i \Big(\log\Big(|f_i^{numerical}(r_{match})|\Big) - \log\Big(|f_i^{expansion}(r)|\Big) \Big)^2, \tag{A.9}
$$

where $f_i \in \{a, \, b, \, c, \, f\}$, $f_i^{numerical}$ refers to the solution obtained by forward integration and $f_i^{expansion}$ refers to the UV expansion. We then minimize m using **NMinimize** and **AccuracyGoal** $= 20$. If the minimization procedure yields a small value ($m \leq 10^{-4}$) this setup determines the UV parameters R_1, q_1, h_1 for which our numerical solution has the required UV behavior. Some sample solutions obtained with this procedure are presented in figure 2. Note that we choose to normalize the dilaton such that

$$
(g_s N)^{3/4} e^{2\phi_0/3} = 1 \tag{A.10}
$$

where $\phi_0 \equiv \phi(r = 0)$.

A natural question to ask is to what extent integrating back with the parameters found through the minimization procedure will reproduce the integrated forward solution. Since the IR expansions are of very high order ($\mathcal{O}(r^{27})$) while the UV expansions are only of order $\mathcal{O}(1/r^9)$ we expect that the UV solution will not be very accurate in the IR. We present plots comparing the backward and forward integrated solutions in figure 5. In order to verify that the small discrepancies in the IR are due to accumulated numerical error we evaluate the residual. Namely, we define a function res_k that evaluates the equation of motion for $k(r)$ using the numerical solution. If the solution were exact res_k should be identically zero. Since it is a numerical solution there will always be certain deviation form zero.

$$
\begin{aligned}
res_a(r) &= \left| \dot{a}_{\text{num}} + \frac{c_{\text{num}}}{2a_{\text{num}}} - \frac{a_{\text{num}}^5 f_{\text{num}}^2}{8b_{\text{num}}^4 c_{\text{num}}^3} \right|, \\
res_b(r) &= \left| \dot{b}_{\text{num}} + \frac{c_{\text{num}}}{2b_{\text{num}}} + \frac{a_{\text{num}}^2(a_{\text{num}}^2 - 3c_{\text{num}}^2)f_{\text{num}}^2}{8b_{\text{num}}^3 c_{\text{num}}^3} \right|, \\
res_c(r) &= \left| \dot{c}_{\text{num}} + 1 - \frac{c_{\text{num}}^2}{2a_{\text{num}}^2} - \frac{c_{\text{num}}^2}{2b_{\text{num}}^2} + \frac{3a_{\text{num}}^2 f_{\text{num}}^2}{8b_{\text{num}}^4} \right|,
\end{aligned}
$$
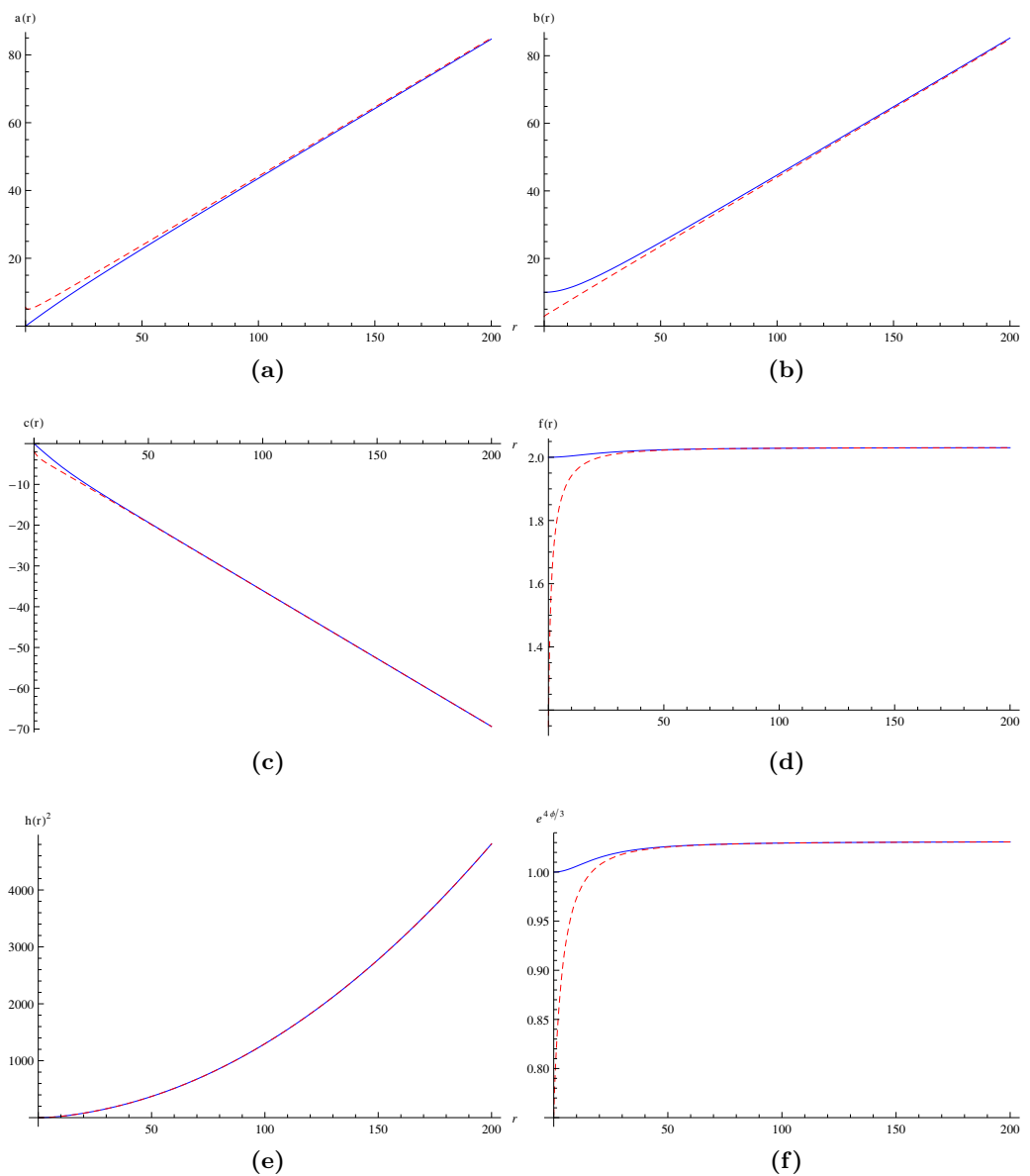
Figure 5. The blue curves are the result of forward integration with $R_0 = 10$, $q_0 = 1/5$. After the minimization procedure we obtain the UV parameters $q_1 = 1.31946$, $R_1 = -2.03087$, $h_1 = -1.9733$ and plot (dashed red lines) the result of integrating back with these parameters to show that it coincides with the forward integration. The small discrepancies in the IR are due to accumulated numerical error. The mismatch function for this solution is $m < 10^{-4}$. We also plot $h(r)^2$ and $e^{4\phi/3}$ defined in (2.10).
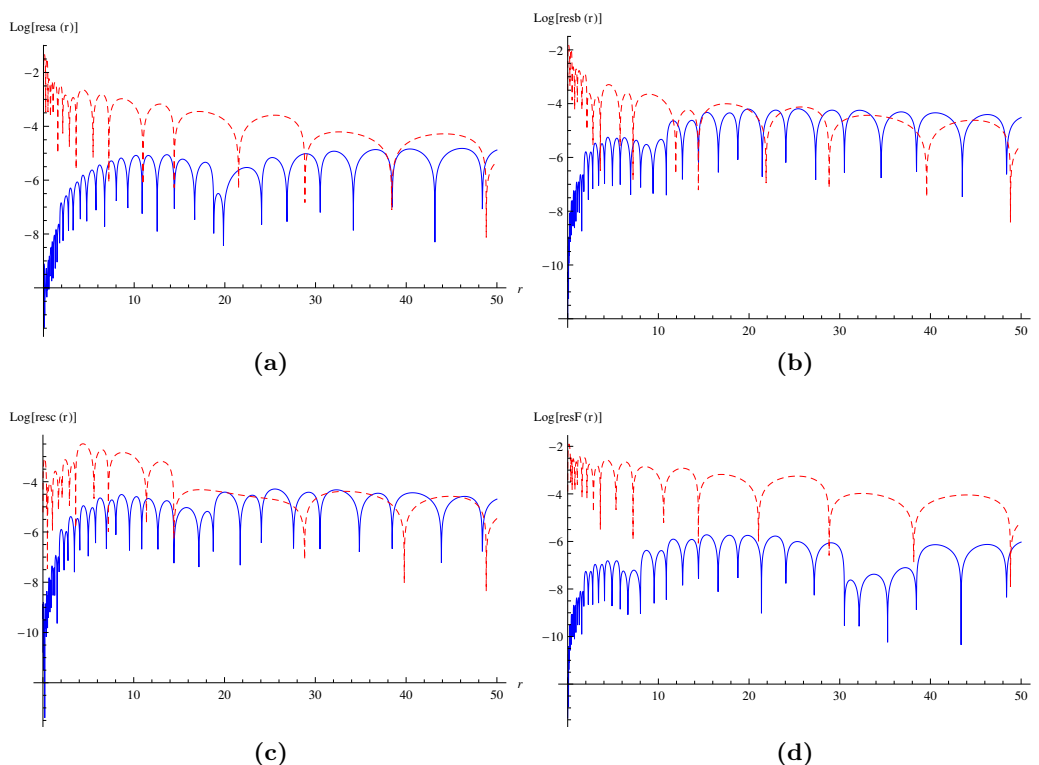
Figure 6. $\log_{10}$ plot of the residuals defined in (A.11). The solid blue line is for the solution obtained by integrating forward (IR to UV) , dashed line is for the solution obtained by integrating from the UV back to the IR.

$$res_f(r) = \left| \dot{f}_{\mathrm{num}} + \frac{a_{\mathrm{num}}^4 f_{\mathrm{num}}^3}{4b_{\mathrm{num}}^4 c_{\mathrm{num}}^3} \right|. \tag{A.11}$$

In figure 6 we see that the integrated forward solution is more accurate for all values of r. Also note, (figure 6 a, b and d) that the integrated back solution fails considerably close to the IR ($resa(r_{IR}) \sim 10^{-2}$) and this explains the differences in figure 5.

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References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[2] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [INSPIRE].

[4] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB resolution of naked singularities*, *JHEP* **08** (2000) 052 [`hep-th/0007191`] [ɪɴSPIRE].

[5] J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure N = 1 super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [`hep-th/0008001`] [ɪɴSPIRE].

[6] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [`hep-th/9803131`] [ɪɴSPIRE].

[7] M. Atiyah, J.M. Maldacena and C. Vafa, *An M-theory flop as a large-N duality*, *J. Math. Phys.* **42** (2001) 3209 [`hep-th/0011256`] [ɪɴSPIRE].

[8] B.S. Acharya and E. Witten, *Chiral fermions from manifolds of G_2 holonomy*, `hep-th/0109152` [ɪɴSPIRE].

[9] M. Atiyah and E. Witten, *M theory dynamics on a manifold of G_2 holonomy*, *Adv. Theor. Math. Phys.* **6** (2003) 1 [`hep-th/0107177`] [ɪɴSPIRE].

[10] A. Brandhuber, *G_2 holonomy spaces from invariant three forms*, *Nucl. Phys.* **B 629** (2002) 393 [`hep-th/0112113`] [ɪɴSPIRE].

[11] M. Cvetič, G.W. Gibbons, H. Lü and C.N. Pope, *A G_2 unification of the deformed and resolved conifolds*, *Phys. Lett.* **B 534** (2002) 172 [`hep-th/0112138`] [ɪɴSPIRE].

[12] A. Brandhuber, J. Gomis, S.S. Gubser and S. Gukov, *Gauge theory at large-N and new G_2 holonomy metrics*, *Nucl. Phys.* **B 611** (2001) 179 [`hep-th/0106034`] [ɪɴSPIRE].

[13] M. Cvetič, G.W. Gibbons, H. Lü and C.N. Pope, *Special holonomy spaces and M-theory*, `hep-th/0206154` [ɪɴSPIRE].

[14] R. Hernandez and K. Sfetsos, *An eight-dimensional approach to G_2 manifolds*, *Phys. Lett.* **B 536** (2002) 294 [`hep-th/0202135`] [ɪɴSPIRE].

[15] J.P. Gauntlett, D. Martelli, S. Pakis and D. Waldram, *G structures and wrapped NS5-branes*, *Commun. Math. Phys.* **247** (2004) 421 [`hep-th/0205050`] [ɪɴSPIRE].

[16] P. Koerber and D. Tsimpis, *Supersymmetric sources, integrability and generalized-structure compactifications*, *JHEP* **08** (2007) 082 [`arXiv:0706.1244`] [ɪɴSPIRE].

[17] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *JHEP* **08** (2004) 046 [`hep-th/0406137`] [ɪɴSPIRE].

[18] L. Martucci and P. Smyth, *Supersymmetric D-branes and calibrations on general N = 1 backgrounds*, *JHEP* **11** (2005) 048 [`hep-th/0507099`] [ɪɴSPIRE].

[19] J. Maldacena and D. Martelli, *The unwarped, resolved, deformed conifold: fivebranes and the baryonic branch of the Klebanov-Strassler theory*, *JHEP* **01** (2010) 104 [`arXiv:0906.0591`] [ɪɴSPIRE].

[20] J. Gaillard, D. Martelli, C. Núñez and I. Papadimitriou, *The warped, resolved, deformed conifold gets flavoured*, *Nucl. Phys.* **B 843** (2011) 1 [`arXiv:1004.4638`] [ɪɴSPIRE].

[21] E. Caceres, C. Núñez and L.A. Pando-Zayas, *Heating up the baryonic branch with U-duality: a unified picture of conifold black holes*, *JHEP* **03** (2011) 054 [`arXiv:1101.4123`] [ɪɴSPIRE].

[22] E. Caceres and S. Young, *Stability of non-extremal conifold backgrounds with sources*, *Phys. Rev.* **D 87** (2013) 046006 [`arXiv:1205.2397`] [ɪɴSPIRE].

[23] S. Bennett, E. Caceres, C. Núñez, D. Schofield and S. Young, *The non-SUSY baryonic branch: soft supersymmetry breaking of $N = 1$ gauge theories*, *JHEP* **05** (2012) 031 [arXiv:1111.1727] [INSPIRE].

[24] D. Elander, J. Gaillard, C. Núñez and M. Piai, *Towards multi-scale dynamics on the baryonic branch of Klebanov-Strassler*, *JHEP* **07** (2011) 056 [arXiv:1104.3963] [INSPIRE].

[25] E. Conde, J. Gaillard, C. Núñez, M. Piai and A.V. Ramallo, *A tale of two cascades: higgsing and Seiberg-duality cascades from type IIB string theory*, *JHEP* **02** (2012) 145 [arXiv:1112.3350] [INSPIRE].

[26] X.C. de la Ossa and F. Quevedo, *Duality symmetries from non-Abelian isometries in string theory*, *Nucl. Phys.* **B 403** (1993) 377 [hep-th/9210021] [INSPIRE].

[27] E. Alvarez, L. Álvarez-Gaumé, J.L.F. Barbon and Y. Lozano, *Some global aspects of duality in string theory*, *Nucl. Phys.* **B 415** (1994) 71 [hep-th/9309039] [INSPIRE].

[28] A. Giveon and M. Roček, *On non-Abelian duality*, *Nucl. Phys.* **B 421** (1994) 173 [hep-th/9308154] [INSPIRE].

[29] Y. Lozano, *Non-Abelian duality and canonical transformations*, *Phys. Lett.* **B 355** (1995) 165 [hep-th/9503045] [INSPIRE].

[30] K. Sfetsos, *Gauged WZW models and non-Abelian duality*, *Phys. Rev.* **D 50** (1994) 2784 [hep-th/9402031] [INSPIRE].

[31] K. Sfetsos and D.C. Thompson, *On non-Abelian T-dual geometries with Ramond fluxes*, *Nucl. Phys.* **B 846** (2011) 21 [arXiv:1012.1320] [INSPIRE].

[32] Y. Lozano, E. O Colgain, K. Sfetsos and D.C. Thompson, *Non-Abelian T-duality, Ramond fields and coset geometries*, *JHEP* **06** (2011) 106 [arXiv:1104.5196] [INSPIRE].

[33] G. Itsios, C. Núñez, K. Sfetsos and D.C. Thompson, *Non-Abelian T-duality and the AdS/CFT correspondence: new $N = 1$ backgrounds*, *Nucl. Phys.* **B 873** (2013) 1 [arXiv:1301.6755] [INSPIRE].

[34] Y. Lozano, E. Ó Colgáin, D. Rodríguez-Gómez and K. Sfetsos, *Supersymmetric AdS_6 via T duality*, *Phys. Rev. Lett.* **110** (2013) 231601 [arXiv:1212.1043] [INSPIRE].

[35] G. Itsios, C. Núñez, K. Sfetsos and D.C. Thompson, *On non-Abelian T-duality and new $N = 1$ backgrounds*, *Phys. Lett.* **B 721** (2013) 342 [arXiv:1212.4840] [INSPIRE].

[36] N.T. Macpherson, *Non-Abelian T-duality, G_2-structure rotation and holographic duals of $N = 1$ Chern-Simons theories*, *JHEP* **11** (2013) 137 [arXiv:1310.1609] [INSPIRE].

[37] E. Gevorgyan and G. Sarkissian, *Defects, non-Abelian T-duality and the Fourier-Mukai transform of the Ramond-Ramond fields*, *JHEP* **03** (2014) 035 [arXiv:1310.1264] [INSPIRE].

[38] Y. Lozano, E.O. Colgain and D. Rodriguez-Gomez, *Hints of 5d fixed point theories from non-Abelian T-duality*, arXiv:1311.4842 [INSPIRE].

[39] S. Zacarías, *Semiclassical strings and non-Abelian T-duality*, arXiv:1401.7618 [INSPIRE].

[40] K. Sfetsos, *Integrable interpolations: from exact CFTs to non-Abelian T-duals*, *Nucl. Phys.* **B 880** (2014) 225 [arXiv:1312.4560] [INSPIRE].

[41] P. Kaste, R. Minasian, M. Petrini and A. Tomasiello, *Kaluza-Klein bundles and manifolds of exceptional holonomy*, *JHEP* **09** (2002) 033 [hep-th/0206213] [INSPIRE].

[42] J. Gaillard and J. Schmude, *The lift of type IIA supergravity with D6 sources: M-theory with torsion*, *JHEP* **02** (2010) 032 [arXiv:0908.0305] [INSPIRE].

[43] J.D. Edelstein and C. Núñez, *D6-branes and M-theory geometrical transitions from gauged supergravity*, *JHEP* **04** (2001) 028 [hep-th/0103167] [INSPIRE].

[44] D. Andriot, *New supersymmetric flux vacua with intermediate* SU(2) *structure*, *JHEP* **08** (2008) 096 [arXiv:0804.1769] [INSPIRE].

[45] R. Minasian, M. Petrini and A. Zaffaroni, *Gravity duals to deformed SYM theories and generalized complex geometry*, *JHEP* **12** (2006) 055 [hep-th/0606257] [INSPIRE].

[46] A. Butti et al., *On the geometry and the moduli space of beta-deformed quiver gauge theories*, *JHEP* **07** (2008) 053 [arXiv:0712.1215] [INSPIRE].

[47] J. McOrist, D.R. Morrison and S. Sethi, *Geometries, non-geometries and fluxes*, *Adv. Theor. Math. Phys.* **14** (2010) [arXiv:1004.5447] [INSPIRE].

[48] J. Gaillard, N.T. Macpherson, C. Núñez and D.C. Thompson, *Dualising the baryonic branch: dynamic* SU(2) *and confining backgrounds in IIA*, *Nucl. Phys.* **B 884** (2014) 696 [arXiv:1312.4945] [INSPIRE].

[49] D. Andriot, *String theory flux vacua on twisted tori and generalized complex geometry*, Ph.D. thesis, Université Pierre et Marie Curie Paris 6, Paris France (2010) [INSPIRE].

[50] M. Petrini and A. Zaffaroni, $N = 2$ *solutions of massive type IIA and their Chern-Simons duals*, *JHEP* **09** (2009) 107 [arXiv:0904.4915] [INSPIRE].

[51] A. Barranco, J. Gaillard, N.T. Macpherson, C. Núñez and D.C. Thompson, *G-structures and flavouring non-Abelian T-duality*, *JHEP* **08** (2013) 018 [arXiv:1305.7229] [INSPIRE].

[52] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large-N limit of theories with sixteen supercharges*, *Phys. Rev.* **D 58** (1998) 046004 [hep-th/9802042] [INSPIRE].

[53] C. Núñez, M. Piai and A. Rago, *Wilson loops in string duals of walking and flavored systems*, *Phys. Rev.* **D 81** (2010) 086001 [arXiv:0909.0748] [INSPIRE].

[54] B.S. Acharya and C. Vafa, *On domain walls of* $N = 1$ *supersymmetric Yang-Mills in four-dimensions*, hep-th/0103011 [INSPIRE].

[55] U. Gürsoy, S.A. Hartnoll and R. Portugues, *The chiral anomaly from M-theory*, *Phys. Rev.* **D 69** (2004) 086003 [hep-th/0311088] [INSPIRE].

[56] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of* $N = 4$ *super Yang-Mills from AdS dynamics*, *JHEP* **12** (1998) 022 [hep-th/9810126] [INSPIRE].

[57] S. Ryu and T. Takayanagi, *Holographic derivation of entanglement entropy from AdS/CFT*, *Phys. Rev. Lett.* **96** (2006) 181602 [hep-th/0603001] [INSPIRE].

[58] I.R. Klebanov, D. Kutasov and A. Murugan, *Entanglement as a probe of confinement*, *Nucl. Phys.* **B 796** (2008) 274 [arXiv:0709.2140] [INSPIRE].

[59] U. Kol, C. Núñez, D. Schofield, J. Sonnenschein and M. Warschawski, *Confinement, phase transitions and non-locality in the entanglement entropy*, *JHEP* **06** (2014) 005 [arXiv:1403.2721] [INSPIRE].

[60] J.L.F. Barbon and C.A. Fuertes, *Holographic entanglement entropy probes (non)locality*, *JHEP* **04** (2008) 096 [arXiv:0803.1928] [INSPIRE].

[61] D. Marolf, *Chern-Simons terms and the three notions of charge*, hep-th/0006117 [INSPIRE].

[62] F. Benini, F. Canoura, S. Cremonesi, C. Núñez and A.V. Ramallo, *Backreacting flavors in the Klebanov-Strassler background*, *JHEP* **09** (2007) 109 [arXiv:0706.1238] [INSPIRE].

[63] F. Benini, F. Canoura, S. Cremonesi, C. Núñez and A.V. Ramallo, *Unquenched flavors in the Klebanov-Witten model*, *JHEP* **02** (2007) 090 [hep-th/0612118] [INSPIRE].

[64] Y. Lozano and N.T. Macpherson, *A new AdS$_4$/CFT$_3$ dual with extended SUSY and a spectral flow*, arXiv:1408.0912 [INSPIRE].

[65] N. Macpherson, C. Nunez, L. Pando-Zayas, V. Rodgers and C. Withing, *Type IIB supergravity solutions with AdS$_5$ from Abelian and non-Abelian T-dualities*, to appear.