

Improving basic math skills through integrated dynamic representation strategies

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Abstract

Background: In this paper, we analyze the effectiveness of the Integrated Dynamic Representation strategy (IDR) to develop basic math skills. **Method:** The study involved 72 students, aged between 6 and 8 years. We compared the development of informal basic skills (numbers, comparison, informal calculation, and informal concepts) and formal (conventionalisms, number facts, formal calculus, and formal concepts) in an experimental group ($n = 35$) where we applied the IDR strategy and in a Control group ($n = 37$) in order to identify the impact of the procedure. **Results:** The experimental group improved significantly in all variables except for number facts and formal calculus. **Conclusions:** It can therefore be concluded that IDR favors the development of the skills more closely related to applied mathematics than those related to automatic mathematics and mental arithmetic.

Keywords: formal competence, informal competence, mathematical competence, Integrated Dynamic Representation.

Resumen

Mejora de las competencias matemáticas básicas mediante estrategias de representación dinámica integrada. **Antecedentes:** con este trabajo se pretende analizar la eficacia de la estrategia Representación Dinámica Integrada (RDI) para desarrollar las competencias matemáticas básicas. **Método:** participaron en el estudio 72 estudiantes, de edades comprendidas entre los 6 y los 8 años. Se comparó el desarrollo de competencias básicas informales (numeración, comparación, cálculo informal y conceptos informales) y formales (convencionalismos, hechos numéricos, cálculo formal y conceptos formales) en un grupo experimental ($n = 35$) al que se aplicó la RDI y un grupo control ($n = 37$) con el fin de identificar el impacto de la intervención. **Resultados:** el grupo experimental mejora significativamente más que el grupo control en todas las variables evaluadas, excepto en hechos numéricos y cálculo formal. **Conclusiones:** se puede concluir, por tanto, que la RDI favorece en mayor medida el desarrollo de las competencias más relacionadas con la matemática aplicada que las relacionadas con la matemática de los automatismos y del cálculo mental.

Palabras clave: competencias formales, competencias informales, competencia matemática, Representación Dinámica Integrada.

The main international assessment tests to evaluate academic performance (TIMSS —*Trends in International Mathematics and Science Study*— International Association for the Evaluation of Educational Achievement, 2011; PISA-OECD, 2010) warn about the existence of mathematical learning difficulties that should be identified in order to address them at the beginning of schooling. Accordingly, the need for early interventions to decrease students' low performance and academic failure during the first educational stages is proposed. According to Gil and Vicent (2009), the first school years are essential to stimulate mathematical development, because this is when the informal competences (acquired outside of the school setting) can become formal knowledge and skills that facilitate acquisition of subsequent mathematical competences, especially taking into account that such competences evolve

hierarchically and integratively (Olkun, Altun, & Deryakulu, 2009). For example, dealing with the concept of numbers provides the basis to subsequently perform more complex tasks such as reasoning and problem solving (Seethaler & Fuchs, 2006).

It is therefore necessary to first know which mathematical competences require a more specific approach to promote intervention programs adapted to the initial learning difficulties. Before beginning Primary Education, children acquire informal knowledge of numbers, mathematical concepts, quantities, forms, and the fundamentals of comparison and classification (Hirvonen, Tolvanen, Aunola, & Nurmi, 2012; Orrantia, Múñez, Fernández, & Matilla, 2012). In the formal teaching context, students learn to use numbers as symbols and are thus able to apply them to the basic mathematical operations. Moreover, mathematics initially includes different competences, such as counting, measurement, calculation, algorithms, and heuristics (Hirvonen et al., 2012). Heuristics constitute one of the most relevant aspects of the syllabus (National Mathematics Advisory Panel, 2008), in spite of many students' difficulties in beginning to solve them (Jacobse & Harskamp, 2009; Montague, Enders, & Dietz, 2011). As these authors note, problem solving is a process in which it is necessary

to implement strategies to understand and interpret the initial statements through internal representations that capture the different proposals and their semantic relations, and to elaborate a situated model (Orrantia, 2003; Timoneda, Pérez, Mayoral, & Serra, 2013; Vicente, Orrantia, & Verschaffel, 2008). For this purpose, it is appropriate to incorporate aids to promote mathematical and situational rewriting, as well as drawings and visual representations to reach the final solution (Arcavi, 2003). Mayer (2001) showed that images that follow a set of principles, such as coherence, contiguity, or personalization, improve text comprehension. In the same vein, Vicente et al. (2008) analyzed the utility of these aids in problem solving in a sample of 152 students from 3rd to 5th grade of Primary Education. They concluded that both rewriting and mathematical drawings increase correct performance of these activities. This line of research has also been studied from the sphere of visualization (Arcavi, 2003) and from an instructional perspective (Aguilar, Navarro, & Alcalde, 2007), concluding that teaching students to schematically represent the mathematical structure of problems facilitates their correct comprehension and final resolution.

Ultimately, many variables are involved in the problem solving process. Whereas some variables are related to comprehension/representation, others are related to resolution/performance (Aguilar et al., 2007). Problem representation assumes a transfer phase, in which the statement is represented and fragmented in order to subsequently be integrated into the final solution.

Accordingly, the development and implementation of procedures to help students acquire this type of strategies will be essential for a good subsequent evolution. Jitendra et al. (2013) underline the benefits provided by teaching explicit strategies. Moreover, Gil and Vicent (2009) indicate that intervention at early ages is essential to stimulate mathematical development, as long as such stimulation facilitates the connection between meaningless symbols and the informal knowledge and skills that children already possess. For this purpose, different intervention programs have been implemented by Aguilar et al. (2007), which allow solving arithmetic problems following a procedure that enhances students' strategic thinking by using response devices with direct feedback, such as learning graphic representations of the components of the problem. Also, with the aim of improving students' metacognitive strategies and problem-solving skills, Jacobse and Harskamp (2009) developed a computer program to stimulate these skills with very positive results. This indicates that guidance during problem solving enhances performance in this type of tasks. Jonassen (2003) elaborated a very effective computer application, in which the subject generates a model of the situation by manipulating the data with static images.

In order to group the different strategies implemented by the above-mentioned investigations (both strategy-based training for problem solving and rewriting and visualization of the concepts in images), in this work, we propose to design and apply a computerized strategy, the *Integrated Dynamic Representation* (IDR) for early learning of basic mathematical competences. According to Álvarez, González-Castro, Núñez and González-Piñeda (2007), IDR is the result of a combination of the models of external (diagrams or drawings), internal and situated representation. For this investigation, it was administered to students aged between 6 and 8 years. Taking into account that knowledge is constructed by means of a discontinuous flow of textual and audiovisual information (Nicoleta, 2011), by dealing with the three types of representation (iconic, combined, and

symbolic), IDR can be implemented from early childhood education, even before mastering lexical processing. Moreover, as the strategy can be applied in computer language, its benefits can be observed not only at the level of mathematical knowledge but also in students' attitudes towards the subject (Delen & Bulut, 2011; Walker et al., 2012).

Taking these antecedents into account, the proposed goal of this work is to analyze the effectiveness of this computerized tool in the early stimulation of basic mathematical skills (informal and formal).

Method

Participants

Seventy two students, aged between 6 and 8 years old ($M = 7.01$, $SD = 0.66$), were enrolled in four classes of the first cycle of Primary Education (1st and 2nd grade).

A convenience sample (Argibay, 2009) was selected in collaboration with the Guidance Department of an educational center. The semi-structured interview for parents Diagnostic Interview Schedule for Children DISC-IV (Shaffer, Fisher, Lucas, Dulcan, & Schwab-Stone, 2000) was applied to rule out possible learning difficulties or associated disorders, and the WISC-IV (Wechsler, 2005) was used to appraise the possible existence of cognitive deficits or high capacities. In each grade, one of the two groups was randomly assigned to one of the experimental conditions. Thus, the EG group was made up of 35 students and the CG included 37 students, whose characteristics are presented in Table 1.

Instruments

The *Diagnostic Interview Schedule for Children Version IV* (DISC-IV; Shaffer et al., 2000; Bravo et al., 2001) is a highly structured interview that allows one to diagnose psychopathological disorders in children/adolescents according to the criteria of the *Diagnostic and Statistical Manual of Mental Disorders-IV* (DSM-IV, American Psychiatric Association, 2000). This interview is currently one of the most extensively employed in international research and child and youth psychiatric epidemiology, as it is well validated, both in its original version and in the Spanish version (Canino et al., 2004). This interview includes the clinical significance of symptoms, but does not use it as a diagnostic criterion. Clinical significance is assessed as a function of the report of distress or disability in diverse development settings that are associated with the presence of symptoms.

Variable	EG	CG	Statistic	P-value
Age (M / SD)	7.03 / 0.67	6.99 / 0.65	$t = 0.270$.788
IQ (M / SD)	94.29 / 6.09	93.97 / 7.35	$t = 0.196$.845
Gender			$\chi^2 = 0.069$.792
Males (n)	20	20		
Females (n)	15	17		

Note: EG = Experimental group, CG = Control group

The *Wechsler Intelligence Scale for Children-IV* (WISC-IV; Wechsler, 2005; Corral, Arribas, Santamaría, Sueiro, & Pereña, 2005) is an individually applied instrument to assess the intelligence of children or adolescents between ages 6 years-0 months and 16 years-11 months. It is made up of 15 subtests (of which 10 are obligatory and 5 are optional), which provide information about the subject's intellectual functioning in specific cognitive areas. The main scores are: total intelligence quotient (TIQ), verbal comprehension index (VC), perceptive reasoning index (PR), working memory index (WM), and processing speed (PS).

The *Test of Early Mathematics Abilities TEMA 3* of Ginsburg and Baroody, (2003) adapted into Spanish by Núñez and Lozano (2010), assesses mathematical competence, distinguishing between informal and formal competences. Informal competences are assessed with four subtests: (a) *counting*, consisting of the identification and flexibility in the use of sequences (basic skill for the representation—internalization—of quantity, while facilitating access to mental calculation); (b) *quantity comparison*, which implies number sense, the knowledge of the order of numbers that is linked to recognition of increasing and decreasing directions; (c) *informal calculation*, which refers to dealing with numbers to resolve simple situations using the operations of addition and subtraction; (d) *informal concepts*, which assesses the concept of number as an aggregate of sets from the enactive viewpoint, differentiating that the part is less than the whole (it includes the conservation of material). Formal competences are assessed by means of four subtests: (a) *conventionalisms*, referring to the capacity to read and write quantities, a coding-decoding task; (b) *number facts* implies knowledge of the result of simple operations of addition, subtraction, and multiplication without needing to perform the calculation at that moment; (c) *formal calculation*, which involves the operations of addition and subtraction of increasing difficulty; (d) *formal concepts*, which assesses the concept of number from the symbolic and iconic viewpoints. The tasks do not require much reading ability; hence, reading difficulties do not affect the result of mathematical competence. Moreover, the instrument provides a general coefficient, the Mathematical Competence Index (MCI), which indicates the student's global performance compared with the corresponding age group ($M = 100$, $SD = 15$).

Procedure

Firstly, we selected the grades in which to test the intervention. Next, we requested active informed consent from the families. Then, we made the initial assessment with the DISC-IV (completed by parents) and the WISC-IV, which allowed us to exclude the students who had associated difficulties and capacities lower than 80 or higher than 130 (in total, 6 students were excluded). Subsequently, the groups were randomly assigned to the experimental conditions. After assignment and the first assessment, the school psychologist evaluated the basic competences with the TEMA-3 in both groups. The intervention program was applied by the teacher-tutor between the months of January and April (45 fifty-minute sessions). The teacher-tutor in charge of administering the intervention in each experimental group had been previously trained by an expert in the use of the program during two 45-minute sessions to learn to use the tool. Finally, the school psychologist reapplied the TEMA-3 to all the groups to verify the possible differential evolution from pre- to posttest in the EG and the CG. In view of the efficacy of the program, the CG also subsequently received the intervention.

A quasi-experimental design with a nonequivalent control group was used. The EG students received the intervention with the IDR program, whereas the CG students followed the classical learning methodology (presentation of the contents by the teacher, using the subject textbook as work material and doing exercises from the same textbook, which were done in paper and corrected by the teacher in class).

The EG students worked with the IDR, developed by Álvarez et al. (2007). This strategy is the key element of heuristic processes, the result of the combination of external and internal representations. It is structured around the following three components (Solaz-Portolés y Sanjosé-López, 2008): fragmented comprehension, fragmented representation, and integration of the representations. It is not a part-whole schema but instead a dynamic sequence of fragmentation-representation-integration. Fragmentation-representation is related to “knowing what,” and the integration of the representations is related to “action schemas” or “knowing how.” The key strategy to address “knowing what” is network processing (Friege & Lind, 2006), whereas integration of the fragmented representations is essential to address “knowing how.”

The administration process was done at four phases or levels of representation. The first level is the *representation of concepts* (selection of the relevant information) in which the key concepts are presented, associated with drawings, the numerical data that accompany them are framed in squares and the verbs are replaced with pictograms. The second level is the *representation of the links* (iconic-symbolic combination) in which, after the key concepts are identified, they are represented in union-intersection sets, whose number of elements is specified by the numerical data. The third level is the *representation of questions* (integration of the representations). At this level, the representations are connected to each other, depending on the types of relation of the links of the statement: union (addition) and intersection (subtraction). The fourth level is the *reversibility of the process* (generalization to other contexts), where the subject is asked to reformulate the problem statement without taking the initial statement into account, on the basis of the integrated representation that leads to the final solution. This strategy favors reversibility and, thereby, generalization of learning.

This multilevel process is carried out with the IDR.com program computer, which follows the logical sequence when applying the competences corresponding to the educational level of the program. A first block includes activities aimed at working on the competence of addition without carrying a number; in a second block, addition carrying a number and subtraction without carrying are introduced; lastly, combined additions and subtractions are worked on. These three blocks, sequenced as a function of the degree of difficulty and in which new competences are gradually introduced, are presented following two types of representation, combined representation (the concepts are associated with images/words) and symbolic representation (the statements are presented exclusively in linear text).

Managing the program is simple at the computer level and it is adapted to student skills according to age and educational level. In the combined representation, students begin by dragging the icons and progressively working on writing. Finally, at the symbolic level, the students represent the data of the problem and the corresponding concepts until they reach the final solution. To generalize these learnings, the program presents the reversibility of the process on a new screen, so that, starting with the graphic representation, the student can rewrite the initial statement, based

exclusively on the representation. At this phase, subjects will not have access to the corresponding feedback unless they finish the representation correctly. All the students of EG worked with IDR during two months in sessions of 45 minutes (two sessions per week). They worked in the computer room and one laptop was shared by two students. Each student (making sure that all participated) took turns to correct the exercises in the interactive white board under the supervision of the teacher.

Data analysis

Firstly, we determined possible group differences in relevant variables such as age, IQ, or gender and in the pretest scores of each dependent variable. For this purpose, the appropriate statistical contrast was used, according to the characteristics of the scores in each variable: Pearson’s chi-square for gender; Student’s *t* for independent samples for the variables age and IQ; Mann-Whitney’s *U* for the pretest scores. Secondly, we compared the groups’ posttest scores in each dependent variable, using the Mann-Whitney *U* statistic. Lastly, we compared the pretest and posttest scores within each group and in each variable by means of Wilcoxon’s signed ranks test. All the analyses were performed using the SPSS 15.0 statistical software for Windows.

Results

Intergroup pretest comparison

Before the intervention, no statistically significant group differences were found in the variables age, IQ, or gender (Table 1). With regard to the pretest scores in TEMA-3, no statistically

significant group differences were found in the MCI or in the global scores of informal or formal competence. However, we did observe statistically significant differences in three specific competences. Specifically, the EG obtained significantly higher scores than the CG in the variables quantity comparison and conventionalisms. The CG obtained significantly higher scores than the EG in the variable informal concepts (Table 2).

Intergroup posttest comparison

After the intervention, statistically significant group differences were found in the posttest means of the MCI and of the global scores of informal and formal competence. More specifically, the EG scored significantly higher in the variables counting, quantity comparison, informal calculation, informal concepts, formal concepts, and conventionalisms (Table 2).

Intragroup Pretest-Posttest Comparison

In order to analyze the pretest-posttest evolution in each experimental condition for all the variables, we used Wilcoxon’s signed ranks test. These analyses revealed statistically significant pretest-posttest differences in the EG in all the TEMA-3 factors (Table 2). After the intervention, the EG obtained higher scores in the MCI, the global scores of informal and formal competence, and all of the eight mathematical competences assessed (Table 2). For the CG, the pretest-posttest differences were statistically significant in MCI, the global scores of informal and formal mathematical competences, and all of the specific mathematical competences assessed, except for conventionalisms and informal concepts (Table 2).

Table 2
Pre- and posttest comparison of means of the dependent variables between the experimental group (N = 35) and the control group (N = 37)

Variable	Group	Pretest	U	Posttest	U	Z
MCI	EG (M / SD)	85.86/8.57	582.500	106.43/10.36	148.000***	-5.145***
	CG (M / SD)	84.16/9.412		91.11/6.73		-5.022***
Informal competences	EG (M / SD)	26.88/4.47	584.500	33.54/4.61	280.500***	-5.182***
	CG (M / SD)	26.13/4.51		28.37/4.08		-4.802***
Counting	EG (M / SD)	16.46 / 3.02	546.500	19.69 / 2.76	292.000***	-5.114***
	CG (M / SD)	15.76 / 3.40		16.57 / 3.11		-4.388***
Quantity comparison	EG (M / SD)	4.14 / 0.73	414.000**	5.20 / 0.76	276.500***	-5.336***
	CG (M / SD)	3.59 / 0.76		4.24 / 0.80		-3.757***
Informal calculation	EG (M / SD)	4.14 / 0.81	575.000	5.26 / 1.15	438.000**	-4.840***
	CG (M / SD)	4.00 / 0.67		4.62 / 0.49		-3.758***
Informal concepts	EG (M / SD)	2.14 / 0.36	307.500***	3.40 / 0.60	411.500**	-5.224***
	CG (M / SD)	2.78 / 0.67		2.95 / 0.62		-1.604
Formal competences	EG (M / SD)	10.02/5.10	640.500	15.34/7.44	424.000*	-5.170***
	CG (M / SD)	9.83/4.28		11.16/4.74		-4.239***
Conventionalisms	EG (M / SD)	5.51 / 1.22	404.000**	6.37 / 1.21	245.000***	-4.182***
	CG (M / SD)	4.70 / 0.85		4.97 / 0.90		-1.908
Number facts	EG (M / SD)	2.00 / 2.21	633.500	3.37 / 2.96	485.500	-4.615***
	CG (M / SD)	1.86 / 1.99		2.24 / 2.17		-3.125**
Formal calculation	EG (M / SD)	1.17 / 1.10	542.000	2.74 / 2.45	530.000	-4.417***
	CG (M / SD)	1.59 / 1.42		1.89 / 1.54		-3.317***
Formal concepts	EG (M / SD)	1.34 / 0.91	508.500	2.86 / 1.22	400.000**	-5.308***
	CG (M / SD)	1.68 / 0.75		2.05 / 0.74		-3.300***

Note: EG = Experimental group; CG = Control group; MCI = Mathematical Competence Index; U = Mann-Whitney U; Z = Wilcoxon’s signed ranks test.
* $p \leq .05$; ** $p \leq .01$; *** $p \leq .001$

Discussion

This aim of this investigation was to analyze the efficacy of the IDR computer tool for early stimulation of basic mathematical competences (informal and formal), in two groups of students from Primary Education (6-8 years). After analysis of the results, it can be concluded that the strategy favored a better development of most of the mathematical competences analyzed, in comparison with the habitual work methodology.

In general terms, the intervention had a more positive impact than the traditional learning methodology on the development of the MCI, as well as on the informal and formal mathematical competences considered globally. The general pattern showed no statistically significant group differences before the intervention, but there were pretest-posttest differences after the intervention, with the EG showing better results than the CG. Two specific mathematical competences, number facts and formal calculation were the only ones that did not show a more positive impact. In both variables, both the EG and the CG improved significantly from pretest to posttest, but no group differences were found after the intervention; thus, the improvement of the CG cannot be due to the intervention program. That the IDR strategy was not significantly better in these two variables than the habitual methodology may be due to the fact that both variables are related to solving basic mathematical operations, a task that is strongly emphasized in the traditional methodology, as reflected in the syllabus proposed by the educational administration for this stage. Both from the manipulative and the mechanical viewpoint, dealing with numbers improved in both groups because these tasks and activities are worked on repeatedly in formal teaching. The noteworthy aspect of IDR is that, without any kind of specific learning, students end up learning by association.

The IDR strategy has reflected a greater impact on the six remaining specific competences than the habitual methodology. In this regard, three patterns of results are identified. The first one corresponds to the variables counting, informal calculation, and formal concepts. In all three cases, the EG and the CG showed no differences before the intervention and, although both groups improved at posttest, the improvement was much more obvious in the EG (at posttest, the group differences became statistically significant in favor of the EG). These three variables, related to mechanical operations and the concept of number, at both a symbolic and iconic level, clearly evolved after training with IDR, because it is based on the manipulation and association of symbols by means of a combined representation. This type of aggregating representation of the image, the word, and the enactive manipulation of both of them facilitates the basic development of these skills.

A second pattern refers to the variables conventionalisms and quantity comparison. In both cases, the EG presented a better performance in these variables than the CG, but after the intervention, the group differences in favor of the EG were more pronounced. Recognition of quantities, as well as their comparison (higher level skills related to numerical reasoning) improved significantly after application of the program. Reading-writing quantities and acquiring their number sense imply implementing recognition skills that, although they are basic, are essential for subsequent numerical reasoning. Through the use of IDR, there is an obvious improvement of knowledge of order and of the differences between quantities because the mental representation

of the numerical sequence is facilitated by the formation of sets and the understanding of the comparative techniques involved.

Lastly, the third pattern of results involves the variable informal concepts. In this case, the EG showed a worse performance than the CG before the intervention, but it improved after the training. The pre-posttest improvement observed in the EG was statistically significant, whereas there were no statistically significant pre-posttest differences in the CG. The concept of number as an aggregate of sets is appraised from the enactive viewpoint (differentiating that the part is less than the whole), and this is one of the aspects that is particularly emphasized in the IDR tool, in which the enactive representation in a situated model facilitates its improvement. Initially, children need to represent all the elements to fix the concept, but the representation gradually ceases to be necessary, and the symbol comes to represent the set of elements. Hence, the importance of sequential, accumulative, and hierarchical stimulation, especially because the development of these competences is also a process of progressive and integrating differentiation (Olkun et al., 2009). In this sense, as mentioned above, informal mathematics are the basis for formal mathematics, which are assessed in the school setting from 6 to 8 years of age, and these, in turn, facilitate future mathematical learning. This reflects the importance of stimulating these skills to prevent subsequent failure and it reveals the ineffectiveness of methodologies focused exclusively on the mechanics of operations and mental calculation to achieve future success.

Moreover, it is important to note that, both in the EG and in the CG, there was a positive pre-posttest evolution (except for conventionalisms and informal concepts in the CG), because both groups received some intervention (either IDR or the habitual methodology). The additional benefits provided by IDR can be explained at a conceptual or procedural level. Firstly, when dealing with skills of fragmented representation and their later integration, we are promoting students' deep comprehension and, as a consequence, the abstract reasoning that guides the final solution. Accordingly, following Vicente et al. (2008) any problem solving strategy should take into account internal representations, the identification of semantic relations, and the elaboration of a situated model to solve the problem and achieve subsequent learning generalization. All these strategic indicators are taken into account in IDR, as it facilitates explicit internal representations, semantic relations, and the elaboration of a situated model (third level), leading to the final resolution and its generalization by means of the reversibility of the process (fourth level). Moreover, all this facilitates students' self-regulated learning, following the model of Zimmerman (2008) according to which self-regulated students direct their own learning by implementing a series of strategies, activating and modifying their cognitive and metacognitive processes and their behavior, before, during, and after the learning takes place. The first phase of the strategy allows planning (by fragmenting the problem), then, performance is facilitated, and, finally, assessment is promoted in the integration phase. This is very relevant, especially taking into account that, as proposed by Cueli, García and González-Castro (2013), the planning phase is what differentiates students in a statistically significant way, based on their academic performance; students with better results place more emphasis on this self-regulatory phase. Hence, the importance of working on it in depth.

In addition, the benefits could be related to how the tool is applied and, therefore, to the advantages of using the new

technologies in mathematics classes. Following Walker et al. (2012), the use of these tools produces positive effects, both at the level of mathematical knowledge and at the level of students' attitudes towards the subject. In this sense, Shin, Sutherland, Norris, and Soloway (2012) have shown that the use of technology in the classroom has positive effects on the students, on their learning and on their implementing arithmetic skills. Azevedo and Jacobson (2008) note that the use of the new technologies is beneficial, especially when oriented toward providing interactive learning environments that enhance the development of the cognitive and metacognitive processes.

Some limitations of this investigation that should be taken into account are the small sample size, the assessment and the absent of mathematical learning disabilities. The sample size is small due to the absence of children with learning difficulties. Therefore, new studies with a bigger sample size, as well as a follow up evaluation that analyzes the long term benefits of the

program, would be appropriate. Another limitation is related to the assessment carried out, which was based only on the result of a test of mathematical competence. In future works, it would be appropriate to assess the processes performed by the students, using protocols such as *Thinking Aloud* or the *Triple Task* for this purpose, whose applications are starting to be applied in the area of mathematics (García & González-Pienda, 2012). Finally, we did not address specific mathematical learning difficulties (MLD), and therefore, another proposed line for future research is to analyze the benefits of IDR in students who present MLD or specific associated disorders such as Attention Deficit with Hyperactivity Disorder (Miranda, Meliá, & Marco, 2009).

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References

- American Psychiatric Association (2000). *Diagnostic and statistical manual of mental disorders*-text revision (4th ed.). Washington, DC: Author.
- Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52, 215-241.
- Aguilar, M., Navarro, J.I., & Alcalde, C. (2007). The use of figurative schemas to help solve arithmetic problems. *Cultura y Educación*, 15(4), 385-397.
- Álvarez, L., González-Castro, P., Núñez, J.C., & González-Pienda, J.A. (2007). *Prácticas de Psicología de la Educación* [Practicums in Educational Psychology: Assessment and psychoeducational intervention]. Madrid: Pirámide.
- Argibay, J.C. (2009). Sample in quantitative research. *Subjetividad y Procesos Cognitivos*, 13(1), 13-29.
- Azevedo, R., & Jacobson, J. (2008). Advances in scaffolding learning with hypertext and hypermedia: A summary and critical analysis. *Education Technology Research and Development*, 56(1), 93-100.
- Bravo, M., Ribera, J., Rubio-Stípec, M., Canino, G., Shorout, P., Ramírez, R., et al. (2001). Test-retest reliability of the spanish version of the diagnostic interview schedule for children (DISC-IV). *Journal of Abnormal Child Psychology*, 29, 433-444.
- Canino, G., Shrout, P.E., Rubio-Stípec, M., Bird, H.R., Bravo, M., Ramírez, R., et al. (2004). The DSM-IV rates of child and adolescent disorders in Puerto Rico. Prevalence, correlates, service use, and the effects of impairment. *Archives of General Psychiatry*, 61(1), 85-93.
- Corral, S., Arribas, D., Santamaría, P., Sueiro, M.J., & Pereña, J. (2005). *Escala de inteligencia de Wechsler para niños WISC-IV (adapted into spanish)* [The Wechsler Intelligence Scale for Children-4th edition]. Madrid: TEA Ediciones.
- Cueli, M., García, T., & González-Castro, P. (2013). Self-regulation and academic achievement in mathematics. *Aula abierta*, 41(1), 39-48.
- Delen, E., & Bulut, O. (2011). The relationship between students' exposure to technology and their achievement in science and math. *Turkish Online Journal of Educational Technology*, 10(3), 311-317.
- Friege, G., & Lind, G. (2006). Types and qualities of knowledge and their relation to problem solving in physics. *International Journal of Science and Mathematics Education*, 4, 437-465.
- García, T., & González-Pienda, J.A. (2012). Assessment of the self-regulated learning process in the area of mathematics using digital blackboards. In J. Dulac-Ibergallartu & C. Alconada-Fernández (Eds.), *3rd Digital Blackboard Congress: Publication of Communications* (pp. 85-92). Madrid: Ediciones Pizarratic.
- Gil, M.D., & Vicent, C. (2009). Comparative analysis of the efficacy of a playful-narrative program to teach mathematics at pre-school level. *Psicothema*, 21(1), 70-75.
- Ginsburg, H.P., & Baroody, A.J. (2003). *The Test of Early Mathematics Ability* (3rd ed.). Austin, TX: Pro Ed.
- Hirvonen, R., Tolvanen, A., Aunola, K., & Nurmi, J.E. (2012). The developmental dynamics of task-avoidant behavior and math performance in kindergarten and elementary school. *Learning and Individual Differences*, 22, 715-723.
- International Association for the Evaluation of Educational Achievement IEA (2011). *Results of the 2011 PIRLS and TIMSS tests in Spain*. Madrid: Ministerio de Educación, Cultura y Deporte.
- Jacobse, A.E., & Harskamp, E.G. (2009). Student-controlled metacognitive training for solving word problems in primary school mathematics. *Educational Research & Evaluation*, 15(5), 447-463.
- Jitendra, A.K., Rodríguez, M., Kanive, R., Huang, J.P., Church, C., Corroy, K.A.Z., et al. (2013). Impact of small-group tutoring interventions on the mathematical problem solving and achievement of third-grade students with mathematics difficulties. *Learning Disability Quarterly*, 36(1), 21-35.
- Jonassen, D.H. (2003). Designing research-based instruction for story problems. *Educational Psychology Review*, 15(3), 267-295.
- Mayer, R.E. (2001). *Multimedia learning*. Cambridge, UK: Cambridge University Press.
- Miranda, A., Meliá, A., & Marco, R. (2009). Mathematical abilities and executive function in children with attention deficit hyperactivity disorder and learning disabilities in mathematics. *Psicothema*, 21(1), 63-69.
- Montague, M., Enders, C., & Dietz, S. (2011). Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly*, 34(4), 262-272.
- National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, D.C.: US Department of Education.
- Nicoleta, S. (2011). How can technology improve math learning process? *Teachers for the Knowledge Society*, 11, 170-174.
- Núñez, M.C., & Lozano, I. (2010). *TEMA-3 Test de Competencia Matemática Básica 3 (adaptación al español)* [TEMA-3 The Test of Early Mathematics Ability 3]. Madrid: TEA Ediciones.
- OECD (2010). *PISA 2009 Results*. Paris: OECD.
- Olkun, S., Altun, A., & Deryakulu, D. (2009). Development and evaluation of a case-based digital learning tool about children's mathematical thinking for elementary school teachers. *European Journal of Teacher Education*, 32(2), 151-165.
- Orrantia, J. (2003). El rol del conocimiento conceptual en la resolución de problemas aritméticos con estructura aditiva [The role of conceptual knowledge in solving addition and subtraction word problems]. *Infancia y Aprendizaje*, 26(4), 451-468.

- Orrantía, J., Múñez, D., Fernández, M., & Matilla, L. (2012). Solving arithmetic problems: Conceptual knowledge and level of competence in mathematics. *Aula Abierta*, 40(3), 23-32.
- Seethaler, P.M., & Fuchs, L.S. (2006). The cognitive correlates of computational estimation skill among third-grade students. *Learning Disabilities Research y Practice*, 21(4), 233-243.
- Shaffer, D., Fisher, P., Lucas, C.P., Dulcan, M.K., & Schwab-Stone, M.E. (2000). Diagnostic Interview Schedule for Children - Version IV (NIMH DISC-IV): Description, differences from previous versions and reliability of some common diagnoses. *Journal of the American Academy of Child and Adolescent Psychiatry*, 39(1), 28-38.
- Shin, N., Sutherland, L.M., Norris, C.A., & Soloway, E. (2012). Effects of game technology on elementary student learning in mathematics. *British Journal of Educational Technology*, 43(4), 540-560.
- Solaz-Portolés, J.J., & Sanjosé-López, V.S. (2008). Knowledge and cognitive in problem-solving processes in sciences: Consequences for teaching. *Magis, Revista Internacional de Investigación en Educación*, 1, 147-162.
- Timoneda, C., Pérez, F., Mayoral, S., & Serra, M. (2013). Diagnosis of reading and writing difficulties and of dyslexia based on the PASS Theory of Intelligence and using the DN-CAS battery: Cognitive origin of dyslexia. *Aula Abierta*, 41(1), 5-16.
- Vicente, S., Orrantía, J., & Verschaffel, L. (2008). Influence of mathematical and situational knowledge in solving verbal arithmetic problems: Textual and graphic aids. *Infancia y Aprendizaje*, 31(4), 463-483.
- Walker, A., Recker, M., Ye, L., Robertshaw, M.B., Sellers, L., & Leary, H. (2012). Comparing technology-related teacher professional development designs: A multilevel study of teacher and student impacts. *Educational Technology Research and Development*, 60(3), 421-444.
- Wechsler, D. (2005). *The Wechsler Intelligence Scale for Children- 4th edition*. London: Pearson Assessment.
- Zimmerman, B.J. (2008). Investigating self-regulation and motivation: Historical, background, methodological developments, and future prospects. *American Educational Research Journal*, 45, 166-183.