The use of response surface methodology to improve the thermal transmittance of lightweight concrete hollow bricks by FEM

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Abstract

In this paper, the response surface methodology (RSM) and finite element method (FEM) have been used to develop predictive models for the simulation and optimization of heat transfer process. The input variables were the material conductivity as well as the radiation emissivity, length and width of the recesses. The response variable considered was the thermal transmittance. In order to minimize the thermal transmittance, the RSM technique and FEM analyses have been developed in combination and applied to optimize this parameter. A central composite design (CCD) as the standard design of the RSM technique was used to evaluate the effects and interactions of the six previous factors. Next, the optimal design obtained from the RSM analysis is exposed, demonstrating its ability to build a quadratic polynomial model and to solve nonlinear thermal problems. Finally, analysis of variance (ANOVA) has been used to check the significance of response surface polynomials and FEM models so that the RSM technique was proved as an appropriate methodology for the nonlinear thermal optimization of lightweight concrete hollow bricks.

Keywords: Response surface methodology; Lightweight concrete hollow brick; Finite element modelling; Nonlinear complex heat transfer; Energy savings; Thermal optimization

1 Introduction

Sustainable construction aims at reducing the environmental impact of a building over its entire lifetime, while its economic viability as well as the comfort and safety of its occupants are optimized. Indeed, while standard building practices are guided by short term economic considerations, the sustainable construction is based on best practices which emphasize the long term affordability, quality and energy efficiency. At each stage of the life cycle of the building, the sustainability increases the comfort and quality of life and decreases the negative environmental impacts so that the economic sustainability of the project is also increased [1]. A building designed and constructed in a sustainable way minimizes the use of water, raw materials, energy, land, etc., over the whole life cycle of the building. By using the adapted insulation and energy savings techniques, it is possible to save up to 80% of a building's energy consumption for heating or cooling.

In this sense, there is a great interest in light building materials with good physical material behaviour from the sustainability point of view for housing and building structures. The use of materials today for buildings, energy, packaging, etc. has increased about 20-fold per capita in many highly industrialised countries from the end of the 19th century until now [2]. The Earth cannot sustain the today's growth rate of about 5% in some regions, without serious impact in the future. In this way, it is necessary a conscious energy and ecological design which fulfil all the strength and serviceability requirements.

Hence, several attempts have been made to improve the sustainability of the concrete and to transform it in a low impact construction material. Aggregates occupy the largest volume fraction in the concrete: the main components in the conventional concrete are usually natural aggregates as well as the crushed river stones [3]. Nowadays, another way to transform concrete in a more sustainable construction material is to substitute natural aggregates with alternative ones, such as the expanded clay termed *lightweight aggregate* giving place to the lightweight concrete (LWC).

Some authors have carried out an exhaustive technical review of the building envelope components and respective improvements from an energy efficiency perspective and as a conclusion, all kinds of LWC walls are particularly useful in countries where the construction using concrete is predominant and the use of insulation in walls is not a common practice. Furthermore, they can be constructed faster using less skilled labour [4].

In this study, the major variables influencing the thermal conductivity of masonry materials are illustrated in this work by taking hollow blocks made up of LWC. The finite element method (FEM) is used for finding accurate solutions of the heat transfer equation for several cases. The conduction and convection phenomena are taking into account in this study for six different hollow brick geometries using the thermal transmittance as objective function. The development of mathematical models for the prediction of heat transfer processes is a valuable tool in the field of heat and mass transfer technology. These models play an important role in the simulation and optimization of complex systems leading to efficient and economical designs of lightweight concrete hollow bricks (LCHB) [5,6].

In order to improve the thermal transmittance of the LCHB, the response surface methodology (RSM) and finite element method (FEM) have been used as modeling tools to determine the influence of individual factors and their interactions. The RSM methodology does not need explicit expressions of the physical meaning of the system under study. The RSM models are able to determine a relationship between design variables and a response or output of the process using a limited number of experimental runs [7]. Commonly, these models are developed using the design variable settings to optimize the response variable (process output): the thermal transmittance in this case.

It is worth quoting that the RSM permits to perform empirical polynomial models for the approximation of numerical solutions based on the finite element method (FEM). It is also known as a universal tool for the functional approximation of nonlinear systems. Both methodologies (RSM and FEM) can offer trustable approximation models to predict the true response function (objective function) of the process [8].

In this paper, the thermal behaviour of lightweight concrete hollow brick walls have been studied by the finite element method (FEM) [9–12] using the design of experiments (DOE) for several parameters such as: conductivity, convection, radiation and the geometry of the recesses. With respect to the material's conductivity, we have used a range of values obtained from experimental results. In case of the convection, the thermal resistance of the brick holes has been taken from the rule UNE-EN ISO 6946 [13]. In general, the thermal resistance is a function that depends on the geometry of the recesses and the temperature. In case of the radiation, we have used a range of values for the radiation emissivity obtained from experimental results. Finally, in case of the geometry of the recesses, we have studied different manufacturing values for the thickness and length.

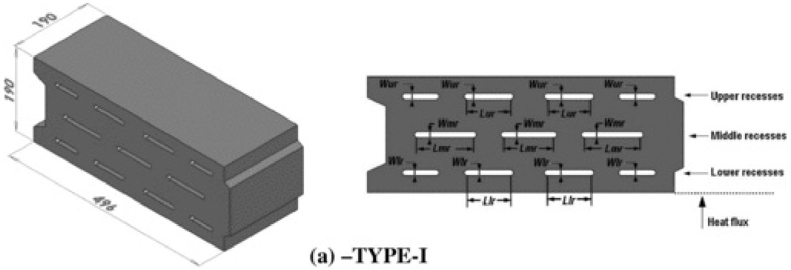
It is clear that finite element solution [9–12] will solve only the selected mathematical model and that all assumptions in this model will be reflected in the predicted response. We cannot expect any more information in the prediction of physical phenomena than the information contained in the mathematical model. Therefore, the choice of an appropriate mathematical model is crucial and this determines completely the insight into the actual physical problem that we can obtain by the analysis. This study has achieved this goal with success.

This research work is structured as follows: firstly, the geometrical models of the six different bricks analysed are described in detail as well as the predictive models using the FEM and RSM are explained in materials and methods section; secondly, the results and discussion of FEM and RSM analyses are presented. Thirdly, the discussion of results is shown in depth and finally, the main conclusions of this research work are exposed.

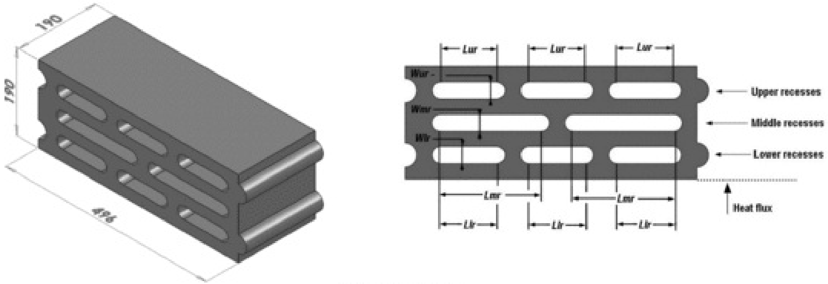
2 Materials and methods

2.1 Geometrical models

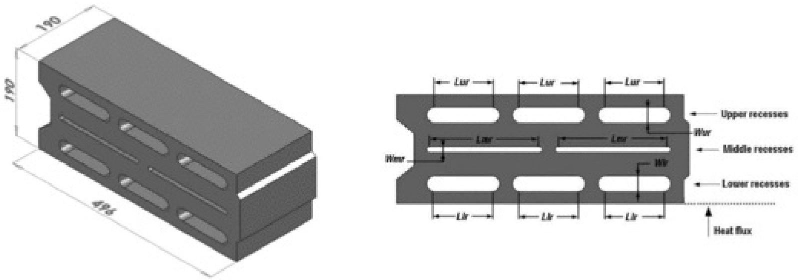
On the one hand, we have modelled a wall made up of three different bricks. All of them with the same thickness (0.190 m) and two different overall lengths: 0.496 m and 0.290 m, respectively. The geometry of cavities is different depending on the kind of brick (see Fig. 1).



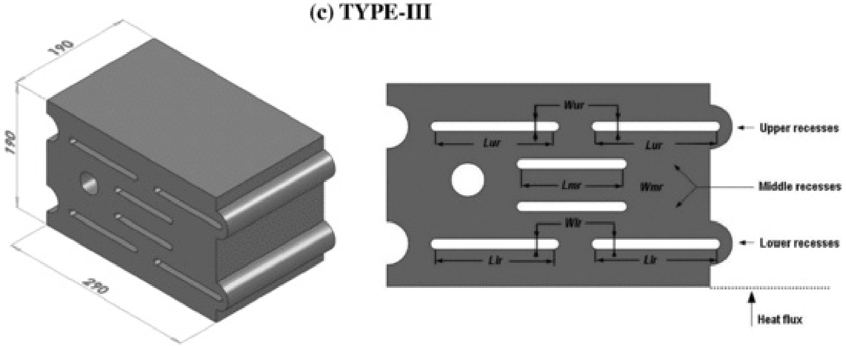
(a) –TYPE-I



(b) TYPE-II



(c) TYPE-III



(d) TYPE-IV

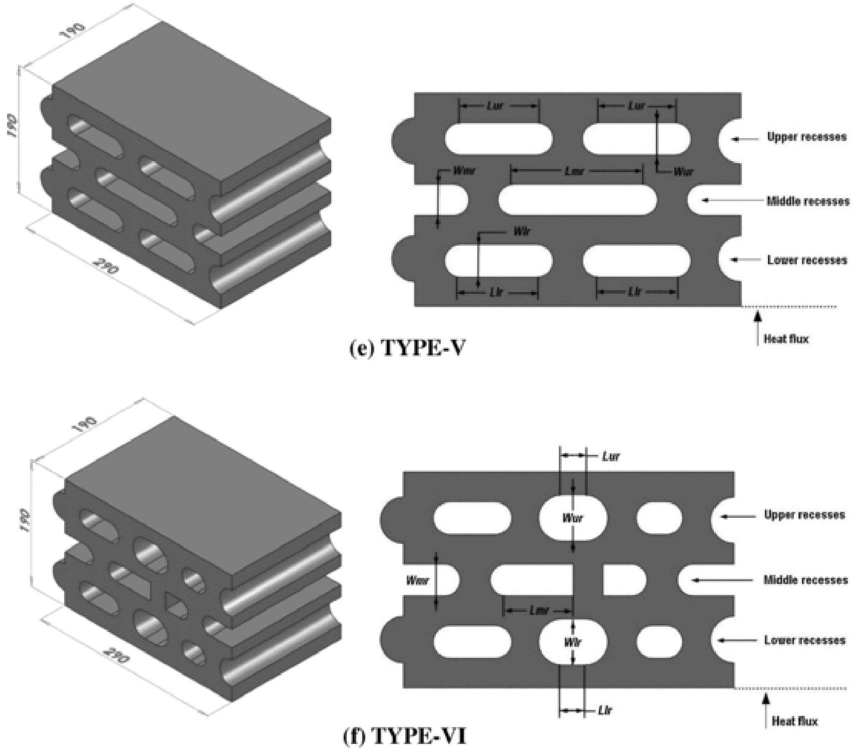


Fig. 1 Geometrical models of the brick types and description of the geometrical variables: (a) brick TYPE-I, (b) brick TYPE-II; (c) brick TYPE-III; (d) brick TYPE-IV; (e) brick TYPE-V; and (f) brick TYPE-VI.

On the other hand, the wall is composed of five equal bricks of the same type. It is also observed that the number of recesses varies in each brick type, as well as the thermal behaviour due to the different shape, distribution and length of the cavities.

The geometrical variables of the inner recesses are:

- *Llr* is the lenght of the lower recesses.
- *Lur* is the lenght of the upper recesses.
- *Lmr* is the lenght of the middle recesses.
- *Wlr* is the widht of the lower recesses.
- *Wur* is the widht of the upper recesses.
- *Wmr* is the widht of the middle recesses.

2.2 Predictive modelling using FEM

In this study, we have tried to analyze the influence of the number and size of brick's recesses in the thermal performance of different walls in order to optimize the shape of the brick. Size and shape thermal optimization problems are normally stated in terms of a minimum weight with an approach based on the temperature constraint.

These traditional statements of the minimum compliance for topology optimization problems offer some obvious advantages, since it is prevented to deal with a large number of highly nonlinear temperature constraints.

The purpose and interest of this research work can be divided in three important lines:

- Construction of simple, reliable and productive procedures allowing the use of optimal design techniques which can be applied in many engineering problems.
- Development of a specific technology for the challenging engineering design problems.
- The advance in the knowledge of general optimization processes. This fact is of fundamental economical importance nowadays.

In this section, we will describe the finite element modelling as well as the obtained results. The use of the finite element method (FEM) [12] shows innumerable advantages of economical and practical order due, on the one hand, to the cost that plays the execution of real tests and, on the other hand, to the technical difficulty of the same, since the elements object of the present study are big in size, i.e. walls made up of lightweight concrete blocks in this case.

The experimental tests have the inconvenience of their important economic cost and the huge execution time, so that only two types of wall were tested. In this paper, we have carried out numerical simulations for the different kind of walls, taking into account the excellent agreement between experimental tests and numerical simulations by using FEM for a certain wall's configuration [14,15].

For the light concrete modelling, we have used a plane element with four nodes named PLANE55 (see Fig. 2a) [16–18]. These 4-node elements can become triangular-shaped elements, well suited to model curved boundaries, such as the recesses and faces of the bricks. This element is also appropriate to reproduce the behaviour of heat transference for conduction in the solid. It has four nodes with a one degree of freedom (temperature) per node.

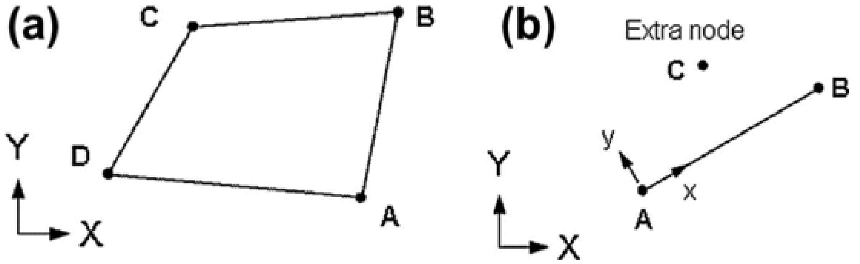


Fig. 2 Finite elements used: (a) Plane55 and (b) Surf151.

In order to simulate the convection and radiation phenomena in the brick's inner recesses, we have used surface elements with 3 nodes (2 nodes +1 extra node), named SURF151 [16–18] (see Fig. 2b), which has one degree of freedom (temperature) per node.

2.2.1 Material properties

The properties of the materials used in the different elements (bricks) that constitute the walls, according to the measurements carried out by means of experimental tests, are shown in Table 1 [19].

Table 1 Material properties.

Kind of brick	Mass of the brick *M* (kg)	Material conductivity $\lambda_{material}$ (W/m K)	Film coefficients for the middle/upper-lower recesses (W/m^2 K)
TYPE-I	12.518	0.3	2.5/2.5
TYPE-II	21.590	0.3	2.5/2.5
TYPE-III	16.144	0.3	1.25/1.25
TYPE-IV	17.410	0.3	2.5/1.25
TYPE-V	9.809	0.3	1.25/1.25
TYPE-VI	10.148	0.3	1.25/1.25

Similarly, an isotropic conductivity without transference of mass has been assumed with the surface resistances according to the standard UNE-EN ISO 6946 [13].

2.2.2 Thermal loads and boundary conditions

In the models studied in this research work, the following parameters are considered: a 10 W/m^2 heat flow in one of the wall sides, a 25 $W/m^2 \cdot K$ external film coefficient and a 273 K ambient temperature.

In order to implement the radiation phenomenon inside recesses, the radiation matrix method has been used [17]. This numerical method requires the previous calculation of the view factors. In this sense, the *hidden method* was used to obtain those ones [16]. The hidden method requires a significant computer time and it is used due to the complexity of the recesses' geometry. In this method, the smaller the meshing size of the radiating surface elements, the more accurate are the view factors.

The problem was solved in a workstation computer with a double-CPU Intel Xeon 5140 @ 2.33 GHz, 24 GB RAM memory and a 4.0 TB hard disk. The mean total estimated elapsed CPU time per case was 500 s and the total number of iterations in order to get the convergence about 50.

2.3 Predictive modelling using RSM

Firstly, the central composite design (CCD) was selected for the optimization of the parameters in the response surface methodology (RSM) procedure [20–22]. Taking into account that the different variables are usually expressed in different units and have different ranges of variation, the importance of their effects on the thermal behaviour can only be compared if they are coded.Secondly, the RSM technique is an optimization's approach permitting to determine the input combination of factors that maximize or minimize a given objective function [23]. According to the DOE and RSM techniques, regression models based on the second order polynomials can be developed to predict the performance of any process or system. Such models are also known as response surface models (RS-models). During the response surface's modelling, the input variables $x_1, x_2, \ldots, x_n$ must be scaled to coded levels. In coded scale the factors vary from (−1) that corresponds to minimum level up to (+1) that suit to maximum level. The second-order models given by RSM are often used to determine the critical points (maximum, minimum, or saddle) and can be written in a general form as [20]:

$$\widehat{Y} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i<j}^{n} \beta_{ij} x_i x_j \quad (1)$$

where $frownY$ denotes the predicted response; x_i refers to the coded levels of the input variables; β_0, β_i, β_{ii}, β_{ij} are the regression coefficients (an offset term, main coefficients, quadratic coefficients and interaction effects, respectively); and n is the total number of designed variables.

To determine the regression coefficients, the ordinary least squares (OLS) method is used. The OLS estimator can be written as follows [20,21]:

$$\vec{\beta}_{OLS} = \left(\vec{X}^T \vec{X}\right)^{-1} \vec{X}^T \vec{Y} \quad (2)$$

where $\vec{\beta}_{OLS}$ is a vector of regression coefficients; $\vec{X}$ is an extended designed matrix of the coded levels of the input variables; and $\vec{Y}$ is a column vector of response determined according to the arrangements points into the experimental design.

Table 2 shows the input variables corresponding to each type of brick as well as their variation ranges: maximum, minimum and initial value of each variable.

Table 2 Input variables and their variation ranges.

Variable	Initial value	Minimum value	Maximum value
TYPE-I			
Lur and *Llr* (mm)	70.00	57.50	82.50
Lmr (mm)	120.00	107.50	132.50
Thermal conductivity, λ (W/mK)	0.30	0.10	0.50
Radiation emissivity, ε (%)	0.88	0.80	0.96
Wmr (mm)	10.00	5.00	15.00
Wur and *Wlr* (mm)	10.00	5.00	15.00
TYPE-II			
Lur and *Llr* (mm)	95.00	85.00	105.00
Lmr (mm)	171.25	160.00	182.50
Thermal conductivity, λ (W/mK)	0.30	0.10	0.50

Radiation emissivity, *ε* (%)	0.88	0.80	0.96
Wmr (mm)	30.00	24.00	34.00
Wur and *Wlr* (mm)	30.00	24.00	34.00
TYPE-III			
Lur and *Llr* (mm)	98.00	86.00	110.00
Lmr (mm)	190.00	175.00	205.00
Thermal conductivity, *λ* (W/mK)	0.30	0.10	0.50
Radiation emissivity, *ε* (%)	0.88	0.80	0.96
Wmr (mm)	10.00	5.00	15.00
Wur and *Wlr* (mm)	30.00	24.00	34.00
TYPE-IV			
Lur and *Llr* (mm)	110.00	99.00	121.00
Lmr (mm)	93.00	84.00	102.00
Thermal conductivity, *λ* (W/mK)	0.30	0.10	0.50
Radiation emissivity, *ε* (%)	0.88	0.80	0.96
Wmr (mm)	10.00	5.00	15.00
Wur and *Wlr* (mm)	10.00	5.00	15.00
TYPE-V			
Lur and *Llr* (mm)	67.50	60.00	75.00
Lmr (mm)	112.50	100.00	125.00
Thermal conductivity, *λ* (W/mK)	0.30	0.10	0.50
Radiation emissivity, *ε* (%)	0.88	0.80	0.96
Wmr (mm)	30.00	24.00	34.00
Wur and *Wlr* (mm)	30.00	24.00	34.00
TYPE-VI			
Lur and *Llr* (mm)	20.00	15.00	25.00
Lmr (mm)	60.00	50.00	70.00
Thermal conductivity, *λ* (W/mK)	0.30	0.10	0.50
Radiation emissivity, *ε* (%)	0.88	0.80	0.96
Wmr (mm)	27.50	20.00	35.00
Wur and *Wlr* (mm)	32.00	26.00	38.00

3 Results and discussion

3.1 Numerical FEM results

The results obtained using FEM [9–12,14,15] are processed in order to obtain the main parameter that define the problem: the overall heat transfer coefficient, U (W/m^2·K). The calculation of this parameter is based on the following equations [24]:

$$U = \frac{\dot{Q}_u}{\Delta T} \tag{3}$$

where $\dot{Q}_u$ is the thermal flow applied in the finite element model (equal to 10 W/m^2 in this case) and ΔT is the difference of temperatures between faces of the central bricks, obtained from the finite element model (K).

Fig. 3 shows the two-dimensional temperature distribution corresponding to walls made up of bricks TYPE-I, TYPE-II, TYPE-III, TYPE-IV, TYPE-V and TYPE-VI.

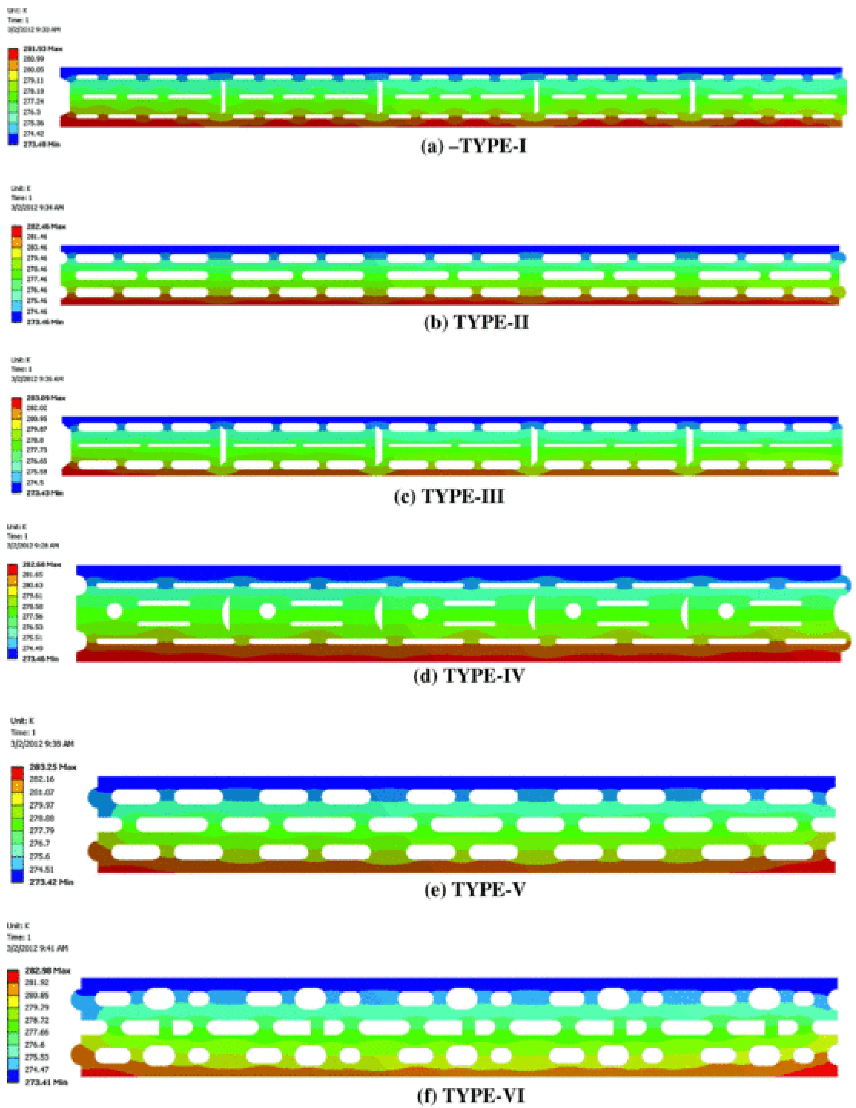


Fig. 3 Numerical FEM results: two-dimensional temperature distribution in walls made up of brick types: (a) TYPE-I; (b) TYPE-II; (c) TYPE-III; (d) TYPE-IV; (e) TYPE-V; and (f) TYPE-VI.

After the exam of the numerical results, it can be assumed that the computer aided simulation constitutes a reasonable approach to describe the thermal behaviour of the system according to other previous research works [5,6]. In summary, the FEM model may reproduce quite accurately the heat transfer in walls made up of lightweight aggregate concrete with open structure and complex shapes with recesses.

3.2 DOE and RSM results

The application of the DOE and RSM techniques leads to development of the predictive response surface. Indeed, the RSM procedure can be used for simulation of heat transfer process and was written in terms of coded variables as follows (TYPE-I, TYPE-II, TYPE-III, TYPE-IV, TYPE-V and TYPE-VI):

$$\begin{aligned}\widehat{Y} = {} & 1.17264 + 0.37107x_1 - 0.02345x_2 - 0.01616x_3 - 0.04947x_4 \\ & + 0.02962x_5 - 0.00768x_6 + 0.03277x_1^2 + 0.00082x_2^2 \\ & + 0.00535x_3^2 + 0.02596x_4^2 + 0.00222x_5^2 + 0.00356x_6^2 \\ & - 0.02088x_1x_2 - 0.01286x_1x_3 - 0.04198x_1x_4 + 0.0125x_1x_5 \\ & - 0.00636x_1x_6 + 0.00018x_2x_3 - 0.00277x_2x_4 + 0.00185x_2x_5 \\ & + 0.00079x_2x_6 + 0.0007x_2x_7 + 0.0021x_3x_5 - 0.00204x_3x_6 \\ & + 0.0084x_4x_5 + 0.00088x_4x_6 + 0.00025x_5x_6\end{aligned} \quad (4)$$

$$\begin{aligned}\widehat{Y} = {} & 1.24552 - 0.03098x_1 - 0.01372x_2 + 0.41259x_3 + 0.02369x_4 \\ & - 0.02036x_5 - 0.04127x_6 + 0.00916x_1^2 + 0.00766x_2^2 \\ & - 0.00884x_3^2 + 0.00841x_4^2 + 0.01401x_5^2 + 0.0183x_6^2 \\ & + 0.00122x_1x_2 - 0.03507x_1x_3 + 0.00178x_1x_4 + 0.0023x_1x_5 \\ & - 0.00257x_1x_6 - 0.01317x_2x_3 + 0.00168x_2x_4 - 0.002x_2x_5 \\ & + 0.00062x_2x_6 + 0.00791x_2x_7 - 0.02014x_3x_5 - 0.03949x_3x_6 \\ & + 0.00175x_4x_5 + 0.00589x_4x_6 + 0.00046x_5x_6\end{aligned} \quad (5)$$

$$\begin{aligned}\widehat{Y} = {} & 1.18563 + 0.25273x_1 + 0.05313x_2 + 0.01002x_3 + 0.01639x_4 \\ & - 0.00669x_5 - 0.01411x_6 - 0.04596x_1^2 + 0.00191x_2^2 \\ & + 0.00364x_3^2 + 0.0049x_4^2 + 0.00235x_5^2 + 0.00229x_6^2 \\ & + 0.01494x_1x_2 - 0.00829x_1x_3 - 0.01749x_1x_4 - 0.00737x_1x_5 \\ & - 0.01836x_1x_6 + 0.00182x_2x_3 + 0.00286x_2x_4 + 0.00193x_2x_5 \\ & + 0.00035x_2x_6 - 0.00019x_2x_7 + 0.00123x_3x_5 - 0.00046x_3x_6 \\ & - 0.00125x_4x_5 + 0.00178x_4x_6 + 0.00185x_5x_6\end{aligned} \quad (6)$$

$$\begin{aligned}\widehat{Y} = 1.19935 - 0.01549x_1 + 0.01985x_2 - 0.01011x_3 - 0.0197x_4 \\ + 0.04547x_5 + 0.25574x_6 + 0.00017x_1^2 + 0.00007x_2^2 \\ - 0.00188x_3^2 + 0.00696x_4^2 + 0.00251x_5^2 - 0.04663x_6^2 \\ + 0.00101x_1x_2 + 0.00179x_1x_3 + 0.00239x_1x_4 + 0.00059x_1x_5 \\ - 0.02203x_1x_6 - 0.00113x_2x_3 + 0.00023x_2x_4 + 0.00511x_2x_5 \\ - 0.02051x_2x_6 - 0.00087x_2x_7 - 0.00102x_3x_5 - 0.00933x_3x_6 \\ + 0.00214x_4x_5 - 0.01504x_4x_6 + 0.0124x_5x_6\end{aligned} \quad (7)$$

$$\begin{aligned}\widehat{Y} = 1.41440 + 0.49497x_1 + 0.05862x_2 - 0.00462x_3 - 0.02708x_4 \\ - 0.00126x_5 - 0.01321x_6 - 0.06527x_1^2 + 0.00742x_2^2 \\ + 0.00940x_3^2 + 0.00502x_4^2 + 0.00710x_5^2 + 0.00521x_6^2 \\ + 0.02301x_1x_2 - 0.03381x_1x_3 - 0.03100x_1x_4 - 0.01768x_1x_5 \\ - 0.01435x_1x_6 + 0.00424x_2x_3 + 0.00315x_2x_4 + 0.00231x_2x_5 \\ + 0.00165x_2x_6 + 0.00179x_2x_7 - 0.00014x_3x_5 + 0.00129x_3x_6 \\ - 0.00039x_4x_5 + 0.00008x_4x_6 + 0.00099x_5x_6\end{aligned} \quad (8)$$

$$\begin{aligned}\widehat{Y} = 1.26977 + 0.38316x_1 + 0.04008x_2 + 0.00035x_3 - 0.01057x_4 \\ + 0.00377x_5 - 0.01505x_6 - 0.02455x_1^2 + 0.00492x_2^2 \\ + 0.00906x_3^2 + 0.00350x_4^2 + 0.00740x_5^2 + 0.00445x_6^2 \\ + 0.01270x_1x_2 - 0.03766x_1x_3 - 0.01872x_1x_4 - 0.01426x_1x_5 \\ - 0.01166x_1x_6 + 0.00637x_2x_3 + 0.00292x_2x_4 + 0.00277x_2x_5 \\ + 0.00178x_2x_6 + 0.00321x_2x_7 + 0.00001x_3x_5 + 0.00082x_3x_6 \\ + 0.00016x_4x_5 + 0.00071x_4x_6 + 0.00086x_5x_6\end{aligned} \quad (9)$$

subjected to $x_i \in \Omega$ with $\Omega = \{x_i | -\alpha \leqslant x_i \leqslant +\alpha\}; \quad \forall i = \overline{1,6}$, and where α denotes the star point (a property of CCD), which delimitates the boundaries of valid region Ω known as *region of experimentation*. In our case, we have six factors (n = 6) and an orthogonal design CCD. It is worth to note that the significance of regression coefficients was tested using the statistical Student's *t*-test [25].

The adequacy of RSM procedure was tested by means of analysis of variance (ANOVA) and the results of the statistical test are shown in Table 3: degrees of freedom, sum of squares of residuals, mean square, *F*-values, *p*-values, the coefficient of determination R^2 and the adjusted coefficient of determination R^2_{adj}.

Table 3 ANOVA table for the RSM procedure predicts the thermal transmittance for the different bricks.

Item	Degrees of freedom	Sum of squares	Mean square	*F*-value	*p*-value	R^2	R^2_{adj}
TYPE-I	27	2.2231	0.0823	1338.29	0.007	0.9988	0.9974
Residual	17	0.0011	0.0001				
Total	44	2.2242					

TYPE-II	27	0.8621	0.0319	301.61	0.001	0.9946	0.9880
Residual	17	0.0018	0.0001				
Total	44	0.8639					
TYPE-III	27	0.8801	0.0326	1465.81	0.009	0.9951	0.9890
Residual	17	0.0017	0.0001				
Total	44	0.8818					
TYPE-IV	27	1.8139	0.0672	3930.69	0.001	0.9996	0.9991
Residual	17	0.0003	0.0001				
Total	44	1.8142					
TYPE-V	27	3.1851	0.1180	1577.42	0.001	0.9990	0.9978
Residual	17	0.0013	0.0008				
Total	44	3.1864					
TYPE-VI	27	1.8980	0.0703	3605.68	0.002	0.9995	0.9990
Residual	17	0.0003	0.0001				
Total	44	1.8983					

The mathematical calculation of the above statistical estimators is extensively known and they can be consulted in many References. [25,26].

The statistical testing of the models was performed with the Fisher's statistical test for analysis of variance (ANOVA) [25,26] and the results of these analyses are shown in Table 2. According to ANOVA, the *F*-values for the models TYPE-I, TYPE-II and TYPE-III are ranging from 301.61 to 1,465.81; and for the models TYPE-IV, TYPE-V and TYPE-VI are ranging from 1,577.42 to 3,930.69. Note that the *F*-value is a measure of the variance of data about the mean.

When the *F*-values are much greater than unity, the input variables explain better the variation in the mean of the data. In our case, these values are sufficiently large to fulfil these requirements. Next, the *p*-values are calculated from the *F*-values and the degrees of freedom. One way to validate the RS-model from the statistical point of view is to observe if the *p*-values are less than 0.05. The *p*-values obtained were 0.007, 0.001 and 0.009 for the models TYPE-I, TYPE-II and TYPE-III, respectively. Similarly, the *p*-values were 0.001, 0.001 and 0.002 for the models TYPE-IV, TYPE-V and TYPE-VI, respectively.Note that the R^2 values are 0.9988, 0.9946 and 0.9951 for models TYPE-I, TYPE-II and TYPE-III, respectively; and 0.9996, 0.9990 and 0.9995 for models TYPE-IV, TYPE-V and TYPE-VI, respectively. Since these values are close to the unity, the goodness of fit of the statistical model indicates that the RS-model fits to the set of observations very well, showing a good prediction for the thermal response. Furthermore, the adjusted coefficient of determination R^2_{adj} is a modification of R^2 that adjusts for the number of explanatory terms in a model and it will always be less than or equal to R^2.

Therefore, all statistical estimators indicate that the RS-model built in this research work is valid from the statistical point of view to explain with accurateness the thermal behaviour of these heat-insulating and lightweight concrete hollow bricks.

4 Discussion of results

In order to obtain a better understanding of the sensitivity analysis and RSM results, the main findings are presented in Figs. 4–9 as a bar graph and three-dimensional response surface plots, respectively.

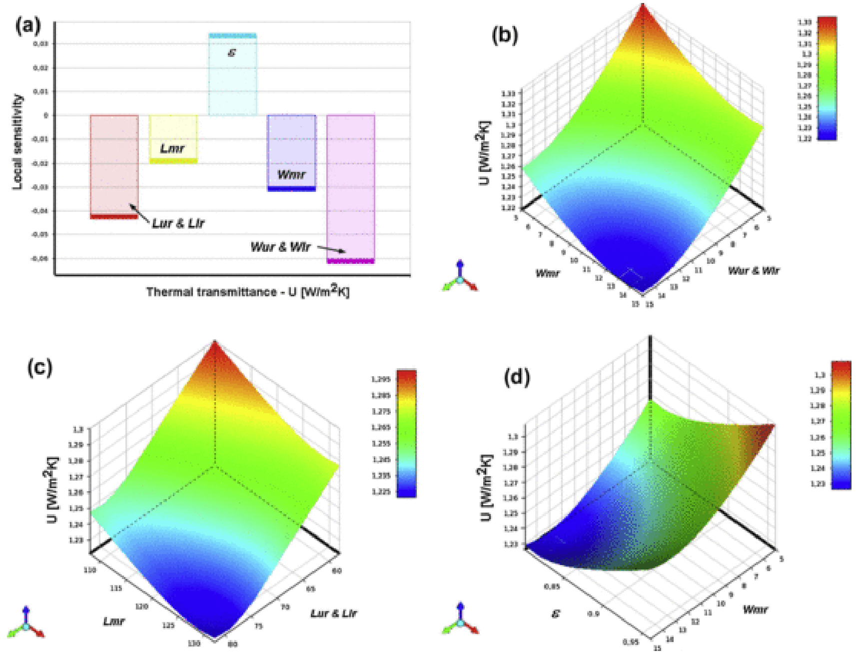


Fig. 4 DOE results for the brick TYPE-I of the overall heat transfer coefficient (*U*): (a) sensitivity analysis; (b) RS plot of *U* as a function of widths of the middle and upper/lower recesses; (c) RS plot of *U* as a function of lengths of the middle and upper/lower recesses; and (d) RS plot of *U* as a function of the radiation emissivity and the width of middle recesses.

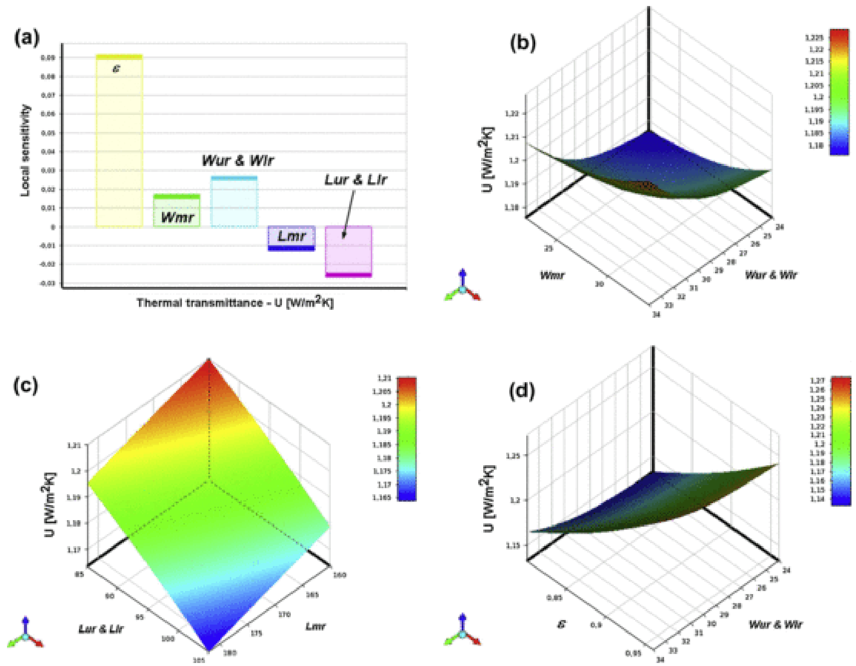


Fig. 5 DOE results for the brick TYPE-II of the overall heat transfer coefficient (*U*): (a) sensitivity analysis; (b) RS plot of *U* as a function of widths of the middle and upper/lower recesses; (c) RS plot of *U* as a function of lengths of the middle and upper/lower recesses; and (d) RS plot of *U* as a function of the radiation emissivity and widths of upper/lower recesses.

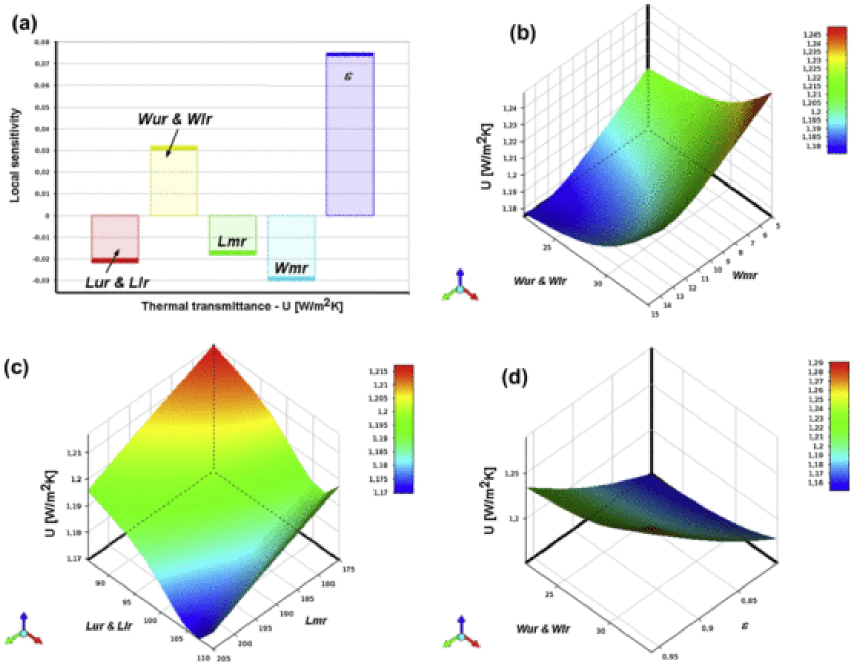


Fig. 6 DOE results for brick TYPE-III of the overall heat transfer coefficient (*U*): (a) sensitivity analysis; (b) RS plot of *U* as a function of widths of the middle and upper/lower recesses; (c) RS plot of *U* as a function of lengths of the middle and upper/lower recesses; and (d) RS plot of *U* as a function of the radiation emissivity and widths of upper/lower recesses.

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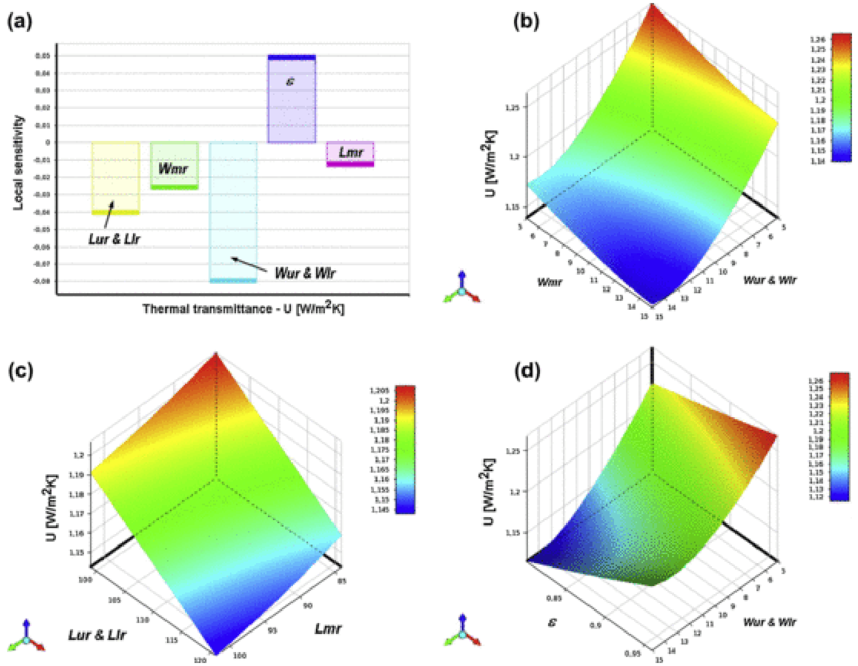

Fig. 7 DOE results for the brick TYPE-IV of the overall heat transfer coefficient (*U*): (a) sensitivity analysis; (b) RS plot of *U* as a function of widths of the middle and upper/lower recesses; (c) RS plot of *U* as a function of lengths of the middle and upper/lower recesses; and (d) RS plot of *U* as a function of the radiation emissivity and widths of upper/lower recesses.

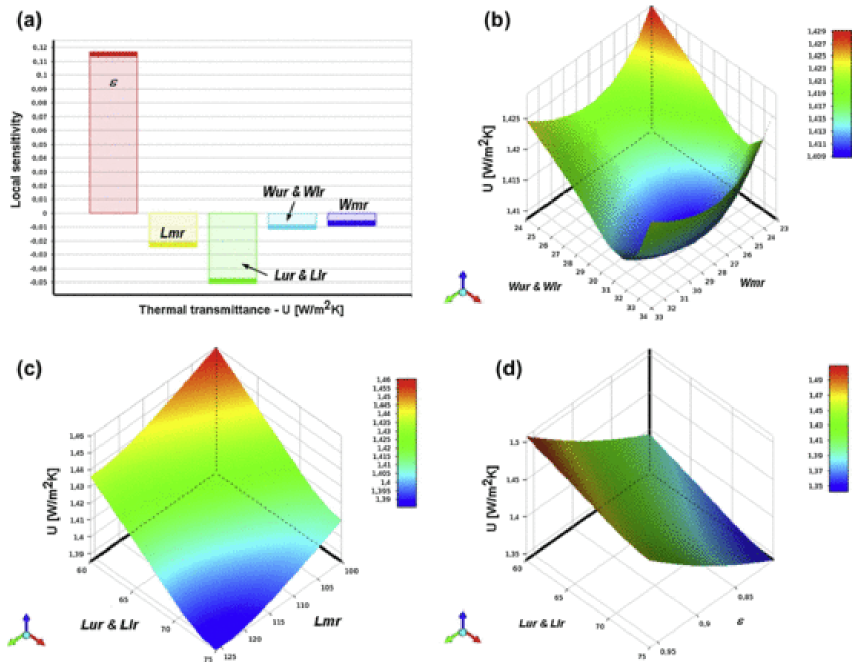

Fig. 8 DOE results for the brick TYPE-V of the overall heat transfer coefficient (*U*): (a) sensitivity analysis; (b) RS plot of *U* as a function of widths of the middle and upper/lower recesses; (c) RS plot of *U* as a function of lengths of the middle and upper/lower recesses; and (d) RS plot of *U* as a function of the radiation emissivity and lengths of the upper/lower recesses.

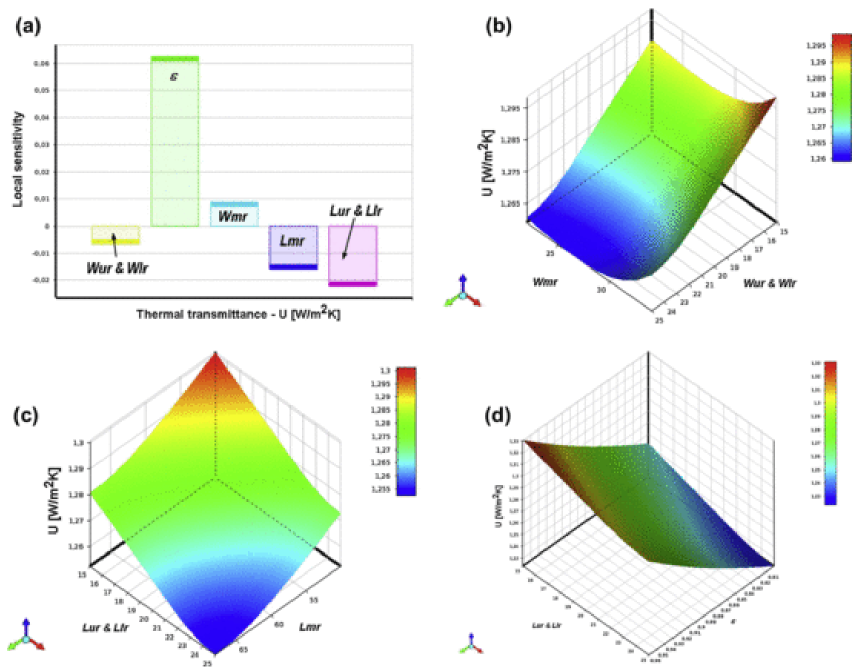

Fig. 9 DOE results for the brick TYPE-VI of the overall heat transfer coefficient (*U*): (a) sensitivity analysis; (b) RS plot of *U* as a function of widths of the middle recesses and the length of upper/lower recesses; (c) RS plot of *U* as a function of lengths of the middle and upper/lower recesses; and (d) RS plot of *U* as a function of the radiation emissivity and lengths of upper/lower recesses.

Firstly, with respect to the brick TYPE-I, Fig. 4a shows the sensitivity diagram excluding the thermal conductivity because it is the most important variable. Note that the hierarchy of the input variables affecting the thermal transmittance is:

- *Wur* and *Wlr* are the most influential variables in an inversely proportional way, that is to say, if *Wur* and *Wlr* are increased, the thermal transmittance is decreased.
- *Lur* and *Llr* are the second most influential variables in an inversely proportional way with respect to the thermal transmittance.
- Radiation emissivity of the material, ε, is the third variable in a directly proportional way, that is to say, if ε is increased, the thermal transmittance is also increased.
- Finally, *Wmr* and *Lmr* are the fourth and fifth variables in an inversely proportional way, with respect to the thermal transmittance.

Fig. 4b–d shows the response surfaces of the overall heat transfer coefficient, *U*, as a function of widths of the middle and upper/lower recesses, as a function of lengths of the middle and upper/lower recesses, and as a function of the radiation emissivity and width of the middle recesses, respectively. In all cases, it is possible to state that the thermal transmittance is monotonically increasing function with respect to the material emissivity.

In consequence, in order to get a minimum value of the overall heat transfer coefficient, it is necessary to increase the width and length of the recesses and decrease the radiation emissivity.

Secondly, with respect to the brick TYPE-II, Fig. 5a indicates the sensitivity diagram excluding the thermal conductivity. In this case, the hierarchy of the input variables affecting the thermal transmittance is as follows:

- The radiation emissivity of the material, ε, is the most influential variable in a directly proportional way, that is to say, if ε is increased, the thermal transmittance is also increased.
- *Lur* and *Llr* are the second influential variables in an inversely proportional way with respect to the thermal transmittance.

- *Wur* and *Wlr* are the third influential variables in a directly proportional way, that is to say, if *Wur* and *Wlr* are increased, the thermal transmittance is also increased.
- *Wmr* is the fourth variable in a directly proportional way, with respect to the thermal transmittance.
- Finally, *Lmr* is the fifth variable in an inversely proportional way, with respect to the thermal transmittance.

Analogously, Fig. 5b–d shows the response surfaces of the overall heat transfer coefficient, *U*, as a function of widths of the middle and upper/lower recesses, as a function of lengths of the middle and upper/lower recesses, and as a function of the radiation emissivity and widths of upper/lower recesses, respectively.

From these response surfaces, it is possible to derive that the thermal transmittance is a monotonically increasing function of the emissivity of the material and widths of recesses. Therefore, to reach a minimum value of the overall heat transfer coefficient, it is advisable that lengths of recesses are increased and the radiation emissivity and widths of recesses are decreased.

Thirdly, with respect to the brick TYPE-III, Fig. 6a shows the sensitivity diagram. In this way, the input variables affecting the thermal transmittance have the following order of priority:

- The radiation emissivity of the material, ε, is the most influential variable in a directly proportional way, that is to say, if ε is increased, the thermal transmittance is also increased.
- *Wur* and *Wlr* are the second influential variables in a directly proportional way, that is to say, if *Wur* and *Wlr* are increased, the thermal transmittance is also increased.
- *Wmr* is the third influential variable in an inversely proportional way, with respect to the thermal transmittance.
- *Lur* and *Llr* are the fourth influential variables in an inversely proportional way with respect to the thermal transmittance.
- Finally, *Lmr* is the fifth variable in an inversely proportional way, with respect to the thermal transmittance.

Furthermore, Fig. 6b–d shows the response surfaces of the overall heat transfer coefficient, *U*, as a function of widths of the middle and upper/lower recesses, as a function of lengths of the middle and upper/lower recesses, and as a function of the radiation emissivity and widths of upper/lower recesses, respectively. From these response surfaces, it is possible to conclude that the thermal transmittance is a monotonically increasing function of the material emissivity and width of upper/middle recesses.

Therefore, in order to reach a minimum value of the overall heat transfer coefficient, it is recommended that lengths of recesses and width of middle recesses are increased and the radiation emissivity and widths of upper/lower recesses are decreased.

Fourthly, with respect to the brick TYPE-IV, Fig. 7a shows the sensitivity diagram. In this way, the input variables affecting the thermal transmittance have the following order of priority:

- *Wur* and *Wlr* are the most influential variables in an inversely proportional way, that is to say, if *Wur* and *Wlr* are increased, the thermal transmittance is decreased.
- The radiation emissivity of the material, ε, is the second influential variable in a directly proportional way, that is to say, if ε is increased, the thermal transmittance is also increased.
- *Lur* and *Llr* are the third influential variables in an inversely proportional way with respect to the thermal transmittance.
- *Wmr* is the fourth influential variable in an inversely proportional way, with respect to the thermal transmittance.
- Finally, *Lmr* is the fifth variable in an inversely proportional way, with respect to the thermal transmittance.

On the one hand, Fig. 7b–d shows the response surfaces of the overall heat transfer coefficient, *U*, as a function of widths of the middle and upper/lower recesses, as a function of lengths of the middle and upper/lower recesses, and as a function of the radiation emissivity and widths of upper/lower recesses, respectively. From these response surfaces, it is possible to conclude that the thermal transmittance is a monotonically increasing function of the material's emissivity.

On the other hand, to reach a minimum value of the overall heat transfer coefficient, it is recommended that all lengths and widths of recesses are increased while the radiation emissivity has to be decreased.

Fifthly, with respect to the brick TYPE-V, Fig. 8a indicates the sensitivity diagram. It is possible to observe in this diagram, the following hierarchy of the input variables with respect to the thermal transmittance:

- The surface radiation emissivity of the recesses, *ε*, is the most influential variable in a directly proportional way, that is to say, if *ε* is increased, the thermal transmittance is also increased.
- *Lur* and *Llr* are the second influential variables in an inversely proportional way with respect to the thermal transmittance.
- *Lmr* is the third variable in an inversely proportional way, with respect to the thermal transmittance.
- *Wur* and *Wlr* are the fourth influential variables in an inversely proportional way, that is to say, if *Wur* and *Wlr* are increased, the thermal transmittance is decreased.
- Finally, *Wmr* is the fifth influential variable in an inversely proportional way, with respect to the thermal transmittance.

Fig. 8b–d shows the response surfaces of the overall heat transfer coefficient, *U*, as a function of widths of the middle and upper/lower recesses, as a function of lengths of the middle and upper/lower recesses, and as a function of the radiation emissivity and lengths of the upper/lower recesses, respectively. From these response surfaces, it is possible to conclude that the thermal transmittance is a monotonically increasing function of the radiation emissivity.

Additionally, a global minimum of the overall heat transfer coefficient is found for a width of the upper/lower recesses of 30 mm and a width of middle recesses of 28 mm. Therefore, to reach a minimum value of the thermal transmittance, it is recommended to take the previous indicated values for the width of recesses, as well as to increase the lengths of recesses and to decrease the radiation emissivity of the recesses' surface.

Finally, with respect to the brick TYPE-VI, Fig. 9a shows the sensitivity graph. It is possible to observe in this diagram the following order of priority for input variables with respect to the thermal transmittance:

- The surface radiation emissivity of recesses, *ε*, is the most influential variable in a directly proportional way, that is to say, if *ε* is increased, the thermal transmittance is also increased.
- *Lur* and *Llr* are the second influential variables in an inversely proportional way, with respect to the thermal transmittance.
- *Lmr* is the third variable in an inversely proportional way, with respect to the thermal transmittance.
- *Wmr* is the fourth influential variable in a directly proportional way, with respect to the thermal transmittance.
- Finally, *Wur* and *Wlr* are the fifth influential variables in an inversely proportional way, that is to say, if *Wur* and *Wlr* are increased, the thermal transmittance is decreased.

Fig. 9b–d shows the response surfaces of the overall heat transfer coefficient, *U*, as a function of widths of the middle recesses and length of the upper/lower recesses, as a function of lengths of the middle and upper/lower recesses, and as a function of the radiation emissivity and lengths of the upper/lower recesses, respectively. From these response surfaces, it is possible to conclude that the thermal transmittance is a monotonically increasing function with respect to the radiation emissivity.

Accordingly, in this kind of brick, the width of the recesses plays a negligible influence on the overall heat transfer coefficient. Therefore, to obtain a minimum value of the thermal transmittance, it is recommended to increase the lengths of recesses and decrease the radiation emissivity of the recesses' surface.

5 Conclusions

The response surface methodology (RSM) and finite element method (FEM) has been implemented in order to build predictive models to study the thermal behaviour of several kinds of walls made up of different types of lightweight concrete hollow bricks.

In order to minimize the thermal transmittance, a full factorial central composite design (CCD) along with the RSM technique were used to optimize the values of the input variables: emissivity of the material, widths of recesses and lengths of recesses. Results reveal the following main findings:

Predicted values of the overall heat transfer coefficient obtained using model equations for the six different bricks were in a good agreement with the FEM values. Indeed, values of the coefficient of determination, R^2, between 0.9946 and 0.9996 were obtained in this research work.

- If the width of the recesses is equal to 10 mm, the increase of the width greater than this previous value gives place to the decrease of the brick thermal conductivity.
- In case of recesses whose width is equal to 30 mm, a width less than this value gives place to the decrease of the brick thermal conductivity.
- An optimum range of the width of recesses is found between 28 and 30 mm for the brick TYPE-V.

- In general, an increase in length of the recesses implies a better thermal behaviour.
- In all analysed cases, a decrease of the recesses' surface radiation emissivity causes a lower thermal transmittance in the brick.

The present study has proved that the response surface methodology (RSM) in combination with the finite element modelling may be applied to optimize the overall heat transfer coefficient in an efficient way in order to get important energy savings. Furthermore, this methodology is an economical way to obtain the best thermal behaviour of different bricks with the fewest number of experiments.

In summary, for new constructed buildings there is a great interest in light building materials with good physical material properties with respect to a conscious energy saving and ecological design and which fulfil all the strength and serviceability requirements. From this point of view, engineers and architects can use the main results shown in this research work in order to obtain the best recesses configuration for new brick designs.

Acknowledgements

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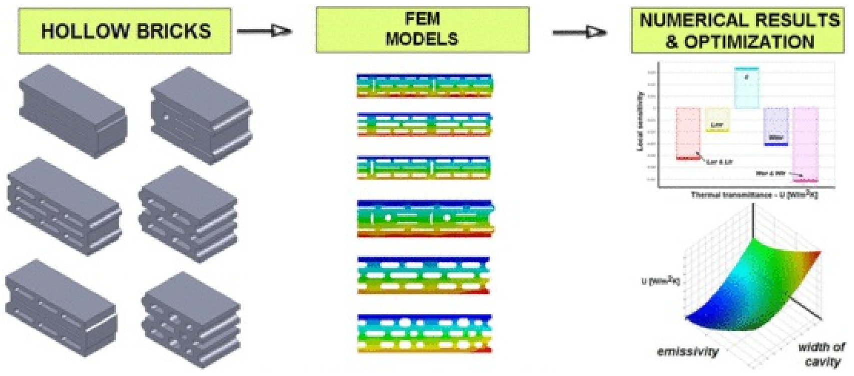
Graphical abstract



Highlights

- Predictive models for simulation and optimization of heat transfer process has been developed.
- Material conductivity, radiation emissivity, length and width of the recesses have been considered as input variables.
- The optimal design of six hollow bricks from the RSM analysis is exposed.
- Response surfaces and sensitivity analysis for each brick type is exposed.

Queries and Answers

Query: Please confirm that given name(s) and surname(s) have been identified correctly.

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Conductivity, emissivity and size of the recesses have been considered as input variables