UNIVERSIDAD DE OVIEDO

PROGRAMA DE DOCTORADO EN
INGENIERÍA INFORMÁTICA

LINGUISTIC MODELING OF COMPLEX PHENOMENA

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RESUMEN DEL CONTENIDO DE TESIS DOCTORAL

1.- Título de la Tesis	
Español/Otro Idioma: Modelado Lingüístico de Fenómenos Complejos	Inglés: Linguistic Modeling of Complex Phenomena

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RESUMEN (en español)

Actualmente, es posible adquirir y almacenar grandes volúmenes de datos sobre diferentes fenómenos complejos en muchas áreas importantes. Con el fin de ser útil, esta información debe ser explicada de la manera más comprensible posible, incluyendo los conocimientos previos disponibles sobre el fenómeno que se estudia.

Estos objetivos sólo pueden lograrse mediante el uso de lenguaje natural, especialmente si la información final va a ser utilizada por personas no expertas. Por lo tanto, esta información debe ser explicada de manera comprensible a través de los modelos lingüísticos. La formulación de modelos lingüísticos puede ser vista como una tarea no trivial, y muchas veces, el modelado lingüístico contribuye a una mejor comprensión de los fenómenos, proporcionando una novedosa e inédita visión de los mismos. En esta tesis, nos basamos en el paradigma de la computación con palabras y percepciones desarrollado por Zadeh con el fin de extender la Teoría Computacional de Percepciones. La idea consiste en extender la Lógica Borrosa para crear modelos de sistemas basados en la forma en que los seres humanos hacen descripciones utilizando el lenguaje natural. El objetivo es utilizar las estructuras complejas del lenguaje natural para hacer modelos imprecisos y robustos de los fenómenos complejos, cuyas principales ventajas son la incorporación de las capacidades creativas, abstractas y de adaptación del ser humano, y reducir al mínimo los aspectos no deseados tales como la imprevisibilidad, la incoherencia, la subjetividad y la inestabilidad temporal. Nuestro objetivo es hacer uso de una relación simbiótica entre el diseñador y el ordenador, de tal manera que la motivación y la creatividad de los diseñadores se vean reforzadas por la gran capacidad de almacenamiento y rendimiento del ordenador.

Hemos ampliado el concepto de Máquina de Estados Finitos Borrosos para abordar el problema de modelar cada fenómeno complejo específico sobre la base de un diseño l lingüístico y guiado por el ser humano. Además, dado que la definición de los parámetros de la Máquina de Estados Finitos Borrosos es, en cada caso particular, una tarea compleja para los expertos, hemos propuesto una metodología que consiste en un método de aprendizaje automático para definir los parámetros del modelo. Esta metodología se basa en la hibridación de las Máquinas de Estados Finitos Borrosos y los Algoritmos Genéticos, que conducen a la Máquina Genética de Estados Finitos Borrosos. La Máquina Genética de Estados Finitos Borrosos es capaz de aprender automáticamente las reglas borrosas y las funciones de pertenencia, mientras que un experto define los posibles estados y las transiciones permitidas entre los estados. A continuación, hemos desarrollado el Modelo Lingüístico Granular de un Fenómeno, que es el modelo necesario para interpretar los datos de entrada de una forma jerárquica. El Modelo Lingüístico Granular de un Fenómeno es capaz de combinar diferentes fuentes de conocimiento, en combinación con la expresividad del paradigma de modelado de las Máquinas de Estados Finitos Borrosos. Una vez que la Máquina de Estados Finitos Borrosos es capaz de modelar cada fenómeno complejo, el Modelo Lingüístico Granular

de un Fenómeno es capaz de producir descripciones lingüísticas sobre él y su evolución en el tiempo.

Finalmente, se ha validado la metodología propuesta con varias aplicaciones en el mundo real. Primero, hemos sido capaces de modelar la marcha y la actividad humana con nuestro enfoque de modelado lingüístico mediante conocimiento experto y conocimiento inducido. A continuación, hemos desarrollado un sistema capaz de modelar la calidad de la marcha y producir descripciones lingüísticas sobre ella. También hemos mostrado cómo nuestra propuesta funciona correctamente en el campo de los Sistemas Inteligentes de Transporte, donde hemos sido capaces de modelar y generar descripciones lingüísticas de la evolución del tráfico en carreteras.

RESUMEN (en Inglés)

Nowadays, it is possible to acquire and store vast volumes of data about different complex phenomena in many crucial areas. In order to be useful, this information must be explained in an understandable way, including facts that may be derived from the data and the background knowledge available about the phenomena under study. This can only be achieved by using natural language, especially if the final information is going to be used by non-experts. Therefore, this information must be explained in an understandable way by means of linguistic models. The formulation of linguistic models can be seen as a non-trivial task, and many times, linguistic modeling contributes to a better understanding of phenomena, providing a novel and previously unseen view of them. In this thesis, we follow Zadeh's computing with words and perceptions paradigm in order to extend the Computational Theory of Perceptions. The idea consists of extending Fuzzy Logic to create system models based on the way that humans make descriptions using natural language. The aim is to use complex structures of natural language to make robust imprecise models of complex phenomena, whose main advantages are the incorporation of creative, abstract and adaptive human capabilities, while minimizing undesirable aspects such as unpredictability, inconsistency, subjectivity and temporal instability. Our aim is to make use of a symbiotic relationship between the designer and the computer, in such a way that designer's motivation and creativity are strengthened by the computer's greater memory storage and higher computational performance.

We have extended the concept of Fuzzy Finite State Machine to deal with the problem of modeling each specific complex phenomenon on the basis of a linguistic, human-guided design. Moreover, since the definition of details of the Fuzzy Finite State Machine in each particular case is a complex task for experts, we have proposed a methodology which consists of a machine learning method to define the model parameters. This methodology is based on the hybridization of Fuzzy Finite State Machines and Genetic Algorithms leading to Genetic Fuzzy Finite State Machines. The Genetic Fuzzy Finite State Machine automatically learns the fuzzy rules and membership functions of the model, while an expert defines the possible states and allowed transitions between states.

Then, we have developed the Granular Linguistic Model of a Phenomenon paradigm, which is the model needed to interpret the input data in a hierarchical fashion. The Granular Linguistic Model of a Phenomenon is able to merge different sources of knowledge in combination with the expressiveness of the Fuzzy Finite State Machine modeling paradigm. Once the Fuzzy Finite State Machine is able to model each complex phenomenon, the Granular Linguistic Model of a Phenomenon is able to produce linguistic descriptions about it and its evolution in time.

Finally, we have validated the proposed methodology with several real world applications. We have been able to model the human gait and the human activity using

Agradecimientos

En primer lugar, me gustaría agradecer a mi director Gracián Triviño la oportunidad de realizar esta tesis doctoral bajo su guía. Durante estos últimos cuatro años hemos compartido no sólo el trabajo diario sino la pasión por innovar y descubrir cosas nuevas.

Me gustaría también agradecer al European Centre for Soft Computing haberme brindado la oportunidad de haber realizado esta tesis doctoral, siempre estaré agradecido a todas y cada una de las personas que forman y han formado parte de este centro, porque en mayor o menor medida han contribuido al desarrollo de esta tesis doctoral.

Así mismo, me gustaría destacar que parte del trabajo desarrollado durante la realización de esta tesis doctoral ha sido financiada por los proyectos del Ministerio de Ciencia e Innovación "Compuatación con Palabras y Percepciones en Entornos Inteligentes" (TIN2008-06890-C02-01) y "Descripción Lingüística de Fenómenos Complejos" (TIN2011-29827-C02-01).

Por último, me gustaría agradecer a mi familia todo el apoyo recibido durante estos últimos años. En especial, a mis abuelos Rogelia, Luis, Teresa y Marcelino; a mis padres Elena y Luis Alberto; a mi hermana Lucía; y a mi mujer Carmen. Sin ellos, nada de esto hubiera sido posible.

We must not forget that when radium was discovered no one knew that it would prove useful in hospitals. The work was one of pure science. And this is a proof that scientific work must not be considered from the point of view of the direct usefulness of it. It must be done for itself, for the beauty of science, and then there is always the chance that a scientific discovery may become like the radium a benefit for humanity.
Marie Curie (1867 - 1934)

Abstract

Nowadays, it is possible to acquire and store vast volumes of data about different complex phenomena in many crucial areas. In order to be useful, this information must be explained in an understandable way, including facts that may be derived from the data and the background knowledge available about the phenomena under study. This can only be achieved by using natural language, especially if the final information is going to be used by non-experts. Therefore, this information must be explained in an understandable way by means of linguistic models. The formulation of linguistic models can be seen as a non-trivial task, and many times, linguistic modeling contributes to a better understanding of phenomena, providing a novel and previously unseen view of them. In this thesis, we follow Zadeh's computing with words and perceptions paradigm in order to extend the Computational Theory of Perceptions. The idea consists of extending Fuzzy Logic to create system models based on the way that humans make descriptions using natural language. The aim is to use complex structures of natural language to make robust imprecise models of complex phenomena, whose main advantages are the incorporation of creative, abstract and adaptive human capabilities, while minimizing undesirable aspects such as unpredictability, inconsistency, subjectivity and temporal instability. Our aim is to make use of a symbiotic relationship between the designer and the computer, in such a way that designer's motivation and creativity are strengthened by the computer's greater memory storage and higher computational performance.

We have extended the concept of Fuzzy Finite State Machine to deal with the problem of modeling each specific complex phenomenon on the basis of a linguistic, human-guided design. Moreover, since the definition of details of the Fuzzy Finite State Machine in each particular case is a complex task for experts, we have proposed a methodology which consists of a machine learning method to define the model parameters. This methodology is based on the hybridization of Fuzzy Finite State Machines and Genetic Algorithms leading to Genetic Fuzzy Finite State Machines. The Genetic Fuzzy Finite State Machine automatically learns the fuzzy rules and membership functions of the model, while an expert defines the possible states and allowed transitions between states.

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paradigm. Once the Fuzzy Finite State Machine is able to model each complex phenomenon, the Granular Linguistic Model of a Phenomenon is able to produce linguistic descriptions about it and its evolution in time.

Finally, we have validated the proposed methodology with several real world applications. We have been able to model the human gait and the human activity using our linguistic modeling approach using expert and induced knowledge. Then, we have developed a system capable of modeling the gait quality and producing linguistic descriptions about it. We have also showed the generality of our proposal, showing how it works in a completely different field, namely, intelligent transportation systems, where we have been able to model and generate linguistic descriptions of the traffic evolution in roads.

Resumen

Actualmente, es posible adquirir y almacenar grandes volúmenes de datos sobre diferentes fenómenos complejos en muchas áreas importantes. Con el fin de ser útil, esta información debe ser explicada de la manera más comprensible posible, incluyendo los conocimientos previos disponibles sobre el fenómeno que se estudia.

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transiciones permitidas entre los estados.

A continuación, hemos desarrollado el Modelo Lingüístico Granular de un Fenómeno, que es el modelo necesario para interpretar los datos de entrada de una forma jerárquica. El Modelo Lingüístico Granular de un Fenómeno es capaz de combinar diferentes fuentes de conocimiento, en combinación con la expresividad del paradigma de modelado de las Máquinas de Estados Finitos Borrosos.Una vez que la Máquina de Estados Finitos Borrosos es capaz de modelar cada fenómeno complejo, el Modelo Lingüístico Granular de un Fenómeno es capaz de producir descripciones lingüísticas sobre él y su evolución en el tiempo.

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General scheme

This memory entitled "Linguistic Modeling of Complex Phenomena", presented to obtain the degree of Doctor by the "Universidad de Oviedo", is organized into two different parts apart from the previous "Abstract" and "Resumen":

- The "Report" is presented in the first part, which is organized as follows. Chapter 1 contains the introduction to the topics developed in this thesis. Chapter 2 describes the main objectives of the thesis. The discussion of results is presented in Chapter 3. Chapters 4 and 5 draw some conclusions, both in English and Spanish, respectively. Finally, at the end of this part, the relevant bibliography is included.

- The second part, "Publications", is organized as follows. Chapter 6 includes a complete copy of the presented publications, including their bibliographic references. Chapter 7 consists of a report on the impact factor of each of the presented publications. Then, Chapter 8 includes the whole list of the candidate's publications. Finally, in Chapter 9, some other publications are included due to their high relationship with the work and topics developed during this Ph.D. Thesis.

Contents

Part I

Report

Chapter 1

Introduction

There is nothing more difficult to take in hand, more perilous to conduct or more uncertain in its success than to take the lead in the introduction of a new order of things.
Niccolo Machiavelli (1469 - 1527)

Currently, new technologies allow to acquire and store vast volumes of data about different time-evolving complex phenomena in many crucial areas, like Economy, Science, and industrial processes. Examples in Economy include the evolution of every kind of economical indicators at local or global levels, like stock funds; electricity, gas or water consumption; price of basic products; etc. In Science, the amount of information collected by researchers is overwhelming and ever growing, including astronomical observations by radio telescopes, space probes, and data collected from experiments in diverse scientific fields among others. Finally, in the context of industrial applications, there are different amounts of data such as the ones related to the supply chain or those ones produced during the whole industrial process.

In order to be useful, these data must be explained in an understandable way, including facts that may be derived from data and the background knowledge available about each phenomenon under study. This objective can only be achieved by using natural language (NL), especially if the final information is going to be used by non-experts. This thesis seeks to contribute with some fundamental basis and practical tools to overcome this problem. The formulation of linguistic models can be seen as a non-trivial task, and many times, linguistic modeling contributes to a better understanding of the phenomenon, providing a novel and previously unseen view of it.

In the literature, the task of modeling complex phenomena is usually called System Identification (SI) [Sod 94, Lju 98]. It is worth noting that the concept of SI and its formal definition were introduced by Zadeh [Zad 56]. According to Zadeh's definition [Zad 62], given a class of models, SI involves finding a model which may be regarded as equivalent to the objective system with respect to input-output data. The field of

SI uses statistical methods in order to build mathematical models of dynamical systems from measured data. SI also includes the optimal design of experiments for efficiently generating informative data for fitting such models. A dynamical mathematical model in this context, is a mathematical description of the dynamic behavior of a system or process in either the time or frequency domain.

Traditionally in SI, engineers use differential equations to build white-box models based on first principles to model the behavior of real-world systems [Oga 67, Lju 98, Nel 00, Ise 09]. In this approach, engineers can choose among several paradigms to represent system models. One of the most expressive model structure is the state space representation [Oga 67, Lju 98]. In this approach, the designer must find out the necessary and sufficient subset of state variables $(x_1, x_2, \ldots x_n)$ to represent the entire state $X[t]$ of the system at the time instant t. The designer uses her/his creativity and personal experience to choose the adequate set of state variables regarding the system goals. This set of variables emphasizes the relevant aspects of the system and hides the irrelevant ones. When the system evolves in time, the current state $X[t]$ follows a trajectory in the state space. The general form of the model of a time-invariant discrete system in the state space is formulated by the following set of equations:

$$\begin{cases} X[t+1] = f(X[t], U[t]) \\ Y[t] = g(X[t], U[t]) \end{cases} \tag{1.1}$$

where:

- U is the input vector of the system: $(u_1, u_2, ..., u_{n_u})$, with n_u being the number of input variables.

- X is the state vector: $(x_1, x_2, ..., x_{n_x})$, with n_x being the number of states and $X_0 = X[t = 0]$ the initial state of the system.

- Y is the output vector: $(y_1, y_2, ..., y_{n_y})$, with n_y being the number of output variables.

- f is the function which calculates the state vector at time instant $t + 1$.

- g is the function which calculates the output vector at time instant t.

Unfortunately, for many systems in our environment, it is not feasible, or it is very costly, to obtain and to solve these equations. This situation is described by the Zadeh's Principle of Incompatibility: *"as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics"* [Zad 73]. This is to say that, when the system to be modeled grows in complexity, the number of variables and equations becomes intractable and we have no other option but to work with alternative models.

A different approach is therefore to start from measurements of the behavior of the system and the external influences (inputs to the system) and try to determine a mathematical relation between them without going into the details of what is actually happening inside the system. These models are known as black-box models. In these models, no prior knowledge is available. Examples of these models are neural networks [Hay 94] or autoregressive linear models [Sea 97, Sta 09].

An intermediate approach is the grey-box modeling, where, although the peculiarities of what is going on inside the system are not entirely known, a certain model based on both insight into the system and experimental data is constructed. Grey-box modeling is also known as semi-physical modeling. In the last forty years, there have been several attempts to deal with this type of problems by means of models based on Fuzzy Logic (FL) [Zad 73, Sug 88, Ped 93, Bar 95, Dri 96, Dri 97, Ped 99], which have become a good alternative to deal with those systems where obtaining and solving the appropriate state equations is a difficult or impossible task. Based on the Principle of Incompatibility, Zadeh suggested linguistic analysis in place of quantitative analysis. He suggested the use of the so-called linguistic variables [Zad 75a, Zad 75b, Zad 75c] instead of or in addition to numerical values, the characterization of simple relations between variables by conditional fuzzy statements, and the characterization of complex relations by fuzzy algorithms. Zadeh's original purpose in introducing the fuzzy sets [Zad 65] was to provide a tool to help in the modeling of complex phenomena, especially, but not restricted to, those involving human agents. By permitting a certain amount of imprecision in our models, we provide a robustness that allows us to model complex situations [Yag 94]. According to De Kleer [Kle 84], *"the behavior of a physical system can be discussed by the exact values of its variables (forces, velocities, positions, pressures, etc.) at each time instant. Such description, although complete, fails to provide much insight into how the system functions. Our longterm goal is to develop an alternative physics in which these same concepts are derived from far simpler, but nevertheless formal qualitative basis. Our proposal is to reduce the quantitative precision of the behavioral descriptions but retain the crucial distinctions"*. In essence, the fundamental reason for a high level approach is to provide transparent models that can be understood and used by practitioners in the relevant fields [Law 04].

Fuzzy modeling is one of the most important issues in FL and it is interpreted as a qualitative modeling scheme by which we describe the system behavior using NL [Sug 93]. On the one hand, semantic expressiveness, using linguistic variables [Zad 75a, Zad 75b, Zad 75c] and rules [Mam 74, Mam 75, Mam 77], is quite close to NL which reduces the effort of expert knowledge extraction. On the other hand, being universal approximators [Buc 93, Cas 95] fuzzy inference systems are able to perform nonlinear mappings between inputs and outputs.

During the last years, models based on FL have grown in complexity as a consequence of the modeling requirements in terms of accuracy and interpretability. The number of variables and the number of needed rules to create a fuzzy model have grown up until making models difficult to understand, and consequently, difficult to apply. Currently, researchers in the field work to establish the formalism that will make the designed fuzzy models more human friendly [Cas 03a, Cas 03b, Alo 07, Alo 08, Alo 11a, Alo 11b].

Figure 1.1 shows the relation between the topics on which the thesis is based (in yellow) and the topics produced during the development of the thesis (in green). We follow Zadeh's computing with words and perceptions paradigm [Zad 99] in order to extend the Computational Theory of Perceptions [Zad 01]. The idea consists of extending FL to create system models based on the way that humans make descriptions using NL. The aim is to make use of complex structures of NL to make robust imprecise models of complex systems. We have considered Fuzzy Finite State Machines (FFSMs) to deal with the problem of modeling the evolution in time of each specific complex phenomenon and we have developed the so-called Granular Linguistic Model of a Phenomenon (GLMP), which is the model needed to create a granular description of it. The GLMP must manage granular and imprecise information in a hierarchical fashion. Therefore, once the FFSM is able to model the evolution of each phenomenon, the GLMP will be able to produce a linguistic description about it and its evolution in time.

The theoretical foundations that established the first FFSMs were introduced by Santos [San 68] and developed by Moderson [Mor 02] among others. Our model of FFSM is inspired by the concepts of fuzzy state and fuzzy system developed by Zadeh [Zad 96a, Zad 96b]. More specifically, it can be considered an implementation of the general idea of input-output fuzzy models of dynamic systems proposed by Yager [Yag 94], where the set of state equations is implemented using a set of fuzzy rules. However, the detailed definition of each of the elements that form part of the FFSM model is a complex task for experts. Therefore, we have proposed a methodology which consists of a machine learning method to define the model parameters, this methodology is based on the hybridization of FFSMs and Genetic Algorithms (GAs) leading to Genetic Fuzzy Finite State Machines (GFFSMs). This Genetic Fuzzy System (GFS) [Cor 01, Her 08] automatically learns the fuzzy rules and membership functions of the FFSM, while an expert defines the possible states and allowed transitions between states. Our aim is that the expert can combine her/his knowledge about the phenomenon in the form of states similar to the state space approach in order to produce an accurate linguistic description of the phenomenon under study. One of the main advantages of linguistic modeling is to develop systems that incorporate the creative, abstract and adaptive attributes of a human, while minimizing the undesirable aspects such as unpredictability, inconsistency, subjectivity and temporal instability [Bro 94]. Inspired by Licklider [Lic 68], our aim is to make use of a symbiotic relationship between the user and the computer, in such a way that human motivation and creativity is strengthened by the computer's greater memory storage and higher computational performance.

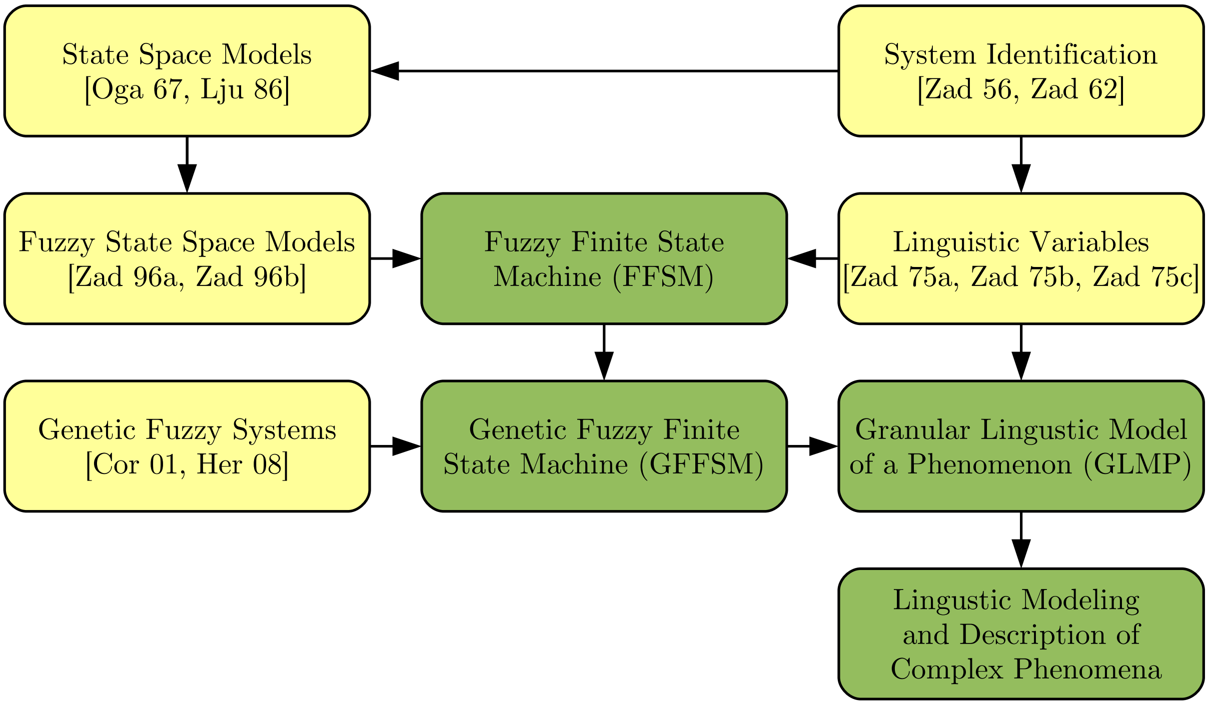

Figure 1.1: Graphical representation that shows the relation between the topics on which the thesis is based (in yellow) and the topics arising from the development of the thesis (in green).

Chapter 2

Objectives

Failure comes only when we forget our ideals and objectives and principles.
Jawaharlal Nehru (1889 - 1964)

Despite the existence of several computational models for modeling complex phenomena, a linguistic approach for developing models of complex phenomena in a hierarchical fashion is missing. Such kind of models are usually developed, and used, in a somehow ad-hoc way without the possibility of allowing the expert to add her/his knowledge or to interpret or to understand the proposed model.

To the best of our knowledge, the extensive literature on computational approaches for modeling complex phenomena concentrates in accuracy disregarding interpretability issues, and do not facilitate the expert to incorporate her/his knowledge. Therefore, we have designed the following set of objectives in order to create linguistic models of complex phenomena:

O1. Contribute to the System Identification field by developing Fuzzy State Space Models leading to the FFSM modeling paradigm.

O2. Develop a general methodology for the construction of the FFSM model based on merging expert and induced knowledge by the use of GAs, leading to the GFFSM model.

O3. Contribute to the Computational Theory of Perceptions by developing the concept of GLMP.

O4. Propose a granular and hierarchical architecture within the GLMP that allows to merge different sources of information or knowledge in combination with the expressiveness of the FFSM modeling paradigm.

O5. Generate linguistic reports of the complex phenomena under study based on the proposed linguistic models.

O6. Validate the proposed methodology with several real world applications.

In order to achieve our objectives, we have proposed a model of complex phenomena evolving in time based on the basis of a linguistic, human-guided design, plus the later incorporation of machine learning mechanisms within the FFSM modeling paradigm. Then we have introduced the FFSM paradigm in combination with the GLMP. Our hypothesis here was that FL and approximate reasoning tools are suitable for modeling complex phenomena and for incorporating expert knowledge in a grey-box modeling approach.

Chapter 3

Discussion of the results

If you want different results, do things differently.
Albert Einstein (1879 - 1955)

In this chapter, we show the results achieved during these last four years of research that demonstrate the fulfillment of all the objectives established in Chapter 2.

Figure 3.1 summarizes the results achieved. It extends the information showed in Figure 1.1, related to the topics on which the thesis was based with their main references (in yellow) and the topics developed during the thesis (in green), by including the produced publications. The publications highlighted in red are those ones presented in Chapter 6 which are the core of the thesis. Moreover, the ones remarked in blue are those additional publications included in Chapter 9. We have also indicated the objectives of the thesis, presented in Chapter 2, fulfilled by each publication.

In the following lines, we include six different sections whose headings are each one of the objectives presented in Chapter 2. Each section explains the work carried out and describes the publications produced during the development of the thesis related to each objective.

O1. Contribute to the System Identification field by developing Fuzzy State Space Models leading to the FFSM modeling paradigm

First, in order to contribute to the System Identification field by the development of the FFSM modeling paradigm, and according to this objective, we established a theoretical framework for the FFSM. In [Alv 09], we created the first formal definition of our FFSM model and applied it to the human gait modeling problem, which consists of studying the biomechanics of this human movement, and it is very difficult to obtain an accurate model due to the number of variables that take part in the process. We analyzed the accelerations produced during this phenomenon, which are quasi-periodic signals, from a linguistic and comprehensible approach.

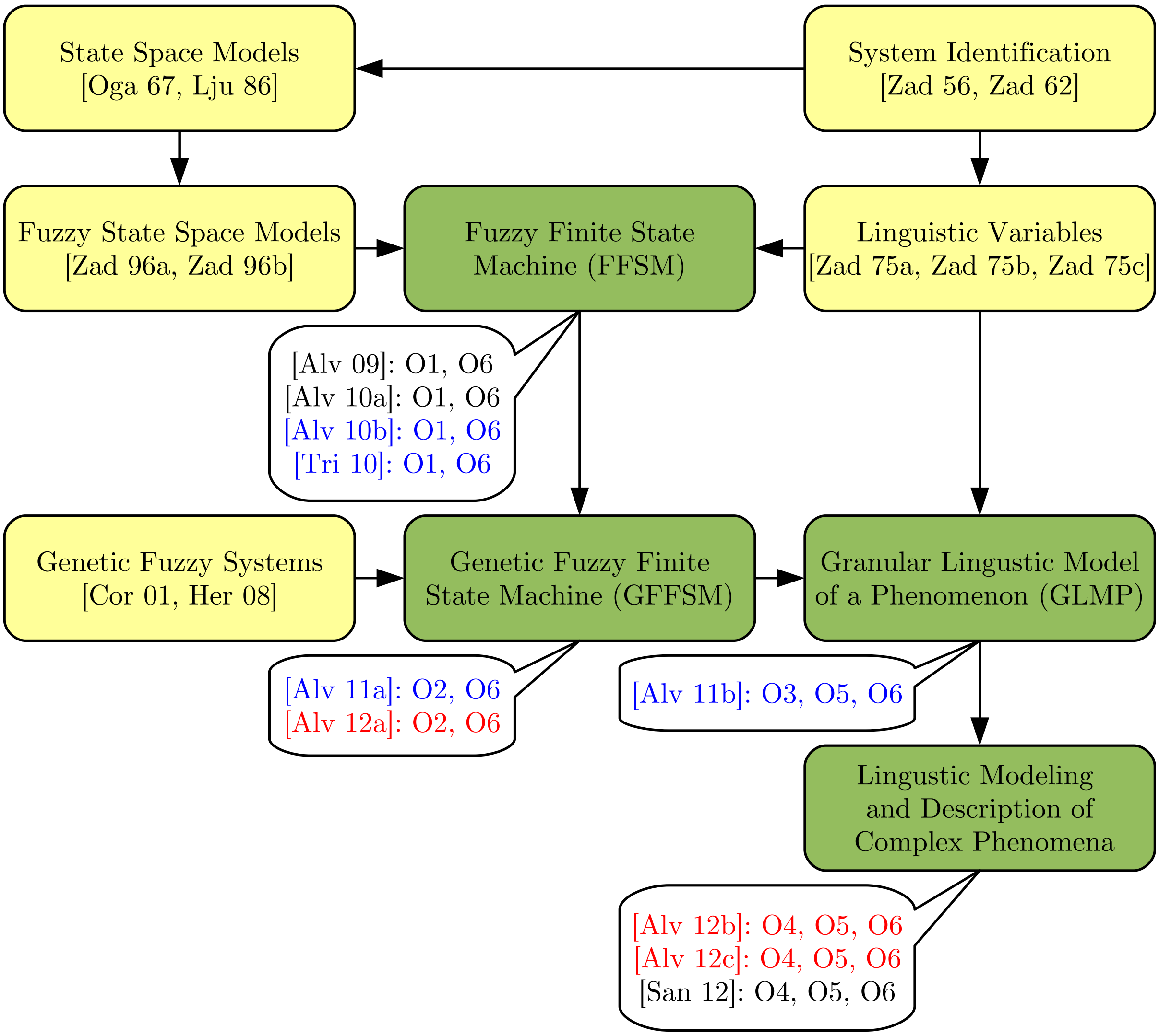

Figure 3.1: Graphical representation of the topics and publications produced during the development of the thesis.

Once we were able to model this phenomena, in [Tri 10], we explored the possibilities of distinguishing between the different phases of the gait in order to extract relevant features that serve as a biometric measure for human gait pattern recognition. The model was easily understood and provided good results, where a practical demonstration with an equal error rate of 3% was included. A complete copy of this article can be found in Chapter 9 as an additional selected publication.

Then, we proposed the application of our modeling paradigm to recognize between different activities in a working environment based on the body posture and the position of the user in the environment [Alv 10a]. This proposal materialized in a successful application that was presented in [Alv 10b], which can be found in Chapter 9 as an additional selected publication. In this article, we applied our framework for the linguistic modeling of the human activity and demonstrated its capabilities for fusing information from different sources of knowledge. Firstly, a WiFi localization system implemented as a Fuzzy Rule-based Classifier was used to obtain an approximate position at the level of discrete zones. Secondly, a FFSM was used for human body posture recognition. Finally, another FFSM combined both WiFi localization and posture recognition to obtain a robust, reliable, and easily understandable activity recognition system. We included a practical application in an office environment with real data that showed the goodness of our proposal.

O2. Develop a general methodology for the construction of the FFSM model based on merging expert and induced knowledge by the use of GAs, leading to the GFFSM model

The common characteristic in all of the previous works was that all the FFSM models were based on expert knowledge, i.e., the states, allowed transitions, membership functions and fuzzy rules that represented the modeled phenomena were defined by the expert; thus making the design of new models a difficult and tedious task. Therefore, and according to this second objective, we decided to incorporate machine learning mechanisms in the FFSM modeling paradigm in order to merge expert and induced knowledge. This goal was achieved by means of GAs, leading to the concept of GFFSM.

In [Alv 12a], which is the first publication presented in Chapter 6, was the first work where we introduced machine learning capabilities to our FFSM modeling tool in order to improve its accuracy keeping its interpretability due to the fact that the number of states and allowed transitions were defined by the expert. In this case, we designed a GFFSM for the human gait modeling problem.

The first part of the article motivated the use of linguistic modeling for complex phenomena. Then, it presented all the theoretical details of our FFSM modeling paradigm, highlighting their usefulness to model dynamical processes which change in time, becoming an extension of classical finite state machines (FSMs), but with the main advantage that their fuzziness allows them to handle imprecise and uncertain data, which are inherent to real-world phenomena. Then, it explained how the definition of details of the FFSM model in each particular case is a complex task for experts and motivated the development of a general methodology for the construction of the FFSM model based on both expert and induced knowledge leading to the concept of GFFSM. This new GFS automatically learned the fuzzy rules and membership functions of the FFSM, while an expert defined the possible states and allowed transitions between states.

Our final goal was to obtain a specific model for each person's gait in such a way that it can generalize well with different gaits of the same person. The obtained model has become an accurate and human friendly linguistic description of this phenomenon, with the capability of identifying the relevant phases of the process, which were the states of the FFSM.

This article included a complete experimentation to test the performance of our proposal when dealing with datasets of 20 different people. It comprised a detailed analysis of results, which discussed our proposal in terms of accuracy, interpretability, computational cost, and importance of the use of expert knowledge. Our proposal was also compared against other state of the art technologies for modeling these types of complex phenomena, namely neural networks and autoregressive linear models. Therefore, we have concluded that the GFFSM model presented in this article constituted an innovative application of fuzzy set theory since it was the first time that GAs were used to design a FFSM; and it outperformed other standard computational intelligence techniques for modeling complex phenomena, by allowing us to produce a linguistic description of the human gait while identifying the relevant phases of the process with an accurate and human friendly model which allowed the designer to introduce her/his own knowledge about the phenomenon.

Once we showed the accuracy improvement of our GFFSM model while keeping its interpretability, in [Alv 11a], we explored the possibility of applying this model for modeling the body posture in such a way that the final model was an accurate and human friendly linguistic description of this phenomenon, with the capability of identifying the posture of the user accurately. A complete experimentation was developed to test the performance of this proposal in terms of accuracy and interpretability. This article was awarded with the IEEE Computational Intelligence Society 2011 Outstanding Paper Award in the 5th IEEE International Workshop on Genetic and Evolutionary Fuzzy Systems held in Paris (France). A complete copy of this article can be found in Chapter 9 as an additional selected publication.

O3. Contribute to the Computational Theory of Perceptions by developing the concept of GLMP

Once we were able to model complex phenomena based on expert and induced knowledge, we introduced the concept of GLMP. This model contributed to the Computational Theory of Perceptions and allowed us to interpret the input data and manage granular and imprecise information in a hierarchical fashion.

In a first approach developed in [Alv 11b], we designed a simple GLMP and explained the elements that form part of it, namely the concept of computational perception (CP) and the concept of perception mapping (PM). A CP is the computational model of a unit of information acquired by the designer about the phenomenon to be modeled, it corresponds to particular details of the phenomenon at different degrees of granularity (from basic CPs that are the numerical input data to complex CPs that describes the phenomenon evolution). The PM is the tool used to create and aggregate CPs.

In this article, we applied our GLMP for the linguistic description about an static phenomenon, which consisted of the linguistic modeling and description about relevant features of the Mars' Surface. This article was motivated by the existence of tens of Satellites in the orbit of Mars planet that provide us with thousands of images of its surface. Typically, these images are analyzed by experts that select the relevant features and generate textual reports containing the result of their observations. However, the number of images is increasing and this procedure is not effective enough. Therefore, we created a computational application able to generate simple linguistic descriptions of circular structures on the Mars' Surface, including several examples and analysis of the obtained results. This article was awarded with the Best Special Session Student Paper Award in the 11th International Conference on Intelligent Systems Design and Applications held in Córdoba (Spain). A complete copy of this article can be found in Chapter 9 as an additional selected publication.

O4. Propose a granular and hierarchical architecture within the GLMP that allows to merge different sources of information or knowledge in combination with the expressiveness of the FFSM modeling paradigm

Once we introduced the concept of GLMP and tested it in a real application, we combined this granular and hierarchical architecture with the expressiveness of the FFSM modeling paradigm.

First, in the second publication presented in Chapter 6 [Alv 12c], we designed a GLMP to create a basic linguistic model of the human gait quality, we did this by using our FFSM model as a PM within the GLMP. The FFSM model was in charge of identifying the phases of each gait, while the GLMP created a hierarchical structure where NL concepts such as symmetry and homogeneity of the gait were created based on basic CPs associated to the gait phases. In summary, this paper presented important results in the field of developing computational systems able to model and generate linguistic descriptions of complex phenomena. It also showed how the new version of GLMP including a FFSM is an expressive tool to represent the behavior of this type of phenomenon in a human friendly way.

Once we checked that the combination of the GLMP and the FFSM worked in the field of human gait modeling and gait quality assessment, we explored the possibility of applying this same combination in a completely different field: the field of intelligent transportation systems. In this field, one important challenge consists of maintaining updated the electronic panels installed in roads with relevant information expressed in NL. Moreover, it also includes the problem of generating linguistic reports to assist traffic managers that must take their decisions based on large amounts of quickly evolving information.

In the third publication presented in Chapter 6 [Alv 12b], we were not only able to model a phenomenon as complex as the traffic in a road, but we were also capable of facing the challenge of modeling its evolution in time following the steps previously presented in [San 12]. In this third publication, we introduced our developments to create a basic GLMP of the traffic evolution in roads based on the traffic density and the speed of the vehicles. These data were obtained from video signals produced by cameras installed in the scenario under study. We used this model to generate a human friendly linguistic description of this phenomenon focused on describing the traffic evolution, where the FFSM model was a PM within the GLMP that modeled the level of service in a road. Moreover, in order to model the evolution of the level of service, we focused on the perception of change and explored possibilities to perform linguistic descriptions of how the traffic evolves in time. We researched on how to model the meaning of sentences such as *"the phenomenon is changing from state A to state B"*. In order to model the evolution of phenomena in time, we used our previous works on FFSMs and we extended the use of the FFSM's output function to be used with this aim. Our perception of temporal evolution of phenomena was modeled by means of three different types of CPs, namely, the perception of the current state (assertive CP), the perception of the trend to evolve (derivative CP) and the summary of accumulated perceptions (integrative CP). The assertive CP was associated with a linguistic expression of the current state of the phenomenon, e.g., "the traffic density is high". The derivative CP corresponded to trend analysis information and gives insight into how the phenomenon is evolving in time, e.g., "the traffic density is decreasing". Finally, the integrative CP represented the accumulated perception of the phenomenon over a period of time, e.g., "the traffic density during the last two hours has been low".

Therefore, this paper contributed to the field of developing computational systems able to model and generate linguistic descriptions of complex phenomena by introducing the perception of change. We showed how the FFSM used as a PM inside the GLMP was not only an expressive tool to represent the level of service in a road in a human friendly way but it was also capable of modeling its evolution in time. Moreover, the introduction of the three different types of CPs allowed us to differentiate between the current state of the phenomenon, its trend, and its evolution over a certain period of time.

O5. Generate linguistic reports of the complex phenomena under study based on the proposed linguistic models

Thanks to the expressiveness of our GLMP model, we could produce linguistic reports in NL about complex phenomena under study thus fulfilling this objective. As explained above, first, we were able to create linguistic descriptions of circular structures on the Mars' Surface [Alv 11b]. Then, we created a basic linguistic model of the human gait, and we used this model to generate a human friendly linguistic description of this phenomenon focused on the assessment of the gait quality [Alv 12c]. Finally, we were able to model and create linguistic summaries about the traffic evolution in roads [Alv 12b, San 12].

O6. Validate the proposed methodology with several real world applications

This objective was fulfilled in the majority of the works and publications developed during this Ph.D. Thesis due to the practical nature of the problems addressed. Moreover, the generality of our proposal was demonstrated by showing how it works in several completely different fields such as intelligent transportation systems, human gait modeling or activity recognition, among others.

Specifically, the three publications presented in Chapter 6 have a strong practical component since all the data that took part in the experimentation were collected using accelerometer sensors (in the case of the human gait) or video cameras (in the case of the traffic) in a real world scenario.

The work related to the first article presented in Chapter 6 [Alv 12a] included a big experimental phase where we collected accelerometer data from twenty different people, where ten different datasets of ten complete gait cycles were collected for each person resulting in a total of 200 different datasets.

The second article presented in Chapter 6 [Alv 12c], includes a real world practical application where we analyzed the gait quality of healthy individuals and people with lesions in their limbs (knee and ankle) before and after their lesions.

Finally, in the experimental part of the third article presented in Chapter 6 [Alv 12b] we used digital image processing techniques to obtain real data from the video cameras installed in the road. Moreover, in order to analyze all the possible situations in a road, we developed a simulator based on the Monte Carlo method, where simulated data followed a normal distribution that was defined by the mean and the standard deviation parameters. These parameters were modified depending on the period of the day in such a way that we were able to generate data that recreates the traffic behavior in all of the possible different situation types. The results showed how our proposal was able to model and describe linguistically the traffic evolution in a real road and with the simulated data.

Chapter 4

Conclusions and future works

You can know the name of a bird in all the languages of the world, but when you're finished, you'll know absolutely nothing whatever about the bird. So, let's look carefully at the bird and let's learn something about it.
Richard Feynman (1918 - 1988)

During the development of this Ph.D. Thesis we have faced the challenge of proposing a framework for linguistic modeling of complex phenomena. We have explored to what extent the existing models are just black-box models and how this choice affects the interpretability and usability of the model. Therefore, we have reviewed the current models and existing technologies for modeling complex phenomena. Then, we have motivated the use of linguistic modeling by showing its advantages based on FL and approximate reasoning tools, which are suitable approaches for modeling complex phenomena and for incorporating expert knowledge in a grey-box modeling approach.

We have fulfilled all the objectives established in Chapter 2. First, we have contributed to the System Identification field by developing Fuzzy State Space Models leading to the FFSM modeling paradigm, highlighting their applicability to model dynamical processes which change in time. We have also developed a general methodology for the design of the FFSM model based on merging expert and induced knowledge by the use of GAs, leading to the GFFSM model. We have also contributed to the Computational Theory of Perceptions by developing the concept of GLMP in order to merge different sources of information or knowledge in combination with the expressiveness of the FFSM modeling paradigm. Then, we have used the expressiveness of the GLMP model to generate linguistic reports of the complex phenomenon under study. Finally, we have validated the proposed methodology with several real world applications. We have been able to model the human gait using our linguistic modeling approach by merging expert and induced knowledge. Then, we have developed a system capable of modeling the human gait quality and producing linguistic descriptions about it. We have also showed how our proposal works in the field of intelligent transportation systems, where we presented successful results in the field of developing computational systems able to model and generate linguistic descriptions of complex phenomena evolving in time by introducing the perception of change.

In the current stage of development, we have been able to model and generate linguistic descriptions that correspond to the context of a laboratory experimental setup. In future projects, we will deep into the specific application field in order to improve the meaning and, therefore, the usability of these texts. We will apply these results to generate expressions close to NL in the context of each specific application, e.g., to assess the risk of falling in elderly people, to monitor the recovery process in physiotherapy, and to set up a complete system for monitoring and control the traffic in a real world scenario. From the theoretical point of view, we will continue exploring how to model the meaning of different linguistic expressions that will allow us to model and describe complex phenomena from different perspectives that do not only include the validity of the NL expression, e.g., the truthfulness, relevance or importance of the current state of the phenomenon under study.

The promising results obtained during the development of this thesis have pushed us to the development of an exciting entrepreneurial project: the creation of a spin-off promoted also from the European Centre for Soft Computing. The business idea of this spin-off consists of offering the possibility of creating linguistic descriptions of data and complex phenomena using the techniques and models developed during this thesis in order to facilitate their interpretation by individuals as well as the development of commercial applications that can interact and communicate with users in NL. Currently, we are working on the first prototype which consists of an Android application installed in a smartphone and a server that implement a personal trainer that summarizes the user's daily activity. This application will measure the accelerometer data of the user provided by the sensors embedded in the smartphone, then, it will communicate with the server and produce a summary in NL on the progression and performance of daily physical activity of this user.

Chapter 5

Conclusiones y trabajo futuro

Tú puedes saber el nombre de un pájaro en todos los idiomas del mundo, pero cuando hayas terminado, no sabrás absolutamente nada sobre el pájaro. Por tanto, miremos atentamente a ese pájaro y aprendamos algo de él.
Richard Feynman (1918 - 1988)

Durante el desarrollo de esta Tesis Doctoral nos hemos enfrentado al desafío de proponer un marco para el modelado lingüístico de fenómenos complejos, hemos explorado hasta qué punto los modelos existentes son sólo modelos de caja negra y cómo esta elección afecta a la interpretabilidad y usabilidad del modelo. Por lo tanto, hemos revisado los modelos actuales y las tecnologías existentes para el modelado de fenómenos complejos. A continuación, hemos motivado el uso de los modelos lingüísticos, mostrando sus ventajas basadas en la Lógica Borrosa y el razonamiento aproximado, que son herramientas adecuadas para la modelización de fenómenos complejos mediante la incorporación de conocimiento experto en un enfoque de de caja gris.

Hemos cumplido todos los objetivos establecidos en el Capítulo 2. En primer lugar, hemos contribuido en el campo de la Identificación de Sistemas mediante el desarrollo de Modelos Borrosos en el Espacio de Estados que nos llevan al paradigma de modelado de las Máquinas de Estados Finitos Borrosos, destacando su utilidad para modelar los procesos dinámicos que evolucionan en el tiempo. También hemos desarrollado una metodología general para la construcción de las Máquinas de Estados Finitos Borrosos basado en la fusión de concimiento experto e inducido mediante el uso de los Algoritmos Genéticos, que conduce al modelo de Máquina Genética de Estados Finitos Borrosos. También hemos contribuido a la Teoría Computacional de Percepciones mediante el desarrollo del concepto de Modelo Lingüístico Granular de un Fenómeno con el fin de combinar diferentes fuentes de información o conocimiento, en combinación con la expresividad del paradigma de modelado de las Máquinas de Estados Finitos Borrosos. A continuación, hemos utilizado la expresividad del Modelo Lingüístico Granular de un Fenómeno para generar informes lingüísticos acerca del fenómeno complejo que se estudia. Finalmente, se ha validado la metodología propuesta con varias aplicaciones del mundo real. Hemos sido capaces de modelar la marcha humana con nuestro enfoque de modelado lingüístico mediante conocimiento experto y conocimiento inducido. A continuación, hemos desarrollado un sistema capaz de modelar la calidad de la marcha y producir descripciones lingüísticas sobre la misma. También hemos mostrado cómo nuestra propuesta se puede aplicar en el campo de los Sistemas Inteligentes de Transporte, donde se presentan resultados importantes en el campo de desarrollo de sistemas computacionales capaces de modelar y generar descripciones lingüísticas de los fenómenos complejos que evolucionan en el tiempo mediante la introducción de la percepción asociada al cambio de un fenómeno.

En la etapa actual de desarrollo, hemos sido capaces de modelar y generar descripciones lingüísticas que corresponden a un contexto de laboratorio. En futuros proyectos, profundizaremos en cada campo específico de aplicación con el fin de mejorar el significado y, por lo tanto, la utilidad de estos textos. Vamos a aplicar estos resultados para generar expresiones en lenguaje natural en el contexto específico de cada aplicación, por ejemplo, para evaluar el riesgo de caídas en personas de edad avanzada, para supervisar el proceso de recuperación en fisioterapia, y la creación de un sistema completo para el seguimiento y control del tráfico en un escenario real. Desde el punto de vista teórico, vamos a seguir estudiando la forma de modelar el significado de diferentes expresiones lingüísticas que nos permitirán modelar y describir fenómenos complejos desde diferentes perspectivas que no sólo incluyen el grado de validez de la expresión en lenguaje natural, sino también otros como la veracidad, la relevancia o la importancia del estado actual del fenómeno bajo estudio.

Los prometedores resultados obtenidos durante la elaboración de esta tesis nos han llevado a desarrollar un proyecto empresarial ilusionante: la creación de una spin-off promovida también por el European Centre for Soft Computing. La idea de negocio de esta spin-off consiste en ofrecer la posibilidad de crear descripciones lingüísticas de los datos y fenómenos complejos utilizando las técnicas y modelos desarrollados en esta tesis con el fin de facilitar su interpretación por parte de las personas, así como el desarrollo de aplicaciones comerciales que puedan interactuar y comunicarse con los usuarios en lenguaje natural. En la actualidad, estamos trabajando en el primer prototipo, que consiste en una aplicación para Android instalada en un smartphone y un servidor que implementan un entrenador personal que resumen la actividad diaria del ususario. Esta aplicación medirá los datos de las aceleraciones del usuario proporcionadas por los sensores integrados en el smartphone, a continuación, se comunicará con el servidor y producirá un resumen en lenguaje natural con la progresión y el rendimiento de la actividad física diaria del usuario.

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Part II

Publications

Chapter 6

Presented publications

Better to write for yourself and have no public, than to write for the public and have no self.
Cyril Connolly (1903 - 1974)

This chapter contains a complete copy of the presented publications. It is divided into three different Sections corresponding to each article together with their bibliographic reference.

6.1 Human gait modeling using a genetic fuzzy finite state machine

A. Alvarez-Alvarez, G. Trivino, and O. Cordón. "Human Gait Modeling Using a Genetic Fuzzy Finite State Machine". *Fuzzy Systems, IEEE Transactions on*, Vol. 20, No. 2, pp. 205–223, April 2012.

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Human Gait Modeling Using a Genetic Fuzzy Finite State Machine

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***Abstract*—Human gait modeling consists of studying the biomechanics of this human movement. Its importance lies in the fact that its analysis can help in the diagnosis of walking and movement disorders or rehabilitation programs, among other medical situations. Fuzzy finite state machines can be used to model the temporal evolution of this type of phenomenon. Nevertheless, the definition of details of the model in each particular case is a complex task for experts. In this paper, we present an automatic method to learn the model parameters that are based on the hybridization of fuzzy finite state machines and genetic algorithms leading to genetic fuzzy finite state machines. This new genetic fuzzy system automatically learns the fuzzy rules and membership functions of the fuzzy finite state machine, while an expert defines the possible states and allowed transitions. Our final goal is to obtain a specific model for each person's gait in such a way that it can generalize well with different gaits of the same person. The obtained model must become an accurate and human friendly linguistic description of this phenomenon, with the capability to identify the relevant phases of the process. A complete experimentation is developed to test the performance of the new proposal when dealing with datasets of 20 different people, comprising a detailed analysis of results, which shows the advantages of our proposal in comparison with some other classical and computational intelligence techniques.**

***Index Terms*—Fuzzy finite state machines, fuzzy systems, genetic algorithms (GAs), genetic fuzzy systems, human gait modeling.**

I. Introduction

HUMAN gait modeling consists of studying the biomechanics of this human movement and can help in the detection of gait disorders, identification of balance factors, and assessment of clinical gait interventions and rehabilitation programs [1]. Typically, in human gait modeling there are a large number of variables such as height, limb length, walking speed, acceleration along axes, foot forces, etc., which are obtained by means of different measurement techniques, thus making the obtaining of an accurate model a very complex task.

Traditionally in system identification, engineers use differential equations to model the behavior of real-world systems (white-box models) [2]–[5]. However, when the system grows in complexity, the number of variables and equations becomes intractable. In the last 40 years, fuzzy logic (FL)-based models [6]–[9] have become a good alternative to deal with those systems, where to obtain the appropriate differential equations is difficult or impossible.

FL is widely recognized for its ability for linguistic concept modeling and its use in system identification. On the one hand, semantic expressiveness, using linguistic variables [10]–[12] and rules [13], is quite close to natural language (NL), which reduces the effort of expert knowledge extraction. On the other hand, being universal approximators [14] fuzzy inference systems are able to perform nonlinear mappings between inputs and outputs. Thanks to these advantages, FL has been successfully applied in classification [15], [16], regression [17], [18], control [7], [19], [20], and system modeling [8], [9] achieving a good interpretability accuracy tradeoff.

Fuzzy finite state machines (FFSMs) are specially useful tools to model dynamical processes which change in time, becoming an extension of classical finite state machines (FSMs) [21], [22]. The main advantage of FFSMs is that their fuzziness allows them to handle imprecise and uncertain data, which are inherent to real-world phenomena, in the form of fuzzy states and transitions. The theoretical basics of FFSMs were established in [23] and later developed in [24]–[26]. In previous studies, we have learned that FFSMs are suitable tools to model signals that follow an approximately repetitive pattern. In [27], we explored the possibilities to use an FFSM to create the linguistic description of the temporal evolution of a signal by using a skin conductivity meter and accelerometers to model the activity of a person. Once we had checked the ability of FFSMs to deal with temporal data, we analyzed the chance to consider FFSMs for pattern recognition tasks such as human gait recognition [28], [29] and gesture recognition [30]. Finally, in [31], we used an FFSM to fuse information related to body posture and WiFi positioning [32], which consists of recording and processing signal strength information of WiFi networks to obtain the estimated position in indoor environments.

As any fuzzy system, FFSMs require the definition of a knowledge base (KB). It is well known that this is a complex task for experts as it was the case in the previous applications of FFSMs. In addition, the dynamic nature of FFSMs increases the complexity of the process. For this reason, in this contribution we consider the design of an automatic learning method for the fuzzy KB of FFSMs. In particular, we will take the use of genetic algorithms (GAs) [33] as a base, which have proven

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largely their effectiveness and efficiency for this task during the last two decades in the so-called genetic fuzzy systems (GFSs) area [34]–[38].

In our approach, the fuzzy states and transitions will still be defined by the expert in order to keep the knowledge that she/he has over the whole system, while the fuzzy rules and membership functions (MFs) that regulate the state changes will be derived automatically by the GFS, making a robust, accurate, and human friendly model, which is called genetic FFSM from now on. In addition, the use of this expert knowledge and the prefixed structure of the FFSM allows us to learn only the MFs and part of the rules to build its fuzzy KB, dealing with a reduced search space.

In the presented application, we show how the expert can combine her/his knowledge about human gait dynamics with the numerical data of the acceleration signals in order to produce an accurate linguistic description of the phenomenon. According to Licklider [39], our aim is to create a symbiotic relationship between the user and the computer in such a way that human motivation and creativity are strengthened by the computer's greater memory storage and higher computational performance.

In the experimental phase, we have worked with gaits of 20 different people. Regarding to the human gait modeling problem, the goal is to obtain a specific model (FFSM) for each person in such a way that this FFSM can generalize well with different gaits of the same person. Each FFSM will be composed of a small set of linguistic fuzzy IF-THEN rules in the transition function producing a linguistic description of the gait of this person while identifying the relevant states of the model. The design of the FFSM will be tackled in an automatic fashion by the proposed GFS. The performance of the obtained FFSMs will then be benchmarked against other system identification approaches.

To our mind, this research constitutes an innovative application of fuzzy set theory since, to the best of our knowledge, i) it is the first time that GAs are used to design an FFSM, and thus, it is also the first time that human gait modeling is tackled by means of an intelligent system of this kind, and ii) it outperforms other standard and nonfuzzy computational intelligence techniques, allowing us to produce a linguistic description of the human gait while identifying the relevant phases of the process with an accurate and human friendly model.

The remainder of this paper is organized as follows. Section II presents the human gait modeling problem. Section III describes how to use FFSMs to model the temporal evolution of a phenomenon. Section IV explains how to build FFSMs to model the human gait. The automatic method to learn the fuzzy KB of these FFSMs based on GAs is presented in Section V. Section VI describes the experimentation carried out, comparing the obtained results with other system identification tools. Finally, Section VII draws some conclusions and introduces some future research works.

II. Human Gait Modeling

Human gait modeling consists of studying the biomechanics of this human movement aimed to quantify factors that govern the functionality of the lower extremities. Gait is a complex integrated task that requires precise coordination of the neural and musculoskeletal system to ensure correct skeletal dynamics [40]. Therefore, its analysis can help in the diagnosis and treatment of walking and movement disorders, identification of balance factors, and assessment of clinical gait interventions and rehabilitation programs [1], [41].

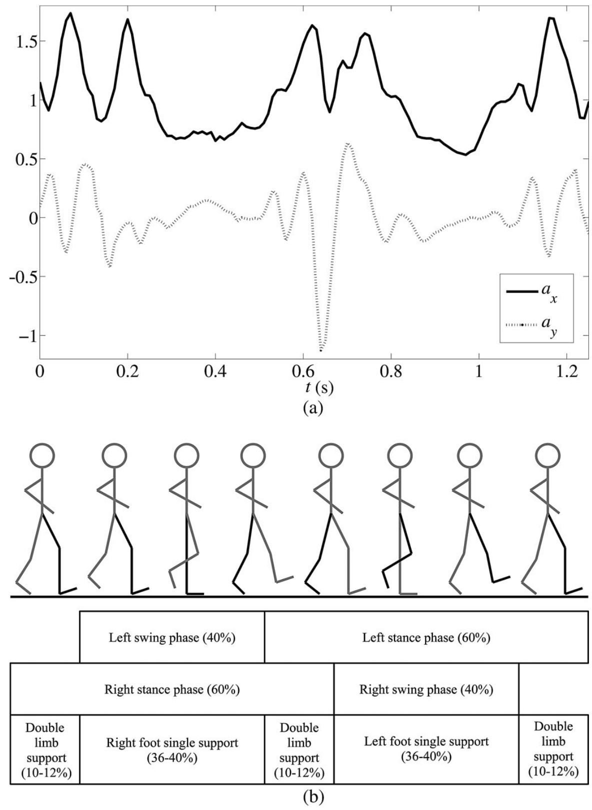

Fig. 1. One gait cycle illustrating various phases and events and the dorsoventral a_x and mediolateral a_y accelerations.

The gait cycle is a periodical phenomenon which is defined as the interval between two successive events (usually heel contact) of the same foot [42]. It is characterized by a stance phase (60% of the total gait cycle), where at least one foot is in contact with the ground, and a swing phase (40% of the total gait cycle), during which one limb swings through the next heel contact (see Fig. 1). These phases can be quite different between individuals but when normalized to a percentage of the gait cycle they maintain close similarity, indicating the absence of disorders [43].

Typically, in human gait modeling there are a large number of variables that are obtained by means of different measurement techniques. Most gait parameters can be categorized as anthropometric data which include height, weight, or limb length; spatiotemporal data that comprise variables such as walking speed, step length, or phases times; kinematic data of measurements of joint angles, displacement, or acceleration along axes; kinetic data variables that include foot force and torques; or electromyographic data which measure the muscle activation levels.

Our approach consists of identifying the relevant phases of the gait based on the accelerations that are produced during the process, i.e., we will develop human gait modeling by means of kinematic data. We have measured the accelerations using an accelerometer which is placed in the waist and centered in the back of the person that provides us with the dorsoventral acceleration a_x, the mediolateral acceleration a_y, and the anteroposterior acceleration a_z at each instant of time. In this contribution, we only use a_x and a_y because a_z has to do with the walking speed, and this speed can vary for the same person.

Fig. 1 shows three different synchronized pictures. The first one (at the top) illustrates the dorsoventral acceleration a_x and the mediolateral acceleration a_y that are obtained from the three-axial accelerometer. The middle picture plots a sketch of a person who represent the different phases of the gait with the right limb boldfaced. Finally, the picture at the bottom represents the time period from one event (usually initial contact) of one foot to the subsequent occurrence of initial contact of the same foot.

III. Fuzzy Finite State Machines

In system identification, designers can choose among several paradigms to represent system models. One of the more expressive model structures is the state space representation [2]. In this approach, the designer must find out the necessary and sufficient subset of state variables $(x_1, x_2, \ldots x_n)$ to represent the entire state $X[t]$ of the system at the time instant t.

The designer uses her/his creativity and personal experience to choose the adequate set of state variables regarding the system goals. This set of variables emphasizes the relevant aspects of the system and hides the irrelevant ones.

When the system evolves in time, the current state $X[t]$ follows a trajectory in the state space. The general form of the model of a time-invariant discrete system in the state space is formulated by the following set of equations [2]:

$$\begin{cases} X[t+1] = f(X[t], U[t]) \\ Y[t] = g(X[t], U[t]) \end{cases} \tag{1}$$

where
1) U is the input vector of the system: $(u_1, u_2, \ldots, u_{n_u})$, with n_u being the number of input variables;
2) X is the state vector $(x_1, x_2, \ldots, x_n)$, with n being the number of states, and X_0 is the initial state of the system, i.e., $X_0 = X[t = 0]$;
3) Y is the output vector: $(y_1, y_2, \ldots, y_{n_y})$, with n_y being the number of output variables;
4) f is the function that calculates the state vector at time step $t + 1$;
5) g is the function that calculates the output vector at time step t.

Unfortunately, for many systems in our environment, we are unable, or it is very costly, to obtain the differential equations corresponding to the functions f and g. This situation is described by Zadeh's Principle of Incompatibility: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics" [6].

This is to say that, when systems to be modeled grow in complexity, we have no other option but to work with imprecise models. There have been several attempts to deal with this type of problems by means of linguistic fuzzy models, which are models where at least one variable is fuzzy [44].

During the last few years, FL-based models have grown in complexity as a consequence of the modeling requirements in terms of accuracy and interpretability. The number of variables and the number of needed rules to create a fuzzy model have grown up until making models difficult to understand and, consequently, difficult to apply. Currently, researchers in the field work to establish the formalism that will make the designed fuzzy models more human friendly [45]–[49].

In this paper, we follow Zadeh's computing with words and perceptions paradigm [50]. The idea consists of extending FL to create system models based on the way that humans make descriptions using NL. The aim is the use of complex structures of NL to make robust imprecise models of complex systems.

As said, we will consider an FFSM to deal with the human gait modeling problem. The initial concept of FFSM was introduced by Santos [23] and developed by different authors (see, e.g., [24]). This family of FFSMs was characterized by having fuzzy states but crisp inputs. Later, this initial model was extended to have fuzzy inputs [25], [26]. Although the basic concept of FFSM that is used in this paper is much related to the latter one, the initial conception is quite different. The model of FFSM presented is inspired by the concepts of the fuzzy state and fuzzy system developed by Zadeh [51], [52]. More specifically, it can be considered an implementation of the general idea of *input–output fuzzy models of dynamic systems* proposed by Yager [53], where the set (1) is implemented using sets of fuzzy rules. In addition, we focus our contribution on the practical challenge to develop a mechanism to learn automatically the set of rules and MFs of the FFSM.

In this section, we introduce the main concepts and elements of our paradigm for system modeling allowing experts to build comprehensible linguistic fuzzy models in an easier way. In our framework, an FFSM is a tuple $\{Q, U, f, Y, g\}$, where
1) Q is the set of states of the system;
2) U is the set of input vectors of the system;
3) f is the transition function that calculates the set of states of the system;
4) Y is the set of output vectors of the system;
5) g is the output function that calculates the set of output vectors of the system.

Each of these components is described in detail in the following sections. See [28], [29], and [31] to find several previous applications of this FFSM model.

A. Fuzzy States (Q)

Q is the set of states of the system, which is defined as a linguistic variable [10]–[12] that takes its values in the set of linguistic labels $\{q_1, q_2, \ldots, q_n\}$, with n being the number of fuzzy states. Every fuzzy state represents the pattern of a repetitive situation. The concept of fuzzy state was introduced by Zadeh in [6]. Numerically, the state of the FFSM is represented by a state activation vector:

$S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$, where $s_i[t] \in [0, 1]$.

S_0 is defined as the initial value of the state activation vector, i.e., $S_0 = S[t = 0]$. The FFSM implementation verifies $\sum_{i=1}^{n} s_i[t] = 1$ in such a way that we maintain compatibility with classical FSMs where only one state can be activated with degree 1 at each time instant. Hence, in order to maintain the latter characteristic in our FFSM model, the activation degree of the states must sum up to 1 for any system input. This restriction has been applied in previous fuzzy extensions of crisp phenomena such as fuzzy clustering [54], [55], where the sum of the membership value of a pattern to different clusters must also equal to 1. This decision of design is easily implemented using (2) as is shown in Section III-C1.

B. Input Vectors (U)

U is the set of input vectors: $(u_1, u_2, \ldots, u_{n_u})$, with n_u being the number of input variables. U is a set of linguistic variables that are obtained after fuzzification of numerical data. Typically, u_i can be directly obtained from sensor data or by applying some calculations to the raw measures, e.g., the derivative or integral of the signal, or the combination of several signals.

The expert summarizes the domain of numerical values representing them by a set of linguistic labels which define all the possible values that u_i can take.

$A_{u_i} = \{A^1_{u_i}, A^2_{u_i}, \ldots, A^{n_i}_{u_i}\}$, with n_i being the number of linguistic labels of the linguistic variable u_i.

C. Transition Function (f)

f is the state transition function that calculates, at each time instant, the next value of the state activation vector: $S[t+1] = f(U[t], S[t])$.

This function is implemented by means of a fuzzy KB. Once the expert has identified the relevant states in the model, she/he must define the fuzzy rules that govern the temporal evolution of the system among these states.

We can distinguish between rules R_{ii} to remain in a state q_i, and rules R_{ij} to change from state q_i to state q_j. Fuzzy rules will only be associated with allowed transitions, i.e., if a transition is forbidden in the FFSM, it will have no fuzzy rules associated.

A generic expression of a rule R_{ij} is formulated as follows.

R_{ij}: **IF** ($S[t]$ is q_i) **AND** C_{ij} **THEN** $S[t+1]$ is q_j

where we have the following.

1) The first term in the antecedent ($S[t]$ is q_i) involves the computation of the degree of activation of the state q_i in the time instant t, i.e., $s_i[t]$. With this mechanism, we only allow the FFSM to change from the state q_i to the state q_j (or to remain in the state q_i, when $i = j$) if it was previously in that state.
2) The second term in the antecedent C_{ij} describes the constraints imposed on the input variables that are required to change from the state q_i to the state q_j (or to remain in state q_i, when $i = j$). For example, $C_{ij} = (u_1[t]$ is $A^3_{u_1})$ **AND** $(u_2[t]$ is $A^4_{u_2}$ **OR** $A^5_{u_2})$[1].
3) Finally, the consequent of the rule is the next value of the state activation vector $S[t+1]$. It consists of a vector with a zero value in all of its components but in $s_j[t]$, where it takes value 1.

1) Fuzzy Reasoning Mechanism: The next value of the state activation vector is calculated as a weighted average of the individual rules. The weight of a rule k is calculated from its firing degree ω_k. To calculate the value of ω_k, we use the minimum *t*-norm ($\top_{\min}(a, b) = \min\{a, b\}$) for the AND operator and the bounded sum of the Łukasiewicz *t*-conorm ($\bot_{\text{Luk}}(a, b) = \min\{a + b, 1\}$) for the OR operator [60], e.g., the constraint $C_k = (u_1$ is $A^3_{u_1})$ **AND** $(u_2$ is $A^4_{u_2}$ **OR** $A^5_{u_2})$ will produce a firing degree $\omega_k = \min\{A^3_{u_1}(u_1), \min\{1, A^4_{u_2}(u_2) + A^5_{u_2}(u_2)\}\}$.

As we have explained earlier, a certain output of a rule k predicting state q_i will be of the form $(0, \ldots, s_i[t] = 1, \ldots, 0)_k$. To calculate the total output of the rules and therefore, the state activation vector ($S[t+1]$), a weighted average of the individual outputs of each rule is computed as defined in

$$S[t+1] = \begin{cases} \dfrac{\sum_{k=1}^{\#\text{Rules}} \omega_k \cdot (s_1, \ldots, s_n)_k}{\sum_{k=1}^{\#\text{Rules}} \omega_k} & \text{if } \sum_{k=1}^{\#\text{Rules}} \omega_k \neq 0 \\ S[t] & \text{if } \sum_{k=1}^{\#\text{Rules}} \omega_k = 0. \end{cases} \tag{2}$$

This expression is a typical defuzzification mechanism that is applied to a set of Mamdani-type fuzzy rules where the linguistic labels of the consequent are singletons (see, e.g., [53]). With this fuzzy reasoning mechanism, the state activation vector always verifies the two constraints that are demanded in Section III-A: $s_i[t] \in [0, 1]$ and $\sum_{i=1}^{n} s_i[t] = 1$. Moreover, it keeps the system in its previous state if no rule is fired.

Notice that the similarity between the FFSM's fuzzy rule structure and a fuzzy classification rule can easily be recognized. Among the three existing fuzzy classification rule structures, which mainly differ on the composition of the consequent, the simplest one is based on the use of a single class (the other two variants either include the class and a certainty degree or a certainty degree for each possible class) [16], [61]. Besides, a significant relation can be identified between the fuzzy reasoning mechanism used by the FFSM and that usually applied in fuzzy-rule based classification systems based on the latter kind of rules [61]. In fact, the computation of the next state for the FFSM can be considered as a classification problem, where the set of possible fuzzy states are taken as the classes, and the fuzzy system provides a membership degree to each of them by means of a single selection or an aggregation of the firing degree of the fuzzy rules matching the class and the input pattern. Nevertheless, the main difference between both fuzzy reasoning mechanisms is that, while the membership degree to all the possible fuzzy states must sum up to 1 in any case in an FFSM,

[1] Notice that this fuzzy rule structure corresponds to a disjunctive normal form (DNF), which has been largely used in fuzzy modeling and fuzzy classification [56]–[59].

there is no such restriction for the existing class labels in a fuzzy rule-based classification system.

D. Output Vectors (Y)

Y is the set of output vectors: $(y_1, y_2, \ldots, y_{n_y})$, with n_y being the number of output variables. Y is a summary of the characteristics of the system evolution that are relevant for the application, i.e., each y_i can be obtained as the average of certain parameters of the system, while the model remained in the state q_i.

E. Output Function (g)

g is the output function: $Y[t] = g(U[t], S[t])$. It calculates, at each time instant, the value of the output vector $Y(t)$. The most simple implementation of g is $Y[t] = S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$. In this contribution, the output is the current fuzzy state of the system that is represented by the state activation vector. An application example of a complex output function can be found in [29].

IV. Fuzzy Finite State Machine for Human Gait Modeling

This section presents the design of the main elements that are needed to build an FFSM to model the human gait.

A. Fuzzy States

As stated in Section III-A, every state represents the pattern of a repetitive situation. According to the diagram at the bottom of Fig. 1 and using our own knowledge about the process, we can define four different fuzzy states which explain when double limb support, right limb single support, or left limb single support are produced. Therefore, we can easily define the possible set of fuzzy states as follows.

1) $q_1 \rightarrow$ the right foot is in stance phase, and the left foot is in stance phase (double limb support).
2) $q_2 \rightarrow$ the right foot is in stance phase, and the left foot is in swing phase (right limb single support).
3) $q_3 \rightarrow$ the right foot is in stance phase, and the left foot is in stance phase (double limb support but different of q_1 because the feet position).
4) $q_4 \rightarrow$ the right foot is in swing phase, and the left foot is in stance phase (left limb single support).

B. Input Variables

As we have explained in Section II, we only use two of the three available accelerations: a_x and a_y. Therefore, the set of input variables is $U = \{a_x, a_y\}$. We will build two different FFSMs, where each input variable will have three (FFSM 3) or five (FFSM 5) associated linguistic labels because, as we will show in the experimental results, they are enough to achieve a good accuracy keeping a high interpretability. The linguistic labels for each linguistic variable in FFSM 3 are $\{S_{a_x}, M_{a_x}, B_{a_x}\}$ and $\{S_{a_y}, M_{a_y}, B_{a_y}\}$, where S, M, and B are linguistic terms that represent small, medium, and big, respectively. While the linguistic labels for each linguistic variable in the FFSM 5 are $\{VS_{a_x}, S_{a_x}, M_{a_x}, B_{a_x}, VB_{a_x}\}$ and $\{VS_{a_y}, S_{a_y}, M_{a_y}, B_{a_y}, VB_{a_y}\}$, where the additional terms VS and VB are linguistic terms that represent very small, and very big, respectively.

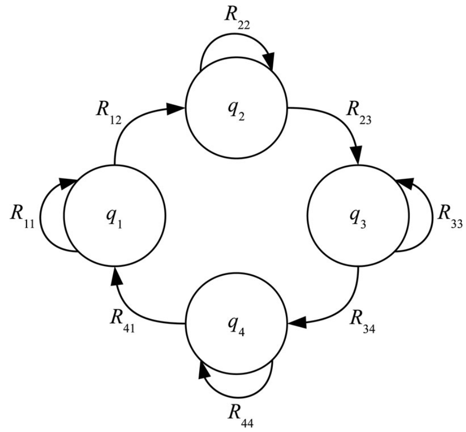

Fig. 2. State diagram of the FFSM for the human gait cycle.

C. Transition Function

As shown in Section III-C, the only thing required to determine the structure of the fuzzy rule base (RB) is the definition of which transitions are allowed and which are not. This is easily represented by means of a state diagram. Fig. 2 shows the proposed state diagram of the FFSM for the human gait cycle. This state diagram is very simple because the accelerations that are produced during the human gait are quasi-periodic, i.e., they are repeated with approximately similar values and periods. Moreover, all the states are correlative, i.e., they always follow the same activation order. Therefore, it is rather easy to define the allowed transitions and the forbidden ones.

From the state diagram that is represented in Fig. 2, it can be recognized that there are eight fuzzy rules in total in the system: four rules to remain in each state and other four to change between states. Therefore, the RB will have the following structure.

$$
\begin{array}{l}
R_{11}\text{:}\textbf{IF } (S[t] \text{ is } q_1) \textbf{ AND } C_{11} \textbf{ THEN } S[t+1] \text{ is } q_1 \\
R_{22}\text{:}\textbf{IF } (S[t] \text{ is } q_2) \textbf{ AND } C_{22} \textbf{ THEN } S[t+1] \text{ is } q_2 \\
R_{33}\text{:}\textbf{IF } (S[t] \text{ is } q_3) \textbf{ AND } C_{33} \textbf{ THEN } S[t+1] \text{ is } q_3 \\
R_{44}\text{:}\textbf{IF } (S[t] \text{ is } q_4) \textbf{ AND } C_{44} \textbf{ THEN } S[t+1] \text{ is } q_4 \\
R_{12}\text{:}\textbf{IF } (S[t] \text{ is } q_1) \textbf{ AND } C_{12} \textbf{ THEN } S[t+1] \text{ is } q_2 \\
R_{23}\text{:}\textbf{IF } (S[t] \text{ is } q_2) \textbf{ AND } C_{23} \textbf{ THEN } S[t+1] \text{ is } q_3 \\
R_{34}\text{:}\textbf{IF } (S[t] \text{ is } q_3) \textbf{ AND } C_{34} \textbf{ THEN } S[t+1] \text{ is } q_4 \\
R_{41}\text{:}\textbf{IF } (S[t] \text{ is } q_4) \textbf{ AND } C_{41} \textbf{ THEN } S[t+1] \text{ is } q_1
\end{array}
$$

D. Output Vector and Output Function

In this contribution, we simply consider $Y[t] = S[t]$, i.e., the output of the FFSM is the degree of activation of each state.

V. Genetic Fuzzy System

Fuzzy systems have showed their ability to deal with a large number of applications. In most of cases, the key for the success was the ability of fuzzy systems to incorporate human expert knowledge [47], [62], [63]. However, the lack of learning capabilities has generated a strong interest for the study of fuzzy systems with added learning capabilities. One of the most popular approaches is the hybridization between FL and artificial neural networks [64] leading to the well-known area of neuro-fuzzy systems [65], [66]. Another very extended hybrid computational intelligence system is based on the use of GAs (and, in general, evolutionary algorithms) to learn the components of a fuzzy system leading to the field of GFSs [34]–[38]. This section introduces a new fusion framework of FFSMs, a fuzzy system type, and GAs, which will be called genetic fuzzy finite state machines (GFFSMs) from now on. Basically, a GFFSM is an FFSM augmented by a learning process that is based on a GA. In particular, this section is devoted to present the GFS that is developed to learn the KB of the FFSM designed for human gait modeling.

When using a GA to learn a rule-based system, we can cover different levels of complexity according to the structural changes that are produced in the learning system by the search algorithm [67], i.e., we can do parameter optimization which is the simplest case or we can learn the complete rule set of a fuzzy rule-based system (FRBS). The KB is usually the object of study in the GFS framework. From the view point of optimization, the task to find an appropriate KB for a particular problem is equivalent to parameterize the KB and to find those parameter values that are optimal with respect to the optimization criterion. The KB parameters constitute the search space, which is transformed into a suitable genetic representation on which the search process operates [35], [37].

As seen in Section III, the FFSM is a fuzzy system and, more specifically, a FRBS as the transition function is implemented by means of fuzzy IF-THEN rules. Therefore, we can define a GFS to learn the main components of this fuzzy system.

In our approach, we allow the expert to introduce her/his own knowledge over the whole system by defining the states and transitions and specifying the general structure of the fuzzy rules that define the state transitions. The fuzzy rules themselves and the MFs of the input variables' linguistic labels will be automatically derived by the GFS, thus making a robust, accurate, and human friendly model. Therefore, according to the different approaches presented in [34], [35], and [37], we will develop a complete learning of the KB, i.e., our GFS will learn the MF shapes associated with the linguistic terms and the fuzzy rules simultaneously, although dealing with a reduced search space thanks to the incorporated expert knowledge.

The joint learning of the RB and the MFs associated with the input variables in the database (DB) can be used as a cooperative way to obtain an FFSM that is not only accurate but also compact. We have opted this genetic learning scheme since we consider that the joint learning of DB and RB deals with the interactions existing between both KB components in a better way than following a multistage learning based on first deriving the RB and later refining the preliminary DB definition [34]–[37]. Moreover, in real complex problems, most of the effort developed in an RB learning problem is typically devoted to increase the performance of some wrong rules rather than to improve the performance of the overall system by performing a complex MF parameter learning process [68]. Nevertheless, learning the DB and the RB concurrently can make the search space so large that suboptimal models are generated [69]. Fortunately, in our case the combination of the use of expert knowledge and the prefixed structure of the FFSM allows us to deal with a more reduced search space size, thus allowing the derivation of good performing KBs.

The following sections will describe in detail the structure of the different components of our GFS to learn the KB of FFSMs devoted to human gait modeling.

A. Representation Scheme and Initial Population Generation

Since we are developing a complete learning of the KB, we have divided the representation scheme into two parts: the RB part and the DB part. In the following, we explain each of these representations.

1) RB Part: Once we have the complete rule set defined in Section IV-C, we codify the whole rule set in a chromosome following the Pittsburgh approach [70] because the evaluation of the FFSM requires a complete execution cycle. Moreover, the fixed size and structure of the rules (where the consequent and the first term of the antecedent are known) and the predefined structure of the constraints imposed on the input variables shown in Section III-C allow us to use the classical DNF representation based on a binary string coding [58], [59] to codify only the remaining part of the antecedent. For each of the two input variables a_x and a_y, the rule representation consists of a binary substring of the same length as the number of labels that refers to its linguistic term set. Each bit has a 1 (0) which denotes the presence (absence) of each linguistic term in the rule. Moreover, the feature selection capability of this representation is used since an input variable is omitted in the rule if all of its bits in the representation become 0s or 1s.

As an example of how this representation is developed in the GFFSM 3, let us define a rule R_k with the following constraint over the input variables:

$$C_k = (a_x[t] \text{ is } M_{a_x}) \textbf{ AND } a_y[t] \text{ is } M_{a_y} \textbf{ OR } B_{a_y}).$$

Therefore, the representation of this DNF fuzzy rule will be of the form $\{010 : 011\}$, where in the first substring the second digit indicates the presence of the linguistic term M_{a_x}, and the zeros indicate the absence of the terms S_{a_x} and B_{a_x}. The second substring has 1s in the second and third positions indicating the presence of the linguistic terms M_{a_y} and B_{a_y}, and a zero in the first position, indicating the absence of the linguistic term S_{a_y}.

The RB part of the chromosome will, thus, be composed of 8 rules $\times$ 2 linguistic variables $\times\, l = 16 \times l$ binary-coded genes, being l the number of linguistic terms per input variable.

2) DB Part: Once we have decided the number of linguistic terms for each input variable (see Section IV-B), we can show how to represent the DB part of the KB, i.e., the representation of the MFs definition.

We have used strong fuzzy partitions (SFPs) [54] to define the fuzzy partitions. In an SFP, the membership degree forms

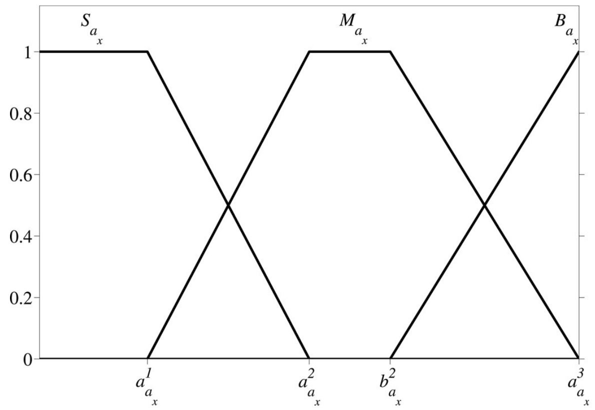

Fig. 3. Parameters that form all the linguistic labels of the linguistic variable a_x in GFFSM 3, which are trapezoidal or triangular MFs.

a partition of unity. SFPs allow us to reduce the number of parameters to tune in such a way that the normalization constraint is easily satisfied by only coding the modal points of the MFs (one point for triangular MFs and two points for trapezoidal shapes). Moreover, when we calculate the OR between two linguistic labels using Łukasiewicz's bounded sum as explained in Section III-C1, the resulting linguistic label will be a convex fuzzy set without sawtooth shapes that would be produced if we use the maximum. From the interpretability point of view, SFPs satisfy semantic constraints and help to get comprehensible fuzzy partitions [47].

We have used trapezoidal SFPs that are defined in the whole domain of discourse of the input variable. Since the fuzzy partition of each input variable is generically comprised by l linguistic labels, we have to code $2 \times [(l-2) \times 2 + 2]$ real parameters, $(l-2) \times 2 + 2$ per input variable where one parameter is enough to codify the first and last linguistic labels and two parameters are needed to codify each intermediate linguistic label. In particular, working with three or five linguistic labels, the DB part of the chromosome will be composed of 8 or 16 real-coded genes, respectively:

$$\text{GFFSM3}\begin{cases} a_x \rightarrow \{a^1_{a_x}, a^2_{a_x}, b^2_{a_x}, a^3_{a_x}\} \\ a_y \rightarrow \{a^1_{a_y}, a^2_{a_y}, b^2_{a_y}, a^3_{a_y}\} \end{cases}$$

$$\text{GFFSM5}\begin{cases} a_x \rightarrow \{a^1_{a_x}, a^2_{a_x}, b^2_{a_x}, a^3_{a_x}, b^3_{a_x}, a^4_{a_x}, b^4_{a_x}, a^5_{a_x}\} \\ a_y \rightarrow \{a^1_{a_y}, a^2_{a_y}, b^2_{a_y}, a^3_{a_y}, b^3_{a_y}, a^4_{a_y}, b^4_{a_y}, a^5_{a_y}\}. \end{cases}$$

Fig. 3 shows the graphical representation of the fuzzy partition related to the linguistic input variable a_x in GFFSM 3. For the first linguistic label S_{a_x}, we only need one parameter $a^1_{a_x}$. The same stands for the last one B_{a_x} whose parameter is $a^3_{a_x}$. For the intermediate linguistic label, we need two parameters $a^2_{a_x}$ and $b^2_{a_x}$. Note that we have chosen trapezoidal MFs because triangular MFs are a particular case of trapezoidal MFs, i.e., the linguistic label B_{a_x} will be of triangular shape when the value $a^3_{a_x}$ reaches the limits of the domain of discourse of the input variable a_x.

We should remark that this learning problem demands a real-coded representation, and therefore, we have to implement real-coded crossover and mutation genetic operators. Moreover, to define the variation interval of each allele we have considered that each parameter can be only modified within the interval that is defined by its previous and next parameter, i.e., in Fig. 3, the definition/variation interval of the parameter $a^2_{a_x}$ is $[a^1_{a_x}, b^2_{a_x}]$, while that of the parameter $a^3_{a_x}$ is $[b^2_{a_x}, \max(a_x)]$ (with $\max(a_x)$ being the maximum value taken by the input variable a_x).

Hence, the final chromosome encoding a candidate problem solution will be comprised by $48 + 8 = 56$ genes in GFFSM 3, and $80 + 16 = 96$ genes in GFFSM 5. Fig. 4 shows the shape of the complete chromosome encoding the RB and DB parts of GFFSM 3.

We have initialized the first population by generating all the individuals at random. However, in order to include our previous knowledge about the problem, the DB part of the first individual of the population will encode uniform fuzzy partitions for both linguistic variables a_x and a_y. Then, the following individuals are created at random to introduce diversity.

B. Fitness Function

The fitness function measures the quality of the candidate problem solution encoded in each chromosome. In the case of our GFFSM for human gait modeling, the dependence of the next state on the previous state makes it strictly necessary to test the FFSM over the whole dataset and for each chromosome, which is very computationally expensive. This problem also appears when learning FL controllers, where the fitness measure must be evaluated by simulating how the plant is controlled [71]–[73].

We have chosen the minimization of the mean absolute error (MAE) defined in (3) as the fitness function

$$\text{MAE} = \frac{1}{n} \cdot \frac{1}{T} \cdot \sum_{i=1}^{n} \sum_{j=0}^{T} |s_i[j] - s^*_i[j]| \tag{3}$$

where

1) n is the number of states, i.e., $n = 4$ for the human gait modeling problem (see Section IV);
2) T is the dataset size (i.e., the considered time interval duration);
3) $s_i[j]$ is the degree of activation of state q_i at time $t = j$;
4) $s^*_i[j]$ is the expected degree of activation of state q_i at time $t = j$.

The MAE is a very informative measure of the quality of the candidate solution because it directly measures the difference between the expected state activation vector ($S^*[t]$) and the obtained one ($S[t]$). However, we need to define an expected activation vector $S^*[t]$ for each input dataset that we want to learn, i.e., a training dataset in the context of a supervised learning problem to design our human gait FFSM-based model. This definition could be problematic and must be done carefully because sometimes it must be defined at each time instant more than one state, activated with certain degree in the interval $[0, 1]$. In Section V-C, this issue is explained in detail.

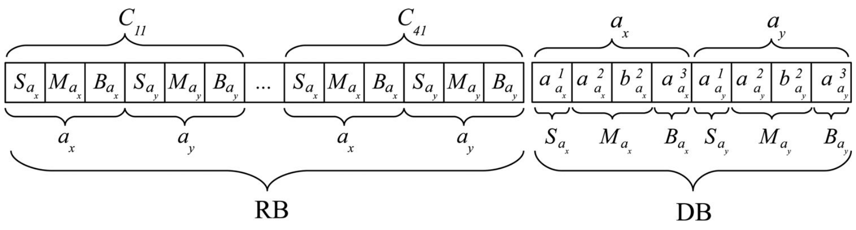

Fig. 4. Chromosome which codifies the RB and DB parts of GFFSM 3.

C. Defining the Training Dataset

As described in Section V-B, in order to learn an FFSM for different gaits of the same person, there is a need to define an expected activation vector $S^*[t]$ for each one of the gaits we want to learn. Hence, we have to create a training vector that consists of $a_x[t]$, $a_y[t]$, and $S^*[t]$, i.e., $(a_x[t], a_y[t], s_1^*[t], s_2^*[t], s_3^*[t], s_4^*[t])$.

To define the training vector, we have developed a user-friendly graphical interface that allows the expert to select the relevant points where each state starts and ends using her/his knowledge about the human gait process. For instance, "the double limb support that comes after the right single support starts just after the heel contact" can be translated as "state q_1 must start when a_x increases drastically and a_y tends to decrease" [74]. The fuzzy definition of the states is based on the imprecision of the expert when defining the start and the end of each state which she/he must identify and label within the time series associated with the measured signals. We have defined the training vectors for datasets which consist of five complete cycles of the human gait. For each state q_i, we will have ten different points: five comprising the beginning (b_i^m) and another five comprising the end (e_i^m) of each state, with $m = 1, \ldots, 5$. In the current FFSM that involves four fuzzy states, the expert will have to tag each sample of five cycles with 40 points: $\{b_1^1, b_1^2, \ldots, e_4^4, e_4^5\}$.

As an example, let us consider the definition of the degree of activation of state q_2 specified by (4). Between the end time of q_1 (e_1^m) and the start time of q_2 (b_2^m), the activation of the state q_2 is rising from 0 to 1. Between the start (b_2^m) and the end time (e_2^m) of q_2 which is defined by the user, the activation has the maximum of 1, and afterward, the activation drops to zero at the start of q_3 (b_3^m). Otherwise, the activation is zero

$$s_2^*[t] = \begin{cases} \dfrac{t - e_1^m}{b_2^m - e_1^m} & \text{if } e_1^m < t < b_2^m \\ 1 & \text{if } b_2^m \le t \le e_2^m \\ \dfrac{b_3^m - t}{b_3^m - e_2^m} & \text{if } e_2^m < t < b_3^m \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Fig. 5 shows an example of how a part of the training vector is labeled based on the beginning and the end points given by the expert.

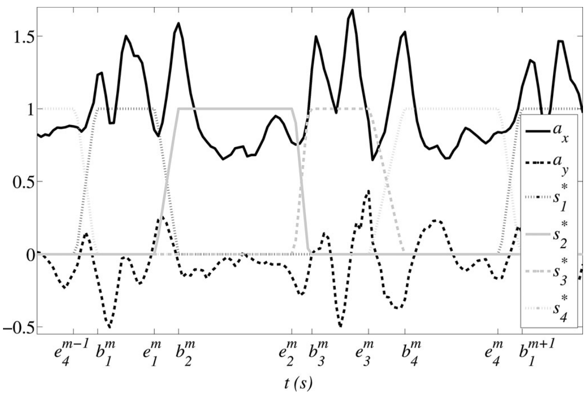

Fig. 5. Construction of the vectors of training data based on the start and end points given by the user.

D. Genetic Operators

The definition of the genetic operators considered in our GFFSM for human gait modeling is shown as follows.

1) Selection Mechanism: To select the parents that will undergo crossover and mutation, a binary tournament selection is considered. This operator is very useful since it does not require any global knowledge of the population [33]. The idea is to select at random two parents and choose the best one with respect to the fitness function, repeating this process until a complete set of parents is built.

2) Crossover: The classical two-point crossover has been used for the RB (binary-coded) part of the chromosome and BLX-α crossover [75] for the DB (real-coded) part. The BLX-α crossover is applied twice over a pair of parents in order to obtain a new pair of chromosomes. When a pair of chromosomes is chosen for crossover that is based on a single crossover probability, we separately crossover the binary part and the real part. Notice that the proposed genetic operators can be independently applied in both chromosome parts ensuring the obtaining of an offspring encoding a coherent FFSM KB definition. That does not always happen when working with GFSs learning the whole KB using a representation scheme based on two information levels (the DB and RB parts) since those two parts can be related so that the action of a genetic operator in one of them can cause the appearance of meaningless chromosomes because the information encoded in the other part is no longer valid (see, for example, [76] and [77]). Nevertheless, that is not the case in the current coding scheme.

3) Mutation: For the binary-coded RB part, the classical bitwise mutation has been selected. For the real-coded DB part, the corresponding mutation operator which is called uniform mutation [33] has been chosen. It consists of changing the allele value of each gene randomly within its definition interval. As for the crossover, the same mutation probability defined at gene level is considered for both chromosome parts.

4) Replacement Mechanism: In our approach, we directly replace the current population by the offspring one (generational replacement) keeping elitism.

5) Termination Condition: In this contribution, we have implemented three different termination conditions. First, the search is stopped when the algorithm has obtained a fitness value equal to zero, which is the best value that the fitness function can take. However, this condition is almost impossible to be reached. Therefore, we have decided to set the maximum number of evaluations and also to stop the search when, for a certain number of evaluations, the fitness value of the best individual is not improved.

VI. Experimentation

This section presents the experimentation which is carried out to validate our proposal. First, the experimental setup, which comprises the data acquisition and the GFFSM parameter values, is explained. Section VI-B contains a brief description of two alternative modeling approaches used for human gait modeling. Finally, Sections VI-C and D report the obtained results and their analysis, respectively.

A. Experimental Setup

1) Data Acquisition: To evaluate the proposed approach, we have collected the acceleration signals of 20 different people in order to create a specific FFSM to model the gait of each person. The group of people consisted of healthy adults, 5 women and 15 men, with ages ranging between 23 and 52 years (with an average age of 30 years) and weights between 45 and 97 kg (with an average weight of 76 kg).

We attached to a belt, which is centered in the back of the person, a three-axial accelerometer with Bluetooth capabilities that provided measurements of the three orthogonal accelerations with a frequency of 100 Hz. We programmed a personal digital agenda (PDA) to receive the data via a Bluetooth connection and to record them with a timestamp. Therefore, every record contained the information (t, a_x, a_y, a_z), where t is each instant of time, a_x is the dorsoventral acceleration, a_y is the mediolateral acceleration, and a_z is the anteroposterior acceleration. As explained in Section II, in this study, we only use a_x and a_y.

We asked each person to walk a certain distance at a self-selected walking speed which comprises around ten complete gait cycles in such a way that we were able to extract five complete gait cycles discarding the first and last steps which are not very stable. This process was repeated ten times for each person that produces a total of ten datasets of five complete cycles for each person. These datasets were then processed as explained in Section V-C in order to define all the fuzzy states. Therefore, once we captured and tagged all the signals, we had ten different datasets for each person with the following components:

$$(a_x[t], a_y[t], s_1^*[t], s_2^*[t], s_3^*[t], s_4^*[t])$$

where
1) $a_x[t]$ is the dorsoventral acceleration at time instant t;
2) $a_y[t]$ is the mediolateral acceleration at time instant t;
3) $s_1^*[t]$ is the expected degree of activation of state q_1 at time instant t;
4) $s_2^*[t]$ is the expected degree of activation of state q_2 at time instant t;
5) $s_3^*[t]$ is the expected degree of activation of state q_3 at time instant t;
6) $s_4^*[t]$ is the expected degree of activation of state q_4 at time instant t.

2) Parameter Values for the Genetic Fuzzy Finite State Machine: Two different granularity levels have been considered for the fuzzy partitions: 3 and 5 (noted as GFFSM 3 and GFFSM 5, respectively). The parameter values that are used by both GFFSMs are as follows. Quite standard values are considered, and a preliminary experimentation was developed to check their good performance:
1) population size $\rightarrow$ 100 individuals;
2) crossover probability $\rightarrow p_c = 0.8$;
3) value of alpha (BLX-α parameter) $\rightarrow \alpha = 0.3$;
4) mutation probability per bit $\rightarrow p_m = 0.02$;
5) termination conditions:
   a) fitness value reached $\rightarrow$ MAE $= 0$;
   b) maximum number of evaluations $\rightarrow$ 40 000 for GFFSM 3 and 60 000 for GFFSM 5;
   c) evaluations without improvement of the fitness function $\rightarrow$ 10 000.

B. Alternative Modeling Approaches

In order to compare the two GFFSM results with other system identification approaches, we have considered two different techniques which are commonly used in system modeling of time-dependent systems: autoregressive linear models (ARX) [3] and neural networks (NNs) [64].

1) Autoregressive Linear Models: We have defined a multiple-input multiple-output (MIMO) ARX model with the structure defined by

$$Y[t] = A_1 \cdot Y[t-1] + \cdots + A_{n_A} \cdot Y[t-n_A] + B_0 \cdot U[t] + \cdots + B_{n_B} \cdot U[t-n_B] \quad (5)$$

where
1) $Y[t] = (s_1[t], s_2[t], s_3[t], s_4[t])$ is the current output vector;
2) $Y[t-1], \ldots, Y[t-n_A]$ are the previous output vectors on which the current output vector depends;
3) $U[t] = (a_x[t], a_y[t]), \ldots, U[t-n_B]$ are the current and delayed input vectors on which the current output vector depends;

TABLE I
PARAMETER VALUES CONSIDERED FOR THE DIFFERENT ARX MODELS AND THEIR COMPLEXITY

MODEL	n_A	n_B	COMPLEXITY
ARX 1	1	1	0
ARX 2	2	2	0.010
ARX 3	5	5	0.040
ARX 4	10	10	0.091
ARX 5	20	20	0.192
ARX 6	25	25	0.242
ARX 7	50	50	0.495
ARX 8	75	75	0.747
ARX 9	80	80	0.798
ARX 10	100	100	1

4) n_A is the number of previous output vectors on which the current output vector depends;
5) n_B is the number of previous input vectors on which the current output vector depends.
6) $A_1, \ldots, A_{n_A}, B_0, \ldots, B_{n_B}$ are the matrices that define the models. They are estimated using the least-squares method.

We have tested the performance of this model for ten different values of the parameters n_A and n_B in order to obtain several models with a different accuracy–complexity tradeoff.

For the first parameter values, we have selected a simple model similar to the delay of our GFFSM, i.e., $n_A = n_B = 1$ resulting in the ARX model number 1 defined by

$$Y[t] = A_1 \cdot Y[t-1] + B_0 \cdot U[t]. \tag{6}$$

Then, another nine different values (with the maximum delay of 100) were used to progressively increase the complexity of the model. A linear complexity index is defined in such a way that the complexity of the basic model with $n_A = n_B = 1$ is 0 and the complexity for the most complex model with $n_A = n_B = 100$ is 1. The different parameter values for each model together with the model complexity are shown in Table I.

2) Neural Networks: As for the ARX models, we have built ten different feed-forward NN architectures that represent different levels of complexity. The first and simplest one (NN 1) consists of two neurons in the input layer which represent the two input variables $a_x[t]$ and $a_y[t]$, one hidden layer, and four output neurons in the output layer that correspond to the four components of the state activation vector $(s_1[t], s_2[t], s_3[t], s_4[t])$.

The other NN models, which represent different levels of complexity, are determined by the number of delayed input variables. Moreover, in order to avoid NNs with a large number of input neurons (which leads to overfitting and big training times), we have considered delayed input variables that are separated by a fixed interval of ten samples. For example, the second NN architecture (NN 2) has $a_x[t], a_x[t-10], a_y[t]$, and $a_y[t-10]$ as inputs, while the most complex one (NN 10) has 20 inputs that cover a delay of 90 samples: $a_x[t], a_x[t-10], \ldots, a_x[t-90]$, and $a_y[t], a_y[t-10], \ldots, a_y[t-90]$.

The NN weights have been estimated using the Levenberg–Marquardt method during 500 epochs. The number of neurons in the hidden layer was chosen to minimize the test error of each specific architecture. The architectures of the two extreme NNs are represented in Fig. 6.

Similar to the ARX models, a complexity index is defined in such a way that the complexity of the NN with 2 inputs (NN 1) is 0 and the complexity of the NN with 20 inputs (NN 10) is 1. The different models, their input variables, and their complexity are shown in Table II.

C. Results

To test the performance of the two GFFSMs and the alternative modeling approaches, we have done a leave-one-out cross validation [78] for each of the ten datasets of each person. As an example, Table III shows the MAE that is obtained for each fold of the leave-one-out corresponding to the first person's experiments. It also depicts the average value of the MAE and its standard deviation for the ten folds.

As a global summary of the results that are obtained, Table IV reports, for each of the leave-one-out applications for the ten datasets of each person, the average (MEAN) and standard deviation (STD) of the MAE for eight different models: those two corresponding to our proposal (GFFSM 3 and GFFSM 5), three ARX models comprising a good tradeoff model (ARX 7) and the two extreme ones (ARX 1 and ARX 10), and three NN models comprising a good tradeoff model (NN 4) and the two extreme ones (NN 1 and NN 10).

To select the best accuracy–complexity tradeoff model for both NN and ARX models, we compute 1000 random weights $\omega_i \in [0, 1]$. We calculate the average MAE for each model for the 20 people ($\overline{\text{MAE}}$) and normalize the resulting set of MAEs in the interval $[0, 1]$. We take the average value of the aggregation function of both the normalized MAE ($\widetilde{\text{MAE}}$) and the complexity index value (COMPLEXITY) of each model as shown in (7). Finally, the model with the lowest aggregated value is selected as that with the best tradeoff

$$Q_{\text{MODEL}} = \sum_{i=1}^{1000} \omega_i \cdot \widetilde{\text{MAE}}_{\text{MODEL}} + (1 - \omega_i) \cdot \text{COMPLEXITY}_{\text{MODEL}}. \tag{7}$$

Moreover, since our final goal is to obtain a specific model (FFSM) for each person's gait, Table V shows the average of the MAE for each one of the person's models (FFSMs) generated during the leave-one-out procedure when the input data are the whole set of gaits of each person. The aim of these results is to check if the generated models are significantly fitted to the specific person's gait than to other persons' gaits, as expected and desired.

As can be seen, that is clearly the case. For example, notice that models GFFSM 3 and GFFSM 5 for the first person (P1) corresponding to the first two rows get an average MAE (boldfaced) of 0.088 and 0.076, respectively, with its own person's gait (first row, first column), while they get large average MAE values for the gaits of the rest of the people (the rest of the columns in the first row). This fact can also be checked for the models of the rest of the people. In addition, the last column of Table V (MEAN^-) shows boldfaced the average value of all the MAEs obtained by each person's FFSM model with the gaits of

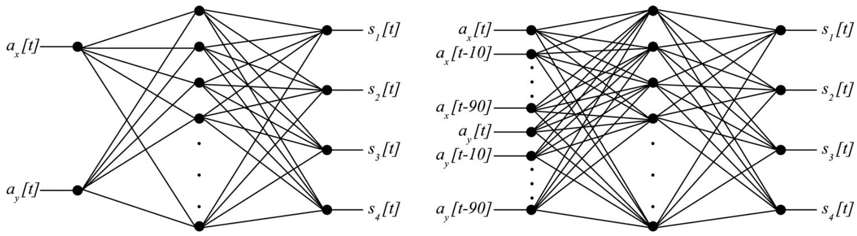

Fig. 6. Architectures of the simplest neural network (NN 1) and the most complex one (NN 10) designed for human gait modeling. All of them have a single hidden layer and the difference arises in the number of (delayed) inputs.

TABLE II
TEN DIFFERENT NN ARCHITECTURES WITH THEIR INPUT VARIABLES AND COMPLEXITIES

MODEL	INPUT VARIABLES	COMPLEXITY
NN 1	$a_x[t], a_y[t]$	0
NN 2	$a_x[t], a_x[t-10], a_y[t], a_y[t-10]$	0.111
NN 3	$a_x[t], a_x[t-10], a_x[t-20], a_y[t], a_y[t-10], a_y[t-20]$	0.222
NN 4	$a_x[t], a_x[t-10], \dots, a_x[t-30], a_y[t], a_y[t-10], \dots, a_y[t-30]$	0.333
NN 5	$a_x[t], a_x[t-10], \dots, a_x[t-40], a_y[t], a_y[t-10], \dots, a_y[t-40]$	0.444
NN 6	$a_x[t], a_x[t-10], \dots, a_x[t-50], a_y[t], a_y[t-10], \dots, a_y[t-50]$	0.556
NN 7	$a_x[t], a_x[t-10], \dots, a_x[t-60], a_y[t], a_y[t-10], \dots, a_y[t-60]$	0.667
NN 8	$a_x[t], a_x[t-10], \dots, a_x[t-70], a_y[t], a_y[t-10], \dots, a_y[t-70]$	0.778
NN 9	$a_x[t], a_x[t-10], \dots, a_x[t-80], a_y[t], a_y[t-10], \dots, a_y[t-80]$	0.889
NN 10	$a_x[t], a_x[t-10], \dots, a_x[t-90], a_y[t], a_y[t-10], \dots, a_y[t-90]$	1

TABLE III
MAE OF THE LEAVE-ONE-OUT FOR THE DATASETS OF THE FIRST PERSON, WITH THE AVERAGE (MEAN) AND STANDARD DEVIATION (STD) FOR EACH OF THE EVALUATED MODELS

FOLD	GFFSM 3	ARX 1	ARX 2	ARX 3	ARX 4	ARX 5	ARX 6	ARX 7	ARX 8	ARX 9	ARX 10
1	0.089	0.337	0.316	0.293	0.262	0.201	0.172	0.068	0.065	0.066	0.068
2	0.066	0.340	0.316	0.293	0.261	0.192	0.154	0.058	0.051	0.051	0.058
3	0.135	0.341	0.320	0.296	0.267	0.224	0.162	0.060	0.044	0.044	0.060
4	0.108	0.343	0.318	0.291	0.256	0.212	0.166	0.054	0.049	0.048	0.054
5	0.133	0.337	0.320	0.298	0.259	0.217	0.215	0.054	0.039	0.040	0.053
6	0.078	0.338	0.315	0.300	0.258	0.184	0.169	0.059	0.056	0.057	0.058
7	0.101	0.345	0.316	0.299	0.256	0.219	0.182	0.058	0.051	0.048	0.058
8	0.149	0.345	0.317	0.298	0.274	0.234	0.164	0.107	0.105	0.105	0.107
9	0.086	0.340	0.311	0.288	0.253	0.241	0.221	0.117	0.117	0.115	0.116
10	0.081	0.335	0.313	0.291	0.259	0.195	0.196	0.070	0.079	0.079	0.070
MEAN	0.103	0.340	0.316	0.295	0.261	0.212	0.180	0.070	0.065	0.065	0.070
STD	0.028	0.003	0.003	0.004	0.006	0.019	0.023	0.023	0.027	0.026	0.023
FOLD	GFFSM 5	NN 1	NN 2	NN 3	NN 4	NN 5	NN 6	NN 7	NN 8	NN 9	NN 10
1	0.107	0.218	0.138	0.098	0.084	0.077	0.076	0.065	0.061	0.060	0.059
2	0.081	0.209	0.114	0.087	0.068	0.067	0.057	0.056	0.048	0.049	0.046
3	0.077	0.221	0.125	0.089	0.072	0.065	0.056	0.051	0.053	0.043	0.041
4	0.085	0.219	0.123	0.096	0.084	0.070	0.066	0.053	0.054	0.058	0.050
5	0.085	0.221	0.132	0.092	0.074	0.073	0.060	0.061	0.055	0.050	0.051
6	0.063	0.219	0.146	0.110	0.087	0.084	0.069	0.060	0.062	0.055	0.058
7	0.083	0.209	0.123	0.088	0.078	0.070	0.063	0.061	0.061	0.054	0.051
8	0.142	0.236	0.144	0.113	0.085	0.089	0.079	0.073	0.075	0.068	0.074
9	0.076	0.212	0.131	0.107	0.085	0.077	0.074	0.074	0.066	0.057	0.065
10	0.080	0.207	0.119	0.080	0.067	0.062	0.057	0.056	0.050	0.045	0.046
MEAN	0.088	0.217	0.129	0.096	0.078	0.073	0.066	0.061	0.059	0.054	0.054
STD	0.022	0.009	0.011	0.011	0.008	0.009	0.008	0.008	0.008	0.008	0.010

all the people, except its own input gaits. It can be easily seen that these values are much greater than the ones obtained with the gaits of each person's model (boldfaced in the diagonal cells of the table).

D. Discussion

This section aims to present a discussion about four different issues of our proposed model: its accuracy, its interpretability, its computational cost, and the importance of the use of expert knowledge.

1) Accuracy Analysis: The results given in Tables III and IV show that the GFFSM models exhibit better accuracies when compared with the simplest competing models, namely ARX 1 and NN 1. Besides, it can be seen how the best tradeoff model ARX 7 is able to outperform our proposal, although it needs a big delay of 50 samples to do so. In contrast, the best tradeoff model NN 4 shows a similar accuracy to our models. Its results are slightly better than those of GFSSM 3 and slightly worse than those of GFFSM 5. In fact, GFFSM 5 is better than GFFSM 3 for the majority of the people because of its higher granularity in the number of linguistic labels, which provides it with additional freedom degrees for the modeling task.

TABLE IV
AVERAGE (MEAN) AND STANDARD DEVIATION (STD) OF THE MAE FOR EACH ONE OF THE LEAVE-ONE-OUT FOR THE 10 DATASETS OF EACH PERSON

PERSON	GFFSM 3		GFFSM 5		ARX 1		ARX 7		ARX 10		NN 1		NN 4		NN 10	
	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD	MEAN	STD		
1	0.103	0.028	0.088	0.022	0.340	0.003	0.070	0.023	0.070	0.023	0.217	0.009	0.078	0.008	0.054	0.010
2	0.063	0.025	0.055	0.020	0.379	0.011	0.064	0.027	0.064	0.027	0.194	0.007	0.061	0.009	0.053	0.008
3	0.091	0.018	0.077	0.022	0.331	0.007	0.103	0.041	0.102	0.041	0.238	0.010	0.080	0.010	0.053	0.008
4	0.150	0.100	0.143	0.040	0.328	0.006	0.110	0.050	0.109	0.050	0.257	0.007	0.118	0.012	0.078	0.011
5	0.071	0.047	0.055	0.031	0.254	0.006	0.071	0.016	0.071	0.016	0.248	0.005	0.093	0.008	0.063	0.010
6	0.106	0.030	0.117	0.022	0.323	0.014	0.107	0.040	0.106	0.040	0.258	0.006	0.115	0.014	0.077	0.011
7	0.170	0.045	0.159	0.034	0.355	0.008	0.074	0.021	0.074	0.021	0.255	0.009	0.094	0.012	0.052	0.007
8	0.065	0.019	0.067	0.034	0.352	0.004	0.065	0.020	0.065	0.020	0.211	0.003	0.072	0.005	0.048	0.006
9	0.098	0.041	0.121	0.075	0.319	0.009	0.115	0.076	0.115	0.076	0.243	0.010	0.120	0.020	0.088	0.016
10	0.121	0.056	0.098	0.050	0.382	0.012	0.088	0.030	0.088	0.030	0.237	0.005	0.108	0.008	0.068	0.008
11	0.101	0.039	0.110	0.032	0.376	0.004	0.094	0.018	0.090	0.025	0.221	0.003	0.109	0.012	0.088	0.010
12	0.303	0.143	0.229	0.131	0.339	0.002	0.078	0.018	0.076	0.028	0.301	0.021	0.261	0.048	0.257	0.065
13	0.281	0.104	0.263	0.126	0.348	0.022	0.083	0.046	0.086	0.059	0.282	0.029	0.268	0.085	0.296	0.094
14	0.059	0.020	0.066	0.044	0.370	0.005	0.070	0.029	0.066	0.026	0.204	0.005	0.062	0.011	0.046	0.014
15	0.279	0.125	0.209	0.127	0.339	0.020	0.089	0.028	0.080	0.030	0.290	0.026	0.249	0.075	0.253	0.082
16	0.059	0.038	0.066	0.064	0.233	0.009	0.085	0.069	0.070	0.020	0.238	0.010	0.093	0.012	0.055	0.007
17	0.215	0.135	0.153	0.120	0.333	0.004	0.114	0.038	0.124	0.051	0.297	0.034	0.233	0.110	0.233	0.124
18	0.088	0.046	0.070	0.015	0.343	0.006	0.069	0.026	0.073	0.020	0.210	0.009	0.077	0.009	0.052	0.008
19	0.105	0.029	0.106	0.041	0.384	0.007	0.101	0.044	0.104	0.044	0.243	0.006	0.110	0.012	0.080	0.012
20	0.142	0.053	0.120	0.036	0.366	0.018	0.117	0.056	0.101	0.039	0.270	0.005	0.099	0.012	0.062	0.007
MEAN	0.133	0.057	0.119	0.054	0.340	0.009	0.088	0.036	0.087	0.034	0.246	0.011	0.125	0.025	0.103	0.026

TABLE V
AVERAGE OF THE MAE FOR EACH ONE OF THE PERSON'S FFSM MODELS WHEN THE INPUT DATA ARE THE WHOLE SET OF GAITS OF EACH PERSON

	INPUT																				
MODEL	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	MEAN^{-}
$\mathrm{P1}_{\mathrm{GFFSM\ 3}}$	**0.088**	0.276	0.347	0.336	0.373	0.357	0.319	0.167	0.335	0.331	0.288	0.335	0.335	0.417	0.325	0.343	0.349	0.357	0.361	0.346	**0.331**
$\mathrm{P1}_{\mathrm{GFFSM\ 5}}$	**0.076**	0.341	0.370	0.333	0.357	0.380	0.348	0.260	0.353	0.357	0.301	0.366	0.360	0.361	0.366	0.312	0.360	0.366	0.358	0.364	**0.348**
$\mathrm{P2}_{\mathrm{GFFSM\ 3}}$	0.400	**0.060**	0.331	0.415	0.405	0.419	0.393	0.402	0.394	0.344	0.400	0.395	0.362	0.404	0.392	0.435	0.405	0.399	0.417	0.407	**0.396**
$\mathrm{P2}_{\mathrm{GFFSM\ 5}}$	0.405	**0.047**	0.305	0.394	0.403	0.418	0.388	0.400	0.390	0.362	0.395	0.383	0.341	0.351	0.386	0.410	0.386	0.404	0.404	0.406	**0.386**
$\mathrm{P3}_{\mathrm{GFFSM\ 3}}$	0.407	0.319	**0.082**	0.400	0.375	0.407	0.397	0.410	0.405	0.383	0.224	0.381	0.369	0.425	0.374	0.400	0.386	0.398	0.447	0.399	**0.384**
$\mathrm{P3}_{\mathrm{GFFSM\ 5}}$	0.380	0.321	**0.072**	0.377	0.363	0.384	0.386	0.312	0.399	0.373	0.254	0.380	0.369	0.409	0.382	0.403	0.382	0.386	0.429	0.367	**0.371**
$\mathrm{P4}_{\mathrm{GFFSM\ 3}}$	0.284	0.309	0.409	**0.103**	0.279	0.287	0.333	0.300	0.358	0.374	0.323	0.347	0.313	0.325	0.342	0.224	0.383	0.365	0.243	0.372	**0.325**
$\mathrm{P4}_{\mathrm{GFFSM\ 5}}$	0.252	0.315	0.360	**0.109**	0.345	0.285	0.290	0.310	0.400	0.389	0.316	0.340	0.308	0.342	0.355	0.249	0.365	0.369	0.251	0.376	**0.327**
$\mathrm{P5}_{\mathrm{GFFSM\ 3}}$	0.410	0.399	0.467	0.345	**0.046**	0.360	0.369	0.372	0.380	0.352	0.395	0.362	0.395	0.229	0.398	0.360	0.376	0.386	0.347	0.382	**0.373**
$\mathrm{P5}_{\mathrm{GFFSM\ 5}}$	0.378	0.368	0.409	0.373	**0.039**	0.319	0.356	0.411	0.352	0.345	0.317	0.365	0.387	0.355	0.381	0.364	0.372	0.363	0.366	0.347	**0.365**
$\mathrm{P6}_{\mathrm{GFFSM\ 3}}$	0.380	0.362	0.388	0.327	0.242	**0.091**	0.339	0.334	0.400	0.423	0.429	0.348	0.305	0.382	0.316	0.219	0.364	0.327	0.252	0.329	**0.340**
$\mathrm{P6}_{\mathrm{GFFSM\ 5}}$	0.363	0.298	0.360	0.342	0.256	**0.085**	0.345	0.285	0.419	0.418	0.410	0.349	0.331	0.406	0.363	0.259	0.354	0.347	0.261	0.330	**0.342**
$\mathrm{P7}_{\mathrm{GFFSM\ 3}}$	0.278	0.215	0.229	0.287	0.381	0.267	**0.135**	0.213	0.379	0.396	0.240	0.335	0.317	0.411	0.350	0.302	0.333	0.310	0.328	0.378	**0.313**
$\mathrm{P7}_{\mathrm{GFFSM\ 5}}$	0.306	0.230	0.284	0.302	0.389	0.237	**0.124**	0.248	0.354	0.374	0.276	0.338	0.317	0.376	0.355	0.317	0.339	0.295	0.310	0.371	**0.317**
$\mathrm{P8}_{\mathrm{GFFSM\ 3}}$	0.264	0.239	0.327	0.332	0.327	0.333	0.311	**0.061**	0.345	0.369	0.322	0.339	0.336	0.397	0.332	0.349	0.337	0.350	0.371	0.328	**0.332**
$\mathrm{P8}_{\mathrm{GFFSM\ 5}}$	0.216	0.275	0.315	0.337	0.380	0.335	0.317	**0.056**	0.337	0.395	0.291	0.358	0.358	0.395	0.340	0.374	0.331	0.378	0.370	0.361	**0.340**
$\mathrm{P9}_{\mathrm{GFFSM\ 3}}$	0.402	0.409	0.438	0.379	0.379	0.411	0.374	0.379	**0.073**	0.356	0.387	0.332	0.373	0.251	0.365	0.377	0.340	0.357	0.359	0.378	**0.371**
$\mathrm{P9}_{\mathrm{GFFSM\ 5}}$	0.384	0.426	0.447	0.384	0.381	0.393	0.362	0.408	**0.075**	0.306	0.408	0.322	0.373	0.208	0.367	0.374	0.348	0.398	0.368	0.385	**0.371**
$\mathrm{P10}_{\mathrm{GFFSM\ 3}}$	0.395	0.375	0.385	0.385	0.354	0.432	0.341	0.392	0.252	**0.093**	0.390	0.345	0.382	0.330	0.311	0.415	0.352	0.401	0.432	0.332	**0.369**
$\mathrm{P10}_{\mathrm{GFFSM\ 5}}$	0.342	0.362	0.407	0.368	0.323	0.362	0.341	0.367	0.236	**0.073**	0.334	0.321	0.347	0.353	0.311	0.360	0.367	0.383	0.358	0.318	**0.345**
$\mathrm{P11}_{\mathrm{GFFSM\ 3}}$	0.279	0.254	0.243	0.312	0.334	0.305	0.357	0.251	0.404	0.389	**0.075**	0.341	0.322	0.393	0.366	0.353	0.355	0.418	0.354	0.363	**0.337**
$\mathrm{P11}_{\mathrm{GFFSM\ 5}}$	0.258	0.265	0.266	0.320	0.323	0.300	0.311	0.210	0.354	0.337	**0.072**	0.332	0.309	0.337	0.348	0.318	0.331	0.373	0.324	0.341	**0.314**
$\mathrm{P12}_{\mathrm{GFFSM\ 3}}$	0.325	0.335	0.315	0.344	0.335	0.317	0.335	0.311	0.298	0.331	0.338	**0.174**	0.317	0.351	0.319	0.292	0.329	0.342	0.316	0.340	**0.326**
$\mathrm{P12}_{\mathrm{GFFSM\ 5}}$	0.348	0.362	0.317	0.348	0.336	0.298	0.348	0.281	0.365	0.345	0.351	**0.133**	0.307	0.402	0.313	0.287	0.319	0.316	0.347	0.325	**0.332**
$\mathrm{P13}_{\mathrm{GFFSM\ 3}}$	0.333	0.380	0.342	0.308	0.369	0.350	0.344	0.328	0.360	0.340	0.337	0.333	**0.188**	0.393	0.320	0.318	0.311	0.325	0.369	0.353	**0.343**
$\mathrm{P13}_{\mathrm{GFFSM\ 5}}$	0.330	0.338	0.286	0.322	0.341	0.335	0.320	0.284	0.359	0.356	0.325	0.336	**0.179**	0.384	0.319	0.358	0.322	0.310	0.318	0.344	**0.331**
$\mathrm{P14}_{\mathrm{GFFSM\ 3}}$	0.383	0.416	0.451	0.406	0.358	0.415	0.388	0.385	0.318	0.342	0.382	0.383	0.388	**0.053**	0.401	0.399	0.394	0.384	0.372	0.399	**0.388**
$\mathrm{P14}_{\mathrm{GFFSM\ 5}}$	0.414	0.412	0.444	0.410	0.385	0.411	0.401	0.426	0.296	0.347	0.419	0.387	0.398	**0.041**	0.402	0.386	0.380	0.392	0.366	0.398	**0.393**
$\mathrm{P15}_{\mathrm{GFFSM\ 3}}$	0.368	0.401	0.372	0.329	0.360	0.328	0.353	0.375	0.303	0.378	0.351	0.312	0.344	0.398	**0.180**	0.352	0.333	0.313	0.327	0.360	**0.350**
$\mathrm{P15}_{\mathrm{GFFSM\ 5}}$	0.363	0.375	0.369	0.306	0.358	0.319	0.335	0.319	0.327	0.422	0.306	0.309	0.327	0.411	**0.137**	0.331	0.332	0.319	0.301	0.334	**0.340**
$\mathrm{P16}_{\mathrm{GFFSM\ 3}}$	0.327	0.360	0.363	0.293	0.322	0.342	0.338	0.318	0.359	0.369	0.336	0.330	0.329	0.359	0.353	**0.044**	0.349	0.339	0.233	0.338	**0.334**
$\mathrm{P16}_{\mathrm{GFFSM\ 5}}$	0.292	0.363	0.342	0.319	0.330	0.339	0.340	0.301	0.354	0.348	0.315	0.329	0.334	0.356	0.354	**0.034**	0.347	0.342	0.235	0.344	**0.331**
$\mathrm{P17}_{\mathrm{GFFSM\ 3}}$	0.299	0.358	0.261	0.340	0.423	0.358	0.361	0.267	0.380	0.386	0.260	0.339	0.312	0.349	0.330	0.346	**0.134**	0.294	0.310	0.393	**0.335**
$\mathrm{P17}_{\mathrm{GFFSM\ 5}}$	0.319	0.327	0.254	0.353	0.416	0.355	0.362	0.190	0.368	0.390	0.260	0.359	0.306	0.433	0.353	0.342	**0.121**	0.320	0.323	0.401	**0.338**
$\mathrm{P18}_{\mathrm{GFFSM\ 3}}$	0.323	0.336	0.337	0.366	0.372	0.377	0.374	0.344	0.385	0.409	0.383	0.350	0.351	0.399	0.349	0.326	0.368	**0.070**	0.341	0.336	**0.359**
$\mathrm{P18}_{\mathrm{GFFSM\ 5}}$	0.394	0.343	0.362	0.352	0.416	0.399	0.361	0.366	0.393	0.412	0.389	0.378	0.352	0.388	0.363	0.326	0.405	**0.061**	0.284	0.391	**0.372**
$\mathrm{P19}_{\mathrm{GFFSM\ 3}}$	0.303	0.350	0.387	0.291	0.350	0.343	0.353	0.310	0.372	0.367	0.365	0.371	0.328	0.391	0.385	0.312	0.364	0.287	**0.082**	0.352	**0.346**
$\mathrm{P19}_{\mathrm{GFFSM\ 5}}$	0.298	0.322	0.375	0.300	0.343	0.332	0.346	0.316	0.383	0.342	0.359	0.377	0.352	0.403	0.384	0.310	0.373	0.303	**0.079**	0.343	**0.345**
$\mathrm{P20}_{\mathrm{GFFSM\ 3}}$	0.369	0.408	0.385	0.386	0.314	0.404	0.419	0.366	0.315	0.276	0.385	0.333	0.367	0.341	0.331	0.373	0.329	0.343	0.388	**0.107**	**0.360**
$\mathrm{P20}_{\mathrm{GFFSM\ 5}}$	0.383	0.428	0.395	0.396	0.329	0.380	0.429	0.379	0.318	0.294	0.386	0.313	0.361	0.314	0.310	0.376	0.365	0.365	0.392	**0.086**	**0.364**

TABLE VI
p-VALUES OBTAINED FOR THE THREE DIFFERENT WILCOXON SIGNED-RANK TESTS AND THEIR NULL HYPOTHESES

GFFSM 3				
ALG	$H_0 : \mu_{\text{GFFSM 3}} = \mu_{\text{MOD}}$	$H_0 : \mu_{\text{GFFSM 3}} < \mu_{\text{MOD}}$	$H_0 : \mu_{\text{GFFSM 3}} > \mu_{\text{MOD}}$	CONCLUSION
GFFSM 5	0.021	0.010	0.990	[-]
ARX 1	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 2	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 3	$3.81 \cdot 10^{-6}$	1	$1.91 \cdot 10^{-6}$	[+]
ARX 4	$2.67 \cdot 10^{-5}$	1	$1.34 \cdot 10^{-5}$	[+]
ARX 5	0.004	0.998	0.002	[+]
ARX 6	0.231	0.892	0.115	[=]
ARX 7	0.018	0.009	0.991	[-]
ARX 8	0.005	0.002	0.998	[-]
ARX 9	0.004	0.002	0.998	[-]
ARX 10	0.010	0.005	0.995	[-]
NN 1	$1.20 \cdot 10^{-4}$	1	$6.01 \cdot 10^{-5}$	[+]
NN 2	$8.20 \cdot 10^{-5}$	1	$4.10 \cdot 10^{-5}$	[+]
NN 3	0.121	0.939	0.061	[=]
NN 4	0.232	0.116	0.884	[=]
NN 5	0.022	0.011	0.989	[-]
NN 6	0.003	0.002	0.999	[-]
NN 7	0.002	$8.01 \cdot 10^{-4}$	0.999	[-]
NN 8	$5.93 \cdot 10^{-4}$	$2.96 \cdot 10^{-4}$	1	[-]
NN 9	$1.68 \cdot 10^{-4}$	$8.39 \cdot 10^{-5}$	1	[-]
NN 10	$3.22 \cdot 10^{-4}$	$1.61 \cdot 10^{-4}$	1	[-]
GFFSM 5				
ALG	$H_0 : \mu_{\text{GFFSM 5}} = \mu_{\text{MOD}}$	$H_0 : \mu_{\text{GFFSM 5}} < \mu_{\text{MOD}}$	$H_0 : \mu_{\text{GFFSM 5}} > \mu_{\text{MOD}}$	CONCLUSION
GFFSM 3	0.021	0.990	0.010	[+]
ARX 1	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 2	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
ARX 3	$8.84 \cdot 10^{-5}$	1	$4.42 \cdot 10^{-5}$	[+]
ARX 4	$8.84 \cdot 10^{-5}$	1	$4.42 \cdot 10^{-5}$	[+]
ARX 5	$2.10 \cdot 10^{-4}$	1	$1.05 \cdot 10^{-4}$	[+]
ARX 6	0.040	0.980	0.020	[+]
ARX 7	0.033	0.017	0.983	[-]
ARX 8	0.005	0.003	0.997	[-]
ARX 9	0.005	0.002	0.998	[-]
ARX 10	0.016	0.008	0.992	[-]
NN 1	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
NN 2	$1.91 \cdot 10^{-6}$	1	$9.54 \cdot 10^{-7}$	[+]
NN 3	$2.61 \cdot 10^{-4}$	1	$1.31 \cdot 10^{-4}$	[+]
NN 4	0.179	0.911	0.089	[=]
NN 5	0.601	0.300	0.700	[=]
NN 6	0.255	0.127	0.873	[=]
NN 7	0.151	0.075	0.925	[=]
NN 8	0.073	0.037	0.963	[=]
NN 9	0.042	0.021	0.979	[-]
NN 10	0.090	0.045	0.959	[=]

In order to assess whether significant differences exist among the results of all models, we use the Wilcoxon signed-rank test [79] for pairwise comparison between our models (GFFSM 3 and GFSSM 5) and the rest of competing models. We choose this test because it does not assume normal distributions and because it has been commonly used to compare performance of methods in computational intelligence [80], [81]. To perform the test, we use the standard confidence level of $\alpha = 0.05$.

We have run the Wilcoxon signed-rank test for three different hypotheses: if the average MAEs of our proposed approaches (μ_{GFFSM3} and μ_{GFFSM5}) are *equal to*, *less than*, or *greater than* those obtained by the other modeling techniques (μ_{MOD}). We conclude that our proposal is better (denoted by [+]) if the test rejects both null hypotheses $H_0 : \mu_{\text{GFFSM}} > \mu_{\text{MOD}}$ and $H_0 : \mu_{\text{GFFSM}} = \mu_{\text{MOD}}$. We conclude that our proposal is worse (denoted by $[-]$) if the test rejects both null hypotheses $H_0 : \mu_{\text{GFFSM}} < \mu_{\text{MOD}}$ and $H_0 : \mu_{\text{GFFSM}} = \mu_{\text{MOD}}$. In all other cases, we draw no conclusions (denoted by [=]).

Table VI shows the obtained *p*-values and the drawn conclusions. The results particularly indicate that the GFFSM 3 model is significantly better than the first five ARX models and the first two NNs (while for ARX 6, NN 3, and NN 4, the testings do not provide clear conclusions). The GFFSM 5 model is significantly better than the GFFSM 3, the first six ARX models, and the first three NNs (while for NN 4, NN 5, NN 6, NN 7, NN 8, and NN 10, the testings do not provide clear conclusions). In view of these results, the accuracy of the GFFSMs, in particular that of GFFSM 5, is competitive with almost every NN (all but NN 9) and the first six ARX models.

The good accuracy of our model is also illustrated in Fig. 7. The vertical axis depicts each component of the state activation vector for a gait of the first person obtained using our proposal ($S[t]^{\text{GFFSM3}}$ and $S[t]^{\text{GFFSM5}}$), the THRIFT-FFSM model ($S[t]^{\text{THRIFT}}$; see Section VI-D4), the best tradeoff ARX model (ARX 7) ($S[t]^{\text{ARX7}}$), and the best tradeoff NN architecture (NN 4) ($S[t]^{\text{NN4}}$). The actual values ($S^*[t]$) are reported in the bottom line for comparison. The activation value is represented by means of a gray intensity scale (black means 1 and white means 0). Notice that both ARX 7 and NN 4 calculate the activation vector after the first 0.5 and 0.3 s, respectively, because of the fact that they need the first 50 and 30 samples to operate (0.5 and 0.3 s with a sampling frequency of 100 Hz).

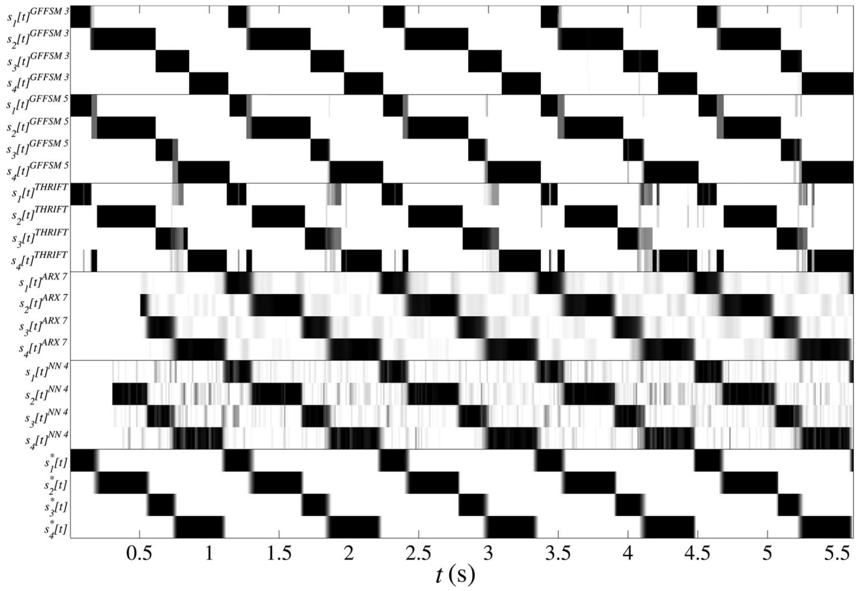

Fig 7. Comparison of the state activation vector ($s_1[t], s_2[t], s_3[t], s_4[t]$) values obtained by means of the two GFFSMs, the THRIFT-FFSM, the ARX 7 model, and the NN 4 model, with the actual values ($s_1^*[t], s_2^*[t], s_3^*[t], s_4^*[t]$) with respect to time.

It can be seen how GFFSM 3 and GFFSM 5 are able to follow the appropriate sequence of states with the correct activation degree.

2) Interpretability Analysis: From the interpretability point of view, both NNs and the ARX models are black-box models which are difficult to be understood by human experts and even more if they have a big number of delayed input variables or a high number of inputs. Nevertheless, our GFFSMs are able to describe and model the human gait phenomenon by means of only eight linguistic fuzzy IF-THEN rules (whose input variables have only three or five associated linguistic labels) achieving a good interpretability–accuracy tradeoff. As an example of how our proposal is describing linguistically the temporal evolution of the accelerations produced during the human gait, a complete RB learned for GFFSM 3 in one of the executions of the GA is shown as follows:

R_{11}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x})
THEN $S[t+1]$ is q_1
R_{22}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is B_{a_x}) **AND** ($a_y[t]$ is M_{a_y} **OR** B_{a_y})
THEN $S[t+1]$ is q_2
R_{33}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is M_{a_y})
THEN $S[t+1]$ is q_3
R_{44}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is S_{a_x} **OR** B_{a_x}) **AND** ($a_y[t]$ is S_{a_y} **OR** M_{a_y})
THEN $S[t+1]$ is q_4
R_{12}: **IF** ($S[t]$ is q_1) **AND** ($a_x[t]$ is S_{a_x}) **AND** ($a_y[t]$ is S_{a_y} **OR** M_{a_y})
THEN $S[t+1]$ is q_2
R_{23}: **IF** ($S[t]$ is q_2) **AND** ($a_x[t]$ is M_{a_x}) **AND** ($a_y[t]$ is S_{a_y})
THEN $S[t+1]$ is q_3
R_{34}: **IF** ($S[t]$ is q_3) **AND** ($a_x[t]$ is M_{a_x} **OR** B_{a_x}) **AND** ($a_y[t]$ is M_{a_y} **OR** B_{a_y})
THEN $S[t+1]$ is q_4
R_{41}: **IF** ($S[t]$ is q_4) **AND** ($a_x[t]$ is S_{a_x} **OR** M_{a_x}) **AND** ($a_y[t]$ is B_{a_y})
THEN $S[t+1]$ is q_1

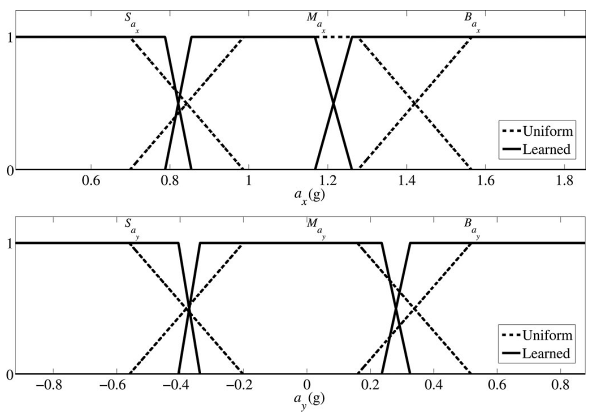

Fig. 8. MFs which comprise the learned DB compared with the original uniformly distributed MFs.

Fig. 8 shows the graphical representation of the learned DB associated with this RB. The initial DB is also plotted, which consists of uniformly distributed MFs. In both cases, the use of SFPs produces comprehensible fuzzy partitions which allow us to get an interpretable fuzzy system.

As mentioned earlier, the main advantage of our model of human gait is its interpretability. The chance to properly understand the obtained model can report a large number of benefits for the designer. For example, in [29], we took advantage of this interpretability to create a model which is aimed to compare the characteristics of different human gaits, which is a different but related problem. With this aim, we elaborated upon two relevant measures of human gait based on the degree of activation and duration of successive states. We called these measures symmetry and homogeneity. Symmetry is a measure of the similarity among accelerations that are produced by steps given by the right leg (states q_1 and q_2) and accelerations that are produced by steps of the left leg (states q_3 and q_4). Homogeneity is a measure of how the same pattern of accelerations is repeated on time, i.e., it is a measure of similarity between each two steps and the following ones. Empirically, we have observed that these measures are characteristic of the style of walking of each person. In the paper mentioned earlier, we showed how to use these parameters to authenticate one person among 11 individuals (see a research work with the same goal but with a different approach in [82]). As in [1] and [41], we think that the model presented in that paper could be used to detect and analyze pathological disorders in the gait in the same way. It seems evident that symmetry and homogeneity will be affected by the presence of gait disorders, i.e., we can check this point measuring the symmetry of gait when a person is carrying a heavy bag in one hand and when she/he is free of that heavy unbalancing load.

Focusing on the current contribution, the expert analysis of the RB and the DB obtained of the human gait GFFSM model constitutes another approach to detect gait disorders. The antecedents of the learned rules in conjunction with the MFs of each variable can provide relevant information about the quality of the gait of a person, i.e., by showing abnormal membership values of the dorsoventral acceleration (a_x) or inconsistent rules not compatible with the expert's knowledge. Moreover, regarding the topic of gait modeling, it is worth noting that the interpretability of the model allows us to calculate relevant temporal features of the gait, i.e., the duration of the states and their temporal sequence. With this information, we can easily count the number of steps and the duration of each of them and therefore the instantaneous walking speed. This is a significant issue in gait disorder analysis because of the fact that, e.g., patients tend to alter speed in order to accommodate loads that are applied on the knee.

3) Computational Cost Analysis: We have already compared the different algorithms in terms of the complexity and accuracy. Nevertheless, it is also interesting to evaluate their computational cost. The average times needed to build GFFSM 3 and GFFSM 5 models were 4076 and 6022 s, respectively, while the ARX 1, ARX 7, and ARX 10 models took 0.25, 72, and 630 s, respectively. NN 1, NN 4, and NN 10 took 40, 118, and 314 seconds, respectively. All the methods were run in a single computer, with 4 GB RAM and an Intel Core 2 Quad Q8400 with 2.66 GHz.

As expected, the GFFSMs spent a larger run time as they do not only involve parameter estimation but also structure identification. As said in Section V-B, the dependence of the next state on the previous state in our GFFSM makes it strictly necessary to test the FFSM over the whole dataset for each chromosome evaluation, which is very computationally expensive. Nevertheless, the additional interpretability advantage makes this computational cost increase worthy. In addition, while NNs and ARX models are implemented in well-established and -optimized libraries, the GFFSMs were programmed in not optimized MATLAB code (more refined implementations could be done in the future).

4) Importance of the Use of Expert Knowledge Analysis: Our GFFSMs are designed to take advantage from the available expert knowledge, exploiting the power of fuzzy systems which are capable of integrating this knowledge with machine learning techniques. The possibility to merge expert information with the information derived from data using GAs allows us to obtain a rough linguistic description of the gait, i.e., the final set of fuzzy rules obtained provides a linguistic description of the phenomenon. In the current GFFSMs, the designer has chosen a model of human gait with four basic fuzzy states that are easily recognized when we observe a walking person (see Fig. 1). Applying this constraint in the model, the designer makes the model easily understandable. Then, the GA explores possibilities into this restricted framework to define the final model structure and to estimate its parameters.

Even so, we have also decided to check whether the proposed GFFSM method is powerful enough to handle the overall learning problem, i.e., to extract the whole model (fuzzy rule set) structure from scratch along with the relevant labels and MFs in the case of three linguistic labels per input variable. We have assumed full ignorance of the RB and tried to build an FFSM using the classical genetic learning method proposed by Thrift [83] to derive the RB of the FFSM keeping the previous derivation of the DB based on a GA with real-coded chromosomes (from now on, this method will be called THRIFT-FFSM).

Thrift's RB derivation method is based on encoding all the cells of the complete decision table in the chromosomes. In our case, we have three antecedents: the current state (with four different possibilities corresponding to the number of possible states, the only information provided by the expert in the current experiment, together with the granularity of the fuzzy partitions), the input variable a_x (with three different linguistic labels corresponding to S_{a_x}, M_{a_x}, and B_{a_x}), and the input variable a_y (with another three different linguistic labels corresponding to S_{a_y}, M_{a_y}, and B_{a_y}). Therefore, the decision table will be a 3-D structure of size $4 \times 3 \times 3$ consisting of a total of 36 possible rules. Each cell of the decision table will represent the output of each fuzzy rule by means of an integer coding scheme represented by the set $\{0, 1, 2, 3, 4\}$, where 0 indicates the absence of the rule and $1, 2, 3$, or 4 indicates that the next state will be q_1, q_2, q_3, or q_4, respectively. Hence, we substitute the first part of the chromosome in Fig. 4 (RB part), which is composed of 48 binary genes (see Section V-A), by an integer-coded array of size 36, encoding the consequents for each possible rule. The resulting coding scheme has, thus, 44 genes (36 for the RB part plus 8 for the DB part).

We have used the same genetic operators for the GA as explained in Section V and the same parameter values shown

TABLE VII
MAE OF THE LEAVE-ONE-OUT FOR THE DATASETS OF THE FIRST AND SECOND PERSON, WITH THE AVERAGE (MEAN) AND STANDARD DEVIATION (STD) FOR THE TWO EVALUATED MODELS

FOLD	PERSON 1		PERSON 2	
	THRIFT-FFSM	GFFSM 3	THRIFT-FFSM	GFFSM 3
1	0.115	0.089	0.054	0.049
2	0.085	0.066	0.091	0.072
3	0.115	0.135	0.072	0.065
4	0.102	0.108	0.107	0.047
5	0.205	0.133	0.092	0.072
6	0.172	0.078	0.107	0.037
7	0.083	0.101	0.169	0.048
8	0.111	0.149	0.051	0.066
9	0.152	0.086	0.079	0.124
10	0.083	0.081	0.121	0.046
MEAN	0.122	0.103	0.094	0.063
STD	0.041	0.028	0.035	0.025

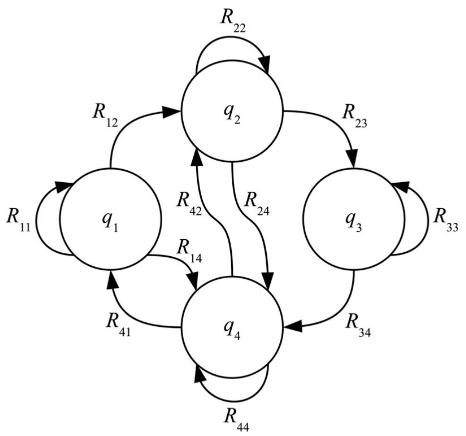

Fig. 9. State diagram obtained with Thrift's RB derivation.

in Section VI-A2, except the bitwise mutation (designed for binary-coded chromosomes) which was replaced by the original Thrift's mutation operator that randomly adds or subtracts 1 (with equal probability) to the current value of the allele within the set $\{0, 1, 2, 3, 4\}$.

Table VII shows the MAE that is obtained for each fold of the leave-one-out corresponding to the first and second person using Thrift's RB derivation keeping the DB derivation (THRIFT-FFSM) compared with our expert information-based proposal (GFFSM 3). It also depicts the average value of the MAE and its standard deviation for the ten folds. As can be seen, the lack of expert knowledge pays the cost of larger test errors. Moreover, the average training time for each THRIFT-FFSM model is 10 855 s, while the original GFFSM takes an average of 4076 s.

We can also examine whether the RB extracted by the THRIFT-FFSM model resembles to the expert knowledge-based one. As an example, the RB obtained for the seventh fold of the first person (with a MAE of 0.083) is shown as follows:

R_{11}^{1}: **IF** $(S[t]$ is $q_1)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $S_{a_y})$
THEN $S[t+1]$ is q_1
R_{11}^{2}: **IF** $(S[t]$ is $q_1)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_1
R_{11}^{3}: **IF** $(S[t]$ is $q_1)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_1
R_{11}^{4}: **IF** $(S[t]$ is $q_1)$ **AND** $(a_x[t]$ is $M_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_1
R_{22}^{5}: **IF** $(S[t]$ is $q_2)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_2
R_{33}^{6}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_3
R_{33}^{7}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $M_{a_x})$ **AND** $(a_y[t]$ is $S_{a_y})$
THEN $S[t+1]$ is q_3
R_{33}^{8}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_3
R_{33}^{9}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $M_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_3
R_{33}^{10}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_3
R_{44}^{11}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_4
R_{12}^{12}: **IF** $(S[t]$ is $q_1)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $S_{a_y})$
THEN $S[t+1]$ is q_2
R_{14}^{13}: **IF** $(S[t]$ is $q_1)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_4
R_{23}^{14}: **IF** $(S[t]$ is $q_2)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $S_{a_y})$
THEN $S[t+1]$ is q_3
R_{24}^{15}: **IF** $(S[t]$ is $q_2)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_4
R_{34}^{16}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_4
R_{34}^{17}: **IF** $(S[t]$ is $q_3)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_4
R_{41}^{18}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is $M_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_1
R_{41}^{19}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_1
R_{42}^{20}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $S_{a_y})$
THEN $S[t+1]$ is q_2
R_{42}^{21}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is $B_{a_x})$ **AND** $(a_y[t]$ is $M_{a_y})$
THEN $S[t+1]$ is q_2
R_{42}^{22}: **IF** $(S[t]$ is $q_4)$ **AND** $(a_x[t]$ is $S_{a_x})$ **AND** $(a_y[t]$ is $B_{a_y})$
THEN $S[t+1]$ is q_2

It consists of 22 rules (14 rules were automatically discarded) which, as can be seen in the state diagram shown in Fig. 9, are not able to capture the expert knowledge represented by the state diagram of the human gait shown in Fig. 2. It presents some weird transitions as that represented in rule number 13 from the state q_1 to the state q_4 or the transitions between states q_2 and q_4 in rules 15, 20, 21, and 22. The effects of these transitions are reflected in Fig. 7, where the state activation vector corresponding to the THIRFT-FFSM model ($S[t]^{\mathrm{THRIFT}}$) activates q_4 when going from the state q_1 to the state q_2.

In summary, it is clear that the full consideration of the expert knowledge is the best way to design an FFSM for the human gait modeling problem by means of GAs.

VII. CONCLUDING REMARKS

We have presented a practical application where we described how to build FFSMs to model the human gait of a set of people by using GAs and expert knowledge. We have defined the principal elements of the human gait cycle and developed a genetic learning procedure for FFSMs to model the gait cycle for each person. It has been shown how this GFS can obtain automatically the fuzzy rules and the fuzzy MFs associated with the linguistic terms of the FFSM, while the states and transitions are defined by the expert, thus maintaining the knowledge that she/he has about the problem. To incorporate this expert knowledge, we have designed a user-friendly graphical interface to define the fuzzy states of the human gait. The results obtained showed the goodness of our proposal.

We have increased the capabilities of FFSMs with a novel GA-based procedure for the automatic definition of its KB. Therefore, a great number of opportunities arise. We can set out new applications of system modeling by means of GFFSMs. The ability of our proposal to combine the available expert knowledge with the accuracy achieved by the learning process can be used to study several signals where the human interaction is demanded. Examples of application could range from biomedical engineering (e.g., electroencephalogram or electrocardiogram signals) to other time series analysis (e.g., econometrics or natural processes).

Our next research work in this direction consists of developing a model of the human gait where gait symmetry and homogeneity can be analyzed in detail. This work will include the automatic generation of linguistic reports about the human gait quality.

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Prof. Cordón received the UGR Young Researcher Career Award in 2004. In July 2010, he received the IEEE Computational Intelligence Society (CIS) Outstanding Early Career Award in its 2011 edition: the first such award conferred. In 2011, his developments on soft computing methods in forensic identification were recognized with the International Fuzzy Systems Association Award for Outstanding Applications of Fuzzy Technology and with the granting of the European Society for Fuzzy Logic and Technologies Best Ph.D. Thesis Award to Dr. Ibañez's Ph.D. dissertation, co-advised by Dr. Cordón. He is in the Editorial Board of ten international journals and was recognized as the Outstanding Associate Editor of the IEEE TRANSACTIONS ON FUZZY SYSTEMS in 2008. He is a reviewer for more than 30 international journals. Since 2004, he has taken many different representative positions with EUSFLAT and the IEEE CIS. He is currently a member of the IEEE CIS AdCom (term 2010–2012).

6.2 Linguistic description of the human gait quality

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Linguistic description of the human gait quality

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The human gait is a complex phenomenon that is repeated in time following an approximated pattern. Using a three-axial accelerometer fixed in the waist, we can obtain a temporal series of measures that contains a numerical description of this phenomenon.

Nevertheless, even when we represent graphically these data, it is difficult to interpret them due to the complexity of the phenomenon and the huge amount of available data. This paper describes our research on designing a computational system able to generate linguistic descriptions of this type of quasi-periodic complex phenomena.

We used our previous work on both, Granular Linguistic Models of Phenomena and Fuzzy Finite State Machines, to create a basic linguistic model of the human gait. We have used this model to generate a human friendly linguistic description of this phenomenon focused on the assessment of the gait quality. We include a practical application where we analyze the gait quality of healthy individuals and people with lesions in their limbs.

1. Introduction

Human beings describe phenomena in our environment using natural language (NL). In order to perform this task, we interpret the available data using our experience in both, namely, the field of knowledge that allow us to recognize the phenomena and our experience on using NL.

Our research line deals with the design and development of a new family of computational systems capable of generating linguistic descriptions of complex phenomena, i.e., these computational systems obtain data from a phenomenon and provide linguistic descriptions that are relevant for specific users in specific contexts. This type of systems will be used in supervision and control applications and especially in the development of user interfaces based on the use of NL.

Human gait is a quasi-periodic phenomenon which is defined as the interval between two successive events (usually heel contact) of the same foot (Begg et al., 2007). This process is characterized by a stance phase (that approximately takes 60% of the total gait cycle), where at least one foot is in contact with the ground, and a swing phase (approximately 40% of the total gait cycle), during which one limb swings through the next heel contact. Gait phases can be quite different between individuals but when normalized to a percentage of the gait cycle they maintain close similarity, indicating the absence of disorders (Perry, 1992). Fig. 1 shows two different synchronized pictures. The top picture plots a sketch of a person representing the different phases of the gait with the right limb boldfaced. The picture at the bottom represents the time period from one event (usually initial contact) of one foot to the subsequent occurrence of initial contact of the same foot.

Due to the fact that human gait is a complex integrated task which requires precise coordination of the neural and musculoskeletal system to ensure correct skeletal dynamics (Winter, 1990), its analysis can help in the diagnosis and treatment of walking and movement disorders, identification of balance factors, and assessment of clinical gait interventions and rehabilitation programs (Hamacher et al., 2011; Lai et al., 2009; Moustakidis et al., 2010; Sant'Anna et al., 2011; Wren et al., 2011).

In human gait analysis, there are a huge number of variables obtained by means of different measurement techniques. Most gait parameters can be categorized as anthropometric data which include height, weight, or limb length; spatiotemporal data comprising variables such as walking speed, step length, or phases time span; kinematic data of measurements of joint angles, displacement, or acceleration along axes; kinetic data variables including foot force and torques; or electromyographic data which measures the muscle activation levels.

Two of the most common approaches to manage and analyze human gait kinematic data are the computer vision approach (Tafazzoli and Safabakhsh, 2010) and the sensor-based one. The main advantage of the computer vision approach is the avoidance of placing sensors on the user's body. However, an expensive and

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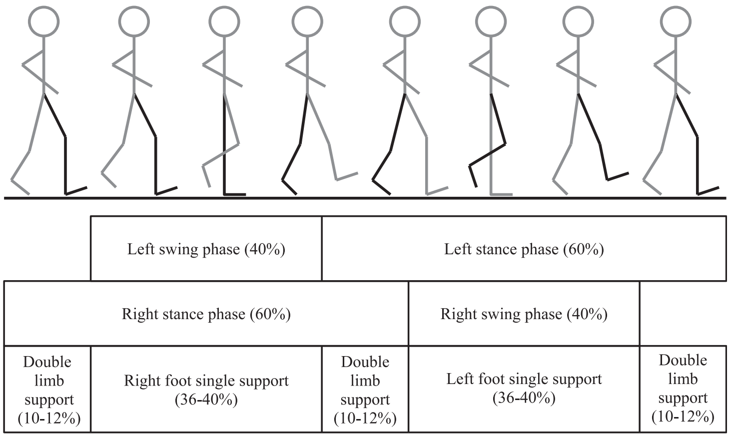

Fig. 1. One gait cycle illustrating the four main phases.

complex system for capturing images is needed. Moreover, these methods usually work in lab but fail in real world scenarios due to clutter, variable light intensity and contrast. On the other hand, the sensor-based approach consists of using small sensors (usually accelerometers) placed in the body of the person. This solution provides a smart solution to the problem of capturing the signal, where data can be obtained anywhere by means of a smartphone. Moreover, they can be used in the dark and provide three-dimensional data. This line of research has attracted an important number of researchers that focus the problem of human gait modeling from different perspectives (see, e.g., Alaqtash et al., 2011; Najafi et al., 2003).

Our approach is based on the Computational Theory of Perceptions (CTP). This field was introduced in the Zadeh's (1999) seminal paper "From computing with numbers to computing with words—from manipulation of measurements to manipulation of perceptions" and further developed in subsequent papers. CTP provides a framework to develop computational systems with the capacity of computing with the meaning of NL expressions, i.e., with the capacity of computing with imprecise descriptions of the world in a similar way that humans do it. In CTP, a granule is a clump of elements which are drawn together by indistinguishability, similarity, proximity or functionality (Zadeh, 1979). The boundary of a granule is fuzzy. Fuzziness of granules allow us to model the way in which human concepts are formed, organized and manipulated in an environment of imprecision, uncertainty, and partial truth (Zadeh, 1997). A granule underlies the concept of a linguistic variable (Zadeh, 2008). A linguistic variable is a variable whose values are words or sentences in NL (Zadeh, 1975a,b,c).

In this paper, we do an extensive use of our previous research, contributing to the human gait quality analysis field by providing a new technique for modeling this type of phenomenon. We have developed a computational application that uses a single three-axial accelerometer to generate linguistic descriptions for assessing the human quality. Here, we develop upon our previous research on the Granular Linguistic Model of a Phenomenon (GLMP) improving its expressiveness by introducing a new type of components based on the concept of Fuzzy Finite State Machine (FFSM). First, we identify the relevant phases of the gait based on the accelerations produced during the process. Once the phases are recognized, we use two relevant features of the human gait (homogeneity and symmetry) to evaluate the gait quality corresponding to a specific person. Finally, we develop a method for producing a linguistic report about the quality of the gait in terms of the homogeneity and the symmetry. This type of reports could be used to analyze the evolution of the human gait, e.g., after a recovery treatment and also for preventing falls in elderly people.

The remainder of this paper is organized as follows. Section 2 presents the main concepts of our approach to linguistic description of complex phenomena evolving in time. Section 3 describes how to use these concepts for the linguistic description of the human gait quality. Section 4 describes the experimentation carried out, by describing the experimental setup and discussing the results. Finally, Section 5 draws some conclusions and introduces some future research works.

2. Linguistic description of phenomena evolving in time

Our approach to computational model of phenomena is based on subjective perceptions of a domain expert that we call the designer. The more experienced designer, with better understanding and use of NL in the application domain, the richer the model with more possibilities of achieving and responding to final users' needs and expectations. The designer uses the resources of the computer, e.g., sensors, to acquire data about a phenomenon and uses her/his own experience to interpret these data and to create a model of the phenomenon. Then the designer uses the resources of the computer to produce the linguistic utterances.

In this section, we introduce the components of the GLMP, our approach based on CTP for developing computational systems able to generate linguistic descriptions of phenomena (Alvarez-Alvarez et al., 2011a; Eciolaza and Trivino, 2011; Mendez-Nunez and Trivino, 2010; Trivino et al., 2010b).

2.1. Computational perception (CP)

A *CP* is the computational model of a unit of information acquired by the designer about the phenomenon to be modeled. In general, *CP*s correspond to particular details of the phenomenon at certain degrees of granularity. A *CP* is a couple (A, W) where:

$A=(a_1,a_2,\ldots,a_n)$ is a vector of n linguistic expressions (words or sentences in NL) that represents the whole linguistic domain of the *CP*. Each a_i describes the value of the *CP* in each situation with specific granularity degree. These sentences can be either simple, e.g., a_i="*The dorso-ventral acceleration is high*" or more complex, e.g.,

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a_i="*The homogeneity during the double limb support of the reference foot is low*".

$W=(w_1,w_2,\ldots,w_n)$ is a vector of validity degrees $w_i \in [0,1]$ assigned to each a_i in the specific context. The concept of validity depends on the application, e.g., it is a function of the truthfulness of each sentence in its context of use.

2.2. Perception mapping (PM)

We use *PM*s to create and aggregate *CP*s. There are many types of *PM*s and this paper explores several of them and contributes to this research line including a new one. A *PM* is a tuple (U, y, g, T) where:

U is a vector of input *CP*s, $U=(u_1,u_2,\ldots,u_n)$, where $u_i=(A_{u_i},W_{u_i})$ and n is the number of input *CP*s. In the special case of the first order perception mappings (1-*PM*s), these are the inputs to the GLMP, which are values $z \in \mathbb{R}$ provided either by a sensor or obtained from a database.

y is the output *CP*, $y=(A_y,W_y)$.

g is an aggregation function employed to calculate the vector of validity degrees assigned to each element in y, $W_y=(w_1,w_2,\ldots,w_{n_y})$. It is an aggregation of input vectors, $W_y=g(W_{u_1},W_{u_2},\ldots,W_{u_n})$, where W_{u_i} are the validity degrees of the input perceptions. In Fuzzy Logic, many different types of aggregation functions have been developed. For example, g might be implemented using a set of fuzzy rules. In the case of 1-*PM*s, g is built using a set of membership functions as follows:

$$W_y=(\mu_{a_1}(z),\mu_{a_2}(z),\ldots,\mu_{a_{n_y}}(z))=(w_1,w_2,\ldots,w_{n_y})$$

where W_y is the vector of degrees of validity assigned to each a_y, and $z \in \mathbb{R}$ is the input data.

T is a text generation algorithm that allows generating the sentences in A_y. In simple cases, T is a linguistic template, e.g., "*The dorso-ventral acceleration is* {*low*|*medium*|*high*}".

2.3. Granular Linguistic Model of a Phenomenon

The GLMP consists of a network of *PM*s. Each *PM* receives a set of input *CP*s and transmits upwards an output *CP*. We say that each output *CP* is explained by the *PM* using a set of input *CP*s. In the network, each *CP* covers specific aspects of the phenomenon with certain degree of granularity. Fig. 2 shows an example of a GLMP. In this example, the phenomenon can be described at a very basic level in terms of three variables providing values z_1, z_2, and z_3 respectively at a certain instant of time.

Using different aggregation functions and different linguistic expressions, the GLMP paradigm allows the designer to model computationally her/his perceptions. In the case of Fig. 2, other two higher-level descriptions of the phenomenon are provided. These descriptions are given in the form of computational perceptions CP_4 and CP_5. The second order perception mappings (2-*PM*s) PM_4 and PM_5 indicate that CP_4 and CP_5 can be explained in terms of CP_1, CP_2, and CP_3, i.e., how the validity of each item in CP_4 and CP_5 is explained by those of CP_1, CP_2, and CP_3. Finally, the top-order description of the phenomenon is provided, at the highest level of abstraction, by CP_6, explained by PM_6 in terms of CP_4 and CP_5. Notice that, using this structure, one can provide not only a linguistic description of the phenomenon at a certain level, but also an explanation in terms of linguistic expressions at a lower level.

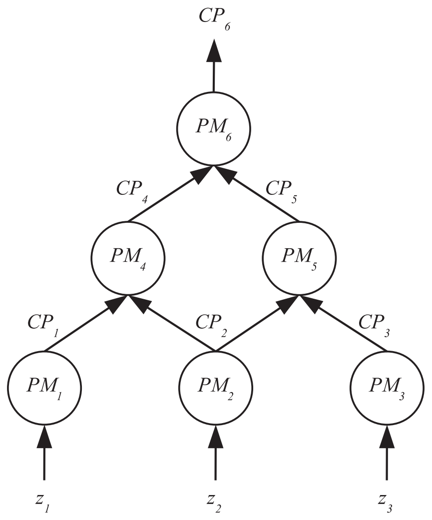

Fig. 2. Example of a GLMP.

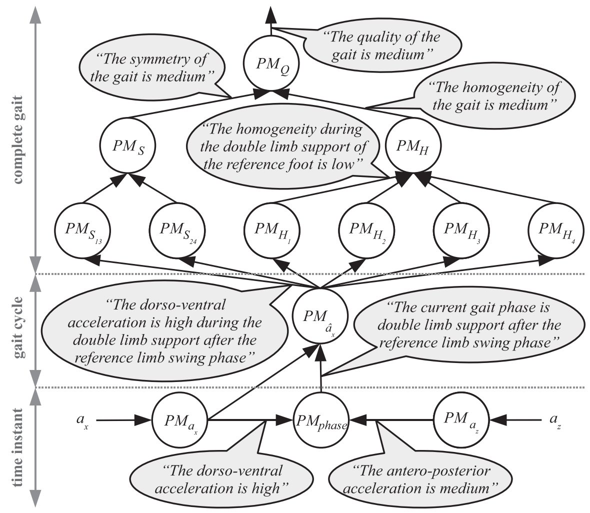

Fig. 3. GLMP for the linguistic description of the human gait quality divided into three different levels of granularity.

2.4. Report generation

Once the GLMP is fed with input data, the aggregation functions are used to calculate the weights corresponding to potentially hundreds of linguistic expressions. Now, the challenge consists of choosing the more adequate combination of these sentences to generate a useful linguistic description of the phenomenon evolution including the current state. The design of this report requires a deep analysis of the application domain of language and, therefore, the collaboration of the specific final user.

In this paper, we provide a simple solution to this problem. Here, we are focused on a demonstration of concept that can be solved with a simple report. For this first version of our human gait report generator, we will choose the linguistic expressions

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with the highest validity degree and we will include detailed explanations of a perception using the conjunction "because".

3. Linguistic description of the human gait quality

Fig. 3 shows the GLMP designed for the linguistic description of the human gait quality. The grey speech bubbles show different examples of linguistic expressions associated to several *CP*s.

We use the dorso-ventral acceleration (a_x) and the antero-posterior acceleration (a_y) to identify the relevant phases of the human gait. Then, using the information provided by the human gait phases, we analyze the homogeneity and the symmetry of each phase based on the dorso-ventral acceleration (a_x) during each phase. Finally we use the total homogeneity and symmetry of the gait to obtain its quality. In the following subsections, we explain the different *PM*s and related *CP*s in the model.

It is worth remarking that the trapezoidal membership functions and the sets of rules used below were designed empirically after an important experimental effort. Nevertheless, in order to apply this model in practice, these parameters should be tuned according with the criteria of the specific final user. Moreover, we must say the same regarding with the definition of the most suitable set of sentences for describing each *CP*.

3.1. Dorso-ventral acceleration perception mapping (PM_{a_x})

It is a 1-*PM* whose input is the numerical value of the dorso-ventral acceleration ($a_x \in \mathbb{R}$) that is normalized by subtracting its average.

The output *CP* y_{a_x} includes the following set of NL sentences:

$a_{1_{a_x}} \rightarrow$ "*The dorso-ventral acceleration is low*"
$a_{2_{a_x}} \rightarrow$ "*The dorso-ventral acceleration is medium*"
$a_{3_{a_x}} \rightarrow$ "*The dorso-ventral acceleration is high*"

The validity degrees ($w_{1_{a_x}}, w_{2_{a_x}}, w_{3_{a_x}}$) are obtained by means of the aggregation function g_{a_x}, which uses a set of trapezoidal membership functions, i.e., to obtain these validity degrees from the input variable a_x, we fuzzify the numerical values using three linguistic labels which consist of uniformly distributed trapezoidal membership functions: $\{L_{a_x}, M_{a_x}, H_{a_x}\}$, where L_{a_x}, M_{a_x} and H_{a_x} are linguistic terms representing low, medium, and high respectively in such a way that validity degrees are directly $w_{1_{a_x}} = L_{a_x}(a_x)$, $w_{2_{a_x}} = M_{a_x}(a_x)$, and $w_{3_{a_x}} = H_{a_x}(a_x)$.

3.2. Antero-posterior acceleration perception mapping (PM_{a_z})

This 1-*PM* is similar to PM_{a_x}. It has the numerical value of the antero-posterior acceleration ($a_z \in \mathbb{R}$) as input. This input variable a_z is also normalized by subtracting its average value.

The output *CP* y_{a_z} includes the following set of NL sentences:

$a_{1_{a_z}} \rightarrow$ "*The antero-posterior acceleration is low*"
$a_{2_{a_z}} \rightarrow$ "*The antero-posterior acceleration is medium*"
$a_{3_{a_z}} \rightarrow$ "*The antero-posterior acceleration is high*"

The validity degrees ($w_{1_{a_z}}, w_{2_{a_z}}, w_{3_{a_z}}$) are also directly obtained from trapezoidal membership functions: $w_{1_{a_z}} = L_{a_z}(a_z)$, $w_{2_{a_z}} = M_{a_z}(a_z)$, and $w_{3_{a_z}} = H_{a_z}(a_z)$.

3.3. Gait phase perception mapping (PM_{phase})

This 2-*PM* has two 1-*CP*s as inputs: the dorso-ventral acceleration and the antero-posterior acceleration. Therefore, the set of input *CP*s is $U = (u_{a_x}, u_{a_z})$.

According to the diagram of Fig. 1 and using our own knowledge about the process, we define four different phases which explain when double limb support, reference limb single support, or opposite limb single support are produced. Therefore, the output *CP* y_{phase} identifies different four gait phases having the following set of four possible sentences:

$a_{1_{phase}} \rightarrow$ "*The current gait phase is double limb support after the reference limb swing phase*"
$a_{2_{phase}} \rightarrow$ "*The current gait phase is reference limb single support and swing phase of the opposite limb*"
$a_{3_{phase}} \rightarrow$ "*The current gait phase is double limb support after the opposite limb swing phase*"
$a_{4_{phase}} \rightarrow$ "*The current gait phase is opposite limb single support and swing phase of the reference limb*"

The aggregation function (g_{phase}) calculates, at each time instant, the next value of the validity degrees for each sentence based on the previous validity degrees and current input *CP*s. The aggregation function is, therefore, an expert knowledge based FFSM. In a previous work, we have used a model of the human gait based on a FFSM to recognize the gait pattern of a specific person (Trivino et al., 2010a). Our model differs significantly from others, e.g., based on machine learning techniques, because we use a linguistic model to represent the subjective designer's perceptions of the human gait process. This model is easily understood and does not require high computational cost. Nevertheless, there exists the possibility of making use of an automatic machine learning technique to tune the elements of the FFSM as explained in Alvarez-Alvarez et al. (in press).

Due to the characteristics of the human gait as a quasi-periodic process, there are eight fuzzy rules in total in the system, four rules to remain in each phase and other four to change between phases. We chose the phase 1 as the initial phase, i.e., the sentence "*The current gait phase is double limb support after the reference limb swing phase*" will initially have a validity degree of 1. In this way, the FFSM will synchronize with the gait, without the need of doing previous segmentation of the signal, when the conditions to be in that phase are fulfilled. We defined the conditions over the input *CP*s to remain in a state or to change between states by combining the information obtained from the sensors and the available expert knowledge about the human gait. The rule base of g_{phase} is as follows:

R_{11}: IF $a_{1_{phase}}$ AND $a_{3_{a_x}}$ AND ($a_{1_{a_z}}$ OR $a_{2_{a_z}}$) AND $T_{stay_1}(d_1)$ THEN $a_{1_{phase}}$
R_{22}: IF $a_{2_{phase}}$ AND ($a_{1_{a_x}}$ OR $a_{2_{a_x}}$) AND $T_{stay_2}(d_2)$ THEN $a_{2_{phase}}$
R_{33}: IF $a_{3_{phase}}$ AND $a_{3_{a_x}}$ AND ($a_{1_{a_z}}$ OR $a_{2_{a_z}}$) AND $T_{stay_3}(d_3)$ THEN $a_{3_{phase}}$
R_{44}: IF $a_{4_{phase}}$ AND ($a_{1_{a_x}}$ OR $a_{2_{a_x}}$) AND $T_{stay_4}(d_4)$ THEN $a_{4_{phase}}$
R_{12}: IF $a_{1_{phase}}$ AND $a_{3_{a_z}}$ AND $T_{change_1}(d_1)$ THEN $a_{2_{phase}}$
R_{23}: IF $a_{2_{phase}}$ AND $a_{3_{a_x}}$ AND $T_{change_2}(d_2)$ THEN $a_{3_{phase}}$
R_{34}: IF $a_{3_{phase}}$ AND $a_{3_{a_z}}$ AND $T_{change_3}(d_3)$ THEN $a_{4_{phase}}$
R_{41}: IF $a_{4_{phase}}$ AND $a_{3_{a_x}}$ AND $T_{change_4}(d_4)$ THEN $a_{1_{phase}}$

Where

- The first term in the antecedent computes the previous validity degree of the sentence $a_{i_{phase}}$, i.e., $w_{i_{phase}}$. With this mechanism, we only allow the FFSM to change from the phase i to the phase j (or to remain in phase i, when $i=j$). For example, in R_{11}, it is computed the validity degree of the sentence "*The current gait phase is double limb support after the reference limb swing phase*" ($w_{1_{phase}}$).
- The second term in the antecedent describes the constraints imposed on the dorso-ventral acceleration input *CP* (u_{a_x}).

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It computes the validity degree of one or two of the three possible sentences that this *CP* has, e.g., "*the dorso-ventral acceleration is low*" ($w1_{a_x}$).

- The third term in the antecedent describes the constraints imposed on the antero-posterior acceleration input *CP* (u_{a_z}). It computes the validity degrees of one or two of the three possible sentences that this *CP* has, e.g., "*the antero-posterior acceleration is low or medium*" ($w1_{a_z}$ OR $w2_{a_z}$).
- The fourth term in the antecedent describes the conditions that constrain the phases duration. To control this duration, we define two linguistic labels for each phase *i*: T_{stay_i} (which is the maximum time that the phase *i* is expected to lasts) and T_{change_i} (which is the minimum time that phase *i* is expected to lasts before changing to phase *j*). For example, in R_{11}, we calculate the membership degree of d_1 to the linguistic label T_{stay}, where d_1 is the time that $w1_{phase} > 0$. Fig. 4 shows an example of the linguistic labels T_{stay_1} and T_{change_1} used to define the temporal constraints of phase 1. In agreement with our knowledge about the typical human gait cycle, we assign to each phase a duration according to its percentage of the gait period *T*, which is calculated using the Fast Fourier Transform (FFT) (Brigham and Morrow, 1967) over the antero-posterior acceleration (a_z).
- Finally, the consequent of the rule defines the next phase. To calculate the validity degrees of the sentences associated with each phase *j* ($w_{j_{phase}}$), a weighted average using the firing degree of each rule R_{ij} (ϕ_{ij}) is computed as defined in Eq. (1):

$$w_{j_{phase}} = \frac{\sum_{i=1}^{4} \phi_{ij}}{\sum_{i=1}^{4} \sum_{j=1}^{4} \phi_{ij}} \tag{1}$$

where ϕ_{ij} is calculated using the minimum for the AND operator and the bounded sum of Łukasiewicz (Alsina et al., 2006) for the OR operator.

Note that each rule of this set is, therefore, a complete linguistic expression as can be seen in the following expanded expression of the rule R_{11} to remain in phase 1: "*If the previous gait phase is double limb support after the reference limb swing phase, and the dorso-ventral acceleration is low, and the antero-posterior acceleration is low or medium, and it is time to remain in this phase. Then, the current gait phase is double limb support after the reference limb swing phase*".

As an example of the performance of our proposal for human gait modeling, Fig. 5 represents the validity degrees of each sentence together with the dorso-ventral acceleration input variable (u_{a_x}) and the antero-posterior acceleration input variable (u_{a_z}). It shows how this set of fuzzy rules is able to model linguistically the four phases of the human gait.

It is worth noting that this is an especial type of *PM*, which is applied here for the first time, i.e, in this paper, we contribute to this research field by combining both of our previous results, namely, GLMP and FFSM. The interested reader could see our previous papers on FFSM for a more detailed description of this paradigm and its applications (Alvarez-Alvarez et al., 2010, 2011b, in press; Trivino et al., 2010a).

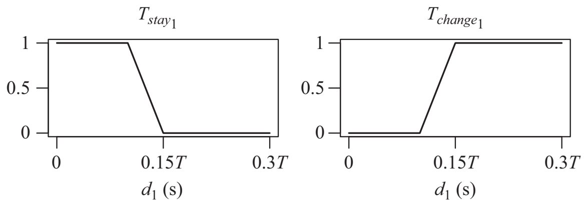

Fig. 4. Temporal conditions for phase 1.

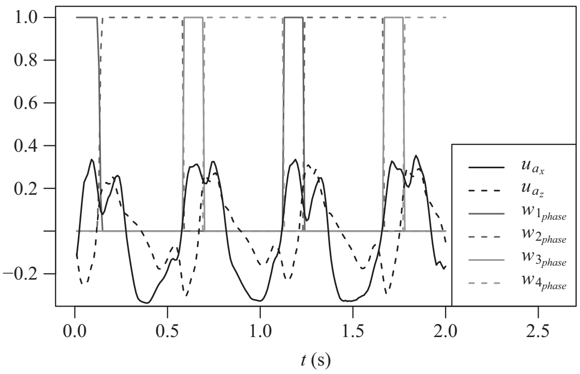

Fig. 5. Graphical representation of the validity degrees of each sentence together with the evolution of the dorso-ventral acceleration input variable (u_{a_x}) and the antero-posterior acceleration input variable (u_{a_z}).

3.4. Dorso-ventral acceleration during each gait phase perception mapping ($PM_{\hat{a}_x}$)

This *PM* belongs to an upper level of granularity (gait cycle level). Its output *CP* is calculated for each gait cycle instead of being calculated at each time instant. As can be seen in Fig. 3, it has two *CP*s as inputs: the dorso-ventral acceleration and the gait phase, i.e., $U=(u_{a_x}, u_{phase})$.

The output *CP* $y_{\hat{a}_x}$ includes the following set of NL sentences:

$a11_{\hat{a}_x}$ → "*The dorso-ventral acceleration is low during the double limb support after the reference limb swing phase*"
$a12_{\hat{a}_x}$ → "*The dorso-ventral acceleration is low during the reference limb single support and swing phase of the opposite limb*"
$a13_{\hat{a}_x}$ → "*The dorso-ventral acceleration is low during the double limb support after the opposite limb swing phase*"
$a14_{\hat{a}_x}$ → "*The dorso-ventral acceleration is low during the opposite limb single support and swing phase of the reference limb*"
$a21_{\hat{a}_x}$ → "*The dorso-ventral acceleration is medium during the double limb support after the reference limb swing phase*"
$a22_{\hat{a}_x}$ → "*The dorso-ventral acceleration is medium during the reference limb single support and swing phase of the opposite limb*"
$a23_{\hat{a}_x}$ → "*The dorso-ventral acceleration is medium during the double limb support after the opposite limb swing phase*"
$a24_{\hat{a}_x}$ → "*The dorso-ventral acceleration is medium during the opposite limb single support and swing phase of the reference limb*"
$a31_{\hat{a}_x}$ → "*The dorso-ventral acceleration is high during the double limb support after the reference limb swing phase*"
$a32_{\hat{a}_x}$ → "*The dorso-ventral acceleration is high during the reference limb single support and swing phase of the opposite limb*"
$a33_{\hat{a}_x}$ → "*The dorso-ventral acceleration is high during the double limb support after the opposite limb swing phase*"
$a34_{\hat{a}_x}$ → "*The dorso-ventral acceleration is high during the opposite limb single support and swing phase of the reference limb*"

The aggregation function ($g_{\hat{a}_x}$) calculates, for each gait cycle *k*, the validity degrees $w_{ij_{\hat{a}_x}}$ of each sentence. This function is defined by Eq. (2), which merges the validity degrees of the input *CP*s at each time instant t during the total duration T of each cycle k:

$$w_{ij_{\hat{a}_x}}[k] = \frac{\sum_{t=0}^{T} w_{i_{\hat{a}_x}}[t] \cdot w_{j_{phase}}[t]}{\sum_{t=0}^{T} w_{j_{phase}}[t]} \tag{2}$$

3.5. *Symmetry of the phases perception mapping ($PM_{S_{13}}$, $PM_{S_{24}}$)*

In Fig. 3, we can see that these *PM*s are on an upper level of granularity (complete gait level) compared to the previous one because their output *CP*s are calculated for a complete gait instead of being calculated for each gait cycle. These *PM*s have the dorso-ventral acceleration during each gait phase as input *CP*. Therefore, the set of input *CP*s is $U=(u_{\hat{a}_x})$. The symmetry of a gait is obtained by comparing the movement of both legs. We can analyze the symmetry during the double limb support phases by comparing the dorso-ventral acceleration during phases 1 and 3, or during the swing phases by comparing the dorso-ventral acceleration during phases 2 and 4. Each *PM* has a set of three NL sentences as output *CP*, e.g., the output *CP* of $PM_{S_{13}}$ ($y_{S_{13}}$) includes the following set of NL sentences:

$a_{1_{S_{13}}} \rightarrow$ "*The symmetry during the double limb support phase is low*"
$a_{2_{S_{13}}} \rightarrow$ "*The symmetry during the double limb support phase is medium*"
$a_{3_{S_{13}}} \rightarrow$ "*The symmetry during the double limb support phase is high*"

The validity degrees ($w_{1_{S_{13}}}, w_{2_{S_{13}}}, w_{3_{S_{13}}}$) are obtained by means of the aggregation function $g_{S_{13}}$. This function makes use of the Jaccard index (Hamers et al., 1989) as similarity function $J(x,y)$, which is defined using Eq. (3), in order to compare the dorso-ventral acceleration of the different limbs during the same gait phase

$$J(x,y) = \begin{cases} 1 & \text{if } x=y=0 \\ \dfrac{\min(x,y)}{\max(x,y)} & \text{otherwise} \end{cases} \tag{3}$$

First, we calculate three similarities for each gait cycle k:

- The similarity between the validity degrees of the sentences $a_{11_{\hat{a}_x}}$ and $a_{13_{\hat{a}_x}}$: $J(w_{11_{\hat{a}_x}}[k], w_{13_{\hat{a}_x}}[k])$, which refer to a low dorso-ventral acceleration during these phases.
- The similarity between the validity degrees of the sentences $a_{21_{\hat{a}_x}}$ and $a_{23_{\hat{a}_x}}$: $J(w_{21_{\hat{a}_x}}[k], w_{23_{\hat{a}_x}}[k])$, which refer to a medium dorso-ventral acceleration during these phases.
- The similarity between the validity degrees of the sentences $a_{31_{\hat{a}_x}}$ and $a_{33_{\hat{a}_x}}$: $J(w_{31_{\hat{a}_x}}[k], w_{33_{\hat{a}_x}}[k])$, which refer to a high dorso-ventral acceleration during these phases.

Then, we calculate the average value of the three similarities by means of Eq. (4), which gives us a value of the symmetry during the phases 1 and 3 ($symmetry_{13}[k]$) for each gait cycle k:

$$symmetry_{13}[k] = \frac{J(w_{11_{\hat{a}_x}}[k], w_{13_{\hat{a}_x}}[k]) + J(w_{21_{\hat{a}_x}}[k], w_{23_{\hat{a}_x}}[k]) + J(w_{31_{\hat{a}_x}}[k], w_{33_{\hat{a}_x}}[k])}{3} \tag{4}$$

Once we have a complete set of symmetry values during the total number of available cycles ($symmetry_{13}$), we apply the ordered weighted averaging operator (OWA) (Dubois and Prade, 1985; Yager, 1988) showed in Eq. (5), over the lowest three values of $symmetry_{13}$ in order to obtain a conservative symmetry value which ensures that the gait symmetry is low when any of the steps is not symmetric during phases 1 and 3

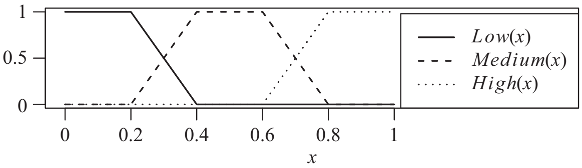

Fig. 6. Trapezoidal membership functions used to calculate the validity degree of *low*, *medium*, and *high* values of the symmetry and the homogeneity.

$$symmetry'_{13} = \begin{bmatrix} 3/6 \\ 2/6 \\ 1/6 \end{bmatrix} \cdot symmetry_{13} \tag{5}$$

Finally, the validity degrees of the NL sentences associated to the output *CP* $y_{S_{13}}$, are calculated using Eqs. (6)–(8), which make use of the trapezoidal membership functions showed in Fig. 6. These membership functions convert the numerical value of $symmetry'_{13} \in [0,1]$ into three fuzzy linguistic values (low, medium, and high)

$$w_{1_{S_{13}}} = low\ (symmetry'_{13}) \tag{6}$$

$$w_{2_{S_{13}}} = medium\ (symmetry'_{13}) \tag{7}$$

$$w_{3_{S_{13}}} = high\ (symmetry'_{13}) \tag{8}$$

In the same way, the output *CP* of $PM_{S_{24}}$ ($y_{S_{24}}$) includes the following set of NL sentences:

$a_{1_{S_{24}}} \rightarrow$ "*The symmetry during the swing phase is low*"
$a_{2_{S_{24}}} \rightarrow$ "*The symmetry during the swing phase is medium*"
$a_{3_{S_{24}}} \rightarrow$ "*The symmetry during the swing phase is high*"

Whose validity degrees ($w_{1_{S_{24}}}, w_{2_{S_{24}}}, w_{3_{S_{24}}}$) are obtained in a similar way to the ones associated to the output *CP* $y_{S_{13}}$, i.e., first we obtain a complete set of symmetry values during the total number of available cycles ($symmetry_{24}$) using the Jaccard index as showed in Eq. (4), then we apply the OWA operator to get the value $symmetry'_{24}$, and finally the validity degrees are calculated using Eqs. (9)–(11)

$$w_{1_{S_{24}}} = low\ (symmetry'_{24}) \tag{9}$$

$$w_{2_{S_{24}}} = medium\ (symmetry'_{24}) \tag{10}$$

$$w_{3_{S_{24}}} = high\ (symmetry'_{24}) \tag{11}$$

3.6. *Homogeneity of the phases perception mapping (PM_{H_1}, PM_{H_2}, PM_{H_3}, PM_{H_4})*

These *PM*s, as the previous ones, are on an upper level of granularity (complete gait level) because their output *CP*s are calculated for a complete gait and they also have the dorso-ventral acceleration during each gait phase as input *CP*. Therefore, the set of input *CP*s is $U=(u_{\hat{a}_x})$. The homogeneity of a gait is obtained by comparing a gait with itself in subsequent instants of time and it is calculated for each phase using two consecutive gait cycles. Each *PM* has a set of three NL sentences as output *CP*, e.g., the output *CP* of

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PM_{H_1} (y_{H_1}) includes the following set of NL sentences:

$a_{1_{H_1}}$ → "*The homogeneity during the double limb support after the reference limb swing phase is low*"
$a_{2_{H_1}}$ → "*The homogeneity during the double limb support after the reference limb swing phase is medium*"
$a_{3_{H_1}}$ → "*The homogeneity during the double limb support after the reference limb swing phase is high*"

The validity degrees ($w_{1_{H_1}}$,$w_{2_{H_1}}$,$w_{3_{H_1}}$) are obtained by means of the aggregation function g_{H_1}. This function is similar to the one explained for $PM_{S_{13}}$ and $PM_{S_{24}}$, however, there is an important difference: while in the previous *PM*s we are comparing the dorso-ventral acceleration during phases 1 and 3 or during phases 2 and 4 for each gait cycle *k*, here, we compare the dorso-ventral acceleration of a single phase *i* during the current gait cycle *k* and the previous one $k-1$. Therefore, we calculate three similarities for each gait cycle *k* (starting in the second gait cycle):

- The similarity between the validity degrees of the sentences $a_{11_{\ddot{a}_x}}[k-1]$ and $a_{11_{\ddot{a}_x}}[k]$: $J(w_{11_{\ddot{a}_x}}[k],w_{11_{\ddot{a}_x}}[k-1])$, which refer to a low dorso-ventral acceleration during phase 1 in the current gait cycle (k) and the previous one ($k-1$).
- The similarity between the validity degrees of the sentences $a_{21_{\ddot{a}_x}}[k-1]$ and $a_{21_{\ddot{a}_x}}[k]$: $J(w_{21_{\ddot{a}_x}}[k],w_{21_{\ddot{a}_x}}[k-1])$, which refer to a medium dorso-ventral acceleration during phase 1 in the current gait cycle (k) and the previous one ($k-1$).
- The similarity between the validity degrees of the sentences $a_{31_{\ddot{a}_x}}[k-1]$ and $a_{31_{\ddot{a}_x}}[k]$: $J(w_{31_{\ddot{a}_x}}[k],w_{31_{\ddot{a}_x}}[k-1])$, which refer to a high dorso-ventral acceleration during phase 1 in the current gait cycle (k) and the previous one ($k-1$).

Then, we calculate the average value of the three similarities by means of Eq. (12), which gives us a value of the homogeneity of the phase 1 ($homogeneity_1[k]$) for each gait cycle *k*

$$homogeneity_1[k] = \frac{J(w_{11_{\ddot{a}_x}}[k],w_{11_{\ddot{a}_x}}[k-1])+J(w_{21_{\ddot{a}_x}}[k],w_{21_{\ddot{a}_x}}[k-1])+J(w_{31_{\ddot{a}_x}}[k],w_{31_{\ddot{a}_x}}[k-1])}{3} \tag{12}$$

Once we have a complete set of homogeneity values of phase 1 during the total number of available cycles ($homogeneity_1$), we apply the OWA operator over the lowest three values of $homogeneity_1$ in order to obtain a conservative homogeneity value of the phase 1 as showed in Eq. (13)

$$homogeneity'_1 = \begin{bmatrix} 3/6 \\ 2/6 \\ 1/6 \end{bmatrix} \cdot homogeneity_1 \tag{13}$$

Finally, the validity degrees of the NL sentences associated to the output *CP* y_{H_1}, are calculated using Eqs. (14)–(16), which make use of the trapezoidal membership functions shown in Fig. 6

$$w_{1_{H_1}} = low(homogeneity'_1) \tag{14}$$

$$w_{2_{H_1}} = medium(homogeneity'_1) \tag{15}$$

$$w_{3_{H_1}} = high(homogeneity'_1) \tag{16}$$

The rest of *PM*s, have the output *CP*s y_{H_2}, y_{H_3}, and y_{H_4}; which include NL sentences expressing if the homogeneity is low, medium, or high during each gait phase. Their aggregation functions work similar to g_{H_1}, they compare the dorso-ventral acceleration during each single phase, then they get an homogeneity value of the complete gait for each phase using the OWA operator, and finally this value is qualified as low, medium and high using the trapezoidal membership functions.

3.7. Symmetry of the gait perception mapping (PM_S)

This *PM* has two *CP*s as inputs: the symmetry during the double limb support phases and the symmetry during the swing phases. Therefore, the set of input *CPs* is $U = (u_{S_{13}}, u_{S_{24}})$. This *PM* includes a set of three NL sentences in its output *CP* (y_S)

a_{1_S} → "*The symmetry of the gait is low*"
a_{2_S} → "*The symmetry of the gait is medium*"
a_{3_S} → "*The symmetry of the gait is high*"

The aggregation function (g_S) calculates the validity degrees (w_{i_S}) for each sentence. This function is defined by Eq. (17), which calculates the average value of each pair of validity degrees of the input *CP*s associated with a low, medium, and high symmetry

$$w_{i_S} = \frac{w_{i_{S_{13}}} + w_{i_{S_{24}}}}{2} \tag{17}$$

3.8. Homogeneity of the gait perception mapping (PM_H)

This *PM* has four *CP*s as inputs: the homogeneities of each of the four phases. Therefore, the set of input *CP*s is $U = (u_{H_1}, u_{H_2}, u_{H_3}, u_{H_4})$. This *PM* has a set of three NL sentences as output *CP* (y_H)

a_{1_H} → "*The homogeneity of the gait is low*"
a_{2_H} → "*The homogeneity of the gait is medium*"
a_{3_H} → "*The homogeneity of the gait is high*"

The aggregation function (g_H) calculates the validity degrees (w_{i_H}) for each sentence. This function is defined by Eq. (18), which calculates the average value of the four validity degrees of the input *CP*s associated with a low, medium, and high homogeneity

$$w_{i_H} = \frac{w_{i_{H_1}} + w_{i_{H_2}} + w_{i_{H_3}} + w_{i_{H_4}}}{4} \tag{18}$$

3.9. Quality of the gait perception mapping (PM_Q)

The top *PM* has two *CP*s as inputs: the symmetry and the homogeneity of the gait. Therefore, the set of input *CP*s is $U = (u_S, u_H)$. We have defined five different levels of quality: very low, low, medium, high, and very high. Therefore, the output *CP* y_Q has the following set of five possible sentences:

a_{1_Q} → "*The gait quality is very low*"
a_{2_Q} → "*The gait quality is low*"
a_{3_Q} → "*The gait quality is medium*"
a_{4_Q} → "*The gait quality is high*"
a_{5_Q} → "*The gait quality is very high*"

The aggregation function (g_Q) is an expert knowledge based fuzzy rule-based system, with one rule for each sentence:

R_1: IF a_{1_S} AND a_{1_H} THEN a_{1_Q}
R_2: IF (a_{1_S} AND a_{2_H}) OR (a_{2_S} AND a_{1_H}) THEN a_{2_Q}
R_3: IF (a_{1_S} AND a_{3_H}) OR (a_{3_S} AND a_{1_H}) OR (a_{2_S} AND a_{2_H}) THEN a_{3_Q}
R_4: IF (a_{2_S} AND a_{3_H}) OR (a_{3_S} AND a_{2_H}) THEN a_{4_Q}
R_5: IF a_{3_S} AND a_{3_H} THEN a_{5_Q}

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The consequents of the rules define the gait quality. To calculate the validity degrees of the sentences associated with the different levels of gait quality (w_{i_Q}), a weighted average using the firing degree of each rule R_i (ϕ_i) is computed as defined in Eq. (19)

$$w_{i_Q} = \frac{\phi_i}{\sum_{i=1}^{5} \phi_i} \tag{19}$$

Each rule of this set is a complete linguistic expression as can be seen in the following expanded expression of the rule R_4 that predicts a high value of the gait quality: "*If the gait symmetry is medium and the gait homogeneity is high, or if the gait symmetry is high and the gait homogeneity is low. Then the gait quality is high*".

4. Experimentation

In this section, we present the experimental results obtained with our proposal. First, Section 4.1 presents the experimental setup, which includes the data acquisition details. Then, the next subsection shows and discusses the obtained results for different people's gait and different gaits of the same person.

4.1. Experimental setup

The data acquisition was done using a sensor device including a three-axial accelerometer and Bluetooth communication capabilities. It was attached to a belt, centered in the back of each person providing measurements of the three orthogonal accelerations with a frequency of 100 Hz. We programmed a personal digital agenda (PDA) to receive the data via a Bluetooth connection and to record it with a timestamp. Therefore, every record contained the information: (t, a_x, a_y, a_z) where t is each instant of time, a_x is the dorso-ventral acceleration, a_y is the medio-lateral acceleration, and a_z is the antero-posterior acceleration. As explained in Section 3, in this work we only use a_x and a_y. We asked each person to walk a certain distance at a self-selected walking speed which comprises around 10 complete gait cycles. This process was repeated 10 times for each person producing a total of 10 datasets for each person.

To evaluate the proposed approach, we collected the acceleration signals of a set of 17 different people in order to assess the gait quality of each person. One set of people consisted of 15 healthy adults, 3 women and 12 men, with ages ranging between 23 and 51 years (with an average age of 30 years) and weights between 45 and 95 kg (with an average of 74 kg). The remaining two individuals have different lesions that modify their gait quality.

The first injured person was a 28 years old man with a weight of 88 kg that was not previously in our database. He suffered a medial malleolus (which is the prominence on the inner side of the ankle) fracture in the left limb. After that, he was undergo under parallel screw fixation of the medial malleolus surgery, and followed a rehab treatment during one and half months. We only had two gait data sets obtained after one month of rehab treatment and when this treatment was finished.

The other injured individual was a 39 years old man with a weight of 93 kg whose gaits were in our database as a healthy individual for another study only related to gait modeling (Alvarez-Alvarez et al., in press), but one month after capturing his data he seriously injured his left knee (meniscus tear) playing football. After that, he was undergo under a meniscus removal (meniscectomy) using arthroscopic surgery, and followed a rehab treatment during one month. Therefore, we have a complete database of different gaits of this person that will show the gait quality evolution.

4.2. Results and discussion

This section presents the results obtained for each person. It shows the different summaries obtained about the gait quality of the people. We have divided the results into three parts: first, the results related to healthy people are showed; second, we analyze the obtained sentences related to the person injured in his ankle; and finally, we focus on the gait quality evolution of the man injured in his knee by comparing his healthy gaits versus the gait data obtained after the lesion.

4.2.1. Healthy people

Table 1 shows the validity degrees of the sentences associated with the gait qualities of the 15 healthy individuals. Those validity degrees whose value is the maximum for each attribute are boldfaced. Therefore, the generated sentences are those ones which have the maximum validity degrees as can be seen in the following examples:

- "The gait quality of person 2 is very high because the gait symmetry is high and the gait homogeneity is high".
- "The gait quality of person 6 is medium because the gait symmetry is medium and the gait homogeneity is medium".
- "The gait quality of person 15 is high because the gait symmetry is medium and the gait homogeneity is high".

It can be clearly seen how the quality of these gaits is always medium or greater than medium, being high for six people and very high for two people.

Table 1
Validity degrees of the sentences associated with the gait qualities of the healthy people.

Person	w_{1_Q}	w_{2_Q}	w_{3_Q}	w_{4_Q}	w_{5_Q}	w_{1_S}	w_{2_S}	w_{3_S}	w_{1_H}	w_{2_H}	w_{3_H}
1	0.00	0.00	0.11	0.31	**0.58**	0.00	0.35	**0.65**	0.00	0.12	**0.88**
2	0.00	0.00	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**
3	0.00	0.04	0.40	**0.42**	0.14	0.05	**0.78**	0.17	0.00	0.49	**0.51**
4	0.02	0.02	**0.49**	0.25	0.22	0.02	**0.69**	0.29	0.02	**0.64**	0.34
5	0.00	0.00	0.36	**0.42**	0.22	0.00	**0.73**	0.27	0.00	0.47	**0.53**
6	0.09	0.21	**0.66**	0.04	0.00	0.24	**0.76**	0.00	0.10	**0.85**	0.05
7	0.00	0.04	**0.40**	0.34	0.22	0.05	**0.65**	0.30	0.00	**0.55**	0.45
8	0.00	0.00	0.29	**0.38**	0.33	0.00	**0.53**	0.47	0.00	0.42	**0.58**
9	0.12	0.22	**0.49**	0.17	0.00	0.28	**0.72**	0.00	0.15	**0.63**	0.22
10	0.02	0.04	**0.51**	0.24	0.19	0.02	**0.74**	0.24	0.05	**0.65**	0.30
11	0.00	0.00	0.28	**0.40**	0.32	0.00	**0.55**	0.45	0.00	0.39	**0.61**
12	0.11	0.15	0.24	**0.26**	0.24	0.17	**0.44**	0.39	0.23	0.37	**0.40**
13	0.15	0.16	**0.56**	0.10	0.03	0.20	**0.77**	0.03	0.19	**0.68**	0.13
14	0.00	0.00	**0.48**	0.41	0.11	0.00	**0.87**	0.13	0.00	**0.54**	0.46
15	0.00	0.01	0.40	**0.49**	0.10	0.02	**0.88**	0.10	0.00	0.45	**0.55**

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Table 2
Validity degrees of the sentences associated with the gait quality of the man injured in his ankle.

Sentence	After 30 days of rehab treatment	After 45 days of rehab treatment
w_{1_Q}	0.23	0.15
w_{2_Q}	**0.44**	0.16
w_{3_Q}	0.31	**0.66**
w_{4_Q}	0.02	0.03
w_{5_Q}	0.00	0.00
w_{1_S}	**0.59**	0.17
w_{2_S}	0.41	**0.83**
w_{2_S}	0.00	0.00
w_{1_H}	0.30	0.18
w_{2_H}	**0.67**	**0.78**
w_{3_H}	0.03	0.04
$w_{1_{S_{13}}}$	**1.00**	0.06
$w_{2_{S_{13}}}$	0.00	**0.94**
$w_{3_{S_{13}}}$	0.00	0.00
$w_{1_{S_{24}}}$	0.18	0.29
$w_{2_{S_{24}}}$	**0.82**	**0.71**
$w_{3_{S_{24}}}$	0.00	0.00
$w_{1_{H_1}}$	0.00	0.00
$w_{2_{H_1}}$	**0.89**	**0.92**
$w_{3_{H_1}}$	0.11	0.08
$w_{1_{H_2}}$	0.06	0.40
$w_{2_{H_2}}$	**0.94**	**0.60**
$w_{3_{H_2}}$	0.00	0.00
$w_{1_{H_3}}$	**0.92**	0.00
$w_{2_{H_3}}$	0.08	**0.92**
$w_{3_{H_3}}$	0.00	0.08
$w_{1_{H_4}}$	0.23	0.32
$w_{2_{H_4}}$	**0.77**	**0.68**
$w_{3_{H_4}}$	0.00	0.00

4.2.2. Man injured in his ankle

Table 2 shows the validity degrees of the sentences associated with the gait quality of the man injured in his ankle. As explained above, the generated sentences are those ones which have the maximum validity degrees. Thanks to the hierarchical fashion of the GLMP, the final recipient of the report can choose the granularity level which better fits to her/his desired detail. In this case, the report not only details the quality, symmetry and homogeneity of the gait but also details the symmetry and homogeneity during each gait phase

- "After 30 days of rehab treatment, the gait quality of this person is low because the gait symmetry is low and the gait homogeneity is medium. The gait symmetry is medium because the symmetry during the double limb support phase is low and the symmetry during the swing phase is low. The gait homogeneity is medium because the homogeneity during the double limb support after the reference limb swing phase is medium, the homogeneity during the reference limb single support and swing phase of the opposite limb is medium, the homogeneity during the double limb support after the opposite limb swing phase is low, and the homogeneity during the opposite limb single support and swing phase of the reference limb is medium".
- "After 45 days of rehab treatment, the gait quality of this person is medium because the gait symmetry is medium and the gait homogeneity is medium. The gait symmetry is medium because the symmetry during the double limb support phase is medium and the symmetry during the swing phase is medium. The gait homogeneity is medium because the homogeneity during the double limb support after the reference limb swing phase is medium, the homogeneity during the reference limb single support and swing phase of the opposite limb is medium, the homogeneity during the double limb support after the opposite limb swing phase is medium, and the homogeneity during the opposite limb single support and swing phase of the reference limb is medium".

4.2.3. Man injured in his knee

Finally, Table 3 shows the validity degrees of the sentences associated with the gait quality of the man injured in his knee. In this case, we have three different situations: one set of gaits before the lesion (second column), different gait sets taken at different days after the knee lesion (columns three, four, and five), and different gait sets after the surgery (columns six and seven). The short versions of the reports (without the details about symmetry and homogeneity during each gait phase) are listed as follows:

- "Before the knee lesion, the gait quality is high because the gait symmetry is medium and the gait homogeneity is high".
- "28 days after the knee lesion, the gait quality is very low because the gait symmetry is low and the gait homogeneity is low".
- "35 days after the knee lesion, the gait quality is low because the gait symmetry is low and the gait homogeneity is medium".
- "42 days after the knee lesion, the gait quality is low because the gait symmetry is low and the gait homogeneity is low".
- "71 days after the knee lesion and 27 days after the surgery, the gait quality is medium because the gait symmetry is low and the gait homogeneity is medium".
- "195 days after the knee lesion and 151 days after the surgery, the gait quality is high because the gait symmetry is medium and the gait homogeneity is medium".

As can be seen in these results, this person had a high gait quality before this lesion, which drastically was reduced after the lesion. Our proposal is able to identify correctly the gait quality during these different phases.

Moreover, thanks to the granularity of our proposal, we can directly describe the main details for each gait quality level in each period of time. For example, the gait quality is high 195 days after the knee lesion and 151 days after the surgery, but the homogeneity during the second phase (reference limb single support and swing phase of the opposite limb) is medium in contrast with the gaits obtained before the knee lesion (where it was high). This can be explained at the gait cycle level (see Fig. 3) by analyzing the sentences related to the dorso-ventral acceleration during each gait phase $CP\,(\hat{a}_x)$, in this case, the validity degrees of the sentence "*The dorso-ventral acceleration is low during the reference limb single support and swing phase of the opposite limb*" ($w_{12_{\hat{a}_x}}$) are zero in some cycles while during another cycles are greater than zero, thus reducing the homogeneity of the gait during this phase. Therefore, we can produce a linguistic expression that explains the causes at gait cycle level, e.g., "*the homogeneity during the reference limb single support and swing phase of the opposite limb is medium because the dorso-ventral acceleration during this phase sometimes is low while other times is not*".

5. Conclusions and future works

This paper presents important results of a long term research project aimed to develop computational systems able to generate

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Table 3
Validity degrees of the sentences associated with the gait quality of the man injured in his knee.

Sentence	Before lesion	28 days after the lesion	35 days after the lesion	42 days after the lesion	71 days after the lesion and 27 days after the surgery	195 days after the lesion and 151 days after the surgery
w_{1_Q}	0.00	**0.51**	0.27	0.30	0.21	0.00
w_{2_Q}	0.00	0.35	**0.43**	**0.32**	0.36	0.08
w_{3_Q}	0.25	0.14	0.28	0.31	**0.36**	0.30
w_{4_Q}	**0.53**	0.00	0.02	0.07	0.07	**0.43**
w_{5_Q}	0.22	0.00	0.00	0.00	0.00	0.19
w_{1_S}	0.00	**0.84**	**0.61**	**0.50**	**0.50**	0.10
w_{2_S}	**0.71**	0.16	0.39	0.50	0.50	**0.64**
w_{3_S}	0.29	0.00	0.00	0.00	0.00	0.26
w_{1_H}	0.00	**0.59**	0.38	**0.45**	0.28	0.00
w_{2_H}	0.32	0.41	**0.59**	0.45	**0.62**	0.41
w_{3_H}	**0.68**	0.00	0.03	0.10	0.10	**0.59**
$w_{1_{S_{13}}}$	0.00	**0.68**	0.22	0.00	0.00	0.00
$w_{2_{S_{13}}}$	0.47	0.32	**0.78**	**1.00**	**0.99**	0.48
$w_{3_{S_{13}}}$	**0.53**	0.00	0.00	0.00	0.01	**0.52**
$w_{1_{S_{24}}}$	0.00	**1.00**	**1.00**	**1.00**	**1.00**	0.21
$w_{2_{S_{24}}}$	**0.96**	0.00	0.00	0.00	0.00	**0.79**
$w_{3_{S_{24}}}$	0.04	0.00	0.00	0.00	0.00	0.00
$w_{1_{H_1}}$	0.00	**0.79**	0.17	0.00	0.00	0.00
$w_{2_{H_1}}$	0.07	0.21	**0.83**	**0.72**	**0.72**	0.13
$w_{3_{H_1}}$	**0.93**	0.00	0.00	0.28	0.28	**0.87**
$w_{1_{H_2}}$	0.00	**0.61**	**0.61**	**1.00**	0.48	0.00
$w_{2_{H_2}}$	0.33	0.39	0.39	0.00	**0.52**	**0.72**
$w_{3_{H_2}}$	**0.67**	0.00	0.00	0.00	0.00	0.28
$w_{1_{H_3}}$	0.00	0.23	0.00	0.00	0.00	0.00
$w_{2_{H_3}}$	0.40	**0.77**	**0.90**	**0.88**	**0.87**	0.44
$w_{3_{H_3}}$	**0.60**	0.00	0.10	0.12	0.13	**0.56**
$w_{1_{H_4}}$	0.00	**0.72**	**0.75**	**0.79**	**0.66**	0.00
$w_{2_{H_4}}$	0.47	0.28	0.25	0.21	0.34	0.34
$w_{3_{H_4}}$	**0.53**	0.00	0.00	0.00	0.00	**0.66**

linguistic descriptions of complex phenomena. Here, we have used the human gait as an interesting example of complex phenomenon evolving in time. We have shown that the new version of GLMP including a FFSM is an expressive tool to represent the behavior of this type of phenomena in a human friendly way.

In the current stage of development, we have generated linguistic descriptions that correspond to the context of a laboratory experimental setup. In future projects, we will deep into two important fields, namely, Linguistics in order to improve the generated texts, and the specific application field, in order to improve the meaning and, therefore, the usability of these texts. We will deal with applying these results to generate NL expressions in the context of specific applications, e.g., to assess the risk of falling in elderly people and to monitor the recovery process in physiotherapy.

The main contribution of this paper is the practical result of a user friendly model of the human gait. Moreover, this is an example of other possible linguistic models of complex quasi-periodic phenomena, e.g., other biological cycles such as the breath rhythm or the electrocardiogram signals, or artificial cycles such as the ones produced during the manufacturing processes of many products. In this paper, we show results that demonstrate the feasibility of these future projects.

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6.3 Automatic linguistic report of traffic evolution in roads

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Automatic linguistic report of traffic evolution in roads

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ABSTRACT

In the field of intelligent transportation systems, one important challenge consists of maintaining updated the electronic panels installed in roads with relevant information expressed in natural language. Currently, these messages are produced by human experts. However, the amount of data to analyze in real time and the number of available experts are imbalanced and new computational tools are required to assist them in this work. Moreover, the same problem appears when we deal with automatically generating linguistic reports to assist traffic managers that must take their decisions based on large amounts of quickly evolving information.

In this paper, we contribute to solve this problem by designing a computational application based on our research in the field of computational theory of perceptions. Here, we present an application where we generate linguistic descriptions of the traffic behavior evolving in time and changing between different levels of service. We include some results obtained with both, simulated and real data.

1. Introduction

Intelligent transportation systems (ITS) aim to get safer traffic conditions and comfort in transportation, and also to increase the road traffic efficiency by improving the functionality of cars and roads (Angulo, Romero, Garca, Serrano-Guerrero, & Olivas, 2011; Bas, 2007; Button & Hensher, 2001).

Due to increasing social demands of mobility and safety in road transportation and the increasing computer capabilities, the need of automatic, economic and real-time solutions for reliable traffic flow analysis becomes a priority for many governments. In this context, one goal of automatic traffic analysis is the detection and tracking of vehicles driving through a controlled area in order to discover abnormal events such as traffic congestions, speed violations, some other illegal behavior of drivers or even the detection of accidents (Atkiciounas, Blake, Juozapavicius, & Kazimianec, 2005; Durduran, 2010). The availability of new suitable computational applications certainly will improve the efficiency of roads, assisting in the quick detection of traffic alarms, and also helping to foresee some problems when traffic is normal in road and highways.

An interesting and paradigmatic problem consists of generating dynamically the most adequate natural language (NL) messages to communicate with drivers using electronic panels installed in the roads. Currently, these messages are produced by human experts but this task can be tough and tedious. Moreover, the balance between the amount of changing data to analyze and the number of experts available is getting worse dramatically. This situation causes the need of computational systems that can interpret and describe linguistically the large amount of available information.

In Drane and Rizos (1998), we can found a survey of technologies for locating the position of vehicles on the road. In this direction, the works by Wen show an intelligent traffic management expert system with radio frequency identification (RFID) technology (Wen, 2010) and a dynamic and automatic traffic light control expert system for solving road congestion problems (Wen, 2008). In Messelodi et al. (2009), authors present a technology to collect and organize data about the vehicles moving in a road network. Nevertheless, to the best of our knowledge, currently, a technology able to generate automatic linguistic descriptions of the traffic behavior in a granular fashion is not available.

In this paper, we aim to contribute to this field by presenting a computational application able to generate linguistic descriptions in real-time about the traffic evolution. Our approach is based on the use of Fuzzy Logic (FL), which is widely recognized for its ability for linguistic concept modeling and its use in system identification (Quek, Pasquier, & Lim, 2009). On the one hand, semantic expressiveness, using linguistic variables (Zadeh, 1975a, 1975b, 1975c) and rules (Mamdani, 1977; Zadeh, 1973), is quite close to NL. On the other hand, being universal approximators (Castro, 1995) fuzzy inference systems are able to perform nonlinear

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mappings between inputs and outputs. More specifically, our approach is based on the computational theory of perceptions (CTP) introduced in the Zadeh's seminal paper "From computing with numbers to computing with words - from manipulation of measurements to manipulation of perceptions" (Zadeh, 1999) and further developed in subsequent papers. CTP provides a framework to develop computational systems with the capability of computing with the meaning of NL expressions, i.e., with the capacity of computing with imprecise descriptions of the world in a similar way that humans do it.

In previous works on this line, we have generated linguistic descriptions of different types of phenomena. For example, we generated financial reports from data taken from the Spanish securities market commission (CNMV) (Mendez-Nunez & Trivino, 2010) and linguistic descriptions about relevant features of the Mars' Surface (Alvarez-Alvarez, Sanchez-Valdes, & Trivino, 2011a). Specifically, in the field of ITS, we generated linguistic reports about the traffic on roundabouts (Trivino et al., 2010b) and we generated assessing reports in truck driving simulators (Eciolaza & Trivino, 2011; Eciolaza, Trivino, Delgado, Rojas, & Sevillano, 2011).

In this work, we focused on the perception of change. We explored possibilities to perform linguistic descriptions of how the traffic evolves in time. We have researched on how to model the meaning of sentences such as "*the phenomenon is changing from state A to state B*". In order to model the evolution of phenomena in time, we have used our previous works on fuzzy finite state machines (FFSMs). Here, we have extended the use of the FFSM's output function to be used with this aim. With a different approach, see in Pouzols, Barriga, Lopez, and Solano (2008) how this idea has also been explored with the aim of summarizing network flow statistics.

The remainder of this paper is organized as follows. Section 2 presents the main concepts of our approach to linguistic description of complex phenomena evolving in time. Section 3 describes how to use these concepts for the linguistic description of the traffic behavior. Afterwards, Section 4 describes the experimentation carried out. Finally, Section 5 draws some conclusions and introduces some future research works.

2. Linguistic description of complex phenomena

In this section, we present several basic concepts of our contribution to CTP aimed to develop computational systems able to generate linguistic descriptions of phenomena. According to Zadeh, the object of perceptions are not only the attributes of objects, e.g., the distance, velocity and angle. The object of perceptions can be the whole systems, e.g., a person parking a car, the traffic in a roundabout, the air-conditioned system in buildings, etc. In this way, we use the term phenomenon to represent an object, or a set of interrelated objects, that is perceived in the computer environment. Phenomena are located in certain context and evolve in time among different situation types.

The Granular linguistic model of a phenomenon (GLMP) is based on subjective perceptions of a domain expert that we call designer. The more experienced designer, with better understanding and use of NL, the richer the model with more possibilities of achieving and responding to final users' needs and expectations. The designer uses the resources of the computer, e.g., sensors, to acquire data about a phenomenon and uses her/his own experience to interpret these data and to create the model. Then, the designer uses the resources of the computer to produce the linguistic utterances. In the following subsections, we introduce the main elements of our architecture for the linguistic description of complex phenomena.

2.1. Computational perception (CP)

A *CP* is the computational model of a unit of information acquired by the designer about the phenomenon to be modeled. In general, *CP*s correspond to particular details of the phenomenon at certain degrees of granularity. A *CP* is a couple (A,W) where:

$A = (a_1, a_2, \ldots, a_n)$ is a vector of n linguistic expressions (words or sentences in NL) that represents the whole linguistic domain of the *CP*. Each a_i describes the value of the *CP* in each situation with specific granularity degree. These sentences can be either simple, e.g., a_i = "*Traffic density is high*" or more complex, e.g., a_i = "*Usually, at midday, the traffic density increases in this part of the road*".
$W = (w_1, w_2, \ldots, w_n)$ is a vector of validity degrees $w_i \in [0,1]$ assigned to each a_i in the specific context. The concept of validity depends on the application, e.g., it is a function of the truthfulness and relevance of each sentence in its context of use.

In this application paper, in order to model our perception of temporal evolution of phenomena, we applied a paradigm composed of three types of *CP*, namely, the perception of the current state (assertive *CP*), the perception of the trend to evolve (derivative *CP*) and the summary of accumulated perceptions (integrative *CP*). The assertive *CP* is associated with a linguistic expression of the current state of a characteristic of the phenomenon, e.g., "*the traffic density is high*". The derivative *CP* corresponds to trend analysis information and gives insight into how the phenomenon is evolving in time, e.g., "*the traffic density is decreasing*". Finally, the integrative *CP* represents the accumulated perception of the phenomenon over a period of time, e.g., "*the traffic density in the last period has been low*".

2.2. Perception mapping (PM)

We use *PM*s to create and aggregate *CP*s. A *PM* is a tuple (U,y,g,T) where:

$U = (u_1, u_2, \ldots, u_n)$ is a vector of n input *CP*s $u_i = (A_{u_i}, W_{u_i})$. In the special case of first order perception mappings (1-*PM*s), these are the inputs to the GLMP and they are values $z \in \mathbb{R}$ being provided either by a sensor or obtained from a database.
$y = (A_y, W_y)$ is the output *CP*.
$W_y = g(W_{u_1}, W_{u_2}, \ldots, W_{u_n})$ is an aggregation function employed to calculate $W_y = (w_1, w_2, \ldots, w_{n_y})$ from the input *CP*s. In FL, many different types of aggregation functions have been developed. For example, g might be implemented using a set of fuzzy rules. In the case of 1-*PM*s, g is built using a set of membership functions $\mu_{a_i}(z)$ as follows: $W_y = (\mu_{a_1}(z), \mu_{a_2}(z), \ldots, \mu_{a_{n_y}}(z)) = (w_1, w_2, \ldots, w_{n_y})$
T is a text generation algorithm that allows generating the sentences in A_y. In simple cases, T is a linguistic template, e.g., "*road vehicle density is* {*high*|*medium*|*low*}".

There are many types of *PM*s. In this paper, we contribute to this research line by exploring two of them focused on describing how phenomena evolve on time. In Section 3.2.8, we introduce a *PM* based on fuzzy quantifiers and in Section 3.2.9, we introduce a *PM* based on a FFSM.

2.3. Granular linguistic model of a phenomenon (GLMP)

The GLMP consists of a network of *PM*s. Each *PM* receives a set of input *CP*s and transmits upwards a *CP*. We say that each output *CP* is explained by the *PM* using a set of input *CP*s. In this network, each *CP* covers specific aspects of the phenomenon with certain degree of granularity. Using different aggregation functions and different linguistic expressions, the GLMP paradigm allows the designer to model computationally her/his perceptions.

Fig. 1 shows an example of a GLMP. In this example, we describe the phenomenon at a very basic level in terms of three input variables that provide values z_1, z_2, and z_3 respectively at a certain instant of time. These variables are introduced in the perception mappings PM_1, PM_2 and PM_3, providing CP_1, CP_2 and CP_3. Using these three 1-*CP*s, we use the perception mappings PM_4 and PM_5 to explain CP_4 and CP_5. Finally, a top-order description of the phenomenon is provided, at the highest level of abstraction, by CP_6, explained by PM_6 in terms of CP_4 and CP_5. Notice that, by using this structure, one can provide not only a linguistic description of the phenomenon at a certain level, but an explanation in terms of linguistic expressions at lower levels.

2.4. Report generator

Fig. 2, shows the basic architecture of our report generator. The main processing modules of this computational system are, namely, the data acquisition (DAQ) module, the validity module, and the expression module that are described in the following sections.

2.4.1. DAQ module

This processing module provides the data needed to feed the 1-*CP*s. The data acquisition module provides the interface with the application physical environment. This module could include either sensors or access to information in a database. In this paper we generate examples of these data using both, a road traffic simulator and an image processing module.

2.4.2. Validity module

Once a sample of input data is available, the validity module uses the aggregation functions in the GLMP to calculate the validity

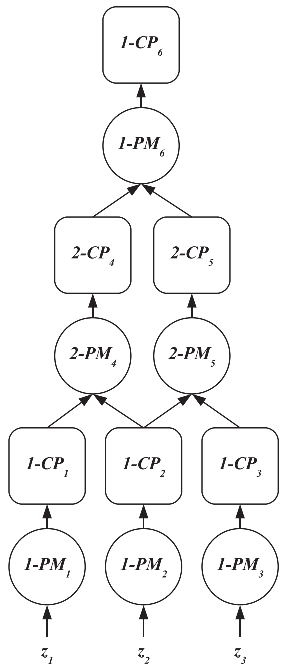

Fig. 1. Example of a simple GLMP.

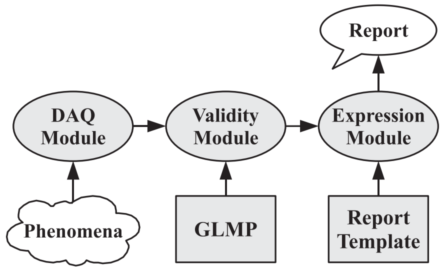

Fig. 2. Main components of the proposed architecture of our report generator.

degree of each *CP*. Therefore, this module provides as output a collection of linguistic clauses together with associated degrees of validity.

2.4.3. Expression module

Provided a set of valid linguistic clauses, the goal is to combine this information to build a linguistic report. This module deals with generating the most relevant linguistic report by choosing and connecting the adequate linguistic clauses based on a report template data structure.

3. Linguistic description of traffic behavior

This section describes how to apply our approach for linguistic description of complex phenomena to the analysis of the traffic behavior. We explain the relevant modules needed to produce the linguistic description of traffic behavior.

From the study of several sources, including the Highway Capacity Manual (Board, 1985), we obtained a first list of parameters about the traffic behavior that our reports should contain, namely, the speed of vehicles, traffic density and level of service in road (*LOS*). These parameters allow us to report the traffic behavior and to study anomalous situations that may occur, e.g., a vehicle traveling at a speed too high or too low, a vehicle circulating in opposite direction. To enrich the report, the linguistic description should include a time reference to place each event in the fraction of time in which it occurred.

3.1. DAQ module

Here, we used basic measures of traffic parameters that can be obtained from different type of sensors, e.g., video cameras, radar, pressured hoses and inductive burial loops. These basic measures are, namely, the vehicles speed (*vs*), the average road speed (*rs*), that is calculated as the average of speeds at each moment, and the traffic density (*td*), which is calculated as the percentage of road that is occupied at each time instant.

3.2. Validity module

Fig. 3 shows a GLMP which tries to summarize and highlight the relevant aspects of the traffic behavior. In the following subsections, we describe each of the *PM*s and associated *CP*s in this model.

3.2.1. Road speed (1-PM_{RS})

It is an assertive 1-*PM* whose input is the numerical value of the average road speed ($rs \in \mathbb{R}$). The output *CP* y_{RS} includes the following set of NL sentences:

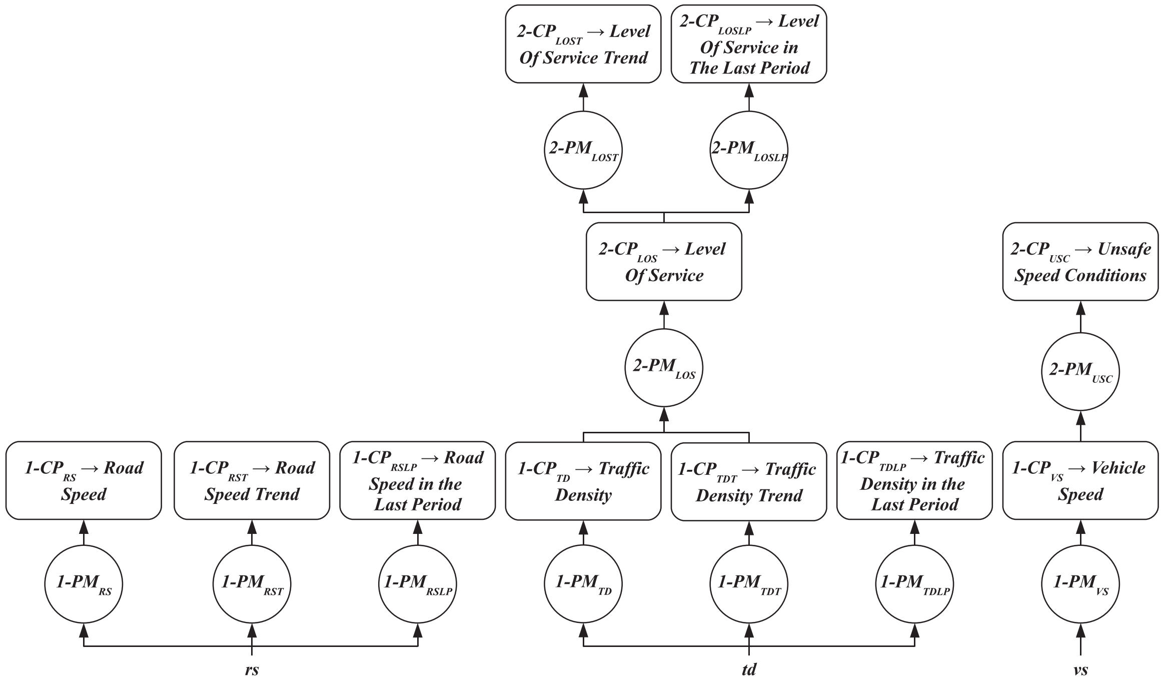

Fig. 3. GLMP for the linguistic description of the traffic behavior. The circles represent perception mappings while the rectangles stand for computational perceptions.

a_{RS_1} → "*The average road speed is very low*"
a_{RS_2} → "*The average road speed is low*"
a_{RS_3} → "*The average road speed is medium*"
a_{RS_4} → "*The average road speed is high*"
a_{RS_5} → "*The average road speed is very high*"

As in the rest of 1-*PM*s described in the following subsections, the validity degrees are obtained by means of a set of uniformly distributed trapezoidal membership functions (MFs) forming a strong fuzzy partition (SFP) (Ruspini, 1969). Here, e.g., the validity degrees are directly $w_{RS_1} = VL_{RS}(rs)$, $w_{RS_2} = L_{RS}(rs)$, $w_{RS_3} = M_{RS}(rs)$, $w_{RS_4} = H_{RS}(rs)$, and $w_{RS_5} = VH_{RS}(rs)$.

3.2.2. Road speed trend (1-PM$_{RST}$)

This derivative 1-*PM* has validity degrees obtained from the numerical derivative of *rs* (drs/dt). It allows to generate the following set of NL sentences:

a_{RST_1} → "*The average road speed is decreasing*"
a_{RST_2} → "*The average road speed is keeping constant*"
a_{RST_3} → "*The average road speed is increasing*"

3.2.3. Road speed in the last period (1-PM$_{RSLP}$)

It is an integrative 1-*PM* whose input is the numerical value of the average road speed ($rs \in \mathbb{R}$). The output *CP* y_{RSLP} includes the following set of NL sentences:

a_{RSLP_1} → "*The average road speed in the last period was very low*"
a_{RSLP_2} → "*The average road speed in the last period was low*"
a_{RSLP_3} → "*The average road speed in the last period was medium*"
a_{RSLP_4} → "*The average road speed in the last period was high*"
a_{RSLP_5} → "*The average road speed in the last period was very high*"

The validity degrees are obtained by means of the aggregation function g_{RSLP}, which calculates the average value of *rs* during a period ($\overline{rs}$) defined empirically.

3.2.4. Traffic density perception mapping (1-PM$_{TD}$)

It is an assertive 1-*PM* that produces the following set of NL sentences:

a_{TD_1} → "*The traffic density is extremely low*"
a_{TD_2} → "*The traffic density is very low*"
a_{TD_3} → "*The traffic density is low*"
a_{TD_4} → "*The traffic density is high*"
a_{TD_5} → "*The traffic density is very high*"
a_{TD_6} → "*The traffic density is extremely high*"

3.2.5. Traffic density trend (1-PM$_{TDT}$)

This derivative 1-*PM* has validity degrees obtained from the numerical derivative of *td* (dtd/dt). It allows to generate the following set of NL sentences:

a_{TDT_1} → "*The traffic density is decreasing*"
a_{TDT_2} → "*The traffic density is keeping constant*"
a_{TDT_3} → "*The traffic density is increasing*"

3.2.6. Traffic density in the last period (1-PM$_{TDLP}$)

It is an integrative 1-*PM* that produces the following set of NL sentences:

a_{TDLP_1} → "*The average road speed in the last period was very low*"
a_{TDLP_2} → "*The average road speed in the last period was low*"
a_{TDLP_3} → "*The average road speed in the last period was medium*"
a_{TDLP_4} → "*The average road speed in the last period was high*"
a_{TDLP_5} → "*The average road speed in the last period was very high*"

The validity degrees are obtained by means of the aggregation function g_{TDLP}, which calculates the average value of td during a period ($\overline{td}$) defined empirically.

3.2.7. Vehicle speed (1-PM$_{VS}$)

This assertive 1-*PM* produces, for each detected vehicle, the following set of NL sentences:

a_{VS_1} → "*The vehicle speed is very low*"
a_{VS_2} → "*The vehicle speed is low*"
a_{VS_3} → "*The vehicle speed is medium*"
a_{VS_4} → "*The vehicle speed is high*"
a_{VS_5} → "*The vehicle speed is very high*"

3.2.8. Unsafe speed conditions (2-PM$_{USC}$)

This integrative 2-*PM* aggregates the information provided by 1-*CP*$_{VS}$ during a period of time. Its output includes the following set of NL sentences where we combine crisp quantifying expressions with imprecise quantifying expressions:

a_{USC_0} → "*Zero vehicles speeding*"
a_{USC_1} → "*One vehicle speeding*"
a_{USC_2} → "*Two vehicles speeding*"
a_{USC_3} → "*Three vehicles speeding*"
a_{USC_4} → "*Four vehicles speeding*"
a_{USC_5} → "*Several vehicles speeding*"
a_{USC_6} → "*Many vehicles speeding*"

The validity degrees are obtained by means of the aggregation function g_{USC}, which is based on the α-cuts method proposed by Delgado, Sánchez, and Vila (2000). For example, using the validity degree w_{VS_5} of "*The vehicle speed is very high*", we calculate the percentage of vehicles with a very high speed contained at each α-level (N_α) by means of Eq. (1), with $\alpha \in A = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

$$N_\alpha = \frac{1}{n}\sum_{i=1}^{n} F_\alpha(w_{VS_5}) \tag{1}$$

where:

$$F_\alpha(w_{VS_5}) = \begin{cases} 1 & \text{if} \quad w_{VS_5} > \alpha \\ 0 & \text{if} \quad w_{VS_5} \leqslant \alpha \end{cases} \tag{2}$$

Then, we calculate the membership degree of each N_α to each element of the set of linguistic quantifiers: $\{Q_0, \ldots, Q_6\}$ = $\{Zero, One, Two, Three, Four, Various, Many\}$, e.g., $\mu_{Q_3}(N_\alpha) = Three(N_\alpha)$. Fig. 4 shows these linguistic labels defined on the domain of the number of vehicles n.

The last step is to calculate the average value of the membership degrees obtained for each α-level using Eq. (3). The number of elements in the set A is the resolution degree, i.e., here, $|A| = 10$.

$$w_{USC_i} = \frac{1}{|A|}\sum_{\forall \alpha \in A} \mu_{Q_i}(N_\alpha) \tag{3}$$

This final value contains the relevant information about the amount of vehicles circulating at a very high speed, e.g., the validity degree of the sentence "*Three vehicles speeding*" (w_{SA_3}) will be determined by Eq. (4):

$$w_{USC_3} = \frac{1}{|A|}\sum_{\forall \alpha \in A} Three(N_\alpha) \tag{4}$$

3.2.9. Level of service (2-PM$_{LOS}$)

The level of service (*LOS*) is a measure used by traffic engineers to determine the effectiveness of elements of transportation infrastructure. *LOS* is most commonly used to analyze highways by categorizing traffic flow with corresponding safe driving conditions. The Highway Capacity Manual (Board, 1985) distinguishes between six levels of service: *A*, *B*, *C*, *D*, *E*, and *F*. Therefore, we have defined an assertive output *CP* (y_{LOS}) that identifies these six levels having the following set of possible sentences:

a_{LOS_1} → "*The level of service is A. Free-flow operation*"
a_{LOS_2} → "*The level of service is B. Reasonably free flow, the ability to maneuver is only slightly restricted and the effects of minor incidents still are easily absorbed*"
a_{LOS_3} → "*The level of service is C. Stable flow, speeds at or near free-flow and queues may form*"
a_{LOS_4} → "*The level of service is D. Approaching unstable flow, speeds decline slightly with increasing flows while density increases more quickly*"
a_{LOS_5} → "*The level of service is E. Unstable flow, with operation near or at capacity and no usable gaps in the traffic stream*"
a_{LOS_6} → "*The level of service is F. Forced or breakdown flow, queues form behind breakdown points and demand is greater than capacity*"

This 2-*PM* has two 1-*CP*s as inputs: the traffic density (1-*CP*$_{TD}$) and its trend (1-*CP*$_{TDT}$).

The aggregation function (g_{LOS}) calculates, at each time instant (t), the value of the validity degrees for each sentence based on the previous validity degrees (time instant $t-1$) and current input *CP*s. Therefore, the aggregation function is a FFSM. For a more detailed description of this paradigm and its applications, the interested reader could see our previous papers (Alvarez-Alvarez, Trivino, & Cordón, 2012, 2011b, 2010; Trivino, Alvarez-Alvarez, & Bailador, 2010a). Fig. 5 shows how we use a FFSM to define constraints on the possibilities to change of *LOS*. Using this state diagram, we identify 16 fuzzy rules: 6 rules (R_{ii}) to remain in each *LOS* and other 10 rules (R_{ij}) to change between different *LOS*. This rule base is defined using expert knowledge based on the descriptions of the Highway Capacity Manual (Board, 1985), which links terms related to traffic density and its evolution along time. It is clear how the system evolution is given by the traffic density, which has a different associated linguistic term for each *LOS*; and its trend, which governs the change to a better or worse state if the density trend is negative or positive, respectively. These rules are listed as follows:

R_{11}: IF a_{LOS_1} AND a_{TD_1} AND a_{TDT_2} THEN a_{LOS_1}
R_{22}: IF a_{LOS_2} AND a_{TD_2} AND a_{TDT_2} THEN a_{LOS_2}
R_{33}: IF a_{LOS_3} AND a_{TD_3} AND a_{TDT_2} THEN a_{LOS_3}
R_{44}: IF a_{LOS_4} AND a_{TD_4} AND a_{TDT_2} THEN a_{LOS_4}
R_{55}: IF a_{LOS_5} AND a_{TD_5} AND a_{TDT_2} THEN a_{LOS_5}
R_{66}: IF a_{LOS_6} AND a_{TD_6} AND a_{TDT_2} THEN a_{LOS_6}
R_{12}: IF a_{LOS_1} AND a_{TD_2} AND a_{TDT_3} THEN a_{LOS_2}
R_{23}: IF a_{LOS_2} AND a_{TD_3} AND a_{TDT_3} THEN a_{LOS_3}
R_{34}: IF a_{LOS_3} AND a_{TD_4} AND a_{TDT_3} THEN a_{LOS_4}
R_{45}: IF a_{LOS_4} AND a_{TD_5} AND a_{TDT_3} THEN a_{LOS_5}
R_{56}: IF a_{LOS_5} AND a_{TD_6} AND a_{TDT_3} THEN a_{LOS_6}
R_{21}: IF a_{LOS_2} AND a_{TD_1} AND a_{TDT_1} THEN a_{LOS_1}
R_{32}: IF a_{LOS_3} AND a_{TD_2} AND a_{TDT_1} THEN a_{LOS_2}
R_{43}: IF a_{LOS_4} AND a_{TD_3} AND a_{TDT_1} THEN a_{LOS_3}
R_{54}: IF a_{LOS_5} AND a_{TD_4} AND a_{TDT_1} THEN a_{LOS_4}
R_{65}: IF a_{LOS_6} AND a_{TD_5} AND a_{TDT_1} THEN a_{LOS_5}

where:

- The first term in the antecedent computes the previous validity degree of the sentence a_{LOS_i}, i.e., w_{LOS_i}. With this mechanism, we only allow the FFSM to change from the *LOS* i to the *LOS* j (or to

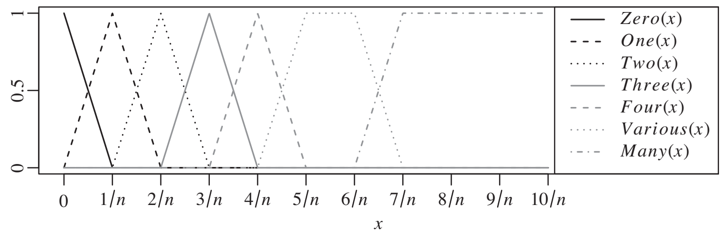

Fig. 4. Linguistic labels that represent the linguistic quantifiers "*Zero*", "*One*", "*Two*", "*Three*", "*Four*", "*Various*", or "*Many*" vehicles speeding.

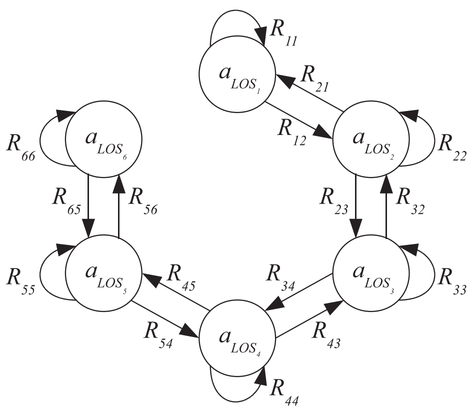

Fig. 5. State diagram of the FFSM for *LOS* modeling.

remain in *LOS i*, when $i = j$). For example, in R_{11}, it is computed the validity degree of the sentence "*The level of service is A. Free-flow operation*" (w_{LOS_1}).

- The second term in the antecedent describes the constraints imposed on the traffic density 1-CP_{TD}, e.g., "*the traffic density is extremely low*" (w_{TD_1}).
- The third term in the antecedent describes the constraints imposed on the traffic density trend 1-CP_{TDT}, e.g., "*the traffic density is keeping constant*" (w_{TDT_2}).
- Finally, the consequent of the rule defines the next *LOS*. To calculate the validity degrees of the sentences associated with each LOS_j (w_{LOS_j}), a weighted average using the firing degree of each rule $R_{ij}(\phi_{ij})$ is computed as defined in Eq. (5):

$$w_{LOS_j} = \frac{\sum_{i=1}^{6} \phi_{ij}}{\sum_{i=1}^{6}\sum_{j=1}^{6} \phi_{ij}} \tag{5}$$

where ϕ_{ij} is calculated using the minimum for the AND operator.

Note that each rule of this set, is therefore, a complete linguistic expression as can be seen in the following expanded expression of the rule R_{11} to remain in the *LOS A*: "*If the previous level of service was A, and the traffic density is extremely low, and the traffic density is decreasing. Then, the current level of service is A. Free-flow operation*".

3.2.10. Level of service trend (2-PM_{LOST})

During the design of this derivative 2-*PM*, we explored different ways of expressing linguistically the perception of change and, therefore, how to calculate their validity degree. This derivative 2-*PM* has the level of service (2-CP_{LOS}) as input. The output *CP* y_{LOST} includes four types of NL propositions for each *LOSi* at each time instant *t*:

$a_{LOST_{1i}}$ → "*The level of service is keeping constant in level i*"
$a_{LOST_{2ij}}$ → "*The level of service is changing from level i to level j*"
$a_{LOST_{3j}}$ → "*The level of service of the road has changed to level j*"
$a_{LOST_{4i}}$ → "*The level of service of the road has returned to level i. The change has not been completed*"

Here, the aggregation function g_{LOST} calculates the trend of each *LOS* by analyzing the derivative of the validity degrees of each sentence $a_{LOS_i}(w_{LOS_i})$. At each time instant we determine if a certain *LOS i* is decreasing (D_{LOST_i}), keeping constant (KC_{LOST_i}), or increasing (I_{LOST_i}) by fuzzifying its derivative. We also defined that there has been a change in the level of service (denoted by C_{ij}), when a certain level *i*, which had a higher validity degree than other level *j*, becomes smaller than *j*. This binary indicator takes value 0 when there is not a change and 1 when a change is produced. After careful experimentation, we have defined the validity degrees of each type of sentence at each time instant *t* as follows:

$w_{LOST_{1i}}[t] = \min(KC_{LOST_i}, 1 - C_{ij})$. A level *i* is keeping constant when two conditions are satisfied. The first condition involves that the current level must be keeping constant (KC_{LOST_i}). The second one implies that the previous level must be the same ($C_{ij} = 0$).
$w_{LOST_{2ij}}[t] = \min(D_{LOST_i}, I_{LOST_j}, 1 - C_{ij})$. A level *i* is changing to a level *j* when three conditions are satisfied. The first condition involves that the current level *i* must be decreasing (D_{LOST_i}). The second one implies that the next level *j* must be increasing (I_{LOST_j}). Finally, the third condition is that the change between levels must not have been completed yet ($C_{ij} = 0$).
$w_{LOST_{3j}}[t] = C_{ij}$. A *LOS* has recently changed to a level *j* when one condition is satisfied: the *LOS* of the previous time instant must have been different to the current one ($C_{ij} = 1$).
$w_{LOST_{4i}}[t] = \min(I_{LOST_i}, D_{LOST_j}, w_{LOST_{2ij}}[t-1])$. In some many cases, a *LOS* could have been changing but the change was not completed. For example, the *LOS* could be changing from level *E* to level *F* but suddenly the density decreases and the change is stopped, keeping at level *E*. This case is recognized when the system was changing from level *i* to level $j(w_{LOST_{2ij}}[t-1])$ but the level *i* starts to increase (I_{LOST_i}) and the expected level *j* starts to decrease (D_{LOST_j}).

3.2.11. Level of service in the last period (2-PM_{LOSLP})

Here, we experiment with another example of 2-*PM*. This integrative 2-*PM* has the Level of service (2-CP_{LOS}) as input *CP*. The output *CP* y_{LOSLP} includes four types of NL propositions for each *LOS i* that summarize the amount of times that each *LOS* has been activated during a certain period of time:

$a_{LOSLP_{1i}}$ → "*In the last period, the level of service has never been i*"
$a_{LOSLP_{2i}}$ → "*In the last period, the level of service has been few times i*"
$a_{LOSLP_{3i}}$ → "*In the last period, the level of service has been sometimes i*"
$a_{LOSLP_{4i}}$ → "*In the last period, the level of service has been many times i*"

These sentences are based on the summarizers proposed by Yager (1995): "*Q the LOS has been i*". Where *Q* is a fuzzy quantifier (Zadeh, 1983) applied on the cardinality of the perception "*the LOS has been R*". And *R* is the summarizer, in this case the set of possible *LOS*. The set of linguistic labels for each quantifier are uniformly distributed trapezoidal SFPs denoted by the expressions *never*, *few times*, *sometimes* and *many times*.

This information is really important to summarize traffic behavior in a certain amount of time because it allows to compare the state and trend of traffic in a specific road whose study is interesting to obtain conclusions such as checking the need to redirect the traffic. The aggregation function (g_{LOSLP}) calculates the validity degrees for each sentence based on the cardinality values (*Card*) of the validity degrees of each sentence $a_{LOS_i}(w_{LOS_i})$ during the desired period duration:

$$a_{LOSLP_{1i}} = never[Card(w_{LOS_i})]$$
$$a_{LOSLP_{2i}} = few\ times[Card(w_{LOS_i})]$$
$$a_{LOSLP_{3i}} = sometimes[Card(w_{LOS_i})]$$
$$a_{LOSLP_{4i}} = many\ times[Card(w_{LOS_i})]$$

3.3. Expression module

Apart from the goal of obtaining suitable texts to be showed to drivers, the linguistic reports can be used by traffic experts with the aim of understanding changes in traffic and foreseeing its future behavior. Using the set of available *CP*s in the GLMP, namely, the evolution of the *LOS*, vehicles speed, road speed trend, extraordinary speed conditions and so on, the developed application provides two different types of linguistic description reports: an specific report which describes the instantaneous state of the traffic, and a periodical report that summarizes traffic behavior throughout a specific period of time. In both cases, we have applied basic report templates, see in Alvarez-Alvarez et al. (2011a) an example of a template that change the structure of the report depending on the validity degrees of the sentences.

3.3.1. Specific report

A specific time instant or eventual report about the traffic behavior is given. The periodicity of these reports depends on the final user's needs (one minute, five minutes, ten minutes, etc). Each report informs about the *LOS* trend (changing, recently changed or keeping constant), the traffic density and road speed in the last period of time, traffic density trend and road speed trend.

The report template is represented in Fig. 6. It uses the sentences provided by the traffic density and its trend *CP*s (1-CP_{TD} and 1-CP_{TDT}), the *LOS* and its trend *CP*s (2-CP_{LOS} and 2-CP_{LOST}), and the road speed and its trend *CP*s (1-CP_{RS} and 1-CP_{RST}). The sentences with the highest validity degree are chosen at each time instant for each *CP*. This type of information shows us that it is possible that some *CP*s are changing while the *LOS* is keeping constant, e.g., the traffic density can be increasing inside the level *C* but this does not mean that *LOS* is changing from level *C* to level *D*. This is the difference between the changes of the traffic density during a certain *LOS* and the changes between different *LOS*. One possible specific traffic report may be as follows: "*currently, the traffic density is low and it is increasing. The level of service is changing from level B to level C, stable flow, speeds at or near free-flow and queues may form. the road speed is medium and it is decreasing*".

3.3.2. Periodical report

This report summarizes traffic behavior throughout a period of time, e.g., a full day or other sets of periods that can give relevant information about the traffic progress. Traffic experts decide how many periods they want to analyze separately, in order to verify the differences existing among them. This type of information allows them to extract conclusions and to implement appropriate measures to improve the quality of traffic (improving infrastructure, notice drivers and so on). For example, one type of differentiation could be analyzing separately sunrise, morning, midday, afternoon, evening and night. In the same way, the final user could decide to segment the day into smaller periods and to extract information about periods of different sizes. The traffic summary report also gives information relative to the average traffic density and the average level of service in each period of time.

Therefore, a different report template must be used. It is represented in Fig. 7 and it uses the sentences provided by the traffic density in the last period *CP* (1-CP_{TDLP}), the *LOS* in the last period *CP* (2-CP_{LOSLP}), the road speed in the last period *CP* (1-CP_{RSLP}), and the unsafe speed conditions *CP* (1-CP_{USC}). Similarly to the specific report, the sentences with the highest validity degree are chosen for each *CP*. One possible global traffic report throughout the afternoon may be as follows: "*In the afternoon, the Traffic Density has been medium. The level of service has never been A and B; and sometimes C, D, E and F. The road speed has been low. There were not vehicles speeding*".

4. Experimentation

4.1. Simulated traffic data

In order to deal with a broad number of situation types, we have designed a simulator after analyzing several databases of

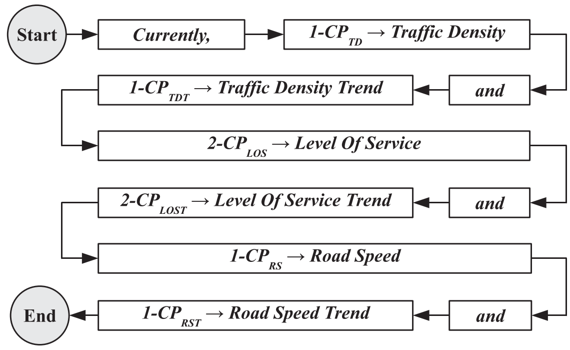

Fig. 6. Template for the specific report.

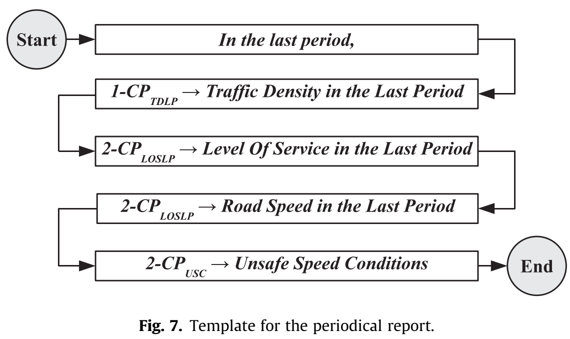

Fig. 7. Template for the periodical report.

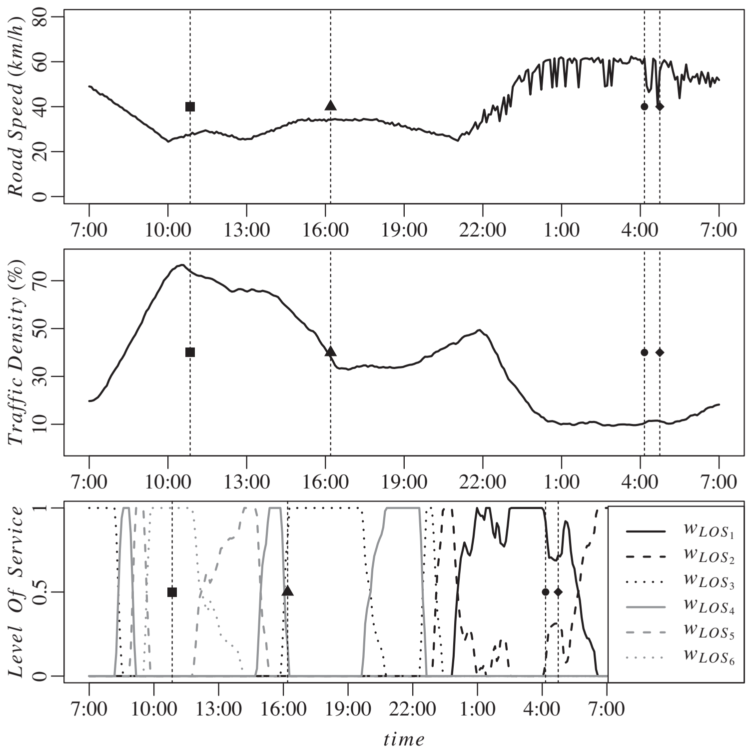

Fig. 8. Graphical representation of the traffic density, road speed, and the validity degrees of the sentences associated to each level of service.

traffic control centers of important cities, such as Madrid, Valencia, and Sevilla (http://www.trajano.com, 2012). The simulator is based on the Monte Carlo method where simulated data (number of cars, its size and its speed) follow a normal distribution. Each normal distribution is defined by its mean and its standard deviation, e.g., these parameters vary depending on the period of the day. This simulator allow us to generate data that recreates the traffic behavior in different situation types providing data each five minutes of simulated time. Fig. 8 shows an example of simulation of traffic density and road speed along a typical working day, and includes the validity degrees obtained for the sentences related to the *LOS*.

In the following, we show several examples of specific (represented with the symbols ■, ▲, •, and ⧫ in Fig. 8) and periodic traffic reports associated to these simulated data. Every report, specific or periodic, is accompanied by its reference of time, either the period of the day or the specific time of measurement. Results are consistent and show accurately the simulated situations.

- **Specific reports:**
  ■ "*At 10:55, the traffic density is extremely high and it is decreasing. The level of service is keeping constant in level F, forced or breakdown flow, queues form behind breakdown points and demand is greater than capacity. The road speed is low and it is keeping constant*"
  ▲ "*At 16:15, the traffic density is medium and it is decreasing. The level of service has changed to level C, stable flow, speeds at or near free-flow and queues may form. The road speed is low and it is keeping constant*"
  • "*At 4:10, the traffic density is extremely low and it is increasing. The level of service is changing from level A to level B, reasonably free flow, the ability to maneuver is only slightly restricted and the effects of minor incidents still are easily absorbed. The road speed is high and it is decreasing*"
  ⧫ "*At 4:45, the traffic density is extremely low and it is keeping constant. The level of service has returned to level A, free-flow operation. The road speed is medium and it is increasing*"
- **Periodical reports:**
  – "*In the morning (from 7:00 to 10:00), the traffic density has been extremely high. The level of service has never been A and B; few times D; sometimes C and E; and many times F. The road speed has been low. There were 4 vehicles speeding*"
  – "*In the afternoon (from 13:00 to 22:00), the traffic density has been low. The level of service has never been A and B; and sometimes C, D, E and F. The road speed has been low. There were not vehicles speeding*"
  – "*At night (from 22:00 to 7:00), the traffic density has been extremely low. The level of service has never been E and F; few times C and D; sometimes B; and many times A. The road speed has been medium. There were many vehicles speeding*"
  – "*During the whole day, the traffic density has been medium. The level of service has been few times B, D, E and F; and sometimes A and C. The road speed has been low. There were many vehicles speeding*"

4.2. Video camera real data

In order to check the performance and effectiveness of our application in a real situation, we have used digital image processing techniques to acquire the input data from a video camera. In a

Fig. 9. Traffic tracking visual results on two different sample frames (the red numbers are the vehicle identifiers). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

previous work (Trivino et al., 2010b), we have already used the images of a video stream, where we used an overhead camera to acquire real images and generate successfully linguistic descriptions about traffic conditions in a roundabout. Here, we used images obtained from the right side of the road. Here, we acquired information from live video recordings, recognizing vehicles and their different features (speed, position and size).

In order to recognize the image content, we considered enter and exit regions, rail regions, image perspective and obstacles that could appear in images. In our model, each vehicle moving along the analyzed road region is characterized by a unique identification number that is assigned after it is first detected passing by any enter region of the road. Fig. 9 shows graphically a typical situation in which we can observe a set of vehicles identified by a number and highlighted with its bounding box.

The specific traffic reports obtained for these two frames were as follows: "*currently, the traffic density is high and it is decreasing. The level of service has returned to level D, approaching unstable flow, speeds decline slightly with increasing flows while density increases more quickly. The road speed is low and it is decreasing*" and "*currently, the traffic density is low and it is decreasing. The level of service is keeping constant in level C, stable flow, speeds at or near free-flow and queues may form. The road speed is low and it is increasing*".

5. Concluding remarks

During the last few years, we have developed an extension of CTP that allows generating linguistic descriptions of complex phenomena. In this work, our goal consists of exploring a practical application in the field of ITS that led us to design new types of *PM*s, i.e., we aimed to improve the number of available types of linguistic expressions and therefore the versatility of our technology. Together with the practical application, in this paper, we have contributed to the development a new general approach to produce linguistic descriptions of complex phenomena evolving in time. Specifically, we have designed several 2-*PM*s demonstrating how to model the meaning of several different linguistic expressions belonging to the specific application domain of language. Moreover, we have showed that depending on the user requirements, our approach allows us to generate a great variety of customizable linguistic reports.

However, there is still too much work to do. Indeed, NL has a limitless potential of meaning. From the theoretical point of view, we will continue exploring how to model the meaning of different linguistic expressions, i.e., new *CP*s and *PM*s. From the practical point of view, we will continue developing practical applications, e.g., we will try to set up a complete system for monitoring and control the traffic in a real world scenario. The experiments performed using simulated data have allowed us to demonstrate the great expressiveness of the presented resources. The experiments performed using data obtained from real video images have allowed us to demonstrate the viability of our approach to create real industrial applications.

Acknowledgments

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Chapter 7

Impact factor of the presented publications

There is no safety in numbers, or in anything else.
James Thurber (1894 - 1961)

The impact factor (IF), is a measure reflecting the average number of citations to recent articles published in science and social science journals. It is frequently used as a measure of the relative importance of a journal within its field, with journals with higher impact factors deemed to be more important than those with lower ones. IFs are calculated yearly for those journals that are indexed in Thomson Reuters Journal Citation Reports ®.

In the following sections, The Journal Citation Report (JCR) of each of the journals, where the articles of Chapter 6 were published, is presented.

7.1 JCR of IEEE Transactions on Fuzzy Systems

ISI Web of Knowledge℠

Journal Citation Reports®

2011 JCR Science Edition

Journal: IEEE TRANSACTIONS ON FUZZY SYSTEMS

Journal Impact Factor

Cites in 2011 to items published in: 2010 = 340
2009 = 546
Sum: 886

Number of items published in: 2010 = 93
2009 = 115
Sum: 208

Calculation: Cites to recent items / Number of recent items = 886 / 208 = **4.260**

Journal Ranking

For **2011**, the journal **IEEE TRANSACTIONS ON FUZZY SYSTEMS** has an Impact Factor of **4.260**.

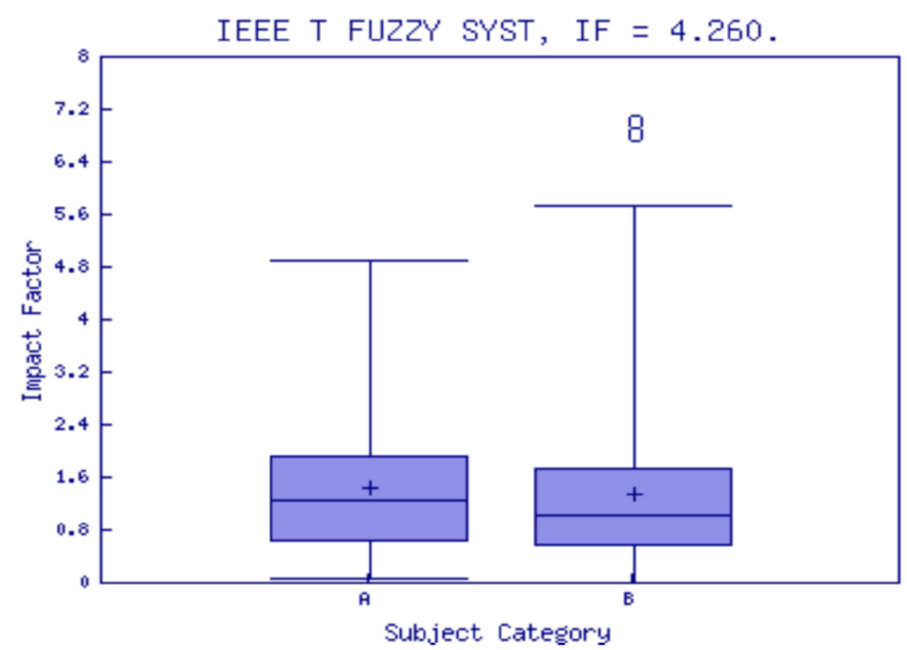
This table shows the ranking of this journal in its subject categories based on Impact Factor.

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE	111	5	Q1
ENGINEERING, ELECTRICAL & ELECTRONIC	244	8	Q1

Category Box Plot

For **2011**, the journal **IEEE TRANSACTIONS ON FUZZY SYSTEMS** has an Impact Factor of **4.260**.

This is a box plot of the subject category or categories to which the journal has been assigned. It provides information about the distribution of journals based on Impact Factor values. It shows median, 25th and 75th percentiles, and the extreme values of the distribution.



Key

A - COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE

B - ENGINEERING, ELECTRICAL & ELECTRONIC

7.2 JCR of Engineering Applications of Artificial Intelligence

ISI Web of Knowledge℠

Journal Citation Reports®

2011 JCR Science Edition

Journal: ENGINEERING APPLICATIONS OF ARTIFICIAL INTELLIGENCE

Journal Impact Factor

Cites in 2011 to items published in: 2010 = 149	Number of items published in: 2010 = 130	
2009 = 274	2009 = 124	
Sum: 423	Sum: 254	

Calculation: Cites to recent items / Number of recent items = 423 / 254 = **1.665**

Journal Ranking

For **2011**, the journal **ENGINEERING APPLICATIONS OF ARTIFICIAL INTELLIGENC...** has an Impact Factor of **1.665**.

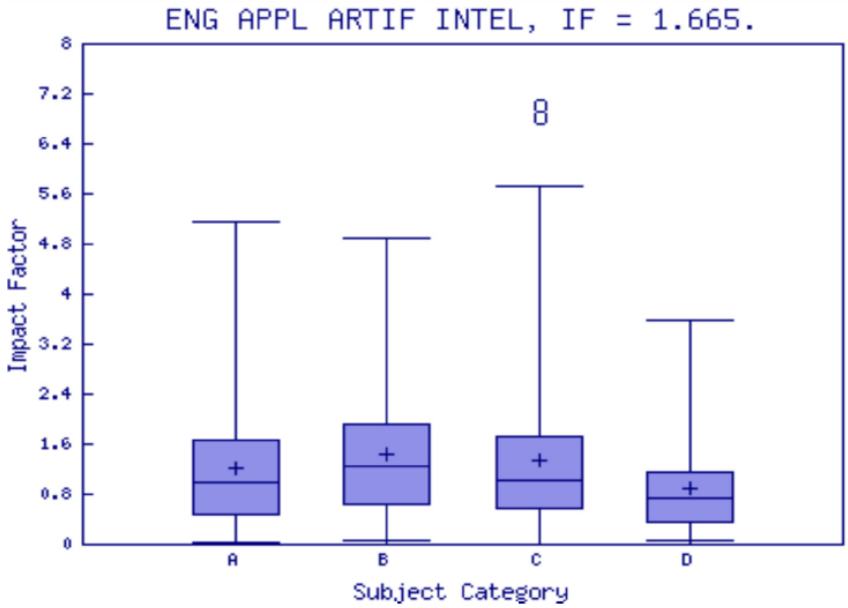
This table shows the ranking of this journal in its subject categories based on Impact Factor.

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
AUTOMATION & CONTROL SYSTEMS	58	15	Q2
COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE	111	34	Q2
ENGINEERING, ELECTRICAL & ELECTRONIC	244	65	Q2
ENGINEERING, MULTIDISCIPLINARY	90	10	Q1

Category Box Plot

For **2011**, the journal **ENGINEERING APPLICATIONS OF ARTIFICIAL INTELLIGENC...** has an Impact Factor of **1.665**.

This is a box plot of the subject category or categories to which the journal has been assigned. It provides information about the distribution of journals based on Impact Factor values. It shows median, 25th and 75th percentiles, and the extreme values of the distribution.



Key

A - AUTOMATION & CONTROL SYSTEMS

B - COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE

C - ENGINEERING, ELECTRICAL & ELECTRONIC

D - ENGINEERING, MULTIDISCIPLINARY

7.3 JCR of Expert Systems with Applications

ISI Web of Knowledge℠

Journal Citation Reports® **2011 JCR Science Edition**

Journal: EXPERT SYSTEMS WITH APPLICATIONS

Journal Impact Factor

Cites in 2011 to items published in:	2010 = 1422	Number of items published in:	2010 = 1010
	2009 = 3855		2009 = 1385
	Sum: 5277		Sum: 2395

Calculation: $\frac{\text{Cites to recent items}}{\text{Number of recent items}} = \frac{5277}{2395}$ = **2.203**

Journal Ranking

For **2011**, the journal **EXPERT SYSTEMS WITH APPLICATIONS** has an Impact Factor of **2.203**.

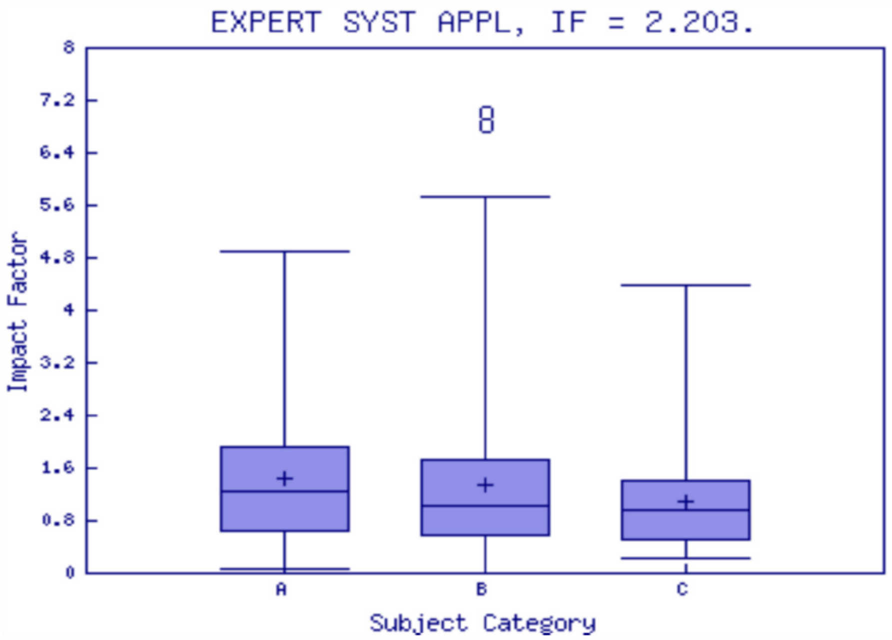
This table shows the ranking of this journal in its subject categories based on Impact Factor.

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE	111	22	Q1
ENGINEERING, ELECTRICAL & ELECTRONIC	244	41	Q1
OPERATIONS RESEARCH & MANAGEMENT SCIENCE	77	5	Q1

Category Box Plot

For **2011**, the journal **EXPERT SYSTEMS WITH APPLICATIONS** has an Impact Factor of **2.203**.

This is a box plot of the subject category or categories to which the journal has been assigned. It provides information about the distribution of journals based on Impact Factor values. It shows median, 25th and 75th percentiles, and the extreme values of the distribution.



Key

A - COMPUTER SCIENCE, ARTIFICIAL INTELLIGENCE

B - ENGINEERING, ELECTRICAL & ELECTRONIC

C - OPERATIONS RESEARCH & MANAGEMENT SCIENCE

Chapter 8

Full list of publications

I find television very educational. The minute somebody turns it on, I go to the library and read a good book.
Groucho Marx (1890 - 1977)

This chapter details the whole list of the author's publications. It contains the full bibliographic references of the articles divided into three sections. Sections 8.1 and 8.2 detail the articles publishes in national and international Conferences, while Section 8.3 enumerates the articles publishes in International Journals.

8.1 National conferences

- **A. Alvarez-Alvarez**, J. M. Alonso, G. Trivino, N. Hernández, F. Herranz and M. Ocaña. "Towards people indoor localization combining WiFi and human motion recognition". In: *Actas XV Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF), Huelva, Spain*, pp. 7–12, February 2010.

- D. Sanchez-Valdes, **A. Alvarez-Alvarez** and G. Trivino. "Linguistic description of temporal traffic evolution in roads". In: *Actas XVI Congreso Español sobre Tecnologías y Lógica Fuzzy (ESTYLF), Valladolid, Spain*, pp. 614–619, February 2012.

8.2 International conferences

- **A. Alvarez-Alvarez** and G. Trivino. "Comprehensible model of a quasi-periodic signal". In: *Proceedings of the 9th International Conference on Intelligent Systems Design and Applications (ISDA), Pisa, Italy*, pp. 450–455, December 2009.

- **A. Alvarez-Alvarez**, J. M. Alonso, G. Trivino, N. Hernández, F. Herranz, A. Llamazares and M. Ocaña. "Human Activity Recognition applying Computational Intelligence techniques for fusing information related to WiFi positioning and body posture". In: *Proceedings of the 2010 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Barcelona, Spain*, pp. 1881–1885, July 2010.

- **A. Alvarez-Alvarez** and G. Trivino. "Automatic linguistic report on the quality of the gait of a person". In: *1st International Open Workshop Fuzziness and Medicine (FUZZ-MED), Mieres, Spain*, March 2011.

- **A. Alvarez-Alvarez**, G. Trivino and O. Cordón. "Body Posture Recognition By Means Of A Genetic Fuzzy Finite State Machine". In: *Proceedings of the 5th IEEE International Workshop on Genetic and Evolutionary Fuzzy Systems (GEFS), Paris, France*, pp. 60–65, April 2011.

- **A. Alvarez-Alvarez**, D. Sanchez-Valdes and G. Trivino. "Automatic Linguistic Description about Relevant Features of the Mars' Surface". In: *Proceedings of the 11th International Conference on Intelligent Systems Design and Applications (ISDA), Córdoba, Spain*, pp. 154–159, November 2011.

8.3 International journals

- G. Trivino, **A. Alvarez-Alvarez** and G. Bailador. "Application of the computational theory of perceptions to human gait pattern recognition". *Pattern Recognition*, Vol. 43, No. 7, pp. 2572–2581, July 2010.
- **A. Alvarez-Alvarez**, G. Trivino and O. Cordón. "Human gait modeling using a genetic fuzzy finite state machine". *Fuzzy Systems, IEEE Transactions on*, Vol. 20, No. 2, pp. 205–223, April 2012.
- **A. Alvarez-Alvarez**, D. Sanchez-Valdes, G. Trivino, A. Sánchez and P. D. Suárez. "Automatic linguistic report about the traffic evolution in roads". *Expert Systems with Applications*, Vol. 39, No. 12, pp. 11293–11302, September 2012.
- **A. Alvarez-Alvarez** and G. Trivino. "Linguistic description of the human gait quality". *Engineering Applications of Artificial Intelligence*, 2012. In press. DOI: 10.1016/j.engappai.2012.01.022
- **A. Alvarez-Alvarez**, J. M. Alonso and G. Trivino. "Human activity recognition in indoor environments by means of fusing information extracted from intensity of WiFi signal and accelerations". *Submitted to Information Sciences.*
- D. Sanchez-Valdes, **A. Alvarez-Alvarez** and G. Trivino. "Linguistic description about relevant features of the Mars' surface". *Submitted to Applied Soft Computing.*

Chapter 9

Additional selected publications

My thoughts are my company; I can bring them together, select them, detain them, dismiss them.
Walter Savage Landor (1775 - 1864)

This chapter contains a complete copy of four additional publications that, although they were not presented in Chapter 6 as the core of the thesis, they are very related to the topics and works developed during the thesis. It is divided into four different Sections corresponding to each article.

9.1 Application of the computational theory of perceptions to human gait pattern recognition

G. Trivino, A. Alvarez-Alvarez, and G. Bailador. "Application of the computational theory of perceptions to human gait pattern recognition". *Pattern Recognition*, Vol. 43, No. 7, pp. 2572–2581, July 2010.

Pattern Recognition 43 (2010) 2572–2581

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Application of the computational theory of perceptions to human gait pattern recognition

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Fuzzy logic
Fuzzy finite state machine
Computational theory of perceptions
Authentication

ABSTRACT

This paper aims to contribute to the field of human gait pattern recognition by providing a solution based on the computational theory of perceptions. Our model differs significantly from others, e.g., based on machine learning techniques, because we use a linguistic model to represent the subjective designer's perceptions of the human gait process. This model is easily understood and provides good results. We include a practical demonstration with an equal error rate of 3%.

1. Introduction

Currently, industry demands new techniques for user authentication. Authentication based on biometrics is one area that has grown over the last few years. There are two types of biometric characteristics that are useful in this field: physiological characteristics, such as fingerprints [1] or DNA, and behavioral characteristics like signature [2], voice [3] or gait.

Gait analysis has been explored thoroughly during the last decade as a behavioral biometric measurement. Some areas of application include: access control, surveillance, activity monitoring and clinical analysis.

Most research is based on computer vision [4–10]. The main advantage of this approach is that there is no need to wear sensors, therefore allowing identification from a distance. For some applications, the main drawbacks of these methods are: dependence on illumination, misinterpretations due to shadows, need of a complex system for capturing images and its computational cost.

Nevertheless, solutions based on accelerometers [11–15] provide a smart solution to the problem of capturing the signal and its practical implementation as a commercial product. They can be used in the dark and provide 3-D data whereas computer vision systems produce 2-D projections. However, the user must wear sensors and this makes the solution invalid for certain applications.

Regarding algorithms used for gait pattern recognition, the most frequent are based on neural networks [16] or hidden Markov models [17,18]. However, the published results do not yet demonstrate the availability of a sufficiently robust method for a marketable product.

In this paper, we make emphasis in modeling the knowledge acquired by a human observer of the system. For example, it is interesting to consider how a human observer has not difficulties recognizing the gait as a quasi-periodic process, i.e., the signal evolves in time approximately repeating its shape and period. Moreover, a human observer is clearly able to separate the relevant from the irrelevant features in the observed signal.

Although, we consider that both procedures are complementary, our approach is based on modeling the designer's perception of the system in contrast with a procedure based on machine learning.

We aim to contribute to the pattern recognition field by providing a technique for modeling this type of human perceptions. We present a new method for human gait recognition involving analysis of the accelerations produced during a complete gait cycle. We used a fuzzy finite state machine (FFSM) [19] to model the perception of the signal evolution, where each state was established using our knowledge about the physiological phases of the human gait.

The model was implemented using fuzzy linguistic variables and rules to describe a set of states that the signal undergoes during its evolution in time. This type of model provides sufficient flexibility to represent the variations in both, signal amplitude and states time span. The model is expressed using linguistic terms that make its interpretation easier with a low computational cost.

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Once the model was available, we used three relevant features of the human gait (homogeneity, symmetry and the relation weight/legs length) to recognize the gait style corresponding to a specific person. In the demonstration, we explain how to solve the problem of authentication of one person among 11 people with an equal error rate (EER) of 3%.

We have limited the scope of this paper to the case of using accelerometers to obtain the signal, but also the described method could be applied to signals that were obtained by computer vision.

2. Computing with the meaning of human perceptions

The computational theory of perceptions (CTP) was outlined in the Zadeh's seminal paper "From computing with numbers to computing with words—from manipulation of measurements to manipulation of perceptions" [20] and further developed in subsequent papers [21]. The general goal of CTP is to develop computational systems with the capability of computing with the meaning of natural language (NL) expressions, i.e., with the capability of computing with imprecise descriptions of the world in a similar way that humans do it.

In this section, we introduce a set of definitions including and developing ideas taken from CTP. We focus our effort on exploring the possibilities of this theory in the field of pattern recognition.

2.1. Perceptions

In CTP, the computational model of a physical model is based on the subjective perceptions of a person that we will call the *designer*.

A perception (p) is a unit of information acquired by the *designer* about different parts of the system and its environment.

The *designer*'s perceptions are described using granules. A granule is a clump of elements which are drawn together by indistinguishability, similarity, proximity or functionality [22].

In CTP, the boundary of a granule is fuzzy. Fuzziness of granules allows us to model the way in which human concepts are formed, organized and manipulated in an environment of imprecision, uncertainty and partial truth [23].

The concept of linguistic variable is essential in the formal description of perceptions. Informally, a linguistic variable is a variable whose values are words or sentences in a NL [26]. For example, the linguistic variable *Age*, with possible values {*very old*, *old*, *quite new*, *new*}, can be used to describe a subjective perception of the age of an automobile.

The attributes of a perception are linguistic variables with values defined using fuzzy sets. The *designer* describes his/her perceptions using constraints [27], i.e., defining a set of relevant attributes and the sets of their possible linguistic values.

2.2. First-order perceptions

The *designer* uses first-order perceptions to define the maximum level of granularity in a model.

Typically, the *designer* obtains a first-order perception (p^1) using data provided by a sensor.

There are two forms of representing (p^1):

The linguistic representation, e.g.:

p^1 : " The Temperature is High"

And the formal representation:

$$p^1 : T = \mu_{A_i}(z)$$

where:

- T is a linguistic variable (e.g., temperature).
- A_i is a linguistic term belonging to the set of possible linguistic values of T (e.g., {Low, Warm, High}).
- $\mu_{A_i}(z)$ is the membership function associated with the linguistic term A_i.
- z is a numerical value obtained from the sensor (e.g., 45 °C).

2.3. Second-order perceptions

The concept of granularity allows the *designer* to create a hierarchy of perceptions. In this structure, the *designer* uses a set of lower order perceptions to *explain* a higher order perception.

For example, two first-order perceptions:

p_1^1: "The Temperature is Warm".
p_2^1: "The Humidity is Medium".

could be used to *explain* the perception of Comfort in a room:

p^2: "The Room is Comfortable".

Typically this *explanation* has the form of a set of fuzzy rules such as

IF p_1^1 AND p_2^1 THEN p^2

The network in this example can be extended easily by considering additional perceptions like "Acoustic noise" or "Number of persons in the room" to *explain* the perception of "Comfort". Furthermore, this perception can be used to *explain* a higher order perception, e.g., "Efficiency" of an air conditioning system (see Fig. 1).

A granular network represents the *explanation* of a perception with certain level of granularity. For example, we summarize the description of an object by hiding the irrelevant granules and remarking the relevant ones. In CTP, the model of a generic perception is called a Protoform [27]. Here, the value of the attributes of a second-order perception changes dynamically when the first-order perceptions change, e.g., when the values provided by the sensors change.

2.4. Perception of a system evolving in time

The most of practical applications concern with the perception of systems that evolve in time. The model of the perception of a system evolving in time is a protoform that describes how the system changes between the different states.

If, for the sake of simplicity, we restrict our attention to time invariant discrete-time systems, the equations that represent the evolution of a system in time are

$$\begin{cases} x[t+1] = f(x[t], u[t]) \\ y[t] = g(x[t], u[t]) \end{cases}$$

where

- $x[t]$ is a vector that represents the state of the system at time t as it is perceived by the *designer*.
- $y[t]$ is a vector that represents the system output as it is perceived by the *designer*.
- $u[t]$ is a vector that represents the system input as it is perceived by the *designer*.
- $f(x[t], u[t])$ is an *explanation* of how the system state evolves in time.
- $g(x[t], u[t])$ is an *explanation* of how the system output evolves in time.

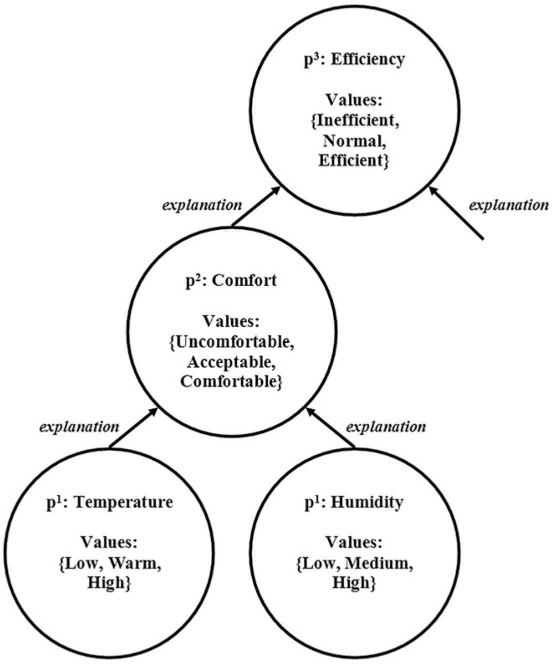

Fig. 1. Granular network corresponding to a higher-order perception.

We say that the system described by these equations is a fuzzy system when at least one of the variables is fuzzy [23]. It is important to remark that, from the point of view of our approach, the model of a system consists of a particular description of the *designer*'s perception of the system. This description contains, in certain degree of detail, an *explanation* of how the perceived outputs could be caused by the perceived inputs.

3. Fuzzy finite state machine

In the rest of this paper, we focus our attention on a specific type of computational models that provide a linguistic summary of data obtained by sensors. Here the *designer* must:

- Define a set of first-order perceptions ($u[t]$) using values provided by sensors.
- Define a second-order perception ($y[t]$) of this information with a level of granularity suitable for the final user purposes.
- Design an *explanation* of the system evolution consisting of a set of fuzzy rules that allows obtaining the values of the output linguistic variables in function of the inputs. This *explanation* includes a set of intermediate order perceptions $x[t]$ representing the relevant internal states of the system.

In a preliminary research, we have learnt that fuzzy finite state machines are suitable tools for modeling signals which evolve approximately following a repetitive pattern [29–31]. We will see that finite state machines provide an interesting paradigm to design the sets of fuzzy rules that allow us to implement the functions $f(x[t], u[t])$ and $g(x[t], u[t])$ for modeling this type of signals.

A fuzzy finite state machine (FFSM) is a tuple:

$$\{X, U, Y, f, g, X_0\}$$

where:

- X is the set of states $\{x_1, x_2, \ldots, x_{n_x}\}$. Every state represents the pattern of a repetitive situation. We say that, the system is in a specific state, when the current input variables and the previous state activations fulfill certain conditions. The activation of a state is a matter of degree, i.e., the FFSM could be partially in several states simultaneously. We will denote $\mathcal{X}_i \in [0,1]$ the degree of activation of a state. Defining the states includes determining their temporal order, i.e., the sequence with which the system follows the different relevant states.
- U is the input vector $\{u_1, u_2, \ldots, u_{n_u}\}$. U is a set of first-order perceptions where each variable u_i takes its value in a domain defined with a set of linguistic labels $\{A_{i1}, A_{i2}, \ldots, A_{in}\}$. Fig. 3 shows an example of these linguistic labels over the vertical axis.
- Y is the output vector $\{y_1, y_2, \ldots, y_{n_y}\}$. Y represents a summary of the values taken by the inputs while staying in a specific state.
- f is the state transition function $X[t+1]=f(U[t], X[t])$. This function can be implemented using a set of fuzzy rules:
  - Rules that constrain the signal amplitude. We distinguish between rules to stay in a state x_i (R_{ii}) and rules to change from the state x_i to the state x_j (R_{ij}):

    $$R_{ii}: \text{ IF } \mathcal{X}_i(t) \vee (u \text{ is } C_i) \text{ THEN } \mathcal{X}_i(t+1)$$
    $$R_{ij}: \text{ IF } \mathcal{X}_i(t) \wedge (u \text{ is } C_j) \text{ THEN } \mathcal{X}_j(t+1)$$

    where C_i and C_j represent the conditions of amplitude for the state x_i and x_j, respectively.

    We used $\vee$ in R_{ii} to introduce an inertia to change of state. This makes the FFSM more robust against spurious in the input. This OR is typically implemented using the maximum operator.

    We used $\wedge$ in R_{ij} to define the conditions to change more sharply. This AND is typically implemented using the minimum operator.
  - Rules that constrain the signal time span. We did this using two additional linguistic labels (see Fig. 4):

    T_{stay_i}: is the maximum time that the signal is expected to remain in state x_i.

    $T_{change_{ij}}$: is the minimum time that the signal is expected to remain in state x_i before changing to state x_j.

    Therefore, adding the temporal conditions:

    $$R_{ii}: \text{ IF } \mathcal{X}_i(t) \vee (u = C_i) \vee (d_i = T_{stay_i}) \text{ THEN } \mathcal{X}_i(t+1)$$
    $$R_{ij}: \text{ IF } \mathcal{X}_i(t) \wedge (u = C_j) \wedge (d_i = T_{change_{ij}}) \text{ THEN } \mathcal{X}_j(t+1)$$

    where d_i is the duration of the state x_i.
- g is the output function $Y[t]=g(U[t], X[t])$. The output variables are obtained as a summary of the values of the inputs while the system remained in the considered state, e.g., using the average or the standard deviation (see an example in the next section).
- X_0 is the initial state.

4. Human gait pattern recognition

The following sections describe how to apply the introduced above concepts in the field of human gait pattern recognition.

4.1. Linguistic terms in the application domain

Before embarking on a description of the different phases of the human gait, it is needed to introduce a small set of terms belonging to the domain of language.

- Reference foot: One foot.
- Opposite foot: The other foot.
- Stance phase: It begins when the heel contacts the ground and ends when the toes rise off the ground.
- Swing phase: It covers the period when the foot is not in contact with the ground.

The human gait is a quasi-periodic process with peculiarities that allow identifying a specific person. We used three characteristics to distinguish among different human gait styles:

- Symmetry: The degree with which the movement of a leg is similar to the other one.
- Homogeneity: The degree with which the whole gait profile repeats in time.
- The estimated proportion between legs length and weight.

We have designed a model of the human gait, i.e., a protoform where these characteristics appear remarked whereas the irrelevant aspects remain hidden.

4.2. Input variables

We attached a sensor in the belt, centered in the back, that provided measurements of three orthogonal accelerations every 100 ms. We programmed a PDA to receive the data via a Bluetooth connection and to record them with a timestamp. Every record contained the following information:

$(Timestamp, a_x, a_y, a_z)$

where:

- a_x is the vertical acceleration.
- a_y is the lateral acceleration.
- a_z is the acceleration in the progress direction.

During a first analysis of data, we realized that a_x and a_y were indicative for the states we wanted to distinguish. a_z was more difficult to use because it has to do with the walking speed and this speed can vary for the same person. Therefore, we used the two first accelerations as input to the fuzzification process.

4.2.1. Normalization

As an initial step, we normalized the signals. First, we subtracted the average making them to be centered on zero. Then, we rescaled them in the range given by their standard deviations. This allowed us to perform the analysis at the scale that gives us more information about the signal changes. Fig. 2 shows an example of the evolution of these two accelerations during one cycle and a half.

4.2.2. Fuzzification

This step allows defining the first-order perceptions. In this level of granularity, the linguistic variables take a value belonging to the set $\{\text{Negative}, \text{Zero}, \text{Positive}\}$. Fig. 3 shows the drawing of these trapezoidal linguistic labels over the vertical axis. Note that, thanks to the normalization step, each trapezoidal linguistic label covers one third of the total amplitude.

4.3. Set of rules

The states were defined as follows:

- x_1: Reference foot is in stance phase and opposite foot is in stance phase (double limb support).
- x_2: Reference foot is in stance phase and opposite foot is in swing phase (reference limb single support).
- x_3: Reference foot is in stance phase and opposite foot is in stance phase (double limb support but different of x_1 because the feet position).
- x_4: Reference foot is in swing phase and opposite foot is in stance phase (opposite limb single support).

We used a set of fuzzy rules to explain the signal evolution between the different states. In contrast to machine learning techniques, we derived the rules from the *designer*'s perceptions about the human gait acceleration signals. The use of linguistic rules allows the *designer* to include his/her experience about the human gait in a easy way.

The model is able to synchronize without the need of doing previous segmentation of the signal. We chose the initial state $X_0 = \{x_1\}(\mathcal{X}_1 = 1$ and $\mathcal{X}_i = 0\ \forall i \neq 1)$, i.e., the FFSM synchronizes with the signal when the conditions of x_1 are fulfilled.

4.3.1. Conditions of amplitude

We defined the conditions of amplitude to remain in a state or to change between states by combining the information obtained from the sensors and the available generic knowledge about the human gait. We defined eight rules (four to remain in each state and four to change between states).

We aggregated the conditions to remain in a specific state using OR to make the system more robust against spurious in the input. For example, the lateral acceleration (a_y) can fluctuate in the state x_3 while the vertical acceleration (a_x) matches its condition.

We aggregated the conditions to change between states using AND, trying to make the changes of state as sharp as possible:

- R_{11}: IF $\mathcal{X}_1(t) \vee (a_x = P) \vee (a_y = P)$ THEN $\mathcal{X}_1(t+1)$
- R_{22}: IF $\mathcal{X}_2(t) \vee (a_x = N) \vee (a_y = Z)$ THEN $\mathcal{X}_2(t+1)$
- R_{33}: IF $\mathcal{X}_3(t) \vee (a_x = P) \vee (a_y = N)$ THEN $\mathcal{X}_3(t+1)$
- R_{44}: IF $\mathcal{X}_4(t) \vee (a_x = N) \vee (a_y = Z)$ THEN $\mathcal{X}_4(t+1)$
- R_{12}: IF $\mathcal{X}_1(t) \wedge (a_x = N) \wedge (a_y = Z)$ THEN $\mathcal{X}_2(t+1)$
- R_{23}: IF $\mathcal{X}_2(t) \wedge (a_x = P) \wedge (a_y = N)$ THEN $\mathcal{X}_3(t+1)$
- R_{34}: IF $\mathcal{X}_3(t) \wedge (a_x = N) \wedge (a_y = Z)$ THEN $\mathcal{X}_4(t+1)$
- R_{41}: IF $\mathcal{X}_4(t) \wedge (a_x = P) \wedge (a_y = P)$ THEN $\mathcal{X}_1(t+1)$

4.3.2. Temporal conditions

We applied self-correlation analysis to the vertical acceleration to obtain an approximation of the signal period (T). In agreement with our knowledge about the typical human gait cycle, we assigned to each state approximately a 25% of T. Fig. 4 shows the linguistic labels T_{stay} and T_{change} used to define the temporal constraints. T_{stay} was tuned manually with the conservative criteria of ensuring that the state will span at least the 25%. T_{change} was designed to allow the change of state as soon as the conditions for changing are fulfilled.

Adding these temporal conditions, the rules were formulated as follows:

- R_{11}: IF $\mathcal{X}_1(t) \vee (a_x = P) \vee (a_y = P) \vee (d_1 = T_{stay})$ THEN $\mathcal{X}_1(t+1)$
- R_{22}: IF $\mathcal{X}_2(t) \vee (a_x = N) \vee (a_y = Z) \vee (d_2 = T_{stay})$ THEN $\mathcal{X}_2(t+1)$
- R_{33}: IF $\mathcal{X}_3(t) \vee (a_x = P) \vee (a_y = N) \vee (d_3 = T_{stay})$ THEN $\mathcal{X}_3(t+1)$
- R_{44}: IF $\mathcal{X}_4(t) \vee (a_x = N) \vee (a_y = Z) \vee (d_4 = T_{stay})$ THEN $\mathcal{X}_4(t+1)$
- R_{12}: IF $\mathcal{X}_1(t) \wedge (a_x = N) \wedge (a_y = Z) \wedge (d_1 = T_{change})$ THEN $\mathcal{X}_2(t+1)$

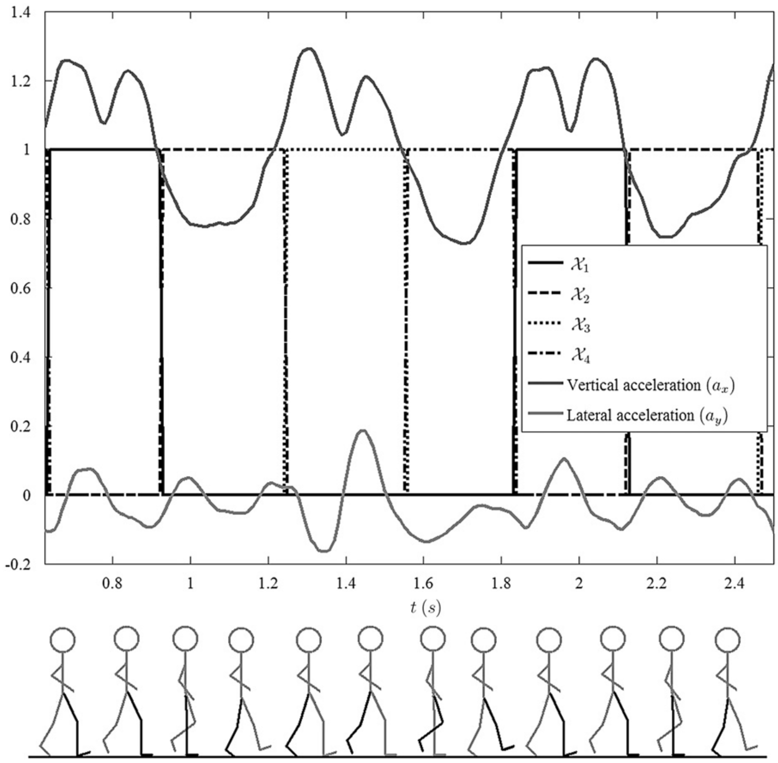

Fig. 2. Vertical and lateral acceleration in g units during the four states of the human gait cycle.

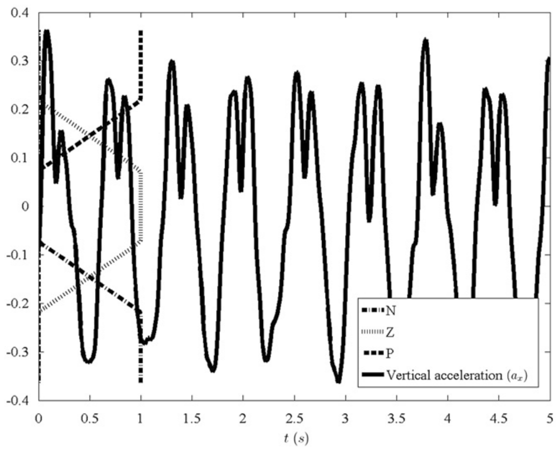

Fig. 3. Trapezoidal linguistic labels for the normalized vertical acceleration.

- R_{23}: IF $\mathcal{X}_2(t) \wedge (a_x = P) \wedge (a_y = N) \wedge (d_2 = T_{change})$ THEN $\mathcal{X}_3(t+1)$
- R_{34}: IF $\mathcal{X}_3(t) \wedge (a_x = N) \wedge (a_y = Z) \wedge (d_3 = T_{change})$ THEN $\mathcal{X}_4(t+1)$
- R_{41}: IF $\mathcal{X}_4(t) \wedge (a_x = P) \wedge (a_y = P) \wedge (d_4 = T_{change})$ THEN $\mathcal{X}_1(t+1)$

where d_i is the duration of the state x_i.

Fig. 2 represents the degree of activation $\mathcal{X}_i$ of the four states of the FFSM following the evolution of the human gait. It shows how this set of fuzzy rules is able to separate efficiently the four phases of the human gait.

4.4. Output variables

After some experimentation, we realized that, although a_y is useful to distinguish between the states x_1 and x_3, it does not provide relevant information for our purpose. Also, as mentioned above, the acceleration in the direction of march a_z depends on the person's walking speed.

Indeed, the use of these variables or other additional signals, e.g. gyroscopes, could be considered. However, the algorithms would grow up in complexity and we should lose the advantage of the simplicity.

Therefore, once identified the four phases in the signal, we focused on the characteristics of the vertical acceleration a_x. This acceleration provided sufficient information for our purpose. Here in after, for simplicity, we denote the vertical acceleration as a.

Fig. 5 shows the evolution of a along the four phases. The four rectangles represent graphically the relevant characteristics of each cycle of a specific gait. The dimensions of every rectangle summarize the values of the acceleration while staying in each state, i.e. they are a graphical representation of a protoform of the human gait. Therefore, the output of the FFSM is a vector:

$$y_i = (\overline{t_i}, \overline{a_i}, \sigma_{t_i}, \sigma_{a_i})$$

The elements of this vector are:

- $\overline{t_i}$: The horizontal coordinate of the center of each rectangle is the temporal "center of mass" of the vertical acceleration in the state x_i. Note that the "mass" in every instant t is calculated as the vertical acceleration $a(t)$ weighted by the degree of activation $\mathcal{X}_i(t)$ of the state x_i.

$$\overline{t_i} = \frac{\sum_{t=0}^{T} t \cdot a(t) \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} a(t) \cdot \mathcal{X}_i(t)}$$

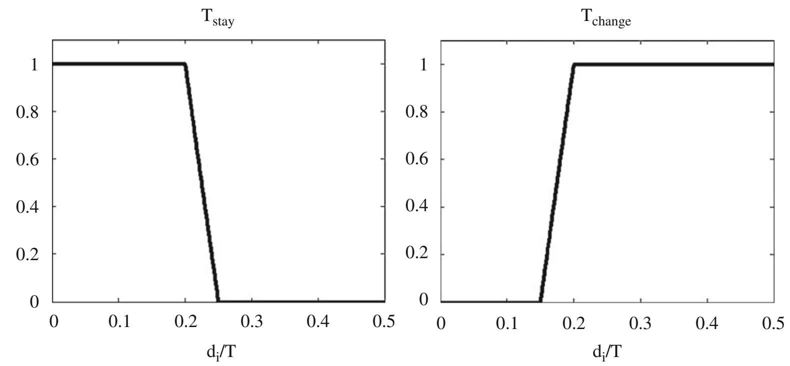

Fig. 4. Temporal conditions for the state x_i.

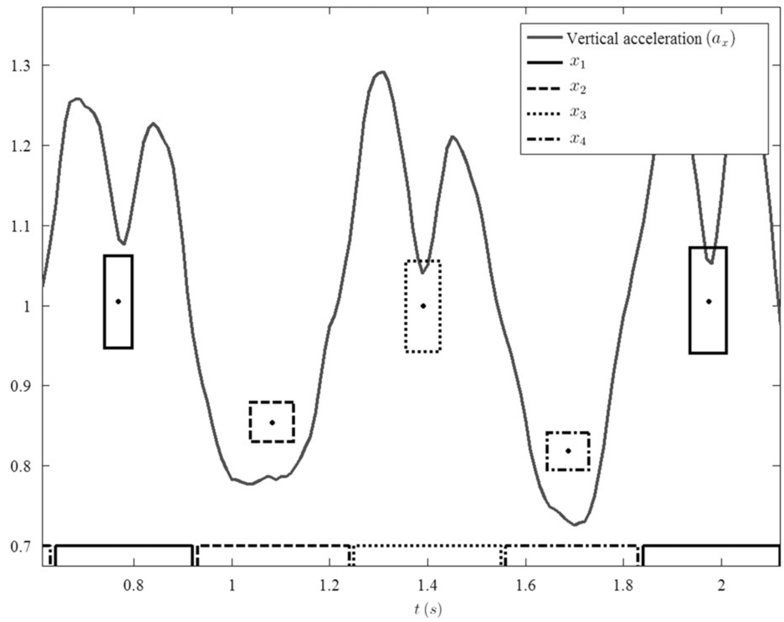

Fig. 5. Characteristic rectangles and vertical acceleration in g units during the human gait cycle.

- $\overline{a_i}$: The vertical coordinate of the center of each rectangle is the average of the vertical acceleration during the state x_i.

$$\overline{a_i} = \frac{\sum_{t=0}^{T} a(t) \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} \mathcal{X}_i(t)}$$

- σ_{t_i}: The width of each rectangle is the standard deviation of the temporal distribution of the vertical acceleration weighted by the degree of activation $\mathcal{X}_i(t)$ of the state x_i.

$$\sigma_{t_i}^2 = \frac{\sum_{t=0}^{T} (t-\overline{t_i})^2 \cdot a(t) \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} a(t) \cdot \mathcal{X}_i(t)}$$

- σ_{a_i}: The height of each rectangle is the standard deviation of the vertical acceleration during the state x_i.

$$\sigma_{a_i}^2 = \frac{\sum_{t=0}^{T} (a(t)-\overline{a_i})^2 \cdot \mathcal{X}_i(t)}{\sum_{t=0}^{T} \mathcal{X}_i(t)}$$

where:

- $a(t)$ is the vertical acceleration at the instant t.
- $\mathcal{X}_i(t)$ is the degree of activation of the state x_i at the instant t.
- T is the duration of a complete cycle.

5. Relevant characteristics

Using this model, we were able to analyze the differences among gaits of different people. We used the areas of the rectangles to distinguish the peculiarities of each specific gait.

In agreement with the section above, each cycle was modeled using four rectangles (states) and each rectangle was represented by the vector $y_i = (\overline{t_i}, \overline{a_i}, \sigma_{t_i}, \sigma_{a_i})$.

As mentioned above, we used three characteristics of the human gait that are useful to recognize the gait style corresponding to a specific person, namely, homogeneity, symmetry and the relation between the weight and the length of legs.

5.1. Homogeneity

The homogeneity ($\mathcal{H}$) was obtained by comparing a gait with itself. The homogeneity is based on the standard deviation of the sequence of rectangles of each state.

The homogeneity of the state x_i ($\mathcal{H}_i$) was formulated as follows:

$$\mathcal{H}_i = \begin{cases} \dfrac{\overline{\mathcal{A}_i} - std(\mathcal{A}_i)}{\overline{\mathcal{A}_i}} & \text{if } std(\mathcal{A}_i) < \overline{\mathcal{A}_i} \\ 0 & \text{if } std(\mathcal{A}_i) \geq \overline{\mathcal{A}_i} \end{cases}$$

where:

- $\mathcal{A}_i$ are the areas of the rectangles corresponding to the state x_i in the total number of available cycles.
- $\overline{\mathcal{A}_i}$ is the mean of this sequence of areas.
- $std(\mathcal{A}_i)$ is the standard deviation of this sequence of areas.

This equation provides a value $\mathcal{H}_i \in [0,1]$. A low standard deviation indicates similar areas, i.e., homogeneity close to 1. A high standard deviation indicates differences, i.e., homogeneity close to 0.

The total homogeneity ($\mathcal{H}$) summarizes the homogeneities of the four states as follows:

$$\mathcal{H} = \frac{1}{4}\sum_{i=1}^{4} \mathcal{H}_i$$

5.2. Symmetry

Symmetry ($\mathcal{S}$) was obtained by comparing the movement of both legs. Symmetry is based on comparing the areas of the states x_1 and x_2 (stance and swing phase of the reference foot) versus the areas of x_3 and x_4 (stance and swing phase of the opposite foot). A gait will be symmetric if the areas of the states x_1 and x_2 are similar to the areas of the states x_3 and x_4. The symmetry in a cycle j ($\mathcal{S}_j \in [0,1]$) was formulated as follows:

$$\mathcal{S}_j = \begin{cases} \dfrac{\mathcal{A}_{j3} + \mathcal{A}_{j4}}{\mathcal{A}_{j1} + \mathcal{A}_{j2}} & \text{if } \mathcal{A}_{j1} + \mathcal{A}_{j2} \geq \mathcal{A}_{j3} + \mathcal{A}_{4j} \\ \dfrac{\mathcal{A}_{j1} + \mathcal{A}_{j2}}{\mathcal{A}_{j3} + \mathcal{A}_{j4}} & \text{if } \mathcal{A}_{j1} + \mathcal{A}_{j2} < \mathcal{A}_{j3} + \mathcal{A}_{j4} \end{cases}$$

where:

$\mathcal{A}_{j1}$, $\mathcal{A}_{j2}$, $\mathcal{A}_{j3}$, $\mathcal{A}_{j4}$ are the areas of the rectangles corresponding to states x_1, x_2, x_3, x_4 in the cycle j.

The total symmetry ($\mathcal{S}$) was calculated as the average of the symmetries of the sampled cycles M:

$$\mathcal{S} = \frac{1}{M}\sum_{j=1}^{M} \mathcal{S}_j$$

5.3. The fourth root model

The "fourth root model" is an empiric model for estimating the distance covered with a number of steps measuring the vertical acceleration [32]. This model includes an experimental parameter ($\mathcal{C}$) that is practically invariant for a certain person, but that varies significantly among different people. $\mathcal{C}$ is related with the proportion between the legs length and the weight. It is calculated experimentally making a person to walk a known distance. The stride length is given by the formula:

$$Step = \mathcal{C} \cdot (a_{max} - a_{min})^{1/4}$$

where:

- $Step$ is the stride length.
- $\mathcal{C}$ is the experimental parameter.
- a_{max} is the maximum of the vertical acceleration.
- a_{min} is the minimum of the vertical acceleration.

The stride length can also be calculated as the product between the walking speed of the person (V) and the period of the corresponding cycle (T):

$$\left.\begin{aligned} Step &= \mathcal{C} \cdot (a_{max} - a_{min})^{1/4} \\ Step &= V \cdot T \end{aligned}\right\} \Rightarrow \frac{C}{V} = \frac{T}{(a_{max} - a_{min})^{1/4}}$$

We considered that V is approximately constant for each person as a function of his/her selection (see Section 6). Therefore, we assumed that $\mathcal{C}/V$ was a constant $\mathcal{K}$:

$$\mathcal{K} = \frac{T}{(a_{max} - a_{min})^{1/4}}$$

where:

- T is the mean of the period during various cycles.
- a_{max} is the mean of the maximum vertical accelerations during various cycles.
- a_{min} is the mean of the minimum vertical accelerations during various cycles.

We used this constant $\mathcal{K}$ as a third invariant characteristic of the human gait.

5.4. Authentication

We applied these formulas to obtain a vector of characteristics ($\mathcal{H}, \mathcal{S}, \mathcal{K}$) for each person in a database. Empirically, we tested that this vector provided sufficient separation among the gaits of different persons whereas the samples of the same person were distributed randomly around a center of gravity. Therefore, we used Gaussian membership functions to represent the distribution of values of these variables on the axes of a three-dimensional domain.

These membership functions were formulated as follows:

$$\mu_{\mathcal{H}} = e^{-(\mathcal{H} - \overline{\mathcal{H}})^2/2 \cdot \sigma_{\mathcal{H}}^2}$$
$$\mu_{\mathcal{S}} = e^{-(\mathcal{S} - \overline{\mathcal{S}})^2/2 \cdot \sigma_{\mathcal{S}}^2}$$
$$\mu_{\mathcal{K}} = e^{-(\mathcal{K} - \overline{\mathcal{K}})^2/2 \cdot \sigma_{\mathcal{K}}^2}$$

where:

- $\overline{\mathcal{H}}$ is of the mean the homogeneity values of a person and $\sigma_{\mathcal{H}}$ the standard deviation.
- $\overline{\mathcal{S}}$ is the mean of the symmetry values of a person and $\sigma_{\mathcal{S}}$ the standard deviation.
- $\overline{\mathcal{K}}$ is the mean of the $\mathcal{K}$ values of a person and $\sigma_{\mathcal{K}}$ the standard deviation.

Fig. 6 shows an example of two clusters with their respective membership functions.

The process of authentication of a person was performed using a sample of his/her gait ($\mathcal{H}^*, \mathcal{S}^*, \mathcal{K}^*$). We calculated the membership values ($\mu_{\mathcal{H}^*}, \mu_{\mathcal{S}^*}, \mu_{\mathcal{K}^*}$). And then, the intersection of these conditions was formulated as follows:

$$Score = \min(\mu_{\mathcal{H}^*}, \mu_{\mathcal{S}^*}, \mu_{\mathcal{K}^*})$$

where $Score$ represents the degree of membership of this sample to the cluster associated to the authenticated person. If $Score > \lambda$

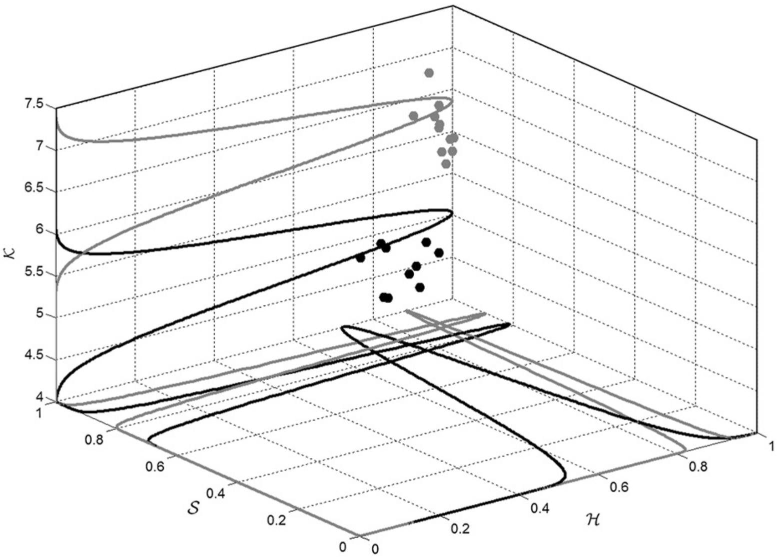

Fig. 6. Gaussian membership functions for two different people.

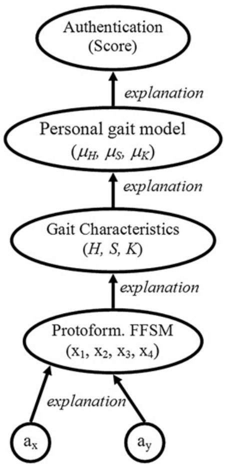

Fig. 7. The granular network explaining the *n* th-order perception needed for authenticate a gait.

the human gait is accepted, being λ a threshold that depends on the application.

In summary, Fig. 7 shows a granular network that *explains* the perception that allows the *designer* to identify a person using the human gait.

6. Experimentation

We tested this pattern recognition method by authenticating one person among 11 people. Subjects were instructed to walk at a self-selected, comfortable walking speed. Each subject walked 20 samples of 10 steps each, so we obtained a total of 220 samples (20 genuine and 200 impostors).

Each sample was tested using the leave-one-out cross validation (K-fold cross-validation with K being equal to the number of observations in the original sample) against the remaining training data.

The equal error rate (EER) is obtained from the intersection between the false acceptance rate (FAR) and the false rejection rate (FRR) versus the threshold λ (see Fig. 8). With our experimental data, we obtained the values: $\lambda = 0.0118$ and EER = 3%.

In order to show the advantage of our approach, we studied the results obtained by other researchers (see Table 1). It is worth to remark that these equal error rates are relatively comparable. These works use different number of subjects, different sensor configuration and different methods to analyze the signal.

We have summarized the differences and similarities of these works as follows:

Ailisto et al. [11] authenticate users of portable devices from the accelerations obtained by a three-axis accelerometer placed on the belt, at back. They divide the signal into one step long parts using the maximum and the minimum of the signal. They assume that the right and left steps are not necessarily symmetrical. They used 36 subjects that walked in their normal, fast and slow speed. They perform three different analysis: correlation, frequency domain and they use two variants of data distribution statistics method. They obtaining an EER of 7%, 10%, 18% and 19%, respectively.

Gafurov et al. [12] authenticate 22 subjects walking in their normal speed wearing a three-axial accelerometer in their hip. They normalize each gait cycle in time. They use a cycle length analysis method to obtain an EER of 16%. In a subsequent paper [13] they authenticate 21 subjects with the accelerometer fixed on the ankle. They use the same on time-normalized cycle length method and histogram similarity analysis to obtain an EER of 9% and 5%, respectively. Finally, in [14] they authenticate 50 subjects with the accelerometer placed in the trousers pocket. They perform the same preprocessing and use four methods: absolute distance, correlation, histogram and higher order moments. The best result (with the absolute distance) is an EER of 7%.

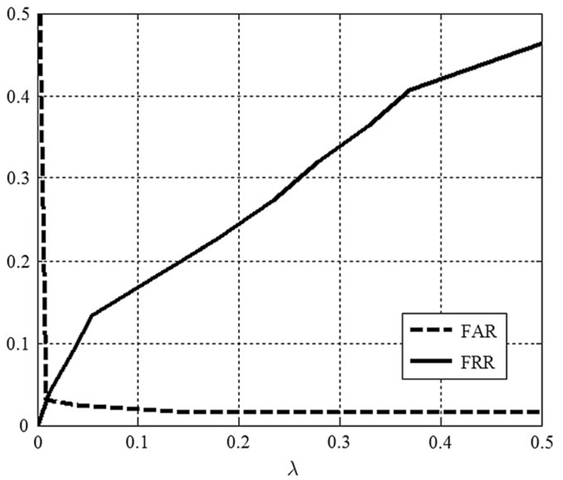

Fig. 8. FAR and FRR versus the threshold λ.

Table 1
Summary of accelerometer-based gait pattern recognition Works.

Work	# Subjects	EER%
Ailisto et al. [11]	36	7, 10, 18, 19
Gafurov et al. [12]	22	16
Gafurov et al. [13]	21	5, 9
Gafurov et al. [14]	50	7.3, 9.2, 14, 20
Rong et al. [15]	21	5.6, 21.1
This paper	11	3

Rong et al. [15] use a three-axis accelerometer fixed on the user's waist to obtain the signal of 21 subjects. They perform a preprocessing of the signal that includes wavelet denoising, gait cycles dividing and dynamic time warping. They analyze the signal in time and frequency domains obtaining an EER of 5.6% and 21.1%, respectively.

In our approach, the segmentation or preprocessing of the signal was not needed. The FFSM was able to divide the acceleration signals into cycles and to synchronize with the signal automatically.

The main difference of our contribution is the use of a fuzzy linguistic model for describing the human gait. The flexibility of this paradigm allows the *designer* to focus the model on the relevant characteristics of the signal for an specific purpose.

7. Conclusions

In this paper, we have contributed to the field of pattern recognition presenting a new method of signal analysis based on the CTP. We have explored the possibility of using a model of the perceptions of a human observer as a complement to the well established automatic machine learning procedures.

We have focused our effort on modeling the perception of a quasi-periodic signal. Specifically, we have proposed a flexible model of the human gait that allows representing linguistically the relative variations of period and amplitude of this type of signal.

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9.2 Human activity recognition applying computational intelligence techniques for fusing information related to WiFi positioning and body posture

A. Alvarez-Alvarez, J. M. Alonso, G. Trivino, N. Hernández, F. Herranz, A. Llamazares, and M. Ocaña. "Human Activity Recognition applying Computational Intelligence techniques for fusing information related to WiFi positioning and body posture". In: *Proceedings of the 2010 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Barcelona, Spain*, pp. 1881–1885, July 2010.

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Human Activity Recognition applying Computational Intelligence techniques for fusing information related to WiFi positioning and body posture

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Abstract— **This work presents a general framework for people indoor activity recognition. Firstly, a Wireless Fidelity (WiFi) localization system implemented as a Fuzzy Rule-based Classifier (FRBC) is used to obtain an approximate position at the level of discrete zones (office, corridor, meeting room, etc). Secondly, a Fuzzy Finite State Machine (FFSM) is used for human body posture recognition (seated, standing upright or walking). Finally, another FFSM combines both WiFi localization and posture recognition to obtain a robust, reliable, and easily understandable activity recognition system (working in the desk room, crossing the corridor, having a meeting, etc). Each user carries with a personal digital agenda (PDA) or smart-phone equipped with a WiFi interface for localization task and accelerometers for posture recognition. Our approach does not require adding new hardware to the experimental environment. It relies on the WiFi access points (APs) widely available in most public and private buildings. We include a practical experimentation where good results were achieved.**

I. INTRODUCTION

People activity recognition provides interesting applications in many areas, e.g., to filter the phone calls depending on different circumstances, personal navigation assistance, personal security, etc. We are mainly interested in indoor security applications (for instance sending warnings when someone gets into a dangerous area in order to reduce the occupational health and safety risk) and/or people assistance (for instance helping elderly or handicapped people).

Our activity recognition system is mainly based on Fuzzy Logic (FL) [1] because it allows to combine several heterogeneous sources of knowledge (mainly expert knowledge and knowledge automatically extracted from experimental data provided by sensors), dealing with vague information, and its interaction with humans demands the design of an easily understandable system. FL is widely recognized for its ability for linguistic concept modeling and its use in system identification. On the one hand, FL semantic expressivity, using linguistic variables [2] and linguistic rules [3], is quite close to natural language what reduces the effort of expert knowledge extraction. On the other hand, being universal approximators [4] fuzzy inference systems (FIS) are able to perform nonlinear mappings between inputs and outputs. Thus, there are lots of fuzzy machine learning methods for knowledge induction from experimental data [5].

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There are other recent works [6], [7] that show the advantages of using FL for modeling and monitoring human activity. They are mainly based on fusing video sensors what means installing additional hardware (HW) like video cameras in the environment under study. On the contrary, our approach takes profit from pre-existent HW and avoids adding new devices to the environment.

In indoor environments, the use of the network infrastructure to estimate user's location is quite common. Local network based systems are sometimes based on pre-existing networks like ZigBee networks designed for home control applications [8]. However, the most used systems are based on WiFi networks which are able to provide indoor absolute localization. In contrast, the main drawback is the need of a complete network infrastructure in the whole building where we want to localize a person. Luckily, this technology is quickly growing of coverage. Currently, there are WiFi Access Points (APs) in most public buildings like hospitals, libraries, universities, museums, etc. Moreover, measuring the WiFi signal level is free even for private WiFi networks. As a result, WiFi technology is a good choice for indoor global localization systems yielding a good accuracy-cost trade-off [9].

Regarding human activity recognition it is important to know the place where a person is located but it is not enough. We propose taking into account also information related to the human body posture. It can be estimated by means of sensor based systems which provide absolute information (e.g., magnetic compass, ultrasonic or infrared sensors) or relative information (e.g., inertial measurement units or pressure sensors). One low-cost inertial sensor is the accelerometer, based on the Micro Electro Mechanical Systems (MEMS) technology that has allowed its integration in small and low energy consumption devices. Accelerometers can be used as step length estimators; furthermore they let us to obtain some information about body posture [10]. In previous works we have already shown how human activity can be analyzed in terms of combining one accelerometer with a skin conductivity meter [11]. This work focuses on exploiting the fusion of WiFi signal and accelerations.

This paper is organized as follows. Section II describes how to design a Fuzzy Rule-based Classifier (FRBC) while Section III formalizes the notion of the Fuzzy Finite State Machine (FFSM). Afterwards, Section IV explains our proposal related to people activity recognition. It combines one FRBC and two FFSMs. Then, Section V shows the

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experimental results. And finally, Section VI expounds the conclusions and future works.

II. Fuzzy Rule Base Classifiers

A FRBC is a fuzzy system able to select one output class from a pre-defined set of classes C=$\{C^1, C^2, \ldots, C^{NC}\}$. Given an n-dimensional input space (X $\subseteq R^n$), a fuzzy inference mechanism yields an activation degree associated to each class C^i. Of course, several classes can be activated at the same time with activation degree greater than zero.

Our FRBC is designed following the fuzzy modeling methodology called HILK (Highly Interpretable Linguistic Knowledge) [12]. It focuses on building comprehensible fuzzy classifiers, i.e., classifiers easily understandable by human beings. Useful pieces of knowledge are automatically extracted from experimental data and represented by means of linguistic variables and rules under the FL formalism. The whole modeling process is made up of three steps:

- **Partition design**. The readability of fuzzy partitioning is a prerequisite to build interpretable FRBCs. Therefore, it is based on the use of Strong Fuzzy Partitions (SFPs) which are the best ones from the comprehensibility point of view.
- **Rule base learning**. Linguistic rules are automatically extracted from data keeping in mind the comprehensibility goal. Therefore, we have chosen Fuzzy Decision Tree (FDT) [13] as rule induction method. It generates a neuro-fuzzy decision tree which is translated into quite general incomplete rules where only a subset of input variables is considered. In addition, inputs are sorted according to their importance (minimizing the entropy). FDT is a fuzzy version of the popular decision trees defined by Quinlan [14]. Rules are of form **If** *Premise* **Then** *Conclusion*, where both *Premise* and *Conclusion* use linguistic terms previously defined for expressing linguistic propositions that describes the system behavior.

  $$R\text{: }\textbf{If}\ \underbrace{\underbrace{I_1 \text{ is } A_1^i}_{Premise\ P_1}\ \textbf{AND} \ldots \textbf{AND}\ \underbrace{I_{NI} \text{ is } A_{NI}^j}_{Premise\ P_{NI}}}_{Premise}\ \textbf{Then}\ \underbrace{Y_R \text{ is } C^i}_{\text{Conclusion}}$$

  where given a rule R, rule premises are made up of tuples (*input variable*, *linguistic term*) where I_a is the name of the input variable a, while A_a^i represents the label i of such variable, with a belonging to $\{1, ..., NI\}$ and being NI the number of inputs. In the conclusion part, C^i represents one of the possible output classes.
- **Knowledge base improvement**. It is an iterative refinement process that comprises both rule base simplification and fuzzy partition optimization. The former increases interpretability while keeping high accuracy. The later gets higher accuracy without penalizing the high interpretability previously achieved.

Designed FRBCs are endowed with the usual fuzzy classification structure based on the Max-Min inference scheme, and the winner rule fuzzy reasoning mechanism:

$$y_{FRBC}(x^p) = C^i \Leftrightarrow \mu_{C^i}(x^p) = \max_{k=1,\ldots,NC} \mu_{C^k}(x^p)$$

$$\mu_{C^k}(x^p) = \max_{R=1,\ldots,NR} \mu_R(x^p) \Leftrightarrow Y_R \text{ is } C^k$$

$$\mu_R(x^p) = \min_{i=1,\ldots,NI} \mu_{A_i^j}(x_i^p)$$

where given an input vector $x^p = \{x_1^p, \ldots, x_{NI}^p\}$, the output class C^i is derived from the highest $\mu_{C^i}(x^p)$ which is the membership degree of x^p to the class C^i. It is computed as the maximum firing degree of all rules yielding C^i as output class. For each rule, the firing degree is computed as the minimum membership degree of x^p to all the attached A_i^j fuzzy set, for all the NI inputs.

III. Fuzzy finite state machines

In previous studies, we have showed that FFSM are suitable tools for modeling phenomena that follow an approximately repetitive pattern [15], [16], [17], [18]. During the development of these works, the concept of FFSM has grown up in clarity and usability. In the following, we will introduce the current version of this paradigm for system modeling.

A FFSM is a tuple $\{Q, S, S_0, U, Y, f, g\}$. We will describe each one of its components in the next subsections.

A. Fuzzy States

Every state represents the pattern of a repetitive situation. The fuzzy state of the system (Q) is a linguistic variable [2] that takes its values in the set of linguistic labels $\{Q_1, Q_2, \ldots, Q_n\}$, where n is the number of states. Numerically, the fuzzy state of the FFSM is represented with a state activation vector:

$S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$, where $s_i \in [0, 1]$.

Moreover, the FFSM implementation verifies $\sum_{i=1}^{n} s_i = 1$.

We require the state activation vector to fulfill the previous relation for two reasons; first, we want that the degree of activation works like a quantity that is distributed among the different states keeping the total degree as a constant equal to one; second, the state activation vector may be used as input of a second FFSM (serial connection), so we want that these input values are normalized in the interval $[0, 1]$ in such a way that we do not need to renormalize them.

We define, S_0 as the initial value of the state activation vector at $t = 0$, i.e., $S_0 = S[t = 0]$.

B. Input variables

U is the input vector $(u_1, u_2, \ldots, u_{NI})$. Typically, U is a set of linguistic variables obtained after fuzzification of numerical measures obtained from sensors. Moreover, u_i can be directly obtained from sensor data and also applying some calculations, e.g., the derivative or integral of the signal, or by combination of several signals.

The designer summarizes the domain of the possible numerical values provided by sensors representing them by a small set of fuzzy intervals.

$A_{u_i} = \{A^1_{u_i}, A^2_{u_i}, \ldots, A^{n_a}_{u_i}\}$ is the set of all the possible values that u_i can take, being n_a the number of linguistic labels of the linguistic variable u_i.

C. Transition function f

The next value of the state activation vector is obtained by means of the transition function f:

$S[t+1] = f(U[t], S[t])$.

This function is implemented by means of a set of expert fuzzy rules. Once the designer has identified the relevant states in the model, he/she must define the rules that govern the temporal evolution of the system among these states (e.g., see Figure 2).

We can distinguish between rules R_{ii} to remain in a state Q_i, and rules R_{ij} to change from the state Q_i to the state Q_j. To design the allowed transitions and the forbidden ones, we follow a simple procedure: the allowed transitions have explicitly associated fuzzy rules while there are not rules associated with the forbidden transitions.

1) Rules to remain in a state: The designer uses these rules to express the conditions of the system to remain in a specific state. The generic expression of R_{ii} is formulated as follows:

R_{ii}: **If** $S[t]$ is Q_i **AND** C_{ii} **Then** $S[t+1]$ is Q_i

where:

- The antecedent ($S[t]$ is Q_i) calculates the degree of activation of the state Q_i in the instant of time t, i.e., $s_i(t)$. Note that the FFSM cannot remain in the state Q_i if it is not in this state previously.
- The antecedent C_{ii} describes the constraints over the input variables to remain in the state Q_i. For example: C_{ii} = (u_1 is $A^3_{u_1}$) AND (u_2 is $A^1_{u_2}$) OR (u_3 is $A^5_{u_3}$).
- Finally, the consequent of the rule is the next value of the state activation vector $S[t+1]$. It consists of a vector with a zero in all of its components except in s_i, where it has a one.

2) Rules to change of state: The designer uses these rules to express the conditions that make the system change from state Q_i to state Q_j. Here, the generic expression of R_{ij} is formulated as follows:

R_{ij}: **If** $S[t]$ is Q_i **AND** C_{ij} **Then** $S[t+1]$ is Q_j

where:

- The antecedent ($S[t]$ is Q_i) calculates the degree of activation of the state Q_i in the instant of time t, i.e., $s_i(t)$. Note that the FFSM cannot change from the state Q_i to the state Q_j if it is not in Q_i previously.
- The antecedent C_{ij} describes the constraints over the input variables to change from the state Q_i to the state Q_j. In a first approach, these conditions could coincide with the amplitude conditions to remain in the destination state of the transition, i.e., $C_{ij} = C_{jj}$. Then, some tuning could be needed to express different conditions to change.
- Finally, the consequent of the rule is the next value of the state activation vector $S[t+1]$. It consists of a vector with a zero in all of its components except in s_j, where it has a one.

D. Output variables

Y is the output vector $(y_1, y_2, \ldots, y_{n_y})$, where n_y is the number of output variables. Y is a summary of the characteristics of the system evolution that are relevant for the application, e.g., each y_i can be obtained as the average of certain parameters of the system while the model remained in the state Q_i.

E. Output function g

The output function $g(U[t], S[t])$ calculates the value of the output vector $Y(t)$. E.g., a possible implementation of g is doing $Y[t] = S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$. Here, the output is the current fuzzy state of the system represented by the state activation vector.

F. Computational implementation

In order to implement the transition function, we use the Takagi-Sugeno-Kang (TSK) approach [19]. The advantage of using TSK is that it provides directly the numerical values of $S[t]$.

Using the TSK implementation, the transition function (f) of the FFSM is formulated as follows:

R^1_{ii}: **If** $S[t]$ is Q_i **AND** C^1_{ii}
Then $S[t+1]^1 = (0, \ldots, s_i = 1, \ldots, 0)$

...

R^r_{ij}: **If** $S[t]$ is Q_i **AND** C^r_{ij}
Then $S[t+1]^r = (0, \ldots, s_j = 1, \ldots, 0)$

The state activation vector ($S[t+1]$) will be the weighted average of the individual outputs:

$$S[t+1] = \begin{cases} \dfrac{\sum_{k=1}^{r} \omega_k \cdot S[t+1]^k}{\sum_{k=1}^{r} \omega_k} & \text{if } \sum_{k=1}^{r} \omega_k \neq 0 \\ S[t] & \text{if } \sum_{k=1}^{r} \omega_k = 0 \end{cases}$$

where ω_k is the degree of firing of the rule k using the minimum for the AND operator. This formulation keeps the system in its previous state when no rule is fired. Moreover, it makes $s_i \in [0,1]$ and $\sum_{i=1}^{n} s_i = 1$.

IV. Proposal

This section introduces the proposed fusion framework for human activity recognition. It is made up of three main modules as illustrated in Figure 1. Each block will be described in the following subsections. First, subsection IV-A focuses on building a FRBC devoted to estimate the location of a person in an indoor environment by means of processing WiFi strength signal levels (SLi). Then, subsection IV-B describes the FFSM1 in charge of human body posture recognition. Finally, subsection IV-C gives the details related to the FFSM2 that combines both WiFi positioning and posture recognition yielding the desired human activity recognition.

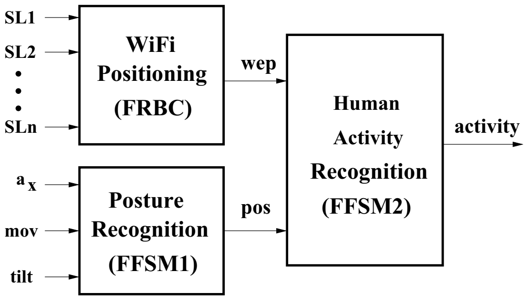

Fig. 1. Scheme of the proposed fusion framework

A. WiFi positioning module (FRBC)

WiFi localization systems use 802.11b/g network infrastructure to estimate a device position without using additional hardware. The received SL from each AP depends on the distance but also on the obstacles between the emitter and the receiver. Therefore, the simplest method for estimating the device position consists of applying a triangulation algorithm. Unfortunately, in indoor environments SL is strongly affected by the well-known multipath effect that comprises reflection, refraction and diffraction. Thus, SL becomes a complex function of the distance that dynamically changes with time because it is affected by every modification made in the environment [20].

Only approximate solutions are able to get nice results. Authors of [21] propose the use of a priori radio map storing the received SL of each AP belonging to an interest region. The radio map is built during the training stage. Then, in the estimation stage, a vector with received SL of each AP is created and compared with the radio map to obtain the estimated position. We have previously proposed the use of fuzzy classification for WiFi localization inspired on the radio map method, handling the signal measure uncertainty and getting small localization errors [9]. In this contribution we propose the use of an enhanced version of such WiFi localization system, yielding room-level localization. Notice that, the output of the FRBC will be one zone of the environment along with an activation degree which is understood as a degree of confidence on the system output. Of course, the interpolation ability of fuzzy systems makes possible to define a hierarchical localization system where the position may be refined as much as desired. In a first level it is possible to identify the floor of the building, in a second level it points out the room where the person is located, but in a third level (depending on the application) it may be interesting giving also the position inside the room. Thus, thanks to this approach, a FRBC made up of a small number of rules is used for each level, keeping a good trade-off between accuracy and interpretability. Although we have measured the SL in many points of each room, we will only consider the second level of this hierarchy, i.e., we work at the room level with all rooms located at the same floor of the building.

As an illustrative example, let us suppose that two zones A and B are one close to the other (with a common wall) and one person is inside zone A but near the wall. The FRBC is made up of rules like **If** *Signal received from APi is High* **AND** *Signal received from APj is Low* **Then** *The person is close to Position P which belongs to zone Z*. In this example, at least two rules may be fired yielding as output an activation degree of 0.7 related to zone A and 0.3 regarding zone B. Output is computed as the result of a fuzzy inference that takes into account all defined variables and rules.

First of all, we need to identify the zones of interest in a map of the environment under analysis. The number of zones determines the number of classes of the FRBC. Second, we have to find out the APs visible in such environment. The number of APs determines the number of input variables of the FRBC. Then, in the training stage we build the radio map of the environment. To do so, we collect a training data set (LRN) with the SL measures (from all visible APs) carried out in several locations for each of the zones of interest. Then, HILK methodology [12] is applied (as it was introduced in section II) on LRN in order to automatically generate a FRBC with a good accuracy-interpretability trade-off. All input variables (one per each AP visible in the environment) are characterized by strong fuzzy partitions of nine linguistic terms (*extremely low*, *very low*, *low*, etc). In addition, linguistic rules are automatically generated from data by means of the algorithm FDT. Finally, the simplification procedure provided by HILK is run getting a more compact and general FRBC, keeping high accuracy while increasing even more its interpretability. Notice that, the FRBC follows the usual fuzzy classification structure and the winner rule fuzzy reasoning mechanism. For further information the interested reader is referred to the cited papers.

Thanks to its flexibility and adaptability the designed FRBC can be used whenever the environment does not suffer a great modification, i.e., when some access points are switched off. In such case, the system should be re-adjusted, but usually these things do not happen and the fuzzy system is able to deal with slight modifications like people moving in the environment or changes in the state of the doors.

B. Posture recognition module (FFSM1)

In previous works, we have shown how a FFSM is able to synchronize with the acceleration signal produced during the human gait and to extract the relevant characteristics suitable for our purpose [16]. In the following, we explain how to design a FFSM for body posture recognition:

1) Fuzzy States: Here, the fuzzy states are defined to recognize different body postures and human activity. So, we have identified three fuzzy states: $\{Q_1$: Seated, Q_2: Upright, Q_3: Walking$\}$.

2) Input variables: The set of linguistic variables U, as stated in the definition of the FFSM, can be directly obtained from sensors. In this case, we have used a three-axial accelerometer tight with a belt in the middle of the back, therefore, the numerical values that we obtain from the sensor are: the dorso-ventral acceleration (a_x), the medio-lateral acceleration (a_y) and the antero-posterior acceleration (a_z). With these numerical values, and in order to distinguish between the three different states, we have created three linguistic variables $\{a_x, mov, tilt\}$:

- a_x is the dorso-ventral acceleration as it was obtained from the sensor.
- mov measures the movement, it is the sum of the difference between the maximum and minimum of a_x, a_y and a_z respectively contained in a interval of 1 second.
- $tilt$ is a variable that measures the tilt of the body, it is calculated as the sum of the absolute value of the medio-lateral acceleration (a_y) and the absolute value of the antero-posterior acceleration (a_z), i.e., $|a_y|+|a_z|$.

The linguistic labels, that summarize the domain of each linguistic variable, are uniform strong fuzzy partitions based on trapezoidal or triangular membership functions in order to achieve a good interpretability, satisfying semantic constraints on membership functions in order to respect semantic integrity within the partitions. They are defined for each linguistic variable in the intervals defined by their maximum and minimum values taken by their numeric values, i.e., they are adapted for each user in an off-line process. The possible values of the three linguistic variables are summarized as follows:

- $a_x = \{L_{a_x}, H_{a_x}\}$ which corresponds to the terms *Low* and *High* respectively.
- $mov = \{L_{mov}, M_{mov}, H_{mov}\}$ which corresponds to the terms *Low*, *Medium* and *High* respectively.
- $tilt = \{L_{tilt}, H_{tilt}\}$ which corresponds to the terms *Low* and *High* respectively.

The input vector (U), with the set of its possible values, represents the system input with lower granularity than the domain of numerical values directly obtained from sensors.

3) Transition function f: As we have stated previously, we will obtain the next value of the activation vector using the transition function: $S[t+1] = f(U[t], S[t])$.

Figure 2 shows how we use the FFSM to define constraints on the possibilities to change of state. More specifically, we force the model to pass by the state Upright (Q_2) when the subject passes from Seated (Q_1) to Walking (Q_3). The subject cannot be seated and start walking, first he/she must get upright. This restriction makes the system more robust.

- *Rules to remain in a state*. Using the generic expression of R_{ii} explained in section III-C.1, we can define the three constraints over the input variables to remain in each state:
  C_{11} = a_x is L_{a_x} AND mov is L_{mov} AND $tilt$ is H_{tilt}
  C_{22} = a_x is H_{a_x} AND mov is L_{mov} AND $tilt$ is L_{tilt}
  C_{33} = a_x is H_{a_x} AND mov is (M_{mov} OR H_{mov})
- *Rules to change of state*. In a first approach, the constraints over the input variables to change of state could be the same as the constraints over the input variables to remain in the destination state of the transition, i.e., C_{ij} = C_{jj}. But, as we have stated, some tuning is needed to express different conditions to change:
  C_{12} = a_x is H_{a_x} AND mov is L_{mov} AND $tilt$ is L_{tilt}
  C_{21} = a_x is L_{a_x} AND mov is L_{mov} AND $tilt$ is H_{tilt}
  C_{23} = a_x is H_{a_x} AND mov is H_{mov} AND $tilt$ is L_{tilt}
  C_{32} = a_x is H_{a_x} AND mov is L_{mov} AND $tilt$ is L_{tilt}
  C_{31} = a_x is L_{a_x} AND mov is H_{mov} AND $tilt$ is H_{tilt}

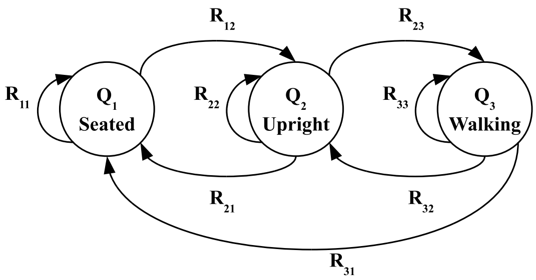

Fig. 2. Diagram of states of the FFSM1 (body posture recognition)

4) Output variables: Since we are going to use the output of this FFSM1 as input in the FFSM2, we can use as output variable the state activation vector, i.e., $Y[t] = S[t]$.

5) Output function g: The output function, as we have stated above, is simply: $g = S[t]$.

C. Human activity recognition module (FFSM2)

Currently, a new generation of smart-phones and PDAs including capabilities for WiFi communications and accelerometers is available. We use a PDA to obtain the information that our system requires for inferring the user activity. In the following, we explain how to design a FFSM for combining the WiFi Positioning module and the Posture Recognition module to achieve a Human Activity Recognition system:

1) Fuzzy States: The system must be adapted to each specific user. We manage linguistic descriptions of the different activities daily performed by the user. For example we distinguish among the following fuzzy states of activity in an office:

- Q_1: Walking. Typical body movement detected by accelerometers
- Q_2: Working at his/her desk. Usually, the user is seated, in specific WiFi coordinates, the most of time.
- Q_3: Visiting a colleague. Seated or standing upright, in several possible WiFi coordinates, for little time.
- Q_4: Having coffee. Seated or standing upright, in specific WiFi coordinates, some time.

- Q_5: Having a meeting. Seated in specific WiFi coordinates for some time.

2) Input variables: In order to distinguish among the different states, we have created two linguistic variables $\{wep, pos\}$ that characterize the outputs of the two previous modules (FRBC and FFSM1):

- wep is the WiFi estimated position (FRBC computed it as explained in section IV-A).
- pos is the posture estimation obtained from the posture recognition module (FFSM1 described in section IV-B).

These variables are characterized by the following linguistic labels which are defined in the interval [0,1]:

- $wep = \{WAA, MC, WAB, WO, CA, MR\}$, which are the zones of our experimental scenario (see later section V).
- $pos = \{Seated, Upright, Walking\}$ which corresponds to the three different states of the FFSM1.

3) Transition function f: As in the FFSM1, we will obtain the next value of the activation vector using the transition function $S[t+1] = f(U[t], S[t])$.

Since we have already identified the relevant states in the model, we can represent the fuzzy rules that govern the temporal evolution of the system among these states. Figure 3 shows the transition diagram of the FFSM2. There are five rules to remain in a state (R_{ii}) and eight rules to change of state (R_{ij}). In this application not all the possible transitions are allowed, the majority of the states are connected to the state Q_1 (*Walking*).

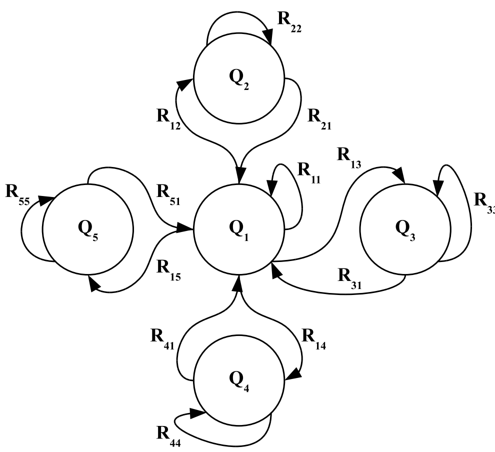

Fig. 3. Diagram of states of the FFSM2 (human activity recognition)

- *Rules to remain in a state*: Using the generic expression of R_{ii} explained in section III-C.1, we can define the constraints over the input variables to remain in each state.

C_{11} = pos is $Walking$
C_{22} = pos is $Seated$ AND wep is WAA
C_{33} = pos is ($Seated$ OR $Upright$) AND wep is (WO OR WAB)
C_{44} = pos is ($Seated$ OR $Upright$) AND wep is CA
C_{55} = pos is $Seated$ AND wep is MR

- *Rules to change of state*: Using the generic expression of R_{ij} explained in section III-C.2, we can define the constraints over the input variables to change of each state.

C_{12} = pos is $Seated$ AND wep is WAA
C_{13} = pos is ($Seated$ OR $Upright$) AND wep is (WO OR WAB)
C_{14} = pos is ($Seated$ OR $Upright$) AND wep is CA
C_{15} = pos is $Seated$ AND wep is MR
C_{21} = pos is $Walking$ AND wep is (WAA OR MC)
C_{31} = pos is $Walking$ AND wep is WAB
C_{41} = pos is $Walking$ AND wep is MC
C_{51} = pos is $Walking$ AND wep is MR

4) Output variables: We can use as output variable the state activation vector, i.e., $Y[t] = S[t]$. But we have to give a crisp description of the activity of the person. Therefore, we can consider as output the state with the maximum degree of activation at each instant of time t. However, this selection will make the FFSM very sensitive to noise and spurious in the signal, and that is precisely what we want to avoid. Therefore, the output is designed as the state which has had the maximum average degree of activation over the last second.

5) Output function g: The output function $g(U[t], S[t])$ that calculates the value of the output variables is, as we have stated above, the average operator in an interval of one second combined with the maximum operator to make the decision.

V. Experiments

The experimentation took place at the premises of the European Centre for Soft Computing (ECSC). The layout of ECSC environment is shown in Figure 4. It has a surface of 440 m^2 illustrated on the top picture. We have identified six zones (look at the bottom picture): *WAA* (working area A), *MC* (main corridor), *WAB* (working area B), *WO* (working office), *CA* (coffee area), and *MR* (meeting room).

The user carried a HP iPAQ hw6910/hw6915 PDA. It has a WiFi interface with a maximum acquisition frequency of 4Hz, i.e., it is able to capture up to four samples per second. In addition, an external accelerometer (WiTilt v2.5) with acquisition frequency of 100Hz was connected to our PDA through Bluetooth. The user wore the accelerometer tight with a belt in the middle of his back. Our program measures both WiFi signal and accelerations in the same cycle with the aim of keeping synchronization. Notice that, each 25 measures provided by the accelerometer correspond to only one WiFi measure.

As it can be seen at the top picture in Figure 4, there are four APs in the environment covering most of the zones. Inside each zone we have set several training fixed positions. They are represented by filled circles at the bottom picture in the Figure. For each of them, we collected 100 samples from all the four APs. The resultant data set was taken as LRN for training the WiFi positioning module as explained in section IV-A. The FRBC contains four inputs (one per AP). First, we set strong fuzzy partitions with nine linguistic terms per input. Second, linguistic rules were induced with FDT. Third,

Fig. 4. Discretization of the ECSC environment

TABLE I
DESCRIPTION OF THE EXPERIMENTAL SCENARIO

Length (s)	Description	Activity
60	Seated and typing	Working at the desk
30	Standing up and walking towards the coffee area	Walking
75	Staying up in front of the coffee machine. Sitting and having the coffee	Having a coffee
25	Standing up and walking until the office of a colleague	Walking
50	Staying up and waiting for a colleague	Visiting a colleague
30	Walking towards the meeting room	Walking
100	Seated in the meeting room	Having a meeting
40	Standing up and walking back to the work-desk	Walking
100	Seated and typing	Working at the desk

TABLE II
PERCENTAGE OF MISCLASSIFIED SAMPLES

	Mean (%)	St. Deviation (%)
FRBC	13.8	4.7
FFSM1	1.2	0.2
FFSM2	1.5	0.7

simplification was carried out. As a result, the final FRBC is made up of 14 rules with a total of 41 premises.

Table I gives the description of our experimental scenario that tries to summarizes a normal day at the work. Of course, this is a simplified scenario where we have set a reduced time for the different tasks. For example, *Having a meeting* lasts less than 2 minutes. Notice that we wanted to test how our system is able to recognize all defined states of activity. The whole experiment takes about 9 minutes because the time walking is approximated. Furthermore, the same user has repeated eleven times the same experiment yielding more than one hour and a half of experimentation. There may be a slight time lag between different repetitions of the experiment when the user is walking. The first trial was used for tuning the FFSMs. Then, another different day, we run in a row the rest of ten executions which have been used for testing the previously designed system.

Table II includes the test averaged results for the ten repetitions of the experiment. We have reported results (in terms of misclassified samples) for all the three modules that constitute the system (look at Figure 1). The first row shows the percentage of error (about 14%) for the FRBC module. The two last rows are related to the two FFSMs. In both cases, the percentage of misclassified samples is very small (1.2 and 1.5% respectively). This is due to the characteristic memory effect of FFSMs which define the new state taking into account the transition conditions but also the previous state. In addition, output of FFSM2 is averaged in an interval of only one second what makes feasible the use of our system in real-time applications. It is also important to remark how FFSM2 is able to absorb and correct the errors produced by the FRBC due to the high variability in the WiFi signal. Note how the error is dramatically reduced from 14 to 1.5%.

Figure 5 plots the system output for the worse trial of our experiments, the one yielding the largest percentage of error. The figure is made up of three pictures. The first one (at the top) illustrates in the vertical axis each component of the state activation vector $S[t]$, while its activation value is printed by means of the gray intensity (black means one and white means zero). The picture below plots the output vector $Y[t]$ obtained from the output function g, i.e. the state which has had the maximum average degree of activation over the last one second. Finally, the picture at the bottom represents the expected output vector $Y[t]$, i.e., the right output of the FFSM at each instant of time defined in our experiment (as detailed in Table I). As it can be appreciated, most errors correspond to the situations of *Having a coffee* and *Visiting a colleague*. Indeed, it seems that the user was slightly moving while waiting for the coffee and for his colleague. In consequence, the state *Walking* is activated for a few seconds.

VI. CONCLUSIONS

In this paper we have described a system able to detect some simple tasks carried out by a human in a usual working day. The main contributions can be summarized as follows:

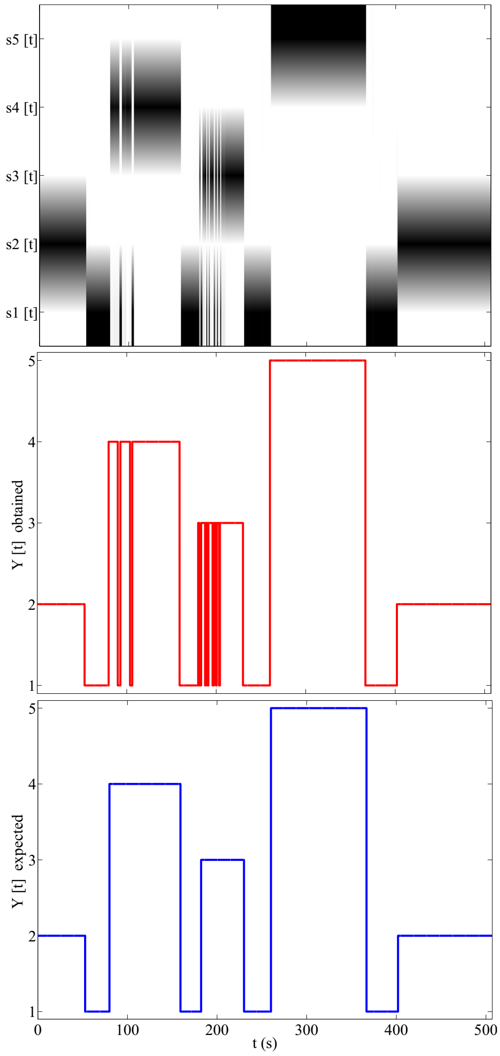

Fig. 5. System output (human activity recognition)

(1) the FRBC is mainly based on automatically induced knowledge yielding the approximated location of the user; (2) the FFSMs allow the designer to introduce constraints in the model based on the available expert knowledge about the activity states; (3) they also allow to fuse data from different sources and to compensate the partial errors, with the aim of producing a robust, reliable and easily understandable activity recognition module; and (4) the whole system is quite interpretable because it comprises several sets of linguistic variables and rules. In the future we will extend our model with the aim of detecting more complex human activities.

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9.3 Body posture recognition by means of a genetic fuzzy finite state machine

A. Alvarez-Alvarez, G. Trivino, and O. Cordón. "Body Posture Recognition By Means Of A Genetic Fuzzy Finite State Machine". In: *Proceedings of the 5th IEEE International Workshop on Genetic and Evolutionary Fuzzy Systems (GEFS), Paris, France*, pp. 60–65, April 2011.

Body Posture Recognition By Means Of A Genetic Fuzzy Finite State Machine

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***Abstract*—Body posture recognition is a very important issue as a basis for the detection of user's behavior. In this paper, we propose the use of a genetic fuzzy finite state machine for this real-world application.**

Fuzzy finite state machines (FFSMs) are an extension of classical finite state machines where the states and inputs are defined and calculated by means of a fuzzy inference system, allowing them to handle imprecise and uncertain data. Since the definition of the knowledge base of the fuzzy inference system is a complex task for experts, we use an automatic method for learning this component based on the hybridization of FFSMs and genetic algorithms (GAs). This genetic fuzzy system learns automatically the fuzzy rules and membership functions of the FFSM devoted to body posture recognition while an expert defines the possible states and allowed transitions.

We aim to obtain a specific model (FFSM) with the capability of generalizing well under different subject's situations. The obtained model must become an accurate and human friendly linguistic description of this phenomenon, with the capability of identifying the posture of the user. A complete experimentation is developed to test the performance of the new proposal, comprising a detailed analysis of results which shows the advantages of our proposal in comparison with another classical technique.

I. Introduction

Body posture recognition consists of identifying the different poses of a human being. This research field has attracted considerable attention as a basis for the detection of user's behavior, which could provide new context aware services. Example of applications range from proactive care for elderly people to safety applications based on fall detection.

There are two well distinguished approaches to tackle this problem: the sensor-based and the computer vision approach. The sensor-based approach consists of using small sensors (usually accelerometers) placed in the body of the person. In [1], the authors showed how acceleration data can aid the recognition of pace and incline. The main advantages of this approach are the possibilities of embedding these sensors into clothes or electronic devices such as mobile phones due to the advances in miniaturization, the capabilities of communication between sensors through wireless connections, and the low cost and energy consumption thanks to to the Micro Electro Mechanical Systems (MEMS) technology. Its principal drawback is the user's need of wearing the sensors.

The computer vision approach is based on the use of video cameras installed in the scenario under study [2]. While the sensor-based approach made the user to wear sensors, in this case, the additional hardware must be installed in each room of the environment. This approach usually works in lab but fails in real world scenarios due to clutter, variable light intensity, and contrast. Moreover, the video cameras are sometimes perceived as invasive and threatening by some people. Another important drawback is the computational cost of working with video signals.

In this work, we propose the use of fuzzy finite state machines (FFSMs) for body posture recognition within the sensor-based approach. FFSMs are specially useful tools for modeling dynamical processes which change in time, becoming an extension of classical finite state machines. The main advantage of FFSMs is the use of Fuzzy Logic (FL), which provides semantic expressiveness by the use of linguistic variables [3] and rules [4] close to natural language (NL). Moreover, being universal approximators [5], fuzzy inference systems are able to perform nonlinear mappings between inputs and outputs, allowing FFSMs to handle imprecise and uncertain data, which is inherent to real-world phenomena, in the form of fuzzy states and transitions.

In previous studies [6], [7], we have learnt that FFSMs are suitable tools for modeling signals that follow an approximately repetitive pattern. As any fuzzy system, FFSMs require the definition of a knowledge base (KB). It is well known that this is a complex task for experts as it was the case in these previous works. In addition, the dynamic nature of FFSMs increases the complexity of the process. For that reason, in [8], we proposed a new automatic learning method for the fuzzy KB of FFSMs based on the use of genetic algorithms (GAs) [9]. GAs have proven largely their effectiveness and efficiency for the latter task in the last two decades in the so-called genetic fuzzy systems (GFSs) area [10], [11], [12]. In our approach, the fuzzy states and transitions are defined by the expert in order to keep the knowledge that she/he has over the whole system while the fuzzy rules and membership functions regulating the state changes will be derived automatically by the GFS. This combined action thus results in a robust, accurate, and human friendly model called genetic fuzzy finite state machine (GFFSM) [8].

In this contribution, we propose the use of a GFFSM for the body posture recognition problem. Our final goal is to obtain a specific model (FFSM) in such way that this FFSM can generalize well under different subject's situations. Moreover, the obtained FFSM will result in an accurate and human friendly linguistic description of this phenomenon, with the capability of identifying the posture of the user. A complete experimentation is developed to test the performance of the new proposal, comprising a detailed analysis of results which shows the advantages of our proposal in comparison with another classical technique. Furthermore, we will also compare this new proposal against a FFSM previously developed for body posture recognition, whose KB had been defined by the expert in [13].

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The remainder of this paper is organized as follows. Section II presents how to use FFSMs for modeling the temporal evolution of a phenomenon. Section III explains how to build FFSMs for recognizing the body posture. The automatic method for learning the KB of these FFSMs based on GAs is introduced in Section IV. Section V presents the experimentation carried out, comparing the obtained results with another system identification tool. Finally, Section VI draws some conclusions and future research works.

II. Fuzzy Finite State Machines

In this section, we introduce the main concepts and elements of our paradigm for system modeling allowing experts to build comprehensible fuzzy linguistic models in an easier way. In our framework, a FFSM is a tuple $\{Q, U, f, Y, g\}$, where:

- Q is the state of the system.
- U is the input vector of the system.
- f is the transition function which calculates the state of the system.
- Y is the output vector of the system.
- g is the output function which calculates the output vector.

Each of these components are described in the following subsections. Furthermore, the interested reader can refer to [6], [7], [8], [13] for a more detailed description.

A. Fuzzy States (Q)

The state of the system (Q) is defined as a linguistic variable [3] that takes its values in the set of linguistic labels $\{q_1, q_2, \ldots, q_n\}$, with n being the number of fuzzy states. Every fuzzy state represents the pattern of a repetitive situation and it is represented numerically by a state activation vector:

$S[t] = (s_1[t], s_2[t], \ldots, s_n[t])$, where $s_i[t] \in [0,1]$ and $\sum_{i=1}^{n} s_i[t] = 1$.

S_0 is defined as the initial value of the state activation vector, i.e., $S_0 = S[t=0]$.

B. Input Vector (U)

U is the input vector $(u_1, u_2, ..., u_{n_u})$, with n_u being the number of input variables. U is a set of linguistic variables obtained after fuzzification of numerical data. Typically, u_i can be directly obtained from sensor data or by applying some calculations to the raw measures, e.g., the derivative or integral of the signal, or the combination of several signals. The domain of numerical values that u_i can take is represented by a set of linguistic labels, $A_{u_i} = \{A^1_{u_i}, A^2_{u_i}, \ldots, A^{n_i}_{u_i}\}$, with n_i being the number of linguistic labels of the linguistic variable u_i.

C. Transition Function (f)

The transition function (f) calculates, at each time instant, the next value of the state activation vector: $S[t+1] = f(U[t], S[t])$. It is implemented by means of a fuzzy KB.

Once the expert has identified the relevant states in the model, she/he must define the allowed transitions among states. There are rules R_{ii} to remain in a state q_i, and rules R_{ij} to change from state q_i to state q_j. If a transition is forbidden in the FFSM, it will have no fuzzy rules associated.

A generic expression of a rule is of the form:

R_{ij}: **IF** ($S[t]$ is q_i) **AND** C_{ij} **THEN** $S[t+1]$ is q_j

where:

- The first term in the antecedent ($S[t]$ is q_i) computes the degree of activation of the state q_i in the time instant t, i.e., $s_i[t]$. With this mechanism, we only allow the FFSM to change from the state q_i to the state q_j (or to remain in state q_i, when $i = j$).
- The second term in the antecedent C_{ij} describes the constraints imposed on the input variables in disjunctive normal form (DNF) [10]. For example: C_{ij} = ($u_1[t]$ is $A^3_{u_1}$) **AND** ($u_2[t]$ is $A^4_{u_2}$ **OR** $A^5_{u_2}$).
- Finally, the consequent of the rule defines the next value of the state activation vector $S[t+1]$. It consists of a vector with a zero value in all of its components but in $s_j[t]$, where it takes value one.

To calculate the next value of the state activation vector ($S[t+1]$), a weighted average using the firing degree of each rule k (ω_k) is computed as defined in Equation 1:

$$S[t+1] = \begin{cases} \dfrac{\sum_{k=1}^{\#Rules} \omega_k \cdot (s_1, \ldots, s_n)_k}{\sum_{k=1}^{\#Rules} \omega_k} & \text{if } \sum_{k=1}^{\#Rules} \omega_k \neq 0 \\ S[t] & \text{if } \sum_{k=1}^{\#Rules} \omega_k = 0 \end{cases} \tag{1}$$

where (ω_k) is calculated using the minimum for the AND operator and the bounded sum of Łukasiewicz [14] for the OR operator.

D. Output Vector (Y)

Y is the output vector: $(y_1, y_2, ..., y_{n_y})$, with n_y being the number of output variables. Y is a summary of the characteristics of the system evolution that are relevant for the application.

E. Output Function (g)

The output function (g) calculates, at each time instant, the next value of the output vector: $Y[t] = f(U[t], S[t])$. The most simple implementation of g is $Y[t] = S[t]$.

III. Fuzzy Finite State Machine for Body Posture Recognition

This section presents the design of the main elements needed to build a FFSM for body posture recognition [13].

A. Fuzzy States

In this application, we have defined three different fuzzy states which directly describe the body posture:

$$\{q_1 \rightarrow \text{Seated},\ q_2 \rightarrow \text{Upright},\ q_3 \rightarrow \text{Walking}\}$$

B. Input Vector

In our experiments, we have used a three-axial accelerometer tight with a belt in the middle of the back. Therefore, the numerical values that we obtain from the sensor are the dorso-ventral acceleration (a_x), the medio-lateral acceleration (a_y), and the antero-posterior acceleration (a_z). In order to distinguish between the three different states, we have created three linguistic variables $\{a_x, mov, tilt\}$ with these numerical values:

- a_x is the dorso-ventral acceleration as it was obtained from the sensor.

- mov measures the amount of movement. It is the sum of the difference between the maximum and minimum of a_x, a_y, and a_z, respectively, contained in an interval of 1 second.
- $tilt$ is a variable that measures the tilt of the body. It is calculated as the sum of the absolute value of the medio-lateral acceleration (a_y) and the absolute value of the antero-posterior acceleration (a_z), i.e., $|a_y|+|a_z|$.

The term sets for each linguistic variable are: $\{S_{a_x}, M_{a_x}, B_{a_x}\}$, $\{S_{mov}, M_{mov}, B_{mov}\}$, and $\{S_{tilt}, M_{tilt}, B_{tilt}\}$, where S, M, and B are linguistic terms representing small, medium, and big, respectively.

C. Transition Function

The definition of which transitions are allowed and which are not can be easily represented by means of the state diagram. Figure 1 shows how we use the FFSM to define constraints on the possibilities to change of state. More specifically, we force the model to pass by the state Upright (q_2) when the subject passes from Seated (q_1) to Walking (q_3). This restriction makes the system more robust.

From the state diagram represented in Figure 1, it can be recognized that there are 8 fuzzy rules overall in the system: 3 rules to remain in each state and other 5 to change between states.

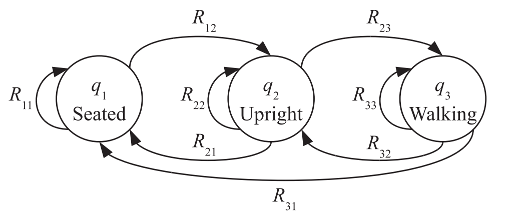

Fig. 1. State diagram of the FFSM for body posture recognition.

Therefore, the RB will have the following structure:

R_{11}: **IF** ($S[t]$ is q_1) **AND** C_{11} **THEN** $S[t+1]$ is q_1
R_{22}: **IF** ($S[t]$ is q_2) **AND** C_{22} **THEN** $S[t+1]$ is q_2
R_{33}: **IF** ($S[t]$ is q_3) **AND** C_{33} **THEN** $S[t+1]$ is q_3
R_{12}: **IF** ($S[t]$ is q_1) **AND** C_{12} **THEN** $S[t+1]$ is q_2
R_{21}: **IF** ($S[t]$ is q_2) **AND** C_{21} **THEN** $S[t+1]$ is q_1
R_{23}: **IF** ($S[t]$ is q_2) **AND** C_{23} **THEN** $S[t+1]$ is q_3
R_{32}: **IF** ($S[t]$ is q_3) **AND** C_{32} **THEN** $S[t+1]$ is q_2
R_{31}: **IF** ($S[t]$ is q_3) **AND** C_{31} **THEN** $S[t+1]$ is q_1

where C_{ij} could be: (a_x is S_{a_x}) **AND** (mov is M_{mov}) **AND** ($tilt$ is M_{tilt} **OR** B_{tilt}).

D. Output Vector and Output Function

In this contribution, we simply consider $Y[t] = S[t]$, i.e., the output of the FFSM is the degree of activation of each state.

IV. Genetic Fuzzy System

The current section reviews the fusion framework between FFSMs and GAs developed in [8], which will be considered to solve a new application in the current contribution. In this case, we will learn the KB of the FFSM designed for body posture recognition. The KB is comprised by the data base (DB), which contains the linguistic labels' membership functions (MFs); and the RB, which collects the fuzzy if-then rules. The following subsections describe the structure of the different components of this GFS.

A. Representation Scheme and Initial Population Generation

We have divided the representation scheme into two parts: the RB part and the DB part. In the following, we explain each of these representations.

1) RB part: We codify the whole rule set in a chromosome following the Pittsburgh approach [15]. For each of the three input variables a_x, mov, and $tilt$, the rule representation consists of a binary sub-string of length 3 that refers to its linguistic term set $\{S_{a_x}, M_{a_x}, B_{a_x}\}$, $\{S_{mov}, M_{mov}, B_{mov}\}$, and $\{S_{tilt}, M_{tilt}, B_{tilt}\}$, respectively. Only the non-fixed part of the DNF rule antecedent (see Section III-C) is encoded. Each bit has a one (zero) which denotes the presence (absence) of each linguistic term in the rule. Moreover, the feature selection capability of this representation is used since an input variable is omitted in the rule if all of its bits in the representation become zeros or ones. The RB part of the chromosome will thus be composed of 8 rules $\times$ 9 linguistic terms (3 per input variable) = 72 binary-coded genes.

2) DB part: We have considered the use of trapezoidal strong fuzzy partitions (SFPs) [16] because they allow us to reduce the number of parameters to tune, in such way that the normalization constraint is easily satisfied by only coding the two modal points of each MF. Therefore, we have to code 12 real parameters, 4 per input variable where one parameter is enough to codify the first and third linguistic label and two parameters are needed to codify the second linguistic label. Therefore, the DB part of the chromosome will be composed of 12 real-coded genes:

$$\{a^1_{a_x}, a^2_{a_x}, b^2_{a_x}, a^3_{a_x}, a^1_{mov}, a^2_{mov}, b^2_{mov}, a^3_{mov}, a^1_{tilt}, a^2_{tilt}, b^2_{tilt}, a^3_{tilt}\}$$

We use a real-coded representation. The variation interval of each allele is defined within the interval defined by its previous and next parameter. Figure 2 shows the graphical representation of the fuzzy partition related with the linguistic input variable mov. Notice that, the parameters a^1_{mov} and a^3_{mov} are enough to codify the first and third linguistic labels, S_{mov} and B_{mov} respectively, while two parameters a^2_{mov} and b^2_{mov} are needed to codify the intermediate linguistic label M_{mov}.

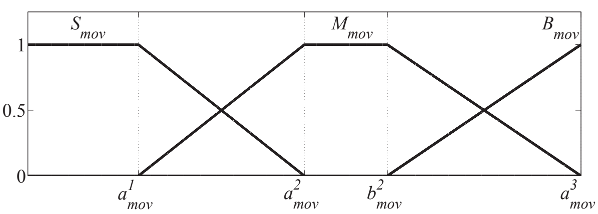

Fig. 2. Parameters that form all the trapezoidal linguistic labels of the linguistic variable mov.

Hence, the final chromosome encoding a candidate problem solution will be comprised by $72+12 = 84$ genes, with the first 72 being binary-coded genes corresponding to the RB part, and the last 12 being real-coded genes associated to the DB part. We have initialized the first population by generating all the individuals at random, except the DB part

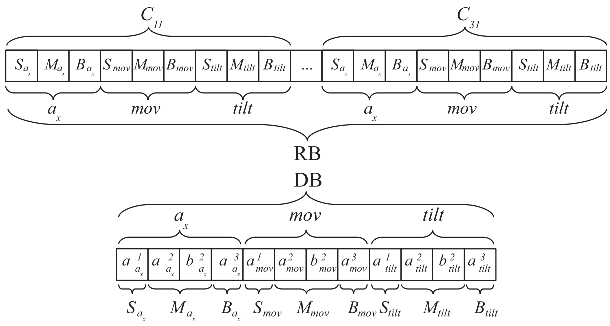

Fig. 3. Chromosome structure encoding the RB and DB part.

of the first individual of the population that encodes uniform fuzzy partitions for each linguistic variable. Figure 3 shows the structure of the complete chromosome encoding the RB and DB part.

B. Fitness Function

Since the computation of the next state is based on the previous state, we need to evaluate the tentative FFSM definition encoded in each chromosome over the whole data set. We have chosen as fitness function the mean absolute error (MAE) measure, defined in Equation 2:

$$\text{MAE} = \frac{1}{n} \cdot \frac{1}{T} \cdot \sum_{i=1}^{n} \sum_{j=0}^{T} |s_i[j] - s_i^*[j]| \tag{2}$$

where:

- n is the number of states, i.e., $n = 3$.
- T is the dataset size (i.e., the considered time interval duration).
- $s_i[j]$ is the degree of activation of state q_i at time $t = j$.
- $s_i^*[j]$ is the expected degree of activation of state q_i at time $t = j$.

The MAE directly measures the difference between the actual state activation vector ($S^*[t]$) and the obtained one ($S[t]$). However, we need to define $S^*[t]$ for each input data set that we want to learn. This definition could be problematic and must be done carefully because, more than one state can be defined at each time instant, each of those states activated with certain degree in the interval $[0, 1]$. In the following subsection, this issue is explained in detail.

C. Defining the Training Data Set

We have to create a training vector which consists of $a_x[t]$, $a_y[t]$, $a_z[t]$ and $S^*[t]$, i.e., $(a_x[t], a_y[t], a_z[t], s_1^*[t], s_2^*[t], s_3^*[t])$. To define it, we have developed a user-friendly graphical interface that allows the expert to select manually the relevant points where each state starts and ends using her/his knowledge about body posture and the duration of each part of the experiment. The fuzzy definition of the states is based on the imprecision of the expert when defining those relevant points. For each state q_i, there are different points comprising the beginning (b_i^j) and the end of each state (e_i^j), with $j \in \mathbb{N}$.

As an example, let us consider the definition of the actual degree of activation of state q_3 when there is a transition from state q_2 to state q_3 and then from state q_3 to state q_1. The actual value of $s_3^*[t]$ is then specified by Equation 3. Between the end time of q_2 (e_2^j) and the start time of q_3 (b_3^j), the activation of the state q_3 is rising from 0 to 1. Between the start (b_3^j) and the end time (e_3^j) of q_3, defined by the user, the activation has the maximum of 1. Afterwards, the activation drops till zero at the start of q_1 (b_1^j). Otherwise, the activation is zero.

$$s_3^*[t] = \begin{cases} \frac{t - e_2^j}{b_3^j - e_2^j} & \text{if} \quad e_2^j < t < b_3^j \\ 1 & \text{if} \quad b_3^j \leq t \leq e_3^j \\ \frac{b_1^j - t}{b_1^j - e_3^j} & \text{if} \quad e_3^j < t < b_1^j \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The interested reader is referred to [8] for a deeper description on the human definition of the activation degrees for the fuzzy states in FFSMs.

D. Genetic Operators

A binary tournament selection and a generational replacement with elitism are considered. The classical two-point crossover has been used for the RB (binary-coded) part of the chromosome and BLX-α crossover [17] for the DB (real-coded) part. The BLX-α crossover is applied twice over a pair of parents in order to obtain a new pair of chromosomes. The classical bitwise mutation has been selected for the binary-coded RB part, while uniform mutation has been chosen for the real-coded DB part.

In this contribution, we have implemented three different termination conditions. First, the search is stopped when the algorithm has obtained a fitness value equal to zero, which is the best value that the fitness function can take. Moreover, we have decided to set a maximum number of generations and also to stop the search when, for a certain number of generations, the fitness value of the best individual is not improved.

V. EXPERIMENTATION

This section presents the experimentation carried out. First, the experimental setup, which comprises the data acquisition and the parameters of the GA, is explained. The second part contains a brief description of an alternative model used for body posture recognition. Finally, the third part presents and analyzes the results obtained.

A. Experimental Setup

1) Data acquisition: We have used a wireless three-axial accelerometer attached to a belt, centered in the back of the person. It provided measurements of the three orthogonal accelerations with a frequency of 100 Hz. Therefore, every record contained the information: (t, a_x, a_y, a_z) where t is each instant of time, a_x is the dorso-ventral acceleration, a_y is the medio-lateral acceleration, and a_z is the antero-posterior acceleration.

We asked this person to perform a variety of activities, such as sitting at her/his desk, having a coffee, visiting a colleague, having a meeting, etc. In this simplified scenario, we have set a reduced time for the different tasks because we wanted to test how our system is able to recognize all defined states related to body posture. This process was repeated ten times producing ten different datasets. These datasets were then processed as explained in Section IV-C getting the following structure:

$$(a_x[t], a_y[t], a_z[t], s_1^*[t], s_2^*[t], s_3^*[t])$$

where:

- $a_x[t]$ is the dorso-ventral acceleration at time instant t.
- $a_y[t]$ is the medio-lateral acceleration at time instant t.
- $a_z[t]$ is the antero-posterior acceleration at time instant t.
- $s_1^*[t]$ is the expected degree of activation of state q_1 at time instant t.
- $s_2^*[t]$ is the expected degree of activation of state q_2 at time instant t.
- $s_3^*[t]$ is the expected degree of activation of state q_3 at time instant t.

2) Parameters of the GA:

- Population size $\rightarrow$ 30 individuals.
- Crossover probability $\rightarrow p_c = 0.8$.
- Value of alpha (BLX-α parameter) $\rightarrow \alpha = 0.3$.
- Mutation probability per bit $\rightarrow p_m = 0.02$.
- Termination conditions:
  - Fitness value reached $\rightarrow$ MAE $= 0$.
  - Maximum number of generations $\rightarrow$ 200.
  - Generations without improvement of the fitness function $\rightarrow$ 50.

B. Autoregressive Linear Models

In order to benchmark the GFFSM results, we have considered another technique commonly used in system modeling of time-dependent systems: autoregressive linear models (ARX) [18]. We have defined a multiple-input multiple-output (MIMO) ARX model with the structure defined by Equation 4:

$$\begin{aligned} Y[t] = A_1 \cdot Y[t-1] + \ldots + A_{n_A} \cdot Y[t-n_A] \\ + B_0 \cdot U[t] + \ldots + B_{n_B} \cdot U[t-n_B] \end{aligned} \tag{4}$$

where:

- $Y[t] = (s_1[t], s_2[t], s_3[t])$ is the current output vector.
- $Y[t-1], \ldots, Y[t-n_A]$ are the previous output vectors on which the current output vector depends.
- $U[t] = (a_x[t], mov[t], tilt[t]), \ldots, U[t-n_B]$ are the current and delayed input vectors on which the current output vector depends.
- n_A is the number of previous output vectors on which the current output vector depends.
- n_B is the number of previous input vectors on which the current output vector depends.
- $A_1, \ldots, A_{n_A}$ and $B_0, \ldots, B_{n_B}$ are the matrices that define the models. They are estimated using the least squares method.

The performance of this model has been tested with values of $n_A = n_B = 20$, resulting in the ARX model defined by Equation 5:

$$\begin{aligned} Y[t] = A_1 \cdot Y[t-1] + \ldots + A_{20} \cdot Y[t-20] \\ + B_0 \cdot U[t] + \ldots + B_{19} \cdot U[t-19] \end{aligned} \tag{5}$$

C. Results

To test the performance of the GFFSM and the ARX model, we have done a leave-one-out cross validation for each of the 10 datasets. Table I shows the MAE obtained for each fold of the leave-one-out in training and test. It also depicts the average value of the MAE (MEAN) and its standard deviation (STD) for the ten results of the procedure.

TABLE I
MAE FOR EACH DATASET OF THE LEAVE-ONE-OUT.

FOLD	GFFSM		ARX	
	TRAIN	TEST	TRAIN	TEST
1	0.010	0.016	0.071	0.083
2	0.009	0.007	0.072	0.093
3	0.010	0.009	0.076	0.064
4	0.009	0.010	0.078	0.059
5	0.010	0.013	0.076	0.072
6	0.009	0.012	0.075	0.073
7	0.010	0.010	0.075	0.081
8	0.011	0.009	0.070	0.104
9	0.008	0.010	0.077	0.065
10	0.009	0.009	0.076	0.072
MEAN	0.009	0.011	0.074	0.077
STD	0.001	0.002	0.003	0.014

TABLE II
MAE OBTAINED FOR EACH DATASET BY THE FFSM DEFINED BY THE EXPERT AND OBTAINED IN TEST WITH THE LEAVE-ONE-OUT.

DATASET	FFSM	GFFSM	ARX
1	0.023	0.016	0.083
2	0.027	0.007	0.093
3	0.016	0.009	0.064
4	0.020	0.010	0.059
5	0.022	0.013	0.072
6	0.028	0.012	0.073
7	0.022	0.010	0.081
8	0.030	0.009	0.104
9	0.017	0.010	0.065
10	0.018	0.009	0.072
MEAN	0.022	0.011	0.077
STD	0.005	0.002	0.014

In addition, we have evaluated the FFSM manually defined by the expert in [13] (where no training data has been used) over these ten datasets. Table II shows the MAE obtained for each dataset by the expert FFSM and the MAE obtained in test with the leave-one-out procedure for the GFFSM and the ARX model.

It can be easily observed that our proposal (GFFSM) and the FFSM defined by the expert obtain better results than the autoregressive linear model (ARX). Moreover, ARX models are black-box models not understandable by the human expert while our GFFSM is able to describe and

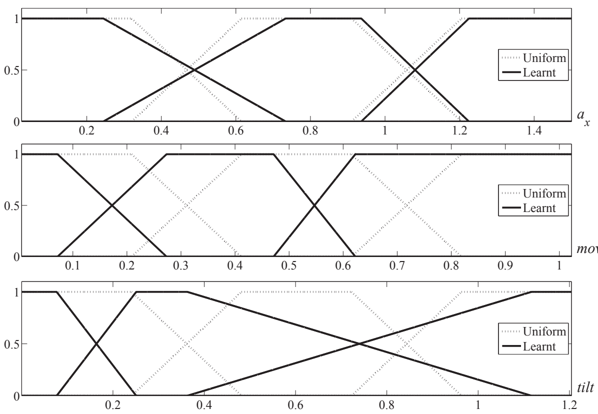

Fig. 4. Linguistic labels' trapezoidal MFs of each linguistic variable which comprise the learnt DB compared with the uniformly distributed original ones.

model the body posture by means of linguistic fuzzy if-then rules achieving a good interpretability-accuracy tradeoff.

Note that, with the application of the genetic learning procedure proposed in [8], we have increased the accuracy of the FFSM defined by the expert (by reducing the average MAE from 0.022 to 0.011) keeping her/his knowledge about this application, and producing a RB and a DB that have the same interpretability as the former.

As an example of how our novel proposal is describing linguistically the temporal evolution of the body posture, a complete set of constraints imposed on the input variables (which forms the RB as explained in III-C) learned for the second fold of the leave-one-out procedure is shown as follows:

C_{11} = (a_x is M_{a_x}) **AND** (mov is S_{mov} **OR** B_{mov}) **AND** ($tilt$ is B_{tilt})
= (a_x is M_{a_x}) **AND** (mov is $\neg M_{mov}$)[1] **AND** ($tilt$ is B_{tilt})
C_{22} = (a_x is B_{a_x})
C_{33} = (mov is M_{mov})
C_{12} = (a_x is $\neg S_{a_x}$) **AND** (mov is $\neg S_{mov}$) **AND** ($tilt$ is $\neg B_{tilt}$)
C_{21} = (a_x is S_{a_x}) **AND** (mov is $\neg M_{mov}$) **AND** ($tilt$ is B_{tilt})
C_{23} = (a_x is B_{a_x}) **AND** (mov is $\neg S_{mov}$) **AND** ($tilt$ is S_{tilt})
C_{32} = (mov is S_{mov}) **AND** ($tilt$ is $\neg B_{tilt}$)
C_{31} = (a_x is S_{a_x}) **AND** (mov is S_{mov}) **AND** ($tilt$ is M_{tilt})

Figure 4 shows the graphical representation of the learnt DB associated with this RB. The initial DB is also plotted, which consists of uniformly distributed MFs.

VI. Concluding Remarks

We have presented a practical application where we described how to build a FFSM to recognize the body posture in a dynamical environment. We defined three different states related to the body posture and applied the FFSM genetic learning procedure proposed in [8] to recognize these states.

This GFS can obtain automatically the fuzzy rules and fuzzy MFs associated to the linguistic terms of the FFSM while the states and transitions are defined by the expert, thus maintaining the knowledge that she/he has about the application. The results obtained by the GFFSM showed the goodness of our proposal. Moreover, its ability to combine the handling of the available expert knowledge with the accuracy achieved by the learning process can be used to study several phenomena where the human interaction is demanded.

Acknowledgment

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9.4 Automatic linguistic report about relevant features of the Mars' surface

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Automatic Linguistic Description about Relevant Features of the Mars' Surface

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***Abstract*—Satellites in the orbit of Mars planet provide thousands of images of its surface. Typically these images are analyzed by experts that select relevant features and generate textual reports containing the result of their observations. Nevertheless, the database of images has grown up and currently this procedure is not effective enough.**

As a result of our research on Computational Theory of Perceptions, we describe a computational application able to generate simple linguistic descriptions of circular structures on the Mars' surface. We include several examples and analysis of the obtained results.

***Keywords*-linguistic data summarization; computational theory of perceptions; image description;**

I. Introduction

New technologies allow us to acquire and store a vast array of data about complex phenomena in many areas of science and technology. However, to convert data into knowledge it is necessary to interpret and represent the data in an understandable way, giving in each type of situation, their relationship with data context and, in general, with information related with each specific phenomenon. Currently, this type of descriptions are reports that contain text and graphics produced by human experts. However, the relation between the amount of data to analyze and the number of experts available is growing dramatically. This situation causes a strong demand for computational systems that can interpret and describe linguistically the large amount of information that is being generated in many areas.

This paper was motivated by the existence of a huge database of thousands of images of the Mars' surface produced by satellites and is part of a collaboration project with the Spanish National Institute for Aerospace Technology (INTA). These images are usually analyzed by a small number of experts on Martian geology and provide important applications, e.g., in [1] infrared to visible wavelength images of the Mars' surface were analyzed to obtain relevant geological information that may help in the location of water frost. The seek of water is also the final goal in [2], where the measurements of elevations yielded a highly accurate global map of the topography of Mars that determines an upper limit of the present surface water inventory.

In this work, we will use digital image processing techniques [3], [4], in order to obtain automatically relevant features of each image. Our approach is based on the use of Fuzzy Logic (FL), which is widely recognized for its ability for linguistic concept modeling and its use in system identification. On the one hand, semantic expressiveness, using linguistic variables [5] and rules [6], [7], is quite close to natural language (NL). On the other hand, being universal approximators [8] fuzzy inference systems are able to perform nonlinear mappings between inputs and outputs. Thanks to these advantages, FL has been successfully applied to classification, regression, and system modeling.

From the viewpoint of describing images in NL using FL, there are recent works such as [9], where authors proposed a hierarchical fuzzy segmentation of the image and a collection of linguistic features able to describe each region. In [10], the authors explained how a Fuzzy Object-Relational Database Management System can be employed to implement and integrate the different elements needed for the linguistic description of images, briefly ontology, concept representation and language generation. Our research develops the Computational Theory of Perceptions (CTP) introduced by Zadeh [11], [12]. In previous works on this line, we have generated linguistic descriptions about the traffic on roundabouts [13], we generated financial reports from data taken from the Spanish Securities Market Commission (CNMV) [14] and we assessed reporting in truck driving simulators [15]. This first prototype on describing images of Mars' surface is limited to create a report on detected circular structures, i. e., volcanoes or meteorite impacts. If these structures exist in the image, the report should provide information about their size and relative position. The paper deals with the challenge of creating human like useful reports that could help experts to analyze the huge database of available images. Here, we include several contributions to this research line.

This paper is organized as follows. Section II describes the architecture of a system able to create linguistic descriptions of phenomena while Section III explains how to apply it to our proposal to describe the Mars' surface. Afterwards, Section IV shows the experimentation carried out and the validation. And finally, Section V expounds some concluding remarks.

II. Architecture

In this paper, we develop results of previous research. We face the challenge of linguistic description of data with the basic architecture depicted in Fig. 1. Here, we develop upon the concept of Granular Linguistic Model of a Phenomenon including in the architecture new elements and performing new experimentation. The main processing modules of this computational system are, namely, the Data Acquisition (DAQ) module, the Validity module, and the Expression module. We describe these modules and the associated data structures in the following subsections.

A. Granular Linguistic Model of a Phenomenon

The kernel of the report generator is the Granular Linguistic Model of a Phenomenon (GLMP). The system designer creates the GLMP as a representation of her/his own perceptions of the monitored phenomenon organized in several levels of granularity.

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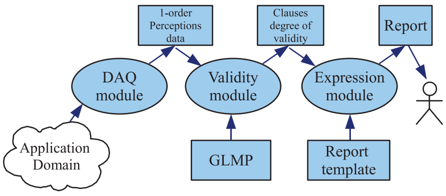


Fig. 1. Main components of the proposed computational system for linguistic description of data.

The basic element of the GLMP is called Computational Perception (CP). A CP is the computational model of a unit of information acquired by the designer about the phenomenon to be modeled. A CP is a couple $(A, W) = \{(a_1, w_1), (a_2, w_2), \ldots, (a_n, w_n)\}$ where:

- A is a set of NL sentences that is a linguistic representation of the meaning of CP. These sentences can be either simple, e.g., $a_i =$*"The circle is big"* or more complex, e.g., $a_i =$*"The image contains a big circle, many medium circles and three small circles"*.
- W $\in [0, 1]$ is the vector of validity degrees assigned to each a_i in the specific context. The concept of validity depends on the application, e.g., it is a function of the truthfulness and relevancy of the sentence in its context of use.

We call first-order perception mapping (1-PM) to a function that allows the designer to define the first-order computational perceptions (1-CPs), i.e., her/his interpretation of input data (u). A 1-PM is a tuple (u, y, g, T) where:

- u is a variable defined in the input data domain, e.g., the value $u \in \mathbb{R}$ which represents the radius of a circle in pixels.
- y is an output CP, e.g., the size of a circle in the image. It contains values $y = (A_y, W_y) = \{(a_1, w_1), (a_2, w_2), \ldots, (a_{n_y}, w_{n_y})\}$.
- g is built using a set of membership functions (MFs) to fuzzify the input data
$$W_y = (w_1, w_2, \ldots, w_{n_y}) = g(u) = (\mu_{a_1}(u), \mu_{a_2}(u), \ldots, \mu_{a_{n_y}}(u))$$
where W_y is the vector of validity degrees assigned to each a_i, and $\mu_{a_i}(u)$ is the membership degree of the input variable u to the fuzzy set a_i.
- T Here, it is typically a simple template that allows generating the elements in A_y, e.g., *"The circle is* $\{$*small* $|$ *medium* $|$ *big*$\}$*"*.

Computational perceptions whose meaning is based on other subordinate perceptions are called second-order computational perceptions (2-CPs). They are obtained using second-order perception mappings (2-PMs). A 2-PM is a tuple (U, y, g, T) where:

- U is a set of input CPs $(u_1, u_2, \ldots, u_n)$.
- y is the output CP with values $y = (A_i, W_i) = \{(a_1, w_1), (a_2, w_2), \ldots, (a_{n_y}, w_{n_y})\}$.
- g is the aggregation function.
$$W_y = g(W_{u_1}, W_{u_2}, ..., W_{u_n})$$
where W_y is a vector $(w_1, w_2, ..., w_{n_y})$ of validity degrees assigned to each element in y and W_{u_i} are the degrees of validity of the input perceptions. The designer chooses the most adequate aggregation function to each case. In FL, many different types of aggregation functions have been developed. In this application, we present a new aggregation function.
- T is a text generation algorithm that allows generating the sentences in A_y.

The designer uses a network of PMs to create a description of the monitored phenomenon with different levels of granularity that constitute the GLMP. The GLMP corresponding to the practical application can be seen in Fig. 3, it will be throughly explained in Section III-B.

B. *Validity Module*

Once a sample of input data is available, the Validity module uses the aggregation functions in the GLMP to calculate the degree of validity of each CP. Therefore this module provides as output a collection of linguistic clauses together with associated degrees of validity.

C. *DAQ Module*

This module provides the data needed to feed the 1-CPs. The Data Acquisition module provides the interface with the application physical environment. This module could include either sensors or access to information in a database. In the Section III-A, we present an example of application where the DAQ module takes the information from satellite images and implements an image processing algorithm.

D. *Expression module*

Provided a set of CPs, the goal is to combine this information to build a linguistic report. This module deals with generating the most relevant linguistic report by choosing and connecting the adequate linguistic clauses. In this paper we develop a new technique to perform this task by the introduction of the concept of Fuzzy Tree of Choices (FTC). It is a mechanism to represent part of the constraints imposed to the linguistic report which consists of a directed graph including choices and the linguistic expressions to be linked together.

III. Linguistic Description of the Mars' Surface

In this section, we will describe the relevant modules needed to produce a linguistic description of the Mars' surface.

A. *DAQ Module: Image Processing*

This module is in charge of recognizing circles in the image. The recognition of patterns is an open issue of research in the field of automatic image processing. In this first prototype, in order to extract information about the presence of circular structures, we have applied classical filtering techniques to remove the background, techniques of edge detection, and the generalized Hough transform which is particularly suitable for detecting the presence of circles [4], [16].

In the first task, known as pre-processing, everything that is not interesting in the image is "deleted". This is a procedure that transforms the image slightly to eliminate any noise, imperfections, shine, etc.

Then, we detect the edges in the image that can be defined as transitions between two significantly different levels of color intensity. This provides valuable information on the borders of objects that can be used for image segmentation. In our application, we used a modified version of the Sobel operator [4] as boundary filter which consists of two arrays whose size is 5×5 pixels.

Finally, the search for circles in the image is tackled by means of the Hough Transform [17], which was initially concerned with the identification of lines in the image, but later it has been extended to identify arbitrary shapes such as circles or ellipses. To get good results with this procedure, the pre-processing stage is essential, since the procedure is mainly based on the color jump produced at the edge of either meteorite impacts or volcano craters. The great difficulty of this procedure lies in the analysis of those photographs in which, by its nature, the land has many irregularities, which accumulates inaccuracies in the analysis and, therefore, we must assume a margin of error in the obtained results. Moreover, since we do not know in advance the size and shapes of the relevant objects, we have to work with a generalized Hough transform that enables detection of objects whose shapes and dimensions are, initially, unknown.

As an example of the performance of this module, Fig. 2 shows the circles detected for a specific image.

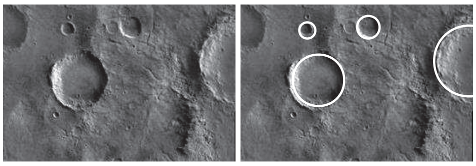


Fig. 2. Example of detection of circles in an image.

B. GLMP for the Linguistic Description of the Mars' Surface

In this application, the designer has built a GLMP which tries to summarize and highlight the relevant aspects of the data obtained from the DAQ module. In the following subsections, we will explain in detail how each CP is built based on the definition given above.

1) 1-CPs: These CPs are obtained from the output of the DAQ Module, which recognizes circles in the image. We distinguish three different 1-CPs, namely, the size of the circles (the radius of each circle in pixels), the position in the X coordinate, and the position in the Y coordinate. The template T defines three different NL propositions for each 1-CP. In Fig. 3, the possible values of each 1-CP can be seen. The terms S, M and B denotes small, medium and big and are calculated using trapezoidal MFs over the value of the radius. The same procedure is done to get the terms T, C, and B which denote top, center, and bottom and the terms L, C, and R which denote left, center, and right; the MFs are uniformly distributed along the vertical and the horizontal axes.

2) 2-CPs: As explained in Section II-A, these CPs are calculated based on subordinate CPs. For this application, we defined five different 2-CPs which describe the circles in the image at different levels of detail. In this GLMP, we can distinguish between two types of 2-CPs: there are 2-PMs that aggregate (Σ) the information from the same subordinate CP (2-CP$_1$, 2-CP$_2$ and 2-CP$_3$) and 2-PMs which combine (Π) information from different subordinate CPs (2-CP$_4$ and 2-CP$_5$).

TABLE I
DOMAIN OF POSSIBLE VALUES (A, W) OF 1-CP$_3$ FOR EACH CIRCLE.

Circle	small	medium	big
1	(a_1^1, w_1^1)	(a_2^1, w_2^1)	(a_3^1, w_3^1)
2	(a_1^2, w_1^2)	(a_2^2, w_2^2)	(a_3^2, w_3^2)
...	...	...	...
n	(a_1^n, w_1^n)	(a_2^n, w_2^n)	(a_3^n, w_3^n)

TABLE II
SET OF POSSIBLE SENTENCES ASSOCIATED WITH 2-CP$_3$.

	small circles	medium circles	big circles
Zero	(a_{01}, w_{01})	(a_{02}, w_{02})	(a_{03}, w_{03})
One	(a_{11}, w_{11})	(a_{12}, w_{12})	(a_{13}, w_{13})
Two	(a_{21}, w_{21})	(a_{22}, w_{22})	(a_{23}, w_{23})
Three	(a_{31}, w_{31})	(a_{32}, w_{32})	(a_{33}, w_{33})
Four	(a_{41}, w_{41})	(a_{42}, w_{42})	(a_{43}, w_{43})
Various	(a_{51}, w_{51})	(a_{52}, w_{52})	(a_{53}, w_{53})
Many	(a_{61}, w_{61})	(a_{62}, w_{62})	(a_{63}, w_{63})

C. Validity Module

The implementation of the aggregation function (g) of the 2-PMs that combine information from different subordinate CPs calculates the product of the validity degrees of these CPs, e.g., the sentence "A big circle in the bottom left part" will have as validity degree the product of the validity degrees of the sentences "A big circle", "A circle in the bottom part", and "A circle in the left part".

The implementation of the aggregation function (g) of the 2-PMs which aggregate information from the same subordinate CP is less straightforward. As an example, we show here how we implement the 2-PM$_3$ that calculates the validity degrees of the sentences associated with 2-CP$_3$. These validity degrees are calculated using 1-CP$_3$ for the total of the n circles in an image. The domain of possible values (A, W) of 1-CP$_3$ for each circle is represented in Tabel I. On the other hand, the set of possible sentences associated with 2-CP$_3$ is represented in Table II.

We used the $\alpha-$cut based method proposed by Delgado et al. [18] to define the validity degree of each sentence associated with 2-CP$_3$. For each fuzzy set j (small, medium and big), we calculate the percentage of circles contained at each $\alpha-$level (N_α^j) by means of Eq. 1, with $\alpha \in \mathbf{A} = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

$$N_\alpha^j = \frac{1}{n} \sum_{i=1}^{n} F_\alpha(w_j^i) \qquad (1)$$

It is worth noting that we are considering strict $\alpha-$cuts, as can be seen in Eq. 2.

$$F_\alpha(w_j^i) = \begin{cases} 1 & \text{if} \quad w_j^i > \alpha \\ 0 & \text{if} \quad w_j^i \leq \alpha \end{cases} \qquad (2)$$

Then, we calculate the membership degree of each N_α^j to each element of the set of linguistic quantifiers: $\{Q_0, \ldots, Q_6\} = \{Zero, One, Two, Three, Four, Various, Many\}$, e.g., $\mu_{Q_3}(N_\alpha^j) = Three(N_\alpha^j)$. The shapes of these linguistic labels are determined by the total number of circles n as can be seen in Fig. 4.

The last step is to calculate the average value of the membership degrees obtained for each $\alpha-$level using Eq.

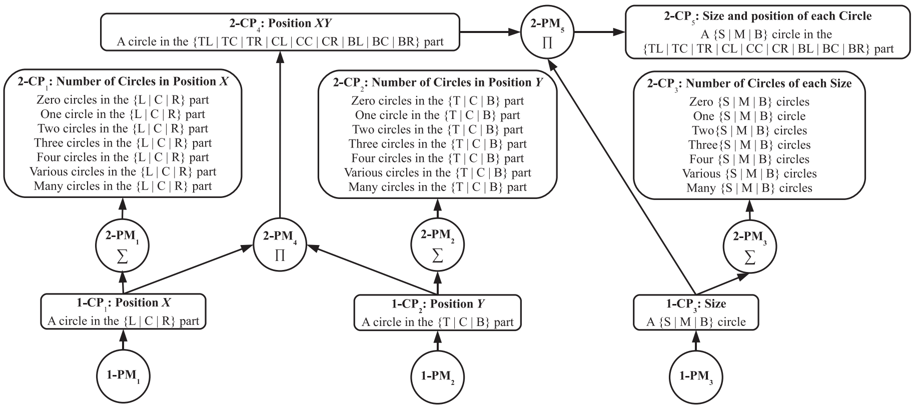


Fig. 3. GLMP for the linguistic description of the Mars' surface. The circles represent perception mappings while the rectangles stand for computational perceptions.

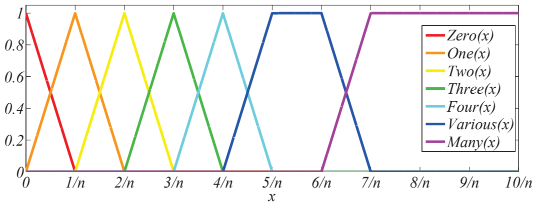


Fig. 4. Linguistic labels that represent the linguistic quantifiers "Zero", "One", "Two", "Three", "Four", "Various", or "Many" circles.

3. The number of elements in the set $\mathbf{A}$ is the level of resolution, i.e., $|\mathbf{A}| = 10$ in this particular case.

$$w_{kj} = \frac{1}{|\mathbf{A}|} \sum_{\forall \alpha \in \mathbf{A}} \mu_{Q_k}(N_\alpha^j) \tag{3}$$

This final value contains the relevant information about the amount of circles belonging to each fuzzy set (small, medium or big). For example, the validity degree of the sentence "Three medium circles" (w_{32}) will be determined by Eq. 4:

$$w_{32} = \frac{1}{|\mathbf{A}|} \sum_{\forall \alpha \in \mathbf{A}} Three(N_\alpha^2) \tag{4}$$

D. Expression Module

In this application, we developed the FTC that can be seen in Fig. 5. It contains four choices made over 2-CP$_3$. The choices order is determined by the relevance of the sentences for the final user. Here, our aim is to emphasize the presence of big circles, then medium circles and finally the small circles. Therefore, the four choices are as follows:

- q_1 Does the image contain circles?
- q_2 How many big circles does the image contain?
- q_3 How many medium circles does the image contain?
- q_4 How many small circles does the image contain?

It is worth noting that, in general, choices in the FTC have not a crisp response but a fuzzy one. This means that we need to analyze every combination of branches in the tree until we can find out the sequence that accumulates the highest degree of validity calculated using Eq. 5, which calculates the product of all the possible perceptions that take part in that sequence:

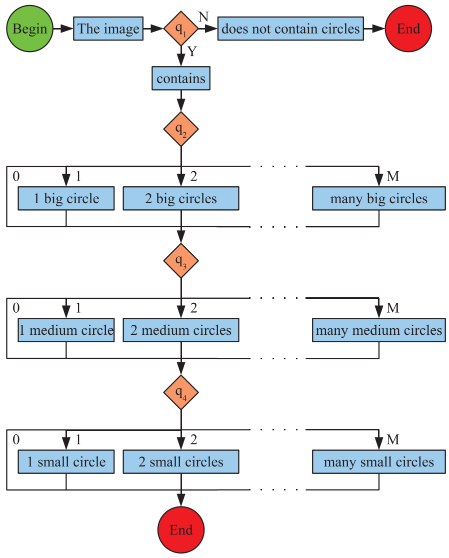
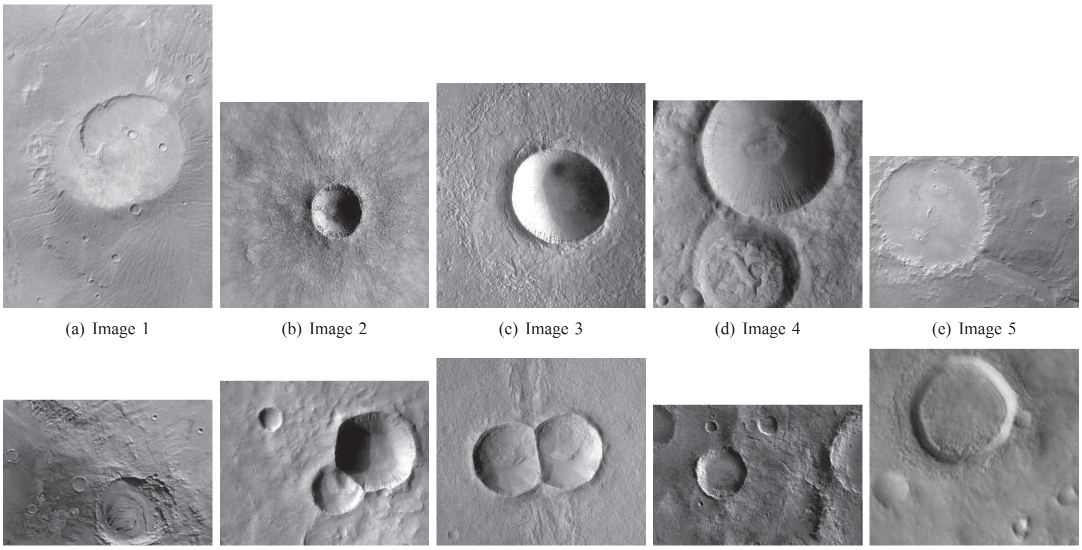


Fig. 5. This FTC is part of the constraints imposed to the linguistic description.

$$w_{\text{sequence}} = \prod_{\forall i \in \text{sequence}} w_i \tag{5}$$

The FTC represents the user's preferences regarding with the format of the report. Nevertheless, in general, we will apply additional constraints. In our application example, an obvious limitation is that the finally chosen sequence must speak about the total number of circles in the image, e.g., if an image has a total of 3 circles, we cannot choose the report: "the image contain one big circle, two medium circles and many small circles", because it does not fulfill the compatibility with the total number of circles. Moreover, once we have determined the sequence that expresses the total number of circles according to their size, we used 2-



(a) Image 1 (b) Image 2 (c) Image 3 (d) Image 4 (e) Image 5

(f) Image 6 (g) Image 7 (h) Image 8 (i) Image 9 (j) Image 10

Fig. 6. The ten analyzed images of the Mars' surface.

CP_5 to specify the position of the big circle(s) or the position of a single circle (independently of its size), if there is only one circle in the image. Of course, different perceptions and different possibilities can be expressed making different FTCs.

IV. Experimentation

We worked with a total of 10 different images that can be seen in Fig. 6.

A. Descriptions Obtained

The linguistic description for each image is presented. The obtained sequence is the one with the highest degree of validity that fulfills the compatibility requirements of the image.

1) Image 1: "The image contains one big circle, many medium circles and four small circles. The big circle is in the center of the image."

2) Image 2: "The image contains one medium circle in the center."

3) Image 3: "The image contains one big circle in the center."

4) Image 4: "The image contains two big circles and two medium circles. The two big circles are in the bottom center and in the top center parts of the image."

5) Image 5: "The image contains one big circle, three medium circles and various small circles. The big circle is in the center left part of the image."

6) Image 6: "The image contains one big circle and many medium circles. The big circle is in the bottom center part of the image."

7) Image 7: "The image contains one big circle and two medium circles. The big circle is in the center right part of the image."

8) Image 8: "The image contains two big circles. The two big circles are in the center of the image."

9) Image 9: "The image contains one big circle and three medium circles. The big circle is in the center right part of the image."

10) Image 10: "The image contains one big circle and three medium circles. The big circle is in the top center part of the image."

B. Evaluation of the Results

Assessing the performance of a a system which aims to summarize data using NL is a challenging task. The meaning of NL sentences is determined by its context of use, including the personal experience of the writer and the reader [19]. In order to contribute to solve this problem, we have used a straightforward strategy: we have used the computer to create a list of sentences to build a basic domain of meaning.

To measure the quality of the obtained descriptions, we have done a survey which consisted of five different descriptions for each of the ten images. These five different descriptions were those compatible sequences which got the best validity degrees, including, of course, the best one which is considered the actual output of our system. We asked a total of 22 different people to choose the best of these five descriptions. In Table III, we show, for each image, the average percentage of agreement of these people with our system (System agreement), i.e., the percentage of times that the choice of the people is the same as the description obtained by our system; and we also show the average percentage of agreement of these people among them (People agreement), i.e., the average percentage of times that the choice of each person is the same as the description obtained by the rest of the people. The global average values for all the images are also represented at the bottom.

The system agreement gets an average for all the images of 59.5% which is indeed greater than the average people agreement (45.3%). The highest values correspond to the images number 4, 8, and 3, which have a low number of

TABLE III
PERCENTAGE OF AGREEMENT OF THE HUMAN OBSERVERS WITH OUR SYSTEM (SYSTEM AGREEMENT) AND PERCENTAGE OF AGREEMENT OF THESE HUMAN OBSERVERS AMONG THEM (PEOPLE AGREEMENT).

Image	System agreement (%)	People agreement (%)
1	59.1	37.7
2	54.5	41.1
3	72.7	56.2
4	90.9	82.3
5	54.5	33.8
6	59.1	45.9
7	54.5	34.6
8	77.3	63.2
9	18.2	20.8
10	54.5	37.7
All images	59.5	45.3

circles easy to identify. Moreover, the system agreement is greater than the people agreement for all of the images except the number 9, which means that each person agrees, in average, more with our system than with the descriptions chosen by the rest of the people. This fact can be explained focusing on Fig. 2, where the four recognized circles of the image number 9 are represented. The description made by our system based on these four circles is: "The image contains one big circle and three medium circles. The big circle is in the center right part of the image". However, a human observer who tends to include the big circle in the center right part of the image, tends also to include the circle at the top left corner that is not recognized by the DAQ Module. Therefore, not only is the description obtained by our report generator extremely dependent on the results of the DAQ Module, but also on the subjectivity of the people.

V. CONCLUDING REMARKS

This paper presents a contribution to solve the important challenge of generating linguistic reports from data. We have developed upon our previous works with two new contributions:

1) The introduction of the concept of Fuzzy Tree of Choices to describe the template of a linguistic report.
2) The proposal of an evaluation test to obtain a partial measure of the quality of the generated linguistic reports.

In human generated linguistic reports, the use of NL depends on the application context and specifically of the writer experience and intentions. Here, the chosen linguistic expressions should capture the subjectivity of the human beings participating in the process of designing the computational system, namely, the expert on Martian geology that will provide the functional requirements and the designer that will try to implement that functionality. However, this procedure can also be applied to different fields, the difficulties of this adaptation will depend on the complexity of the desired linguistic reports.

In future works, the idea is to improve the DAQ Module in order to recognize different structures that allow us to provide a more complex description of the Mars' surface.

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