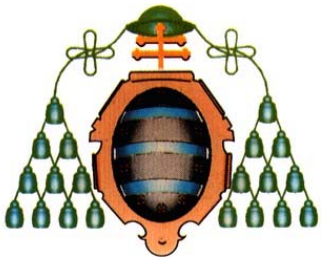
UNIVERSIDAD DE OVIEDO
FACULTAD DE FORMACIÓN DEL PROFESORADO Y EDUCACIÓN

ASTURIAS
CAMPUS DE EXCELENCIA
INTERNACIONAL
| AD FUTURUM |



Universidad de Oviedo

Facultad de Formación del Profesorado y Educación

Trabajo Fin del Máster en Enseñanza Integrada de Lengua Inglesa y Contenidos: Educación Infantil y Primaria

MODEL FOR THE TEACHING AND LEARNING OF BILINGUAL MATHEMATICS IN A PRIMARY SCHOOL

Marcos Zurbano Fernández

Tutores:

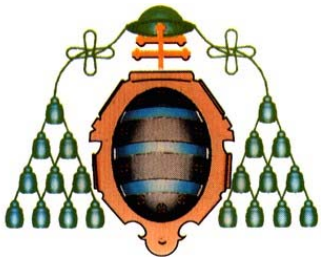
Robert Deuchar

Daniel García Velasco

Julio 2014

UNIVERSIDAD DE OVIEDO
FACULTAD DE FORMACIÓN DEL PROFESORADO Y EDUCACIÓN

ASTURIAS
CAMPUS DE EXCELENCIA
INTERNACIONAL
| AD FUTURUM |



Universidad de Oviedo

Facultad de Formación del Profesorado y Educación

Trabajo Fin del Máster en Enseñanza Integrada de Lengua Inglesa y Contenidos: Educación Infantil y Primaria

MODEL FOR THE TEACHING AND LEARNING OF BILINGUAL MATHEMATICS IN A PRIMARY SCHOOL

Marcos Zurbano Fernández

Tutores:

Robert Deuchar

Daniel García Velasco

Julio 2014

INDEX

1. INTRODUCTION AND JUSTIFICATION.

An analysis of a proposition for a planning of teaching and learning of Bilingual Mathematics in a Spanish Primary school is developed through this master's thesis, under the Máster en Enseñanza Integrada de Lengua Inglesa y Contenidos from the Universidad de Oviedo. That is the most important part of the work and the largest one by far, although there are other introductory and concluding sections. The main intention through this thesis is to determine a possible model for the total implementation of this subject in Primary (although only one hypothetical school is initially taken as reference to do this analysis), proposing new ideas, considering other ones related to the traditional subjects of Mathematics and English, focusing on theoretical and practical aspects, and analyzing advantages and difficulties.

All of these ideas, which are taken into account to carry out this project, are developed based on the Content and Language Integrated Learning principles (CLIL is the main approach on which the Bilingual Education is based in Spain and in other countries) and on related aspects acquired during my time as a student of the master, on my experience as a teacher in a public Primary school through the Practicum and on information extracted from different sources, such us one particular Maths textbook that is widely used, and above all, the current Spanish Curriculum in Mathematics and English, and the *National Numeracy Strategy. Framework for Teaching Mathematics from Reception to Year 6* UK document, known as NNS. This last one was considered in the British Primary schools after 1999 and included an outline of expected teaching in Mathematics for all pupils at this stage.

This theme (the idea to introduce the subject of Bilingual Mathematics) has been chosen for different reasons. On the one hand, traditional Mathematics has always been taught, as it is one of the most important subjects in any country. On the other hand, the English language has gained special importance in the Spanish education system, such an extent that Bilingual Education has been strongly implemented inrecent years. And above all, in contrast to this, the main reason is that the subject of Bilingual Mathematics has hardly been taught in the Spanish Primary Education system because it is thought that the level of difficulty would be too high. Considering all these facts, researching into the idea of implementing this new subject can be really challenging and interesting in my opinion: there is much to explore, there are supposed to be more difficulties to overcome than when implementing for example Art, Science or PE, and this option may be useful for the future, provided that proper planning is established, which is essential. It may be useful because it is a new bilingual subject that includes Mathematics and English, so that all of the important contents from

these two basic disciplines would be acquired simultaneously for the first time in almost all Primary schools (at present, this subject is taught in very few ones). This would be a new important advance in Bilingual Education, but it would be vital to be extremely careful when developing it because of the possible difficulties that can be found.

Thus, this thesis aims to establish guidelines in order to develop attractive proposals to students. The idea is to perform a useful planning avoiding the traditional rejection from part of the Spanish pupils to the subject of English and, above all, to Mathematics.

Finally, it is necessary to indicate that the thesis is organized in different parts, which are listed now:

- ✓ Apart from a cover sheet and an index, this general introduction.
- ✓ Some first sections referring to the process of learning and teaching of traditional Mathematics and the introduction of Bilingual Education.
- ✓ A section in which references about the main documents that are used in the proposal are made.
- ✓ The main part: the proposal. It will be based on the hypothetical case that I was a new teacher in a Spanish bilingual school (upper-middle class students, good previous level of English) and that other educators and I had to teach Bilingual Mathematics for the first time, that being a new subject in this school. However, it would be an evolution from the traditional subject of Mathematics (and a substitute for it). The assumption would be that my students were the second cycle ones. The idea would be that, as the subject would be newly introduced, the other teachers and I thought it useful to establish all together a base programme for it, from which each specialist would develop its teaching individually, and that would be useful for future years and educators. That is my intention. A programme that would be used as a guide, that would emphasize the main introductory and general aspects of the subject, that would indicate how the basic competences are developed and that would establish the objectives to reach and the methodology to follow, in addition to the contents (which are organized in blocks) and the assessment criteria to consider in each cycle. In my case, as I would only teach third and fourth year students, I would only have to focus on the second cycle contents and evaluation criteria. The teachers of the children from the other cycles would be the responsible ones for detailing the contents and criteria that correspond to them. A programme that, although it would be planned as a single guide for a bilingual school, will include references to the first sections of this work,

as it is part of this thesis and it is connected to previously developed information. Then, I will also establish a practical proposition to apply and justify many of these aspects, containing a lesson plan with sessions (similar to a didactic unit) to develop in one of those groups (fourth year group) in which I would work hypothetically. A recent fourth year Mathematics textbook will be taken as reference. And, as it has just been mentioned, many theoretical aspects developed in the programme will be justified and put into practice, but not all of them (for example, it is impossible to refer to all of the objectives, contents and other criteria of the whole cycle if I develop a lesson plan with certain sessions for the fourth year). The main and final objective is to determine a new and hypothetical subject that could be very interesting for the future (for this actual future, in which the Bilingual Education is being introduced in Spain). Thus, my idea is to explain the characteristics that I consider interesting for this bilingual subject through the consideration of this hypothetical case.

- ✓ A section to analyze possible advantages and disadvantages.
- ✓ A conclusion.
- ✓ The bibliography.

2. TRADITIONAL (NON-BILINGUAL) MATHEMATICS IN PRIMARY SCHOOLS. LEARNING AND TEACHING GENERAL ASPECTS.

The subject of Mathematics is one of the classic ones that have always been taught. In fact, it is considered essential at all levels, and for that reason, it is compulsory and given in the six courses throughout Primary Education. Like the rest of the subjects, it is planned based on the curriculum standards. The objectives, the contents (which are organized in blocks), the evaluation criteria and the methodological and assessment guidelines change in Primary according to each cycle, although there are still similarities between them.

In this second section of the thesis, general aspects related to the learning and teaching of Mathematics in Primary schools are taken into account. Reference to general ideas of certain authors and to certain works or documents is also made. And although it is true that these considerations were and have been initially determined to be applied in the teaching and learning of the traditional and non-bilingual Mathematics (simply because the bilingual ones have hardly been implemented), they would be equally useful even if the subject became a bilingual one (although new aspects related to the learning and teaching through the introduction of the CLIL approachwould also be incorporated). The objectives and

mathematical content must remain the same although the current subject was adapted as a bilingual one, as well as many methodological and assessment aspects, such as the criteria. Thus, the two subjects would be very similar, and many considerations of teaching and learning must be taken into account in both of them (after all, Mathematics will remain the subject). Therefore, some ideas about teaching and learning are mentioned in the next paragraphs because they will be considered during the development of the bilingual proposition throughout the following sections.

When the methodology of this new subject is analyzed in detail and the practical proposition is developed, references to these general considerations relating to learning and teaching will be made again, so it will be clear that they will be taken into account. It will be shown that they can be applied, both in the traditional subject of Mathematics, and in the Bilingual Mathematics one.

2.1. ASPECTS RELATED TO THE LEARNING OF MATHEMATICS IN PRIMARY.

Discussing the aspects related to the process of learning involves, above all, referring to the theories of this one.

Historically, two main types of theories, known as behaviorism and constructivism, have been developed. Both are quite opposite, although it has been tried to reconcile some aspects through recent works. The fact of adopting this classification does not mean that there are only two theories of learning: both names are like families hosting a multitude of very different theories, but with certain common characteristics that allow us tounify them.

Today, more importance is given to constructivism than to behaviourism when learning Mathematics. This does not imply that the behaviorist theory is discarded, but in this case that one is not the main one. Constructivism is not only so considered in Mathematics or in Primary Education, but these are the subject and the stage analyzed through this thesis. And although traditional Mathematics give way to the bilingual ones, these constructivist ideas will continue to be taken into account, as it will be demonstrated through the development of the bilingual proposition in future sections.

Referring to constructivism means referring to authors. And although there were several who developed this theory, I will focus on two: Piaget, one of the fathers of it, and Bruner, author with interesting considerations from a mathematical point of view.

Piaget, the first one, considers that the construction of knowledge is possible because of the constant interaction of the internal cognitive factors and environmental factors. According to him, this construction is given through a process in which the information that can be acquired and apprehended is determined largely by what is already known. Thus, the acquisition of new knowledge is achieved.

In general, it is accepted that each child evolves following a concrete sequence of stages, through a transition that is extended from the initial reliance on the real and concrete concepts to the ability to understand the meaning of abstract propositions that are formulated in a symbolic way.

And Bruner, the other one, is a cognitive psychologist whose research, related to the formation of concepts, is developed on the basis of the genetic epistemology of Piaget. Cognitive structures are considered by Bruner as combinations of concepts acquired and skills thought, so that in the field of mathematics the basic objects are discovered experimentally in the medium in which the student is. In this way, simple structures formed by few concepts are obtained. And later, more elaborate structures are formed, while incorporating new concepts through a process of knowledge organization.

For this author, the most important aspects in the development of thought are the forms of representation that are used, which involve three stages of learning:

- ✓ Enactive: it is an essentially manipulative experimental stage. Real objects are used and the representation is carried out through the action.
- ✓ Iconic: manipulative action gives way to a graphical language, through drawings.
- ✓ Symbolic: symbolic forms of a higher level are used, such as mathematical signs.

2.2. ASPECTS RELATED TO THE TEACHING OF MATHEMATICS IN PRIMARY.

The learning and teaching of Mathematics in Primary Education go hand in hand, so that both are equally important. From quite a general point of view of the teaching of Mathematics, without going into very precise methodological details relative to the traditional subject or into those others which will be analyzed later with the introduction of the bilingual subject, I consider it important to mention in this section a known British mathematician (Cockcroft), author with other specialists of the reference book *Mathematics Counts* (Cockcroft Committee,1982) (known as Cockcroft Report), document from which interesting information on this subject is extracted. Subsequently, other aspects of teaching based on

information from the *Diseño Curricular Base* (MEC, 1989) and the current Curriculum of Mathematics (MECD, 2007) are also mentioned. Thus, reference to general ideas about teaching gradually collected over the years that are considered interesting today is made. And these are also aspects that will be taken into account in the proposition of the bilingual subject, and that is the reason why they are mentioned.

The Cockcroft Report is considered as one of the most important documents related to the teaching of Mathematics. It consists of many sections, two of which are going to be taken into account: numbers 242 and 243.

When Cockcroft discusses the issues related to classroom practice, he makes two assertions in these two paragraphs of his work. On the one hand, he claims that there is not any optimal methodology. On the other one, he also considers that there are certain elements that must be present in all successful teaching of Mathematics. These points are taken from that publication and mentioned below:

'We are aware that there are some teachers who would wish us to indicate a definitive style for the teaching of Mathematics, but we do not believe that this is either desirable or possible'.

'Mathematics teaching at all levels should include opportunities for:

- ✓ *Exposition by the teacher.*
- ✓ *Discussion between teacher and pupils and between pupils themselves.*
- ✓ *Appropriate practical work.*
- ✓ *Consolidation and practice of fundamental skills and routines.*
- ✓ *Problem solving, including the application of Mathematics to everyday situations.*
- ✓ *Investigational work'.*

I mention these two ideas because I consider that they are basic when teaching Mathematics, in the eighties and now. These are also aspects to be taken into account at all levels, although the stage analyzed through this thesis is the Primary Education one.

Thus, referring only to this phase, references to considerations related to the teaching process developed in Spain years after the Cockcroft Report have been made. In 1989, through the *Área de Matemáticas para la Educación Primaria* of the *Diseño Curricular Base*, it is proposed that the process of construction of the mathematical knowledge should begin considering each student's own basic experience. Then, it has to be continued with a

reflection on each activity, so that later the student will be led to the more abstract, formal and deductive levels of mathematical knowledge. These are the guidelines recommended:

- ✓ Give priority to practical and oral work, introducing the decontextualized activities and written work (use of symbolic notation) only when students demonstrate an understanding of the mathematical concepts and an interest in them.
- ✓ Give priority to mental work (especially to mental calculations), in order to go in-depth on the intuitive mathematical knowledge, just before moving into their formalization.
- ✓ Pay particular attention to the development of personal strategies of problem solving, promoting the inclusion in these ones of the mathematical knowledge that will be acquired (graphs and numerical representations, registration of alternatives explored, simplification of problems, etc.).
- ✓ Use the various areas of experiences of students (in and out of school) as a source of mathematical experiences.

Nowadays, all of the aspects related to the teaching of Mathematics that have been indicated are still taken into account. However, other new considerations have also been made over the last few years.

In Spain, different methodological guidelines have been determined through the different Mathematics Curriculums established based on each of the educational laws. Today, the law in force is the *Ley Orgánica de Educación* (LOE) (2006). In relation, considering the Mathematics Curriculum of Primary Education (MECD, 2007), which was defined when this law came into effect, the methodological issues that are included in it are mentioned below. Reference to some aspects already mentioned and to other new ones is made:

- ✓ Much importance is given to two features of mathematics that are considered preponderant; the instrumental aspect and the formative aspect. On the one hand, the mathematical content which is acquired by the students, as well as the procedures and strategies used, have to be useful for them to solve simple problems of daily life and in the communication processes in other curricular areas. On the other hand, as it has already been mentioned, a cognitive development of students is intended, giving importance to the inductive and deductive logical reasoning, and to the promotion of rigor and precision.
- ✓ It is also highlighted the necessity to start the students off in simple mathematical investigations, something already mentioned by Cockcroft. This should be given

through guided learning, providing the resources and reinforcements that may be required.

- ✓ In addition, reference to the need to teach through contextualized situations is made. The higher the contextualization of the subject, the better. And the importance of using manipulative materials is also mentioned.
- ✓ But the most important aspect is the incorporation of the information and communication technologies (ICT). These can be as useful as other traditional resources to research, analyze and evaluate information, to do works, or to record mathematical data.

Thus, the processes and resources for the teaching and learning of Mathematics have been adapted over the past decades. Through these previous paragraphs, these were referred to from a general point of view and based on the traditional subject, but the next sections will be focused on the proposal to introduce Bilingual Mathematics in the field of Primary Education, which is also the central topic of this thesis. However, all of these ideas will continue to be taken into account and even developed in future sections (especially in those ones relating to education and methodology). Otherwise, they would not have been mentioned. As it was pointed out, the subject will continue to be Mathematics, based on the current and traditional one that is taught today.

3. INTRODUCTION OF THE BILINGUAL EDUCATION IN SPAIN.

As it is already known, the importance of the acquisition of any foreign language is essential for life, and as English is the most spoken language in the Western world, it is logical that it should be the most learned.

For that reason, it is fundamental to consider some basic aspects in order to start teaching this language properly:

- ✓ To introduce the students in the use of English from the earliest ages. Young children acquire, learn and imitate easier than older ones, as their minds are really open and absorb information quickly.
- ✓ To plan programs based on each cycle, on each course and even on each student, adapting to modern times and introducing new tools, but without leaving aside the traditional useful programs.

- ✓ As in the subject of Mathematics, to give a greater importance to constructivism than to other theories when setting the learning model. It is also essential to develop a creative and active methodology that allows them to lose their fear of using the foreign language.
- ✓ To consider the importance of the qualification of the teachers: a high level is basic.

Well, based on these general ideas about how the learning and teaching of English in schools should be developed, and considering the undisputed global importance of language, initiatives to enhance its teaching have been developed in Spain in recent years.

Although these considerations of the second paragraph of this section have been, or are beginning to be taken into account, Spain has always been distinguished by the low level of language learning through generations. The Spanish education system is not the most reputed, since the level of English traditional classes has been too superficial in general. And to make matters worse, if we add that few students continue gaining it after finishing high school or college education, and that there is a tendency not to study it abroad, the situation turns worrying in a world increasingly globalized.

To try to solve this problem, Bilingual Education has been implemented in Spain in recent times. It is considered as a solution to the lack of education of these decades, in full technological revolution. It was introduced in the late nineties and the process is long, gradual and not simple, and it is still being incorporated to Primary and Secondary Education. Moreover, as it has happened with the traditional subjects and education, the contents, the criteria and the methodology have been adapted to each age. Its importance is absolute as the CLIL approach, which has been considered in order to establish this type of education in Spain, opens new horizons for languages, working on the basis of key competencies and merging different learning styles (San Isidro, Domínguez Cuña & Barreiro, 2009).

As it was pointed out, CLIL stands for Content Language Integrated Learning. Different definitions of CLIL have been established, although one of the clearest ones is this one in my opinion: it is a dual-focused educational approach in which a different and new language is used for the teaching and learning of both language and content (Coyle, Hood & Marsh, 2010). The term CLIL was coined in 1994 by Marsh.

This method has been developed in many countries over recent years. The main characteristics that justify its importance are the next ones:

- ✓ A learner – centered and flexible teaching is developed, which means a great involvement of the students.
- ✓ A learning which is focused on the development of processes and tasks through the use of multiple resources, such as ICT, is also developed.
- ✓ A greater contact with the foreign language is established. Moreover, this language is acquired by the pupils through meaningful and more motivating contexts.
- ✓ And a more complete preparation of the student in the face of an increasingly multicultural world is given. This implies the development of intercultural communication skills and multilingual interests and attitudes.

Finally, it is necessary to refer to the subjects that have been incorporated into the Bilingual Program in Primary Education: Physical Education, Knowledge of the World and Art are the subjects usually taught. Obviously, Spanish Language and Literature cannot be integrated, and Bilingual Mathematics has barely become relevant. As it was considered that Mathematics may also be included in this type of education, the analysis and propositions over the next sections of this master's thesis are based on the possibility of incorporating this new subject to the Spanish education system definitively.

4. MAIN SOURCES ON WHICH I AM GOING TO BASE THE BILINGUAL PROPOSITION.

As it was already mentioned in the introduction, the ideas for this thesis are developed based on the knowledge about CLIL acquired during the master, on my own experience as a teacher in a public Primary school through the Practicum and on information extracted from different sources.

Some sources (such as the Curriculum of Mathematics from the LOE or the *Mathematics Counts* report) have already been consulted in specific moments for the development of the previous sections. And many other books, texts and a web page will be also taken as reference from now, as the proposition on the bilingual subject is developed through the main sections of this thesis. Those sources and other ones that will be used in the final sections do not have to be mentioned now, as all of them will be referenced throughout the thesis and in the final bibliography considering the APA system. However, I think it is important to refer to three of them, because they are basic to the development of the main proposition. Moreover, one of them is completely unknown in Spain, so I would like to make

a special reference to it. That is the reason why this section of the thesis is included. The information related to these three sources that is considered in the next paragraphs cannot be detailed through the bibliography.

These most used ones are:

- ✓ The Curriculum of Mathematics from the LOE, which was already consulted.
- ✓ The Curriculum of English from the same law.
- ✓ A specific Primary Education Mathematics textbook. It is a book for fourth year students which was published in 1993 by 'sm'
- ✓ *The National Numeracy Strategy. Framework for Teaching Mathematics from Reception to Year 6.*, by the UK Department For Education And Employment.

4.1. CURRICULUM FROM THE LOE. MATHEMATICS AND ENGLISH.

In previous sections of the thesis, some reference has been made to the Curriculum of Mathematics. The new subject of Bilingual Mathematics will be based on the traditional subject mainly, and that is the reason why the Curriculum should be considered. As it was also said, the mathematical objectives, contents and assessment criteria, as well as many aspects of the methodology and evaluation contained in this document, will still be taken into account.

However, the Curriculum of English will also be handled. Although it is not as important as the Mathematics one (as it was pointed out, the new subject will be based on the current Mathematics rather than on the subject of English), this language will be introduced, and as it will be determined, some objectives, assessment criteria, and ideas about methodology and evaluation from this Curriculum will also be considered.

The information is extracted from these two current curriculums because the subject of Bilingual Mathematics is hypothetical, and for this reason there is not a curriculum for it upon which the bilingual proposition can be developed.

4.2. TEXTBOOK.

The textbook will be taken as a reference to develop the practical proposition. The lesson plan with sessions established through that proposition will be based on a concrete lesson of that book, in order to prepare that bilingual lesson based on a previous and approved

model from a textbook. The idea is to show those two examples as a comparison: both of them will be similar from a mathematical point of view, but in the second one (the one I propose) the foreign language will be introduced.

4.3. UK NATIONAL NUMERACY STRATEGY (NNS)

The number and the measurements are the basis on which mathematics are founded and built. In this sense, the capacity to use those mathematical concepts confidently in different situations of everyday life is the foundation of the mathematical competence.

In Spain there is not a specific term for the aforementioned capacity, although some authors use the term *competencia numérica*. Meanwhile, in the UK it is called numeracy, which could be translated into Spanish as *alfabetización numérica*, but its scope is wider than the one of a simple *alfabetización*.

Taking into account this, the *The National Numeracy Strategy. Framework for Teaching Mathematics from Reception to Year 6.* UK document was developed in the UK in 1999. As it was noted in the introduction, it included an outline of expected teaching in Mathematics for Primary Education pupils. References to this capacity (numeracy) were made and, above all, many aspects related to methodology and evaluation were established. In this line, I am convinced that these considerations may also be useful, which is why many of them will be mentioned as my proposition is carried out.

Thus, the model determined will be based on different sources, but, above all, on these three main ones.

5. MAIN PROPOSITION: MODEL OF IMPLEMENTATION, TEACHING AND LEARNING OF BILINGUAL MATHEMATICS IN A PRIMARY SCHOOL.

This proposition is developed through this section and it is based on the hypothetical case which was mentioned when the reference to the parts of the thesis was made: the consideration of the fact that I am a teacher in a bilingual school in which the subject of Bilingual Mathematics has just been implemented and in which the development of a help document to guide the educator teaching processes is going to be taken. As it was pointed out, my real objective is to explain the ideas that I consider interesting for this bilingual subject through the consideration of this hypothetical case, while applying these ideas through the

subsequent practical proposition that I have established. Thus, the first half includes the theoretical aspects which are set out in the guide, while the practical example is developed through the second half. And everything is adapted to the context of this thesis, relating it to the information from the preceding paragraphs if needed.

The sections of the guide, to which reference has already been made, are the following ones:

- ✓ General considerations.
- ✓ Blocks of contents of the subject at all levels (these are the same in all cycles).
- ✓ Contribution to the development of the basic competences (from a global point of view of the subject in Primary Education).
- ✓ Objectives of the subject at all levels, from a global perspective also.
- ✓ Contents and evaluation criteria (focusing on the cycle of my hypothetical students: the second one).
- ✓ Methodological aspects (detailed in a more global way).

Therefore, the guide will be organized taking into account the structure and sections of the current curriculums, such as the Mathematics and the English ones. It is determined in this way because I consider that it is a good scheme that covers the basic aspects. If the subject of Bilingual Mathematics existed, this hypothetical guide would probably be based on the curriculum of that subject (possibly similar to the current ones of Mathematics and English), but that does not apply. And if the case of the guide and the educators who teach this subject were real, considering the existence of that curriculum, this document would be similar to it, but more accessible, helpful and concrete in that school because it would have been written by them.

However, it is important to remark again that anything related to the bilingual subject, its curriculum or the guide already exists: I establish this imaginary case in order to let the characteristics of the model I thought about be known. This is a model that could be applicable in the future in my opinion. And I prefer to make reference to this hypothetical case and create that guide rather than, for example, determine a possible curriculum for the subject, because that would probably be too ambitious an option.

On the other hand, leaving aside these theoretical aspects, the second part of this main proposition of the thesis (the practical one) will consist of:

- ✓ Introduction.

- ✓ Timing.
- ✓ Objectives.
- ✓ Contents.
- ✓ Contribution to the development of basic competences.
- ✓ Methodological aspects.
- ✓ Resources.
- ✓ Attention to diversity.
- ✓ Organization of the sessions.
- ✓ Methodological aspects.

5.1. GUIDE FOR THE IMPLEMENTATION OF THE SUBJECT. THEORETICAL ASPECTS.

5.1.1. General considerations.

The general considerations are based primarily on information extracted from the current English and Mathematics Curriculums, although it is obviously adapted (the origin of the subject is the traditional Mathematics, the English subject also exists, but Bilingual Mathematics is new, replacing this traditional and non-bilingual one and including that foreign language). The considerations to take into account are developed from the next paragraph:

The command of foreign languages in this multilingual and multicultural world is essential. For this reason, the national and European commitments in order to develop language incentive programs are maximum. Spain is committed to promoting the knowledge of other languages of the European Union, like the rest of the countries that are part of this community. And through the Common European Framework of Reference for Languages, guidelines for the learning of languages and for the valuation of the competence in the different languages of each speaker are established.

The implementation of bilingual subjects (especially through English) considering the CLIL approach over the last years has been continuous and one area that can be adapted to Bilingual Education through this language is the area of Mathematics. Mathematics is a key subject at all levels.

Mathematics is a set of basic knowledge related to numbers and forms, which is completed progressively until a valuable way to analyze different situations is developed. Its importance in daily life should be highlighted, as it helps to know the reality, structuring, analysis and collection of new information about it.

In this way, mathematics not only facilitate the use of numbers and geometric shapes, but also the fact of asking oneself questions, of obtaining models and of identifying relationships and structures, acquiring new information and conclusions when analyzing reality and, therefore, when facing with the situations that may arise. The need to learn Mathematics in school is evident.

So, one of the basic roles of Primary Education is to achieve an effective mathematical literacy. The mathematical literacy is the ability to identify and understand the role of discipline in the world, to make well-founded judgments and to obtain useful information from situations involving numbers and their relationships. The most important objective is to ensure that children are competent in using the mathematical content in the different spheres of the social life.

In fact, the knowledge of the mathematical algorithms is not enough to achieve real literacy. It is also essential to be able to use them whenever it is necessary and to identify the basic relations that are given among them.

Students should learn Mathematics through the development of functional contexts related to daily life situations, so that more complex knowledge from experience and previous knowledge is acquired. This idea was highlighted through the reference to the teaching of traditional Mathematics in the second section of the thesis, and it is still taken into account in this new subject. In this sense, the need to prioritize learning through experience is understood. It is also important to emphasize the basic role of problem solving processes, through which many basic skills such as reading, comprehension or the planning, review and discussion of what is worked are used. In this way, when solving problems, students can cope with difficult situations in the environment through mathematical models, acquiring basic life concepts.

On the other hand, through the incorporation of the subject of Mathematics to the Bilingual Education system, as well as purely mathematical aspects, additional ones concerning the foreign language have to be considered. In fact, the emphasis on the language is translated into numerous modifications in relation to the traditional subject.

The development of the mathematical literacy continues being a key objective, but not the only one. From a linguistic point of view, the ability to train people on speaking, listening, reading and writing in relation to the mathematical concepts becomes basic. That is, the need to learn to express oneself mathematically in a foreign language is considered fundamental. For that reason, teaching Bilingual Mathematics from the early ages is essential.

Although the students who complete their studies in early childhood education have acquired a certain basis in the English language, the first year is started from an elementary level, as this subject of Bilingual Mathematics may be absolutely new for many of them. For this reason, the importance of working progressively according to the needs, abilities and preferences of the students is basic, taking into account the previous knowledge and experiences in order to start the learning of the mathematical language, or to reinforce it.

The fact of expressing oneself correctly using the English language involves its knowledge from four main points of view: speaking, listening, writing and reading. Through Bilingual Mathematics, the evolution is continuously produced in the four senses.

Oral expression is developed through mathematical communication situations. Thanks to the use of words and the construction of meanings and structures, the knowledge of concepts related to the discipline is acquired. Aspects such as intonation and pronunciation should be taken into account.

Learning to listen is also needed, which involves understanding and processing the information that is captured. The development of the ability to draw conclusions about what is acquired is equally important.

The writing of universal mathematical terms, such as numbers or symbols, and of other statements and structures through the foreign language should also be developed.

And finally, proper reading implies the identification of mathematical texts, statements and numbers and the understanding of the information received, whether it is in Spanish or English.

Thus, it is necessary to find a balance between the learning of the foreign language and the acquisition of mathematical knowledge. In order to achieve this, a careful planning of the subject should be established, considering aspects such as the methodology, the objectives, the contribution to the development of the basic competences and the assessment criteria, and organizing the contents in different blocks.

5.1.2. Blocks of contents.

As it was pointed out, this subject of Bilingual Mathematics replaces the traditional one of Mathematics and not the traditional one of English, being based on the first one although the foreign language is introduced. For that reason, the blocks of contents are the same as in the current subject of Mathematics (the ones which are determined through the LOE). The blocks are four, following the general model from the LOE and not the established ones by certain Comunidades Autónomas which are based on this general one (for example, five blocks of contents are considered in Asturias).

These are *Numbers and operations*, *The measure: estimation and calculation of magnitudes*, *Geometry* and *Information processing, chance and probability*. Although the contents of the blocks are different, they are all interdependent. In addition, problem solving acts as the backbone that links all blocks. And referring to the acquisition of the mathematical language through English, the progress in the command of the speaking, listening, writing and reading skills will always be basic, regardless of each block.

Through block 1, *Numbers and operations*, the development of the number sense is intended. This is understood as the command of the main numerical relations, which can be expressed through capabilities such as: the use of the structure of the decimal numbering system, the ability to decompose numbers naturally or the use of the properties and relationships between operations in order to perform calculations mentally. The numbers should be used in different contexts, based on previous planning appropriate to each group of students. Throughout this stage, it is intended that students do mathematical calculations and reasonable estimates, trying to strike a balance between the understanding and use of concepts and the acquisition of numeracy. The understanding and expression of the acquired mathematical concepts are developed through the use of the foreign language.

In block 2, *The measure: estimation and calculation of magnitudes*, the objective is to facilitate the understanding of the messages through which magnitudes are quantified. It is also intended to refer to real situations concerning the magnitudes that have to be interpreted correctly by the students. Once the different magnitudes are known, the taking of measurements is carried out. And finally, a progressively greater number of units are used. For the development of this process from the earliest ages, it is recommended to begin through the use of physical (inch, foot) and arbitrary units (string, rods), the standardized measures will be acquired in future sessions. And as in the previous block, the mathematical language development will be carried out through the use of the foreign language.

Through block 3, *Geometry*, contents about geometric shapes and structures are developed. Learning geometry requires the definition, description, analysis and classification of properties, in order to develop the ability to visualize geometrical relationships. This learning is achieved in two main ways: through the use of geometrical materials that help to understand the importance of the real models. And through the use of mathematical computer programs that facilitate the practice of geometry. And, as in the previous blocks, the importance of the foreign language in the content transmission and acquisition processes is basic.

Block 4 is *Information processing, chance and probability*. The full meaning of its contents is acquired when these are presented in connection with activities involving other areas of knowledge. The work developed through this block should have a significant impact on the understanding of the information media. In this way, it is intended to raise interest in these ideas and to help consider the benefits provided by the statistical knowledge when making decisions, usually on issues related to other areas. The attitudinal contents are especially important within this block, because through their acquisition students are better able to present the data orderly and to discover that the knowledge of mathematics can be very helpful to solve everyday problems. And the foreign language will be, again, the cornerstone to express and develop the mathematical concepts acquired.

5.1.3. Contribution to the development of the basic competences.

For the development of this section, the Mathematics and English curriculums and the current basic competences are taken as reference. These basic competences are as follows:

- ✓ *Mathematical competence.*
- ✓ *Competence in linguistic communication.*
- ✓ *Competence in knowledge of and interaction with the physical world.*
- ✓ *Competence in processing information and use of the ICT.*
- ✓ *Learning to learn competence.*
- ✓ *Autonomy and personal initiative competence.*
- ✓ *Competence in social skills and citizenship.*
- ✓ *Cultural and artistic competence.*

The contents of the area are established in order to guarantee the maximum development of the mathematical competence during the acquisition of all of the

mathematical knowledge. But this development is not only reached with the acquisition of this knowledge and its use inside the school environment. A real contribution to the *mathematical competence* is only achieved to the extent that the learning of these contents is related to the utility to deal with the many situations in which children use mathematics outside the classroom.

Besides the importance of the mathematical competence, the contribution to the development of the *competence in linguistic communication* is particularly noteworthy. As happens with the rest of the bilingual subjects, the communication skills in Bilingual Mathematics are undoubtedly completed and enriched by the acquisition of the foreign language, much better than if those skills are developed through the Spanish language. Furthermore, in the case of the mathematical language, two specific aspects must be taken into account: first, the need to incorporate its essential aspects to the familiar contexts and the adequate precision in its use. Second, the importance of the contents associated with the verbal description of the reasoning and the processes.

The competence in *knowledge of and interaction with the physical world* also plays a key role. This is because the development of the mathematical thinking allows for a better understanding and a more accurate description of the environment. First, through the development of visualization, the ability of students to make structures and to manipulate figures in the plane and in the space is improved. Second, through the development of measurements, a better understanding of reality is achieved. In this way, the chances to interact with it and to transmit precise information about quantifiable aspects of the environment are increased. Finally, the skill in the use of graphics in order to interpret the information provides a valuable tool for a better understanding and analysis of the reality.

The contribution of Bilingual Mathematics to the acquisition of the *competence in processing information and use of ICT* is also important. On the one hand, because through the knowledge of a foreign language it is possible to communicate by using a much larger amount of the information that is offered by the technological means, whether or not this is related to the mathematical world. And, on the other hand, because the skills associated with the use of the numbers are provided, such as the comparison, the approximation and the relationships between the different ways of expressing them. In this way, the understanding of information on quantities or measures is easier. In addition, it also contributes to the use of graphical and statistical languages, essential for interpreting the information about reality. And finally, the use of calculators and technology tools to facilitate the understanding of the

mathematical content is also linked to the development of this *competence in processing information and use of the ICT*.

The *learning to learn competence* is also developed. On the one hand, the learning and use of the foreign language involves a contribution to the improvement of the communication skills. And this contribution, besides being basic, opens many doors. The language is the main vehicle of the human thought and the ultimate learning tool, as it helps to understand and interpret the reality. Thus, children not only learn to use the language, not just to memorize concepts and mathematical expressions used in the classroom, reflecting on what has been learned and on what is going to be learned. Its usefulness in connection with the acquisition of future knowledge and skills is essential. On the other one, the instrumental nature of many of the Bilingual Mathematics content also provides value for the development of this competence. In fact, the ability to use basic mathematical tools is normally a requirement for learning.

The contribution to the development of the *autonomy and personal initiative competence* is also important. On the one hand, through the acquisition of the foreign language in the subject of Bilingual Mathematics, the student is able to improve his language skills independently and to reflect on his own learning. On the other hand, another contribution to this competence is made possible by the development of the content related to problem solving. Problem solving has at least three complementary aspects associated with the development of this competence: the planning, the resource management and the assessment of the results. The planning is related to the understanding of the action proposed in order to find strategies and make decisions. The optimization of the resolution processes is included within the resource management. And through the evaluation of the results, it is possible to cope with new problems and situations with a greater chance of success. Thanks to problem solving and to the development of similar activities, students practice, make choices and follow their own criteria.

The contribution of this area to the acquisition of the *competence in social skills and citizenship* has to be mentioned. First, the use of a foreign language facilitates the communication and the social transmission, since its learning involves the knowledge of features related to other language speaking communities. Moreover, if the learning is well-oriented, students will feel interest to know about societies and interact with other people, speakers or learners of those different languages (English in this case), always taking into account the values of respect, tolerance and acceptance of their differences. Second, from a

more mathematical point of view, the participation in team activities becomes very important if it is learned to cooperate and accept different opinions, especially when the problem solving strategies are used. In this way, the competence in social skills can also be acquired.

Finally, the *cultural and artistic competence* is also developed. Mathematics contributes to this competence from the consideration of the mathematical knowledge as a contribution to the cultural development of the world. Moreover, from the language point of view, the use of English means the acquisition of a language from a different cultural and artistic context. For these reasons, the role of this competence is also important.

5.1.4. Objectives.

In order to determine the objectives to pursue through the processes of learning and teaching of Bilingual Mathematics in Primary Education, the objectives of the traditional subjects of Mathematics and English which are listed in both Curriculums are taken into account. These new objectives are determined below. The first ones are established from a mathematical point of view; the last ones, from the foreign language point of view:

1) Use the mathematical knowledge in order to understand, appreciate and produce information and messages about events and situations of everyday life, apart from recognizing its instrumental character for other fields of knowledge.
2) Appreciate the importance of mathematics in everyday life, enjoy its development and recognize their value.
3) Recognize mathematical situations for which understanding or treatment elementary operations of calculation are required. Formulate and solve these operations by using appropriate and simple mathematical expressions, evaluating the meaning of the results.
4) Gain confidence in mathematical skills in order to cope with the different situations related to the subject, enjoying the practical aspects and taking into account their possible uses.
5) Develop and use mental calculation and measurement tools and strategies, as well as develop spatial orientation procedures in problem solving contexts, valuing the benefits of their use and the consistency of their results in each case.
6) Identify geometric shapes of the natural and cultural environment in order to describe real contexts and develop new chances for action.

7) Use basic techniques of data collection in order to obtain information about real events and situations. Represent the information through numbers and graphs, and make judgements about it.
8) Use the information and communication technologies in an appropriate way in order to do calculations, and to look for and to represent information, acquiring that mathematical knowledge through the foreign language.
9) Express oneself orally through the foreign language in an appropriate way for each age using the mathematical language correctly.
10) Listen and understand the mathematical messages received through the foreign language, using the information provided to perform exercises and problems.
11) Propose and solve mathematical problems and operations using the foreign language and the calculation, measurement, estimation and testing results appropriate procedures.
12) Write concepts, statements and mathematical texts based on previous models through the use of the foreign language.
13) Read and understand concepts, statements and mathematical texts through the use of the foreign language.
14) Use the knowledge and the previous experiences related to the foreign language in order to acquire it in a quick, efficient and independent way.
15) Value the importance of the foreign language and show interest in its learning and use.

5.1.5. Contents and evaluation criteria.

Both are related and depend on the objectives. The previously established content blocks are also considered. And both the contents and the evaluation criteria are determined considering the basis of the traditional subject of Mathematics, but also from the foreign language point of view. The proposal is focused on the analysis of the second cycle. As it was pointed out, it is assumed that I teach third and fourth year students, which is why I detail the content and criteria for these years (as it was said, this guide is planned following the structure of the current curriculums, and their content and assessment criteria are organized according to the cycles). The teachers of the first and third cycle students would do the same with the corresponding contents and evaluation criteria.

- ✓ Second cycle contents.

SECOND CYCLE. Block I. Numbers and operations.
Natural numbers and fractions. Natural numbers (from 1 to 1 million). Decimal Number System. Position value of the figures. Use in real situations. Order and relationship between numbers. Notation. Fractional numbers to express partitions and relationships in real contexts. Comparison between simple fractions. Graphic representation. Introduction to the decimal number: tenths and hundredths. *Use of the foreign language:* primarily on the reading and writing of numbers from 1 to 1 million, of fractional numbers and of decimal numbers.
Operations. Operations with natural numbers: addition, subtraction, multiplication and division between a one-digit numbers. Identification and use of the specific terms of multiplication. Multiplication of a number by a one-digit number followed by zeros Use of simple multiplications in everyday situations, through easy activities and problems. Identification and use of the specific terms of division. Use of numbers and calculation to solve problems in real situations, considering the resolution processes and the results. *Use of the foreign languages:* primarily through the identification of the terms of multiplication and division, through the oral and written expression of operations and through problem solving.
Calculation strategies. Additive and multiplicative decomposition of numbers. Construction and memorization of multiplication tables. Use of standard algorithms, in the context of problem solving, addition, subtraction, multiplication and division by a one-digit number. Development and use of mental calculation strategies. Estimation of the result of an operation between two numbers, taking into account if the answer is reasonable. Use of the calculator in everyday problem solving if the teacher thinks that the calculations are too difficult. *Use of the foreign language:* primarily through the development of mental calculation

strategies, in additive and multiplicative decompositions of numbers and in problem solving.

SECOND CYCLE. Block II. The measure: estimation and calculation of magnitudes.
Length, weight / mass and capacity. Decimal Metric System units and equivalences. Election of the most suitable unit for the expression of a measure. Expression in a simple form of length, capacity or weight measure given in a complex form and vice versa. Comparison and order of units and quantities of the same magnitude. Addition and subtraction of weight, length and capacity measures given in a simple way. Estimation of measures of everyday objects. Development and use of personal strategies to measure. Taking of measurements using measurement instruments and conventional units in everyday contexts. Analysis of the processes and strategies followed in measurements. *Use of the foreign language:* primarily through the reading and writing of measurement units and equivalences, in the estimation of measures and in the analysis of the processes and strategies followed.
Time measurement. Correct reading of analog and digital clocks through the use of time measures (second, minute, hour, day). Simple calculation of equivalences. *Use of the foreign language:* basically in the correct reading of the hours.

SECOND CYCLE. Block III. Geometry.
The situation in space, distances, angles and rotations. Precise location of items in the space. Elemental representation of known spaces: drawings and models. Description of positions and movements in a topographical context. Location of points through the use of the Cartesian coordinate system. Interpretation of sketches and simple drawings. Straight and curve lines. Parallel, perpendicular and oblique lines. Relationship between the concepts of angle and rotation.

Use of the foreign language: primarily on the analysis of elements in the space, of the types of lines and of the concepts of angle and rotation, in the description of positions and numbers and in the interpretation of drawings and sketches.

Plane and spatial forms.

Identification of plane and spatial figures in everyday situations.

Classification of polygons. Sides, vertexes, bases, diagonals, angles, lines of symmetry.

The circumference and the circle. Centre, radius, diameter, chord and arc.

The geometric bodies: cubes, spheres, prisms, pyramids and cylinders. Edges, vertices and faces.

Description of the shape of objects using the basic geometric vocabulary.

Construction of plane geometric figures from the use of data and the knowledge of the geometric shapes.

Comparison and classification of figures and geometric bodies using various criteria.

Comparison and classification of angles: right, acute, obtuse, plane, greater than 180º and complete.

Use of the foreign language: basically through the acquisition of the concepts related to polygons, circles, circumferences, and geometric bodies, and through the description of objects and the types of angles.

Regularities and symmetries.

Metric transformations: translations, rotations and symmetries.

Identification of translations, rotations and symmetries in familiar and natural environments.

Use of the foreign language: primarily through the acquisition of concepts related to metric transformations.

SECOND CYCLE. Block IV. Information processing, chance and probability.

Graphics and tables.

Collection and recording of data related to objects, events and family situations using basic techniques of survey, observation and measurement.

Reading, interpretation and development of two-way tables commonly used in everyday life.

Construction of absolute frequency tables.

Interpretation and description of significant elements from simple graphics related to familiar phenomena.

Development of simple graphics: pictographs and bar graphs.

Use of the foreign language: mainly in the collection and recording of data, and in the interpretation of tables, graphics and their elements.
The randomness of some experiences. Assessment of the results of experiences related to probability. Appreciation of the existence of more and less probable facts and of the impossibility of predicting a concrete result. Introduction to the language of chance. *Use of the foreign language:* mainly through the introduction to the language of chance and through the evaluation of the results of experiences.

The contents that are mentioned are mathematical because this is the subject of Mathematics, although the foreign language is introduced. As it was pointed out, this bilingual subject replaces the traditional Mathematics one and not the traditional English one. However, I consider it basic to highlight the use of the target language: the contents are mathematical, but English is learned every day, as are linguistic contents also acquired in turn. The idea is to practice the foreign language every day for as long as possible and to refer to its importance, I have decided to emphasize in which moments this should be used primarily considering each block of contents. It is a different way to make reference to the language, but today Spanish bilingual subjects are not determined through specific curriculums and there is not a model through which I can guide myself. In addition, this subject has hardly been implemented and the proposition of this thesis does not have to be the only one. This is an example of a proposition and not the final one.

Finally, the contents of the other two cycles would be organized in the same way, but obviously adapted to each age.

- ✓ Evaluation criteria.

1) Read and write the natural numbers less than one million and use them in everyday contexts. Interpret the value of each figure and compare and order numbers on the number line considering the place value. Based on this criterion, it is assessed whether the child is able to:
   - ✓ Represent the numbers less than one million, taking into account the concept of place value.

- ✓ Know how to write the digits of a number in order and how to put that number on the number line.
- ✓ Read and write the natural numbers less than one million.
- ✓ Recognize the place value of the digits of a natural number.
- ✓ Compare and order natural numbers.
- ✓ Recognize fractional numbers to express partitions and relationships in real contexts.
- ✓ Compare and order halves, thirds and fourths.
- ✓ Represent and order simple fractional numbers.

2) Perform numerical calculations with natural numbers in problem solving situations, taking into account the properties of the operations and the decimal number system. It is taken into account if the student is able to:
   - ✓ Consider the structure of the decimal system when carrying out numerical calculations.
   - ✓ Use the properties of operations, being flexible when choosing the most appropriate procedures and dominating the writing of algorithms.
   - ✓ Establish the possible relationships between the terms of the subtraction.
   - ✓ Perform multiplications and divisions of numbers by one or two-digit numbers.
   - ✓ Represent simple multiplications and divisions through drawings.
   - ✓ Use the calculator if the teacher thinks that it is necessary because of the difficulty of the operations.

3) Develop mental mathematical strategies to perform additions, subtractions, multiplications and simple divisions, where the validity of the results is more important than the time taken to do the operations, whether they are accurate or estimated. According to this criterion, it is assessed whether the child is able to:
   - ✓ Calculate additions, subtractions, multiplications and simple divisions mentally.
   - ✓ Apply the relationship between addition and subtraction operations previously studied.

4) Make estimates and measurements in real contexts. Choose and use appropriate tools and units of measurement based on the object to be measured. Based on this criterion, it is considered if the student is able to:
   - ✓ Make estimates of magnitudes in everyday contexts and analyze them.

- ✓ Choose the most appropriate unit of measurement in terms of what it is measured.
- ✓ Knows how to express a simple form of a measure given in a complex form and vice versa.
- ✓ Compare and order different measurement units.
- ✓ Choose the most appropriate measuring instrument and know how to use it.
- ✓ Do operations and exercises with measurement units.

5) Analyze and describe spatial representations considering everyday objects as references. Use the basic notions of geometric movements in order to describe and understand real situations and to consider artistic expressions. It is assessed whether the child is able to:
   - ✓ Get information of plans and sketches in which everyday objects or displacements have been represented.
   - ✓ Use geometric properties, such as alignment, parallelism or perpendicularity, as reference elements in order to describe spatial situations.
   - ✓ Describe and represent movements in the plane using geometric elements such as lines and simple angles.
   - ✓ Make movements by following simple instructions represented in simple plans.
   - ✓ Make sketches of familiar environments and represent objects and everyday items.
   - ✓ Use coordinate systems.
   - ✓ Identify symmetries and translations in real and everyday contexts.
   - ✓ Recognize symmetries and draw symmetrical figures.

6) Recognize and analyze the forms and geometric shapes: polygons, circles, cubes, prisms, spheres and cylinders. According to this criterion, it is considered if the student is able to:
   - ✓ Analyze the differences between plane figures considering their features.
   - ✓ Compare and classify the different types of angles.
   - ✓ Compare the elements of the geometric shapes in order to differentiate them.
   - ✓ Solve simple problems using the most important properties of geometric figures.
   - ✓ Recognize geometric elements in real contexts.

7) Collect and interpret data on daily events using simple counting techniques. Classify these data according to different criteria and represent them through tables or graphics. Relate the different degrees of probability with specific events. It is assessed whether the child is able to:

- ✓ Use data counting techniques.
- ✓ Represent data through the most appropriate statistical graphics.
- ✓ Interpret information and elements that are represented in the graphics.
- ✓ Consider the probability of everyday events.

8) Solve problems related to contents acquired in which it is necessary to apply one or two mathematical operations or to use ideas related to measurement and estimation, geometry or information processing. It is taken into account if the student is able to:
   - ✓ Understand a problem statement, identifying the ideas, the data and the situation to be resolved.
   - ✓ Represent the data through graphics if it is necessary.
   - ✓ Relate the problem with similar ones already made.
   - ✓ Select the appropriate operations to solve the problem.
   - ✓ Check the solutions by using the calculator.
   - ✓ Analyze and understand the process carried out in each case. Discuss the strategy used to solve each problem and the validity of the result.

9) Develop the speaking expression and comprehension of the mathematical language considering the level required for eight and nine year-old students. Use the foreign language every day to communicate and progress, expressing and understanding the ideas, vocabulary and statements related to the mathematical content acquired. The medium level of third and fourth year students should be taken into account. And depending on the specific contents for this cycle, the child should be able to:
   - ✓ Understand and express the following concepts and ideas orally, analyzing and describing their characteristics and their relationships: natural numbers; ordinal numbers from 10^{th}; simple fractions and decimal numbers; estimated results of measurements; units of the metric system and equivalences (length, mass and capacity); time measurement units; location of elements in space; location of points using Cartesian coordinates; description and interpretation of sketches and drawings; types of lines; the circle and the circumference; plane figures and geometric bodies; angles; symmetries; statistical graphics and data tables, and chance and its results.
   - ✓ Explain and understand the processes carried out in the resolution of operations and calculations (addition, subtraction, multiplication and simple divisions), and the

meaning of the data, the processes followed and the resulting solutions obtained through the resolution of problems related to the concepts listed.

- ✓ Take part actively in individual or group activities in which the oral use of the foreign language is required and that are planned according to all of the previous concepts.

10) Develop the written expression and comprehension of the mathematical language through the use of the foreign language. Write numbers, vocabulary, expressions, notions and small texts related to the concepts mentioned above in a correct way. Perform operations and solve problems. Taking also into account the level of the second cycle, the students should be able to:
    - ✓ Understand and write all of the ideas related to the concepts mentioned that should be studied.
    - ✓ Develop and write the processes carried out during the resolution of operations, calculations and problems.
    - ✓ Show interest and participate in individual and group activities in which the comprehensive practice of writing is required.

11) Develop the reading expression and comprehension through the use of the foreign language considering the standards of the second cycle. Read carefully the information related to the concepts studied. Pay attention to the statements of the exercises. Taking into account this criterion, the child should be able to:
    - ✓ Read in a comprehensive way all the figures, notions, sentences and texts related to the concepts that have been mentioned in previous paragraphs and selected for study.
    - ✓ Pay attention to the details of the operations, calculations, problems and other activities whose implementation is required by the teacher, whether they are individual or group exercises and regardless of the type of materials used.

12) Listen in a comprehensive way to capture all the information on the previous concepts that has to be acquired. Based on these criteria and considering the level of the second cycle, the student should be able to:
    - ✓ Pay attention to the explanations of the teacher.
    - ✓ Respect and listen to the other students.

- ✓ Listen to his or her own words and oral interventions in order to improve in the use of the foreign language.
- ✓ Listen to recorded information: CDs, videos, Internet.

5.1.6. Methodological aspects.

The methodological aspects are considered based on the CLIL ideas I acquired during the development of the master (some of which were mentioned in the third section of this work), on my own opinion about this approach, on the Mathematics and English Curriculums and on the other sources which were already mentioned. The ideas about teaching and learning which were developed in the third section of this thesis are also taken into account: as it was pointed out, this information, which was established from the teaching and learning of traditional Mathematics point of view, was collected because it is also interesting from the perspective of the bilingual subject. It is basic to remember that this bilingual subject is born from the traditional one.

The methodology has to be useful, and its planning should suit the needs of the bilingual students in the classroom and the characteristics of the subject, which have been collected in the previous sections of this thesis. Its development implies different aspects to consider, with the most important ones listed below:

- ✓ A modern, oral, interactive and adapted to everybody way of learning and teaching.
- ✓ A real and continuous use of the foreign language.
- ✓ A proper organization of the daily lessons, considering the activities to propose and the materials to use.
- ✓ An adequate assessment, encompassing all of the aspects related to the science of mathematics and to the language of English.

Based on these main characteristics, the rest of the methodological ideas are detailed.

A modern, oral, interactive and adapted to everybody way of learning and teaching.

A modern way of learning and teaching means considering certain important and recent educational aspects. On the one hand, from a learning point of view, it involves giving priority to contemporary theories (the constructivist ones) to the detriment of other traditional ones (for example, the behaviorism theories), in order to acquire the contents of the Bilingual Mathematics subject appropriately. This does not mean that behaviorism is discarded, but the

theory introduced by Piaget and Bruner is preponderant. On the other one, from a teaching point of view, it also means taking into account the aspects extracted from the *Mathematics Counts* report, the *Proyecto Curricular Base* and the Curriculum, which were mentioned in one of the first sections. And, above all, the development of a modern and bilingual subject of Mathematics in Spain involves considering the information related to the CLIL methodology, including ideas taken from the experience in the implementation of the previous bilingual subjects in recent years. This development has to be really careful, as the acquisition of the mathematical content and the English language has to be given properly and at the same time.

The way of learning and teaching to be developed must also be fundamentally interactive, similar to the ones carried out through other bilingual subjects, based on the students and on their participation in the classroom. It is necessary to follow a lively method, in which the fundamental roles are played by the teacher and, above all, by the pupils.

Students are expected to play an active part by answering questions, contributing points to discussions and explaining and demonstrating their ideas to the class (Department For Education And Employment, 1999). And the commitment of the educator has to be maximum, as he or she is the responsible for the efficiency of the method, regardless of the level of each child.

The teacher has to be especially clear when explaining, making sure that students know what to do and drawing attention to the points over which they should take special care. This means focusing on the important aspects, whose information has to be well structured and ordered. In this regard, it is also necessary to use examples and demonstrations through appropriate resources, and references to work or previous methods, in order to facilitate the understanding of the ideas.

But the daily work of the educator is not reduced to the explanations of the contents. Once this process has been carried out, it is necessary to ask questions and discuss, in order to clarify doubts and ensure that all students take part. The information acquired should also be consolidated by maximizing opportunities to reinforce and develop what has been taught. And the answers and progress of the students must be considered in order to identify errors, clarify new doubts and help them. Finally, it is essential to carry out a review of what has been taught and what the pupils have learned.

In this sense, Cockcroft mentioned certain aspects that any teaching of Mathematics should include, which were listed in one of the previous sections of the thesis: expositions,

discussions, appropriate practical work, consolidation and practice of skills and routines, problem solving and investigational work. All of these aspects have to be considered.

Finally, the way of learning and teaching has to be adapted to the characteristics, needs and preferences of each student.

The number of students per classroom is considerable and that means that their levels may be very different. In addition, many children might have difficulties when acquiring the English language and not when learning mathematics, or vice versa, because, although there is only one subject, the contents in both cases and the way to acquire mathematics or English are different. These aspects should be considered.

Firstly, pupils who are very able can be encouraged through the development of differentiated group work, harder activities for homework and new challenges to be done at the end of each unit when other students are doing consolidation exercises. Teachers can also direct some specifically questions towards these students, in order to help them to feel motivated during the development of the lesson (Department For Education And Employment, 1999).

Secondly, pupils with particular needs who may have problems to reach the average level also require special attention. All children can benefit from oral work, and as it has happened with very able students, specific questions can be planned for them when the session is developed. Another idea is to sit them near the teacher, so that they can be helped in a discreet way: for example, when pupils discuss and answer in pairs before responding. Talking about non-oral work, they also have to be supported if it is necessary. It can be fine to minimize certain written explanations on worksheets, exercises and problems, but without forgetting that the foreign language has to be acquired effectively. And the use of supportive materials to help them to pick up the rules quickly can also be very interesting (Department For Education And Employment, 1999).

It is not easy to cater for pupils with special needs. Individual need does not necessarily mean individual attention (Department For Education And Employment, 1999), but many of these students require more time than the rest. Moreover, the subjects of Mathematics and English have traditionally not been easy, and that is why the bilingual subject is going to be more difficult than other similar ones. In order that all students progress at the same time and in the same direction, it is essential for teachers to have the backing of qualified assistants, as some students may need extra support.

Thirdly, other pupils may not require special needs, but emotional or behavioral difficulties. As in the previous cases, these ones might also require additional help.

And, as in the rest of the subjects, it is also essential to create an affective and motivating climate. Every student will take more advantage of the sessions if he or she feels comfortable. Therefore, it is also useful to ask them and know their opinions about the development of the lessons.

Summing up, the way of learning and teaching Bilingual Mathematics has to be characterized by: being based on recent learning and teaching theories and methods (and, above all, based on CLIL) and being oral, interactive and adapted to all students. In the next pages, other basic ideas related to methodology are developed: the importance of using the foreign language in al real and continuous way, how to organize each daily session and how to evaluate.

Real and continuous use of the foreign language.

The real[1] and continuous use of the language has to be promoted as soon as possible, because it is necessary to take advantage of the ease of young children to acquire knowledge. The great curiosity that the youngest students feel must also be considered, so that the duty of the teacher is to create communication situations. Besides, these situations are always given if English is used properly and consistently, regardless of the subject taught.

The use of the language is primarily developed through mathematical situations that can be given in the classroom. English is used by the teacher and acquired by the students through speaking, writing, reading and listening processes when giving explanations, solving doubts, completing and checking activities or doing reviews.

However, the English language is also used in real classroom situations that are not directly related to the science of mathematics. These situations are usually given when the classroom language is used. Examples of classroom language are the greetings to begin and end the class, the possible routines (*What is the weather like?,What date is it today?)* and many other expressions that are useful to catch attention, give orders, ask and answer, carry out explanations, interact with the teacher or with other students, etc..

[1] A real use means a serious and appropriate use of the language through real communication situations. The knowledge of languages in real life is essential and that is why these have to be acquired adequately. Thus, the real and efficient learning and use of the language is basic for future.

Considering the language, it is also necessary to refer to the importance of the non-verbal one. The use of body language through gestures is very interesting, especially when giving explanations and solving doubts.

On this basis, it is clear that English is not only basic when acquiring and using mathematical language through the possibilities and situations that arise in the classroom, but also in different contexts that can be given. The important thing, as it was mentioned, is that language has to be acquired in a real and continuous way.

Finally, the continuous use of English does not mean that the use of Spanish is not allowed. It is true that it is essential to use the foreign language as long as possible, but the use of the mother tongue in certain times, as for example in difficult explanations or other special situations, can be necessary. The code-switching process must be taken into account.

Plan for each lesson: time, activities and materials.

Putting emphasis on the organization of the lessons is essential because the success of each day is determined by the development of good planning. In the following paragraphs, references to the preparation of sessions and to details on the structuring of every day time are made.

The planning for each session has to be carried out previously. Before each lesson, it is important to know which the general aims are, how it is expected to achieve them and how each lesson connects with other lessons, past and future (Brewster, Ellis & Girard, 1991). It is also necessary to consider the needs, characteristics and preferences of the students. After taking into account all these aspects, the skills that will be taught throughout the session can be determined.

In order to clarify the objectives and prepare the lesson, the writing of a lesson plan can be very useful. It is a very interesting tool, as it helps to decide what to do and how to do it. Furthermore, It provides each teacher with a written record of what has been done in each session, so that it can be accessed later to analyze what happened (Brewster et al., 1991).

Its development is not obligatory and there is no one 'correct' way to write it, although a clear picture of what it is intended to do if it is made should be given. However, the daily planning of the sessions is essential, with or without the written plan.

Once the ideas related to the previous planning are mentioned, it is necessary to refer to the organization of the time of each session, which is an essential aspect to consider.To avoid boring, unproductive and too long lessons, it is necessary to establish some limits. Fifty-five minutes is an appropriate duration for each one. And sessions are divided into three parts: a starter of some minutes of oral work, mental calculation or review of previous contents, the main teaching activity and a plenary of five or ten minutes to round off the lesson.

The first part lasts between eight and ten minutes and is begun on a carpet in one of the corners in the classroom. Upon entering the classroom, students sit together there and start listening to the teacher, who is with them. It is an opportunity for them to develop their oral and mental abilities, by practicing mental calculation, reviewing contents through questions and answers, clarifying doubts and facilitating interaction between everyone (everything through the use of the foreign language). As there is not much time, not all children will participate every day. However, it is essential that all of them are focused and involved. If they do not intervene in that moment, they will do it later. The idea is that all students have the opportunity to use the foreign language throughout the lesson.

The plan of the day is also explained during these first minutes by the teacher, who uses a small board to write it on. After clarifying the contents of the session, all students sit at their desks, which can be arranged in a U-shape or in groups of three or four. It is now time to keep remembering previous knowledge by correcting the homework, which is sent every day. It is recommended that this process be developed by students and through the use of the board. That way, they pay more attention (they know they will probably have to take part and if the teacher does not follow an order when asking them to go to the board, they will not know when to stand up), lose fear of public speaking and continue learning (students acquire more knowledge if they correct their own exercises than if the teacher does it, although the teacher must always be ready to help them and to solve doubts).

Next, the main teaching activity is developed. This section of the lesson provides time for consolidating previous knowledge or extending it and lasts between thirty and forty minutes. Throughout this part, teachers explain the contents of the work for that day (making links to previous lessons, highlighting the important ideas and through the use of the board, flashcards, the computer, realia…), encourage their pupils to take part by asking them questions to involve them in the development of the lesson, ask them to do related work, identify and correct mistakes, and provide feedback. And in the case of students, they listen,

clarify their doubts, do activities, solve problems and develop investigational work, acquiring mathematical knowledge through the learning and use of the foreign language.

The plenary lasts eight or ten minutes and rounds off the lesson. Its importance is unquestionable. This part of the session is useful to review content, solve new doubts, present and express the work done, provide homework (that does not have to be always written work. For example: students can be asked to think about how to solve a problem, to learn some number facts or multiplication tables by heart, to gather information to use in the next lesson, etc. (Department For Education And Employment, 1999)), provide more feedback, highlight the progress made or refer to future sessions.

And once the lesson is finished, it is useful to allow some time to reflect on what happened in class. This is helpful for future planning and for building up one's self-confidence. The facts of making an analysis of the development of each session and evaluating the work done as a teacher objectively are recommended in order to improve.

The work propositions are based on the objectives, contents and other criteria related to the subject, but, as it was pointed out, the characteristics of each of the students must also be taken into account. The teacher must plan varied and useful activities designed to be done inside or outside the classroom and in an individual way, in pairs or in groups. The development of adequate tasks is basic, as CLIL subjects have to be focused on the development of processes and tasks.

If the students work individually, it will be necessary to keep them doing related activities, exercises or problems (or the same ones). If they do it in pairs, it will be important to encourage discussion and co-operation between them. And if they are organized in groups, other aspects will have to be taken into account: to have a manageable number of groups in order to know what each of them should be doing at any moment, to decide how groups will be introduced to tasks and how the group work will end, to avoid interruption by students by making sure that they can work independently in groups, that they know what to do before asking for help and if they finish early and that they know where to find the resources, to provide activities that do not involve children in a long wait for turns, and to sit and work intensively with one or two groups (without flitting between them all) (Department For Education And Employment, 1999).

All of these types of activities, which usually have to be developed based on contextualized situations, have to allow the development of all of the basic competences and

the possibility to make the students know the different strategies in each case, helping them to consider the different procedures to deal with each situation in order to choose the right ones. And all without losing sight of the acquisition of the second language and the mathematical content.

It is basic for this to use different materials. The textbook is important but not the only resource. In fact, the manipulation of real objects helps a lot in the understanding of mathematics. The same happens with the use of school library books and, especially, with the use of the information and communication technologies, which open up a huge range of possibilities. The teacher's duty is to look for, develop and adapt materials according to each situation.

The preparation of useful and varied materials is essential, as they are a basic element of the teaching and learning processes. For this reason, the development of the lessons depends largely on the materials that are used, which have to be carefully selected. The most important ones to consider when preparing the Bilingual Mathematics sessions are mentioned throughout the following paragraphs.

The textbook is a very interesting tool. It is a useful learning aid for the pupil, it helps to identify what should be taught or learned and the order in which it should be taught or learned, and it can provide many of the materials needed, such as CDs, DVDs, flashcards or worksheets (Brewsteret al.,1991). However, the course book is not essential. Sessions can be developed without using it, as there are many other complementary and supplementary tools.

These other tools should always be considered, regardless of whether the textbook is followed or not. They can be very numerous, very different and applied to most of the teaching or learning situations that can arise. For this reason, it is essential to know how to choose them depending on the moment. The most important are highlighted below.

It is inevitable to refer to the individual materials of each pupil, as they are used consistently. The pencil, rubber, scissors, glue, colored pencils, rule or compass are essential tools.

The use of realia is also very important, which is why it has to be frequent. Abacuses, bundles of addition, subtraction, multiplication and division cards for number bonds and one

hundred squares[2] help to count and do operations, and measurement equipment such as scales, rules or quantities, are used to calculate masses, distances or capabilities. The sets of plane figures or the construction kits help to recognize their basic characteristics and the same happens with the use of plastic geometric solids when analyzing their elements and differences. Even some kind of food can be useful: the ounces of chocolate or the mandarin segments serve to explain the most basic features of the fractions.

What is clear is that there are many real materials that can be adapted to each situation, as the mentioned above are just some of them. However, there are more tools. Worksheets and games are useful to review or to apply what has been studied, and library books help to complete information.

Finally, special mention must be made to information and communication technologies. Its development over the past decades has been vertiginous. Not only CDs and DVDs are used, as the Internet provides access to countless interesting information, such as videos and online exercises and problems. Furthermore, it should be remembered that the technological revolution has only just begun.

Assessment considerations.

Planning an adequate evaluation process is as important as developing the aspects mentioned in previous paragraphs. The assessment of the students learning refers to the systematic and continuous process through which the degree to which the learning objectives are being achieved is determined. A complete assessment should include all of the data related to the evolution of the student in a clear way, and both the mathematical and the foreign language aspects must be taken into account. Moreover, it has to be developed from the first day to the last one without interruption.

There are different evaluation methods. Most of them are developed by teachers, who have to monitor the progress of students and, on that basis, evaluate their own lesson plans for the future. However, students can also play an active role in these cases, through the development of self-assessment and peer-assessment processes.

[2] The one hundred squares are large squares composed of a hundred of smaller squares. In them, numbers from one to ten are written in order. There are ten rows and ten columns. These are useful tools to practice two-digit number additions or subtractions.

With reference to the evaluation of the students by the teacher, three types of processes are differenced: initial, formative and summative assessments. Students are constantly assessed by the educator in the short, medium and long term.

Through the initial assessment, their level in terms of knowledge of mathematics and of the foreign language is checked. It is developed at the beginning of the course, so that the teacher can decide if the students are able to advance without problems or if they need reinforcement. It could also be conducted at the beginning of each term.

The summative evaluation of the course is conducted through written exams or tests. An exam or test is established at the end of each of the units that are given.

And the formative assessment is continuously developed day by day and it is defined as the type of assessment which is "carried out during the instructional process for the purpose of improving teaching or learning. (…) What makes formative assessment formative is that it is immediately used to make adjustments so as to form new learning" (Shepard, 2008, p.281). It is useful for teachers to assess the daily work of their students, as it is basic to obtain information about them, to indicate whether their evolution is correct or not and to give them feedback for future if it is necessary, but also to draw conclusions about the changes that have to be produced for future sessions, making decisions for a better development of the future students learning. Finally, formative assessment is composed of three stages: the collection of information concerning the student's progress and difficulties, the interpretation of these data and the diagnosis of the factors that cause difficulties, and the adaptation of future activities. (Allal, Cardinet & Perrenoud, 1979).

When referring to the formative evaluation of a bilingual subject, many aspects have to be taken into account, so that it is not an easy process to perform without a suitable planning. The most important measurable ones are these ones: oral expression and comprehension, written expression and comprehension, listening and reading skills, attendance, daily study, homework and behaviour. All of these aspects have to be valued on the basis of each academic year, each group and even each person. There is obviously some minimum knowledge to be reached, but not all of the students are at the same level or evolve simultaneously.

An idea is to use tables to write individual marks related to all of these points. And once collected the information for each child, it is time to carry out their analysis. It is not a simple process.

On the one hand, it is basic to guide students in order to help them to improve for the rest of the school year. The positive and negative aspects have to be detected and parent meetings are useful to analyze the evolution of each child.

On the other hand, the teacher has to consider if the objectives are being achieved or if there are areas for improvement. For example, if a large number of students read with difficulties, do not show interest or misbehave, it is obvious that a thorough review of the situation should be made. And it is also necessary to take notes at the end of each unit, term or school year, establishing considerations in the medium and long term.

Given these ideas, it is clear that the formative assessment is not as simple as it might seem. In fact, the final mark of the exam and the achievement of the objectives depend on the daily progress, and for that reason its importance is unquestionable. Moreover, the individual and progressive notes and marks also have to be considered when establishing the final mark of each term, as tests are not the only assessment tool. In this case, not all of the marks established along the lessons must have the same value for this final mark, but all of them count.

Finally, as it was mentioned above, students can also play an active role in the evaluation process. On the one hand, children can evaluate themselves in the most objective possible way, reflecting on what was learned and what can be improved. And on the other one, it would be useful to provide them the opportunity to evaluate any of their classmates, correcting him or her and helping him or her to improve.

5.2. PRACTICAL PROPOSITION. DIDACTIC UNIT.

5.2.1. Introduction

As it was pointed out, the practical application of the theoretical considerations which were determined in previous sections is carried out through this proposition. My purpose is to develop an attractive model that is motivating for students and useful for the acquisition of the mathematical content and the foreign language.

This program is developed for a hypothetical four-year primary classroom with eighteen students.

The contents of the proposition are related to the measures of capacity. The main unit of capacity (litre) is remembered and the multiples and submultiples of the litre are studied. The planning consists of eight sessions in which theoretical aspects, exercises and problems related to this topic are developed. And, as this subject is bilingual, the acquisition of the foreign language (through the learning of mathematical vocabulary and expressions and through the use of the classroom language) is also basic.

Moreover, as this own model is based on a unit from a real primary textbook, the pages of this one which are used are scanned in order to show a clear comparison.

Finally, it is true that any other topic could have been chosen to develop the unit. The reason to consider the one related to capacities is that I enjoyed more the acquisition of content related to measures of magnitudes than the study of other mathematical units and concepts during my time as a student. In addition, I also believe that more interesting real materials than the ones that are recommended to teach other mathematical content can be used in this case.

5.2.2. Timing.

The objectives, contents and criteria related to the measurement of magnitudes are included in block 2 in both cases. For this reason, the eight sessions are developed at the end of the first term or at the beginning of the second one. Each session lasts fifty-five minutes.

5.2.3. Objectives.

Considering the topic of measures of capacity and some of the general objectives which were proposed previously, the specific objectives of this unit are the following ones:

- ✓ To know the main unit of capacity, its multiples and its submultiples.
- ✓ To develop and use tools and strategies to measure capacities.
- ✓ To estimate capacities in everyday containers.
- ✓ To choose the most suitable unit for the expression of a measure.
- ✓ To compare and order units of capacity and quantities.
- ✓ To express in a simple form of capacity a measure given in a complex form and vice versa.

- ✓ To add and subtract measures of capacity through mental calculation, easy written operations and mathematical problems, using the appropriate procedures.
- ✓ To develop the learning processes taking into account the previous knowledge and experiences.
- ✓ To use the language constantly and take part in interactive communication situations when remembering previous experiences related to the unit, asking and answering questions, understanding concepts, solving doubts, doing activities, establishing statements, doing reviews of contents, etc. That means the use of the foreign language through speaking, listening, writing and reading processes.

5.2.4. Contents.

The contents of the unit are determined based on the general ones which were established through the theoretical sections and on the objectives considered in the previous list. They are the next ones:

- ✓ Knowledge of the main unit of capacity, its multiples and its submultiples.
- ✓ Estimation of capacities in everyday containers.
- ✓ Election of the most suitable unit for the expression of a measure.
- ✓ Comparison and order of units of capacity and quantities.
- ✓ Expression of a simple form of capacity a measure given in a complex form and vice versa.
- ✓ Addition and subtraction of capacity measures.

And all of them are considered through the use of the foreign language.

5.2.5. Contribution to the development of the basic competences.

In the next paragraph, all the basic competences are listed and the way in which these are developed is mentioned. The general ideas related to these basic competences which were picked up in the theoretical section are considered.

1) *Mathematical competence:* it is the most developed of all of them, as the name of the subject indicates the evidence. However, as it was pointed out, a real development is given only if the mathematical contents which are learned are useful for real life.

2) *Competence in linguistic communication:* it is undoubtedly developed through the acquisition of the foreign language (classroom language, and vocabulary and expressions related to capacity measures, to capacity containers and to the resolution of operations, estimations and comparisons). Moreover, the essential aspects of the mathematical language that is learned have to be accurately used (for example, when describing and reasoning processes) and incorporated to the familiar contexts.
3) *Competence in knowledge and interaction with the physical world:* it is also developed. It is taken into account the importance of the knowledge and use of measures of capacity in order to understand aspects of the physical world.
4) *Treatment of information and digital competence:* its development is given in different ways: first, the knowledge of English provides access to the information from the media and its acquisition, and this foreign language is constantly used throughout this unit of measures of capacity. Second, the skills associated with the use of the numbers are or have been provided, and that facilitates the understanding of information on measures. Finally, the use of technology tools is also useful to teach and learn, and a PowerPoint is used in this sense.
5) *Learning to learn competence:* as it was pointed out, the instrumental nature of many of the Bilingual Mathematics content provide value to the development of this competence. The knowledge of the capacity measures and its use through operations and problems is useful for future subjects and for real life (for example, the knowledge of the measurement units is necessary when buying liquid products in a supermarket). And from a linguistic point of view, students acquire English because of its usefulness for life. The learning of vocabulary related to the capacity measures can also be interesting for future situations.
6) *Autonomy and personal initiative competence:* it is also developed. When learning about measurement, students do exercises and solve problems following their own criteria, and acquire the foreign language, developing their abilities and thinking about their knowledge.
7) *Competence in social skills and citizenship:* this competence is also developed. English is used and its acquisition always facilitates the communication and social transmission: the classroom language can be very useful for social life and the vocabulary and expressions related to the measures of capacity might also be important. Moreover, students are asked to do individual and group work, respecting, cooperating and accepting different opinions.
8) *Cultural and artistic competence:* it is developed because the mathematical knowledge is considered as a contribution to the cultural development of the world and the learning of

concepts related to the capacity measurements means the acquisition of certain knowledge. And also because a language from a different context is acquired.

5.2.6. Resources.

Although my proposition is created based on a real textbook, any book is used in any of the sessions. The materials which are needed are the next ones:

- ✓ Different containers.
- ✓ Worksheets.
- ✓ Flashcards.
- ✓ PowerPoint (A screen in the classroom is needed).
- ✓ An online game.
- ✓ Individual tools from each student.
- ✓ A dictionary.

5.2.7. Attention to diversity.

As it was determined in the theoretical section, it is essential that each teacher can be helped by another adult in all of the sessions, and the role that is played by the teacher assistant through the support and reinforcement of students is basic. In this case, I consider that there are not very able students, but one of the children of the hypothetical group of students suffers a mild dyscalculia, sits next to the teacher and receives additional individual tutoring from a third and different teacher, but without leaving any of the group sessions developed by the main teacher and the assistant one (the student would receive that single individual lesson in a different session of the week used by his/her peers to do their homework). Moreover, I also consider that the other students are obviously not identical and that they may need the assistant help at certain times. Thus, the importance of this second professional is unquestionable, in order to help the students and the teacher.

5.2.8. Organization of the sessions.

Just before starting the unit, the main teacher writes a personal lesson plan in order to decide what to teach in each session and how to do it. Then, he shares that document with the assistant.

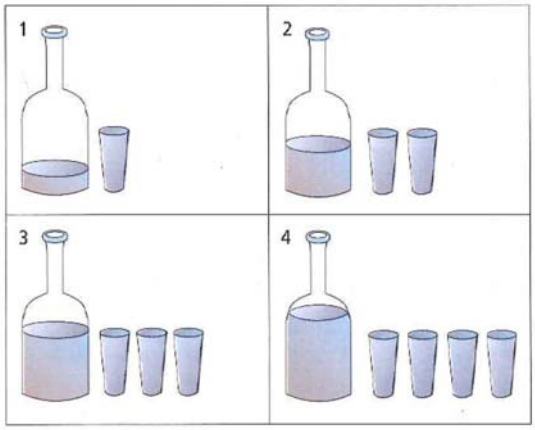
1st session: the main unit of capacity: the litre.

1. LA UNIDAD PRINCIPAL DE CAPACIDAD: EL LITRO

Para medir la capacidad de un recipiente, la comparamos con la de otro. Por ejemplo, podemos medir la capacidad de una botella comparándola con la de un vaso.

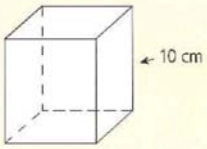
Si la botella queda llena después de verter en ella el contenido de 4 vasos de agua, decimos que la botella tiene una capacidad de 4 vasos.

Para que podamos entendernos mejor, sin tener que explicar cómo es el vaso o el frasco que empleamos para medir, se utiliza una unidad fija: el **litro**.



La unidad fundamental de capacidad es el **litro**. Se escribe **l**.

Un litro es la capacidad de un cubo de 10 cm de arista.



Ejemplo:

Para llenar una piscina portátil se han necesitado 10 viajes de un camión cisterna. En cada viaje, el camión descargaba 600 litros de agua. ¿Cuántos litros caben en la piscina?

Será 600 litros 10 veces: $600 \times 10 = 6000$.

En la piscina caben 6000 litros.

1. De los siguientes objetos y productos, subraya aquellos que, normalmente, puedes encontrar envasados en recipientes de un litro:

   naranjada, aceite, vinagre, jarabe, frasco de gotas nasales, perfume, suavizante, colonia, licores, colirios, pintura, esmalte de uñas, alcohol, leche, agua, tinta china, champú

2. Con una botella de refresco de un litro de capacidad llenamos 2 jarritas exactamente iguales. ¿Qué capacidad tiene cada una? ¿Y si llenamos 4 jarritas iguales?

3. Llenamos un barreño utilizando un cazo de cocina de 2 l. Si echamos 8 cazos, ¿qué capacidad tiene el barreño?

4. Si una vaca da unos 8 litros diarios de leche, ¿cuántos litros obtendrá un granjero en una semana si en su granja hay cinco vacas? Calcula cuánto cobrará por dicha leche si la vende a 45 PTA cada litro.

5. De un depósito de 12 000 l de agua se extrae la tercera parte de su contenido, y después, la cuarta parte. ¿Cuántos litros quedan todavía?

a) Starter (8 min)[3].
   - ✓ It starts on the carpet in the corner.
   - ✓ Starter routines (1 min). The lessons are always begun with two questions from the teacher: *what day is it today?*, *what is the weather like?*. The teacher, who tries to use body language constantly (in all explanations and in order to facilitate the understanding to everyone), could also ask in certain cases *what is the first /second question that I always ask?*, giving the students the chance to ask it. When answering, in some cases they do it all together and in other ones in an individual way (in order to stimulate the shiest students).
   - ✓ Introduction to this new unit (7 min). The pupils are asked about their previous knowledge related to capacities. The topic of the unit is written on a small board that is next to the teacher. It should be remembered that some characteristics of capacities and the unit of litre are taught in the previous years. The idea is to interact with them helping them to develop the foreign language. For this reason, this part of the session is entirely oral.

b) Main part (38 min).
   - ✓ PowerPoint (15 min). It is developed to introduce the unit, and the computer and the screen of the classroom are used. Each person is in each desk, forming an U-shape all together. The idea is not only to show the information, but also to let them take part, discover and learn through the use of the foreign language. Some real containers full of water (a jar (1l), a bottle (1/2l) and a glass (1/4l) are also taken to explain to the students the variation of capacities. The starting point is the

[3] The duration of the parts of the sessions is approximate, as it may vary depending on factors such as the degree of difficulty or the interest of the students in each of these parts.

main unit of capacity. Pupils have the chance to ask and answer questions in order to solve doubts and the educator also tries to encourage interaction. And each student receives a copy of the PowerPoint which is shown.

As the PowerPoint is also used as an introduction to the unit, it is not necessary to delve into the new concepts that are shown. The idea is to provide a first contact with these ones.

The PowerPoint includes information on: the main unit of measure, the submultiples and multiples of the litre and the relations between all of these units.

- ✓ Oral activity (5 min). *Say examples of products that can be found in one-litre containers.* Students raise their hands and take part. It is essential to encourage all of them to speak. All of the names of possible products are written on the blackboard by the teacher, and the students have to copy them in their notebooks.
- ✓ Example of a problem (5 min). *PROBLEM STATEMENT: A machine extracts 20 litres of water per minute from a pool. If the machine works for one hour. How many litres of water are extracted? SOLUTION: 1 hour = 60 min. 20 l x 60 min = 1200 litres. The machine extracts 1200 litres.* The problem has been solved and is given as an example. As this subject is bilingual, it is basic not only to teach how to reason in order to solve the problem, but also to focus on the linguistic aspects: use of English (the statement of the problems is always read by the students, there could be a discussion in order to find a solution), understanding of concepts (for example, possible new words (*machine*?), etc. The unknown expressions have to be written in the notebook and learned. These can be asked to the teacher or to the assistant, or found in a dictionary. And the same happens with the rest of future problems and activities.

  It is essential to be sure that all students understand the exercise.
- ✓ Math problem solving (13 min). The pupils work in an individual way. The exercises to do are similar to the previous one and are given through a worksheet that has to be completed. Before using worksheets, two students are responsible for distributing them: everybody does it, depending on the day.

  A man fills a tank using a 5-litre bottle. If the man uses the bottle 7 times, what is the capacity of the tank?

  A fuel tank contains 15752 litres. First, an employee removes half of that amount. Then, he extracts other 1100 litres. How much fuel is in the tank at the end?

The exercises are checked on the blackboard. All students use it, it does not matter if it is at the moment or in future activities.

On the one hand, some students could finish the exercises much earlier than the rest. Those pupils are asked to review their own exercises or to solve their doubts without disturbing the others. On the other one, other pupils, as the one who suffers dyscalculia, could spend more time and need additional help from the teacher or the assistant. The same could happen with future exercises and activities again.

c) Plenary (9 min).

- ✓ Homework:

  Two problems which are given (worksheet, half of an A4 sheet):

  A man uses a bottle that is full of milk to fill 2 identical jars. The capacity of the bottle is of 1 litre. What is the capacity of each jar? And what is the capacity of each jar if the man fills 4 identical ones?

  A woman has to distribute 462 litres of oil between three tanks. How many litres of oil are in each tank? If the woman sells all of the oil and each litre costs 2 Euros, how much money does she earn?

- ✓ There is a moment in which the teacher gives back the exams which were completed in the last session of the previous unit. At this moment, the students know about their mistakes and the mark they have.

Once the class is finished, the teacher has to carry out a self-assessment analyzing what has been taught and how has been taught in order to improve. The educator should ask himself or herself about the development of the lesson, analyzing the strengths and trying to correct the possible weaknesses for future lessons. This self-assessment should be developed at the end of each day.

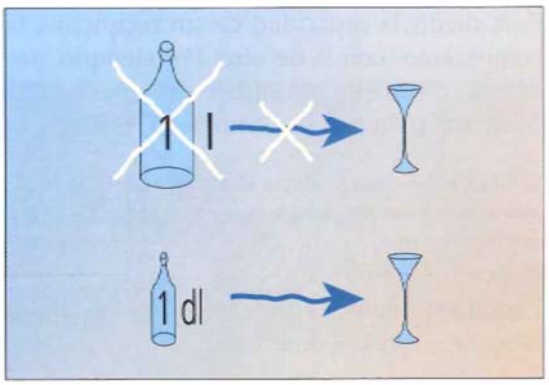
2nd session. The submultiples of the litre.

2. SUBMÚLTIPLOS DEL LITRO

Ya sabes que con el litro puedes medir muchas capacidades, pero no siempre te será del todo útil.

Si quieres saber la capacidad de una copa de agua, la unidad del litro te resultará excesiva; deberás, pues, buscar alguna unidad más pequeña...

Para este y otros casos utilizaremos las unidades inferiores al litro: el decilitro y el centilitro.



6. Enumera cuatro objetos o productos que, normalmente, se vendan en envases inferiores a un litro de capacidad.
7. En un frasco de colonia caben unos 20 cl. Expresa esta capacidad en decilitros.
8. ¿Cuántos decilitros de refresco hay en una botella de medio litro? ¿Y cuántos centilitros?
9. ¿Cuántos centilitros de leche hay en un vaso grande de cuarto litro? ¿Y cuántos decilitros?
10. Con una botella de horchata de litro y medio llenamos vasos de 25 cl de capacidad. ¿Cuántos podremos llenar?
11. Marca con un aspa la unidad más adecuada al medir las capacidades de los siguientes objetos:

	Tacita de café	Olla de caldo	Bombilla	Refresco individual	Media botella de leche
dl					
l					
cl					

a) Starter (11 min).

- ✓ Starter routines (1 min) on the carpet.
- ✓ Correction of the two problems (homework) on the blackboard (10 min). The children leave the carpet and move to their desks. The possible doubts are consulted (the mathematical ones or the related to the foreign language ones, as for example, the difficult words of the statement). And the children are asked to write the main data of the problems on the blackboard before solving them.

b) Main part (35 min).

- ✓ Oral explanation of the importance of using the submultiples of the litre (dl, cl, ml), which are basic to measure small capacities. Consideration of certain equivalences (1l = 10 dl, 1l = 100 cl, 1l = 1000 ml). The pupils also have their copies of the PowerPoint to see these data on their own and ask to solve their doubts (15 min).
- ✓ Oral activity (4 min). *Say examples of containers with capacities expressed in decalitres, centilitres or millilitres.* As in the 1st session, students take part, speak and copy the names of the products.
- ✓ Example of a problem (3 min). *PROBLEM STATEMENT: a waitress uses a 3 – litre water bottle to fill glasses with a capacity of 20 centilitres each. How many glasses can she fill? SOLUTION: 3l = 300 cl. 300 cl / 20 cl = 15. She can fill 15 glasses.*
- ✓ Math problem solving (13 min). The pupils work in an individual way. First, the statements of the problems are copied from the blackboard on the notebooks. Then, they are completed (and the pupils can solve their doubts if they ask the teacher and / or the assistant. If those doubts are related to the understanding of the language, the students, as it was mentioned, always have to copy the unknown words and expressions on their notebooks).

  A container contains 453 litres of fuel. Express that quantity in decilitres and centilitres.

  The capacity of a yogurt is 100 millilitres. The capacity of another recipient is 2 litres. How many yogurts could fit in that container?

  Then, the exercises are checked on the board. And as in previous cases, the students who finish early do reviews of what they have done or solve doubts without disturbing, and the ones who have more difficulties are especially supported.

c) Plenary (9 min).

- ✓ Review of the content acquired during the lesson. The teacher gives his students the opportunity to express their doubts and their opinion through the use of the foreign language. Thus, the teacher realizes what the students have learned and takes notes for future sessions.

- ✓ Homework: ask each student to bring three small containers from home: milk boxes, drink cans and bottles, cream containers, plastic glasses, etc. The capacity of these ones should be written on the packaging.

3rd session. Relations between the submultiples of the litre.

3. RELACIONES ENTRE EL LITRO Y SUS SUBMÚLTIPLOS

La madre de Benito va a comprar la jarra más grande de la tienda. Encuentra tres modelos como los que ves en el dibujo.

¿Cuál de los tres quiere comprar la madre de Benito? Para averiguarlo hay que hacer un pequeño cálculo. La unidad más pequeña que aparece en el dibujo es el centilitro.

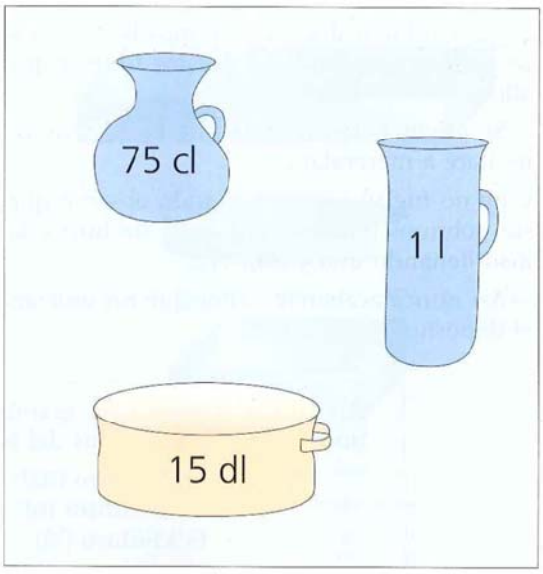
Expresamos todas las capacidades en centilitros:

75 cl
1 l = 100 cl
15 dl = 150 cl

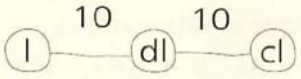
La mayor es la de 150 cl, o sea, 15 dl.

La madre de Benito debe comprar la jarra de 15 dl.

Es muy importante saber relacionar el litro, el decilitro y el centilitro.



1 l = 10 dl → 1 dl = 0,1 l
1 l = 100 cl → 1 cl = 0,01 l
1 dl = 10 cl → 1 cl = 0,1 dl



Ejemplo:

Expresa en litros el contenido de las tres jarras anteriores.

75 cl = 75 centésimas de litro = 0,75 l
15 dl = 15 décimas de litro = 1,5 l

12. Expresa en centilitros las siguientes capacidades:

12 dl 3 l 4,1 dl 1,5 l 0,2 dl

Ordénalas de menor a mayor.

13. En las siguientes frases, indica si son verdaderas o falsas:

a) El litro equivale a 10 decilitros.
b) El centilitro es a centésima del litro.
c) El decilitro equivale a diez litros.
d) El centilitro es la décima del decilitro.
e) El decilitro equivale a cien centilitros.

14. Expresa en litros el refresco que cabe en las botellas que tienen las siguientes capacidades:

10 dl 150 cl 100 cl 50 cl 2,5 dl

15. Juan tiene una caja de 75 cl, Teresa tiene un bote de 12 dl, y Cristina, una botella de 0,4 l.

¿Quién tiene un recipiente de más de medio litro y menos de un litro? ¿Y de más de cuarto de litro pero menos de medio litro?

a) Starter (11 min).
   - ✓ Starter routines (1 min) on the carpet.
   - ✓ Maths game (10 min). It is a two-level online game. The link to the website is listed on the last page of this work. Students are asked to match different pairs of cards that express the same quantities (for example, 1 l and 1000 ml).

     Students take part one by one. They stand up and use the computer to choose the right option. Some children will not participate in this game, but they will do it in future exercises and in the following starter activity. However, everybody can help from each desk if the student who is doing the activity has got doubts.

b) Main part (35 min).
   - ✓ Review of the information related to the submultiples of the litre which was acquired in the previous lesson, including those certain equivalences (1l = 10 dl, 1l = 100 cl, 1l = 1000 ml) (5 min).
   - ✓ Explanation of the other possible relations between the submultiples of the litre and this main unit (to move to the immediate upper unit, it is multiplied by 10; to move to the immediate lower unit, it is divided by 10) (20 min).

     The explanation is started through this example: *PROBLEM STATEMENT: there are four types of plastic boxes for sale in a furniture shop. The capacity of the first one is 12 litres, the capacity of the second one is 200 decilitres, the capacity of the third box is 2500 centilitres and the capacity of the last one is 5000 millilitres. Which one is the smallest one? SOLUTION: 12l = 12000 ml. 200 dl = 20000ml. 2500 cl = 25000 ml. 5000 ml. The fourth box is the smallest one.* As in previous examples, this exercise has been done and the solution is given to the students.

Then, all of the possible equivalences are considered: apart from the ones from the previous session: 1dl = 0,1 l; 1cl = 0,01 l; 1ml = 0,001 l; 1dl = 10 cl; 1cl = 10 ml; 1 dl = 100 ml; 1cl = 0,1dl; 1ml = 0,1 cl; 1ml = 0,01 dl. Many new examples of equivalences with different numbers should be given, interacting with the students. And as this may be quite difficult, the best option is to draw a picture on the blackboard that shows the equivalences and tell them to copy it. Finally, it is essential to solve all possible doubts of pupils.

Activity in pairs (10 min) The containers that the students have brought from home are used. The pupils sit in pairs, take their own containers together and compare them. Then, they get used to the idea of the physical space that can occupy 1 litre of milk, 33 cl of Coke or 200 ml of sun cream. Finally, they draw the objects and add all of the quantities (considering the use of only one unit that they can choose). And the teacher and the assistant move around the desks checking their results.

c) Plenary (9 min).

- ✓ Homework:

  Three exercises have to be completed (through a worksheet which is given):

  Express the following quantities in centilitres: 1,4 ml; 370 l; 0,2 dl; 166 ml; 1 dl.

  Albert has got three bottles. The first one contains 40 centilitres of water, the second one contains 785 millilitres and the third one contains 4,5 decilitres. What bottle contains less water? What bottle contains more than half a little of liquid?

  Are the following statements true or false? a) 10 decilitres and 1 millilitre is the same quantity. b) The millilitre is the tenth part of the centilitre. c) 10 decilitres and 1 litre is the same quantity. d) The centilitre is the tenth part of the litre.

- ✓ New possible doubts.

4th session. The multiples of the litre.

4. MÚLTIPLOS DEL LITRO

El tío de Ignacio y María es dueño de una bodega. Un buen día, sus sobrinos le visitan y se quedan sorprendidos por los toneles que allí hay. Él les dice:

—Si adiviináis la capacidad de este barril, os invitaré a merendar...

Cuál no fue su sorpresa cuando observó que sus sobrinos tomaban la botella de litro y la iban llenando una y otra vez:

—Así nunca acabaréis... ¿Por qué no utilizáis el depósito del decalitro?

Para medir capacidades grandes, utilizaremos las unidades superiores al litro o **múltiplos** del litro:

El **decalitro** (dal) → **1 dal** = 10 l
El **hectolitro** (hl) → **1 hl** = 100 l
El **kilolitro** (kl) → **1 kl** = 1 000 l

Ejemplo:

La capacidad de una piscina olímpica es de 2 100 kl. ¿Cuántos litros de agua caben en una de estas piscinas?

Como 1 kl = 1 000 l, en 2 100 kl habrá:

$$2\,100 \times 1\,000 = 2\,100\,000$$

Por tanto, la piscina puede tener 2 100 000 l de agua.

16. De los siguientes objetos, ¿cuáles deben medirse con los múltiplos del litro?

Una botella familiar de agua, un camión cisterna, un tonel de vino, un pantano, un cubo de fregar, una cantimplora, una copa, un lago, una botella de colonia, el bidé de un lavabo

17. Una piscina portátil tiene una capacidad de 6 kl. Expresa dicha cantidad en hectolitros, decalitros y litros.

18. Una familia gasta cada mes 210 hl de agua. ¿Cuántos litros gasta, aproximadamente, cada día? ¿Cuántos decalitros gasta en un año?

19. En un depósito caben 3 kl y 2 hl. Si ahora hay 1 500 litros, ¿cuántos litros faltan para llenarlo?

20. Un camión cisterna tiene una capacidad de 20 kl; en una gasolinera descarga 130 hl, y en otra, 600 dal. Si partió con la carga llena, ¿cuántos decalitros quedan todavía en el camión?

a) Starter (12 min).

- ✓ Starter routines (1 min) on the carpet.
- ✓ Correction of the three problems (homework) on the blackboard (11 min). As in the previous sessions.

b) Main part (33 min).

- ✓ Oral explanation of the importance of using the multiples of the litre (dal, hl, kl), when measuring big capacities. Consideration of certain equivalences (1dal = 10 l, 1hl = 100 l, 1kl = 1000 l). The pupils also have their copies of the PowerPoint to see these data on their own and ask to solve their doubts (15 min).
- ✓ Oral activity (4 min). *Say examples of structures or containers with capacities expressed in decalitres, hectorlitres or kilolitres.* As in previous sessions, students take part, speak and copy the names of the products.
- ✓ Example of a problem (3 min). *PROBLEM STATEMENT: a water tank contains 17000 hectolitres. How many litres of water are in the tank? SOLUTION: 1hl = 100 l. 17000 hl x 100 = 1700000 litres of water are in the tank.*
- ✓ Math problem solving (11 min). The pupils work in an individual way. As in the 2nd session, the statements of the problems are copied from the blackboard on the notebooks and then they are completed. Then, each student compares his / her result with a colleague and tries to correct any possible mistake. Finally, the activities are checked on the board.
- ✓ *A family spends 84 hectolitres of water per week. How many decalitres do they spend per day? And how many litres do they spend per month?*
- ✓ *A tank contains 30 kilolitres of flammable liquid. If a professional introduces 78 hectolitres into it one day and 120 decalitres another day, how many litres are inside the tank at the end?*

c) Plenary (10 min).

- ✓ Review of the content acquired during the lesson.
- ✓ Information for students: exam at the 8th session.

5th session. Relations between the multiples of the litre.

5. RELACIONES ENTRE EL LITRO Y SUS MÚLTIPLOS

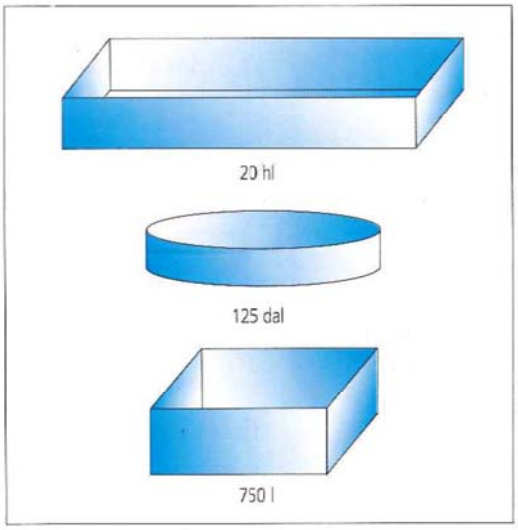
Los padres de Ana quieren comprar una piscina hinchable. Hay tres modelos, como los del dibujo.

A Ana le gusta más la redonda, y su padre se decidió por la más grande.

¿Ha tenido suerte Ana?

Para averiguarlo hay que expresar todas las cantidades en la misma unidad.

Conviene dominar muy bien la relación entre el litro y sus múltiplos.



1 dal = 10 l → 1 l = 0,1 dal
1 hl = 100 l → 1 l = 0,01 hl

Ejemplo:

¿Qué piscina compró el padre de Ana?

Expresamos las capacidades de las piscinas en la misma unidad. Por ejemplo, en decalitros:

125 dal
750 l = 75 dal
20 hl = 200 dal

La mayor cantidad es 200 dal; o sea, 20 hl.

El padre de Ana compró la piscina de 20 hl.

21. Expresa en litros las siguientes capacidades:

14 hl 5 kl 314 dal 2,1 hl 0,2 dal

y ordénalas de mayor a menor.

22. ¿Cuántos litros le faltan a cada cantidad para llegar a un hectolitro?

83 l 7 dal 0,1 kl 3,1 dal 0,9 hl

23. ¿Cuántos litros de agua gasta al mes cada familia?

Familia López: 6 hl Familia González: 21 kl
Familia Pérez: 40 hl Familia Gómez: 74 kl

24. En el ejercicio anterior, localiza a las siguientes familias:

a) Gasta más de medio kilolitro pero menos de uno.
b) Gasta más de 10 hl pero menos de 50 hl.
c) Gasta menos de 5 000 dal pero más de 500 dal.

a) Starter (11 min).

- ✓ Starter routines (1 min) on the carpet.

  Starter game (10 min). Some flashcards are shown. Some of them contain pictures (a tank, a lake, a jug of water…) and the other ones contain different numbers (different capacities: 50 hl, 750000 kl, 0,5 dal…). All of the flashcards are on a table and placed against the wall. In this way, their position is almost vertical and children can see them. The flashcards are mixed and the students have to put them in pairs (a tank = 50 hl, a lake = 750000 kl, a jug of water = 0,5 dal…). Each pupil stands up one by one and puts the cards in pairs. Those children who did not take part in the online game participate now. And, as in that online game, everybody can help from each desk if the student who is doing the activity has got doubts.

b) Main part (34 min).

- ✓ Review of the information related to the multiples of the litre which was acquired in the previous lesson, including those certain equivalences (1dal = 10 l, 1hl = 1000 l, 1kl = 1000 l) (4 min).
- ✓ Explanation of the other possible relations between the multiples of the litre and this main unit (to move to the immediate upper unit, it is multiplied by 10; to move to the immediate lower unit, it is divided by 10) (20 min).

  The explanation is started through this example: *PROBLEM STATEMENT: a winemaker wants to buy a barrel for his business. He finds three types of barrels: the capacity of the first one is 150 litres, the capacity of the second one is 7 hectolitres and the third one can contain 27 decalitres. The winemaker wants to by the smallest one. What barrel does he have to buy? SOLUTION: it is basic to*

express all quantities using the same unit. 150 l = 15 dal. 7hl = 70 dal. 27 dal. He has to by the first barrel. As in previous examples, this exercise has been done and the solution is given to the students.

Then, all of the possible equivalences are considered: apart from the ones from the previous session: 1dal = 10l; 1hl = 100l; 1kl = 1000 l; 1kl = 10 hl; 1kl = 100 dal; 1 hl = 10 dal; 1hl = 0,1kl; 1dal = 0,01 kl; 1dal = 0,1 hl. As in the 3rd session, new examples of equivalences with different numbers are given and a picture that shows these equivalences is copied. And pupils ask to solve their doubts.

- ✓ Exercises (10 min). As in previous cases, these are copied on the notebook from the board (and checked on this one).

  Express the following quantities in kilolitres: 21hl; 137 dal; 496 l; 56 dal; 4 hl.

  Express the following quantities in decalitres: 19 l; 240 kl; 8 hl; 166 l; 4,1 hl.

c) Plenary (10 min).

- ✓ Homework.

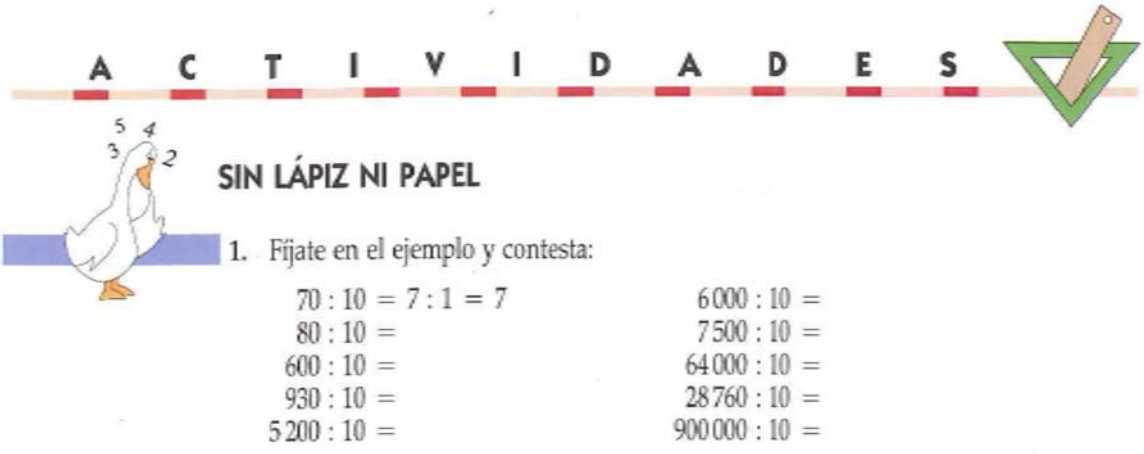
  Put these quantities in order from lowest to highest. 31 dal; 0,9 kl; 2100 l; 1hl; 4 l.

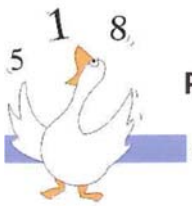
  How many litres of fuel does each truck carry? Truck A, 0,6 kl. Truck B, 40 dal. Truck C, 127 l. Truck D, 8 hl.

  9 dal = 0,6 hl + ... l. *0,07 kl = 300 l + ... hl.* *400 l = 2 kl - .....hl.*

- ✓ New possible doubts.

6thsession. Review and reinforcement activities (I)





PARA PRACTICAR

4. Clasifica las capacidades de los siguientes objetos según la unidad más adecuada (kilolitro, litro, centilitro) para medirlas.

Una botella de leche	Un frasco de jarabe
Un lago	Una garrafa de agua
Un frasco de perfume	Una lata de refresco
Una piscina	Una olla de cocina
Un yogur	Un camión cisterna

5. En una bodega hay 12 cajas de refresco; en cada caja hay 16 botellas de litro y medio cada una. ¿Cuántos litros de refresco hay almacenados?
6. Convierte a hectolitros y decalitros la capacidad de una piscina de 1 500 kl.
7. Una farmacia ha vendido 8 botellas de 50 cl y 4 botellas de medio litro de alcohol. ¿Cuántos decilitros de alcohol se han vendido?
8. ¿Cuánto habrá costado todo el alcohol vendido del problema anterior si cada litro valía 60 pesetas?
9. En una granja hay una cisterna de agua para la bebida de los animales. Si en ella caben 2 kl, 8 hl y 8 dal, y cada día se consumen 32 dal de agua, ¿durante cuántos días podrán beber los animales?
10. ¿Cuántos vasos de 25 cl harían falta para llenar un barreño de 20 l?
11. Para fregar un suelo se utiliza un desinfectante de manera que por cada decalitro de agua es necesario echar 5 cl de dicho desinfectante. Si finalmente se han utilizado 20 cl de desinfectante, ¿cuántos litros de agua se habrán consumido para fregar?

a) Starter (15 min).
   - ✓ Starter routines (1 min).
   - ✓ Correction of the three activities (homework) on the blackboard (14 min).

b) Main part (30 min).
   - ✓ Review and reinforcement activities.

   Three oral activities (the teacher asks one by one, everybody takes part).

   Look at the example and complete the exercises: 90:10 = 9:1 = 9.

   60: 10 =200:10 = 450:10 = 6280:10 = 32460:10 = 914000:10 =

   Look at the example and complete the exercises: 90:100 = 9:10 = 0,9.

   70:100 = 600:100 = 2600:100 = 27000:100 = 28300:100 = 436200:100 =

   Look at the example and complete the exercises: 90:1000 = 9:100 = 0,09.

   60:1000 = 800:1000 = 71000:1000 =

108000:1000 = 2140000:1000 = 18560000:1000 =

Choose from one of these units (kilolitre, litre, centilitre) to measure the capacities of the following things: swimming pool, barrel, sun cream container, milk box, wine bottle, perfume container, can of coke, lake.

Written exercises. The students receive a worksheet that contains the activities. Then, these are checked on the board. Finally, possible new doubts are solved.

Complete this table.

kl	*hl*	*dal*	*l*	*dl*	*cl*	*ml*
			0,8			
						99000
		3,7				

A man buys 6 litres of olive oil and 4,5 litres of sunflower oil. How many decilitres of oil does he buy?

Albert and Alan buy 350 litres of milk. Each litre costs 82 cents. If Albert takes 16.500 cl and Alan the rest, how much money does each person pay?

c) Plenary (10 min).

- ✓ Homework.

  Four problems (worksheet).

  A farmer has got 6 cows. Each cow produces 206 decilitres of milk. If each litre of milk costs 40 cent., how many Euros does he earn?

  How many decilitres is the fourth part of a hectolitre?

  The capacity of a tank is 12,5 kilolitres. How many decilitres can be introduced in 5 tanks?

  186 children take a shower everyday at a youth camp. If each child uses 0,28 hectolitres of water per day in his / her shower, how many litres of water will they spend in 10 days?

- ✓ New possible doubts.

7th session. Review and reinforcement activities (II)

A C T I V I D A D E S

PARA RESOLVER

12. El pantano de Montijo (Badajoz), en época de sequía, tenía 1 100 000 kl de agua. ¿Cuántos camiones cisterna de 22 000 l cada uno podrían llenarse con toda el agua del pantano?

13. A partir del 1 de enero de 1992, la gasolina súper costaba en España 92 PTA cada litro. Si el depósito de un coche es de 4,5 dal, ¿cuánto cuesta llenarlo?

14. En 1990, el precio del litro de leche de vaca que cobraban los ganaderos era de 38 PTA. ¿Cuántos centilitros nos darían si comprásemos leche por valor de 2 052 PTA?

15. El zumo que contiene una naranja es de unos 8 dl, aproximadamente. Si exprimimos tres de ellas, ¿cuántos litros de zumo obtendremos?

16. Un balón reglamentario de fútbol tiene una capacidad de 5,8 l. Si lo llenásemos de agua, ¿cuántos vasos de 2 dl cada uno podríamos completar?

17. La pirámide de Keops (Egipto) tiene una capacidad de unos 2 415 770 kl, mientras que la de Kefrén es de 7 253 000 l. ¿Cuántas veces es mayor una que otra, aproximadamente?

a) Starter (15 min).
   - ✓ Starter routines (1 min).
   - ✓ Correction of the four activities (homework) on the blackboard (14 min).

b) Main part (30 min).
   - ✓ More written activities. (A4 worksheet with spaces to complete the tasks). These tasks are also checked.

     One boat carries 414 kilolitres of fuel. Another boat carries 1296 hectolitres and 485litres of oil. What is the difference in litres of cargo between the two boats?

A worker loads barrels of oil in a train. Each barrel contains 200 litres. If he has to load 5,4 kilolitres of oil, how many barrels does he have to load?

A lorry carries 10 tanks. Each tank has 0,4 kilolitres and 12 decalitres of milk. How many litres of milk does the lorry carry?

A basketball ball has a capacity of 6,4 litres. If a child fills it with water, how many glasses of 4 dl can he fill?

The human heart provides 25 centilitres of blood to the brain per minute. How many centilitres and decilitres does it provide in one day?

c) Plenary (10 min).

- ✓ Review of the vocabulary and expressions which have been studied (litre, submultiples, tenth part, barrel, tank, etc). As it was mentioned when developing the first session, every child has to ask or look for the meaning of the unknown and new words, copy them on the notebook and learn them.
- ✓ Clarification of all possible doubts.
- ✓ No homework.

8th session. Exam.

a) Starter (5 min).

- ✓ Starter routines (1 min).
- ✓ Possible doubts before the exam (4 min).

b) Main part (40 min).

- ✓ It is time for the test. It is easy and all of the activities are similar to the ones which were already made. Some vocabulary is asked, too. The teacher explains the exercises and the students have got 35 min. to complete it. This is the exam: the activities are listed and the number between parentheses is the score of the exercise (out of 10). The student who suffers mild dyscalculia could be helped in certain moments.

1) *Fill the gaps using vocabulary from this unit of measures of capacity.*
   hl = …… ml = …… 1/100 = One …… part.
   dal = …… The kl, the hl and the dal are the…… of the litre.
2) *Express these quantities in dl: 1,8 hl; 147ml; 2,1 dal; 166 ml; 1,98 l.*
3) *Express these quantities in hl: 97 dal; 4,1kl; 0,34l; 871 ml; 14dl.*
4) *The capacity of a tank is 8,5 kl. How many l can be introduced in 6 tanks?*

5) *One lemon contains 9 dl of juice. How many litres of juice do 3 lemons contain?*
6) *An owner of a bar sells 6 bottles with alcohol of 40 cl each and 3 bottles of 5 dl each. How many cl of alcohol does the owner sell?*
7) *One lorry carries 128 dal of milk. Another one carries 23,1hl of the same product. What is the difference in litres of cargo between the two lorries?*

1st activity: (1,5). 2nd activity: (1,5). 3rd activity: (1,5). 4th activity: (1). 5th activity: (1). 6th activity: (1,75). 7th activity: (1,75).

c) Plenary (10 min).
   - ✓ The test is checked on the blackboard.
   - ✓ No homework.

5.2.9. Methodological aspects.

This section on methodological aspects, which is the last one of this practical proposition, is carried out after the development of the sessions in order to be able to justify this theory with examples from those sessions. At the same time, this theory is developed on the basis of the theoretical ideas of the first sections.

The first theoretical part of this thesis is the one about the learning and teaching of Mathematics in Primary Education. From a learning point of view, more importance is given to the constructivist ideas than to the behaviorist ones, and this is considered through the practical proposal. On the one hand, Piaget highlighted the role of the environmental factors and the consideration of what is already known when acquiring new knowledge. And the measures of capacity are learned on the basis of certain previous ideas (this is not a new topic, as students learn basic aspects related to this in earlier school years) and of information extracted from the environment, from their experience in life. Moreover, he also considered that each student follows a sequence of evolution that goes from the reliance on real concepts to the understanding of the meaning of abstract propositions: and these pupils do exercises, solve problems and draw conclusions after acquiring some basic concepts (for example, the theory of the first session).

On the other one, Bruner referred to the need to acquire mathematics through discovery processes and in an experimental way. In this sense, for example, all of the students are asked to look for containers and touch them, compare them and experiment with them. And the three stages of learning that he mentioned (or two of them are least) are considered in many

activities: for example, in the task in pairs of the third section in which these containers are used: every student touches, draws and does calculations.

From a teaching point of view, Cockcroft thought that there was not a unique method to teach mathematics (this practical proposal is only a possible example, not the only one) and considered that this teaching should include opportunities for: exposition by the teacher (in this case, the teacher always explains the new concepts before asking to do activities related to them), discussions (there are interactions in all sessions), appropriate practical work (related to the theory and adapted to the age), consolidation, practice and problem solving (for example, through the reinforcement activities) and investigational work (for example, when the pupils are asked to look for containers at home).

The ideas from the *Diseño Curricular Base* which are mentioned are also put into practice. A priority to the oral and practical work is given (the introductory sessions are more active and communicative, and apart from that, many exercises and problems are completed during all of them) and also to the development of strategies of problem solving (for example, with the resolution of examples of problems by the teacher in class) and to the use of the own experiences of each student as a source of mathematical experiences (this lesson on measures of capacity would be taught through the consideration of certain previous experiences).

And some ideas from the LOE which were also highlighted are developed: the importance of the instrumental and formative aspect of the subject (the measures of capacity are taught because they are essential for life), the need to teach through contextualized situations (for example, most of the problems represent real and possible situations), the inclusion of real materials when teaching (for example, the use of jars in the first session can be very helpful to understand new contents) and the need to incorporate the information and communication technologies (ICT) (in this case, the use of the PowerPoint or the enjoyment of the online games are some clear examples).

From a mathematical point of view, as it can be seen, the theoretical ideas are applied in the practical proposal. And the same happens with the linguistic aspects which were collected in the third section of this thesis and in the first part of this main proposal (in the hypothetical guide).

The CLIL methodology is followed. Much importance is given to the development of processes and tasks (although the theory is explained and the doubts are solved through a clear use of English and through gestures, the practical activities are preponderant), a greater

contact with the foreign language is undoubtedly made (in fact, English is used constantly and in a real way in a relatively new bilingual subject through an oral and interactive method) and a more complete preparation of the student for future is also developed (it is basic to the learn the measures of capacity in Spanish, but do it in English could open more doors: for example, when going abroad). Besides, the lessons are also adapted to everybody (the number of students is not high in order to control them more thoroughly and the role of the assistant is essential), the sessions are organized on the basis of the considerations of the hypothetical guide (starter, main part and plenary) and a use of varied materials is given, such as worksheets, the dictionary, the aforementioned containers, (the jars of the teacher or the ones brought from home by each student), the online games, etc.

Finally, a reference to the assessment processes was also made within the theoretical section about methodology in the hypothetical guide. Considering the determinations which were established, the way to evaluate the students is established in the following table:

Summative assessment (60%)	Final test (the one in the 8th session).
Formative assessment (40%)	Oral expression and comprehension, written expression and comprehension, listening and reading through the English language, attendance, daily study, homework and behavior. (The information must be written in a table to be able to collect all data on each student)
Assessment criteria	Knows and uses the main unit of capacity, its multiples and its submultiples. Knows how to estimate capacities of everyday containers and how to analyze them. Knows how to choose the most appropriate unit when expressing a measure of capacity. Compare and order units of capacity and quantities. Knows how to express a simple form of capacity a measure given in a complex form and vice versa. Does simple addition and subtraction of capacity measures.

The evaluation process of the pupils by the teacher is developed in this way. However, this person also carries out an assessment at the end of each session (this importance aspect is only highlighted at the end of the information related to the 1st session, but it is always necessary) analyzing what has been taught and how has been taught in order to improve. And in the case of the pupils, they also have the opportunity to evaluate other classmates during the correction of certain exercises (for example, after finishing the problem that is proposed in the 4th session).

6. ADVANTAGES AND DISADVANTAGES OF THE INTRODUCTION OF THE PROPOSITION ON BILINGUAL MATHEMATICS.

6.1. ADVANTAGES.

The bilingual subjects implemented in Spain have been introduced through the consideration of the CLIL methodology. My proposition is determined based on the search for information from various sources, but especially taking into account the CLIL methodology. For this reason, refer to the benefits related to the introduction of the Bilingual Mathematics proposition involves considering all of the positive aspects about CLIL, aspects which have already been listed in the previous section.

Moreover, it is also necessary to emphasize two other aspects related to the subject of Mathematics specifically. On the one hand, its introduction in the bilingual system is an important advance in order to develop the full implementation of this method, which is an essential step as the traditional subject has hardly been adapted. On the other one, learning to communicate mathematically is basic (Moschkovich, 2002), which means that the importance of knowing how to express oneself mathematically in the foreign language is absolute.

6.2. DISADVANTAGES.

The CLIL method involves numerous advantages, which are considered a solid approach to take into account when educating. However, there is not a perfect model, and that is why the CLIL proposal also entails a certain number of disadvantages.

From a general point of view of CLIL education, a serious problem is the lack of trained teachers in many educational areas. For example, the model has been recently introduced in Spain and the English language is hardly mastered by many of the professionals who are working in the schools of this country. The differences that may be given between teachers from linguistic and non-linguistic areas can also be a negative aspect. Moreover, there might be difficulties related to the level and amount of contents (both should not be reduced) and when evaluating all of the different aspects. And above all, referring only to Spain, the educational system is not the most complete one and the teaching and learning of foreign languages leaves much to be desired in many cases. This is the main obstacle to consider and to overcome in my opinion.

From a more concrete point of view related to the introduction of the Bilingual Mathematics through CLIL, the implementation of this subject has not been very clear so far. It is not one of the easiest subjects, since a significant number of students could have trouble when acquiring knowledge. However, although in some contexts multilingual pupils tended to underachieve in Mathematics (Cockingand & Chipman, 1988, p.20), many other authors have argued that multilingualism does not impede its learning (Setati, 2005). Nevertheless, I consider that the introduction of the subject of Bilingual Mathematics would be difficult in many current Spanish schools due to the aforementioned low level in teaching and learning of foreign languages.

7. CONCLUSION.

I chose this proposition on Bilingual Mathematics because it is a subject that has barely been implemented. As it was pointed out, it is not easy to establish it. The challenge of finding a new and useful model is greater, and this aspect has encouraged me to address this topic.

Through this work, my planning is determined, but it does not have to be the only possible one. However, I consider that this proposition can be perfectly applicable, and that is why I am satisfied with my final master thesis, whose information could be very useful when implementing the Bilingual Mathematics in the Spanish education system. Although it is also true that, as it was said, it could be difficult to establish that subject today.

The introduction of Bilingual Education in this country is very positive, but I think that some significant changes at a social and educational level should be made in order to create a

really solid model. Some of these changes are already being produced, but more progress is needed. If the aspects which are mentioned in the following paragraphs are not considered, the implementation of a serious Bilingual Mathematics subject will not be possible in my opinion.

First, a financial investment in education is essential. Education cutbacks as the current ones are the opposite of what should be done. It is impossible to try to improve the Spanish educational system without spending money.

Second, it is necessary to implement a serious educational law that makes reference to the Bilingual Education and that is not completely changed every four years. Stability is essential when establishing a long-term solid project.

It is also important to develop the implementation projects of this process in due time. It is true that the efforts for the introduction of Bilingual Education in schools should be valued positively, but the disadvantages related to CLIL and to the implementation of the Bilingual Mathematics which were mentioned in the previous section have to be considered. Taking them into account, the imminent introduction of the Bilingual Mathematics can be negative. For that reason, it would probably be interesting to wait a few years until the level of learning and use of the foreign language is higher than now and, when the time comes, to implement a transition phase to the new subject in certain educational contexts only if it is considered that there could be more problems in these contexts than in others when introducing the Bilingual Mathematics.

In relation to this, equally significant is the fact of taking special care in establishing bilingual propositions. The foreign language has to be developed properly and all of the current contents have to be kept, so that the planning is a considerable challenge.

And another essential requirement is the introduction of bilingualism in early childhood education. It is very important that students who have access to Primary Education have acquired some level of English. Quality foreign language teaching must be given from the earliest years.

These are the main changes that ought to take place in order to achieve a successful implementation of the bilingual subjects, including Mathematics. Some advances have been produced and, in my opinion, it is perfectly possible to reach the objectives, but a real social awareness is also needed in this regard. It cannot be forgotten that English is and will be the global language par excellence.

8. BIBLIOGRAPHY.

Allal, L., Cardinet, J., & Perrenoud, D. (1979). *L'évaluation formative dans un enseignement différencié*. Berna: Peter Lang.

Brewster, J., Ellis, G., & Girard, D. (1991). *The Primary English Teacher's Guide*. London: Penguin Group.

Bujanda, M., Panellas, M., Castellví, A., Roig, A., Rubio, S., & Solves, N. (1993). *Matemáticas. 4º curso de Primaria.* Madrid: SM.

Cocking, R. R., & Chipman, S. (1988). Conceptual Issues Related to Mathematics Achievement of Language Minority Children. In Cocking, R. R., and Maestre, J.P. (Ed.), *Linguistic and Cultural Influences on Learning Mathematics* (pp. 17-46). Nueva Jersey: Lawrence Erlbaum Associates.

Cockcroft, Committee (1982*). Mathematics Counts. Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr WH Cockcroft*. London: HMSO.

Coyle, D., Hood, P., & Marsh, D. (2010). *Content and Language Integrated Learning*. Cambridge: Cambridge University Press.

Department For Education And Employment (1999). *The National Numeracy Strategy. Framework for Teaching Mathematics from Reception to Year 6.* London: Department for Education and Employment (DfEE).

MEC (1989). *Diseño Curricular Base (Área de Matemáticas).* Madrid: MEC.

MECD (2007). *Orden ECI/2211/2007, de 12 de julio, por la que se establece el currículo y se regula la ordenación de la Educación primaria.* Madrid: MECD.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, 4, 189–212.

San Isidro, F.X., Domínguez Cuña, A. & Barreiro, M.I. (2009). *CLIL: integrando linguas 'a través' do currículo.* Consellería de Educación e Ordenación Universitaria: Xunta de Galicia.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education*, 36(5), 447-446.

Shepard, L.A. (2008). Formative assessment: Caveat emptor. In C. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning* (pp. 279 – 303). New York: Lawrence Erlbaum Associates.

9. WEB PAGE.

http://mathsframe.co.uk/en/resources/resource/21/card_game