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" BARIONES Y GRAVITONES GIGANTES EN EL MARCO DE LA CORRESPONDENCIA ADS / CFT "

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Capítulo 1

Introducción

Uno de los resultados más notables encontrados en Teoría de Cuerdas es la existencia de una dualidad, o correspondencia, entre ciertas teorías de cuerdas en backgrounds muy específicos (espacios AdS), y teorías gauge conformes (CFT). Esta dualidad es de tipo holográfico, ya que se codifica la misma información en un espacio de D dimensiones (el "lado gravitatorio"donde tenemos teoría de cuerdas) y otro espacio de $D-1$ dimensiones (el "lado gauge"). Este resultado, que es una conjetura (no está completamente demostrada) permite establecer un diccionario de correspondencias entre los resultados que se obtienen en la parte gravitatoria y la parte gauge. Pero estos resultados son producto de un modelo previo para la dualidad.

Es decir, es necesario construir configuraciones dentro, por ejemplo, del lado gravitatorio, cuya dinámica nos permita obtener resultados traducibles mediante la dualidad a resultados correspondientes en el lado gauge. También ocurre esto a la inversa: resultados en el lado gauge la dualidad son traducibles a configuraciones en el lado gravitatorio. Es posible así probar la consistencia del modelo gravitatorio propuesto (si ya conocemos previamente cuál debe ser el dual gauge) pero también producir resultados nuevos, podríamos denominarlos predicciones, siempre dentro de la dualidad (no son predicciones del tipo teoría - experimento).

En esta tésis se presentan tres artículos publicados reciéntemente en los que hemos construido y analizado dos tipos de configuraciones en el lado gravitatorio de la dualidad AdS/CFT: vértices bariónicos y gravitones gigantes.

Los vértices bariónicos se corresponden con estados ligados de quarks en el lado gauge. En el lado gravitatorio se construyen a partir de branas enrollando subespacios del background, y atadas a cuerdas fundamentales que conectan estas branas con la frontera del espacio, que es donde tenemos la teoría gauge, de forma que se corresponden con los quarks en la teoría de campos (cada quark es el extremo de una de las cuerdas que conectan la frontera del espacio con la brana en el interior). El hecho significativo que hemos estudiado en nuestro trabajo es que existen estos estados ligados no singlete: el número de quarks no es igual al rango del grupo gauge. Nuestro estudio ha consistido en restringir lo más posible cuánto de carácter no singlete tienen estas configuraciones, obteniendo rangos para el número de quarks que las constituyen. Aparte de esta descripción general, el estudio de vértices bariónicos en los backgrounds que hemos tratado dan lugar a resultados teóricos de por sí interesantes, como la aparición de nuevos acoplos en la acción efectiva de D-branas no predichos antes en la literatura.

Los gravitones gigantes son configuraciones en cierto modo similares a los vértices bariónicos en el sentido de que consisten básicamente en branas enrollando subespacios del background. Pero en este caso no existen cuerdas atadas a la brana que en el lado gauge nos puedan dar lugar a una interpretación como estado ligado de quarks. En este caso, los gravitones gigantes sirven como realización del principio de exclusión en teoría de cuerdas: el momento que adquieren no supera cierto límite que depende del rango del grupo gauge.

Aparte de que cada configuración por separado da lugar a resultados interesantes (bariones no singlete, principio de exclusión), utilizando estas configuraciones podemos explorar la región de acoplo de 't Hooft finito. Para ello hacemos uso de la posibilidad de describir microscópicamente las branas enrolladas como un conjunto de branas de dimensiones menores expandiéndose dieléctricamente mediante efecto Myers. Este fenómeno puramente gravitatorio conduce de forma muy directa a una interpretación microscópica que reproduce exactamente los resultados macroscópicos en el límite en que el número de branas que se expanden es muy grande. El rango de aplicabilidad de esta descripción microscópica es complementario al rango macroscópico (supergravedad) y podríamos acceder así al rango de acoplo de 't Hooft finito, si tuviésemos en cuenta las correcciones a α' (lo cual no hemos hecho en nuestros trabajos).

Podemos así decir que el tema central de esta tésis consiste en la descripción de dos tipos de configuraciones que aprovechan la topología del background, para reproducir aspectos de teorías gauge ya conocidos, o bien obtener resultados nuevos que intentamos analizar en contextos cada vez más restringidos. Además, utilizando el efecto dieléctrico de Myers podemos describir microscópicamente estas configuraciones y, finalmente, nos es posible explorar la región de acoplo de 't Hooft finito, complementaria a la región de probe aproximation apropiada en supergravedad para la descripción macrosópica.

A continuación se comentan algunos resultados y trabajos previos que han servido para desarrollar la tésis, y que consituyen la bibliografía de los artículos que se presentan.

Estabilidad del barión en ABJM y el efecto Myers

En los dos primeros trabajos de esta tésis, utilizamos una dualidad AdS_4/CFT_3 que relaciona la teoría de supercuerdas tipo IIA en el background $AdS_4 \times CP^3$ con una teoría Chern-Simons con materia $\mathcal{N} = 6$ con grupo gauge $U(N)_k \times U(N)_{-k}$ conocida como modelo ABJM [1].

La posibilidad de enrollar branas en ciclos no triviales en este background se estudió también en el artículo original. En concreto, la brana D6 que enrrolla el CP^3 completo es el análogo del vértice bariónico descrito por Witten [2]. También es posible enrollar branas D2 y D4 que son duales al operador monopolo de 't Hooft y al operador dibarión, respectivamente [3, 4].

Es posible extender la interpretación de vértice bariónico a la brana D4, si añadimos flujos gauge no triviales en el worldvolume de la brana. En este caso, también estas, como la D6 o la D2, atrapan un flujo y desarrollan un tadpole. La necesaria cancelación de este tadpole (la carga neta debe ser cero) hace necesario introducir cuerdas en la configuración. Esta configuración: brana enrollada más cuerdas atadas a la misma que se extienden hasta la frontera de AdS, no es exclusiva de la D6. Estas generalizaciones han sido estudiadas en [5], y en el contexto de AdS/CMT en [6, 7]. También se ha explorado la conexión entre la existencia de límites en el campo magnético introducido y el principio de exclusión de cuerdas [16], lo cual también sería esperable a partir de los resultados obtenidos en nuestros trabajos.

Como ya se dijo, un resultado no previsto en los vértices bariónicos es el hecho de que existen configuraciones no singletes. Es decir, el número de quarks en el lado gauge no es igual al rango del grupo. De hecho, el número de quarks está dentro de un intervalo que depende del rango del grupo. Este hecho ya se encontró en $AdS_5 \times S^5$ [8, 9]. A su vez, el estudio de la estabilidad de estas configuraciones confirmó que son estables también para un número de quarks dentro de cierto intervalo, si bien más restringido que el dado por su existencia. Este trabajo fue realizado por dos de nuestros colaboradores [10], y es un análisis que hemos realizado también en nuestra configuraciones: en todas ellas analizamos su estabilidad siguiendo el mismo procedimiento. También obtenemos expresiones para su tamaño que tienen similitud con el caso de los vértices bariónicos en $AdS_5 \times S^5$ y el sistema quark-antiquark [20, 21]. Obtenemos también la energía de ligadura de estas configuraciones, señalando su similitud con resultados previos en la literatura en relación a cómo es la fuerza y el comportamiento dictado por invariancia conforme [23, 24, 25, 26, 27, 28, 29, 30, 31].

Otro resultado de gran importancia que hemos utilizado en nuestro trabajo es el efecto dieléctrico de Myers [13, 32]. La expansión de un conjunto de branas en espacios fuzzy nos permite elaborar descripciones microscópicas de las configuraciones que hemos construido. Y éstas a su vez nos permiten explorar las regiones de acoplo de 't Hooft finito. Las branas dieléctricas expandiéndose en espacios de tipo coset fuzzy ya se discutieron en la literatura en diferentes contextos [33, 34]. También incluimos en la bibliografía trabajos que describen los espacios fuzzy y cómo se construyen [37, 38, 42, 51].

Así mismo hemos encontrado necesario dar cuenta de la anomalía de Freed-Witten en el dibarión, la cual exige introducir un campo B_2 semientero, ya a nivel clásico [14, 15]. También en nuestras configuraciones es necesario introducir este campo. Como resultado de ello, aparecen acoplos nuevos en las acciones de n D-branas coincidentes no presentes en la literatura anterior. En concreto, construiremos una versión microscópica de los acoplos de alta curvatura [17, 18, 19]. Este tipo de acoplos pueden generalizar acoplos anómalos ya descritos antes, pero para una única D-brana [45]. Este es un resultado notable de nuestro trabajo.

5-branas dieléctricas y gravitones gigantes en ABJM

En la bibliografía para los gravitones gigantes [1] observamos que ya se señaló que son duales a Polinomios de Schur [2, 3, 4] y esto permite extraer muchas propiedades tales como el principio de exclusión de cuerdas [5] o detalles sobre las geometrías locales y globales de las branas duales que están codificados en la teoría gauge [6, 7, 8, 9, 10].

Ya se han construido ejemplos particulares de gravitones gigantes en la dualidad entre teoría IIA en $AdS_4 \times CP^3$ y la teoría ABJM [12, 13, 14, 15, 16, 17, 18], así como a partir de branas D2 [19, 20, 22]. Ya conocemos por tanto la existencia de estas configuraciones y su interpretación dual. Aquí tratamos de construir configuraciones similares, tanto macroscópica como microscópicamente.

Para la construcción del gravitón gigante hacemos uso de las isometrías del background. Ya conocemos la acción de una M5 enrollada en una dirección isométrica [23]. A partir de esta acción podemos analizar cuándo esta M5 se comporta como gravitón gigante. A su vez, como hemos hecho con los vértices bariónicos, construimos una descripción microscópica de estos gravitones aprovechando el efecto dieléctrico de Myers. Esta descripción permite explorar la región de acoplo de 't Hooft finito. Para esto, utilizamos la acción para gravitones coincidentes [27].

Bariones no singlete en backgrounds menos supersimétricos

Para el estudio de bariones no singlete en backgrounds menos supersimétricos, tratamos de analizar estas configuraciones en backgrounds más restrictivos, que o bien son menos supersimétricos y/o confinantes [16, 17] o en backgrounds deformados [21, 22]. Es por eso que hacemos uso de las descripciones de estos backgrounds y estudios ya realizados de configuraciones tipo vértice bariónico en los mismos [9, 10, 11, 12, 13, 14, 15, 18, 19, 20]. Este estudio es similar al caso del background $AdS_4 \times CP^3$, y también aquí analizamos la estabilidad de las configuraciones.

En el siguiente capítulo abordaremos más en detalle los objetivos de nuestros trabajos, así como la descripción de los backgrounds utilizados en el tercero de ellos.

Capítulo 2

Objetivos

Como hemos explicado en la parte introductoria, la tésis consiste en el análisis de configuraciones de branas de dos tipos (vértice bariónico, y gravitón gigante) dentro del contexto de la dualidad entre teoría de supercuerdas tipo IIA en $AdS_4 \times CP^3$ y la teoría ABJM. Además, con la intención de analizar el carácter no singlete de los vértices bariónicos, realizamos el estudio de estas configuraciones en otros backgrounds más restrictivos que son variaciones de $AdS_5 \times S^5$. Completamos el trabajo con una descripción microscópica de estas configuraciones.

2.1. Vértices bariónicos y su estabilidad en ABJM

Consideramos configuraciones en $AdS_4 \times CP^3$ de tipo vértice bariónico cargado magnéticamente con un número reducido de quarks. Estudiamos si estas configuraciones son soluciones de las ecuaciones de movimiento clásicas y analizamos su estabilidad y cómo esta afecta al número de quarks permitido. El hecho de que el flujo magnético disuelva carga de brana D0 hace posible dar una descripción microscópica en términos de branas D0 expandiéndose en espacios CP^n fuzzy mediante efecto dieléctrico de Myers. Los objetivos son:

1. Resumir las propiedades de las configuraciones tipo vértice bariónico magnetizado construidas en trabajos previos. Calcular la energía de estas configuraciones. Analizar sus acciones Chern-Simons para deducir la cantidad de carga de branas D0 y D2 disuelta, así como la carga de cuerda fundamental inducida por el background (el denominado tadpole) que exige añadir a la configuración cuerdas fundamentales atadas a la brana (de forma que la carga total neta sea cero).

2. Reducir el número de quarks en estas configuraciones y encontrar los valores para los que aún existen configuraciones clásicas. Se trata de variar el número de cuerdas en la configuración, estudiar la existencia de soluciones clásicas y deducir los límites en el número de cuerdas posible en la configuración. Analizar los casos extremos. Calcular y analizar el radio de la configuración y la energía de ligadura.

3. Realizar un estudio de la estabilidad bajo pequeñas fluctuaciones. Deducir de este análisis un límite en el número de cuerdas en la configuración. Comparar con el resultado anterior.

4. Describir microscópicamente estas configuraciones. Escribir la acción para un número de branas D0 coincidentes expandiéndose en un espacio CP^n fuzzy. Describir estos espacios. Introducir el campo B_2. Analizar si esta descripción concuerda con la macroscópica. Deducir de este análisis la necesidad de incluir acoplos de mayor curvatura en la acción.

2.2. Gravitones gigantes en ABJM

Construimos una brana $M5$ con topología de 5-esfera enrollada en una dirección isométrica dentro de $AdS_4 \times S^7/Z_k$. Damos su descripción microscópica en términos de gravitones de Teoría M expandiéndose en espacios fuzzy. Reducimos esta configuración dimensionalmente para obtener una brana NS5 en CP^3 con carga de brana D0, o bien una brana D4 enrollada en un CP^2. Analizamos en qué casos pueden interpretarse como gravitón gigante. Damos a su vez la descripción microscópica de estas configuraciones.

1. Describir el gravitón gigante con topología de 5-esfera en $AdS_4 \times S^7/Z_k$ utilizando la acción para una brana M5 enrollada en una dirección isométrica. Calcular el hamiltoniano y analizar las soluciones de energía mínima y sus duales en la teoría gauge. Añadir flujo magnético para inducir momento a lo largo de la dirección isométrica. Intercambiar el papel de las direcciones S^1/Z_k y S^1, y realizar de nuevo el mismo análisis.

2. Dar la descripción microscópica complementaria en términos de gravitones de Teoría M expandiéndose en subespacios fuzzy de S^7/Z_k debido al efecto dieléctrico de Myers. Obtener el hamiltoniano microscópico y estudiar su concordancia con el análisis microscópico.

3. Reducir dimensionalmente la solución tipo gravitón gigante a partir de una brana M5, para producir una brana NS5 estática expandiéndose en una 5-esfera retorcida dentro de CP^3 con carga de brana D0. Obtener el hamiltoniano y estudiar las configuraciones de menor energía. Estudiar cómo se realiza el principio de exclusión de teoría de cuerdas, así como el caso maximal.

4. Reducir de nuevo la brana M5 enrollada en S^1/Z_k para producir en IIA una brana D4 enrollada en un CP^2 deformado con carga de momento. Analizar el caso maximal.

5. Dar la descripción microscópica complementaria de la configuración con la brana NS5 en términos de gravitones tipo IIA dieléctricos, y bosquejar la descripción de la brana D4 enrollada en el CP^2 en términos de branas D0 con momento angular.

2.3. Vértices bariónicos en backgrounds con menos supersimetría y/o confinantes

Analizamos la descripción holográfica de bariones no singlete en varios backgrounds con menos supersimetría y/o confinantes. Estudiamos su existencia en los backgrounds de la forma $AdS_5 \times Y_5$ con Y_5 un espacio tipo Einstein con flujo de 5-forma, obteniendo un intervalo para el número de quarks en la configuración. También estudiamos estas configuraciones en el background confinante de Maldacena-Núñez y en backgrounds $AdS_5 \times S^5$ deformados. Las generalizamos incluyendo un flujo magnético y damos una descripción microscópica en términos de branas de menores dimensiones expandiéndose en bariones fuzzy. Los objetivos son:

1. Estudiar la dinámica del vértice bariónico en $AdS_5 \times Y_5$, con Y_5 un espacio tipo Einstein con flujo de 5-forma. Obtener la solución clásica y estudiar su estabilidad.

2. Particularizar para las geometrías $AdS_5 \times Y^{p,q}$ y $AdS_5 \times T^{1,1}$ con flujo magnético no nulo. Calcular la carga de brana D1 o D3 disuelta por el campo magnético. Estudiar la estabilidad de estas configuraciones y el límite que dicha estabilidad impone en el número de cuerdas. Comparar con el caso de S^5.

3. Estudiar las configuraciones en el background de Frolov multi-β deformado. Añadir flujo magnético.

4. Estudiar las configuraciones en el background de Maldacena-Núñez. Construir el vértice bariónico a partir de una brana D3. Obtener el límite para el número de cuerdas. Comparar con backgrounds conformes. Analizar su estabilidad. Añadir flujo magnético.

5. Describir microscópicamente los casos anteriores, en términos de branas D1 expandiéndose en un $S^2 \times S^2$ fuzzy, branas D3 expandiéndose en una S^2 fuzzy, o branas D1 expandiéndose en una S^2 fuzzy respectivamente.

Se describen a continuación los diferentes backgrounds que hemos utilizado en este último trabajo, cuál es su origen y qué interés tienen para nosotros.

Backgrounds tipo $AdS_5 \times Y_5$: Klebanov-Witten y Sasaki-Einstein

La conjetura de Maldacena relaciona una teoría gauge $SU(N)$ $\mathcal{N} = 4$ con Teoría de Cuerdas tipo IIB en $AdS_5 \times S^5$. Es interesante generalizar esta dualidad considerando otros backgrounds de IIB de la forma $AdS_5 \times X_5$ donde X_5 es un espacio Einstein 5-dimensional con flujo de 5-forma. Estos backgrounds están relacionados con teorías de campos conformes 4-dimensionales, en general diferentes de la teoría gauge $SU(N)$ $\mathcal{N} = 4$. Esto es obvio, ya que solamente S^5 preserva el número máximo de supersimetrías (32), mientras que otros espacios Einstein dan lugar a menos supersimetrías. Son éstos backgrounds los que nos interesan: aquellos que preservan una fracción de la supersimetría. Sabemos que existen vértices bariónicos cuando $\mathcal{N} = 4$, y pretendemos analizar si también existen para, por ejemplo, $\mathcal{N} = 1$. Se consigue una teoría con $\mathcal{N} = 1$ cuando X_5 es un espacio homogéneo $T^{1,1} = (SU(2) \times SU(2))/U(1)$, con $U(1)$ siendo un subgrupo diagonal del toro maximal de $SU(2) \times SU(2)$. Según el conteo de supersimetrías, esta compactificación debería ser dual a una teoría de campos conforme $\mathcal{N} = 1$ en 4 dimensiones. Esta teoría de campos es el límite IR de la teoría en el worldvolume de D3-branas coincidentes colocadas sobre una singularidad cónica de un CY_3 no compacto.

Backgrounds multi-β deformados

La dualidad gauge/gravedad relaciona teorías de campo y teorías gravitatorias con condiciones de frontera particulares. Si modificamos estas condiciones de frontera, esto da lugar a modificaciones en el lagrangiano de la teoría de campo dual. Una de estas modificaciones se denomina deformaciones β. Por ejemplo, es posible deformar la teoría Yang-Mills $\mathcal{N} = 4$ a una teoría $\mathcal{N} = 1$ que preserve una simetría $U(1)\times U(1)$ que no es simetría R. Esto se consigue redefiniendo el producto de los campos en el lagrangiano, de tal manera que se introducen nuevas fases, modificando así el superpotencial. Esta es la teoría β-deformada. La forma de obtener esta teoría en el lado gravitatorio es como sigue: si la geometría dual de la teoría original tiene dos isometrías asociadas a las dos simetrías globales $U(1)$, es decir, la geometría contiene un 2-toro, solo debemos redefinir τ en términos de un parámetro de deformación γ. Reducimos la teoría 10-dimensional a 8 dimensiones sobre el 2-toro, que es invariante bajo la acción de $SL(2, R)$ en τ. La deformación es un elemento particular de $SL(2, R)$.

Por otra parte, si el parámetro de deformación es real, se puede obtener el background deformado a partir de $AdS_5 \times S^5$ por medio de una transformación de dualidad T sobre una coordenada isométrica ϕ_1(la que corresponde a un $U(1)$), a continuación un shift de otra coordenada isométrica, seguida de otra transformación de dualidad T sobre ϕ_1. En el caso de S^5, que contiene tres toros, podemos realizar esta cadena de transformaciones (TsT) con diferentes parámetros $\hat{\gamma}_i$, generando así una deformación de $AdS_5 \times S^5$. El background de supergravedad 3-paramétrico debería así ser dual a una deformación de SYM $\mathcal{N} = 4$.

Ya que en $AdS_5 \times S^5$ existen y se han analizado los vértices bariónicos y los rangos para el número de cuerdas de la configuración, nosotros hemos analizado si existen también estas configuraciones en deformaciones de $AdS_5 \times S^5$ dentro del marco de la dualidad AdS/CFT. De ahí que tomemos como modelo los backgrounds multi-β deformados.

Background de Maldacena-Núñez

Este background tiene como objeto encontrar una correspondencia similar a la de AdS/CFT para teorías Yang-Mills puras (es decir, sin materia) con menos supersimetría. Describe una geometría que es dual a una teoría de cuerdas IIB en una brana NS5, que se reduce en el IR a Yang-Mills $\mathcal{N}=1$ pura 6-dimensional con 16 supercargas. La NS5 está enrrollada en una S^2 y tenemos un fibrado normal retorcido. La solución de supergravedad se encuentra reduciendo el problema a supergravedad gaugeada en 7 dimensiones, subiéndola posteriormente a 10 dimensiones. En la aproximación de supergravedad la escala de la teoría de cuerdas y la escala de la teoría 4-dimensional son comparables. La solución tiene una simetría $U(1)_R$ rota en el UV a Z_{2N} y la solución completa rompe esta a Z_2 (encontramos N soluciones diferentes). La teoría es confinante y tiene apantallamiento magnético. Tiene paredes de dominio que separa los diferentes vacíos, con las cuerdas terminando en estas paredes. En el límite de desacoplo, la teoría de cuerdas es dual a Yang-Mills $\mathcal{N}=1$. Lo que nos interesa de este background es, aparte de que tenemos $\mathcal{N}=1$, el hecho de que también es confinante. Pretendemos así analizar si existen vértices bariónicos no singletes cuando hay confinamiento.

Capítulo 3

Discusión de los resultados

3.1. Vértices bariónicos y su estabilidad en ABJM

La acción DBI para una Dp-brana en $AdS_4 \times CP^3$ enrollada en un ciclo $CP^{p/2}$ del CP^3 con $p = 2, 4$ y 6, en presencia de un flujo magnético $F = \mathcal{N} J$, con $N \in 2Z$ se puede escribir:

$$S_{DBI}^{D_p} = -Q_p \int d\tau \frac{2\rho}{L} \tag{3.1}$$

con

$$Q_p = \frac{T_p}{g_s} Vol(CP^{p/2}) \left(L^4 + (2\pi)^2(\mathcal{N}-1)^2\right)^{p/4}$$

para $p = 2, 6$ y para $p = 4$

$$Q_4 = \frac{T_4}{g_s} Vol(CP^2) \left(L^4 + (2\pi\mathcal{N})^2\right)$$

El shift en $\mathcal{N}$ se debe a la introducción del campo B_2 que cancela la anomalía Freed-Witten. Será importante reproducir este resultado microscópicamente.

El análisis de la parte CS de la acción nos indica que el flujo magnético disuelve carga de branas D0 y D2 en el worldvolume de la Dp-brana de la configuración. En concreto, para $p = 4$ encontramos carga de D2 (es decir, siendo la D2 fuente del campo C_3, la carga de D2 indica cuántas fuentes de campo C_3 tenemos, el número de branas D2):

$$S_{CS}^{D4} = \frac{\mathcal{N}}{2} T_2 \int C_3$$

y carga de D0:

$$S_{CS}^{D4} = \frac{\mathcal{N}^2}{8} T_0 \int C_1$$

Ya que las branas D4 y D6 tienen branas D2 disueltas enrrollando CP^1, estas branas D2 capturan flujo de F_2, y esto genera un tadpole (carga no compensada).

Para la D4 tenemos:

$$S_{CS}^{D4} = k \frac{\mathcal{N}}{2} T_{F_1} \int dt A_t$$

y para la D6:

$$S_{CS}^{D6} = \left(k \frac{\mathcal{N}^2}{8} + N \right) T_{F_1} \int dt A_t$$

Esta carga no compensada nos indica que hay que añadir cuerdas fundamentales, lo cual junto con la brana Dp completa la configuración.

Para el caso de D6, además, observamos que añadir el campo B_2 debido a la anomalía de Freed-Witten nos obliga a introducir nuevos acoplamientos que también contribuyen a la carga de cuerda fundamental. Parte de estas nuevas contribuciones se cancelan con otros acoplamientos (denominados de alta curvatura). Pero hay otra parte que no se cancela si no es introduciendo de nuevo cuerdas fundamentales. Finalmente, obtenemos una carga total:

$$q_{D6} = N + k \frac{\mathcal{N}(\mathcal{N} - 2)}{8}$$

Será así interesante estudiar cómo se reproduce microscópicamente este resultado. Este estudio conduce a nuevos acoplamientos no estudiados antes en la literatura como veremos.

Estudiamos la existencia de estas configuraciones macroscópicas y encontramos que para las branas D2 y D6, $\mathcal{N}$ debe estar por debajo de cierto límite, y para la D4 existe además un límite inferior. Estos valores extremos definen a su vez un número mínimo de quarks l_{min} (con $\sqrt{1-\beta^2} = \frac{2Q_p}{LqT_{F1}}$):

$$l \geq \frac{q}{2}(1 + \sqrt{1-\beta^2}) = l_{min}$$

Establecemos así un espacio de parámetros $(l, \mathcal{N})$ para el cual la configuración tipo vértice bariónico existe. Los casos límite son aquellos en que los quarks son libres.

Calculamos el radio de la configuración, y observamos que tiene la misma forma que el tamaño del vértice bariónico en $AdS_5 \times S^5$ y el sistema $q\bar{q}$ (en el sentido de tener el mismo tipo de dependencia con los parámetros de la configuración).

Obtenemos la energía on-shell total de la configuración y a partir de ésta la energía de ligadura (con $f(x)$ una función positiva decreciente y ℓ el tamaño de la configuración):

$$E_{bin} = -f(x)\frac{(g_s N)^{2/5}}{\ell}$$

La fuerza es atractiva y creciente en magnitud (algo no esperado necesariamente para estados ligados de quarks). El comportamiento $1/\ell$ es el dictado por invariancia conforme, y la dependencia de tipo no analítica en el acoplo de 't Hooft es el predicho previamente en la literatura. Esto indica que hay un comportamiento universal de la configuración basado en la simetría conforme de la teoría gauge. Este hecho se reafirma al estudiar en el tercer trabajo de la tésis estas configuraciones en otros backgrounds.

El estudio de la estabilidad de estas configuraciones restringe aún más el intervalo de valores permitidos para el número de quarks (con γ_c un número positivo):

$$l \geq \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2})$$

Estas configuraciones, macroscópicas, se han analizado en la aproximación de probe brane, válida en el límite de supergravedad ($L >> 1$ o bien $k << N$) y en la región de acoplo débil ($g_s << 1$, o bien $N << k^5$). Es posible describirlas para acoplo de 't Hooft finito en términos de espacios $CP^{p/2}$ fuzzy construidos a partir de branas D0 expandidas dieléctricamente. Esta es la descripción microscópica complementaria.

Partiendo entonces de la acción que describe n branas D0 coincidentes (dada en la literatura), y utilizando la también conocida versión fuzzy de $CP^{p/2}$, obtenemos para el caso $B_2 = 0$:

$$S^{DBI}_{nD0} = -\frac{n}{g_s}\left(1+\frac{L^4}{16\pi^2 m^2}\right)^{p/4}\int d\tau \frac{2\rho}{L}$$

Aquí, n es la dimensión de la representación irreducible totalmente simétrica de orden m, $(m, 0)$ de $SU(\frac{p}{2}+1)$, siendo $CP^{p/2}$ el espacio coset $SU(\frac{p}{2}+1)/U(\frac{p}{2})$ en la realización fuzzy.

La expresión anterior concuerda con el resultado macroscópico a primer orden en m. También se reencuentra en el cálculo microscópico la condición de cuantización $\mathcal{N} \in 2Z$.

El efecto del campo B_2, excluido antes, es de orden $O(1/m)$. A este orden obtenemos, para la acción de n branas D0:

$$S_{nD0}^{DBI} = -\frac{T_p}{g_s} Vol(CP^{p/2}) \left(L^4 + (2\pi)^2 (2m + \frac{p}{2})^2\right)^{p/4} \int d\tau \frac{2\rho}{L}$$

Este resultado concuerda con el cálculo macroscópico si redefinimos la dependencia de $\mathcal{N}$ con m según:

$$\mathcal{N}_{p=2,6} = 2m + \frac{p}{2} + 1$$
$$\mathcal{N}_{p=4} = 2m + \frac{p}{2}$$

Con estas redefiniciones, no solo se conserva la condición de cuantización para $\mathcal{N}$ sino que reproducimos el shift $\mathcal{N} \to \mathcal{N} - 1$ para $p = 2, 6$. Esto subraya la necesidad de introducir el campo B_2, que en la descripción macroscópica se debió a la anomalía de Freed Witten. Sin embargo, no está claro cómo se construiría en este caso el CP^2 en una descripción microscópica (branas D0 expandiéndose en el CP^2) para el caso de la brana D4, ya que el campo de Freed-Witten no se acopla al worldvolume de las branas D0.

La descripción microscópica de las cuerdas fundamentales concuerda también con los resultados macroscópicos. A partir de la acción CS para n branas D0 coincidentes encontramos acoplos que describen diversas posibilidades: para n branas D0 expandiéndose en un CP^1 fuzzy, obtenemos la carga de cuerda fundamental (en el límite de m grande):

$$q = \frac{2}{p} k \frac{\mathcal{N}^{p/2-1}}{2^{p/2-1}(p/2-1)!}$$

Concuerda con el resultado macroscópico cuando evaluamos la cantidad de carga fundamental debida a las branas D2 disueltas que atrapan flujo de F_2. A esta carga fundamental hay que añadir, por supuesto, la aportación de la brana en que están disueltas estas D2. Así, en el caso de D6, a partir de branas D0 expandiéndose en un CP^3 fuzzy, se obtiene la carga de cuerda para el caso macroscópico de una D6 enrrollada en el CP^3 ($q = N$).

El resto de acoplos presentes en la acción CS para las branas D0 se corresponden a la contribución de B_2. En este caso, ya que este campo contribuye a la carga en orden $O(1/m)$, hace falta añadir a esta nueva carga la contribución que no se tuvo en cuenta al tomar m grande en la carga calculada anteriormente. Teniendo en cuenta estas contribuciones, encontramos que la carga para $p = 2$ es k, para $p = 4$ es $k(m + 1)$ y para $p = 6$, $N + \frac{k}{2}((m + 2)^2 - m - 2 + \frac{1}{4})$. Concuerda con la contribución macroscópica a la carga de cuerda para la D4 y la D6. En este último caso existe una contribución extra $k/8$ que se corresponde a la carga macroscópica que se canceló teniendo en cuenta los acoplos de mayor curvatura. Es obligado entonces, y así hacemos en nuestro trabajo, dar una versión microscópica de estos nuevos acoplamientos, no presentes en la acción CS. Comprobamos que con nuestra versión microscópica podemos reproducir los resultados macroscópicos para estos nuevos acoplos, y señalamos que éstos, no discutidos antes en la literatura, pueden servir para obtener términos dieléctricos que acoplen potenciales RR a derivadas de B_2 y la métrica (mediante dualidad T) que generalicen los términos anómalos ya derivados en la literatura para una única brana Dp.

3.2. Gravitones gigantes en ABJM

Construimos un gravitón gigante a partir de una brana M5 en $AdS_4 \times S^7/Z_k$, enrollada en S^5 dentro de S^7/Z_k y propagándose a lo largo de S^1/Z_k (descomponemos S^5 como un fibrado U(1) sobre CP^2). Tenemos así una dirección isométrica (la de la fibra) y en la literatura disponemos de la acción de la brana M5 para este caso.

Encontramos así el Hamiltoniano (P_τ es el momento asociado a la coordenada en S^1/Z_k):

$$H = \frac{k}{R} P_\tau \sqrt{1 + \tan^2 \mu \left(1 - \frac{N}{P_\tau} \sin^4 \mu\right)^2}$$

La solución de gravitón gigante es

$$\sin \mu = \left(\frac{P_\tau}{N}\right)^{1/4}$$

Su energía satisface:

$$E = \frac{k}{R} P_\tau$$

Además, el gravitón gigante satisface $P_\tau \leq N$, lo cual concuerda con el principio de exclusión de cuerdas. En el caso maximal ($\mu = \pi/2$), $P_\tau = N$ y $E = kN/R$. Se interpreta entonces o bien como un estado ligado de N gravitones con energía k/R, o bien como un estado ligado de k branas M5 enrolladas en S^5 con energía N/R.

Generalizamos la construcción anterior añadiendo flujo magnético para obtener una brana con momento en una dirección χ que no sea la isométrica (y de esa manera su reducción a IIA no es estática). Obtenemos el Hamiltoniano:

$$H = \frac{k}{R} P_\tau \sqrt{1 + \tan^2\mu \left(1 - \frac{Nk\sin^6\mu + P_\chi}{kP_\tau \sin^2\mu}\right)^2}$$

La solución de energía mínima en este caso no satisface el límite BPS (depende sólo de uno de los momentos). Sin embargo, en el caso maximal, las direcciones χ y τ son paralelas, y $P_\tau = N + P_\chi/k$.

Esta solución magnética, en IIA, se corresponde a una NS5 enrollada en una 5-esfera retorcida con carga de brana D0 (P_τ) y momento P_χ. Solo es BPS en el caso maximal.

El siguiente paso ha sido intercambiar las direcciones τ y χ. Tomamos una M5 enrollando τ y las coordenadas de CP^2, propagándose en χ. Al reducir a IIA, esta configuración se convierte en una D4 enrollada en un CP^2 aplastado, propagándose en χ. Pero, de nuevo, no es gravitón gigante excepto en el caso maximal. El Hamiltoniano que encontramos es:

$$H = \frac{P_\chi}{R\sin\mu} \sqrt{1 + \tan^2\mu \left(1 - \frac{N\sin^4\mu + kP_\tau}{P_\chi}\right)^2}$$

Observando el denominador fuera de la raiz se observa que solo existen solucions BPS en el caso maximal ($\mu = \pi/2$). En tal caso, $P_\chi = N + kP_\tau$. Recuperamos el gravitón gigante usual cuando $P_\tau = 0$. En la reducción a IIA la 11ª dirección se reduce a un punto, y la carga N no puede interpretarse como carga de momento. La configuración se interpreta como un dibarión con energía N/L.

Damos una descripción microscópica de las configuraciones anteriores en términos de gravitones expandiéndose mediante efecto Myers (efecto dieléctrico) en una brana M5 fuzzy. Esta es la descripción microscópica de la brana M5 enrollando subespacios clásicos de S^7/Z_k con momento angular. En nuestra construcción este espacio es una S^5 fuzzy definida como un fibrado S^1 sobre un CP^2 fuzzy. Esta dirección S^1 es crucial para encontrar el acoplo dieléctrico que causa la expansión de los gravitones. Eligiendo la dirección 11-dimensional como dirección de movimiento y la dirección isométrica como la de enrollamiento, se llega a un hamiltoniano que concuerda con el resultado macroscópico para un número de gravitones muy grande. Si intercambiamos las direcciones de propagación y enrollamiento obtenemos la descripción microscópica del graviton gigante maximal S^5/Z_k (en este caso, tenemos gravitones expandiéndose en una S^5/Z_k fuzzy). Concuerda con el resultado macroscópico.

La reducción a IIA (en $AdS_4 \times CP^3$) del gravitón gigante 5-esférico da lugar a una brana NS5 expandiéndose en una 5-esfera retorcida dentro de CP^3. El momento en la 11ª dirección se traduce en carga de brana D0, y la NS5 es estática. El Hamiltoniano resultante es (M es el momento conjugado al escalar que se obtiene al reducir la 11ª dimensión):

$$H = \frac{k}{L} M \sqrt{1 + \tan^2\mu \left(1 - \frac{N}{M}\sin^4\mu\right)^2}$$

El gravitón gigante tiene energía $E = kM/L$, es decir, una configuración BPS de M branas D0 con energía k/L. Se satisface además $M \leq N$, realizando así el principio de exclusión de cuerdas en términos de branas D0 expandiéndose en una 5-esfera retorcida dentro de CP^3. En el caso maximal, $M = N$ y la energía puede interpretarse como un estado ligado de N monopolos de 't Hooft con energía k/L, o bien como un estado ligado de k dibariones con energía N/L. En este caso la 5-esfera retorcida se reduce a CP^2, y la NS5 colapsa a una D4 enrollando CP^2. Esto es debido a que en el caso maximal las direcciones τ y χ son paralelas, de forma que la M5 enrollada en χ (que es una NS5 en IIA) y la M5 enrollada en τ (una D4 en IIA) son equivalentes.

Podemos por tanto construir el gravitón gigante maximal en IIA como una D4 enrollada en CP^2 con momento N. Construimos explícitamente esta configuración, con momento P_χ, y obtenemos el Hamiltoniano:

$$H = \frac{P_\chi}{L\sin\mu} \sqrt{1 + \tan^2\mu \left(1 - \frac{N}{P_\chi}\sin^4\mu\right)^2}$$

Solo el caso maximal es BPS, y en tal caso $P_\chi = N$ y $H = N/L$. El espacio en que se enrolla la brana se reduce a un CP^2 y el gravitón es en IIA un dibarión. Este análisis muestra que el dibarión aparece en el lado gravitatorio como caso límite de una brana D4 enrollando el CP^2 aplastado.

Finalmente, proporcionamos la descripción microscópica de estas configuraciones, a partir de la acción para gravitones coincidentes en IIA o branas D0 (ya existente en la literatura) demostrando la concordancia de este resultado con las descripciones macroscópicas.

3.3. Vértices bariónicos en backgrounds con menos supersimetría y/o confinantes

Construimos en primer lugar el vértice bariónico a partir de una brana D5 enrollada en Y_5 (espacio Einstein) dentro de $AdS_5 \times Y_5$. El resultado es totalmente similar al caso $AdS_5 \times S^5$ ya analizado en la literatura. Comprobamos que existen estados no singlete también en este caso, estables, con un número de cuerdas (y por tanto de quarks) $5N/8 < k \leq N$.

A continuación construimos el vértice bariónico en $AdS_5 \times T^{1,1}$ con flujo magnético $F = \mathcal{N}_1 J_1 + \mathcal{N}_2 J_2$. Este flujo disuelve en la configuración carga de brana D3 y D1:

$$S_{D5}^{CS} = \frac{\mathcal{N}_1}{3} \int C_4 + \frac{\mathcal{N}_2}{3} T_2 \int C_4$$
$$S_{D5}^{CS} = \frac{\mathcal{N}_1 \mathcal{N}_2}{9} T_1 \int C_2$$

Analizamos la existencia de soluciones tipo vértice bariónico, y encontramos que el límite para el número de cuerdas permitidas en la configuración coincide con el encontrado para el background $AdS_5 \times S^5$:

$$a > \frac{5}{8}$$

El estudio de la estabilidad de esta configuración muestra un límite aún más restrictivo, igual que ocurrió en la construcción de vértices bariónicos en ABJM. Encontramos $a > 0{,}813$.

Analizamos a continuación el vértice bariónico en backgrounds β-deformados. En este caso, la D5 está enrollada en una S^5 deformada. Captura un flujo $F_5 - F_3 \wedge B_2$ y desarrolla un tadpole que debe cancelarse con N cuerdas:

$$S_{D5}^{CS} = 2\pi T_5 \int P[C_4 - C_2 \wedge B_2] \wedge F = -N \int dt A_t$$

Es decir, en la parte CS de la acción no vemos ningún efecto de la deformación. En la parte DBI tampoco se observa efecto de la deformación. Tampoco se modifican la energía de ligadura, ni los estudios de estabilidad, que resultan igual que en el caso $\mathcal{N} = 4$.

Para analizar los vértices bariónicos en backgrounds con menos supersimetría y confinantes, elegimos el background de Maldacena-Núñez, que es solución de supergravedad tipo IIB dual a una teoría gauge confinante con supersimetría $\mathcal{N} = 1$. En este caso, encontramos resultados más restrictivos respecto a los backgrounds analizados hasta ahora.

En concreto, enrollamos una brana D3 en una 3-esfera contenida en el background. Esta configuración introduce un tadpole que debe cancelarse con N cuerdas fundamentales:

$$S_{D3}^{CS} = 2\pi T_3 \int C_2 \wedge F = -N \int dt A_t$$

Calculamos la acción DBI de la D3 y a partir de aquí el tamaño y la energía de ligadura de la configuración. Estos resultados dependen explícitamente de las características propias del background de Maldacena-Núñez a través de una función $\Lambda(\rho)$ asociada al dilatón (ρ es la coordenada radial en el bulk):

$$e^{2\phi} = \frac{\Lambda\rho}{g_s N}$$

Conocemos mediante el background cuál es la forma explícita de esta función $\Lambda(\rho)$ cuando $\rho >> 1$ y cuando $\rho << 1$. Esto nos permite analizar el resultado de la energía de ligadura en el UV. Analizando la condición de fuerza neta nula sobre la brana, encontramos como límite para el número de quarks:

$$a > \frac{3}{4}$$

Así, en este caso el límite inferior mejora respecto a otros backgrounds. Pero, a diferencia de aquellos, en este caso el estudio de la estabilidad no restringe aún más este límite.

Como en el resto de la tésis, completamos la descripción macroscópica (con flujo magnético) de la configuración con la descripción microscópica complementaria, aprovechando el hecho de que introducir flujo magnético en las configuraciones disuelve carga de branas de menores dimensiones. Esta descripción está dada también en términos de branas de menores dimensiones expandiéndose en bariones fuzzy. El interés de esta descripción es explorar la región de acoplo de t' Hooft finito.

En todos los backgrounds, se parte de la acción de branas de menores dimensiones expandiéndose en subespacios fuzzy. En cada caso el resultado para la parte DBI y la parte CS coincide con el cálculo macroscópico cuando el número de branas que se expanden es muy grande:

- Para el background $AdS_5 \times T^{1,1}$, partimos de n branas D1 coincidentes que se expanden en un espacio $S^2 \times S^2$ fuzzy.
- Para los backgrounds β-deformados, partimos de la acción para branas D3 expandiéndose en un S^2 fuzzy.
- Para el background de Maldacena-Núñez, partimos de un sistema de branas D1 expandiéndose en un S^2 fuzzy.

Si bien estos resultados son completamente satisfactorios, no son completos. Una descripción completa de las branas D3 o D5 más el sistema de cuerdas F1 válido para acoplo de t'Hooft finito requeriría construir spikes fuzzy, de forma que las correcciones α' provenientes de las cuerdas pudieran ser tenidas en cuenta.

Capítulo 4

Conclusiones

Hemos construido y analizado varias configuraciones de branas tipo partícula cargadas magnéticamente en ABJM. Hemos mostrado que las configuraciones tipo vértice bariónico, monopolo de 't Hooft y dibarión con carga magnética y número reducido de quarks, no solo son soluciones clásicas de las ecuaciones del movimiento, sino que además son estables bajo fluctuaciones.

Para el flujo magnético inducido en la configuración, encontramos que debe satisfacer cierto límite superior (e inferior, en el caso del dibarión), de forma que en tal límite se llega a un valor mínimo para el número de quarks, que es función de $\mathcal{N}$ (a través de la función β):

$$l \geq \frac{q}{2}(1 + \sqrt{1 - \beta^2})$$

Para el caso $\beta = 0$, el valor de la energía es mínimo y el número de quarks está máximamente reducido.

El análisis de la estabilidad restringe aún más este límite:

$$l \geq \frac{q}{1 + \gamma_c}(1 + \sqrt{1 - \beta^2})$$

con $\gamma_c = 0{,}538$.

El análisis anterior es macroscópico, en el sentido de basarse en la aproximación de probe brane, válido en el límite de supergravedad $k << N$. Hemos completado esta descripción con la complementaria microscópica, usando el hecho de que podemos disolver de forma consistente branas D0 en las configuraciones. De esta forma, la descripción microscópica alternativa viene dada en términos de branas D0 expandiéndose en espacios $CP^{p/2}$ fuzzy. Esto nos permite explorar la región de acoplo de 't Hooft finito. Esta expansión es un efecto dieléctrico puramente gravitatorio (involucra la parte DBI de la acción). La acción CS es la que nos indica la necesidad de introducir un número de cuerdas fundamentales para cancelar los tadpoles y construir la configuración de vértice bariónico. El análisis microscópico preliminar apunta a la existencia de este tipo de configuraciones para acoplo de 't Hooft finito.

En el análisis de estas configuraciones, como resultado extra y de por sí interesante, encontramos acoplos dieléctricos de mayor curvatura que no han sido considerados previamente en la literatura. Estos acoplos son necesarios para obtener la carga correcta del vértice bariónico en la descripción de múltiples branas D0. Mediante dualidad T es posible predecir este tipo de acoplos para branas Dp, con $p > 0$.

Hemos extendido estas configuraciones también a teorías con menor supersimetría y/o confinantes. En concreto, hemos discutido vértices bariónicos no singlete en varios backgrounds IIB para observar cómo se veía modificada la dependencia del límite impuesto en el número de quarks con la supersimetría y propiedades de confinamiento de la teoría gauge dual.

En la aproximación probe brane, hemos demostrado que este límite es el mismo para todos los espacios de tipo $AdS_5 \times Y_5$ con Y_5 un espacio Einstein con flujo de 5-forma, independientemente del número de supersimetrías preservadas. También ocurre así para backgrounds multi β-deformados. Esto señala un comportamiento universal basado en conformalidad.

El mismo análisis, para un background confinante (Maldacena-Núñez) muestra que esta universalidad se pierde. Pero aún existen configuraciones no singlete, aunque el límite en el número de quarks permitidos es más restringido que en otros backgrounds (esto confirma la expectativa de que teorías más realistas deben contener bariones con menor carácter no singlete). Aunque en este caso, utilizando la aproximación probe brane, se rompen todas las supersimetrías, en la literatura ya se han analizado este tipo de configuraciones en el caso de que no se rompan y no se obtuvieron cambios significativos.

Además, en el caso de backgrounds multi β-deformados, no encontramos dependencia de la configuración en el parámetro de deformación.

Igual que hicimos con ABJM, también para estos backgrounds proporcionamos una descripción microscópica complementaria en términos de branas de menores dimensiones expandiéndose en vértices bariónicos fuzzy.

Sin embargo, si bien la descripción microscópica (para todos los casos) nos permite explorar la región de acoplo de 't Hooft finito (de ahí su interés), no podemos concluir que existan estos bariones no singlete en estas regiones. Deberíamos tener en cuenta no solo las correcciones α' que provienen del análisis microscópico de la brana, sino también las correcciones α' a la acción Nambu-Goto de las cuerdas fundamentales y las correcciones al background. Esto significaría analizar soluciones tipo spike en estos backgrounds.

Completamos la descripción de configuraciones en ABJM que permiten (mediante su descripción microscópica en términos de expansión dieléctrica gravitacional) explorar la región de acoplo de 't Hooft finito, construyendo gravitones gigantes a partir de branas M5 expandiéndose en $AdS_4 \times S^7/Z_k$ y discutiendo su realización al reducir a IIA.

La primera configuración es una M5 enrollando una S^5 propagándose en la dirección S^1/Z_k. En IIA se reduce a una NS5 estática expandiéndose en una 5-esfera retorcida dentro de CP^3. El estado fundamental es una configuración de branas D0 gigantes que satisfacen el principio de exclusión de cuerdas.

La segunda configuración analizada es la extensión de la anterior incluyendo un momento adicional a lo largo de la dirección isométrica, para obtener así en IIA un objeto no estático. Solo el caso maximal es BPS y el momento adicional añadido permite al gravitón moverse con momento angular arbitrario.

La última configuración considerada es una M5 cuya dirección isométrica es S^1/Z_k, propagándose tanto en esta dirección como en la fibra S^1 de la 5-esfera. El estado fundamental es un gravitón gigante con tamaño máximo enrollando S^5/Z_k subespacio de S^7/Z_k. Al reducir a IIA obtenemos una brana D4 enrollada en un CP^2 aplastado dentro de CP^3 propagándose a lo largo de la dirección de la fibra S^1. El estado fundamental es de nuevo el gravitón maximal. Esta configuración se interpreta como un dibarión.

En las configuraciones de gravitón gigante, hemos considerado branas enrollando espacios CP^2 y por tanto sujetos a la anomalía de Freed-Witten. Estas branas llevan por tanto flujo magnético semientero en su worldvolume, que debe compensarse con un campo B_2 adicional. Este campo es el introducido también en los vértices bariónicos y que condujo a la necesidad de introducir acoplos de mayor curvatura, no discutidos antes en la literatura.

Hemos descrito estos gravitones gigantes microscópicamente, de igual forma que los vértices bariónicos, en términos de branas D0 expandiéndose en espacios fuzzy.

Sin embargo, no hemos encontrado un gravitón gigante expandiéndose en el CP^3 en $AdS_4 \times CP^3$, aparte del dibarión. La brana D4 enrollando el CP^2 aplastado dentro de CP^3 con momento no nulo está descrita por un Hamiltoniano muy parecido al que describe las soluciones de gravitón gigante en otros backgrounds. Sin embargo, el estado fundamental corresponde al gigante maximal, es decir, el dibarión. Es posible que induciendo momento angular en la NS5 o la D4 de forma que se pueda construir un gravitón gigante dual giratorio, sea posible obtener el gravitón gigante en $AdS_4 \times CP^3$.

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ABJM baryon stability and Myers effect

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Abstract: We consider magnetically charged baryon vertex like configurations in $AdS_4 \times CP^3$ with a reduced number of quarks l. We show that these configurations are solutions to the classical equations of motion and are stable beyond a critical value of l. Given that the magnetic flux dissolves D0-brane charge it is possible to give a microscopical description in terms of D0-branes expanding into fuzzy CP^n spaces by Myers dielectric effect. Using this description we are able to explore the region of finite 't Hooft coupling.

Keywords: Gauge-gravity correspondence, D-branes, Bosonic Strings, Space-Time Symmetries

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Contents

1 Introduction

The AdS_4/CFT_3 duality relates the Type IIA superstring on $AdS_4 \times CP^3$ to an $\mathcal{N} = 6$ quiver Chern-Simons-matter theory with gauge group $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ known as the ABJM model [1]. Like its AdS_5/CFT_4 counterpart it is a strong/weak coupling duality, with 't Hooft coupling $\lambda = N/k$. Being the superpotential coupling proportional to k^{-2}, an appropriate large k limit $N \ll k^5$ allows for a weak coupling regime. The Type IIA theory is then weakly curved when $k \ll N$.

The CP^3 space has $H^q(CP^3) = \mathbb{R}$ for even q. Therefore it is possible to have D2, D4 and D6 particle-like branes wrapping a topologically non-trivial cycle. In $AdS_4 \times CP^3$ these branes were already discussed in [1], and their interpretation in the context of the CFT dual given. The D6-brane wrapped on the entire CP^3 is the analogous of the baryon vertex in $AdS_5 \times S^5$ discussed by Witten in [2]. Due to the F_6 flux of the background it has a tadpole that has to be cancelled with N fundamental strings ending on it, which correspond to N external quarks on the boundary of AdS_4. Similarly, the D2-brane wrapped on a $CP^1 \subset CP^3$ captures the F_2 flux of the $AdS_4 \times CP^3$ background, and develops a tadpole that has to be cancelled with k fundamental strings ending on it. The field theory interpretation of this brane is as a 't Hooft monopole, realized as a Sym_k product of Wilson

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lines. The D4-brane wrapped on a $CP^2 \subset CP^3$ does not capture any of the background fluxes, and it is gauge invariant. It is dual to the di-baryon operator [3, 4], which has the same baryon charge and dimension to agree with the gravity result.

These gravitational configurations admit a natural generalization by allowing non-trivial worldvolume gauge fluxes [5]. These generalizations have been proposed as candidates for holographic anyons [6] in ABJM [7], and are therefore of potential interest for AdS/CMT applications. Allowing for a non-trivial worldvolume magnetic flux has the effect of adding lower dimensional brane charges to the configurations, in particular D0-brane charge. This modifies how the brane captures the background fluxes in a way that depends on the induced charges, such that, in some cases, additional fundamental strings are required to cancel the worldvolume tadpoles. The D2 and D6-branes are only stable if the induced charges lie below some upper bound. In turn, the D4-brane with flux behaves quite differently from the zero charge case, since it now requires fundamental strings ending on it. Given that in the presence of a non-trivial magnetic flux all these branes require fundamental strings ending on them we will loosely refer to them as baryon vertex like configurations.

In this paper we further generalize these constructions by reducing the number of strings that stretch between the brane and the boundary of AdS_4, i.e. the number of quarks. It was shown in [8, 9] that in $AdS_5 \times S^5$ perfect baryon vertex classical solutions to the equations of motion exist for a number of quarks l satisfying $5N/8 \leqslant l \leqslant N$. Although one would expect that bound states of quarks should be singlets of the gauge group the analysis of the stability against fluctuations confirms that the configurations are stable for a number of quarks $0.813N \leqslant l \leqslant N$ [10]. It is likely that this will not be the case in other theories with reduced supersymmetry.

It is one aim of this paper to perform a similar analysis for magnetically charged baryon vertex like configurations with reduced number of quarks in $AdS_4 \times CP^3$. Our analysis will reveal that also in this case baryon vertex like classical solutions exist that are moreover stable against fluctuations.

In order to be able to use the probe brane approximation in the study of the dynamics we will consider a uniform distribution of strings on a $CP^{\frac{p}{2}}$ geometrical shell, with $p = 2, 4, 6$. This will be our particular profile for the distribution of quarks inside the baryon vertex configuration. Although this choice completely breaks supersymmetry we will be able to ignore the strings backreaction [9, 11, 12].

The fact that the magnetized branes have dissolved D0-branes in their worldvolumes hints at the existence of a microscopical description in terms of non-Abelian n D0-branes polarizing due to Myers dielectric effect [13]. This description allows to explore the configurations in the region where $N \ll n^{\frac{4}{p}}\, k$, and is therefore complementary to the supergravity description in terms of probe branes. We will see that classical stable solutions still exist in this regime. Moreover, we will show that the flat half-integer B_2 field that is required by the Freed-Witten anomaly in the di-baryon [14] has to be introduced already at the classical level so that a CP^2 non-spin manifold can be recovered in the large n limit.

The organization of the paper is as follows: We start in section 2 by summarizing some of the properties of the magnetized baryon vertex like configurations constructed in [5]. In section 3 we reduce the number of quarks and find the values for which classical configurations still exist. In section 4 we perform the stability analysis under small fluctuations. Section 5 is devoted to the microscopical description. This description will confirm the existence of non-singlet classical stable solutions when $N \ll k^5$. An interesting output of this analysis will be the derivation of new higher curvature dielectric couplings not predicted before in the literature. Finally, in section 6 we summarize our results and discuss further directions. We have written appendix A, containing a number of useful results on the $AdS_4 \times CP^3$ background and also appendix B with the computation of the Kähler form for the fuzzy $CP^{\frac{p}{2}}$, used in the main text.

2 Magnetically charged baryon vertex configurations in $AdS_4 \times CP^3$ spaces

It was shown in [5] that it is possible to construct more general monopole, di-baryon and baryon vertex configurations in $AdS_4 \times CP^3$ if the particle like branes carry lower dimensional brane charges induced by a non-trivial magnetic flux $F = \mathcal{N} J$, where J is the Kähler form of the CP^3. For the D2 and D6 branes the effect of the magnetic flux is to allow the construction of similar monopole and baryon vertex configurations with D0-brane charge and a different number of fundamental strings attached. Indeed the study of the dynamics reveals that the configurations are stable if the magnetic flux does not exceed some maximum value, for which the configurations reduce to radial fundamental strings (free quarks) plus the wrapped D-brane.

The di-baryon is more substantially modified by the presence of the magnetic flux, capturing the F_2 flux and developing a tadpole. In this case the study of the dynamics shows that the D4-brane with the fundamental strings attached is stable if the magnetic flux takes values in a given interval, at the limits of which the configuration ceases to be stable and reduces to free quarks plus the D4-brane. This is consistent with the fact that the D4-brane with F-strings does not exist for zero magnetic flux. Moreover, since the D4-brane wraps a non-spin manifold it must carry a half-integer worldvolume magnetic flux due to the Freed-Witten anomaly [15]. In order to still keep its dual interpretation as a di-baryon it was proposed in [14] that a flat half-integer B_2-field should be switched on in the dual background in order to cancel the contribution of the Freed-Witten worldvolume magnetic flux.

A question that remained open after the study in [5] was the interpretation of the magnetized Dp-branes in the field theory. A difficulty comes from the expected lack of SUSY for the D2 and D6-branes. In turn the D4-brane with flux forms a threshold BPS intersection with the D0-branes. Therefore one could expect that a supersymmetric spiky solution exists and one could give an interpretation to the bounds in the gauge theory dual. As shown in [5] the maximum (and minimum, if applicable) values of the magnetic flux are functions of $\sqrt{\lambda}$, with λ the 't Hooft coupling, for all branes. This suggests an origin on the conformal symmetry of the gauge theory. Ultimately one would expect a connection between the existence of these bounds and the stringy exclusion principle of [16].

We summarize next the energies and charges carried by the various branes. In order to set up the notation a short review of the $AdS_4 \times CP^3$ background is given in appendix A. We will use Poincaré coordinates to parameterize AdS_4 throughout the paper.

2.1 Charges and energies

The computation of the energy of a Dp-brane in $AdS_4 \times CP^3$ wrapped on a $CP^{\frac{p}{2}}$ cycle of the CP^3 with $p = 2, 4$ and 6, in the presence of a magnetic flux $F = \mathcal{N}J$, with $\mathcal{N} \in 2\mathbb{Z}$, was done in [5]. We review this result and show that the equations of motion are satisfied for $F = \mathcal{N}J$.

The DBI action is

$$S_p = -T_p \int d^{p+1}\xi\, e^{-\phi} \sqrt{|\det(P[g + 2\pi\mathcal{F}])|}\,, \qquad T_p = \frac{1}{(2\pi)^p}\,, \tag{2.1}$$

where $\mathcal{F} = F + \frac{1}{2\pi}B_2$ and we set $\ell_s = 1$. The equations of motion arising from varying the gauge potential are given by

$$\partial_\alpha \left(\sqrt{|\det P([g + 2\pi\mathcal{F}])|}\ (P[g + 2\pi\mathcal{F}])^{-1[\alpha\beta]} \right) = 0\,, \tag{2.2}$$

where $[\alpha\beta]$ denotes the antisymmetric part. Identifying the world-volume coordinates with the angles of the various CP-cycles as indicated in appendix A and considering static solutions independent of the ξ^i's we find an induced metric

$$ds^2_{\text{ind}} = -\frac{16\rho^2}{L^2} d\tau^2 + L^2 ds^2_{CP^{\frac{p}{2}}}\ . \tag{2.3}$$

Using that in our case $\mathcal{F}$ is proportional to the Kähler form, since $F = \mathcal{N}J$ and $B_2 = -2\pi J$, we can easily prove that the equations of motion are satisfied. If M is an antisymmetric $p \times p$ matrix satisfying $M^2 = -c\,\mathbb{I}$, where for consistency $c = -\frac{1}{p}\text{Tr}(M^2)$, one can show that $(\mathbb{I} + M)^{-1} = \frac{\mathbb{I} - M}{1+c}$, and, moreover, due to the fact that M is antisymmetric: $\det(\mathbb{I} + M) = (1 + c)^{\frac{p}{2}}$. Using these identities we find that $\partial_\alpha(\sqrt{g}J^{\alpha\beta}) = 0$, where g is the metric on $CP^{\frac{p}{2}}$, or equivalently $\nabla_\alpha J^{\alpha\beta} = 0$. The latter is the condition for having a Kähler manifold and therefore it is automatically satisfied. Also we find that the DBI action is given by (we use $c = (2\pi\mathcal{N})^2$)

$$S^{Dp}_{DBI} = -\frac{T_p}{g_s} \int d^{p+1}\xi \sqrt{-\det(g + 2\pi\mathcal{F})} = -Q_p \int d\tau \frac{2\rho}{L}\,, \tag{2.4}$$

where

$$Q_p = \frac{T_p}{g_s}\, \text{Vol}(CP^{\frac{p}{2}})\ \left(L^4 + (2\pi)^2(\mathcal{N} - 1)^2\right)^{\frac{p}{4}}\,, \qquad \text{for} \quad p = 2, 6 \tag{2.5}$$

and

$$Q_4 = \frac{T_4}{g_s}\, \text{Vol}(CP^2)\ \left(L^4 + (2\pi\mathcal{N})^2\right)\,, \tag{2.6}$$

since in this case B_2 cancels the contribution of the Freed-Witten vector field, such that $\mathcal{F} = F_{FW} + \mathcal{N}J + \frac{1}{2\pi}B_2 = \mathcal{N}J$. Also, in this case

$$S^{D4}_{DBI} = -\frac{T_4}{g_s} \int d^5\xi \sqrt{-\det(g + 2\pi\mathcal{F})} = -\frac{T_4}{g_s} \int d^5\xi\, \sqrt{|g_{tt}|}\sqrt{g_{\mathbb{P}^2}} \left(L^4 + 2(2\pi)^2 \mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}\right) \tag{2.7}$$

The volume of the $CP^{\frac{p}{2}}$ is given by

$$\text{Vol}(CP^{\frac{p}{2}}) = \frac{\pi^{\frac{p}{2}}}{(\frac{p}{2})!}\,. \tag{2.8}$$

From (2.5) and (2.6) it is clear that $\mathcal{N}^2$ is comparable to $L^4 \gg 1$.

Analyzing the Chern-Simons actions one can also show that the magnetic flux has the effect of dissolving lower dimensional brane charge in the Dp-branes. For instance the D4-brane has D2 and D0-brane charges dissolved, as can be seen from the couplings:

$$S_{CS}^{D4} = 2\pi\, T_4 \int_{\mathbb{R}\times\mathbb{P}^2} C_3 \wedge F = \frac{\mathcal{N}}{2}\, T_2 \int C_3 \tag{2.9}$$

and

$$S_{CS}^{D4} = \frac{1}{2}(2\pi)^2\, T_4 \int_{\mathbb{R}\times\mathbb{P}^2} C_1 \wedge F \wedge F = \frac{\mathcal{N}^2}{8}\, T_0 \int_{\mathbb{R}} C_1\,, \tag{2.10}$$

respectively. In general the number of Ds-branes dissolved in the worldvolume of a Dp is given by [5]

$$n = \frac{\mathcal{N}^{\frac{p-s}{2}}}{2^{\frac{p-s}{2}}\,(\frac{p-s}{2})!}\,. \tag{2.11}$$

Both the D4 and D6-branes have CP^1 D2-branes dissolved. Therefore in the presence of a magnetic flux they capture the F_2 flux and develop a tadpole with charge

$$q = k\,\frac{\mathcal{N}^{\frac{p}{2}-1}}{2^{\frac{p}{2}-1}(\frac{p}{2}-1)!} \tag{2.12}$$

More explicitly, for the D4-brane we have that

$$\begin{aligned} S_{CS}^{D4} &= \frac{1}{2}(2\pi)^2\, T_4 \int_{\mathbb{R}\times\mathbb{P}^2} P[F_2] \wedge F \wedge A = 2\,(2\pi)^2 T_4\, k\mathcal{N} \int_{\mathbb{R}\times\mathbb{P}^2} J \wedge J \wedge A \\ &= k\,\frac{\mathcal{N}}{2}\, T_{F1} \int dt A_t \end{aligned} \tag{2.13}$$

The analogous coupling for the D6-brane is

$$S_{CS}^{D6} = \frac{1}{6}(2\pi)^3\, T_6 \int_{\mathbb{R}\times\mathbb{P}^3} P[F_2] \wedge F \wedge F \wedge A = k\,\frac{\mathcal{N}^2}{8}\, T_{F1} \int dt A_t\,. \tag{2.14}$$

Note however that for the D6-brane the couplings $\int_{D6} F_2 \wedge B_2 \wedge B_2 \wedge A$ and $\int_{D6} F_2 \wedge F \wedge B_2 \wedge A$ in its CS action contribute as well to its k charge. In the absence of magnetic flux it was shown in [14] that the contribution from $\int_{D6} F_2 \wedge B_2 \wedge B_2 \wedge A$ is cancelled from the higher curvature coupling [17–19]

$$S_{h.c.}^{D6} = \frac{3}{2}(2\pi)^5\, T_6 \int C_1 \wedge F \wedge \sqrt{\frac{\hat{\mathcal{A}}(T)}{\hat{\mathcal{A}}(N)}}\,, \tag{2.15}$$

where $\hat{\mathcal{A}}$ is the A-roof (Dirac) genus

$$\hat{\mathcal{A}} = 1 - \frac{\hat{p}_1}{24} + \frac{7\,\hat{p}_1^2 - 4\,\hat{p}_2}{5760} + \cdots \tag{2.16}$$

and the Pontryagin classes are written in terms of the curvature of the corresponding bundle as

$$\hat{p}_1 = -\frac{1}{8\pi^2}\,\mathrm{Tr}\,R^2\,, \qquad \hat{p}_2 = \frac{1}{256\,\pi^4}\Big((\mathrm{Tr}\,R^2)^2 - 2\,\mathrm{Tr}\,R^4\Big)\,. \tag{2.17}$$

This charge cancellation is consistent with the dual interpretation of the D6-brane as a baryon vertex. For a non-vanishing magnetic flux the term $\int_{D6} F_2 \wedge F \wedge B_2 \wedge A$ contributes however with $-k\mathcal{N}/4$ units of F-string charge, as shown in [5]. Therefore, adding the N units induced by the F_6 flux,

$$S^{D6}_{CS} = 2\pi\,T_6 \int_{R\times\mathbb{P}^3} P[F_6]\wedge A = N\,T_{F1}\int dt A_t\,, \tag{2.18}$$

not captured by the other branes, we find that the total F-string charge carried by the D6-brane is given by

$$q_{D6} = N + k\,\frac{\mathcal{N}(\mathcal{N}-2)}{8} \tag{2.19}$$

Note that this is always an integer due to the quantization condition

$$\frac{1}{2\pi}\int F = \frac{\mathcal{N}}{2} \in \mathbb{Z} \tag{2.20}$$

3 Varying the number of fundamental strings

It was shown in [8, 9] that the baryon vertex in $AdS_5 \times S^5$ can be generalized such that the number of quarks l lies in the interval $5N/8 \leqslant l \leqslant N$. These configurations are not only perfect classical solutions to the equations of motion but for $0.813\,N \leqslant l \leqslant N$ are stable against fluctuations [10]. In this section we generalize the construction in [8, 9] to the baryon vertex like configurations discussed in the previous section. We will see that in all cases there exist configurations with a reduced number of quarks that are solutions to the classical equations of motion.

We consider a classical configuration consisting on a Dp-brane wrapped on $CP^{\frac{p}{2}}$, located at $\rho = \rho_0$, l strings stretching from ρ_0 to the boundary of AdS_4 and $(q-l)$ straight strings that go from ρ_0 to 0. The configuration is depicted in figure 1. Further, we switch on the magnetic flux $F = \mathcal{N}J$, with J the Kähler form of the CP^3. Taking the gauge $\tau = t$, $\sigma = \rho$ for the worldsheet coordinates of the string, the Nambu-Goto action of the l fundamental strings is given by [20]

$$S_{lF1} = -l\,T_{F1}\int dt d\rho\,\sqrt{1 + \frac{16\rho^4}{L^4}r'^2} \tag{3.1}$$

where r is the radius of the configuration at the boundary of AdS_4. The equations of motion then reduce to

$$\frac{16\rho^4 r'}{L^4\sqrt{1+\frac{16\rho^4}{L^4}r'^2}} = c = \frac{4\rho_1^2}{L^2} \tag{3.2}$$

where the constant has been fixed demanding that $r' = \infty$ at the turning point of each string, ρ_1. The turning point is such that $0 \leqslant \rho_1 \leqslant \rho_0$. From (3.2)

$$r' = \frac{L^2\rho_1^2}{4\rho^2\sqrt{\rho^4-\rho_1^4}} \equiv r'_{\rm cl} \tag{3.3}$$

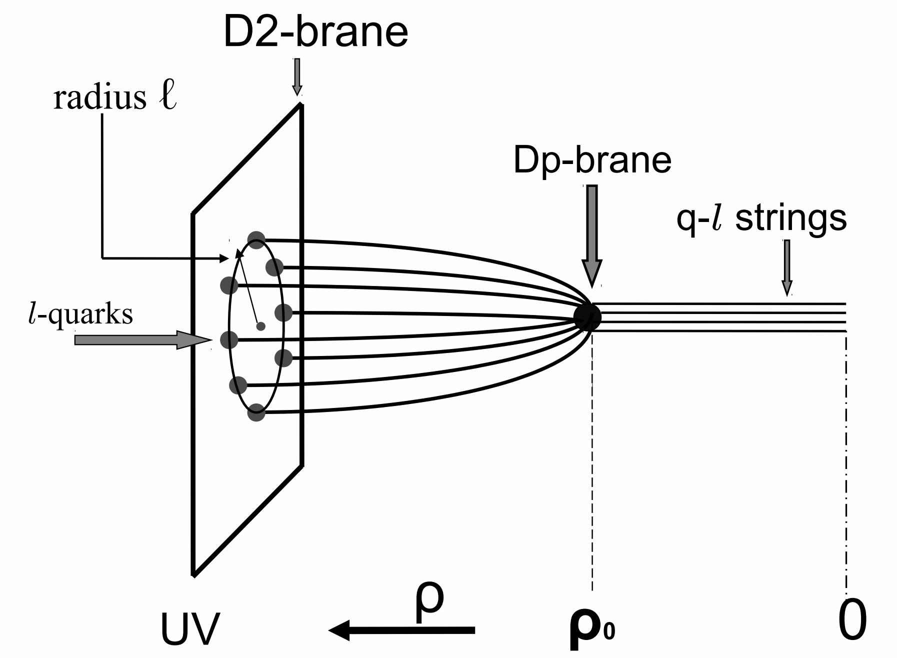

Figure 1. A baryon configuration with l-external quarks placed on a circle of radius ℓ at the boundary of AdS space, each connected to a Dp-brane wrapped on a $CP^{\frac{p}{2}}$ located at $\rho = \rho_0$, and $q - l$ straight strings ending at 0.

Defining $a \equiv \frac{l}{q}$, the boundary equation reads

$$\frac{1}{a}\sqrt{1-\beta^2} + \frac{1-a}{a} = \sqrt{1 - \frac{\rho_1^4}{\rho_0^4}} \tag{3.4}$$

where we defined [5]

$$\sqrt{1-\beta^2} \equiv \frac{2Q_p}{L\, q\, T_{F_1}} . \tag{3.5}$$

We then must have

$$\frac{2Q_p}{L\, q\, T_{F_1}} \leqslant 1 \tag{3.6}$$

in order to find a stable configuration. Since Q_p (and also q, for the D4 and D6-branes), are functions of $\mathcal{N}$ this condition imposes a bound on the magnetic flux that can be dissolved on the worldvolume. For the D2 and D6-branes $\mathcal{N}$ must lie below some upper bound, for which $\beta = 0$. For the D4 the magnetic flux must also lie above a lower bound, for which $\beta = 0$ as well. This is consistent with the fact that the D4-brane with fundamental strings attached only exists for non-zero magnetic flux.

For the values of the magnetic flux allowed by equation (3.6) we must still fulfill the boundary equation (3.4), and this implies that

$$q\sqrt{1-\beta^2} + q - l \leqslant l \qquad \Leftrightarrow \qquad l \geqslant \frac{q}{2}(1 + \sqrt{1-\beta^2}) = l_{\min} \tag{3.7}$$

This condition determines the minimum value of strings that can form the baryon vertex like configuration. Note that $l_{\rm min}$ is a function of the magnetic flux, and is such that it decreases with β. For the D2 and D6-branes β is maximum for zero magnetic flux, for which $l_{\rm min}$ reaches its minimum value: $l_{\rm min}=\frac{q}{2}(1+\frac{1}{2\pi})$, $l_{\rm min}=\frac{q}{2}(1+\frac{1}{6\pi})$, respectively. Recall that for this value of the magnetic flux the configuration is maximally stable [5]. For the D4-brane β is maximum when $\frac{\mathcal{N}}{L^2}=\frac{1}{2\pi}$, which also corresponds to the most stable configuration. For this value of the magnetic flux l/q is minimum[1], and one finds the maximum range of values allowed for l: $\frac{q}{2}(1+\frac{1}{2\pi})\leqslant l\leqslant q$. Again, this range is maximum for the most stable configuration. On the contrary, when $\beta=0$ we can only have $l=q$, and therefore it is not possible to reduce the number of quarks. For this value of the magnetic flux the strings are no longer bounded and the configurations reduce to q free quarks. Indeed, $\beta=0$, $l=q$ implies $\rho_1=\rho_0\to\rho'=\infty$, i.e. the fundamental strings become radial. Note as well that when $l=l_{\rm min}$ the strings become radial for any value of the magnetic flux. The conclusion is that the $(l,\mathcal{N})$ parameter space for which the classical configurations exist is bounded by those values corresponding to the free quarks case.

Equations (3.3) and (3.4) allow to calculate the radius of the configuration,

$$\ell=\frac{L^2\rho_1^2}{12\rho_0^3}\int_1^\infty\frac{dz}{z^2\sqrt{z^4-\frac{\rho_1^4}{\rho_0^4}}}=\frac{L^2\rho_1^2}{12\rho_0^3}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};\frac{\rho_1^4}{\rho_0^4}\right)\,,\qquad\frac{\rho_1^4}{\rho_0^4}=4\frac{l_{\rm min}}{l}\left(1-\frac{l_{\rm min}}{l}\right)\,,\tag{3.8}$$

where we have changed the integration variable as follows $z=\frac{\rho}{\rho_0}$ and ${}_2F_1(a,b,c;x)$ is a hypergeometric function. This expression has the same form than the size of the baryon vertex in $AdS_5\times S^5$ [8, 21] and the $q\bar{q}$ system [20, 22]. Note that the dependence on the location of the Dp-brane, ρ_0, and on L^2 is also the same. This is a non-trivial prediction of the AdS/CFT correspondence for the strongly coupled CS-matter theory. Note as well that (3.8) reduces to the expression found in [5] when $l=q$.

The total on-shell energy is in turn given by

$$\begin{aligned}E&=E_{Dp}+E_{lF1}+E_{(q-l)F1}\\&=l\,T_{F_1}\rho_0\left(\frac{q}{l}\sqrt{1-\beta^2}+\int_1^\infty dz\frac{z^2}{\sqrt{z^4-\frac{\rho_1^4}{\rho_0^4}}}+\frac{q-l}{l}\int_0^1 dz\right)\,.\end{aligned}\tag{3.9}$$

The binding energy can then be obtained by subtracting the (divergent) energy of the constituents. Note that, as we have discussed before, the free quarks configuration is degenerate, since it can be reached in three cases: when the Dp-brane is located at $\rho_0=0$ (at this location the energy of the Dp vanishes), as in [8], when $\beta=0$ ($\Leftrightarrow l=q$) and ρ_0 is arbitrary, and when $l=l_{\rm min}$, for any β and any ρ_0. In all these cases the constituents contribute with an energy $l\,T_{F1}\int_0^\infty d\rho$ and the binding energy is given by:

$$E_{bin}=l\,T_{F1}\,\rho_0\left\{-\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4};4\frac{l_{\rm min}}{l}\left(1-\frac{l_{\rm min}}{l}\right)\right)+2\frac{l_{\rm min}}{l}-1\right\}\,.\tag{3.10}$$

[1]Recall that in this case $q=k\mathcal{N}/2$.

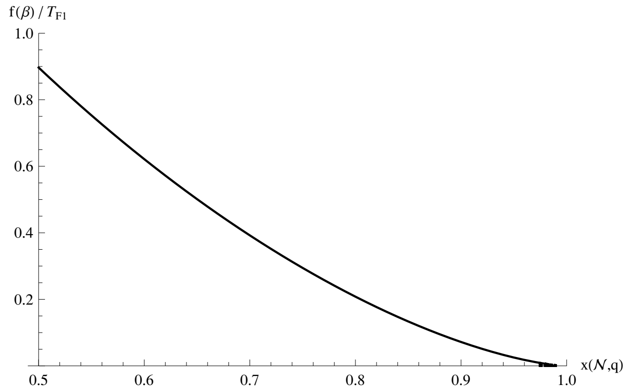

Figure 2. Positivity of $f(x)$ as a function of x

This expression has again the same form than the corresponding expressions in [5, 8, 20–22].[2] Setting $x = l_{\rm min}/l$ the configurations are maximally stable when x is minimum, i.e. when $l = q$ and β reaches its maximum value. This happens for zero magnetic flux for the D2 and D6-branes, and for $\frac{\mathcal{N}}{L^2} = \frac{1}{2\pi}$ for the D4.

From (3.8) and (3.10) we have that for all l and $\mathcal{N}$ the binding energy of the baryon reads

$$E_{bin} = -f(x)\frac{(g_s N)^{2/5}}{\ell} \leqslant 0 \tag{3.11}$$

since $f(x) \geqslant 0$. The behavior of $f(x)$ is depicted in figure 2. Moreover the binding energy satisfies the concavity condition $\dfrac{dE}{dL} \geqslant 0, \dfrac{d^2E}{dL^2} \leqslant 0$. Therefore the force is manifestly attractive and increasing in magnitude. Note however that this was not necessarily expected for baryons, since in this case there is no analogue of the concavity condition for heavy quark-antiquark pairs [23, 24]. The $1/\ell$ behavior is that dictated by conformal invariance, whereas the non-analytical dependence on the 't Hooft coupling λ is the one predicted in [25–31], which hints at a universal behavior based on the conformal symmetry of the gauge theory.

4 Stability analysis

We shall next consider the stability analysis of the classical solution. We know from [10] that the instabilities can emerge only from longitudinal fluctuations of the l strings, since only these possess a non-divergent zero mode, which is a sign of instability. To study the fluctuations about the classical solution we perturb the embedding according to

$$r = r_{\rm cl} + \delta r(\rho) \tag{4.1}$$

and expand the Nambu-Goto action to quadratic order in the fluctuations. δr is then solved from the equation

$$\frac{d}{d\rho}\left(\frac{(\rho^4 - \rho_1^4)^{3/2}}{\rho^2}\frac{d}{d\rho}\right)\delta r = 0\,, \tag{4.2}$$

[2]In this case we have added the on-shell energy of the Dp-brane.

from where we find

$$\delta r = A\int_{\rho}^{\infty} d\rho \frac{\rho^2}{(\rho^4-\rho_1^4)^{3/2}} = \frac{A}{3\rho^3}\, {}_2F_1\left(\frac{3}{2},\frac{3}{4};\frac{7}{4};\frac{\rho_1^4}{\rho^4}\right). \tag{4.3}$$

Supplementing with the boundary condition (eq. (3.12) in [10])

$$\rho_0\gamma^2\delta r' + 2(1+\gamma^2)\delta r = 0 \quad \text{at} \quad \rho=\rho_0 \quad \text{where} \quad \gamma \equiv \sqrt{1-\frac{\rho_1^4}{\rho_0^4}}\,, \tag{4.4}$$

we find that

$${}_2F_1\left(\frac{3}{2},\frac{3}{4};\frac{7}{4};1-\gamma^2\right) = \frac{3}{2\gamma(1+\gamma^2)}\,. \tag{4.5}$$

The numerical result for γ is then $\gamma_c = 0.538$. The critical value for a can be read from (3.4), and we find it is a function of the magnetic flux

$$a_c = \frac{1+\sqrt{1-\beta^2}}{1+\gamma_c} \tag{4.6}$$

Therefore, for the various configurations with magnetic flux there is a bound for the number of F-strings coming from stability

$$l \geqslant \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \tag{4.7}$$

which is more restrictive than the bound imposed by the existence of a classical solution

$$l \geqslant \frac{q}{2}(1+\sqrt{1-\beta^2})\,. \tag{4.8}$$

Note that in fact the stability condition (4.7) imposes a bound on the magnetic flux $1+\sqrt{1-\beta^2} \leqslant 1+\gamma_c$ which is also more restrictive than the one coming from (3.6), since now $\beta \geqslant \sqrt{1-\gamma_c^2}$ and therefore $\beta = 0$, which was setting the condition for the maximum (and minimum, if applicable) magnetic flux, is not reached. Therefore stability further restricts the allowed values for the magnetic flux coming from the analysis of the equations of motion.

Finally, we turn to the fluctuations of the Dp-brane. We perturb the embedding according to

$$x^\mu = \delta x^\mu(t,\theta_\alpha)\,, \qquad \rho=\rho_0\,, \qquad x^\mu = x, y\,, \tag{4.9}$$

leaving the position of the Dp-brane at $\rho=\rho_0$ intact due to the gauge choice $\rho=\sigma$ for the strings. To be more precise, the ρ-fluctuations can be proven to be decoupled from the others both in the equations of motion and in the boundary equations; being periodic in the angles of $CP^{\frac{p}{2}}$. Moreover, leaving the position of the brane at $\rho=\rho_0$ can also be proven to be allowed for spaces for which $g_{tt}\sim\rho^2$ (as in AdS_4) at zero mode of the angular fluctuations, whereas for higher modes the $\delta\rho$ fluctuations are stable. Moreover, for the

CP^1 and CP^2 cases we have kept fixed the D2 and D4 embeddings on the CP^3. We then find that to second order in the fluctuations the expansion of the Dp-brane action reads

$$S_{Dp} = -\frac{T_p}{g_s} L^p (1+c)^{\frac{p}{4}} \int dt\, d\Omega_p \sqrt{-g_{tt}} \sqrt{\gamma} \times \\ \times \left\{ 1 + \frac{g_{\mu\nu}}{2(1+c)} \gamma^{\alpha\beta} \partial_\alpha \delta x^\mu \partial_\beta \delta x^\nu + \frac{g_{\mu\nu}}{2g_{tt}} \delta\dot{x}^\mu \delta\dot{x}^\nu \right\}, \qquad c = (2\pi\mathcal{N})^2 , \tag{4.10}$$

where $c = (2\pi(\mathcal{N}-1))^2$ for the D2 and D6-branes, $c = (2\pi\mathcal{N})^2$ for the D4, $\gamma_{\alpha\beta}$ is the metric of $CP^{\frac{p}{2}}$ and the action is calculated at $\rho = \rho_0$. The subscripts α, μ refer to the angles of $CP^{\frac{p}{2}}$ and to the x, y coordinates, respectively. Expanding the fluctuations in terms of the spherical harmonics of the $CP^{\frac{p}{2}}$ coset manifold[3] as

$$\delta x^\mu(t, \theta_\alpha) = \delta x^\mu(t) \Psi_\ell(\theta_\alpha) , \tag{4.11}$$

we find from the Euler-Lagrange equations for the action that

$$\frac{d^2 \delta x^\mu}{dt^2} + \Omega_\ell^2 \delta x^\mu = 0, \qquad \Omega_\ell^2 = -\frac{g_{tt}}{1+c} \omega_\ell^2 \geqslant 0 \; . \tag{4.12}$$

Note that there are no boundary conditions for these fluctuations, the reason being that the $\mathbb{R} \times CP^{\frac{p}{2}}$ space has no boundary. The conclusion is that the Dp-brane is also stable against fluctuations.

5 The microscopical description

In the previous sections we have described magnetically charged baryon vertex like configurations with varying number of quarks using the probe brane approximation. This description is valid in the supergravity limit $L \gg 1$ (in string units), equivalently when $k \ll N$, and in the weakly coupled region in which $g_s \ll 1$, equivalently when $N \ll k^5$. In this section we show that it is possible to give a description for finite 't Hooft coupling in terms of fuzzy $CP^{\frac{p}{2}}$ manifolds built up out of dielectrically expanded D0-branes.

The fact that the magnetic flux induces D0-brane charge on the Dp-branes wrapped on $CP^{\frac{p}{2}}$ suggests a close analogy with the dielectric effect of [13, 32]. We then expect that a complementary description in terms of coincident D0-branes expanded into fuzzy $CP^{\frac{p}{2}}$ manifolds should be possible. This would be the 'microscopical' realization of the 'macroscopical' Dp-branes wrapping classical $CP^{\frac{p}{2}}$ spaces with magnetic flux. It is well known that the macroscopical and microscopical descriptions have complementary ranges of validity [13]. The first is valid in the supergravity limit $L \gg 1$, whereas the second is a good description when the mutual separation of the expanding D0-branes is much smaller than the string length. For n expanding such branes this is fixed by the condition $L \ll n^{\frac{1}{p}}$. The two descriptions are then complementary for finite n and should agree in the large n limit, where they have a common range of validity. In $AdS_4 \times CP^3$ the regime of validity

[3]Satisfying the eigenvalue equation $\nabla^2_\gamma \Psi_\ell = -\omega_\ell^2 \Psi_\ell$ where ω_ℓ^2 is positive since the Laplace operator is defined on a compact manifold.

of the microscopical description is fixed by the condition that $N \ll n^{\frac{4}{p}} k$. Therefore this description allows to explore the region of finite 't Hooft coupling.

Dielectric branes expanding into fuzzy coset manifolds have been discussed in the literature in different contexts [21, 33–36]. G/H coset manifolds can be described as fuzzy surfaces if H is the isotropy group of the lowest weight state of a given irreducible representation of G [33, 37]. Since different irreducible representations have associated different isotropy subgroups they can give rise to different cosets G/H. For instance, CP^2 has $G = \mathrm{SU}(3)$, $H = \mathrm{U}(2)$, and this is precisely the isotropy group of the SU(3) irreducible representations $(m, 0)$, $(0, m)$, where we parameterize the irreducible representations of SU(3) by two integers (n, m) corresponding to the number of fundamental and anti-fundamental indices. Any other choice of (n, m) has isotropy group $\mathrm{U}(1) \times \mathrm{U}(1)$ and therefore yields a different coset, $\mathrm{SU}(3)/(\mathrm{U}(1) \times \mathrm{U}(1))$. One can also take a more geometrical view more suitable for our purposes. Using the fact that $CP^{\frac{p}{2}}$ spaces can be defined as the submanifolds of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by a given set of $p^2/4$ constraints, a fuzzy version arises by promoting the Cartesian coordinates that embed the $CP^{\frac{p}{2}}$ in $\mathbb{R}^{\frac{p^2}{4}+p}$ to $\mathrm{SU}(\frac{p}{2}+1)$ matrices in the irreducible totally symmetric representations $(m, 0)$ or $(0, m)$. Indeed only for these representations can the set of $p^2/4$ constraints be realized at the level of matrices. The Cartesian coordinates are then taken to play the role of the non-Abelian transverse scalars that couple in Myers action for coincident D-branes. Using this action one can then provide a microscopical description of a Dq-brane wrapped on the classical $CP^{\frac{p}{2}}$ space in terms of D$(q-p)$-branes expanding into a fuzzy $CP^{\frac{p}{2}}$. Exact agreement between the two descriptions is found in the large m limit.

5.1 The DBI action in the microscopical description

The DBI action describing the dynamics of n coincident D0-branes is given by [13]

$$S^{DBI}_{nD0} = -\int d\tau\, \mathrm{STr}\Big\{ e^{-\phi} \sqrt{\left|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^i{}_j E^{jk} E_{k\nu}]\Big)\det Q\right|}\Big\} \tag{5.1}$$

where $E = g + B_2$,

$$Q^i{}_j = \delta^i{}_j + \frac{i}{2\pi}[X^i, X^k] E_{kj}\,, \tag{5.2}$$

and we have set the tension of the D0-branes to 1. We take $g_{\mu\nu}$ to be the metric in $AdS_4 \times CP^3$ and $B_2 = -2\pi J$, as in appendix A. The number of D0-branes, n, is related to the magnetic flux of the macroscopical description by (2.11), with $s = 0$

$$n = \frac{\mathcal{N}^{\frac{p}{2}}}{2^{\frac{p}{2}}(\frac{p}{2})!}\,. \tag{5.3}$$

We now let these D0-branes expand into a fuzzy $CP^{\frac{p}{2}}$ space to build up a Dp-brane. We find that

$$S^{DBI}_{nD0} = -\frac{1}{g_s}\int d\tau \frac{2\rho}{L}\, \mathrm{STr}\sqrt{\det(Q)}\,. \tag{5.4}$$

As we have mentioned, a fuzzy version of $CP^{\frac{p}{2}}$ is well-known. Here we will mainly follow [38]. $CP^{\frac{p}{2}}$ is the coset manifold $\mathrm{SU}(\frac{p}{2}+1)/\mathrm{U}(\frac{p}{2})$, and can be defined by the submanifold of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by the set of $p^2/4$ constraints

$$\sum_{i=1}^{\frac{p^2}{4}+p} x^i x^i = 1\,, \qquad \sum_{j,k=1}^{\frac{p^2}{4}+p} d^{ijk} x^j x^k = \frac{\frac{p}{2}-1}{\sqrt{\frac{p}{4}(\frac{p}{2}+1)}} x^i \tag{5.5}$$

where d^{ijk} are the components of the totally symmetric $\mathrm{SU}(\frac{p}{2}+1)$-invariant tensor. The Fubini-Study metric of the $CP^{\frac{p}{2}}$ is given by

$$ds^2_{CP^{\frac{p}{2}}} = \frac{p}{4(\frac{p}{2}+1)} \sum_{i=1}^{\frac{p^2}{4}+p} (dx^i)^2\,. \tag{5.6}$$

A fuzzy version of $CP^{\frac{p}{2}}$ can then be obtained by imposing the conditions (5.5) at the level of matrices. This is achieved with a set of coordinates X^i $(i = 1, \ldots, \frac{p^2}{4}+p)$ in the irreducible totally symmetric representation of order m, $(m, 0)$, satisfying

$$[X^i, X^j] = i\Lambda_{(m)} f_{ijk} X^k\,, \qquad \Lambda_{(m)} = \frac{1}{\sqrt{\frac{pm^2}{4(\frac{p}{2}+1)} + \frac{p}{4}m}} \tag{5.7}$$

with f_{ijk} the structure constants in the algebra of the generalized Gell-Mann matrices of $\mathrm{SU}(\frac{p}{2}+1)$. The dimension of the $(m, 0)$ representation is given by

$$\dim(m, 0) = \frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!}\,. \tag{5.8}$$

The Kähler form of the fuzzy $CP^{\frac{p}{2}}$ is given by (see appendix B):

$$J_{ij} = \frac{1}{\frac{p}{2}+1}\sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{ijk} X^k\,. \tag{5.9}$$

Substituting this non-commutative ansatz in (5.4) we can compute $\det(Q)$. This is however a difficult computation to perform in general, since $Q^i{}_j = \delta^i{}_j + M^i{}_j$ with M given by

$$M^i{}_j = -\frac{1}{\frac{p}{2}+1}\Lambda_{(m)} f_{ikl} X^l \left(\frac{pL^2}{8\pi}\delta^k{}_j - \sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{kjm} X^m\right), \tag{5.10}$$

and one has to compute traces of powers of M using the constraints above as well as (B). Given this we are going to start by making the comparison with the macroscopical calculation. For this purpose it is enough to work to leading order in m, to which the second term in (5.10), coming from B_2, does not contribute. This should match the macroscopical result for $B_2 = 0$. Indeed, recall from section 2.1 that B_2 contributes to (2.4) to order $O(1/\mathcal{N})$. Already in this case we find that

$$\mathrm{Tr}(M) = 0\,, \qquad \mathrm{Tr}(M^2) = -\frac{p}{2^4\pi^2}\, r\,\mathbb{I}\,, \qquad \mathrm{Tr}(M^3) = -i\,\frac{p(\frac{p}{2}+1)}{2^7\pi^3 L^2}\, r^2\,\mathbb{I}\,, \tag{5.11}$$
$$\mathrm{Tr}(M^4) = \frac{p}{2^8\pi^4}\, r^2\,\mathbb{I} + \frac{p}{2^{10}\pi^4 L^4}\left(\left(\frac{p}{2}+1\right)^2 - 4\right) r^3\,\mathbb{I}\,,$$

with

$$r = \frac{L^4}{m(m+\frac{p}{2}+1)}\,. \tag{5.12}$$

However, in the limit

$$L \gg 1\,, \qquad m \gg 1\,, \quad \text{with} \quad r \simeq \frac{L^4}{m^2} = \text{finite}\,, \tag{5.13}$$

some terms in the traces of higher powers of M drop out, and we find

$$\text{Tr}(M^{2n}) = p\,(-1)^n \left(\frac{r}{16\pi^2}\right)^n \mathbb{I}\,, \qquad \text{Tr}(M^{2n+1}) = 0\;. \tag{5.14}$$

Substituting in (5.4) we then obtain that

$$\det(Q) = \left(1+\frac{r}{16\pi^2}\right)^{\frac{p}{2}} \mathbb{I}\;. \tag{5.15}$$

The DBI action of n D0-branes expanding into a fuzzy $CP^{\frac{p}{2}}$ is then given to leading order in m by

$$S^{DBI}_{nD0} = -\frac{n}{g_s}\left(1+\frac{L^4}{16\pi^2 m^2}\right)^{\frac{p}{4}} \int d\tau \frac{2\rho}{L} \tag{5.16}$$

where $n = \dim(m,0)$ arises as $\dim(m,0) = \text{STr}\,\mathbb{I}$. Note that in the regime of validity of the microscopical description $L \ll n^{\frac{1}{p}} \to L^4 \ll m^2$, and we could expand in powers of $\frac{L^4}{m^2}$. We will see however that the agreement with the macroscopical description still holds for the entire expression in (5.16). We encountered already this situation in the microscopical descriptions of giant gravitons in [34, 36, 39, 40]. Taking into account (5.8) and (5.3) we have that to leading order in m the label of the irreducible representation and the unit of magnetic flux are related through

$$m \sim \frac{\mathcal{N}}{2} \tag{5.17}$$

and (5.16) becomes

$$S^{DBI}_{nD0} = -\frac{T_p}{g_s}\,\text{Vol}(CP^{\frac{p}{2}})\Big(L^4+(2\pi\mathcal{N})^2\Big)^{\frac{p}{4}} \int d\tau \frac{2\rho}{L}\,, \tag{5.18}$$

which exactly matches the result (2.4) of the macroscopical calculation for $B_2 = 0$. Note that $\mathcal{N} \sim 2m$ is in agreement with the quantization condition $\mathcal{N} \in 2\mathbb{Z}$.

Let us now include the effect of the B_2 field. We know from the macroscopical calculation that B_2 produces a shift $\mathcal{N} \to \mathcal{N}-1$ in the D2 and D6-branes, and cancels the contribution of the Freed-Witten worldvolume flux in the D4-brane. Its effect is therefore $O(1/m)$, and this is why we could ignore it in the leading order calculation above. Analytical and numerical results for $B_2 \neq 0$ and the agreement with the macroscopical calculation suggest that the complete expression for the determinant to order $O(1/m)$ can be obtained from the expansion of

$$\det(Q) = \left(\left(1-\frac{1}{2\sqrt{m(m+\frac{p}{2}+1)}}\right)^2 + \frac{r}{16\pi^2}\right)^{\frac{p}{2}}\,. \tag{5.19}$$

This is the exact result for $p = 2$ in the limit (5.13) and correctly matches the macroscopical result to this order for all p. Indeed, using (5.19) we find that

$$S^{DBI}_{nD0} = -\frac{n}{g_s}\left(\left(1 - \frac{1}{2\sqrt{m(m+\frac{p}{2}+1)}}\right)^2 + \frac{L^4}{16\pi^2 m(m+\frac{p}{2}+1)}\right)^{\frac{p}{4}} \int d\tau \frac{2\rho}{L}\,, \qquad (5.20)$$

which to order $O(1/m)$ yields

$$S^{DBI}_{nD0} = -\frac{T_p}{g_s}\,\mathrm{Vol}(CP^{\frac{p}{2}})\left(L^4 + (2\pi)^2\left(2m + \frac{p}{2} + 1 - 1\right)^2\right)^{\frac{p}{4}} \int d\tau \frac{2\rho}{L}\,. \qquad (5.21)$$

Here we have not cancelled the two ones inside the parenthesis to emphasize their different origin, coming from the $1/m$ expansion of the second term in (5.20) (the $+1$) and the B_2 contribution (the -1). Comparing to the macroscopical calculation for $B_2 \neq 0$ this result suggests a redefinition of $\mathcal{N} = \mathcal{N}(m)$ to order $O(1/m)$:

$$\mathcal{N} = 2m + \frac{p}{2} + 1 \qquad \text{for} \quad p = 2, 6 \qquad (5.22)$$

$$\mathcal{N} = 2m + \frac{p}{2} \qquad \text{for} \quad p = 4 \qquad (5.23)$$

With these redefinitions we can, on the one hand, obtain a magnetic flux properly quantized, i.e. such that $\mathcal{N} \in 2\mathbb{Z}$, and, on the other hand, reproduce the expected shift of $\mathcal{N}$, $\mathcal{N} \to \mathcal{N} - 1$, for $p = 2, 6$. The $p = 4$ case is more interesting. Recall that in the macroscopical analysis B_2 was introduced in order to cancel the flux of the (Freed-Witten) vector field required by the Freed-Witten anomaly, such that $\mathcal{F} = F_{FW} + \frac{1}{2\pi}B_2 = 0$.[4] Microscopically we should see, in the absence of B_2, an obstacle to the expansion of the D0-branes into a CP^2, which should be absent for the CP^1 and CP^3. However, since the Freed-Witten field strength cannot couple in the worldvolume of D0-branes it is not clear a priori how exactly a non-vanishing B_2 could allow the construction of the CP^2. We have found through a simple classical computation that B_2 is required in order to get an even $\mathcal{N}$, that is later interpreted as (twice) the units of magnetic flux in the macroscopical description. This clarifies the precise way in which the flat half-integer B_2 allows for the correct construction of the di-baryon with magnetic charge at the microscopical level. We will see in the next section that the analysis of the charges carried by the different branes confirms the redefinitions (5.22), (5.23).

In conclusion, we have seen that it is indeed possible to give a microscopical description of the magnetic baryon vertex like configurations of [5] in terms of D0-branes expanding into fuzzy $CP^{\frac{p}{2}}$. This expansion is caused by the couplings in the Born-Infeld part of the action, and therefore it is entirely due to a gravitational dielectric effect, analogous to the one described in [21, 41]. The regime of validity is fixed by the condition

$$N \ll k \left[\frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!}\right]^{\frac{4}{p}}\,. \qquad (5.24)$$

[4] In fact, the original argument supporting this B_2-field in [14] had to do with the analysis of the supergravity charges, while the analysis of the D4-brane worldvolume dynamics arose as a consistency check. We refer to the original paper for more details.

Therefore for finite m this description allows to explore the region of finite 't Hooft coupling. Note however that for $B_2 \neq 0$ we have not been able to give exact analytical expressions beyond the constant term in a $1/m$ expansion.

5.2 The F-strings in the microscopical description

An essential part of the baryon vertex-like configurations described in this paper are the fundamental strings that stretch from the Dp-brane to the boundary of AdS_4. In this section we show how these strings arise in the microscopic setup.

The CS action for n coincident D0-branes is given by

$$S_{CS} = \int_{\mathbb{R}} \mathrm{STr}\Big\{ P\Big(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q \, e^{B_2} \Big) e^{2\pi F} \Big\} \,. \tag{5.25}$$

In this expression the dependence of the background potentials on the non-Abelian scalars occurs through the Taylor expansion [42]

$$C_q(t,X) = C_q(t) + X^k \partial_k C_q(t) + \frac{1}{2} X^l X^k \partial_l \partial_k C_q(t) + \dots \tag{5.26}$$

and it is implicit that the pull-backs into the worldline are taken with gauge covariant derivatives $D_t X^\mu = \partial_t X^\mu + i[A_t, X^\mu]$.

In the $AdS_4 \times CP^3$ background we have

$$F_2 = \frac{2L}{g_s} J \,, \qquad F_6 = \frac{L^5}{g_s} J \wedge J \wedge J \,, \qquad B_2 = -2\pi J \tag{5.27}$$

with J the Kähler form of the CP^3. Therefore taking into account (5.26) the relevant CS couplings in this background are

$$\begin{aligned} S_{CS} = i \int d\tau \mathrm{STr}\Big\{ \Big[&(i_X i_X) F_2 - \frac{1}{(2\pi)^2} (i_X i_X)^3 F_6 + \frac{i}{2\pi} (i_X i_X)^2 F_2 \wedge B_2 - \\ &- \frac{1}{2} \frac{1}{(2\pi)^2} (i_X i_X)^3 F_2 \wedge B_2 \wedge B_2 \Big] A_\tau \Big\} \,. \end{aligned} \tag{5.28}$$

These terms arise, respectively, from

$$S_{CS} = \int \mathrm{STr}\Big\{ P\Big(C_1 - \frac{1}{2}\frac{1}{(2\pi)^2}(i_X i_X)^2 C_5 + \frac{i}{2\pi}(i_X i_X) C_1 \wedge B_2 - \frac{1}{4}\frac{1}{(2\pi)^2}(i_X i_X)^2 C_1 \wedge B_2 \wedge B_2 \Big) \Big\} \tag{5.29}$$

in (5.25).

The first coupling in (5.28) is non-vanishing when the D0-branes expand into a fuzzy CP^1, which can be that in which a D2-brane is wrapped or any of the CP^1 cycles of a CP^2 D4-brane or a CP^3 D6-brane. Since the Kähler form for a fuzzy $CP^{\frac{p}{2}}$ is given by (see the appendix B)

$$J_{ij} = \frac{1}{\frac{p}{2}+1} \sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{ijk} X^k \tag{5.30}$$

we find that

$$S_{CS_1} = i \int STr\{(i_X i_X) F_2 \wedge A\} = k \Big(m \Big(m + \frac{p}{2} + 1 \Big) \Big)^{-1/2} \frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!} \int d\tau A_\tau \tag{5.31}$$

which gives in the large m limit

$$S_{CS_1} = k\,\frac{m^{\frac{p}{2}-1}}{(\frac{p}{2})!}\int d\tau A_\tau \tag{5.32}$$

Taking into account that the dimension of the irreducible representation is related to the units of magnetic flux of the macroscopical description by $m = \frac{\mathcal{N}}{2}$, as we showed in the previous section, we find that the number of fundamental string charge in each CP^1 is given by:

$$q = \frac{2}{p}\,k\,\frac{\mathcal{N}^{\frac{p}{2}-1}}{2^{\frac{p}{2}-1}(\frac{p}{2}-1)!} \tag{5.33}$$

which is in agreement with the macroscopical result (2.12).

Let us now look at the second term in (5.28). This term is non-vanishing when the D0-branes expand into a fuzzy CP^3, so it should give the fundamental string charge carried by the CP^3 D6-brane in the large m limit. The explicit computation gives

$$S_{CS_2} = -\frac{i}{(2\pi)^2}\int STr\{(i_X i_X)^3 F_6\wedge A\} = N\Big(m(m+4)\Big)^{-3/2}\frac{(m+3)!}{m!}\int d\tau A_\tau \tag{5.34}$$

and, in the large m limit

$$S_{CS_2} = N\int d\tau A_\tau\,, \tag{5.35}$$

in agreement with the macroscopical result.

The third and fourth terms in (5.28) contribute when we take into account the B_2 field that is necessary to compensate the Freed-Witten worldvolume field of the D4-brane. Therefore they contribute to the k charge to order $O(1/m)$ relative to (5.32). We find, explicitly:

$$S_{CS_3} = -\frac{1}{2\pi}\int \mathrm{STr}\Big\{(i_X i_X)^2 F_2\wedge B_2\wedge A\Big\} = -k\left(m\left(m+\frac{p}{2}+1\right)\right)^{-1}\frac{(m+\frac{p}{2})!}{m!\,(\frac{p}{2})!}\int d\tau A_\tau \tag{5.36}$$

and

$$\begin{aligned} S_{CS_4} &= -\frac{i}{2}\frac{1}{(2\pi)^2}\int \mathrm{STr}\Big\{(i_X i_X)^3 F_2\wedge B_2\wedge B_2\wedge A\Big\} \\ &= \frac{3!}{8}k\left(m\left(m+\frac{p}{2}+1\right)\right)^{-3/2}\frac{(m+\frac{p}{2})!}{m!\,(\frac{p}{2})!}\int d\tau A_\tau \end{aligned} \tag{5.37}$$

These yield in the large m limit

$$S_{CS_3} = -k\,\frac{m^{\frac{p}{2}-2}}{(\frac{p}{2})!}\int d\tau A_\tau \tag{5.38}$$

and

$$S_{CS_4} = \frac{3!}{8}\,k\,\frac{m^{\frac{p}{2}-3}}{(\frac{p}{2})!}\int d\tau A_\tau \tag{5.39}$$

respectively. In order to find the total k charge to this (lower) order in m (relative to (5.32)) we have to add the contributions to this order coming from (5.31), that we have ignored in (5.32). Doing this we find that the total F-string charge for $p=2$ is still k, but for $p=4$ and $p=6$ it is given by $k(m+1)$, $N+\frac{k}{2}((m+2)^2-m-2+\frac{1}{4})$, respectively. Taking into account the redefinitions (5.22) and (5.23) we find precisely the $k\mathcal{N}/2$ units of F-string charge of the CP^2 D4-brane and the $N+k\frac{\mathcal{N}(\mathcal{N}-2)}{8}$ units of F-string charge of the CP^3 D6-brane, given respectively by equations (2.12) (for $p=4$) and (2.19). Note that we find in addition a $k/8$ contribution for the D6, coming from S_{CS_4}. Macroscopically we already encountered this charge when computing the contribution of the coupling $\int_{D6} F_2\wedge B_2\wedge B_2\wedge A$ to the D6-brane tadpole. Given that this charge was cancelled from the anomalous higher curvature coupling

$$S_{h.c.}=\frac{3}{2}(2\pi)^5\,T_6\int d^7\xi\, P\left(C_1\wedge\sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}}\right)\wedge F\,, \tag{5.40}$$

a similar cancellation should occur microscopically. We will discuss in the next section how this can be achieved. Coming back to the D4-brane it is interesting that we need again at the classical level a flat half-integer B_2 in order to recover the right fundamental string charge of the macroscopic D4-brane.

5.3 Dielectric higher-curvature terms

In this section we show that generalizing the microscopical Chern-Simons action in [13] to include higher curvature terms [17–19] we can predict the existence of a dielectric higher curvature coupling in the action for multiple D0-branes that exactly cancels the $k/8$ contribution to the D6-brane tadpole that we obtained above.

Generalizing the Chern-Simons action for multiple Dp-branes in [13] to include higher curvature terms we find

$$S_{h.c.}=T_p\int d^{p+1}\xi\ \mathrm{Str}\left[P\left(e^{\frac{i}{2\pi}(i_Xi_X)}\sum_q C_q\, e^{B_2}\,\Omega\right)e^{2\pi F}\right]_{p+1}\,,\quad \Omega=\sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}}\,. \tag{5.41}$$

Keeping the first term in the $\hat{A}$-roof (Dirac) genus expansion, a general term of the previous expression for $D0$-branes has the following form

$$[(i_Xi_X)^nC_q\ (B_2)^k\Omega_4]\wedge F^\ell\,,\quad (n,\ell,k)\in\mathbb{N}\,, \tag{5.42}$$
$$\underbrace{(q+2(k-n)+4)}_{\geqslant 0}+2\ell=1\,,$$

where Ω_4 is given in term of the Pontryagin classes of the normal and the tangent bundle of the three CP^2 circles of the CP^3 manifold [43, 44]; $\Omega_4=3(1-3)\frac{(2\pi)^4}{48\pi^2}J\wedge J$. To find the term of the expansion that contributes for the CP^3 we proceed as follows: We first note that $\ell=0$ and that in the macroscopic limit only terms with $n+1=3\to n=2$ contribute, thus we have to solve $q+2k=1$, which has solution $(k,q)=(0,1)$. Thus the term reads

$$S_{h.c.}=-\frac{1}{2(2\pi)^2}\int_{\mathbb{R}}P[(i_Xi_X)^2C_1\wedge\Omega_4]=-\frac{i}{(2\pi)^2}\int_{\mathbb{R}}[(i_Xi_X)^3(F_2\wedge\Omega_4)]A \tag{5.43}$$

and substituting F_2 and Ω_4:

$$S_{h.c.} = -\frac{\kappa}{8}(m(m+4))^{-3/2}\frac{(m+3)!}{m!}\int_{\mathbb{R}} d\tau A_\tau \simeq -\frac{\kappa}{8}\int_{\mathbb{R}} d\tau A_\tau \,, \tag{5.44}$$

where we took into account that there are three CP^2 circles in CP^3. Thus this higher curvature coupling cancels the S_{CS_4} contribution as in the macroscopical case.

Anomalous dielectric couplings as those predicted by (5.41) have, to the best of our knowledge, not been discussed before in the literature. Furthermore, acting with T-duality on the A-roof in (5.41) one can obtain dielectric terms that couple the RR-potentials to derivatives of B_2 and the metric that generalize the anomalous terms derived in [45, 46] for a single Dp-brane. It would be interesting to confirm the existence of all these new couplings through string amplitude calculations.

5.4 Stability analysis

The study of the stability goes along the same lines than in the macroscopical set-up. Note that also in the microscopical description the DBI action can be written as (2.4), where Q_p depends now on the label of the irreducible representation, m, in the precise way given by (5.20). The number of F-strings that must end on the Dp-brane is in turn given by the sum of the contributions from equations (5.31), (5.34), (5.36) and (5.37), where some of these terms have to be multiplied by the number of CP^1 or CP^2 cycles in the CP^3 as appropriate. Other than these differences we can vary the number of quarks, study the dynamics and the stability exactly along the same lines as in sections 3 and 4. Only now equation (3.6) will impose a bound on m, that is, on the number of D0-branes that can expand into a fuzzy $CP^{\frac{p}{2}}$ by Myers dielectric effect. In the large m limit this is the bound that we encountered for $\mathcal{N}$ in the macroscopical description. As in there the existence of this bound should be related in some way to the stringy exclusion principle of [16], although we have not been able to find a direct interpretation.

The conclusion is that also in the microscopical set-up there exist perfect baryon vertex classical solutions to the equations of motion that are stable against fluctuations.

6 Conclusions

We have analyzed various configurations of magnetically charged particle-like branes in ABJM with reduced number of quarks. We have shown that 't Hooft monopole, di-baryon and baryon vertex configurations with magnetic charge and reduced number of quarks can be constructed which are not only perfect classical solutions to the equations of motion but also stable against small fluctuations.

The magnetic flux has to satisfy some upper bound (also some lower bound for the di-baryon, consistently with the fact that the D4 with fundamental strings only exists for non-zero magnetic flux), and once this bound is fixed it is possible to reduce the number of quarks to a minimum value determined by $\mathcal{N}$ (or β):

$$l \geqslant \frac{q}{2}(1+\sqrt{1-\beta^2})$$

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From here we can see that the number of quarks is maximally reduced when the energy of the configuration is minimum, that is, for those values of the flux for which $\beta = 0$.

The analysis of the stability against small fluctuations reveals that the configurations are stable if

$$l \geqslant \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2})$$

where γ_c is fixed numerically to $\gamma_c = 0.538$. Stability therefore increases the classical lower bound for each value of the magnetic flux. This is the same effect encountered in [10] for asymptotically $AdS_5 \times S^5$ spaces. It is worth mentioning that in fact following [10] it is trivial to extend our analysis to asymptotically $AdS_4 \times CP^3$ backgrounds and non-zero temperature.

The previous analysis is based on a probe brane approximation, and is therefore valid in the supergravity limit $k \ll N$. Using the fact that we can consistently add dissolved D0-branes to the configurations we have given an alternative description in terms of D0-branes expanded into fuzzy $CP^{\frac{p}{2}}$ spaces that allows to explore the finite 't Hooft coupling region. In this description the expansion is caused by a purely gravitational dielectric effect, while the Chern-Simons terms only indicate the need to introduce the number of fundamental strings required to build up the (generalized) vertex. The microscopical analysis confirms the existence of non-singlet classical stable solutions for finite 't Hooft coupling.

An output of this analysis is the prediction of dielectric higher curvature couplings that to the best of our knowledge have not been considered before in the literature. The particular explicit coupling in the action for multiple D0-branes that has come out in our analysis is necessary in order to obtain the right fundamental string charge of the baryon vertex. For the rest of branes they are predicted by T-duality. These couplings imply in turn new couplings of the RR-potentials to derivatives of B_2 and the metric, along the lines in [45, 46], with further implications for other branes via S and U dualities. It would be interesting to explore more closely these implications.

Finally, it would be interesting to extend the existence of non-singlet baryon vertex like configurations like the ones considered in this paper to theories with reduced supersymmetry, like the Klebanov-Strassler backgrounds [47], where the internal geometry is the $T^{1,1}$ conifold.

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A Review of the $AdS_4 \times CP^3$ background

In this appendix we give a short review of the $AdS_4 \times CP^3$ background. In our conventions the $AdS_4 \times CP^3$ metric reads

$$ds^2 = L^2\left(\frac{1}{4}ds^2_{AdS_4} + ds^2_{\mathbb{CP}^3}\right), \tag{A.1}$$

with L the radius of curvature in string units

$$L = \left(\frac{32\pi^2 N}{k}\right)^{1/4} \tag{A.2}$$

and where we have normalized the two factors such that $R_{\mu\nu} = -3g_{\mu\nu}$ and $8g_{\alpha\beta}$ for AdS_4 and CP^3, respectively. The explicit parameterization of AdS_4 we use in the main text is

$$ds^2_{AdS_4} = \frac{16\,\rho^2}{L^2}d\vec{x}^2 + L^2\frac{d\rho^2}{\rho^2}, \quad d\vec{x}^2 = -d\tau^2 + dx_1^2 + dx_2^2\ . \tag{A.3}$$

For the metric on CP^3 we use the parameterization in [48, 49]

$$\begin{aligned} ds^2_{\mathbb{CP}^3} = d\mu^2 + \sin^2\mu \Big[& d\alpha^2 + \frac{1}{4}\sin^2\alpha\left(\cos^2\alpha\,(d\psi - \cos\theta\, d\phi)^2 + d\theta^2 + \sin^2\theta\, d\phi^2\right) \\ & + \frac{1}{4}\cos^2\mu\left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\, d\phi)\right)^2\Big], \end{aligned} \tag{A.4}$$

where

$$0 \leqslant \mu,\, \alpha \leqslant \frac{\pi}{2}, \quad 0 \leqslant \theta \leqslant \pi\,, \quad 0 \leqslant \phi \leqslant 2\pi\,, \quad 0 \leqslant \psi,\, \chi \leqslant 4\pi\ . \tag{A.5}$$

Inside CP^3 there is a CP^1 for $\mu = \alpha = \pi/2$ and fixed χ and ψ and also a CP^2 for fixed θ and ϕ.

In these coordinates the connection in $ds^2_{S^7} = (d\tau + \mathcal{A})^2 + ds^2_{\mathbb{CP}^3}$ reads

$$\mathcal{A} = \frac{1}{2}\sin^2\mu\left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\, d\phi)\right). \tag{A.6}$$

The Kähler form

$$J = \frac{1}{2}d\mathcal{A}\,, \tag{A.7}$$

is then normalized such that

$$\int_{CP^1} J = \pi\,, \qquad \int_{CP^2} J\wedge J = \pi^2\,, \qquad \int_{CP^3} J\wedge J\wedge J = \pi^3\ . \tag{A.8}$$

Therefore,

$$\frac{1}{6}J\wedge J\wedge J = d\mathrm{Vol}(\mathbb{P}^3) \qquad \text{and} \qquad \mathrm{Vol}(\mathbb{CP}^3) = \frac{\pi^3}{6}\ . \tag{A.9}$$

The $AdS_4 \times CP^3$ background fluxes can then be written as

$$F_2 = \frac{2L}{g_s}J\,, \qquad F_4 = \frac{3L^3}{8g_s}\, d\mathrm{Vol}(AdS_4)\,, \qquad F_6 = -(\star F_4) = \frac{6\,L^5}{g_s}\, d\mathrm{Vol}(\mathbb{P}^3)\,, \tag{A.10}$$

where $g_s = \frac{L}{k}$. The flux integrals satisfy

$$\int_{CP^3} F_6 = 32\,\pi^5\,N\,, \qquad \int_{CP^1} F_2 = 2\pi\,k\,. \tag{A.11}$$

The flat B_2-field that is needed to compensate for the Freed-Witten worldvolume flux in the D4-brane is given by [14]

$$B_2 = -2\pi J\ . \tag{A.12}$$

B Computation of the Kähler form for fuzzy $CP^{\frac{p}{2}}$

In this appendix we compute the Kähler form for the fuzzy $CP^{\frac{p}{2}}$ spaces considered in the paper. The Kähler form is given in terms of the exterior derivative of the one form U(1) gauge field [50, 51]

$$\begin{aligned} &J = J_{(i)}X^i\,, && J_{(i)} = \frac{1}{2}\,dA_i\,, && A_i = \sqrt{\frac{p}{\frac{p}{2}+1}}\,L_i, \\ &J \equiv \frac{1}{2}J_{ij}L_i\wedge L_j\,, && L_i = -i\,\mathrm{Tr}(t_i g^{-1}dg)\,, && g\in SU\left(\frac{p}{2}+1\right)\,, \end{aligned} \tag{B.1}$$

where t_i are the generators of SU$(\frac{p}{2}+1)$ in the adjoint representation, $(t_i)_{jk} = -if_{ijk}$. Using that $\mathrm{Tr}(t_it_j) = (\frac{p}{2}+1)\delta_{ij}$, $\mathrm{Tr}(t_it_jt_k) = i\,\frac{\frac{p}{2}+1}{2}f_{ijk}$, which result from the identities [52]

$$\begin{aligned} &f_{ikm}f_{jkm} = N\delta_{ij}, \quad f_{iaj}f_{jbk}f_{kci} = -\frac{N}{2}f_{abc}, \quad d_{iaj}d_{jbk}f_{kci} = \frac{N^2-4}{2N}f_{abc}, \\ &f_{iaj}f_{jbk}f_{kcm}f_{mdi} = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \frac{N}{4}(d_{abe}d_{cde} + d_{ade}d_{bce} - d_{ace}d_{bde})\,, \end{aligned} \tag{B.2}$$

we compute the Kähler form as follows

$$\begin{aligned} J &= \frac{i}{2}\sqrt{\frac{p}{\frac{p}{2}+1}}\,\mathrm{Tr}(t_kg^{-1}dg\wedge g^{-1}dg)X^k\,, \qquad iL_it_i = \left(\frac{p}{2}+1\right)\,g^{-1}dg \\ \Rightarrow \quad J &= \frac{i^3}{2}\sqrt{\frac{p}{\frac{p}{2}+1}}\,\frac{\mathrm{Tr}(t_it_jt_k)}{(\frac{p}{2}+1)^2}\,X^k\,L_i\wedge L_j\,, \\ \Rightarrow \quad J_{ij} &= \frac{1}{\frac{p}{2}+1}\sqrt{\frac{p}{4(\frac{p}{2}+1)}}f_{ijk}X^k \end{aligned} \tag{B.3}$$

Then, for the n $D0$-branes expanding into a fuzzy $CP^{\frac{p}{2}}$ we find that

$$\begin{aligned} &(i_Xi_X)J = X^jX^iJ_{ij} = -\frac{i}{2}\sqrt{\frac{p}{4(\frac{p}{2}+1)}}\,\Lambda_{(m)}\mathbb{I}\,, \\ &(i_Xi_X)^{\frac{p}{2}}\underbrace{J\wedge J\wedge\cdots\wedge J}_{\frac{p}{2}\ \text{terms}} = \left(\frac{p}{2}\right)!\left(-\frac{i}{2}\sqrt{\frac{p}{4(\frac{p}{2}+1)}}\,\Lambda_{(m)}\right)^{\frac{p}{2}}\mathbb{I}\,, \end{aligned} \tag{B.4}$$

so that the interior products are constant.

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Dielectric 5-branes and giant gravitons in ABJM

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Abstract: We construct a supersymmetric NS5-brane wrapped on a twisted 5-sphere expanding in the CP^3 in $AdS_4 \times CP^3$, with D0-brane charge. This configuration provides a realization of the stringy exclusion principle in terms of giant D0-branes. In the maximal case the twisted 5-sphere reduces to a CP^2 and its energy can be accounted for both by a bound state of k D4-branes wrapping the CP^2 and a bound state of N D0-branes, a realization on the gravity side of the symmetry of Young diagrams with N rows and k columns. We discuss some generalizations of this configuration in M-theory carrying angular momentum, some of them with an interpretation as giant gravitons. We provide the microscopical description that allows to explore the region of finite 't Hooft coupling.

Keywords: p-branes, D-branes, AdS-CFT Correspondence, M-Theory

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Contents

1 Introduction

Giant gravitons [1] have proven to be very useful in the context of the AdS/CFT correspondence in matching D-brane configurations in string/M theory with gauge-invariant operators in the dual gauge theory. In particular, the fact that $\frac{1}{2}$ BPS giant gravitons are dual to Schur polynomials [2–4] have helped elucidating that many properties such as the stringy exclusion principle of [5] or details about the global and local geometries of the dual branes are encoded in the gauge theory [6–10].

In the context of the more recent duality between Type IIA in $AdS_4 \times CP^3$ and the 3-dimensional $\mathcal{N} = 6$ Chern-Simons-matter theory with gauge group $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ known as the ABJM theory [11], giant graviton configurations in both the Type IIA and gauge theory sides have been studied in [12–17]. The Penrose limit has been analyzed in [18].

In the gravity side of the Type IIA/ABJM correspondence a D2-brane spherical giant graviton with analogous properties[1] to the usual dual giant gravitons in $AdS_m \times S^n$ backgrounds [19, 20] was constructed in [13, 15]. The corresponding field theory dual was worked out in [14, 21]. A new dual graviton D2-brane solution specific to the $AdS_4 \times CP^3$ background was also constructed in [13]. This D2-brane was obtained by taking the orbifold

[1] See however [15].

reduction of a spinning dual giant graviton in $AdS_4 \times S^7/\mathbb{Z}_k$, constructed by generalizing the usual dual giant graviton ansatz to include a winding number along an angular direction in AdS_4. The angular momentum leads to D0-brane charge after the reduction, whereas the winding number gives F-string charge. The spinning D2-brane can then be regarded as a bound state of a D2-brane, D0-branes and F-strings. Since the D0-branes and F-strings generate magnetic and electric flux, a Poynting vector is generated that produces an angular momentum. This spinning D2-brane preserves some fraction of the supersymmetry, has finite energy for a toroidal topology, and for large angular momentum approaches a ring-like object. The identification of the dual BPS operator in this case is a difficult task, since it should not only encode Gauss' law [6] but also the non-zero genus. Some steps in this direction were taken in [14] (see also [22]).

Supersymmetric states in the dual ABJM theory have also been predicted by reducing on the orbifold giant graviton solutions in $AdS_4 \times S^7/\mathbb{Z}_k$. One such example is the spherical D2-brane with D0-charge contained in the AdS_4 part of $AdS_4 \times CP^3$, constructed in [13]. In this paper we work out another example by reducing on the orbifold the spherical M5-brane giant graviton expanding in $S^7/\mathbb{Z}_k$. This solution gives rise to a supersymmetric static NS5-brane wrapping a twisted 5-sphere with D0-brane charge. In the maximal case the energy of the solution can be accounted for both by a bound state of k D4-branes wrapping the CP^2, which suggests a realization in the dual field theory in terms of k dibaryons, and N D0-branes, which suggests a field theory realization in terms of N 't Hooft monopoles. This is in agreement with the field theory, where we need to consider representations labelled by Young diagrams with N rows and k columns, where a single row gives a D0-brane and a single column a D4-brane [11]. Further, it was pointed out in [11] that this instability could be realized in the string theory side in terms of a NS5-brane instanton that turns the k D4-branes into N D0-branes. Our NS5-brane construction provides an explicit realization of this idea. In the non-maximal case the field theory dual to the NS5-brane with D0-charge should be given in terms of smaller subdeterminants [12].

The paper is organized as follows. In section 2 we describe the 5-sphere giant graviton in $AdS_4 \times S^7/\mathbb{Z}_k$ using the action for an M5-brane wrapped on an isometric direction given in [23]. Given that the M2-branes that end on this brane must also be wrapped on the isometric direction the action does not contain a self-dual worldvolume 2-form but a vector field, and a closed form can be given. For an M5-brane with the topology of an S^5 the isometric direction is the coordinate along the fibre in the decomposition of the S^5 as a U(1) fibre over the CP^2. We provide a generalization of this construction by inducing further angular momenta and by taking the M5-brane wrapped instead on the $S^1/\mathbb{Z}_k$ orbifold direction, this with an aim at getting giant graviton solutions in the reduction to Type IIA. We show however that only the maximal case can be given an interpretation in terms of giant gravitons. Together with this so-called macroscopical description, in terms of spherical M5-branes with momentum charge, we provide in section 3 the complementary microscopical description in terms of M-theory gravitons expanding into fuzzy submanifolds of $S^7/\mathbb{Z}_k$ due to Myers dielectric effect [24]. In Type IIA this description allows to explore the region of finite 't Hooft coupling. In section 4 we dimensionally reduce the M5-brane giant graviton solution to produce a static NS5-brane expanding into a twisted 5-sphere inside the CP^3 with D0-brane charge. This configuration is described macroscopically in terms of the action describing wrapped NS5-branes in Type IIA. We also show in this section that the reduction of the M5-brane wrapped on the $S^1/\mathbb{Z}_k$ direction gives rise to a D4-brane wrapped on a deformed CP^2 with momentum charge which cannot however be interpreted as a giant graviton away from the maximal case. In section 5 we provide the complementary description of the NS5-brane configuration in terms of dielectric Type IIA gravitons, and sketch the description of the D4-brane wrapped on the CP^2 in terms of expanding D0-branes with angular momentum. We discuss these results and future directions in section 6.

2 Giant gravitons in $AdS_4 \times S^7/\mathbb{Z}_k$

In this section we construct the M5-brane giant graviton solution in $AdS_4 \times S^7/\mathbb{Z}_k$ using the action for a wrapped M5-brane given in [23]. In this action the direction in which the M5-brane is wrapped occurs as a special isometric direction, so its use is limited to backgrounds with Abelian isometries. In our particular example we take the M5-brane wrapped on the $S^5 \subset S^7/\mathbb{Z}_k$ and propagating along the $S^1/\mathbb{Z}_k$ direction. The isometry is then that associated to translations along the fibre in the decomposition of the S^5 as a U(1) fibration over the CP^2. In the last section we interchange the role played by the S^1 and $S^1/\mathbb{Z}_k$ directions, and show that the resulting configuration cannot be interpreted as a giant graviton away from the maximal case.

We start by collecting some useful formulae of the $AdS_4 \times S^7/\mathbb{Z}_k$ background.

2.1 The background

In our conventions the $AdS_4 \times S^7/\mathbb{Z}_k$ metric reads:

$$ds^2 = -\left(1+\frac{r^2}{4R^2}\right)dt^2 + \frac{dr^2}{1+\frac{r^2}{4R^2}} + r^2 d\Omega_2^2 + R^2 ds^2_{S^7/\mathbb{Z}_k} \tag{2.1}$$

with R the radius of curvature in Planck units,

$$R = (32\pi^2 Nk)^{1/6} \tag{2.2}$$

This is a good description of the gravity dual of the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ CS-matter theory of [11] when $N \gg k^{1/5}$. Writing the $S^7/\mathbb{Z}_k$ metric in coordinates adapted to its decomposition as an $S^1/\mathbb{Z}_k$ bundle over the CP^3 we have

$$ds^2_{S^7/\mathbb{Z}_k} = \left(\frac{1}{k}d\tau + \mathcal{A}\right)^2 + ds^2_{CP^3} \tag{2.3}$$

where $\tau \in [0, 2\pi]$. The metric of the CP^3 can in turn be written as (e.g. [25])

$$\begin{aligned} ds^2_{CP^3} = d\mu^2 + \sin^2\mu \Big[& d\alpha^2 + \frac{1}{4}\sin^2\alpha \left(\cos^2\alpha\,(d\psi - \cos\theta\, d\phi)^2 + d\theta^2 + \sin^2\theta\, d\phi^2\right) \\ & + \frac{1}{4}\cos^2\mu \left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\, d\phi)\right)^2 \Big] \end{aligned} \tag{2.4}$$

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where

$$0 \le \mu,\, \alpha \le \frac{\pi}{2}\,, \quad 0 \le \theta \le \pi\,, \quad 0 \le \phi \le 2\pi\,, \quad 0 \le \psi,\, \chi \le 4\pi\,, \tag{2.5}$$

while the connection A reads

$$\mathcal{A} = \frac{1}{2}\sin^2\mu\left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\, d\phi)\right). \tag{2.6}$$

Taking the ansatz $\mu =$ constant in $ds^2_{CP^3}$ and redefining $\chi \equiv \chi/2$ we find that $ds^2_{S^7/\mathbb{Z}_k}$ reduces to

$$ds^2_{S^7/\mathbb{Z}_k} = \frac{1}{k^2}d\tau^2 + \frac{2}{k}\sin^2\mu\, d\tau(d\chi + A) + \sin^2\mu\, ds^2_{S^5} \tag{2.7}$$

where now

$$A = \frac{1}{2}\sin^2\alpha\,(d\psi - \cos\theta d\phi) \tag{2.8}$$

and the S^5 is written in coordinates adapted to its decomposition as an S^1 bundle over the CP^2:

$$ds^2_{S^5} = (d\chi + A)^2 + ds^2_{CP^2}\,, \tag{2.9}$$

where $ds^2_{CP^2}$ is the Fubini-Study metric of the CP^2 (e.g. [26]):

$$ds^2_{CP^2} = d\alpha^2 + \frac{1}{4}\sin^2\alpha\left[\cos^2\alpha\,(d\psi - \cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta\, d\phi^2\right] \tag{2.10}$$

The 6-form potential of the $AdS_4 \times S^7/\mathbb{Z}_k$ background reads, in turn:

$$C_6 = \frac{R^6}{k}\sin^6\mu\, d\chi \wedge d\tau \wedge \mathrm{dVol}(CP^2) \tag{2.11}$$

2.2 The 5-sphere giant graviton

Let us now take an M5-brane wrapping the S^5 with radius $R\sin\mu$ in (2.7), located at $r = 0$ in AdS_4, and propagating on the $S^1/\mathbb{Z}_k$ fibre direction, τ:

$$ds^2 = -dt^2 + R^2\left[\frac{1}{k^2}d\tau^2 + \frac{2}{k}\sin^2\mu\, d\tau(d\chi + A) + \sin^2\mu\, ds^2_{S^5}\right] \tag{2.12}$$

Note that χ is an isometric direction, parameterizing the S^1 bundle of the S^5. Therefore, we can use the action constructed in [23] in order to study the M5-brane in this background. This action was successfully used in the description of the 5-sphere giant [1] and dual giant graviton [19, 20] solutions in $AdS_4 \times S^7$ and $AdS_7 \times S^4$, respectively. The action for the Type IIA D4-brane arises from this action when reducing along the isometric direction.

We can describe a wrapped M5-brane in the $AdS_4 \times S^7/\mathbb{Z}_k$ background with the action [23]:

$$S = T_4 \int d^5\xi \left\{ -k\sqrt{\left|\det\left(P[\mathcal{G}] + \frac{2\pi}{k}\mathcal{F}\right)\right|} + P[i_k C_6] + \frac{1}{2}(2\pi)^2 P[k^{-2}k_1] \wedge \mathcal{F} \wedge \mathcal{F} \right\} \tag{2.13}$$

Here k^μ is an Abelian Killing vector that points on the isometric U(1) direction, $\mathcal{G}$ is the reduced metric $\mathcal{G}_{\mu\nu} = g_{\mu\nu} - k^{-2}k_\mu k_\nu$ and $i_k C_p$ denotes the interior product of the

C_p potential with the Killing vector. The action is therefore manifestly isometric under translations along the Killing direction. $\mathcal{F}$ is the field strength associated to M2-branes, wrapped on the isometric direction, ending on the M5-brane: $\mathcal{F} = F + \frac{1}{2\pi}P[i_k C_3]$. When reducing along the isometric direction it gives the BI field strength of the D4-brane. We have denoted T_4 the tension of the brane to explicitly take into account that its spatial worldvolume is effectively 4-dimensional.

$k^{-2}k_1$ is, explicitly, $g_{\mu\chi}/g_{\chi\chi}\, dx^\mu$, in coordinates adapted to the isometry, $k^\mu = \delta^\mu_\chi$, and is therefore identified as the momentum operator in the isometric direction. Momentum along this direction can then be turned on by a convenient choice of $\mathcal{F}$, such that $\int \mathcal{F}\wedge\mathcal{F}$ is non-vanishing. Since our giant graviton solution will only propagate along the $S^1/\mathbb{Z}_k$ direction we will for the moment switch $\mathcal{F}$ to zero.

Note that the 6-form potential of the $AdS_4 \times S^7/\mathbb{Z}_k$ background couples in the action through

$$P[i_k C_6] = C^{(6)}_{\chi\tau\alpha_1\ldots\alpha_4}\dot{\tau} \tag{2.14}$$

where α_i, $i = 1, \ldots, 4$ span the CP^2 directions.

Substituting the background fields in (2.13) and integrating over the CP^2 we find:

$$S = \int dt \Big[-\frac{Nk}{R}\sin^5\mu\sqrt{1-\frac{R^2}{k^2}\cos^2\mu\,\dot{\tau}^2} + N\sin^6\mu\,\dot{\tau}\Big] \tag{2.15}$$

where we have taken units in which the tension of a 0-brane is equal to one.

The Hamiltonian reads, in turn:

$$H = \frac{k}{R}P_\tau\sqrt{1+\tan^2\mu\Big(1-\frac{N}{P_\tau}\sin^4\mu\Big)^2} \tag{2.16}$$

in terms of the conserved τ conjugate momentum, P_τ. Clearly, the minimum energy solution is reached when $\mu = 0$ or

$$\sin\mu = \left(\frac{P_\tau}{N}\right)^{1/4} \tag{2.17}$$

In both cases the BPS bound

$$E = \frac{k}{R}P_\tau \tag{2.18}$$

is reached, and therefore the two solutions correspond, respectively, to the point-like and giant gravitons [1] of the $AdS_4 \times S^7/\mathbb{Z}_k$ background. The giant graviton solution satisfies that $P_\tau \leq N$, as in [1]. Therefore the bound for the angular momentum depends only on the rank of the gauge group, in agreement with the stringy exclusion principle of [5]. The size of the giant graviton is maximal when $\mu = \pi/2$, for which the angular momentum reaches its maximum value $P_\tau = N$, and $E = k\,N/R$. The full energy of the maximal giant graviton can then be accounted for either by a bound state of $P_\tau = N$ gravitons (or M0-branes), with energy k/R, or by a bound state of k M5-branes wrapped on the S^5, with energy N/R. Since in the quotient space the k M5-branes reduce to k D4-branes wrapping the CP^2 the maximal giant is dual to k dibaryons [11, 12, 14].

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As we will see in detail in section 4, the giant graviton M5-brane is realized in Type IIA as an NS5-brane wrapping a twisted 5-sphere inside the CP^3. This NS5-brane is motionless, since the momentum along the M-theory circle gets replaced by D0-brane charge. The stringy exclusion principle of [5] is then realized in the $AdS_4 \times CP^3$ background in terms of giant D0-branes expanded in a twisted NS5-brane.

From the previous discussion it is clear that if we want to obtain a moving brane in Type IIA we need to induce momentum in the eleven dimensional configuration in a direction different from the eleventh direction. In the next section we generalize the previous M5-brane construction to include momentum along the isometric worldvolume direction. Although as we will see the resulting configuration does not behave as a giant graviton away from the maximal case it will be useful in section 3 in order to identify the microscopical set-up that will allow to describe the expanded M5-brane in terms of dielectric gravitons.

2.3 The 5-sphere giant graviton with magnetic flux

Let us now generalize the giant graviton solution constructed before to include a magnetic flux inducing momentum along the isometric direction.

As we discussed in the previous section momentum along this direction can be switched on by a convenient choice of magnetic flux, such that

$$\frac{T_4}{2}(2\pi)^2 \int_{\mathbb{R}\times CP^2} P[k^{-2}k_1] \wedge \mathcal{F} \wedge \mathcal{F} = P_\chi \int_{\mathbb{R}} P[k^{-2}k_1] \tag{2.19}$$

This is achieved with $F = 2\sqrt{2P_\chi}\, J$, with J the Kähler form of the CP^2, $J = \frac{1}{2}dA$. The action (2.15) is then modified according to

$$S = \int dt \Big[-\frac{1}{R\sin\mu}\Big(Nk\sin^6\mu + P_\chi\Big)\sqrt{1 - \frac{R^2}{k^2}\cos^2\mu\,\dot{\tau}^2} + \frac{1}{k}\Big(Nk\sin^6\mu + P_\chi\Big)\dot{\tau}\Big] \tag{2.20}$$

and the new Hamiltonian reads

$$H = \frac{k}{R}P_\tau\sqrt{1 + \tan^2\mu\left(1 - \frac{Nk\sin^6\mu + P_\chi}{kP_\tau\sin^2\mu}\right)^2}\,. \tag{2.21}$$

Written in this way it is clear that $H(\mu)$ is minimum for $\mu = 0$ and $\sin\mu$ satisfying

$$Nk\sin^6\mu - kP_\tau\sin^2\mu + P_\chi = 0\,, \tag{2.22}$$

and that for both these point-like and expanded brane solutions

$$H = \frac{k}{R}P_\tau \tag{2.23}$$

Since for $P_\chi \neq 0$ the energy depends only on one of the two conserved charges the BPS bound is not satisfied. Nonetheless in the maximal case the $S^1/\mathbb{Z}_k$ and S^1 directions become parallel and we have from (2.22) that $P_\tau = N + P_\chi/k$, so in this case there is only one independent conserved charge. A non-vanishing P_χ allows then the maximal giant to propagate with an arbitrary angular momentum on the $S^1/\mathbb{Z}_k$ direction.

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In Type IIA language this expanded solution is realized in terms of a NS5-brane wrapping a twisted 5-sphere submanifold of the CP^3, with D0-brane charge P_τ and momentum P_χ. As before, the energy only depends on the D0-brane charge, and therefore away from the maximal case the configuration is not BPS. Moreover, it does not have an interpretation as a giant graviton. However, inspired by this result we can think of interchanging the role played by the $S^1/\mathbb{Z}_k$ and S^1 directions. Namely, we can take the M5-brane wrapping the submanifold of $S^7/\mathbb{Z}_k$ spanned by τ and the coordinates parameterizing the CP^2, and propagating on χ. This configuration becomes a D4-brane wrapped on a "squashed" CP^2 and propagating on χ in Type IIA. We will see however that it does not have an interpretation as a giant graviton away from the maximal case.

2.4 The giant graviton wrapped on the $S^1/\mathbb{Z}_k$ direction

As we have just mentioned we can similarly consider an M5-brane[2] wrapped on the submanifold of S^7/Z_k with metric

$$ds^2 = R^2\Big[\frac{1}{k^2}d\tau^2 + \frac{2}{k}\sin^2\mu\, A\, d\tau + \sin^2\mu\,(ds^2_{CP^2} + A^2)\Big] \tag{2.24}$$

and propagating on the χ direction. Given that τ is an isometric direction we can still use the action (2.13) with $k^\mu = \delta^\mu_\tau$. For the sake of generality we also switch a magnetic flux $F = 2\sqrt{2P_\tau}\, J$ on, inducing P_τ momentum through the coupling

$$\frac{T_4}{2}(2\pi)^2 \int_{\mathbb{R}\times M_4} P[k^{-2}k_1]\wedge\mathcal{F}\wedge\mathcal{F} \tag{2.25}$$

The configuration simplifies a lot if we also induce an electric flux proportional to the connection on the CP^2: $E_i = R\sin\mu\cos\mu\, A_i$. Then, substituting in the action (2.13) we find:

$$S = \int dt\Big[-\frac{1}{R}\Big(N\sin^4\mu + kP_\tau\Big)\sqrt{1 - R^2\sin^2\mu\cos^2\mu\,\dot\chi^2} + \sin^2\mu\Big(N\sin^4\mu + kP_\tau\Big)\dot\chi\Big] \tag{2.26}$$

and a Hamiltonian

$$H = \frac{P_\chi}{R\sin\mu}\sqrt{1 + \tan^2\mu\Big(1 - \frac{N\sin^4\mu + kP_\tau}{P_\chi}\Big)^2} \tag{2.27}$$

This expression is very similar to the Hamiltonian (2.21) describing the spherical M5-brane with magnetic flux. However in this case one can easily see that there are no solutions for which $E = P_\chi/R$ unless $\mu = \pi/2$, which leads us back to the maximal case. In this case the M5-brane wraps a $S^5/\mathbb{Z}_k$ submanifold of $S^7/\mathbb{Z}_k$ and the induced electric flux vanishes. Also $P_\chi = N + kP_\tau$, and the brane is allowed to move with an arbitrary angular momentum in the S^1 direction. For vanishing P_τ we recover the usual maximal giant graviton with momentum reaching the bound imposed by the stringy exclusion principle, wrapped in this case on a $S^5/\mathbb{Z}_k$ submanifold. In the reduction to Type IIA the S^1 direction shrinks

[2] In our conventions we need to consider in fact an anti-M5-brane with opposite τ-momentum.

to a point, and the N charge can no longer be interpreted as momentum charge. The configuration is instead interpreted as a dibaryon with energy N/L. Our analysis shows that the dibaryon arises in the gravity side as the limiting case of a D4-brane wrapping the "squashed" CP^2 included in

$$ds^2 = -dt^2 + L^2 \sin^2 \mu \left[\cos^2 \mu \, (d\chi + A)^2 + ds^2_{CP^2}\right] \tag{2.28}$$

and propagating along the χ direction.

3 A description in terms of expanding gravitons

In this section we show that the previous configurations can alternatively be described in terms of gravitons expanding into fuzzy M5-branes. This is the microscopical realization of the macroscopical M5-branes wrapping classical submanifolds of $S^7/\mathbb{Z}_k$ with angular momenta. This description is valid in the supergravity limit $R \gg 1$, whereas the microscopical description is good when the mutual separation of the expanding gravitons is much smaller than the string length. For $\mathcal{N}$ expanding gravitons this is fixed by the condition $R \ll \mathcal{N}^{1/4}$. The two descriptions are then complementary for finite $\mathcal{N}$ and should agree in the large $\mathcal{N}$ limit, where they have a common range of validity. In $AdS_4 \times CP^3$ the regime of validity of the microscopical description is fixed by the condition that $N \ll \mathcal{N}k$. Therefore this description allows to explore the region of finite 't Hooft coupling. We will see that the same conclusions regarding the existence of giant graviton configurations that we reached in the macroscopical set-up will hold microscopically.

3.1 The action for M-theory gravitons

We start from the action for coincident gravitons, or gravitational waves, in M-theory constructed in [27]. Using this action it was possible to describe microscopically the giant and dual giant graviton solutions in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ [23, 27], and to derive Matrix theory in the maximally supersymmetric pp-wave background of M-theory [28] with an extra non-perturbative coupling giving rise to the so-far elusive 5-sphere giant graviton [29]. The BI part of the action is given by

$$S_{DBI} = -\int d\xi^0 \, \mathrm{STr}\Big\{k^{-1}\sqrt{|P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^i_k E^{kj} E_{j\nu}]\mathrm{det}Q|}\Big\} \tag{3.1}$$

where $E_{\mu\nu} = \mathcal{G}_{\mu\nu} + k^{-1}(i_k C_3)_{\mu\nu}$, $\mathcal{G}$ is the reduced metric defined in section 2, μ, ν denote spacetime indices and i, j spatial ones, and Q is given by $Q^i_j = \delta^i_j + \frac{ik}{2\pi}[X^i, X^k]E_{kj}$. The CS action reads

$$S_{CS} = \int d\xi^0 \, \mathrm{STr}\Big\{P[k^{-2}k_1] + \frac{i}{2\pi}P[(i_X i_X)C_3] - \frac{1}{2}\frac{1}{(2\pi)^2}P[(i_X i_X)^2 i_k C_6] + \dots\Big\} \tag{3.2}$$

In this action k^μ is an Abelian Killing vector that points on the direction of propagation of the waves. As in action (2.13) this direction is isometric because the background fields are either contracted with the Killing vector or pulled-back in the worldvolume with covariant derivatives relative to the isometry:

$$\mathcal{D}_0 X^\mu = \partial_0 X^\mu - k^{-2} k_\nu \, \partial_0 X^\nu k^\mu \tag{3.3}$$

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In this way the dependence on the isometric direction is effectively eliminated from the action. This action is in fact a gauge fixed action in which the U($\mathcal{N}$) vector field, associated to M2-branes (wrapped on the direction of propagation) ending on the waves, has been taken to vanish. In this gauge U($\mathcal{N}$) covariant derivatives reduce to ordinary derivatives, and gauge covariant derivatives can be defined using ordinary derivatives as in (3.3).

The action given by (3.1) and (3.2) was constructed in [27] by uplifting to eleven dimensions the action for Type IIA gravitational waves derived in [30] using Matrix String theory in a weakly curved background, and then going beyond the weakly curved background approximation by demanding agreement with Myers action for D0-branes when the waves propagate along the eleventh direction. In the action for Type IIA waves the circle in which Matrix theory is compactified in order to construct Matrix String theory cannot be decompactified in the non-Abelian case [30]. In fact, the action exhibits a U(1) isometry associated to translations along this direction, which by construction is also the direction on which the waves propagate. A simple way to see this is to recall that the last operation in the 9-11 flip involved in the construction of Matrix String theory is a T-duality from fundamental strings wound around the 9th direction. Accordingly, in the action we find a minimal coupling to $g_{\mu 9}/g_{99}$, which is the momentum operator $k^{-2}k_1$ in adapted coordinates. Therefore, by construction, the sum of the actions (3.1) and (3.2) is designed to describe BPS waves with momentum charge along the compact isometric direction. It is important to mention that in the Abelian limit, when all dielectric couplings and U($\mathcal{N}$) covariant derivatives disappear, the action can be Legendre transformed into an action in which the dependence on the isometric direction has been restored. This action is precisely the usual action for a massless particle written in terms of an auxiliary γ metric (see [27] and [30] for the details), where no information remains about the momentum charge carried by the particle.

3.2 Expanded graviton configurations

Microscopically the spherical M5-brane described in the previous section is built up of gravitons expanding into a fuzzy 5-sphere through Myers dielectric effect. In our construction the fuzzy S^5 will simply be defined as an S^1 bundle over a fuzzy CP^2. As we will see the existence of the S^1 direction is crucial in order to find the right dielectric coupling that will cause the expansion of the gravitons. This construction of the fuzzy 5-sphere has been successfully used in the microscopical description of various configurations involving 5-spheres (see for instance [23, 29, 31, 32]). In all cases it brings back the right macroscopical description when the number of constituents is large.

The calculation in this section is very similar to the microscopical description of giant gravitons expanding in $AdS_4 \times S^7$ presented in [23].

The fuzzy CP^2 has been extensively studied in the literature in various contexts. In the context of Myers dielectric effect it was first studied in [33] and then in [23, 29, 31, 32, 34]. In general G/H coset manifolds can be described as fuzzy surfaces if H is the isotropy group of the lowest weight state of a given irreducible representation of G [33, 35]. Since different irreducible representations have associated different isotropy groups they can give rise to different cosets G/H. CP^2 has $G = \mathrm{SU}(3)$, $H = \mathrm{U}(2)$, and this is precisely the isotropy group of the SU(3) irreducible representations $(m,0)$, $(0,m)$, where we parameterize the irreducible representations of SU(3) by two integers (m,m') corresponding to the number of fundamental and anti-fundamental indices. One can also take a more geometrical view more suitable for our purposes. Using the fact that CP^n spaces can be defined as the submanifolds of $\mathbb{R}^{n^2+2n}$ determined by a given set of n^2 constraints, a fuzzy version arises by promoting the Cartesian coordinates that embed the CP^n in $\mathbb{R}^{n^2+2n}$ to SU$(n+1)$ matrices in the irreducible totally symmetric representations $(m,0)$ or $(0,m)$. Indeed only for these representations can the set of n^2 constraints be realized at the level of matrices. The Cartesian coordinates are then taken to play the role of the non-Abelian transverse scalars that couple in the action for coincident gravitons.

For the CP^2 the 8 Cartesian coordinates that embed it in $\mathbb{R}^8$ satisfy

$$\sum_{i=1}^{8} x^i x^i = 1 \qquad \sum_{j,k=1}^{8} d^{ijk} x^j x^k = \frac{1}{\sqrt{3}} x^i \tag{3.4}$$

where d^{ijk} are the components of the totally symmetric SU(3)-invariant tensor. In these coordinates the Fubini-Study metric is given by

$$ds^2_{CP^2} = \frac{1}{3}\sum_{i=1}^{8}(dx^i)^2 \tag{3.5}$$

This set of constraints can be implemented at the level of matrices if we choose the set of coordinates X^i $(i=1,\dots 8)$ in the irreducible totally symmetric representation of order m, $(m,0)$, satisfying

$$[X^i, X^j] = i\Lambda_{(m)} f^{ijk} X^k\,, \qquad \Lambda_{(m)} = \frac{1}{\sqrt{\frac{m^2}{3}+m}} \tag{3.6}$$

with f^{ijk} the structure constants of SU(3), $[\lambda^i.\lambda^j] = 2if^{ijk}\lambda^k$. The dimension of the $(m,0)$ representation is given by

$$\mathcal{N} = \frac{(m+2)(m+1)}{2} \tag{3.7}$$

In the $AdS_4 \times S^7/\mathbb{Z}_k$ background we take the gravitons located in $r=0$ and expanding into the fuzzy S^5 with radius $R\sin\theta$ inside $S^7/\mathbb{Z}_k$ described by the metric (2.12), in which we take Cartesian coordinates to parameterize the CP^2. We choose $k^\mu = \delta^\mu_\chi$, so that the gravitons carry by construction P_χ momentum, and take $\tau = \tau(t)$ in order to induce P_τ momentum. We then have that

$$\begin{aligned} k &= R\sin\mu\,, & E_{00} &= -1 + \frac{R^2}{k^2}\cos^2\mu\,\dot\tau^2\,, \\ Q^i_j &= \delta^i_j - \frac{R^3\sin^3\mu}{2\pi\sqrt{m^2+3m}} f_{ijk}X^k\,, & i,j &= 1,\dots,8\,. \end{aligned} \tag{3.8}$$

The determinant of Q can be computed as explained for instance in [34], with the result

$$\det Q = \left(1 + \frac{R^6\sin^6\mu}{16\pi^2(m^2+3m)}\right)^2 \mathbb{I} \tag{3.9}$$

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As explained in [34] this expression is valid in the limit $R \gg 1$, $m \gg 1$, with R^3/m finite.

Substituting in the DBI action (3.1) we find

$$S_{DBI} = -\frac{\mathcal{N}}{R\sin\mu}\left(1 + \frac{R^6\sin^6\mu}{16\pi^2(m^2+3m)}\right)\int dt\,\sqrt{1 - \frac{R^2}{k^2}\cos^2\mu\,\dot{\tau}^2} \tag{3.10}$$

where $\mathcal{N}$ arises as $\dim(m,0) = \mathrm{STr}\,\mathbb{I}$. Using that

$$C_6 = \frac{R^6\sin^6\mu}{2k}d\chi\wedge d\tau\wedge J\wedge J \tag{3.11}$$

and that (see [34])

$$J_{ij} = \frac{1}{3\sqrt{3}}f_{ijk}X^k \tag{3.12}$$

we find the CS action:

$$S_{CS} = \int dt\,\frac{\mathcal{N}}{k}\left(1 + \frac{R^6\sin^6\mu}{16\pi^2(m^2+3m)}\right)\dot{\tau} \tag{3.13}$$

In terms of the conserved conjugate momentum to τ we have the Hamiltonian

$$H = \frac{k}{R}P_\tau\sqrt{1 + \tan^2\mu\left(1 - \frac{\mathcal{N}}{kP_\tau\sin^2\mu}\left(1 + \frac{2Nk\sin^6\mu}{m^2+3m}\right)\right)^2} \tag{3.14}$$

Taking into account that the P_χ momentum of the configuration is by definition the number of gravitons $\mathcal{N}$, in our units in which the tension is set to one, we can rewrite (3.14) as

$$H = \frac{k}{R}P_\tau\sqrt{1 + \tan^2\mu\left(1 - \frac{1}{kP_\tau\sin^2\mu}\left(P_\chi + \frac{2\mathcal{N}}{m^2+3m}Nk\sin^6\mu\right)\right)^2} \tag{3.15}$$

from which it is clear that both Hamiltonians (3.15) and (2.21) exactly agree in the large m limit, where $\mathcal{N} \sim m^2/2$. Moreover, in the limit $P_\chi \to 0$ we can also describe the giant graviton configuration of section 2.2, realized as a spherical M5-brane with just P_τ momentum, with the Hamiltonian

$$H = \frac{k}{R}P_\tau\sqrt{1 + \tan^2\mu\left(1 - \frac{2\mathcal{N}}{m^2+3m}\,\frac{N\sin^4\mu}{P_\tau}\right)^2}\,, \tag{3.16}$$

which agrees exactly with the Hamiltonian (2.16) in the large m limit. Note that the difference between P_χ being zero or not is merely a coordinate transformation, a boost in χ. How to perform coordinate transformations in non-Abelian actions is however an open problem [36–40]. In this case the way in which the limit $P_\chi \to 0$ should be taken is dictated by the agreement with the macroscopical description.

Finally, we can briefly sketch the microscopical description of the $S^5/\mathbb{Z}_k$ maximal giant graviton of section 2.4. In this case we take $k^\mu = \delta^\mu_\tau$, so that the gravitons are wrapped on τ and carry by construction P_τ momentum, and take $\chi = \chi(t)$ in order to induce P_χ

momentum. The fuzzy $S^5/\mathbb{Z}_k$ on which the gravitons expand is then defined as an $S^1/\mathbb{Z}_k$ fibre over a fuzzy CP^2. Substituting in (3.1), (3.2) we find the Lagrangian

$$L = k\mathcal{N}\Big(-\frac{1}{R}+\dot{\chi}\Big)\Big(1+\frac{2N}{k(m^2+3m)}\Big) \tag{3.17}$$

P_χ is simply given by $P_\chi = k\mathcal{N} + \frac{2N\mathcal{N}}{m^2+3m}$ and $H = \frac{P_\chi}{R}$. P_χ gives in the large m limit $P_\chi = k\mathcal{N} + N = kP_\tau + N$, in agreement with the result in section 2.4.

4 Dielectric branes in $AdS_4 \times CP^3$

In this section we give the Type IIA description of the 5-sphere giant graviton solution of section 2.2. We restrict to the zero flux case since we have already seen that away from the maximal case introducing flux ($\leftrightarrow P_\chi$ momentum) does not allow the construction of more general giant graviton solutions. In Type IIA the 5-sphere giant graviton gives rise to a NS5-brane expanding into a twisted 5-sphere inside the CP^3. Momentum along the M-theory circle gets replaced by D0-brane charge, so the NS5-brane is motionless. The energy of the ground state is then accounted for by a bound state of P_τ D0-branes. In the maximal case the ground state is degenerate, and can be accounted for either by a bound state of N D0-branes or by a bound state of k dibaryons. This is a realization on the gravity side of the duality of Young tableaux with N rows and k columns [11]. The stringy exclusion principle is realized in this case in terms of giant D0-branes expanded in a twisted 5-sphere NS5-brane. We also provide the Type IIA realization of the M5-brane wrapped on $S^1/\mathbb{Z}_k$ discussed in section 2.4. This becomes a D4-brane wrapped on a "squashed" CP^2 with angular momentum which can however only be interpreted as a giant graviton in the maximal case. Still, this configuration allows to see the k dibaryons emerging as the limiting case of D4-branes with angular momentum.

4.1 The NS5-brane with D0 charge

The reduction of (2.12) to Type IIA gives the metric of a twisted 5-sphere, with $\cos\mu$ parameterizing the twist between the S^1 and the CP^2:

$$ds^2 = -dt^2 + L^2\sin^2\mu\left[\cos^2\mu\,(d\chi + A)^2 + ds^2_{CP^2}\right]. \tag{4.1}$$

Here A is given by (2.8) and L is the radius of curvature of the CP^3 in string units:

$$L = \left(\frac{32\pi^2 N}{k}\right)^{1/4} \tag{4.2}$$

There is also a RR 1-form field

$$C_1 = k\sin^2\mu\,(d\chi + A)\,, \tag{4.3}$$

a 5-form potential

$$C_5 = L^4 k \sin^6\mu\, d\chi\wedge d\mathrm{Vol}_{CP^2} \tag{4.4}$$

and a dilaton $e^{\phi} = \frac{L}{k}$.

We take the NS5-brane wrapped on the manifold with metric (4.1). Since the background is isometric in the χ direction we can describe the NS5-brane using the action that arises from (2.13) by reducing along a transverse direction. The (bosonic) worldvolume field content of this action consists on 4 transverse scalars, a vector field, associated to D2-branes wrapped on the isometric direction, and a scalar, coming from the reduction of the eleventh direction. This scalar forms an invariant field strength with the RR 1-form potential, and is therefore associated to D0-branes. We can therefore induce D0-brane charge in the configuration through this field. Comparing to the action of the unwrapped NS5-brane in Type IIA constructed in [41, 42] the self-dual 2-form field of the latter has been replaced by a vector, and this allows to give a closed form for the action. The BI part reads:

$$S_{DBI} = -T_4 \int d^5\xi \, e^{-2\phi} \sqrt{k^2 + e^{2\phi}(i_k C_1)^2} \sqrt{\left| \det \left(P[\mathcal{G}] + \frac{e^{2\phi} k^2}{k^2 + e^{2\phi}(i_k C_1)^2} \mathcal{F}_1^2 \right) \right|} \tag{4.5}$$

Here $\mathcal{F}_1$ is the field strength associated to D0-branes "ending" on the NS5-brane: $\mathcal{F}_1 = dc_0 + P[C_1]$, where the pull-back is taken with gauge covariant derivatives, as defined in (3.3). c_0 is the worldvolume scalar whose origin is the eleventh direction. We have ignored for simplicity the contribution of the reduction of $\mathcal{F}$, now associated to D2-branes wrapped on χ, since this field will be vanishing in our background.

The relevant part of the CS action is given by:

$$S_{CS} = T_4 \int i_k C_5 \wedge dc_0 \tag{4.6}$$

Substituting the background fields in the BI and CS actions and integrating over the CP^2 we find:

$$S = \int dt \left[-\frac{Nk}{L} \sin^5\mu \sqrt{1 - \frac{L^2}{k^2} \cos^2\mu \, \dot{c}_0^2} + N \sin^6\mu \, \dot{c}_0 \right] \tag{4.7}$$

where we have taken c_0 time-dependent in order to induce D0-brane charge in the NS5-brane. This action is analogous to (2.15) with $L \leftrightarrow R$, $c_0 \leftrightarrow \tau$. The Hamiltonian is then

$$H = \frac{k}{L} M \sqrt{1 + \tan^2\mu \left(1 - \frac{N}{M} \sin^4\mu\right)^2} \tag{4.8}$$

where we have denoted with M the c_0 conjugate momentum, which is conserved and is now interpreted as D0-brane charge. As for the giant graviton in section 2.2, the minimum energy solution is reached when $\mu = 0$ or

$$\sin\mu = \left(\frac{M}{N}\right)^{1/4} \tag{4.9}$$

In both cases $E = \frac{k}{L} M$, and we find a BPS configuration of M D0-branes with energy k/L. For $\mu = 0$ the NS5-brane is point-like and can carry arbitrary D0-brane charge, while for μ satisfying (4.9) it wraps the twisted 5-sphere described by the spatial part of the metric (4.1) with radius $L\sin\mu = (32\pi^2 M/k)^{1/4}$, and M has to satisfy $M \leq N$. Therefore the stringy exclusion principle is realized in terms of giant D0-branes expanded into a twisted 5-sphere inside the CP^3.

In the maximal case, $\mu = \pi/2$, $M = N$ and the energy can be accounted for both by a bound state of N 't Hooft monopoles, with energy k/L, and a bound state of k dibaryons, with energy N/L. In this case the circle of the twisted 5-sphere shrinks to a point, and the NS5-brane collapses to a D4-brane wrapping the CP^2. This is so because when $\mu = \pi/2$ the τ and χ directions in the eleven dimensional background become parallel, and therefore the M5-brane wrapped on χ, a NS5-brane in Type IIA, and the M5-brane wrapped on τ, a D4-brane, become equivalent. We show explicitly in the next section how the maximal giant in Type IIA can be realized as a D4-brane wrapping the CP^2 with momentum N. As we have already mentioned this configuration cannot however be extended beyond the maximal case to give more general D4-brane giant graviton configurations.

4.2 A D4-brane giant graviton

Let us now use the metric (4.1) to describe a D4-brane wrapped on the "squashed" CP^2 with metric $g_{ij} = g_{ij}^{(CP^2)} + \cos^2\mu\, A_i A_j$, and propagating on the χ direction. This configuration is the IIA realization of the M5-brane wrapped on $S^1/\mathbb{Z}_k$ discussed in section 2.4. As in that case the configuration is greatly simplified if we introduce an electric flux proportional to the connection of the CP^2, $E_i = L\sin\mu\cos\mu\, A_i$. The role of this electric flux is to compensate the contribution of the $\cos^2\mu\, A_i A_j$ part of the metric to the Born-Infeld action. Since the inclusion of a magnetic flux inducing P_τ momentum in eleven dimensions did not allow the construction of more general giant graviton configurations we will take in this section $F_{ij} = 0$. More general configurations with non-vanishing D0-brane charge can be considered if $F_{ij} \neq 0$ which do not have however an interpretation as giant gravitons.

Substituting in the DBI action for a single D4-brane we find

$$S_{DBI} = -T_4 \int d^5\xi\, e^{-\phi}\sqrt{|\det(P[g] + 2\pi F)|} = -\frac{N}{L}\sin^4\mu \int dt \sqrt{1 - L^2\sin^2\mu\cos^2\mu\,\dot{\chi}^2} \tag{4.10}$$

The CS action gives in turn

$$S_{CS} = T_4 \int P[C_5] = N\sin^6\mu \int dt\,\dot{\chi} \tag{4.11}$$

Given that χ is cyclic P_χ is conserved, and the Hamiltonian reads

$$H = \frac{P_\chi}{L\sin\mu}\sqrt{1 + \tan^2\mu\left(1 - \frac{N}{P_\chi}\sin^4\mu\right)^2} \tag{4.12}$$

This is analogous to expression (2.27), with $R \leftrightarrow L$ and $P_\tau = 0$. As in that case the ground state is reached when $\mu = \pi/2$, for which $P_\chi = N$ and $H = N/L$. The $S^5/\mathbb{Z}_k$ expanded manifold reduces to a CP^2 and the giant is simply realized in Type IIA as a dibaryon. This analysis shows that the dibaryon arises in the gravity side as the limiting case of a D4-brane wrapping the "squashed" CP^2 included in

$$ds^2 = -dt^2 + L^2\sin^2\mu\left[\cos^2\mu\,(d\chi + A)^2 + ds^2_{CP^2}\right] \tag{4.13}$$

and propagating along the χ direction.

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5 The microscopical description in Type IIA

We have seen in section 3 that it is possible to describe the giant graviton configurations studied in section 2 in terms of gravitons expanding into fuzzy S^5 or $S^5/\mathbb{Z}_k$ manifolds inside the $S^7/\mathbb{Z}_k$ part of the eleven dimensional background. Reducing to Type IIA we find that the twisted NS5-brane with D0-brane charge is described in terms of Type IIA gravitons with D0-charge expanding into a fuzzy twisted 5-sphere inside the CP^3. The $S^5/\mathbb{Z}_k$ giant graviton is described in turn in terms of D0-branes with χ-momentum expanding into a fuzzy CP^2.

The action describing coincident gravitons in Type IIA was constructed in [43] by reducing along a transverse direction the action for M-theory gravitons reviewed in section 3. Using this action it was possible to reproduce Matrix String theory in various Type IIA pp-wave backgrounds. The BI part of the action reads:

$$S_{DBI} = -\int d\xi^0\, \mathrm{STr}\Bigg\{ \frac{1}{\sqrt{k^2+e^{2\phi}(i_kC_1)^2}} \sqrt{\Big(\mathbb{I}-(k^2+e^{2\phi}(i_kC_1)^2)[c_0,X]^2\Big)\,\mathrm{det}Q} \cdot \sqrt{\left|P[E]+\frac{k^2e^{2\phi}}{k^2+e^{2\phi}(i_kC_1)^2}\mathcal{F}_1^2\right|}\Bigg\} \tag{5.1}$$

where

$$\begin{aligned} E_{\mu\nu} &= \mathcal{G}_{\mu\nu}+\frac{e^{\phi}}{\sqrt{k^2+e^{2\phi}(i_kC_1)^2}}(i_kC_3)_{\mu\nu} \\ Q^i_j &= \delta^i_j + i[X^i,X^k]e^{-\phi}\sqrt{k^2+e^{2\phi}(i_kC_1)^2}E_{kj}\,, \qquad i,j=1,\dots,8 \end{aligned} \tag{5.2}$$

c_0 is the scalar field that comes from the reduction of the eleventh transverse direction and $\mathcal{F}_1$ is its invariant field strength $\mathcal{F}_1 = dc_0 + P[C_1]$, introduced in section 4.1. Therefore c_0 is associated to D0-branes ending on the waves. The first square root in the numerator in (5.1) comes from the reduction of the determinant of the nine dimensional Q matrix, whereas the second square root comes from the reduction of the pull-back of the metric. The terms coming from the reduction of $E_{\mu i}(Q^{-1}-\delta)^i_k E^{kj}E_{j\nu}$ have been omitted from this action, since they will not contribute to the calculation in this section.

The relevant terms in the dimensional reduction of the CS action are

$$S_{CS} = \int d\xi^0\, \mathrm{STr}\Bigg\{P[k^{-2}k_1]+\frac{e^{2\phi}i_kC_1}{k^2+e^{2\phi}(i_kC_1)^2}\mathcal{F}_1-\frac{1}{2}\frac{1}{(2\pi)^2}P[(i_Xi_X)^2i_kC_5]\wedge F\Bigg\} \tag{5.3}$$

In the $AdS_4\times CP^3$ background we take the gravitons located in $r=0$ and expanding into the fuzzy CP^2 with radius $L\sin\theta$ contained in the CP^3, which we parameterize with Cartesian coordinates as in section 3. We choose $k^\mu = \delta^\mu_\chi$ and take c_0 commuting and time-dependent, in order to induce D0-brane charge in the configuration. We then have:

$$\begin{aligned} k &= L\sin\mu\cos\mu\,, \qquad i_kC_1 = k\sin^2\mu\,, \qquad E_{00}=-1 \\ Q^i_j &= \delta^i_j - \frac{kL^2\sin^3\mu}{2\pi\sqrt{m^2+3m}}f_{ijk}X^k\,, \qquad i,j=1,\dots,8 \end{aligned} \tag{5.4}$$

Computing the determinant and substituting in the DBI action we find

$$S_{DBI} = -\frac{\mathcal{N}}{L\sin\mu}\left(1+\frac{k^2L^4\sin^6\mu}{16\pi^2(m^2+3m)}\right)\int dt\,\sqrt{1-\frac{L^2}{k^2}\cos^2\mu\,\dot{c_0}^2} \tag{5.5}$$

where $\mathcal{N}$ arises as $\dim(m,0) = \mathrm{STr}\,\mathbb{I}$. The CS part reads in turn

$$S_{CS} = \int dt\,\frac{\mathcal{N}}{k}\left(1+\frac{k^2L^4\sin^6\mu}{16\pi^2(m^2+3m)}\right)\dot{c}_0 \tag{5.6}$$

In terms of the conserved conjugate momentum to c_0, interpreted as D0-brane charge, M, we have the Hamiltonian

$$H = \frac{k}{L}M\sqrt{1+\tan^2\mu\left(1-\frac{\mathcal{N}}{kM\sin^2\mu}\left(1+\frac{2Nk\sin^6\mu}{m^2+3m}\right)\right)^2} \tag{5.7}$$

As discussed in section 3 we can describe NS5-branes with only D0-brane charge taking the limit $P_\chi \to 0$, which gives

$$H = \frac{k}{L}M\sqrt{1+\tan^2\mu\left(1-\frac{2N\mathcal{N}\sin^4\mu}{M(m^2+3m)}\right)^2} \tag{5.8}$$

in perfect agreememt with the Hamiltonian (4.8) in the large m limit.

Finally, we briefly sketch the microscopical description of the D4-brane maximal giant graviton of section 4.2. In this case we should have D0-branes with χ-momentum charge expanding into a fuzzy CP^2. Substituting in Myers action for $\mathcal{N}$ coincident D0-branes we find

$$L = k\mathcal{N}\left(-\frac{1}{L}+\dot{\chi}\right)\left(1+\frac{2N}{k(m^2+3m)}\right) \tag{5.9}$$

P_χ is simply given by $P_\chi = k\mathcal{N} + \frac{2N\mathcal{N}}{m^2+3m}$ and $H = \frac{P_\chi}{L}$. P_χ gives in the large m limit $P_\chi = k\mathcal{N} + N = kM + N$, in agreement with the result in section 4.2.[3]

6 Conclusions

We have constructed various giant graviton configurations of M5-branes expanded in $AdS_4\times S^7/\mathbb{Z}_k$ and discussed their realization in Type IIA.

The first configuration is an M5-brane wrapping an $S^5 \subset S^7/\mathbb{Z}_k$ and propagating on the $S^1/\mathbb{Z}_k$ direction. This is the trivial extension of the giant graviton in $AdS_4\times S^7$ to the $AdS_4\times S^7/\mathbb{Z}_k$ background. This brane has been described using the action for M5-branes wrapped on an isometric direction constructed in [23]. Since the M2-branes that end on these branes must be wrapped on the isometric direction the self-dual 2-form is replaced by a vector field and the action admits a closed form. This configuration becomes in Type IIA a motionless NS5-brane expanding into a twisted 5-sphere inside the CP^3. The ground state is a BPS configuration of giant D0-branes satisfying the stringy exclusion principle of [5].

[3] In section 4.2 we have taken $M = 0$ but $P_\chi = kM + N$ for $M \neq 0$.

The second configuration that we have analyzed is the extension of the former giant graviton to include an additional momentum along the isometric direction. We have seen however that although the ground state has the energy of a giant graviton propagating on the orbifold direction it is not protected by supersymmetry. In the maximal case the configuration becomes BPS and the additional angular momentum allows the giant graviton to move with an arbitrary angular momentum.

The last configuration that we have discussed is an M5-brane whose isometric direction points on the $S^1/\mathbb{Z}_k$ direction, propagating both on this orbifold direction and the S^1 fibre of the 5-sphere. The ground state corresponds to a giant graviton with maximum size wrapping a $S^5/\mathbb{Z}_k$ submanifold of $S^7/\mathbb{Z}_k$. The reduction to Type IIA gives a D4-brane wrapped on a "squashed" CP^2 inside the CP^3 and propagating along the S^1 fibre direction.[4] The ground state corresponds again to the maximal giant, with the squashed CP^2 becoming an ordinary CP^2 and the direction of propagation collapsing to a point. The configuration is then interpreted as a dibaryon, which arises through our analysis as the limiting case of a D4-brane wrapping a squashed CP^2 inside the CP^3 with non-vanishing angular momentum.

We should point out that in all the cases that we have considered the branes are effectively wrapped on a CP^2 and are therefore subject to the Freed-Witten anomaly [44]. The branes then carry a half-integer worldvolume magnetic flux that is compensated by a flat half-integer B_2-field to produce a vanishing field strength [45].

We have also addressed the microscopical description in terms of gravitons or D0-branes expanding into fuzzy manifolds, which in all cases involve a fuzzy CP^2. This microscopical description is complementary to the macroscopical one in terms of the expanded branes and allows to explore the region with finite 't Hooft coupling. In the microscopical description in terms of 0-branes the worldvolume magnetic flux needed to compensate the Freed-Witten anomaly does not couple in the action, and therefore one has to take into account the effect of the non-vanishing B_2. This was studied recently in [34] in the microscopical description of baryon-vertex like configurations in $AdS_4 \times CP^3$. There it was shown that the contribution of B_2 is subleading in a $1/m$ expansion and has no counterpart in the macroscopical set-up.

We should stress that we have not succeeded in finding a giant graviton expanding on the CP^3 in $AdS_4 \times CP^3$, other than the well-known dibaryon. The D4-brane wrapping the squashed CP^2 inside the CP^3 with non-vanishing momentum studied in section 4.2 is described by a Hamiltonian which is very similar to the one describing giant graviton solutions in other backgrounds. However, the ground state corresponds to the maximal giant, i.e. the dibaryon. It may be that inducing angular momentum on the NS5 or D4 brane configurations that we have analyzed by some mechanism similar to that used in [13] in order to construct the spinning dual giant graviton one may be able to obtain the elusive giant graviton solution in $AdS_4 \times CP^3$. The fact that our NS5-brane wraps a non-trivial twisted 5-sphere seems to be in agreement with the expectations in [46]. It would also

[4] With an additional D0-brane charge coming from the reduction of the momentum on the orbifold direction, although we have set this to zero in our calculation in section 4.2.

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be interesting to explore the connection with the approach taken in [16], which, based on the similarity between the Klebanov-Witten [47] and ABJM theories, tries to build a giant graviton solution similar to the D3-brane graviton blown up on the $T^{1,1}$ of the $AdS_5 \times T^{1,1}$ background. We hope to report progress in these directions in the near future.

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Non-singlet baryons in less supersymmetric backgrounds

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Abstract: We analyze the holographic description of non-singlet baryons in various backgrounds with reduced supersymmetries and/or confinement. We show that they exist in all $\mathrm{AdS}_5 \times Y_5$ backgrounds with Y_5 an Einstein manifold bearing five form flux, for a number of quarks $5N/8 < k \leqslant N$, independently on the supersymmetries preserved. This result still holds for γ_i deformations. In the confining Maldacena-Nuñez background non-singlet baryons also exist, although in this case the interval for the number of quarks is reduced as compared to the conformal case. We generalize these configurations to include a non-vanishing magnetic flux such that a complementary microscopical description can be given in terms of lower dimensional branes expanding into fuzzy baryons. This description is a first step towards exploring the finite 't Hooft coupling region.

Keywords: Gauge-gravity correspondence, D-branes, AdS-CFT Correspondence

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Contents

1 Introduction

Baryon configurations were first suggested in the context of the AdS/CFT [1] correspondence in [2, 3]. The gravitational dual of a bound state of N static external quarks in $\mathcal{N} = 4$ SYM, the so-called baryon vertex, was found in terms of a D5-brane wrapping the S^5 part of the spacetime geometry [2]. If the D5-brane is point-like in the AdS_5 space, its Chern-Simons (CS) action is a tadpole term which can be canceled if we introduce Chan-Paton factors for N-strings, whose endpoints at the boundary of AdS represent the N external quarks. The classical solution corresponding to this configuration was found in [4, 5] using a generalization of the techniques in [6, 7] for the heavy quark-antiquark system. In this approach the influence of the F-strings has to be considered in order to analyze the stability of the baryon vertex in the holographic AdS direction. The energy of the system is then inversely proportional to the distance between the quarks and since the proportionality constant is negative the configuration is stable in the AdS direction.

The description in [4, 5] suffices to deduce the basic properties of the system. However strictly speaking it is only valid when the endpoints of the N F-strings are uniformly distributed on the S^5, so that the latter is not deformed and the probe brane approximation holds. In this approximation all supersymmetries are broken, and this results in a non-vanishing binding energy. In order to have some supersymmetries preserved all strings should end on a point, and then the deformation caused by their tensions and charges should be taken into account. Incorporating the gauge field on the brane the binding energy becomes zero, reflecting the fact that the configuration is supersymmetric [8].

The usual baryon refers to a bound state of N-quarks which form the completely anti-symmetric representation of $\mathrm{SU}(N)$. In the holographic description however it is possible to construct a bound state of k-quarks with $k < N$ (figure 1). The bound state consists of a D5 or D3-brane wrapping the internal space[1] located in the bulk, k strings stretched between the brane and the boundary of AdS representing the quarks, and $N - k$ straight strings that go from the D5 or D3 brane deeper in the bulk to a minimum distance. The bound on how low can the k number go depends on a no-force condition along the AdS direction, and a priori seems to be affected by the geometry of both the internal and the AdS spaces. In the $\mathrm{AdS}_5 \times S^5$ background k should satisfy $5N/8 < k \leqslant N$ [4, 5]. A stability analysis against fluctuations shows that the configurations are stable for a more restricted number of quarks $0.813N \leqslant k \leqslant N$ [9]. An interesting question is what happens to the bound when the supersymmetry is reduced or the conformal invariance is broken and more particularly if confinement is present. A physical expectation would be that at least the lower bound should increase. One of the motivations of this paper is to investigate the bound dependence on the supersymmetry and confinement properties of the gauge theory.

Baryon vertex configurations in $\mathrm{AdS}_5 \times T^{1,1}$ [10] and $\mathrm{AdS}_5 \times Y^{p,q}$ [11–14] geometries have been considered in [16] and [17], respectively. Using the full DBI description it has been shown that they are non-supersymmetric. General properties of baryons in the Klebanov-Strassler [18] and Maldacena-Nuñez [19] models have also been discussed in [20]

[1] A submanifold of it in the case of the D3-brane.

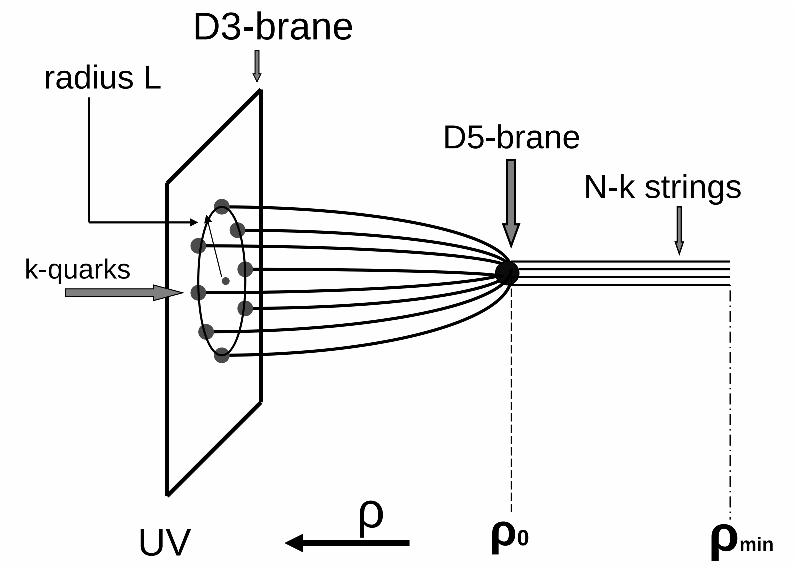

Figure 1. A baryon configuration with k-external quarks placed on a spherical shell of radius L at the boundary of AdS space, each connected to the wrapped Dp-brane located at $\rho = \rho_0$ and $N - k$ straight strings ending at $\rho_{\min}$.

(see also [21, 22]). In these confining backgrounds the baryon is also non-supersymmetric and is significantly different than in the previous cases, with an energy linearly proportional to its size.

In this paper we analyze the dynamics of non-singlet baryons in some of these backgrounds in the probe brane approach. We show that stable configurations exist with non-zero binding energy as long as the number of quarks k satisfies $k_{\min} < k \leqslant N$. The value of $k_{\min} = 5N/8$ for all $\mathrm{AdS}_5 \times Y_5$ backgrounds with Y_5 an Einstein manifold bearing five-form flux, and also for multi-β deformed spaces [23, 24]. The analysis on the deformed spaces basically gives the same undeformed results of $\mathcal{N} = 4$ SYM. This is not unexpected since classical properties like energy and temperature, string configurations, like the 1/4 BPS like Wilson loop, and brane configurations like particular giant gravitons remain also non trivially undeformed [25–27]. A stability analysis confirms that the configurations are stable for a number of quarks $0.813N \leqslant k \leqslant N$, again the same interval found for the $\mathrm{AdS}_5 \times S^5$ background [9]. These findings seem to contradict our expectations that non-singlet states should be more constrained in theories with reduced supersymmetry. Rather, their existence seems to be quite universal and independent on the amount of supersymmetries preserved. We should however keep in mind that the approach taken here breaks all the supersymmetries (see the conclusions for a further discussion on this point). The same analysis for the $\mathcal{N} = 1$ Maldacena-Nuñez background [19] confirms that non-singlet holographic baryons also exist in confining theories. However broken conformal invariance and more particularly confinement increases the minimum number of quarks.

More general baryon vertex configurations with a non-vanishing magnetic flux have been suggested as a first step towards accessing the finite 't Hooft coupling region in the dual CFT [28, 29]. Indeed, showing that these configurations exist for finite λ is of special

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interest when they are not BPS. Allowing for a non-trivial magnetic flux has the effect of adding lower dimensional brane charges to the configuration. This in turn hints at the existence of a microscopical description in terms of non-Abelian lower dimensional branes expanding into the baryon vertex by means of Myers dielectric effect [30]. This description allows to explore the configuration in the region $R \ll n^{1/(r-p)}\, l_s$, where p is the dimensionality and n the number of expanding branes and r the dimensionality of the resulting expanded brane, and is therefore complementary to the supergravity description in terms of probe branes. Thus it is a first step towards exploring the finite 't Hooft coupling region of the dual CFT from the gravity side.

The paper is organized as follows. We start in section 2 with a brief review of the holographic description of baryon vertices and their stability under small fluctuations for a general class of backgrounds. In section 3 we use these results to study the dynamics of the baryon vertex in $\mathrm{AdS}_5 \times Y_5$, with Y_5 an Einstein manifold bearing five-form flux. We particularize to the $\mathrm{AdS}_5 \times Y^{p,q}$ and $\mathrm{AdS}_5 \times T^{1,1}$ geometries, where we switch on a non-vanishing magnetic flux suitable for the microscopical description of the $T^{1,1}$ in section 6. In section 4 the multi-β-deformed Frolov's background is considered. In section 5 we analyze the Maldacena-Nuñez background, where we confirm the existence of non-singlet baryons for a more constrained interval for k due to confinement. We show that in this case the stability requirement does not reduce the allowed interval. In section 6 we perform the microscopical analysis, in terms of D1 or D3-branes, depending on the background. We identify the CS couplings responsible for the F-string tadpoles of the configurations. In section 7 we summarize our results and discuss further directions. Finally, in the appendix we collect some properties of the $Y^{p,q}$ and $T^{1,1}$ geometries relevant for our analysis and address the microscopical description of the baryon vertex in the $Y^{p,q}$ geometries.

2 The holographic baryon vertex construction

In this section we review the holographic description of baryons in the general class of backgrounds presented in [9], as well as the study of their stability against small fluctuations. The first part generalizes the construction in [4, 5] to non-conformal cases like the Maldacena-Nuñez background that we will discuss in section 5.

We consider diagonal metrics of Lorentzian signature of the form

$$ds^2 = G_{tt}dt^2 + G_{xx}(dx^2 + dy^2 + dz^2) + G_{\rho\rho}d\rho^2 + R^2 d\mathbb{M}_p^2\,, \tag{2.1}$$

where x, y and z denote cyclic coordinates and ρ denotes the radial direction playing the role of an energy scale in the dual gauge theory. It extends from the UV at $\rho \to \infty$ down to the IR at some minimum value $\rho_{\min}$ determined by the geometry.
It is convenient to introduce the functions

$$f(\rho) = -G_{tt}G_{xx}\,, \qquad g(\rho) = -G_{tt}G_{\rho\rho}\,, \qquad h(\rho) = G_{xx}G_{\rho\rho}\,, \tag{2.2}$$

which for $\mathrm{AdS}_5 \times \mathbb{M}_5$ with radii R read

$$f(\rho) = \rho^4, \qquad g(\rho) = 1\,, \qquad h(\rho) = 1\,. \tag{2.3}$$

As we have mentioned, a non-singlet baryon is described holographically in terms of a Dp-brane wrapping the internal manifold $\mathbb{M}_p$ with k fundamental strings connecting it to the boundary at $\rho \to \infty$. The remaining $N-k$ straight strings go from the Dp-brane straight up at $\rho_{\min}$. The binding potential energy of the baryon is then given by $e^{-\mathrm{i}ET} = e^{\mathrm{i}S_{\mathrm{cl}}}$, where S_{cl} is the classical action of the holographic baryon. This action consists of three terms, the Nambu-Goto action for the strings stretching from the baryon vertex to the boundary at $\rho \to \infty$, the Nambu-Goto action for the straight strings stretching between the brane and $\rho_{\min}$ and the Dirac-Born-Infeld action for the Dp-brane

$$\begin{aligned} S_{F1} &= -\frac{1}{2\pi}\int d\tau d\sigma \sqrt{-\det P(G_{\alpha\beta})}\,, \\ S_{Dp}^{\mathrm{DBI}} &= -T_p \int_{\mathbb{R}\times\mathbb{M}_p} d^{p+1}\xi \,\sqrt{-\det P(G_{ab}+2\pi F_{ab}-B_{ab})}\,, \end{aligned}$$

where F is the Born-Infeld field strength.
We first fix reparametrization invariance for each string by choosing

$$t=\tau\,, \qquad \rho=\sigma\,. \tag{2.4}$$

For static solutions we consider the embedding of the S^2-sphere on the D3-brane in spherical coordinates (r,θ,ϕ)

$$r=r(\rho)\,, \qquad (\theta,\phi)=\mathrm{const.}\,, \tag{2.5}$$

plus $\mathbb{M}_p$-angles = const., supplemented by the boundary condition

$$\rho(L)=\infty\,. \tag{2.6}$$

Then, the Nambu-Goto action for the strings stretching from the baryon vertex to the boundary of AdS reads

$$S=-\frac{T}{2\pi}\int_{\rho_0}^{\infty} d\rho\,\sqrt{g(\rho)+f(\rho)r'^2}\,, \tag{2.7}$$

where T denotes time and the prime denotes a derivative with respect to ρ. From the Euler-Lagrange equations of motion we obtain

$$\frac{f r'_{\mathrm{cl}}}{\sqrt{g+f r'^2_{\mathrm{cl}}}} = f_1^{1/2} \qquad \Longrightarrow \qquad r'_{\mathrm{cl}} = \frac{\sqrt{f_1 F}}{f}\,, \tag{2.8}$$

where ρ_1 is the value of ρ at the turning point of each string, $f_1 \equiv f(\rho_1)$, $f_0 \equiv f(\rho_0)$ and

$$F=\frac{gf}{f-f_1}\,. \tag{2.9}$$

The $N-k$ strings which extend from the baryon vertex to $\rho=\rho_{\min}$ are straight, since $r'=0$ is a solution of the equations of motion (with $f_1=0$) and satisfies the boundary condition at the vertex. Integrating (2.8) we can express the radius of the spherical shell as

$$L=\sqrt{f_1}\int_{\rho_0}^{\infty} d\rho\frac{\sqrt{F}}{f}\,. \tag{2.10}$$

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Next we fix the reparametrization invariance for the wrapped Dp-brane by choosing

$$t = \tau\,, \qquad \theta_a = \sigma_\alpha\,, \qquad \alpha = 1, 2, \dots, p\,. \tag{2.11}$$

Finally, inserting the solution for $r'_{\rm cl}$ into (2.7) and subtracting the divergent energy of its constituents we can write the binding energy of the baryon as

$$E = \frac{k}{2\pi}\left\{\int_{\rho_0}^{\infty} d\rho\sqrt{F} - \int_{\rho_{\rm min}}^{\infty} d\rho\sqrt{g} + \frac{1-a}{a}\int_{\rho_{\rm min}}^{\rho_0} d\rho\sqrt{g} + \frac{2\pi}{aN}\, E_{Dp}\Big|_{\rho=\rho_0}\right\}, \tag{2.12}$$

where

$$a \equiv \frac{k}{N}\,, \qquad 0 < a \leqslant 1\,. \tag{2.13}$$

The expressions for the length and the energy, (2.10) and (2.12), depend on the arbitrary parameter ρ_1 which should be expressed in terms of the baryon vertex position ρ_0. The most convenient way to find this is to impose that the net force at the baryon vertex is zero [9, 31]

$$\begin{aligned}\cos\Theta &= \frac{1-a}{a} + \frac{2\pi}{aN}\frac{1}{\sqrt{g}}\,\partial_\rho E_{Dp}\Big|_{\rho=\rho_0}\,,\\ \cos\Theta &= \sqrt{1 - f_1/f_0}\,,\end{aligned} \tag{2.14}$$

where Θ is the angle between each of the k-strings and the ρ-axis at the baryon vertex, which determines ρ_1 in terms of ρ_0. An alternative derivation of this expression can be found by demanding that the physical length (2.10) does not depend on the arbitrary parameter ρ_1, in other words

$$\frac{\partial L}{\partial \rho_1} = 0 \quad \Longrightarrow \quad \frac{\partial \rho_0}{\partial \rho_1} = \frac{f'_1}{2\tan\Theta}\sqrt{\frac{f_0}{g_0 f_1}}\int_{\rho_0}^{\infty} d\rho\frac{\sqrt{gf}}{(f-f_1)^{3/2}}\,. \tag{2.15}$$

Minimizing the energy (2.12) with respect to ρ_1 and using (2.15) we find the no-force condition (2.14). Using (2.10), (2.12) and (2.14) it is also possible to see that

$$\frac{dE}{d\rho_0} = \frac{k\sqrt{f_1}}{2\pi}\frac{dL}{d\rho_0} \tag{2.16}$$

which will be useful when we study the Maldacena-Nuñez background.

As we will see in the examples to follow, (2.14) has a solution for a parametric region of (a, ρ_0). However, in order to isolate parametric regions of physical interest a stability analysis of the classical solution should be performed, which further restricts the allowed region. We know from [9] that instabilities can only emerge from longitudinal fluctuations of the k strings, since only these possess a non-divergent zero mode, which is a sign of instability. To study the fluctuations about the classical solution the embedding should be perturbed according to

$$r = r_{\rm cl} + \delta r(\rho)\,, \tag{2.17}$$

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and the Nambu-Goto action should be expanded to quadratic order in the fluctuations. δr is then solved from the equation

$$\frac{d}{d\rho}\left(\frac{gf}{F^{3/2}}\frac{d}{d\rho}\right)\delta r = 0 \tag{2.18}$$

This has to be supplemented with the boundary condition for the δr fluctuations, given by equation (3.12) in [9]

$$2(f-f_1)\delta r' + \delta r\left(2f' - \frac{f'}{f}f_1 - \frac{g'}{g}(f-f_1)\right) = 0 \qquad \text{at} \quad \rho = \rho_0 \tag{2.19}$$

As we will see in the examples to follow these conditions further restrict the parametric region (a, ρ_0) for which a classical non-singlet baryon solution exists.

3 The baryon vertex in $\mathbf{AdS_5 \times Y_5}$ manifolds

The holographic description of the baryon vertex in $\mathrm{AdS}_5 \times Y_5$ backgrounds with Y_5 an Einstein manifold bearing five-form flux is identical, in the probe brane approximation, to that in $\mathrm{AdS}_5 \times S^5$ [4, 5]. Therefore non-singlet states exist for the same number of fundamental strings $5N/8 < k \leqslant N$. Spike solutions associated to the baryon vertices in the $\mathrm{AdS}_5 \times Y^{p,q}$ and $\mathrm{AdS}_5 \times T^{1,1}$ geometries have been discussed in [17] and [16], where it has been shown that they break all the supersymmetries. Therefore we are certain that the bound states found in the probe brane approximation will not become marginal due to supersymmetry once the backreaction is taken into account. In these two geometries we will switch on a magnetic flux that will dissolve D1 and D3-brane charges in the configuration. The vertex will then be described at finite 't Hooft coupling in terms of D1-branes expanding into a fuzzy $S^2 \times S^2$ submanifold of the $T^{1,1}$ for the Klebanov-Witten background and D3-branes expanding into a fuzzy S^2 submanifold of the $Y^{p,q}$ for the Sasaki-Einstein. The detailed microscopical analysis of these configurations will be performed in section 6 and the appendix respectively.

3.1 The D5-brane baryon vertex

In our conventions the $\mathrm{AdS}_5 \times Y_5$ metric reads

$$ds^2 = \frac{\rho^2}{R^2}dx^2_{1,3} + \frac{R^2}{\rho^2}d\rho^2 + R^2 ds^2_{Y_5}\,, \tag{3.1}$$

with R the radius of curvature in string units,

$$R^4 = \frac{4\pi^4 N g_s}{\mathrm{Vol}(Y_5)}\,. \tag{3.2}$$

The $\mathrm{AdS}_5 \times Y_5$ flux is given by $F_5 = (1 + \star_{10})\mathcal{F}_5$, where

$$\mathcal{F}_5 = 4\,R^4\, d\mathrm{Vol}(Y_5)\,. \tag{3.3}$$

A D5-brane wrapping the whole Y_5 captures the F_5 flux, and it requires the addition of N fundamental strings to cancel the tadpole

$$S_{\rm D5}^{\rm CS} = 2\pi\, T_5 \int_{\mathbb{R}\times Y_5} P[C_4]\wedge F = -2\pi\, T_5 \int_{\mathbb{R}\times Y_5} P[F_5]\wedge A = -N\int dt A_t\,, \tag{3.4}$$

where A is the Born-Infeld vector field. The DBI action is in turn given by

$$S_{\rm D5}^{\rm DBI} = -T_5 \int_{\mathbb{R}\times Y_5} d^6\xi\, e^{-\phi}\sqrt{-{\rm det}P(G)} = -\frac{TN}{8\pi}\rho_0\,. \tag{3.5}$$

3.1.1 Classical solution

Given that the energy of the D5-brane is independent of the volume of the Einstein manifold the classical solution in the probe brane approximation is the one found in [4, 5] for $\mathrm{AdS}_5\times S^5$. Making contact with the analysis in the previous section we now have

$$-\,G_{tt} = G_{xx} = G_{\rho\rho}^{-1} = \frac{\rho^2}{R^2}\,. \tag{3.6}$$

The radius and the energy are then given in terms of the position of the D5-brane ρ_0 and the turning point ρ_1 of each string, as

$$L = \frac{R^2\rho_1^2}{3\rho_0^3}\,\mathcal{I}\,, \qquad E = \frac{k\rho_0}{2\pi}\Big(-\mathcal{J} + \frac{5-4a}{4a}\Big)\,, \tag{3.7}$$

with $\mathcal{I}$, $\mathcal{J}$ the hypergeometric functions

$$\mathcal{I} = {}_2F_1\Big(\frac{1}{2},\frac{3}{4},\frac{7}{4};\frac{r_1^4}{r_0^4}\Big)\,, \qquad \mathcal{J} = {}_2F_1\Big(-\frac{1}{4},\frac{1}{2},\frac{3}{4};\frac{r_1^4}{r_0^4}\Big)\,, \tag{3.8}$$

exactly as in $\mathrm{AdS}_5\times S^5$. From (2.14) we find that the no-force condition on the ρ-axis yields

$$\rho_1 = \rho_0(1-\lambda^2)^{1/4}, \qquad \lambda = \frac{5-4a}{4a}\,, \qquad a\equiv\frac{k}{N}\,. \tag{3.9}$$

Given that $\lambda<1$ a baryon configuration exists for $a>a_<$ with $a_< = \frac{5}{8}$. Finally, the binding energy in terms of the physical length of the baryon reads

$$E = -\frac{R^2}{2\pi L}\frac{k\sqrt{1-\lambda^2}}{3}\Big(\mathcal{J} - \frac{5-4a}{4a}\Big)\mathcal{I}\,. \tag{3.10}$$

Thus it has both the expected behavior with $1/L$ dictated by conformal invariance and the non-analyticity of square-root branch cut type in the 't Hooft parameter [4, 6, 7, 29]. We would also like to point out that our string and brane configurations satisfy the Sasaki-Einstein constrains in the way studied in [32–34] and therefore our solutions are in this sense valid.

3.1.2 Stability analysis

Again as in $\mathrm{AdS}_5 \times S^5$ [9] the study of the stability against longitudinal fluctuations gives

$$\delta r(\rho) = A \int_\rho^\infty d\rho \frac{\rho^2}{(\rho^4 - \rho_1^4)^{3/2}} = \frac{A}{3\rho^3}\, {}_2F_1\left(\frac{3}{4}, \frac{3}{2}, \frac{7}{4}; \frac{\rho_1^4}{\rho^4}\right) \tag{3.11}$$

as the solution of equation (2.18). Substituting (3.9) and (3.11) in the boundary equation (2.19) the following transcendental equation must be satisfied

$${}_2F_1\left(\frac{3}{4}, \frac{3}{2}, \frac{7}{4}; 1 - \lambda^2\right) = \frac{3}{2\lambda(1+\lambda^2)}\,. \tag{3.12}$$

Using (3.9) and (3.12) a critical value for a is found numerically, $a \simeq 0.813$, below which the system becomes unstable.

The conclusion of this analysis is that in the probe brane approximation non-singlet baryons with $0.813 < k \leqslant N$ may exist for all Einstein internal manifolds bearing five-form flux. In the next subsection we take the internal manifold to be Sasaki-Einstein and we switch on an instantonic magnetic flux proportional to the Kähler form. The $T^{1,1}$ and S^5 cases will be treated as particular examples, taking due care of the different periodicities. The energy of the D5-brane will depend then on both the magnetic flux and the radius of AdS, and the same calculation above shows that non-singlet states exist as long as the number of quarks is larger than a minimum value that depends now on the volume of the $Y^{p,q}$. In fact the largest minimum value is reached for the S^5, contrary to our expectations that non-singlet baryons would be more restricted in less supersymmetric backgrounds. We review some basic facts about the geometry of $Y^{p,q}$ manifolds suitable for this study in the appendix.

3.2 The baryon vertex in $\mathrm{AdS}_5 \times Y^{p,q}$ with magnetic flux

Let us take the $\mathrm{AdS}_5 \times Y^{p,q}$ geometry and add a magnetic flux

$$F = \mathcal{N} J\,, \tag{3.13}$$

with J the Kähler form of the 4 dimensional Kähler-Einstein submanifold of the $Y^{p,q}$, which solves the equations of motion. As compared to the analysis in the previous subsection the presence of the magnetic flux will turn the parametric region for which a classical solution exists to depend on $(a, \rho_0, \mathcal{N})$.

For a non-vanishing $\mathcal{F}$ as above the energy of the D5-brane wrapped on the $Y^{p,q}$ is modified according to

$$E_{\mathrm{D5}} = \frac{N}{8\pi}\rho_0\left(1 + \frac{4\pi^2\mathcal{N}^2}{R^4}\right) \tag{3.14}$$

where we have used (A.11), and the fact that J is self-dual and the determinant inside the square root is a perfect square.

This magnetic flux dissolves irrational D1-brane charge in the $Y^{p,q}$, as inferred from the coupling

$$S_{\mathrm{D5}}^{\mathrm{CS}} = \frac{1}{2}(2\pi)^2 T_5 \int_{\mathbb{R}\times Y^{p,q}} C_2 \wedge F \wedge F = \frac{\mathcal{N}^2}{8} \frac{q^2[2p + (4p^2-3q^2)^{1/2}]}{p^2[3q^2 - 2p^2 + p(4p^2-3q^2)^{1/2}]}\, T_1 \int_{\mathbb{R}\times S^1_\psi} C_2 \tag{3.15}$$

This implies that the configuration will not allow a complementary description in terms of D1-branes expanding into a fuzzy 4 dimensional submanifold of the $Y^{p,q}$. We will see however that it will be possible to provide such a description in terms of D3-branes expanding into a fuzzy 2-sphere submanifold of the $Y^{p,q}$. In this case the magnetic flux that needs to be switched on will be proportional to the Kähler form on the S^2. We postpone this discussion to the appendix.

In the $T^{1,1}$ case (see appendix A.2 for a brief discussion of the $T^{1,1}$ geometry) our ansatz (3.13) dissolves $\mathcal{N}^2/9$ D1-brane charge in the $T^{1,1}$, as implied by

$$S^{\rm CS}_{\rm D5} = \frac{1}{2}(2\pi)^2 T_5 \int_{\mathbb{R}\times T^{1,1}} C_2 \wedge F \wedge F = \frac{\mathcal{N}^2}{9} T_1 \int_{\mathbb{R}\times S^1_\psi} C_2 \tag{3.16}$$

where we have used the second condition in (A.18). But in this case $\mathcal{N}^2/9$ is an integer due to Dirac quantization condition plus the first equation in (A.18). In this case a microscopical description in terms of expanding D1-branes will make sense, as we will show explicitly in section 6.

Note that in fact for the $T^{1,1}$ we can take a more general ansatz for the magnetic flux, namely $F = \mathcal{N}_1\, J_1 + \mathcal{N}_2\, J_2$, with J_1, J_2 the Kähler forms on each of the S^2's contained in the $T^{1,1}$. In this case the magnetic flux is dissolving $\mathcal{N}_1/3$ and $\mathcal{N}_2/3$ D3-brane charge in each S^2, and $\mathcal{N}_1\mathcal{N}_2/9$ D1-brane charge in $S^2 \times S^2$, as inferred from the couplings

$$S^{\rm CS}_{\rm D5} = 2\pi\, T_5 \int_{\mathbb{R}\times T^{1,1}} C_4 \wedge F = \frac{\mathcal{N}_1}{3}\, T_2 \int_{\mathbb{R}\times S^1_\psi \times S^2_2} C_4 + \frac{\mathcal{N}_2}{3}\, T_2 \int_{\mathbb{R}\times S^1_\psi \times S^1_2} C_4 \tag{3.17}$$

and

$$S^{\rm CS}_{\rm D5} = \frac{1}{2}(2\pi)^2 T_5 \int_{\mathbb{R}\times T^{1,1}} C_2 \wedge F \wedge F = \frac{\mathcal{N}_1\mathcal{N}_2}{9}\, T_1 \int_{\mathbb{R}\times S^1_\psi} C_2\,. \tag{3.18}$$

Therefore $\mathcal{N}_1, \mathcal{N}_2 \in 3\mathbb{Z}$, in agreement with Dirac quantization condition, as implied from (A.18). In this case the energy of the D5 is modified according to

$$E_{\rm D5} = \frac{N}{8\pi}\rho_0 \sqrt{1 + \frac{4\pi^2\mathcal{N}_1^2}{R^4}}\sqrt{1 + \frac{4\pi^2\mathcal{N}_2^2}{R^4}}\,. \tag{3.19}$$

Coming back to the general case for $Y^{p,q}$ manifolds, $F = \mathcal{N} J$, with J the Kähler form of the 4 dimensional Kähler-Einstein submanifold of the $Y^{p,q}$, from (2.14) we find that the no-force condition on the ρ-axis yields

$$\rho_1 = \rho_0 (1 - \lambda_{\rm eff}^2)^{1/4}, \qquad \lambda_{\rm eff} = \frac{5 - 4a_{\rm eff}}{4a_{\rm eff}}\,, \tag{3.20}$$

where $a_{\rm eff}$ includes now the magnetic flux

$$a_{\rm eff} \equiv \frac{a}{1 + \frac{4\pi^2\mathcal{N}^2}{5R^4}}\,. \tag{3.21}$$

Given that $\lambda_{\rm eff} < 1$ a baryon configuration exists for

$$a_{\rm eff} > a_< \qquad \text{with} \qquad a_< = \frac{5}{8} + \frac{\pi^2\mathcal{N}^2}{2R^4}\,. \tag{3.22}$$

In terms of the volume of the $Y^{p,q}$ this reads

$$a_< = \frac{5}{8} + \frac{\mathcal{N}^2}{8\pi^2 N} \mathrm{Vol}(Y^{p,q}) \,, \tag{3.23}$$

so the bound depends now on the volume of the internal manifold. The largest volume given by the $Y^{p,q}$ metrics occurs for the $Y^{2,1}$, for which $\mathrm{Vol}(Y^{2,1}) \approx 0.29\pi^3$. Therefore we have that $\pi^3 = \mathrm{Vol}(S^5) > 16/27\,\pi^3 = \mathrm{Vol}(T^{1,1}) > \mathrm{Vol}(Y^{2,1})$ and $a_<$ is maximum for the S^5, the maximally supersymmetric case. Note that since $a_{\mathrm{eff}} \leqslant 1$ there is also a bound on the instanton number, namely

$$a_< \leqslant 1 \quad \Rightarrow \quad \frac{\mathcal{N}^2}{R^4} \leqslant \frac{3}{4\pi^2} \simeq 0.0761 \,. \tag{3.24}$$

Finally, the binding energy in terms of the physical length of the baryon reads

$$E = -\frac{R^2}{2\pi L} \frac{k\sqrt{1-\lambda_{\mathrm{eff}}^2}}{3} \left(\mathcal{J} - \frac{5 - 4a_{\mathrm{eff}}}{4a_{\mathrm{eff}}} \right) \mathcal{I} \,. \tag{3.25}$$

3.2.1 Stability analysis

The study of the stability against longitudinal fluctuations gives again $\delta r(\rho)$ as in (3.11) where now ${}_2F_1(a,b,c;x)$ must satisfy [9]

$${}_2F_1\left(\frac{3}{4}, \frac{3}{2}, \frac{7}{4}; 1-\lambda_{\mathrm{eff}}^2\right) = \frac{3}{2\lambda_{\mathrm{eff}}(1+\lambda_{\mathrm{eff}}^2)} \,. \tag{3.26}$$

The critical value for a_{eff} that is found numerically is again $a_{\mathrm{eff}} \simeq 0.813$, below which the system becomes unstable. This improves the above bound for the instanton number, in comparison to the 't Hooft coupling, to

$$\frac{\mathcal{N}^2}{R^4} \lesssim 0.00291 \,, \tag{3.27}$$

that should be respected for the classical configuration not only to exist, but also to be perturbatively stable. Thus, the stability analysis sets a low bound for a which is still less than unity.

4 The baryon vertex in β-deformed backgrounds

The description of the baryon vertex in these backgrounds is essentially identical to the one performed in the previous section. Even though the C_2 and B_2 potentials are non-vanishing the tadpole introduced with the brane has still charge N, so it has to be compensated with the same number of fundamental strings attached. Moreover, the energy of the D5-brane wrapped on the deformed S^5 is the same as the one wrapped on the S^5. Therefore we find the same bound for the number of quarks that can form non-singlet baryons.

We discuss the general case of multi $\hat{\gamma}_i$-deformations [24], from which the one- parameter Lunin-Maldacena background [23] is obtained for all $\hat{\gamma}_i$ equal. As we have mentioned, in this background the baryon vertex is described in terms of a D5-brane wrapped on the deformed S^5, with N fundamental strings attached. In the last subsection we will switch on a magnetic flux that on the one hand will increase the parametric space on which classical solutions exist and on the other hand will allow a complementary description in terms of expanding D3-branes suitable for the discussion of the finite 't Hooft coupling regime of the dual gauge theory.

The multi-$\hat{\gamma}_i$ deformed background reads in string frame [24]

$$
\begin{aligned}
ds^2 &= R^2\left[ds^2_{\mathrm{AdS}_5} + \sum_i (d\mu_i^2 + \mathcal{G}\mu_i^2 d\phi_i^2) + \mathcal{G}\mu_1^2\mu_2^2\mu_3^2\Big(\sum_i \hat{\gamma}_i d\phi_i\Big)^2\right], \qquad (4.1)\\
e^{2\phi} &= \mathcal{G}\,, \qquad \mathcal{G}^{-1} = 1 + \hat{\gamma}_3^2\,\mu_1^2\mu_2^2 + \hat{\gamma}_1^2\,\mu_2^2\mu_3^2 + \hat{\gamma}_2^2\,\mu_3^2\mu_1^2\,.\\
B_2 &= R^2\mathcal{G}\left(\hat{\gamma}_3\,\mu_1^2\mu_2^2\,d\phi_1\wedge d\phi_2 + \hat{\gamma}_1\,\mu_2^2\mu_3^2\,d\phi_2\wedge d\phi_3 + \hat{\gamma}_2\,\mu_3^2\mu_1^2\,d\phi_3\wedge d\phi_1\right),\\
C_2 &= -4R^2\omega_1\wedge(\hat{\gamma}_1 d\phi_1 + \hat{\gamma}_2 d\phi_2 + \hat{\gamma}_3 d\phi_3)\,,\\
C_4 &= \omega_4 + 4R^4\mathcal{G}\,\omega_1\wedge d\phi_1\wedge d\phi_2\wedge d\phi_3\,,
\end{aligned}
$$

where μ_i and ϕ_i parameterize a deformed five-sphere, so that we can write:

$$
\begin{aligned}
&\mu_1 = \cos\alpha\,, \qquad \mu_2 = \sin\alpha\cos\theta\,, \qquad \mu_3 = \sin\alpha\sin\theta\,, \qquad \sum_{i=1}^{3}\mu_i^2 = 1\,,\\
&(\alpha,\theta)\in[0,\pi/2]\,, \qquad d\omega_1 = \cos\alpha\sin^3\alpha\sin\theta\cos\theta d\alpha\wedge d\theta\,, \qquad d\omega_4 = \omega_{\mathrm{AdS}_5}\,.
\end{aligned} \qquad (4.2)
$$

For equal $\hat{\gamma}_i$ parameters $\hat{\gamma}_1 = \hat{\gamma}_2 = \hat{\gamma}_3 = \hat{\gamma}$, $\hat{\gamma}$ is related to the deformation parameter β of the gauge theory through [35]:

$$
\hat{\gamma} = R^2\,\beta\,. \qquad (4.3)
$$

4.1 The D5-brane baryon vertex

Let us now consider a D5-brane wrapping the deformed S^5 in (4.1). This brane captures the $F_5 - F_3\wedge B_2$ flux of the background but still requires N fundamental strings to cancel the tadpole

$$
\begin{aligned}
S^{\mathrm{CS}}_{\mathrm{D5}} &= 2\pi\,T_5\int_{\mathbb{R}\times\tilde{S}^5} P[C_4 - C_2\wedge B_2]\wedge F = -2\pi\,T_5\int_{\mathbb{R}\times\tilde{S}^5} P[F_5 - F_3\wedge B_2]\wedge A =\\
&= -N\int dt A_t
\end{aligned} \qquad (4.4)
$$

since

$$
F_5 - F_3\wedge B_2 = \omega_{\mathrm{AdS}_5} + 4R^4\,d\omega_1\wedge d\phi_1\wedge d\phi_2\wedge d\phi_3\,, \qquad (4.5)
$$

as in the $\mathrm{AdS}_5\times S^5$ case. Therefore the CS part of the action is undeformed. The DBI action is in turn given by

$$
S^{\mathrm{DBI}}_{\mathrm{D5}} = -T_5\int_{\mathbb{R}\times\tilde{S}^5} d^6\xi\, e^{-\phi}\sqrt{-\det P(G + 2\pi F - B_2)}\,. \qquad (4.6)
$$

For $F = 0$ the determinant of the pull-back of $G - B_2$ can be written as

$$
\Delta_1 = G_{tt}G_{\alpha\alpha}G_{\theta\theta}\det\Gamma\,, \qquad (4.7)
$$

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where Γ is a 3×3 matrix of the form $G_{ab} - B_{ab}$ for the three U(1) directions. We then get

$$\Delta_1 = G_{tt}\, R^{10} \sin^6 \alpha \cos^2 \alpha \sin^2 \theta \cos^2 \theta\, \mathcal{G}\,, \tag{4.8}$$

such that the DBI action remains also undeformed.

Given that the AdS part of the background remains untouched by the deformation the contribution of the fundamental strings stretching from the D5 to the boundary of AdS is the same as in the undeformed case. The only issue here would be that the binding energy was modified due to the dependence of the D5-brane on the deformation parameter. We have shown however that this dependence drops out both in the CS and DBI actions. Therefore the size and binding energy of the baryon remain undeformed, and coincide with those in $\mathcal{N} = 4$. The classical solution and its stability analysis are therefore identical to those performed in section 3. Last but not least, we should mention that for marginally-deformed backgrounds there are cases on which the classical solution coincides with $\mathcal{N} = 4$, does not depend on the deformation parameter, but the stability analysis even for the conformal case requires an upper value on the imaginary part of the deformation parameter σ as in the case of mesons [36].

4.1.1 Adding a magnetic flux

Finally we can switch on a magnetic flux $F = \mathcal{N} J$, with $\mathcal{N} \in \mathbb{Z}/2$ and J the Kähler form of the S^2 parameterized by (α, θ), dissolving $2\mathcal{N}$ units of D3-brane charge in the baryon. This will allow a microscopical description in terms of expanding D3-branes from which the finite 't Hooft coupling region can be studied. The DBI action changes as

$$S^{\rm DBI}_{\rm D5} = -\frac{TN}{8\pi} \rho_0 \sqrt{1 + \frac{4\pi^2 \mathcal{N}^2}{R^4}}\,. \tag{4.9}$$

In the presence of a magnetic flux the minimum number of quarks forming a non-singlet baryon is modified and there is a maximum on the magnetic flux that can be dissolved in the baryon, in parallel with what we have found in the previous section.

5 The baryon vertex in the Maldacena-Nuñez background

The Maldacena-Nuñez background [19] is a solution to Type IIB supergravity dual to a $\mathcal{N} = 1$ supersymmetric confining gauge theory. It can be obtained as a solution of seven dimensional gauged supergravity [37, 38], uplifted to ten dimensions. Given that this background is confining we expect that the universality of the baryon vertex configurations found in the previous conformal examples (in the absence of a magnetic flux) is lost. This is indeed confirmed by the analysis in this section.

5.1 The Maldacena-Nuñez background

The ten-dimensional metric reads in the string frame

$$ds^2_{10} = e^{\phi} \left[dx^2_{1,3} + g_s N \left(e^{2h} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + d\rho^2 + \frac{1}{4} (w^i - A^i)^2 \right) \right], \tag{5.1}$$

where ϕ is the dilaton, h is a function of the radial coordinate ρ, the one-forms A^i ($i = 1,2,3$) are the components of the non-abelian gauge vector field of the seven-dimensional gauged supergravity,

$$A^1 = -a(\rho)d\theta_1\,, \qquad A^2 = a(\rho)\sin\theta_1 d\phi_1\,, \qquad A^3 = -\cos\theta_1 d\phi_1\,, \tag{5.2}$$

and the w^i's are the right-invariant Maurer-Cartan dreibeins of SU(2), satisfying $dw^i = -\frac{1}{2}\,\varepsilon_{ijk}\,w^j \wedge w^k$. They define a three-sphere that can be parameterized as

$$\begin{aligned} w^1 &= \cos\psi\, d\theta_2 + \sin\psi \sin\theta_2\, d\phi_2\,, \\ w^2 &= -\sin\psi\, d\theta_2 + \cos\psi\sin\theta_2\, d\phi_2\,, \\ w^3 &= d\psi + \cos\theta_2\, d\phi_2\,. \end{aligned} \tag{5.3}$$

The angles $\theta_\alpha, \phi_\alpha\,, \alpha = 1,2$ and ψ take values in the intervals $\theta_i \in [0,\pi]$, $\phi_i \in [0,2\pi]$ and $\psi \in [0,4\pi]$. The functions $a(\rho)$, $h(\rho)$ and the dilaton $\phi(\rho)$ are given by

$$a(\rho) = \frac{2\rho}{\sinh 2\rho}\,, \qquad\qquad e^{2h} = \rho\coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4}\,, \tag{5.4}$$

$$e^{2\phi} = e^{-2\phi_0}\,\frac{\sinh 2\rho}{2e^h} \equiv e^{-2\phi_0}\Lambda(\rho)\,, \qquad e^{2\phi_0} = g_s N\,. \tag{5.5}$$

In particular, $\Lambda(\rho)$ satisfies

$$\Lambda(\rho) \simeq \frac{e^{2\rho}}{4\sqrt{\rho}}\,, \qquad \text{when} \qquad \rho \gg 1 \tag{5.6}$$

and

$$\Lambda(\rho) \simeq 1 + \frac{8\rho^2}{9} + \mathcal{O}(\rho^4)\,, \qquad \text{when} \qquad \rho \ll 1\,. \tag{5.7}$$

The solution also includes a Ramond-Ramond three-form given by

$$F_3 = \frac{g_s N}{4}\Big\{ -(w^1 - A^1)\wedge(w^2 - A^2)\wedge(w^3 - A^3) + \sum_i F^i \wedge (w^i - A^i)\Big\}\,, \tag{5.8}$$

where F^i is the field strength of the SU(2) gauge field A^i, defined as $F^i \equiv dA^i + \frac{1}{2}\varepsilon_{ijk}\,A^j \wedge A^k$.

5.2 The D3-brane baryon vertex

A D3-brane wrapping the 3-sphere parameterized by (θ_2, ϕ_2, ψ) introduces a tadpole that needs to be canceled through the addition of N fundamental strings

$$S_{\rm D3}^{\rm CS} = 2\pi\, T_3 \int_{\mathbb{R}\times S^3} C_2 \wedge F = -2\pi\, T_3 \int_{\mathbb{R}\times S^3} F_3 \wedge A = -N \int dt\, A_t\,. \tag{5.9}$$

The DBI action of this D3-brane is given by:

$$S_{\rm D3}^{\rm DBI} = -T_3 \int_{\mathbb{R}\times S^3} d^4\xi\, e^{-\phi}\sqrt{-\det P(G)} = -\frac{TN}{4\pi}\sqrt{\Lambda(\rho_0)}\,. \tag{5.10}$$

Particularizing to this background the size of the vertex given by (2.10) we find

$$L=\sqrt{g_sN}\int_{\rho_0}^{\infty}\frac{d\rho}{\sqrt{\Lambda(\rho)/\Lambda(\rho_1)-1}}\,,\tag{5.11}$$

which is a decreasing function of ρ_0. The binding energy of the baryon is in turn given by

$$E=\frac{k}{2\pi}\left\{\int_{\rho_0}^{\infty}d\rho\frac{\Lambda(\rho)}{\sqrt{\Lambda(\rho)-\Lambda(\rho_1)}}-\int_{\rho_{\min}}^{\infty}d\rho\,\sqrt{\Lambda(\rho)}+\frac{1-a}{a}\int_{\rho_{\min}}^{\rho_0}d\rho\,\sqrt{\Lambda(\rho)}+\frac{1}{2a}\sqrt{\Lambda(\rho_0)}\right\}.\tag{5.12}$$

Both integrals receive most of their contributions from the region $\rho\approx\rho_1$ so it can be seen that E is linearly proportional to L [20]. Also, from (2.16) we see that E and L share the same dependence on the position of the vertex:

$$\frac{dE}{d\rho_0}=\frac{k\sqrt{\Lambda_1}}{2\pi\sqrt{g_sN}}\frac{dL}{d\rho_0}\,.\tag{5.13}$$

The net-force condition is now

$$\cos\Theta=\frac{1-a}{a}+\frac{1}{4a}\partial_\rho\ln\Lambda(\rho)\Big|_{\rho=\rho_0}\,,\qquad\cos\Theta=\sqrt{1-\frac{\Lambda_1}{\Lambda_0}}\,.\tag{5.14}$$

Taking into account that $\partial_\rho\ln\Lambda(\rho)$ satisfies $\partial_\rho\ln\Lambda(\rho)\lesssim 2-1/(2\rho)+\mathcal{O}(1/\rho^2)$ in the UV we find that $a>a_<$ with $a_<=3/4$. Therefore the minimum value of the number of quarks is restricted with respect to the one found in the previous conformal examples, in agreement with our expectations.

5.2.1 Stability analysis

The study of the stability against longitudinal fluctuations gives

$$\delta r(\rho)=A\,g_sN\int_{\rho}^{\infty}d\rho\,\frac{\Lambda}{(\Lambda-\Lambda_1)^{3/2}}\,,\tag{5.15}$$

as the solution to equation (2.18). Substituting in the boundary equation (2.19) we find

$$2(\Lambda-\Lambda_1)\delta r'+\Lambda'(\rho)\,\delta r=0\qquad\text{at}\quad\rho=\rho_0\,,\tag{5.16}$$

and using (5.14) we can write

$$a\left(\cos\Theta+\frac{a-1}{a}\right)\cos\Theta\,\mathcal{Z}=\frac{1}{2}\tag{5.17}$$

where

$$\mathcal{Z}\equiv\sqrt{\Lambda_0}\int_{\rho_0}^{\infty}d\rho\,\frac{\Lambda}{(\Lambda-\Lambda_1)^{3/2}}\,,\qquad\text{and}\qquad\Lambda_1=\Lambda_0\sin^2\Theta\,.\tag{5.18}$$

From (5.17) we can now solve for a. Note that using (2.15) we find that

$$\frac{\partial\rho_0}{\partial\rho_1}=\frac{1}{2}\mathcal{Z}\cos\Theta\,\partial_\rho\ln\Lambda(\rho)\big|_{\rho=\rho_1}\tag{5.19}$$

from where

$$\mathcal{Z}\cos\Theta=\frac{2}{\partial_{\rho_0}\ln\Lambda(\rho_0)}\in[1,\infty)\,.\tag{5.20}$$

From (5.17) and (5.19) we then find $a\geqslant\frac{1}{2}+\frac{1}{4\mathcal{Z}\cos\theta}\implies a\geqslant\frac{1}{2}+\frac{\partial_{\rho_0}\ln\Lambda(\rho_0)}{8}$. Thus, the stability analysis does not improve the bound imposed by the existence of a classical solution, in contrast to what happened in the conformal examples previously discussed.

5.2.2 Adding a magnetic flux

Finally, in order to compare with the microscopical analysis in section 6.3 we add a magnetic flux to the baryon proportional to the Kähler form on the 2-sphere parameterized by (θ_2, ϕ_2), $F = \mathcal{N} J$, with $\mathcal{N} \in 2\mathbb{Z}$. This flux dissolves $\mathcal{N}/2$ units of D1-brane charge in the S^3. The energy of the baryon is modified according to

$$E_{\rm D3} = \frac{N}{4\pi}\sqrt{\Lambda(\rho_0) + \frac{4\pi^2\mathcal{N}^2}{g_s N}}\,. \tag{5.21}$$

As in the previous cases the magnetic flux changes the minimum bound for the number of quarks in the baryon. Moreover the flux has an upper bound.

6 The microscopical description

In the previous sections we have discussed generalizations of the baryon vertex constructions to allow a magnetic flux dissolving lower dimensional brane charge in the configuration. By analogy with Myers dielectric effect [30, 39] we expect that a complementary description in terms of lower dimensional branes expanding into fuzzy baryons should then be possible. This would be the "microscopical" realization of the "macroscopical" baryons with magnetic flux that we have just described. The interesting thing about the microscopical description is that it allows to explore the finite 't Hooft coupling region, and this is especially relevant in those cases in which the baryons are non-supersymmetric, like those considered in this paper, and are therefore not preserved by a BPS condition.

It is well known that the macroscopical and microscopical descriptions have complementary ranges of validity [30]. While the first is valid in the supergravity limit the second is a good description when the mutual separation of the expanding branes is much smaller than the string length, such that they can be taken to be coincident and therefore described by the U(n) effective action constructed by Myers [30]. For n Dq-branes expanded into an r-dimensional manifold of radius R, the volume per brane can be estimated as R^{r-q}/n, which must then be much smaller than l_s^{r-q}. Thus the condition

$$R \ll n^{\frac{1}{r-q}}\, l_s\,, \tag{6.1}$$

sets the regime of validity of the microscopical description. The macroscopical description is in turn valid when $R \gg 1$. Therefore both descriptions are complementary for finite n, but should agree in the large n limit, where they have a common range of validity. The limit (6.1) is especially appealing in backgrounds with a CFT dual, like the AdS spacetimes that we have considered in this paper. Indeed, in terms of the 't Hooft parameter of the dual CFT the condition (6.1) reads

$$\lambda \ll n^{\frac{4}{r-q}}\,. \tag{6.2}$$

The fact that λ can be finite opens up the possibility of accessing the finite 't Hooft coupling region of the dual CFT through the microscopical study of the corresponding dual brane system.

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Dielectric branes expanding into fuzzy manifolds have been extensively studied in the literature. From (6.2) the lower the dimensionality of the expanding branes the smaller the 't Hooft parameter can get. However for the manifolds that we have discussed in this paper it will not always be possible to provide a description in terms of expanding D1-branes. This is the case for the $Y^{p,q}$ Sasaki-Einstein geometries, in which the natural microscopical description would be in terms of D1-branes wrapped on the Reeb vector direction and expanding into the remaining four dimensional Kähler-Einstein manifold. We are however not aware of a fuzzy realization of these manifolds besides the CP^2 case. Moreover, as we have seen, the number of D1-branes in the macroscopical description is irrational, while this should be an integer in the microscopical description. Still, we will be able to provide a (less) microscopical description in terms of D3-branes expanding into a fuzzy 2-sphere. For the $\hat{\gamma}_i$ deformed backgrounds the natural thing is to dissolve D3-branes wrapped on the $(T^1)^3$ through the addition of a magnetic flux proportional to the Kähler form on the S^2, as we did in section 4.1. The microscopical description will then be in terms of D3-branes expanding into a fuzzy S^2.

We start in section 6.1 with the analysis of the $\mathrm{AdS}_5 \times T^{1,1}$ background, for which a description in terms of D1-branes expanding into a fuzzy $S^2 \times S^2$ manifold can be done. As we will see this description exactly matches the macroscopical description in section 3.2. The extension to arbitrary $Y^{p,q}$ manifolds is more technical and it is postponed to the appendix. In section 6.2 we discuss the $\hat{\gamma}_i$ deformed backgrounds. We end with the Maldacena-Nuñez analysis in section 6.3, in terms of D1-branes expanding into a fuzzy S^2 baryon.

6.1 The $\mathrm{AdS}_5 \times T^{1,1}$ background: D1-branes into fuzzy $S^2 \times S^2$

The DBI action describing the dynamics of n coincident D1-branes is given by [30]

$$S^{\mathrm{DBI}}_{n\mathrm{D1}} = -T_1 \int d^2\xi \, \mathrm{STr}\Big\{ e^{-\phi} \sqrt{\big|\det\big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^i{}_j E^{jk} E_{k\nu}]\big)\det Q\big|} \Big\} \tag{6.3}$$

where $E = G - B_2$ and

$$Q^i{}_j = \delta^i{}_j + \frac{i}{2\pi}[X^i, X^k] E_{kj} \, . \tag{6.4}$$

Let us take the D1-branes wrapped on the U(1) fibre direction ψ in (A.16) and expanding into the fuzzy $S^2 \times S^2$ submanifold parameterized by (θ, ϕ) and (ω, ν).

Using Cartesian coordinates for each S^2 we can impose the condition

$$\sum_{i=1}^{3} (x^i)^2 = 1 \tag{6.5}$$

at the level of matrices if the X^i are taken in the irreducible totally symmetric representation of order m, with dimension $n = m + 1$,

$$X^i = \frac{1}{\sqrt{m(m+2)}} J^i \tag{6.6}$$

with J^i the generators of SU(2), satisfying $[J^i, J^j] = 2i\varepsilon_{ijk} J^k$. Labeling with m_1, m_2 the irreps for each S^2 we have that the total number of expanding branes $n = (m_1+1)(m_2+1)$, and substituting in the DBI action

$$S^{\rm DBI}_{n{\rm D1}} = -T_1 \int d^2\xi \sqrt{-G_{tt}G_{\psi\psi}}\, {\rm Str}\sqrt{\det {\rm Q}} \tag{6.7}$$

we find

$$E_{n{\rm D1}} = \frac{N\rho_0}{8\pi} \frac{(m_1+1)(m_2+1)}{\sqrt{m_1(m_1+2)m_2(m_2+2)}} \sqrt{1+\frac{36\pi^2 m_1(m_1+2)}{R^4}} \sqrt{1+\frac{36\pi^2 m_2(m_2+2)}{R^4}} \tag{6.8}$$

where

$$\det Q = \left(1+\frac{R^4}{36\pi^2 m_1(m_1+2)}\right)\left(1+\frac{R^4}{36\pi^2 m_2(m_2+2)}\right)\mathbb{I} \tag{6.9}$$

and the $(m_1+1)(m_2+1)$ factor comes from computing the symmetrized trace. This expression is exact in the limit

$$R \gg 1\,, \qquad m \gg 1\,, \qquad \text{with} \qquad \frac{R^2}{m} = \text{finite} \tag{6.10}$$

(see section 5.1 of [40] for the detailed discussion). Taking the large m_1, m_2 limit we find perfect match with the macroscopical result given by (3.19) if $m_1 \sim \mathcal{N}_1/3$, $m_2 \sim \mathcal{N}_2/3$, in agreement with (A.18).

6.1.1 The F-strings in the microscopical description

An essential part of the baryon vertex are the fundamental strings that stretch from the Dp-brane to the boundary of AdS$_5$. As we show in this section they arise from the non-Abelian CS action.

The CS action for n coincident D1-branes is given by

$$S_{\rm CS} = \int d^2\xi\, {\rm STr}\left\{ P\left(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q\, e^{-B_2}\right) e^{2\pi F}\right\}. \tag{6.11}$$

In this expression the dependence of the background potentials on the non-Abelian scalars occurs through the Taylor expansion [41]

$$C_q(\xi, X) = C_q(\xi) + X^k\partial_k C_q(\xi) + \frac{1}{2}X^l X^k \partial_l\partial_k C_q(\xi) + \ldots \tag{6.12}$$

and it is implicit that the pull-backs into the worldline are taken with gauge covariant derivatives $D_\xi X^\mu = \partial_\xi X^\mu + i[A_\xi, X^\mu]$.

The relevant CS couplings in the AdS$_5 \times T^{1,1}$ background are

$$S^{\rm CS}_{n{\rm D1}} = \frac{T_1}{2\pi}\int d^2\xi\, {\rm Str}\left(iP[(i_X i_X)C_4] - \frac{1}{2}P[(i_X i_X)^2 C_4]\wedge F\right). \tag{6.13}$$

Taking into account (6.12) and working in the gauge $A_\psi = 0$ these couplings reduce to

$$S^{\rm CS}_{n{\rm D1}} = -\frac{1}{\pi}\int dt\, {\rm Str}[(i_X i_X)^2 i_k F_5] A_t\,, \tag{6.14}$$

where i_k denotes the interior product with $k^\mu = \delta^\mu_\psi$ and we have integrated out ψ, the spatial direction of the D1-branes. Taking into account that in Cartesian coordinates F_5, as given by (A.20), reduces to

$$i_k F_5 = -\frac{R^4}{27} f_{ijm} f_{kln} X^m X^n dX^i \wedge dX^j \wedge dX^k \wedge dX^l, \tag{6.15}$$

where the indices run from 1 to 3 for the first 2-sphere and from 4 to 6 for the second, such that $f_{ijm} = \varepsilon_{ijm}$ for $i, j, m = 1, \ldots 3$ and $i, j, m = 4, \ldots, 6$ and zero otherwise, we finally find

$$S^{\rm CS}_{n{\rm D1}} = -N \frac{(m_1+1)(m_2+1)}{\sqrt{m_1(m_1+2)m_2(m_2+2)}} \int dt\, A_t\,, \tag{6.16}$$

again in perfect agreement with (3.4) in the large m_1, m_2 limit.

To finish this section we would like to point out that more general fuzzy realizations of the $T^{1,1}$ could in principle be considered. For instance one could think of substituting the direct product of the two fuzzy 2-spheres by a Moyal-type of product, $[X^i, X^j] = \imath\theta^{ij}$ where $i = 1, \ldots 3$ refers to the first 2-sphere and $j = 4, \ldots 6$ refers to the second. It is not clear in any case how this would affect the description of the vertex beyond the supergravity limit.

6.2 The β-deformed backgrounds: D3-branes into fuzzy S^2

In this case we start with a system of n coincident D3-branes, whose dynamics is given by the straightforward extension of (6.3) to a four dimensional worldvolume. We take the branes wrapped on the 3-torus and expanding into the 2-sphere in (4.1) parameterized by (α, θ). Given that the expansion is on a fuzzy 2-sphere we take the same ansatz (6.6) as in the previous section. Substituting in the DBI action we have

$$S^{\rm DBI}_{n{\rm D3}} = -T_3 \int d^4\xi\, {\rm Str}\Big[e^{-\phi}\sqrt{-\det P[E_{\mu\nu}]\det Q}\Big] \tag{6.17}$$

with

$$\det Q = \left(1 + \frac{R^4}{\pi^2 m(m+2)}\right)\mathbb{I} \tag{6.18}$$

as in the previous section for each 2-sphere.[2] As explained there this expression is exact in the limit (6.10). The only difference with the calculation in the previous section comes from the fact that $\det P[E_{\mu\nu}]$ depends now on the transverse scalars X^i, and therefore it contributes to the symmetrized trace. In order to compute this contribution we use that

$${\rm Str}(\mu_1\mu_2\mu_3) \simeq \frac{m+1}{4\pi}\int_0^{\pi/2} d\alpha \sin^3\alpha\cos\alpha \int_0^{\pi/2} d\theta \sin\theta\cos\theta = \frac{m+1}{32\pi} \tag{6.19}$$

as implied by equation (4.44) in [42], which is valid in the limit $m \gg 1$. We then find that

$$E_{n{\rm D3}} = \frac{N\rho_0}{8\pi}\frac{m+1}{\sqrt{m(m+2)}}\sqrt{1+\frac{\pi^2 m(m+2)}{R^4}}\,. \tag{6.20}$$

This result for the energy is more approximate than the ones found in the rest of examples, where $\det P[E_{\mu\nu}]$ does not depend on the transverse scalars. Still, it allows to compute $1/m$ corrections to the macroscopical result. Taking the large m limit we find perfect agreement with the macroscopical result (4.9) for $m \sim 2\mathcal{N}$.

[2]The different factor comes from the different radii in the two backgrounds.

6.2.1 The F-strings

The relevant CS couplings in the non-Abelian action for D3-branes in the $\hat{\gamma}_i$ deformed backgrounds are

$$S^{\rm CS}_{n{\rm D3}} = T_3 \int d^4\xi \,{\rm STr}\big(P[C_4] + iP[(i_X i_X)C_4]\wedge F - P[C_2\wedge B_2] - iP[(i_X i_X)(C_2\wedge B_2)]\wedge F\big)\,. \tag{6.21}$$

Using (6.12) and the definition of the gauge covariant pull-backs they reduce to

$$S^{\rm CS}_{n{\rm D3}} = i\,T_3 \int d^4\xi \,{\rm STr}\big(\mathcal{P}[(i_X i_X)F_5] - \mathcal{P}[(i_X i_X)F_3\wedge B_2]\big)A_t\,, \tag{6.22}$$

where $\mathcal{P}$ denotes the gauge covariant pull-back over the spatial directions. Taking the spatial components of the gauge field to vanish and using that

$$F_5 = \frac{1}{8\pi}R^4\,\mathcal{G}\,\varepsilon_{ijk}X^k dX^i\wedge dX^j\wedge d\phi_1\wedge d\phi_2\wedge d\phi_3 \tag{6.23}$$

and

$$F_2 = -\frac{1}{8\pi}R^2\varepsilon_{ijk}X^k dX^i\wedge dX^j\wedge(\hat{\gamma}_1 d\phi_1 + \hat{\gamma}_2 d\phi_2 + \hat{\gamma}_3 d\phi_3)\,, \tag{6.24}$$

the F_5 and $F_3\wedge B_2$ contributions combine to give

$$S^{\rm CS}_{n{\rm D3}} = -N\frac{m+1}{\sqrt{m(m+2)}}\int dt A_t \tag{6.25}$$

which is again in perfect agreement with (4.4) in the large m limit.

6.3 The Maldacena-Nuñez background: D1-branes into fuzzy S^2

Let us now use the action (6.3) to describe n D1-branes wrapped on the ψ direction and expanding into the 2-sphere in (5.1) parameterized by (θ_2,ϕ_2). The expansion is again on a fuzzy 2-sphere, so we take the same non-commutative ansatz (6.6) as in the previous sections. Substituting in the DBI action we have

$$S^{\rm DBI}_{n{\rm D1}} = -T_1\int d^2\xi\sqrt{-G_{tt}G_{\psi\psi}}\,{\rm Str}\sqrt{\det Q} \tag{6.26}$$

as in (6.7), with

$$\det Q = \left(1 + \frac{g_s N\Lambda(\rho_0)}{16\pi^2 m(m+2)}\right)\mathbb{I}\,. \tag{6.27}$$

The regime of validity of the determinant is again fixed by (6.10). Computing the symmetrized trace we finally arrive at

$$E_{n{\rm D1}} = \frac{N}{4\pi}\frac{m+1}{\sqrt{m(m+2)}}\sqrt{\Lambda(\rho_0) + \frac{16\pi^2 m(m+2)}{g_s N}}\,, \tag{6.28}$$

which in the large m limit is in perfect agreement with the macroscopical result (5.21) for $m\sim\mathcal{N}/2$.

6.3.1 The F-strings

The relevant CS couplings are in this case

$$S^{\rm CS}_{n{\rm D1}} = T_1 \int {\rm Str}\left(P[C_2] + iP[(i_X i_X)C_2] \wedge F\right) \tag{6.29}$$

which can be rewritten as

$$S^{\rm CS}_{n{\rm D1}} = 2i \int dt\, {\rm STr}[(i_X i_X) i_k F_3] A_t \tag{6.30}$$

where i_k denotes the interior product with $k^\mu = \delta^\mu_\psi$ and we have integrated over the ψ direction. Using that

$$F_3 = -\frac{N}{4}\varepsilon_{ijk} X^m dX^i \wedge dX^j \wedge d\psi \tag{6.31}$$

we get

$$S^{\rm CS}_{n{\rm D1}} = -N\frac{m+1}{\sqrt{m(m+2)}} \int dt A_t \tag{6.32}$$

in perfect agreement with (5.9) in the large m limit.

The analysis performed in this section shows that the right description for the baryon vertex (with magnetic flux) at finite 't Hooft coupling is in terms of D1- or D3-branes expanding into a $S^1 \times (S^2 \times S^2)_{\rm fuzzy}$ D5-brane, $S^2_{\rm fuzzy} \times T^3$ D5-brane or $S^1 \times S^2_{\rm fuzzy}$ D3-brane. As we have shown these branes introduce tadpoles that need to be cancelled with the addition of fundamental strings. A full description of the D5, or D3, plus F1 system valid at finite 't Hooft coupling would require however the construction of fuzzy spikes, so that the α' corrections coming from the F-strings would also be taken into account. See the conclusions for a further discussion on this point.

7 Conclusions

In this paper we have discussed non-singlet baryon vertices in various Type IIB backgrounds in order to investigate the dependence of the bound imposed on the number of quarks by the existence and stability of the classical solution, on the supersymmetry and confinement properties of the dual gauge theory.

Using the probe brane approximation [4, 5, 20] we have shown that this bound is the same for all $\mathrm{AdS}_5 \times Y_5$ backgrounds with Y_5 an Einstein manifold bearing five form flux, independently on the number of supersymmetries preserved. The same result holds true for β-deformed and even non-supersymmetric multi-β deformed backgrounds, pointing at a universal behavior based on conformality. The same analysis in a confining background, the Maldacena-Nuñez model, shows that universality is lost when confinement is present. In this case although non-singlet baryons still exist, the bound imposed on them is more restrictive, in agreement with our expectations that non-singlet baryons should be more constrained in more realistic gauge theories. It would be interesting to confirm this result in other confining backgrounds, such as the Klebanov-Strassler [18] or the Sakai-Sugimoto models [43].

Although the probe brane analysis has proved to be enough in order to deduce the basic properties of this type of systems (see for instance [4–7, 20]), the fact that all supersymmetries are broken in this approach could imply that it may not be sensitive enough to account for the supersymmetries preserved by the different backgrounds. However previous results in the literature on baryon vertices in $\mathrm{AdS}_5 \times T^{1,1}$, $\mathrm{AdS}_5 \times Y^{p,q}$ and the Klebanov-Strassler and Maldacena-Nuñez backgrounds reveal that even when all fundamental strings are taken to end on the same point of the wrapped D-brane supersymmetry is broken. Therefore significant changes to the probe brane results should not be expected. At any event, the different behaviors based on conformality should represent valid predictions.

We also note that we would expect the baryon analysis in β-deformed Sasaki-Einstein manifolds to provide similar results to the undeformed case. Our string and brane configurations do not seem to depend strongly on the deformation in the way encountered in [44, 45], where important modifications due to the deformation appeared only in the T^3 fibration description.

Using the fact that we can consistently add lower dimensional brane charges we have provided an alternative description of the baryons in terms of lower dimensional branes expanding into fuzzy baryon vertices. This description represents a first step towards the analysis of holographic baryons at finite 't Hooft coupling. In this description the expansion is caused by a purely gravitational dielectric effect, while the Chern-Simons terms only indicate the need to introduce the number of fundamental strings required to cancel the tadpole.

In order to be able to conclude that non-singlet baryons exist at finite 't Hooft coupling we should take into account not only the α' corrections coming from the microscopical analysis of the brane but also the α' corrections to the F-string Nambu-Goto action and the background. This is therefore a difficult program, which we have only begun to explore. An interesting next step in this direction would be to use the microscopical analysis to build up spike solutions in these backgrounds. We expect to report progress in this direction in the near future.

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A The $\mathrm{AdS}_5 \times Y^{p,q}$ background

In this appendix we collect some properties of $Y^{p,q}$ manifolds useful for the description of the baryon vertex in the $\mathrm{AdS}_5 \times Y^{p,q}$ background. The Klebanov-Witten background is described thereof as a particular case.[3] We also provide the detailed microscopical description of the baryon vertex in $\mathrm{AdS}_5 \times Y^{p,q}$ in terms of D3-branes expanding into a fuzzy 2-sphere.

A.1 Some properties of the $\mathrm{AdS}_5 \times Y^{p,q}$ geometry

In our conventions the $\mathrm{AdS}_5 \times Y^{p,q}$ metric reads

$$ds^2 = R^2\big(ds^2_{\mathrm{AdS}_5} + ds^2_{Y^{p,q}}\big) = \frac{\rho^2}{R^2}dx^2_{1,3} + \frac{R^2}{\rho^2}d\rho^2 + R^2 ds^2_{Y^{p,q}}\,, \tag{A.1}$$

with R the radius of curvature in string units,

$$R^4 = \frac{4\pi^4 N g_s}{\mathrm{Vol}(Y^{p,q})}\,. \tag{A.2}$$

For the $Y^{p,q}$ we use the canonical form of the metric [11], given by:

$$\begin{aligned}
ds^2_{Y^{p,q}} &= \frac{1-cy}{6}(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) + \frac{\mathrm{d}y^2}{w(y)q(y)} + \frac{1}{36}w(y)q(y)(\mathrm{d}\beta + c\cos\theta\mathrm{d}\phi)^2 \\
&\quad + \frac{1}{9}[\mathrm{d}\psi + \cos\theta\mathrm{d}\phi + y(\mathrm{d}\beta + c\cos\theta\mathrm{d}\phi)]^2 = \\
&= (e^\theta)^2 + (e^\phi)^2 + (e^y)^2 + (e^\beta)^2 + (e^\psi)^2,
\end{aligned} \tag{A.3}$$

where the fünfbeins read

$$\begin{aligned}
e^\theta &= \sqrt{\frac{1-cy}{6}}\,\mathrm{d}\theta\,, & e^\phi &= \sqrt{\frac{1-cy}{6}}\,\sin\theta\mathrm{d}\phi\,, \\
e^y &= \frac{1}{\sqrt{w(y)q(y)}}\mathrm{d}y\,, & e^\beta &= \frac{\sqrt{w(y)q(y)}}{6}(\mathrm{d}\beta + c\cos\theta\mathrm{d}\phi)\,, \\
e^\psi &= \frac{1}{3}\big(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi + y(\mathrm{d}\beta + c\cos\theta\mathrm{d}\phi)\big)\,, &&
\end{aligned} \tag{A.4}$$

with

$$w(y) = \frac{2(a-y^2)}{1-cy}\,, \qquad q(y) = \frac{a - 3y^2 + 2cy^3}{a-y^2}\,, \tag{A.5}$$

and the metric is normalized such that $R_{\alpha\beta} = 4\,G_{\alpha\beta}$. The ranges of the coordinates (θ, ϕ, ψ) are $0 \leqslant \theta \leqslant \pi$, $0 \leqslant \phi \leqslant 2\pi$ and $0 \leqslant \psi \leqslant 2\pi$. The parameter a is restricted to $0 < a < 1$. By choosing this range the following conditions for y are satisfied: $y^2 < a$, $w(y) > 0$ and $q(y) \geqslant 0$. The coordinate y then ranges between the two smaller roots of the cubic equation $q(y) = 0$, $y_1 \leqslant y \leqslant y_2$. For $c \neq 0$, y can always be rescaled such that $c = 1$ and the parameter a can be written in terms of two coprime integers p and q as:

$$a = \frac{1}{2} - \frac{p^2 - 3q^2}{4p^3}\sqrt{4p^2 - 3q^2}\,. \tag{A.6}$$

[3] With the well-known subtleties regarding the periodicities.

In this case

$$y_1 = \frac{1}{4p}\left(2p - 3q - \sqrt{4p^2-3q^2}\right) < 0\,, \qquad y_2 = \frac{1}{4p}\left(2p + 3q - \sqrt{4p^2-3q^2}\right) > 0\,. \tag{A.7}$$

Finally, β ranges between $-2\pi(6l+c) \leqslant \beta \leqslant 0$, where

$$l = \frac{q}{3q^2 - 2p^2 + p\sqrt{4p^2-3q^2}}\,. \tag{A.8}$$

Note that β needs not be periodic in general. The volume of the $Y^{p,q}$ can be written in terms of p, q as

$$\mathrm{Vol}(Y^{p,q}) = \frac{q(2p+\sqrt{4p^2-3q^2})l\pi^3}{3p^2}\,. \tag{A.9}$$

The canonical metric (A.3) takes the standard form

$$\mathrm{d}s^2_{Y^{p,q}} = \mathrm{d}s^2_{M_4} + \left(\frac{1}{3}\mathrm{d}\psi + \sigma\right)^2, \tag{A.10}$$

where the Killing vector $k^\mu = \delta^\mu_\psi$ is the Reeb vector and $ds^2_{M_4}$ is a local Kähler-Einstein metric with Kähler form

$$J = \frac{1}{2}\mathrm{d}\sigma = \frac{1-cy}{6}\sin\theta\,\mathrm{d}\theta\wedge\mathrm{d}\phi + \frac{1}{6}\mathrm{d}y\wedge(\mathrm{d}\beta + c\cos\theta\mathrm{d}\phi)\,, \tag{A.11}$$

satisfying

$$\int_{M_4} J\wedge J = \frac{3\mathrm{Vol}(Y^{p,q})}{\pi}\,. \tag{A.12}$$

This local property of the metric will be useful in order to induce an instantonic magnetic flux proportional to the Kähler form.

Finally, the $\mathrm{AdS}_5\times Y^{p,q}$ flux reads $F_5 = (1+\star_{10})\mathcal{F}_5$, where

$$\mathcal{F}_5 = 4\,R^4\,\mathrm{dVol}(Y^{p,q}) \tag{A.13}$$

and

$$d\mathrm{Vol}(Y^{p,q}) = e^\theta\wedge e^\phi\wedge e^y\wedge e^\beta\wedge e^\psi = \frac{1}{108}(1-cy)\sin\theta d\theta\wedge d\phi\wedge dy\wedge d\beta\wedge d\psi \tag{A.14}$$

F_5 is then such that

$$\frac{1}{(2\pi)^4 g_s}\int_{Y^{p,q}} F_5 = N\,. \tag{A.15}$$

A.2 The $\mathrm{AdS}_5\times T^{1,1}$ case

As shown in [11] when $c=0$ the metric (A.3) reduces to the local form of the standard homogeneous metric on $T^{1,1}$. Indeed, setting $c=0$ in (A.3), rescaling to set $a=3$ and introducing the coordinates $\cos\omega = y$, $\nu = -\beta$ one gets

$$ds^2_{T^{1,1}} = \frac{1}{9}[d\psi - \cos\theta d\phi - \cos\omega d\nu]^2 + \frac{1}{6}(d\theta^2 + \sin^2\theta d\phi^2) + \frac{1}{6}(d\omega^2 + \sin^2\omega d\nu^2)\,, \tag{A.16}$$

which is the metric of the $T^{1,1}$ in adapted coordinates to its realization as a U(1) bundle over $S^2 \times S^2$ [46], normalized such that $R_{\alpha\beta} = 4\, G_{\alpha\beta}$. Note however that although it is possible to take the period of ν equal to 2π the period of ψ is fixed to 2π, so the manifold that is being described in the $c = 0$ case is the $T^{1,1}/\mathbb{Z}_2$ orbifold. Still, we can study the baryon vertex in $T^{1,1}$ as a particular case of $Y^{p,q}$ geometry if we account for the right periodicity of ψ when relevant.

The Kähler form in the $T^{1,1}$ reads

$$J = \frac{1}{6}(\sin\theta d\theta \wedge d\phi + \sin\omega\, d\omega \wedge d\nu) \tag{A.17}$$

and some properties used in the main text are

$$\int_{S^2} J = \frac{2\pi}{3}\,, \qquad \int_{T^{1,1}} J \wedge J = \frac{3\mathrm{Vol}(T^{1,1})}{2\pi}\,, \tag{A.18}$$

where the volume of the $T^{1,1}$ is given by

$$\mathrm{Vol}(T^{1,1}) = \frac{16\pi^3}{27}\,. \tag{A.19}$$

Finally, the 5-form field strength is $F_5 = (1 + \star_{10})\mathcal{F}_5$, where

$$\mathcal{F}_5 \equiv 4R^4\, \mathrm{dVol}(T^{1,1}) = \frac{R^4}{27} \sin\theta \sin\omega\, d\theta \wedge d\omega \wedge d\psi \wedge d\phi \wedge d\nu \tag{A.20}$$

and satisfies:

$$\frac{1}{(2\pi)^4 g_s} \int_{T^{1,1}} F_5 = N\,. \tag{A.21}$$

A.3 The baryon vertex in AdS$_5 \times Y^{p,q}$ with a magnetic flux proportional to the Kähler form of the S^2

The microscopical description of the baryon vertex in AdS$_5 \times Y^{p,q}$ in terms of D3-branes expanding into a fuzzy 2-sphere is complementary to a macroscopical D5-brane wrapped on the $Y^{p,q}$ with a magnetic flux proportional to the Kähler form on the S^2. This magnetic flux dissolves D3-brane charge, with the D3's spanned on the (y, β, ψ) directions.

The DBI action for the D5-brane in the Sasaki-Einstein background (A.3) reads

$$S^{\rm DBI}_{\rm D5} = -T_5 \int_{\mathbb{R}\times Y^{p,q}} d^6\xi\, e^{-\phi} \sqrt{-\det P(G_{MN} + 2\pi F_{MN})}\,, \tag{A.22}$$

where $M = (\mu; i) = (t, a; i)$, $a = (y, \beta, \psi)$, $i = (\theta, \phi)$. Turning on a magnetic flux proportional to the Kähler form on the S^2 parameterized by θ and ϕ in (A.3) it is easy to prove that

$$\det P(G_{MN} + 2\pi F_{MN}) = G_{tt} I J\,, \tag{A.23}$$
$$\text{with} \quad I = \det[(G + 2\pi F)_{ij}]\,, \qquad J = \det[G_{ab} - G_{ai}(G + 2\pi F)^{-1ij} G_{jb}]\,.$$

The determinant I can be easily computed, with the result

$$I = G_{\theta\theta}G_{\phi\phi}\big(1 + (2\pi\mathcal{N})^2\varepsilon^2\big) \equiv G_{\theta\theta}G_{\phi\phi}\mathcal{I}\,, \qquad \varepsilon^2 \equiv \frac{1}{(1-y)^2R^4}\,. \tag{A.24}$$

For the computation of J we note that $G^{ab}G_{bi} = -\delta_{a\psi}\delta_{i\phi}\cos\theta$. The result reads

$$J = \big(\det[G_{ab}]\big)^2 \det\left[G^{ab} - \frac{\delta_{a\psi}\delta_{b\psi}\cos^2\theta}{G_{\phi\phi}\big(1+(2\pi\mathcal{N})^2\varepsilon^2\big)}\right] \equiv \det[G_{ab}]\mathcal{J}. \qquad \text{(A.25)}$$

Plugging these expressions in the DBI action we finally find

$$E_{\rm D5} = N\frac{\rho_0}{16\pi}\frac{\int_{y_1}^{y_2} dy(1-y)\int_0^\pi d\theta \sin\theta\sqrt{\mathcal{I}\mathcal{J}}}{\int_{y_1}^{y_2} dy(1-y)}\,, \qquad \text{(A.26)}$$

A.4 The microscopical construction

In this appendix we show that the baryon vertex with magnetic flux that we have just discussed can be described at finite 't Hooft coupling in terms of D3-branes expanding into a fuzzy 2-sphere. The geometry of the fuzzy D5-brane is then given by the twisted product of the 3 dimensional manifold spanned by the (y,β,ψ) directions and a fuzzy 2-sphere.

The DBI action describing the dynamics of n coincident D3-branes spanned on the (y,β,ψ) directions and expanding onto the fuzzy S^2 parameterized by θ and ϕ in (A.3) is given by $(I=1,2,3)$

$$S^{\rm DBI}_{n{\rm D3}} = -T_3\int d^4\xi\,{\rm Str}\Big[e^{-\phi}\sqrt{-G_{tt}\tilde{\mathcal{I}}\tilde{J}}\Big]\,, \qquad \text{(A.27)}$$

where

$$\tilde{\mathcal{I}} = \det Q^I{}_J\,, \qquad \tilde{J} = \det P\big[G_{ab} + G_{aI}(Q^{-1}-\delta)^{IJ}G_{Jb}\big]\,. \qquad \text{(A.28)}$$

The determinant of $Q^I{}_J$ can be computed in a similar way as in the previous cases, and the result reads

$$Q^I{}_J = \delta^I{}_J - \frac{\Lambda_{(m)}}{2\pi}\,\varepsilon^{IK}{}_L\,X^L G_{KJ}\,, \qquad \Lambda_{(m)} = \frac{2}{\sqrt{m(m+2)}} \implies \qquad \text{(A.29)}$$

$$\det Q^I{}_J \simeq \left(\frac{\Lambda_{(m)}}{2\pi}\right)^2\varepsilon^{-2}\left(1+\left(\frac{2\pi}{\Lambda_{(m)}}\right)^2\varepsilon^2\right).$$

Next, we consider the determinant $\tilde{J}$ and we note that

$$Q^I{}_J = \delta^I{}_J - \frac{\Lambda_{(m)}}{2\pi}\,\varepsilon^{IK}{}_L\,X^L G_{KJ} = G^{IK}\left(G_{KJ} - \frac{\Lambda_{(m)}}{2\pi}\varepsilon_{KJL}X^L\right) \implies Q = G^{-1}\tilde{Q}\,. \qquad \text{(A.30)}$$

Thus, we have to compute the inverse of $\tilde{Q}$, which in the macroscopical limit $(m \gg 1)$ reads

$$\tilde{Q}_{IJ} = G_{IJ} + \varepsilon_{IJL}v^L \qquad \implies \qquad \tilde{Q}^{-1IJ} = \frac{1}{1+v^2}\big(G^{IJ} + v^I v^J - \varepsilon^{IJ}{}_K v^K\big)\,, \qquad \text{(A.31)}$$

$$v^I = -\frac{\Lambda_{(m)}}{2\pi}X^I, \qquad v^2 = G_{IJ}\,v^I v^J = \left(\frac{\Lambda_{(m)}}{2\pi}\right)^2\frac{(1-y)^2R^4}{36}\,,$$

where the indices are raised using G^{IJ}. Next, we compute $(Q^{-1}-\delta)^{-1IJ}$ which reads

$$(Q^{-1}-\delta)^{-1IJ} \equiv (Q^{-1}-\delta)^{-1I}{}_K\,G^{KJ} = \frac{1}{1+v^2}\big(-v^2G^{IJ} + v^Iv^J - \varepsilon^{IJ}{}_K v^K\big)\,. \qquad \text{(A.32)}$$

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So, using the last equation and $G^{ab}G_{bi} = -\delta_{a\psi}\delta_{i\phi}\cos\theta$ we find

$$\tilde{J} = \left(\det[G_{ab}]\right)^2 \det\left[G^{ab} - \frac{\delta_{a\psi}\delta_{b\psi}\cos^2\theta}{G_{\phi\phi}(1+v^{-2})}\right] \equiv \det[G_{ab}]\tilde{\mathcal{J}}. \tag{A.33}$$

Putting all these ingredients together in (A.27) and using eq. (4.44) of [42]; so to find the leading behavior for $m \gg 1$, we find

$$\begin{aligned}
S^{\rm DBI}_{n{\rm D}3} &= -T_3\int d^4\xi {\rm Str}\left[e^{-\phi}\sqrt{-G_{tt}\tilde{\mathcal{I}}\tilde{J}}\right] \simeq -T_3\frac{\Lambda_{(m)}}{2\pi}\frac{m+1}{4\pi}\int_{S^2} dS^2\int d^4\xi\left[e^{-\phi}\sqrt{-G_{tt}\tilde{\mathcal{I}}\tilde{J}}\right]\\
&= -T_5\frac{m+1}{\sqrt{m(m+2)}}\int d^6\xi\left[e^{-\phi}\sqrt{-G_{tt}\tilde{\mathcal{I}}\tilde{J}}\right] \implies\\
E^{\rm DBI}_{n{\rm D}3} &= N\frac{m+1}{\sqrt{m(m+2)}}\frac{\rho_0}{16\pi}\frac{\int_{y_1}^{y_2} dy(1-y)\int_0^\pi d\theta\sin\theta\sqrt{\tilde{\mathcal{I}}\,\tilde{\mathcal{J}}}}{\int_{y_1}^{y_2} dy(1-y)}\,,
\end{aligned} \tag{A.34}$$

which in the large m limit reproduces the macroscopical result and like as the $T^{1,1}$ case it is given by (A.26) with $m = \mathcal{N}/3$, for which $(\mathcal{I},\mathcal{J}) \Leftrightarrow (\tilde{\mathcal{I}},\tilde{\mathcal{J}})$.

A.4.1 The F-strings

The CS action describing the dynamics of the n coincident D3-branes takes the form

$$S^{\rm CS}_{n{\rm D}3} = T_3\int {\rm Str}\big(P[C_4] + iP[(i_Xi_X)C_4]\wedge F\big) = -i\,T_3\int {\rm Str}\big(P[(i_Xi_X)F_5]\big)\wedge A_t \tag{A.35}$$

F_5 reads, in Cartesian coordinates for the S^2

$$F^{(5)}_{y\beta\psi ij} = \frac{R^4}{27}(1-y)\varepsilon_{ijk}X^k \tag{A.36}$$

Substituting in the action we find that

$$S^{\rm CS}_{n{\rm D}3} = -N\frac{m+1}{\sqrt{m(m+2)}}\int A\,, \tag{A.37}$$

which exactly matches the macroscopical result (3.4) in the large m limit.

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